**Abstract:** In this paper, we deal with the renowned problem of plastic pollution caused by food consumption and its conservation. Specifically, we consider the producer/reseller decision problem of industrial organizations in conditions of perfect competition within small oligopoly clusters. Indeed, very often, one major sustainability problem is that the presence of direct competitors in the same market determines entrepreneurship choices which lower production costs and packaging costs at the expense of the environment and public health. For this purpose, in order to show economic scenarios in which the respect and preservation of the environment and natural resources are quantitatively compatible with profits and economic growth, we present a provisional coopetitive model of the strategic interaction of two food enterprises, in direct duopoly competition, through investments in sustainable-packaging technologies. The macroeconomic goal is to propose possible actions to reduce carbon footprints and the inflow of plastics to the marine environment, following the environmental targets established by the United Nations, also in the presence of direct perfect oligopolistic competition in the same market. From a microeconomic point of view, we assume the existence of two competitors selling a very similar type of food in the same market; therefore, within a competitive interaction, we adopt a classic “Cournot duopoly” core upon which we define a parametric game, namely, a coopetitive game, together with its possible dynamical scenarios and solutions. We should notice that beyond the parameter arising from the cooperation construct, we introduce a matrix of stochastic variables, which we can also consider as the state of the world. Moreover, we numerically examine one possible state of the world to exemplify our model proposal. We determine, analytically and graphically, the optimal investment in the cooperative strategy, the purely coopetitive solution and some super-cooperative solutions. The cooperative strategy represents the common investment chosen to acquire advanced green technologies for innovative packaging, while the fourth component of any solution in the strategy space represents the state of the world at the end of the coopetitive process in which, finally, we can see the profits and costs deriving from the adoption of the green technologies.

**Keywords:** plastic reduction; computational logistic; green packaging; water saving; green coopetition; industrial symbiosis; climate change

**MSC:** 91A05; 91A10; 91A12; 91A25; 91A80

1. Introduction

1.1. United Nations Environmental Goals

The main environmental goals, established by the United Nations for worldwide sustainable development, embrace the sustainable management of water, sustainable energies, actions for combating climate change, sustainable consumption and production patterns, the conservation and sustainable employment of the oceans, and sustainable management of terrestrial ecosystems and forests (see the targets of the 17 United Nations
Sustainable Development Goals at https://www.globalgoals.org/goals/ (last accessed 31 October 2022)).

All these issues are strictly linked to each other and refer to the critical problem of global food production.

In addition, the Courtauld Commitment 2025 agreement (The Courtauld Commitment 2025, available online at https://www.wrap.org.uk/food-drink/business-food-waste/courtauld-2025 (Last access 31 October 2022)) was established to reduce, in the UK, the environmental impact of food, in terms of a reduction in food waste, reduction in the greenhouse-gas intensity of food and reduction in impact associated with water use in the supply chain.

1.2. Motivations


Pollution represents an environmental global risk, with damaging consequences for societies, human health and economic systems. Plastic use still represents a problem for environmental policies, which require multiple instruments addressing externalities determined by the plastic lifecycle. Food and drink supply chains play a significant role in plastic pollution.

Well-addressed actions can also help meet the UN Sustainable Development Goals (SDGs) [1–4]. The Department of Economic and Social Affairs of the UN indicates a clear and robust link between SDG 12, “Sustainable consumption and production”, and SDG 14.1 (Reduce marine pollution) [5]. A reduction in pollution can occur through a choice of new production methods (including food packaging) considering the sustainability of natural resources, as well as waste reduction, see [6]. Furthermore, SDG 12 is linked to 14.4 (Restore fish stocks): changes in consumption behavior affect the demand for food and can increase or decrease pressure on fish stocks.

The OECD emphasizes the need for combining consistent, growth-enhancing policies with well-aligned policy devoted to investment in low-carbon, climate-resilient infrastructures and new technologies [7].

It is essential, thus, to move towards a socio-ecologic transition, with actions that encourage a reduction in carbon footprint and the sustainable use of natural, non-renewable resources. Concerning the marine ecosystem, preventive strategies, i.e., the reduction in the inflow of plastics to the marine environment [8], are the most effective mitigation strategies. To this end, political interventions focused on single-use plastics [9] or recyclable packaging [10] are of primary importance. The Chinese government, for instance, enacted a ban on using multiple single-use plastics and non-degradable food containers [9]. Other possible mitigation policies for a sustainable ocean are ex-post interventions based on marine plastic-waste cleanup with the use of new technologies and strategies of regional cooperation [11–13]. Cooperation is, therefore, the basis for evolutionary success [14], creating to a win-win growth situation.

1.3. The Problem of Global Food Production

Global food production is one of the central issues in recent times, and this is due to increasing demand from the world population and the consequent reduction in natural resources to satisfy this demand. Indeed, when we talk about food production and consumption, we should consider several aspects. The problems caused to the environment by food production include:

- The land used for livestock and crops for human consumption with the associated conversion of forests and grasslands into cropland or pasture.
- Pollution arising from farming (ruminant livestock produces methane through their digestive processes) and fishing (fuel consumption from fishing vessels) and the use of fertilizers.
- The use of large amounts of water for cultivation and irrigation.
Moreover, problems in terms of energy and resource inputs for food processing (converting produce from the farm into final products), transport, packaging and retail have to be considered.

It is essential, therefore, to determine the quantity of food that should be produced—based on demand—in order to avoid surplus and the adoption of methods of food preservation in order to avoid waste deriving from poor conservation.

Indeed, the Food and Agricultural Organization of the United Nations (FAO) has pointed out that one third of food produced is wasted—representing the superfluous use of 1.4 billion hectares of cropland [15]. Moreover, food waste determines methane emissions from landfills, as well as pollution for hauling and putting it in the landfill.

1.4. Possible Actions for Mitigating Global Food Scarcity

Some of these problems can, in part, be mitigated with a more sustainable food diet by eating less meat; see [16,17]. Some resources can be preserved by investing in the development of sustainable machinery with low CO$_2$ emissions or machinery that allow the reuse or recycling of natural resources. Furthermore, it is also possible to think that eating local is the key to a low-carbon diet and, in such a way, transport costs can be reduced by assuming a local production and consumption strategy. According to the above analysis—and following the basic principles of the circular economy (optimization of resources, minimization of waste)—we propose a model of cooperation, between fast-food companies. The proposed model shows possible optimal scenarios concerning the quantity of food production and concerning the investments in sustainable packaging technologies, which guarantee benefits for companies and for the environment.

1.5. Aims of the Paper

In this article, specifically, we consider the coexistence of competing fast-food actors, through green coopetitive agreements among the competitors themselves, and some paper factories. We show possible scenarios in which all participants and the environment could benefit from a green coopetitive interaction.

From an economic-theory point of view, we assume the existence of two competitors selling a very similar type of food on the same market, so that, from a competitive perspective, we construct a classic “Cournot duopoly core” upon which we define a parametric game, namely, a coopetitive game, together with its possible dynamical scenarios and solutions. We should notice that beyond the parameter arising from the cooperation strategy, we introduce a matrix of stochastic variables, which we can also consider as a state vector of the world. Moreover, we examine numerically one possible state of the world to exemplify our model proposal.

2. Methods and Theoretical Background

Since we propose a coopetitive game model using concepts from non-cooperative game theory, quantitative bargaining theory, coopetition, industrial symbiosis, and logistics, we need to present a brief account of the previous theoretical developments adopted here.

2.1. Game-Theory Approach

Much attention has been paid in recent years to trying to reduce the environmental impacts of human activities. To this end, it is important to recycle or reuse waste as much as possible or to create non-polluting or degradable waste. Industrial ecology, for instance, explains how it is possible to use waste from one process as raw materials in another [18]. Game theory is largely used, in the literature, for the study of the deeper interaction between many subjects, such as industries, governments, environment and society, and can explain the macro-level collective dynamics of social systems [19].

There exist several type of the game-theory approach, which are applied to the study of different contexts. Specifically, in cooperative games, the values of the players are calculated with respect to possible coalitions of participants [20], while in non-cooperative finite games,
the goal is to search for possible balances and optimal gains for the participants; see [21]. Another line of research adopts evolutionary game theory for the analysis of growth dynamics over time, in order to understand which components may be winning or losing in the short and medium term; see, for instance, [22].

2.2. Game Theory in Environmental Preservation

Concerning the environmental context, several studies have used game-theory models for solving and optimizing logistics problems, proving that collaboration between players allows the optimization of logistic operations and gains. For instance, recently, Mouatassim et al. adopted a hybridized game-theory model to form coalitions and optimize transport costs in the case of a blood supply chain [23], while [24] proposed an integrated financial game-theory model for humanitarian organizations. Shi and Voß used game theory to model and analyze the behaviors of players in the shipping industry, considering game theory as a decision-making methodology which can usefully analyze interdependencies and interrelations in network-based services [25]. Reyes used game theory for solving the trans-shipment problem for maintaining stable conditions in a logistics network [26].

In the textile industry, Jafari considers some recyclables resources such as the plastic and metal bottles used as substitutable materials for the production of cotton [27]. Li et al. study the problem of pollution from non-biodegradable-plastic food containers, suggesting the use of degradable food packaging through a study of an evolutionary game-theory model between OFD platforms and restaurants [9]. Wang et al. focus on the sustainability problem associated with the takeout-food industry [10]. Indeed, they propose a construction of a recycling industry chain in order to fight the increasing amount of non-degradable waste coming from plastic bags and packing boxes.

2.3. Coopetitive Games

The novelty of the game-theory approach proposed in this article lies in the additional application of the coopetition in studying the problem of recyclable food packaging. However, in this article, we apply not only game theory, but also coopetition. Our approach aligns with the Brandenburger–Nalebuff idea of coopetition (see, e.g., [28,29]). We chose, as a possible model of Brandenburger and Nalebuff’s idea, that the “parametric manifold of non-cooperative games indexed by a shared cooperative strategy space”, introduced by Carfì in [30]. The model of coopetitive games can represent competitive and cooperative interaction simultaneously. This new approach has been applied in the study of different environmental problems (ocean degradation [31] or urban-waste recycling [32]). In addition, recently, Pedreira and Melo have used coopetition in supply chains [33], studying two food manufacturing companies in Brazil and evaluating the quantitative benefits in terms of CO$_2$ emissions and transportation costs.

3. The Economic Model: Coopetitive Agreement between Food Competitors

In this paper, we present a coopetitive interaction, based on a Cournot duopoly, between two food resellers, through investments in innovative paper packaging and reselling of fresh food with low environmental impact.

In more specific terms,

- We define a suitable parametric game with a core “a la Cournot” between two similar large food producers/retailers (fast food) in local competition.
- We assume the presence of innovative factories producing paper, cardboard and new green-effective packaging (for instance, LEIPAflat (see the sustainable packaging solution for fresh food at https://multivac-group.com/it/news-e-eventi/news/detail/2019/04/1012-sustainable-packaging-solution-for-fresh-food/ (last accessed 31 October 2022))).
- The innovative packaging is produced using cardboard and recycled paper, characterized by a negligible amounts of plastic, easily separable by consumers and ready for a quick recycling process.
• In our model, the paper waste from food enterprises is used as raw material for innovative factories.
• In order to compensate for the higher cardboard and paper consumption for their fresh-food packaging, the two enterprises agree to offer a significant percentage of food with low environmental impact.

3.1. General Description
• Our economic model can be considered a two-player non-cooperative family $G$ of games parameterized by a cooperative strategy belonging to a fixed compact interval of the real line.
• The two players are fast-food enterprises and the cooperative strategy consists of possible common investments into innovative green packaging for food products, together with the agreement that a significant percentage of food sold comes from sources with low environmental impact (vegetable-based proteins).

In other words, we are defining a compact smooth parametric curve of non-cooperative games, parameterized by a cooperative compact strategy interval $C$.
• Moreover, we consider curve $G$ as a stochastic curve, defined also upon a real four-dimensional compact state-of-the-world space $M$.
• The elements of the space $M$ are four-dimensional matrices individually representing the actual observable “efficiency” of the cooperative strategy (we simplify efficiency into a pair of interest rates and a pair of cost coefficients).
• Formally, our game $G$ can be defined as a vector function

$$G : S \times M \to \mathbb{R}^2 : (x, y, z, \mu) \mapsto (f^1_\mu(x, y, z), f^2_\mu(x, y, z)),$$

where

- $S$ is the strategy set of the two players, decomposable in the cartesian product

$$S = E \times F \times C;$$
- $M$ is the space of all real $(2,2)$ matrices;
- $\mathbb{R}^2$ is the payoff universe of the game;
- $f^1_\mu$ and $f^2_\mu$ are the two payoff functions of the players, respectively, when the state of the world $\mu$ is realized.

Any stochastic matrix $\mu \in M$ determines how much a common investment $z \in C$ (of the two competing players) is actually influencing the payoffs (revenues and costs determined by the common investment $z$) of the players.

Resuming, the two players of the game are two fast-food enterprises, cooperatively investing in the development of sustainable packaging production, conceived and produced by an innovative paper factory in the nearby area.

Specifically, their cooperative deal consists of the common investment in innovative paper packaging and reselling of low-environmental-impact fresh food.

Concerning the definition of payoff functions, we start from a classic duopoly interaction (Cournot model—see [34]) and we extend the classic core with a non-linear translation depending both upon the chosen common investment and the realized state of the world.

3.2. Strategies
The strategies of the model are:
1. Strategies

$$x \in E := \mathbb{U} = [0, 1],$$

representing any food quantity produced by the first fast-food enterprise (the unity of the above strategy interval represents the Cournot critical quantity defined and discussed in economic literature and in [34]).
2. Strategies

\[ y \in F := \mathbb{U} = [0, 1], \]
representing any food quantity produced by the second fast-food enterprise;

3. Shared strategies

\[ z \in C := z\mathbb{U} = [0, \hat{z}], \]
representing the collective cooperative green strategy space. We simply assume that the chosen strategy set \( C \) is a compact interval and, consequently, we denote by \( \hat{z} \) the maximum of that interval. The two players determine together the strategy space \( C \).

3.3. Cooperative Strategy Space

Any strategy \( z \) in the compact \( C \) represents the aggregate investment for the environmentally sustainable packaging. Specifically,

\[ z \in C = [0, \hat{z}] \]
represents the common investments for adopting advanced innovative alternative packaging from recycled paper waste.

This strategy can be implemented by adopting a compact thermoforming packaging machine allowing the generation of a sustainable, green cardboard composite, used for packing fresh food.

This multilayer composite is made of up to 90 percent renewable raw materials. In order to compensate for the used renewable raw materials, the two producers agree to offer a meaningful percentage of products from low-impact sources.

3.4. Payoff Functions

We propose a classic Cournot-type vector function

\[ c : E \times F \rightarrow \mathbb{R}^2 : (x, y) \mapsto (4x(1 - x - y), 4y(1 - x - y)) \]
perturbed by a non-linear stochastic function \( v \), as the payoff function of our game.

The non-linear stochastic perturbation \( v \) is defined by

\[ v : \mathbb{R}^{2,2} \times C \rightarrow \mathbb{R}^2 \]
with

\[ v_\mu(z) = \mu \begin{pmatrix} z \\ z^2 \end{pmatrix} = \begin{pmatrix} \mu_{11} & \mu_{12} \\ \mu_{21} & \mu_{22} \end{pmatrix} \begin{pmatrix} z \\ z^2 \end{pmatrix} = \begin{pmatrix} \mu_{11}z + \mu_{12}z^2 \\ \mu_{21}z + \mu_{22}z^2 \end{pmatrix}, \]

for every square matrix \( \mu \in \mathbb{R}^{2,2} \) and \( z \in C \).

Setting \( M = \mathbb{R}^{2,2} \), the parametric payoff function of food enterprise 1 is the function \( f_1^1 \) of the parallelepiped

\[ S \times M = E \times F \times C \times \mathbb{R}^{2,2} \]
into the real line, defined by

\[ f_1^1(x, y, z) = 4x(1 - x - y) + \mu_{11}z + \mu_{12}z^2, \quad (1) \]
for every quadruple \( (x, y, z, \mu) \) in \( S \times M \).

Clearly, we are choosing a very specific Cournot core \( c \); we decided to adopt a core \( c \) proportional to the normalized symmetric Cournot duopoly core, by a factor of 4. Here:

- The term \( \mu_{11} > 0 \) represents the interest rate associated with the first player, on the collective investment decided by the two food resellers in the innovative paper packaging (meaning that the term \( \mu_{11}z \) is the net profit of the first player coming from the investment \( z \));
• The term $\mu_{12} \in [-1, 0]$ represents a coefficient-cost relative to the same investment and we assume, in $f^2$, a quadratic dependence upon $z$, just to fix an order of polynomial approximation.

The payoff function of food enterprise 2 is the function $f^2$ of the parallelepiped $S \times M$ into the real line, defined by

$$f^2_\mu(x, y, z) = 4y(1 - x - y) + \mu_{21}z + \mu_{22}z^2$$

for every quadruple $(x, y, z, \mu)$ in $S \times M$. Here:

• The term $\mu_{21} > 0$ represents the interest rate associated with the second player on the collective investment decided by the two resellers in the innovative paper packaging;

• The term $\mu_{22} \in [-1, 0]$ represents a coefficient-cost relative to the same investment and we assume, in $f^2$, a quadratic dependence upon $z$.

Resuming, the payoff function of the coopetitive game $G$ is given by

$$f_\mu(x, y, z) = \left( \begin{array}{c} 4x(1 - x - y) \\ 4y(1 - x - y) \end{array} \right) + \mu \left( \begin{array}{c} z \\ z^2 \end{array} \right) = c(x, y) + v_\mu(z)$$

for every quadruple $(x, y, z, \mu)$ in the strategy parallelepiped $S \times M$.

4. Results

4.1. Study of the Coopetitive Game $G$ by Translations

Now, let us fix, for the moment, a stochastic matrix $\mu$. Let $z$ be a cooperative strategy belonging to the interval $C = [0, \hat{z}]$. The game

$$G_\mu(z) = (p_{\mu, z}, \geq),$$

characterized by the payoff function $p_{\mu, z}$, which is defined on the square $U^2$ through

$$p_{\mu, z}(x, y) = f_\mu(x, y, z),$$

for every $(x, y, z) \in S$, is the translation of the game $G_\mu(0)$ by the vector

$$v_\mu(z) = \mu \left( \begin{array}{c} z \\ z^2 \end{array} \right).$$

Therefore, we can analyze the initial game $G_\mu(0)$ and, then, we translate the features of the initial game $G_\mu(0)$ by the vector $v_\mu(z)$.

4.2. Study of the Game $G_\mu$ for a Specific State of the World $\mu$

From now on, we concentrate on the particular exemplary case in which the state of the word is

$$\mu = \left( \begin{array}{cc} 4 & -3/5 \\ 3/2 & -1/10 \end{array} \right).$$

4.2.1. The Initial Cournot Core

We start with the Cournot payoff space

$$\text{im}(c) = c(E \times F),$$

and the that is the image of the strategy square $E \times F$, under the Cournot core $c$ (Figure 1).
4.2.2. Translation Using the Vector Family $v_\mu$

We translate using the vector family

$$v_\mu = \{v_\mu(z)\}_{z \in C},$$

(see Figure 2), and obtain the coopetitive dynamical path of the initial payoff space, which is the image of the function $f_\mu$:

$$f_\mu(E \times F \times C) = \text{im}(c) + v_\mu(C).$$

In Figure 3, we show the construction of the payoff space of game $G_\mu$. The coopetitive dynamical path of the initial Cournot payoff space is represented in Figure 4.
4.2.3. Choice of the Cooperative Strategy Set C

Above, we chose the maximum of interval $C$ exactly equal to

$$\hat{z} = \frac{20}{3},$$

since $\hat{z} = 20/3$ is the unique positive solution of the equation

$$f_1^{\mu}(0, 0, \hat{z}) = 0.$$

In other terms, we are choosing the value of the shared investment that returns the profits of the first player to the initial state (that is, the non-cooperative state).

4.3. Possible Solutions of the Game $G_\mu$

We propose different types of possible solutions for the parametric game $G_\mu$.

These types of possible solutions fall in two families: the family of purely coopetitive solutions and the family of fully collaborative scenarios.

- The first ones are the solutions in which the only allowed collaboration consists of the cooperative frame determined by the investment, the common investment, in green technologies and green habits).
- The second ones are solutions in which the two enterprises can also collaborate at the level of the initial non-cooperative strategies; that is, at the level of production quantities).
4.3.1. Purely Coopetitive Solutions

We consider the following purely coopetitive payoff solutions:

- The Pareto boundary of the Nash payoff (equilibrium) path;
- The collectively optimal Nash payoff $N'(z^\mu_0)$, which is by itself a purely coopetitive payoff solution;
- A purely coopetitive payoff solution $P_\mu$, collectively equivalent to the optimal Nash equilibrium $N'(z^\mu_0)$, starting from the threat point $N'(0)$.

4.3.2. Fully Cooperative Scenarios

Among the various possible solutions in a fully cooperative payoff scenario, we selected the following ones:

- A super-cooperative payoff solution $K'(z^\mu_0)$;
- A super-cooperative payoff solution $K'_0(z^\mu_0)$.

4.4. The Nash Trajectory of the Coopetitive Game $G_\mu$

In Figure 5, we show the Nash trajectory of the coopetitive game $G_\mu$.

Figure 5. Nash trajectory $N' : z \rightarrow N'(z) = (4/9, 4/9) + \mu(z, z^2)$.

In Figure 6, we show the Pareto boundary of the Nash payoff equilibrium path. That Pareto boundary is the portion of the curve $N'(C)$ constituted by the points $N'$ of $z$ with $z \in [z_0, \hat{z}]$, where

$$z_0 = \frac{\mu_{11}}{2\mu_{12}} = 10/3.$$
4.5. The Collective Optimal Nash Payoff and the Optimal Parameter $z^*_\mu$

We find the collectively optimal Nash payoff $N'(z^*_\mu)$ in Figure 7, where $N'$ is the parametric Nash payoff trajectory and

$$N(z^*_\mu) = (1/3, 1/3, z^*_\mu)$$

is the triple of maximum collective gain within the Nash equilibrium zone.

![Figure 7. Collectively optimal Nash payoff $N'(z^*_\mu)$.](image)

The optimal parameter $z^*_\mu$ is obtained by maximizing the collective payoff function,

$$f_1^\mu + f_2^\mu,$$

of the two players upon the constraint $N'(C)$. Hence, we readily obtain

$$z^*_\mu = -\frac{\mu_{11} + \mu_{21}}{2(\mu_{12} + \mu_{22})} = 55/14.$$

4.6. Analytical Form of Purely Coopetitive Solutions

The unique collectively optimal Nash payoff is

$$N'(z^*_\mu) = N'(0) + z^*_\mu \left( \frac{1}{z^*_\mu} \right),$$

therefore, our unique purely coopetitive solution with maximum collective gain is the Nash equilibrium $N(z^*_\mu)$.

In Figure 8, we show another purely coopetitive sharing, represented by the payoff $P_\mu$, of the maximum collective gain obtained by the two players falling on the purely coopetitive solution $N(z^*_\mu)$. 

![Figure 8. Collectively optimal Nash payoff $N'(z^*_\mu)$.](image)
4.7. Interpretations of Purely Coopetitive Solutions

We emphasize that the above two payoff sharings, \( N'(z^*_\mu) \) and \( P_\mu \), are very different from each other. Indeed, the first solution belongs to the payoff space of the game; on the contrary the second one does not, as shown in Figure 9. Moreover, it is clear that the solution \( N'(z^*_\mu) \) represents a natural economic outcome when the two players know \textit{a priori} the state of the world \( \mu \) and then they fall naturally into the Nash equilibrium \( N(z^*_\mu) \). On the contrary, when the two players do not know \textit{a priori} the state of the world \( \mu \) they are facing, during their coopetitive interaction, it seems quite natural to consider, as the unique possible starting point of the bargaining problem, the very well-known Cournot Nash equilibrium \( N(0) \). In this last scenario, the scenario of an \textit{unknown future}, the only reasonable action for the two players is to agree ex-ante on a fair sharing \((1/2, 1/2)\), of the future payoff profile \( N'(z^*_\mu) \). Practically, they had to choose their respective equilibrium strategy and, after obtaining the optimal Nash payoff, they need to fairly share the gains half and half.

4.8. Super-Cooperative Solutions

In a super-cooperative scenario, we propose the payoff solutions \( K'(z^*_\mu) \) in Figure 10 and \( K'_0(z^*_\mu) \) in Figure 11.

For the above two cooperative solutions, we could repeat word for word the remarks of the above subsection concerning the purely coopetitive scenarios.
Figure 10. Super-cooperative payoff solution $K'(z^*_\mu) = f_\mu(1/4, 1/4, z^*_\mu)$.

Figure 11. Super-cooperative solution $K'_0(z^*_\mu)$, possible sharing of the collective gain realized in $K'(z^*_\mu)$.

4.9. Analytic Determination of the Purely Coopetitive Solutions

Again, in a purely coopetitive mood, we suggest the Nash strategy

$$N(z^*_\mu) = (1/3, 1/3, z^*_\mu),$$

with corresponding payoff profile

$$N'(z^*_\mu) = N'(55/14) = \left(\frac{4/9}{4/9} + \frac{11}{14^2 \times 2} \left(\frac{230}{155}\right)\right).$$

and it is the reciprocal image of the payoff $N'(z^*_\mu)$, which belongs to the space $f(S)$.

The collective gain at the Nash equilibrium $N(z^*_\mu)$ is

$$(f^1_\mu + f^2_\mu)(N(z^*_\mu)) = 8/9 + \frac{385 \times 11}{14^2 \times 2}.$$ 

The Purely Coopetitive Payoff Solution $P_\mu$

From the knowledge of the profile strategy $N(z^*_\mu)$, we can immediately generate another purely coopetitive payoff profile

$$P_\mu = \left(\frac{(f^1_\mu + f^2_\mu)(N(z^*_\mu))}{2}, \frac{(f^1_\mu + f^2_\mu)(N(z^*_\mu))}{2}\right) \approx (5.85, 5.85),$$
obtained from the payoff $N'(z^*_\mu)$ by calculating the collective gain at $N(z^*_\mu)$ and sharing it fairly by a straightforward Kalai–Smorodinsky method, using as a threat point the initial Nash payoff $N'(0)$. Clearly, $N'(z^*_\mu)$ is the solution when we use as a threat point $N'(z^*_\mu)$ itself.

4.10. Analytic Determination of the Super-Cooperative Solutions

From a super-cooperative perspective, considering $N'(z^*_\mu)$ as a threat point (see Figure 10), we can suggest the profile strategy

$$K(z^*_\mu) = \left(\frac{1}{4}, \frac{1}{4}, z^*_\mu\right),$$

which clearly belongs to our strategic space $S$. The corresponding payoff is

$$K'(z^*_\mu) = f(K(z^*_\mu)) = K'(0) + z^*_\mu \mu \left(\frac{1}{z^*_\mu}\right) \approx (6.95, 4.84),$$

where $K'(0)$ is the initial standard Cournot bargaining payoff $(1/2, 1/2)$.

Again, from a super-cooperative perspective, considering $N'(0)$ as a threat point (see Figure 11), we can suggest the profile strategy

$$K'_0(z^*_\mu) = \left(\frac{(f^1_{\mu} + f^2_{\mu})(K(z^*_\mu))}{2}, \frac{(f^1_{\mu} + f^2_{\mu})(K(z^*_\mu))}{2}\right) \approx (5.9, 5.9),$$

which belongs to the straightline of maximum collective gains, but does not belong to the payoff space of the game $G_\mu$.

4.11. The Main Result

We should notice that we determined the closed form for the above solutions and optimal values for any possible state of the world. We recap the obtained results in the following, Theorem 1.

**Theorem 1.** Let $\mu \in \mathbb{R}^{2,2}$ be an invertible matrix with positive first column and negative second column. Then, the coopetitive game defined by

$$G_\mu : S \to \mathbb{R}^2 : (x, y, z) \mapsto f_\mu(x, y, z) = \left(\frac{4x(1 - x - y)}{4x(1 - x - y)} + \mu \left(\frac{z}{z^2}\right)\right) \quad (4)$$

admits one unique purely coopetitive solution with maximum collective gain, the strategy profile

$$N(z^*_\mu) = (1/3, 1/3, z^*_\mu).$$

In particular:

- The optimal investment (the investment of maximum collective gain) in the cooperative strategy is

$$z^*_\mu = \min \left\{-\frac{1}{2} \frac{\mu_{11} + \mu_{21}}{\mu_{12} + \mu_{22}}, \frac{\mu_{11}}{\mu_{12}} - \frac{\mu_{21}}{\mu_{22}}\right\};$$

- The purely coopetitive solution in the payoff space is

$$N'(z^*_\mu) = N'(0) + z^*_\mu \mu \left(\frac{1}{z^*_\mu}\right);$$

- The fifty–fifty sharing of the purely coopetitive solution (in the payoff space) is

$$P_\mu = \frac{1}{2} \left(\frac{f^1_{\mu} + f^2_{\mu}}{f^1_{\mu} + f^2_{\mu}}\right)(N(z^*_\mu)).$$
Moreover, the game $G_\mu$ admits one unique best compromise super-cooperative solution, which is the strategy profile

$$K(z^*_\mu) = (1/4, 1/4, z^*_\mu).$$

In particular:

- The corresponding super-cooperative solution in the payoff space is the pair

$$K'(z^*_\mu) = K'(0) + z^*_\mu \mu \left( \frac{1}{z^*_\mu} \right)$$

where $K'(0)$ is the initial standard Cournot bargaining payoff $4(1/8, 1/8)$;

- The fifty–fifty sharing of the super-cooperative solution (in the payoff space) is

$$K'_0(z^*_\mu) = \frac{1}{2} \left( f^1_{\mu} + f^2_{\mu}, f^1_{\mu} + f^2_{\mu} \right) (K(z^*_\mu)).$$

5. Discussion

5.1. Two Possible Maximum Collective Gains

The results of the analysis prove that we can find, in the strategy space, two win–win maximum collective gain payoff solutions for the firms involved: one purely coopetitive solution and one super-cooperative solution.

We emphasize that these solutions are maximum collective gain solutions in their respective Pareto boundaries (the Pareto boundary of Nash coopetitive equilibria and the Pareto boundary of the entire game).

The above solutions also include advantages for the environment, for human health and for the climate-change crisis.

5.2. Interpretation of the Space $S \times M$

We want to stress that:

- The first component of any strategy profile, which belongs to the strategic interval $[0, 1]$, represents any food quantity produced by the first fast-food enterprise;

- The second component of any strategy profile, which belongs to the strategic interval $[0, 1]$, represents any food quantity produced by the second fast-food enterprise;

- The third component of any strategy profile, which belongs to the strategic interval $C$, represents the common investment chosen to acquire advanced green technologies for innovative packaging derived from recycled paper waste;

- Any parameter belonging to the matrix space $M$—those matrices which are invertible and with a positive first column and negative second column—represents the state of the world at the end of the coopetitive process in which, finally, we can see the profits and costs deriving from the adoption of the green technologies.

5.3. Interpretation of the Payoff Solutions

The two components of the payoff solutions $N'(z^*_\mu), P'_\mu, K'(z^*_\mu)$ and $K'_0(z^*_\mu)$ represent gains. Specifically, the first component represents the income of the first player and the second component represents the income of the second player. However, $P'_\mu$ and $K'_0(z^*_\mu)$ are only indications of how to share the collective gains obtained, respectively, in the payoffs $N'(z^*_\mu)$ and $K'(z^*_\mu)$. Indeed, there exist no strategy profiles leading to those gains directly, while the payoffs $N'(z^*_\mu)$ and $K'(z^*_\mu)$ come directly from $N(z^*_\mu)$ and $K(z^*_\mu)$.

5.4. Advantages for the Environment

The first strategy profile $N(z^*_\mu)$ means that food enterprise 1 decides to produce exactly 1/3 of the Cournot critical quantity production and also food enterprise 2 decides to produce exactly 1/3 of the Cournot critical quantity production. At the same time, the two players decide together to invest the optimal money amount $z^*_\mu$, while facing the state of the world $\mu$. 


Even more respectful for the environment (because of the reduction in required production) is the super-cooperative solution \( K(z^*_\mu) \), which suggests, for food enterprise 1, producing exactly 1/4 of the Cournot critical quantity production and, symmetrically, for food enterprise 2, producing, again, exactly 1/4 of the Cournot critical quantity production. Therefore, the two enterprises can gain a little bit more with respect to the (collectively) optimal Nash solution, producing less food, less waste, less pollution and diminishing the probability and amount of food-waste.

5.5. Stability of the Payoff Solution \( K'_0(z^*_\mu) \)

From a mathematical–economics perspective, once the binding contract is signed, the solution payoff \( K'_0(z^*_\mu) \) is more “stable with respect to convenient small production oscillations” because of the presence of a maximum collective gain plateau (an indifference straight-line segment) in the payoff Pareto boundary of the game \( G_\mu \), just around the profile outcome \( K'(z^*_\mu) \) (see Figure 9).

5.6. Two Possible Types of Sharing

We consider two possible types of solutions, which are very different from each other. The first solution type belongs to the payoff space of the game; on the contrary, the second one does not, as shown in Figure 9. Solutions \( N'(z^*_\mu) \) and \( K'(z^*_\mu) \) represent natural economic outcomes when the two players know a priori the state of the world \( \mu \) they are facing during the coopetitive process and, consequently, they fall naturally to the Nash equilibrium \( N(z^*_\mu) \) or to the associated best compromise solution \( K(z^*_\mu) \). On the contrary, when the two players do not know a priori the state of the world \( \mu \) they are facing during their coopetitive interaction, it seems quite natural to consider, as the unique possible starting point of the bargaining problem, the very well-known Cournot–Nash equilibrium \( N(0) \), which is also equivalent to starting from the conservative strategy profile \((0,0)\). In the latter scenario (the scenario of a fully unknown future), the only reasonable action for the two players remains to agree ex-ante on a fair sharing, \((1/2,1/2)\), of the future payoff profile \( N'(z^*_\mu) \) or \( K'(z^*_\mu) \). Practically, they had to choose their respective optimal strategies and, after obtaining the corresponding optimal payoffs, they need to fairly share the gains half and half.

Figure 9 shows all the proposed solutions in the same graph.

6. Conclusions

6.1. Micro-Economic Point of View: The Sustainability of Natural Resources and Perfect Competition

In this paper, from a microeconomic point of view, we presented a coopetitive model of the strategic interaction of two food enterprises through investments in sustainable-packaging technologies.

Specifically, we consider the producer/reseller decision problem of an industrial organisation in conditions of perfect competition within small oligopoly clusters.

Indeed, very often, one major sustainability problem is that the presence of direct competitors in the same market determines entrepreneurship choices which lower production costs and packaging costs at the expense of the environment and public health.

6.2. Macro-Economic Point of View: Plastic Pollution and Food Marketing

From a macroeconomic point of view, we dealt with the renowned problem of plastic pollution caused by food consumption and its conservation.

For this purpose, in order to show economic scenarios in which the respect and preservation of the environment and natural resources are quantitatively compatible with profits and economic growth, we present a provisional coopetitive model of the strategic interaction of two food enterprises, in direct duopoly competition, through investments in sustainable-packaging technologies.

As a solution of the coopetitive interaction, we determined possible “fair coopetitive agreements”, allowing:
• The actors to increase the gains with respect to a classic situation of non-collaboration;
• A reduction in production costs, by using raw materials;
• A reduction in environmental costs, by using low-carbon new technologies and lowering the inflow of plastics to the environment.

6.3. The Coopetitive Model

We assume the existence of two competitors selling a very similar type of foods on the same market so that, from a competitive perspective, we construct a classic “Cournot duopoly core” upon which we define a parametric game, namely, a coopetitive game, together with its possible dynamical scenarios and solutions.

We should notice that beyond the parameter arising from the cooperation construct, we introduce a matrix of stochastic variables, which we can also consider as a state of the world.

Moreover, we examine numerically one possible state of the world to exemplify our model proposal. We determine analytically and graphically the optimal investment in the cooperative strategy, the purely coopetitive solution and some super-cooperative solutions.

The cooperative strategy represents the common investment chosen to acquire advanced green technologies for innovative packaging, while the fourth component of any solution in the strategy space represents the state of the world at the end of the coopetitive process in which, finally, we can see the profits and costs derived from the adoption of the green technologies.

6.4. Solutions

We showed the complete analysis of our proposed game and we suggested some of its possible solutions. In particular, we propose:
• Two pure coopetitive solutions —solutions in which the only allowed collaboration consists of the cooperative frame determined by the investment, the common investment, in green technologies and green habits;
• Two super-cooperative solutions —solutions in which the two enterprises can also collaborate at the level of the initial non-cooperative strategies (that is, at the level of production quantities) on the coopetitive maximal Pareto boundary of our interaction.

In both cases, in order to quantitatively determine the desired solutions we adopt the Kalai–Smorodinsky method (otherwise, we could use, for example, the classic Nash bargaining solution).

6.5. Economic Interpretation

We emphasise that the non-cooperative part of the solution indicates the classic production quantity while the cooperative part of the solution indicates the necessary common investment needed to obtain the final gains.

We, moreover, emphasise that after the two players train the maximum collective gain on the Nash equilibrium curve or the Parado boundary, they need to cooperate again to share the pie in the most fairly way.

6.6. Environmental Impact of the Model and Its Solutions

We observed that the sustainable packaging suggested by our model and its solutions, and the possible lower production in the super-cooperative scenarios (Cournot model of pure competition requires equilibrium production quantities greater than our super-cooperative production quantities), contribute to the transition towards low-carbon and green economies. The reduction in the environmental impact associated with the more sustainable food packaging persists throughout the supply chain, in terms of lowering the plastic pollution and using renewable raw materials. The lower use of plastic reduces the greenhouse gas emissions in food marketing (use of low-carbon technologies), and creates less impact on oceans (use of the minimal quantity of plastic), less impact on forests (use of recycled paper), and less air pollution, while respecting the market laws.
7. Matlab Code

Here we present the code written for the study of the game, the graphical representation of the payoff space and the proposed solutions (Listing 1).


```matlab
syms('x','y','z')
m11 = 4;
m12 = -0.6;
m21 = 1.5;
m22 = -0.1;
% payoff functions
f1 = 4*x.*(1 - x - y) + m11.*z + m12.*z.^ 2;
f2 = 4*y.*(1 - x - y) + m21.*z + m22.*z.^ 2;
f = [f1; f2];
v = [x y];
J = jacobian(f,v)
D = det(J)
g = solve(D,y) % critical zone
% graphical representation of the payoff space
for z = [0:40/3600:40/6] %z maximum
%AB
x = linspace(0,1)
y = 0
X1 = 4*x.*(1 - x - y) + m11.*z + m12.*z.^ 2;
Y1 = 4*y.*(1 - x - y) + m21.*z + m22.*z.^ 2;
plot(X1,Y1,'b')
hold on
%BC
x = linspace(0,1)
y = 1
X2 = 4*x.*(1 - x - y) + m11.*z + m12.*z.^ 2;
Y2 = 4*y.*(1 - x - y) + m21.*z + m22.*z.^ 2;
plot(X2,Y2,'b')
%CD
x = linspace(0,1)
y = 1
X3 = 4*x.*(1 - x - y) + m11.*z + m12.*z.^ 2;
Y3 = 4*y.*(1 - x - y) + m21.*z + m22.*z.^ 2;
plot(X3,Y3,'b')
%AD
x = linspace(0,1)
y = 0
X4 = 4*x.*(1 - x - y) + m11.*z + m12.*z.^ 2;
Y4 = 4*y.*(1 - x - y) + m21.*z + m22.*z.^ 2;
plot(X4,Y4,'b')
%HK
```

Listing 1. Cont.

```matlab
x = linspace(0,1/2)
X5 = 4*x.*(1 − x − (1/2 − x)) + m11.*z + m12.*z.^2;
Y5 = 4*(1/2 − x).*(1 − x − (1/2 − x)) + m21.*z + m22.*z.^2;
plot(X5,Y5,'b')
end

% Nash curve
a = 40/6
z = linspace(0,a)
x = 1/3
y = 1/3
N1 = 4*x.*(1 − x − y) + m11.*z + m12.*z.^2;
N2 = 4*y.*(1 − x − y) + m21.*z + m22.*z.^2;
plot(N1,N2,'r')

% maximum collective gain Nash equilibrium
z = -(m11 + m21)/(2*(m12 + m22))
x = 1/3
y = 1/3
N1 = 4*x.*(1 − x − y) + m11.*z + m12.*z.^2;
N2 = 4*y.*(1 − x − y) + m21.*z + m22.*z.^2;
plot(N1,N2,'bx')

% Kalai-Smorodinsky curve
z = linspace(0,a)
x = 1/4
y = 1/4
N1 = 4*x.*(1 − x − y) + m11.*z + m12.*z.^2;
N2 = 4*y.*(1 − x − y) + m21.*z + m22.*z.^2;
plot(N1,N2,'go')
```

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