Entropy Generation Due to Magneto-Convection of a Hybrid Nanofluid in the Presence of a Wavy Conducting Wall

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Abstract: The two-dimensional, time-independent conjugate natural convection flow and entropy generation are numerically investigated in three different cases of a wavy conducting solid block attached to the left wall of a square cavity. A hybrid nanofluid with titania (TiO$_2$) and copper (Cu) nanoparticles and base fluid water in the fluid part is considered in the presence of a uniform inclined magnetic field. The leftmost wall of the cavity is the hot one and the rightmost one is the cold one. Radial-basis-function-based finite difference (RBF-FD) is performed on an appropriate designed grid distribution. Numerical results in view of streamlines and isotherms, as well as average Nusselt number in an interface and total entropy generation are presented. The related parameters such as Hartmann number, Rayleigh number, conductivity ratio, amplitude in wavy wall, number of waviness, and inclination angle of magnetic field are observed. Convective heat transfer in the fluid part is an increasing function of $kr$, $Ra$, $\gamma$, while it deflates with the rise in $Ha$ in each case. Total entropy generation increases with the increase in $Ra$ and $kr$ but it decreases with $Ha$ values. Average Bejan number ascends with the rise in $Ha$ and descends with the rise in $Ra$.

Keywords: wavy conducting solid; conjugate natural convection flow; Cu-TiO$_2$/water; radial basis functions; entropy generation

MSC: 76R10; 35Q35;76M22; 80M22; 28D20

1. Introduction

In recent years, there have been many studies on heat transfer (HT) and fluid flow (FF) in enclosures in the presence of different combinations of porous medium, mono and hybrid nanoparticles, bacteria, conducting bodies, and magnetic fields (MF). The main objective in each of these studies is to observe the HT enhancement. The influence of nanoparticles on HT improvement was initially shown by Choi et al. [1]. Since then, many numerical and experimental studies may be found on nanofluids (NF).

Kasaeipoor et al. [2] performed a numerical study by using the Lattice Boltzmann method (LBM) to investigate the entropy generation (EG) due to buoyancy-induced flow in a closed space with refrigerant solid and hybrid NFs. They also made a work to measure the thermophysical properties of the nanofluid MWCNT-MgO (15–85%)/Water. They showed that the configuration of refrigerant rigid body pronounced the effect on EG. Hybrid NFs are used for different HT applications such as solar collectors and radiators in literature due to the controllable viscosity, density, thermal conductivity, and specific heat. These applications are reviewed by Huminic and Huminic [3]. Another review has been performed on HT enhancement techniques using hybrid NFs by Muneshwaran et al. [4]. Dutta et al. [5] studied the impact of hybrid nanoparticles on conjugate mixed convection (MC) of a viscoplastic fluid in a ventilated enclosure. A hybrid NF enhances the HT...
and EG with the rate of HT higher than the rate of EG. Zhang et al. [6] performed a study on conjugate buoyant heat transport in NFs with different nanoparticles. They formulated the generalized Cattaneo law of thermal flux with analytical methods. They found that the heat flux given by the fractional equation leads to the decrease in HT in the solid wall. Priam et al. [7] studied conjugate natural convection (NC) in a vertically divided square-shaped closed space with a corrugated solid partition into air and water regions. They showed that increasing the partition thermal conductivity enhances the thermal performance by up to 25%. Conjugate unsteady NC of air and non-Newtonian fluid in a thick-walled cylindrical closed space partially filled with a porous media was studied by Rodríguez-Núñez et al. [8]. They used finite-volume method (FVM) to solve governing equations. The effects of internal heat generation or absorption on conjugate thermal-NC of a suspension of hybrid NF in a partitioned circular annulus are analyzed by Tayebi et al. [9]. They proposed a new correlation on the mean heat exchange rate in the defined parameters. Entropy generation is an important issue for almost all energy system applications. EG is reviewed in the literature [10] for NC and MC HT. Mondal and Mahapatra [11] solved a numerical problem on magnetohydrodynamics (MHD) double-diffusive MC and EG of NF in a trapezoidal shaped closed space considering the effects of MF. They used the second- and the fourth-order finite difference (FD) approximations to solve the governing equations, and their results show that low MF and low aspect ratio are always preferable to reduce total EG. Korei et al. [12] performed a study on the combined convection and irreversibility analysis under MF for hybrid NF in a partially heated lid-driven enclosure. They used OpenFOAM with a C++ open-source code. They found that the combination of Al2O3 75% and Cu 25% give the highest values of the mean Nusselt (Nu) number and the entropy production. Tayebi et al. [13] performed a study on NC and EG of hybrid-nanoliquid-filled annulus delimited by two elliptic cylinders. Their results showed that hybrid nanoliquid significantly alters the hydrothermal characteristics and EG. Ahrar et al. [14] performed a numerical analysis of NF HT and EG in a closed space by using a novel total variation diminishing hybrid LBM under the MF. They showed that EG can be controlled via MF. Majeed et al. [15] solved the problem of EG in a hexagonally shaped closed space with magnetized hybrid nanomaterials. They used FEM, and their results report that increasing MF’s effects reduces the HT since the conduction motion occupies the motion of the FF. Priyadharsini and Sivaraj [16] studied the entropy production in a ferrofluid filled square closed space with a solid body generating inner heat. They showed that minimum entropy production occurs at the lowest thermal conductivity value. Sachica et al. [17] solved an MHD MC and EG problem by using vorticity, and the stream function form coupled with the energy equation is solved using the control volume method on a nonuniform orthogonal Cartesian grid. They found that the EG is dominated by irreversibilities due to HT for all values of the nanoparticle volume fraction. Inclined magneto-conjugate HT and EG in an inclined domain with a wavy partition are analyzed by Priam and Nasrin [18]. They used FEM and observed that thermal performance and EG are significantly influenced by the MF intensity and closed space inclination. The two-phase mixture model is used to assess the effects of the nanoparticle shape on the hydrothermal aspects and EG of turbulent convection of NF by Alsarraf et al. [19], applying the problem to flat plate solar collector. They examined EG corresponding to different cases and a flow rate. Varol et al. [20] performed a study on the EG due to the conjugate NC in a thick-walled closed space by using FDM for different parameters. Their results demonstrate that EG increases with increasing thermal conductivity ratio and thickness of the walls. EG due to NC in a partially heated triangular closed space was investigated by Varol et al. [21] utilizing FDM. They observed that EG increases but Bejan (Be) number decreases with increasing Ra number. In their other work in [22], they solved the problem of EG for conjugate trapezoid-shaped closed space. They showed that the most important parameters affecting HT and FF are thermal conductivity ratio and dimensionless thickness of the solid wall of the closed space. The conjugate NC flow of SiO2-water nanofluid in the presence of oxytactic bacteria, periodic magnetic field, and Brownian and Thermophoresis
effects is studied in [23]. The results show that convective HT is an increasing function of conductivity ratio. The EG and convection effect on magnetized hybrid nano-liquid flow inside a trapezoidal closed space with a zigzagged wall is studied in [24] and nano-encapsulated phase change particles in a semi-annular cavity in [25]. Magherbi et al. [26] observed HT and FF irreversibilities on unsteady NC flow, utilizing a control-volume finite element method (FEM). In their results, FF irreversibility dominates over HT irreversibility as the Rayleigh (Ra) number rises. Ilis et al. [27] examined total EG in the case of different aspect ratios, performing alternating-direction implicit scheme. They reported that total EG increases with the increase in Ra. A similar aspect ratio analysis is noted in Oliveski et al. [28] using FVM. Parvin et al. [29] implemented FEM for the simulation of NC of Cu–water NF in an odd-shaped cavity while taking EG into account. Their results reveal that HT irreversibility rises with the increase in Ra. Foradanjani et al. [30] studied the radiation effect on NC and EG in a diagonal rectangular cavity involving an NF in the presence of a uniform MF. They concluded that EG increases with increasing radiation parameter.

In the current study, the conjugate (convection and conduction) natural convection flow and entropy generation of a hybrid NF, TiO$_2$-Cu/water, in an enclosure with a wavy conducting solid block is numerically investigated in the presence of an inclined uniform MF. To the best of the authors’ knowledge, the wavy conducting solid part is taken into account for the first time. Numerical results are obtained by performing an in-house implementation of the radial-basis-function-based finite difference method.

2. Problem Formulation

The two dimensional, time-independent natural convection flow in an enclosure involving wavy conducting solid block attached to the left wall is considered. The leftmost wall is the hot wall, while the right vertical straight wall is the cold wall in the enclosure sketched in Figure 1. The top and the bottom walls are adiabatic, where $\partial T/\partial n = 0$. Hybrid NF TiO$_2$-Cu/water exists in the fluid part, while the the solid part of size $d_s$ is thermally conducting. The enclosure is also exposed to a uniform MF with an inclination angle $\gamma$. It is assumed that the nanofluid is Newtonian, the flow is laminar and incompressible, and thermal equilibrium exists between nanoparticles and the base fluid. Induced MF, viscous dissipation, Joule heating, and radiation effects are neglected.

Bearing in mind the single-phase NF model, some physical relations for NF may be listed as

$$\rho_{nf} = (1 - \phi)\rho_f + \phi_1\rho_1 + \phi_2\rho_2$$  \hspace{1cm} (1a)

$$\rho c_p_{nf} = (1 - \phi)(\rho c_p_f) + \phi_1(\rho c_p_1) + \phi_2(\rho c_p_2)$$  \hspace{1cm} (1b)

$$\rho \beta = (1 - \phi)(\rho \beta_f) + \phi_1(\rho \beta_1) + \phi_2(\rho \beta_2)$$  \hspace{1cm} (1c)

$$\mu_{nf} = \mu_f(1 - \phi)^{-2.5}$$  \hspace{1cm} (1d)

$$k_{nf} = k_f \frac{(k_f + 2k_f - 2\phi(k_f - k_{np}))}{(k_{np} + 2k_f + \phi(k_f - k_{np}))}$$  \hspace{1cm} (1e)

$$\sigma_{nf} = \sigma_f \frac{2\phi\sigma_f - \phi\sigma_{np}}{(\sigma_{np} + 2\sigma_f) + \phi\sigma_f - \phi\sigma_{np}}$$  \hspace{1cm} (1f)

where $\phi = \phi_1 + \phi_2$, $k_{np} \equiv (\phi_1 k_1 + \phi_2 k_2)/\phi$, $\sigma_{np} \equiv (\phi_1 \sigma_1 + \phi_2 \sigma_2)/\phi$ and subindices $f, np, hnf$ refer to the host fluid, nanoparticle and hybrid NF, respectively; $\rho$ is the density; $\rho c_p$ is the specific heat at constant pressure; $\beta$ is the thermal expansion coefficient; $\mu$ is the dynamic viscosity (modelled by Brinkman’s model [31]); $k$ is the thermal conductivity; and $\sigma$ is the electrical conductivity ($k_{hf}$ and $\sigma_{hf}$ models are based on Maxwell’s model [32]).

Density variation due to the buoyancy force is treated with Bousinessq approximation. The other thermal and physical properties of water and nanoparticles are constant and are given in Table 1.

[Table 1 is not shown here but would typically contain the values for physical properties like density, thermal conductivity, etc., for water and nanoparticles.]

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Regarding the assumptions, the governing dimensional equations as a combination of continuity equation, momentum equations, and energy equation are as follows [21,30]:
\begin{align}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0, \quad (2a) \\
\mu_{hnf} \nabla^2 u &= \frac{\partial p}{\partial x} + \rho_{hnf} \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) - \sigma_{hnf} B_0^2 \left( v \sin \gamma \cos \gamma - u \sin^2 \gamma \right), \quad (2b) \\
\mu_{hnf} \nabla^2 v &= \frac{\partial p}{\partial y} + \rho_{hnf} \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) - \sigma_{hnf} B_0^2 \left( u \sin \gamma \cos \gamma - v \cos^2 \gamma \right) + (\rho \beta)_{hnf} (T - T_c) g, \quad (2c) \\
\alpha_{hnf} \nabla^2 T &= \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y}, \quad (2d) \\
\nabla^2 s &= 0, \quad (2e)
\end{align}

where \( \nabla \times \) and vorticity is computed using its definition \( \nabla \times u \) in the momentum equations. Velocity components in terms of stream function as \( u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \) satisfy the continuity equation. Thus, dimensionless equations in stream function-vorticity form are

\begin{align}
\nabla^2 \psi &= -w, \quad (4a) \\
\frac{\alpha_{hnf}}{\alpha_f} \nabla^2 T &= u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}, \quad (4b) \\
\nabla^2 s &= 0, \quad (4c) \\
\frac{\mu_{hnf}}{\mu_f} \Pr \nabla^2 \omega &= \frac{\rho_{hnf}}{\rho_f} \left( \frac{\partial \omega}{\partial x} + \frac{\partial \omega}{\partial y} \right) - \frac{\rho \beta_{hnf}}{\beta_f} \frac{\partial T}{\partial x} \\
&\quad - \frac{\sigma_{hnf}}{\sigma_f} \Ha^2 \Pr \left( \frac{\partial u}{\partial x} \sin^2 \gamma - \frac{\partial v}{\partial x} \cos^2 \gamma + 2 \frac{\partial u}{\partial x} \sin \gamma \cos \gamma \right), \quad (4d)
\end{align}

where Prandtl (\( \Pr \)), Rayleigh (\( \Ra \)) and Hartmann (\( \Ha \)) numbers are

\begin{align}
\Pr &= \frac{\mu_f}{\rho_f \alpha_f}, \quad \Ra = \frac{\beta_f \Delta T L^3}{\nu_f \alpha_f}, \quad \Ha = BL \sqrt{\frac{\alpha_f}{\beta_f}}. \quad (5)
\end{align}

On the boundary of the the fluid part, velocity of the fluid is zero, as is stream function, and vorticity is computed using its definition \( \nabla \times u \). For the boundary of the temperature in the entire region, \( T_h = 1 \) on the leftmost wall, while the right wall is the cold wall \( T_c = 0 \). In the interface, temperature condition is carried out as \( \partial T / \partial n = k_r \partial T_s / \partial n \), where \( kr = k_h / k_f \) is the thermal conductivity ratio. The normal gradient of temperature on the top and bottom adiabatic walls is zero.
3. Solution Method

Many researchers have been interested in radial basis functions (RBFs) in recent years because of the dependence of the discretization on the radial distance between points in the considered domain. Novel books [35,36] involve very useful details on RBFs in terms of both theoretical and applied approaches.

As a local method, RBF-FD provides localization using stencils. The paper Flyer et al. [37] explains the method very well. A brief explanation may be written as follows:

Let \( x_i \) be a point with coordinates \((x_i, y_i)\) in the concerned domain \( \Omega \). Let \( \Omega_i \) be a stencil centered at \( x_i \) involving \( ns \) number of points around \( x_i \). In this stencil, the conventional RBF interpolation augmented with polynomial terms is expressed as

\[
u(x) \approx s(x) = \sum_{i=1}^{ns} \chi_i \Gamma(||x - x_i||) + \sum_{i=1}^{m} \eta_k p_k(x),
\]

subject to the constraints

\[
\sum_{i=1}^{m} \chi_i p_k(x_i) = 0, \quad k = 1, 2, \ldots, m,
\]

where \( \Gamma \) is an RBF depending on the radial distance \( r = ||x - x_i|| \) with \( x = (x, y) \), and \( m \) is the total number of augmented polynomial terms.

In an equivalent matrix-vector form, Equation (6) may be written as

\[
(A_{new}) Y = u'(x) \implies Y = (A_{new})^{-1} u'(x),
\]

in which \( Y \) is the vector involving the terms \( \chi_1, \ldots, \chi_{ns}, \eta_1, \ldots, \eta_m \), and \( A_{new} \) is a matrix of size \((ns + m) \times (ns + m)\) involving RBF matrix constructed by \( \Gamma \) with \( ns \times ns \) nodes in a stencil and \( m \) additional polynomial terms added as rows and columns.

Cardinal basis functions are used to express the interpolant in another way as

\[
s_i(x) = \Psi(x) u'(x),
\]

where \( \Psi = [\theta_1(x), \theta_2(x), \ldots, \theta_{ns}(x), \theta_{ns+1}(x), \ldots, \theta_{ns+m}(x)] \), and the cardinal basis function is defined as

\[
\theta_j(x_i) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \quad j = 1, \ldots, ns.
\]

If \( Y \) is known, interpolation at a point \( x \) will be as

\[
s_i(x) = \Phi(x) Y, \quad \text{or} \quad s_i(x) = \Phi(x)(A_{new})^{-1} u'(x),
\]

where \( \Phi(x) = [\Gamma(||x - x_1||), \Gamma(||x - x_2||), \ldots, \Gamma(||x - x_{ns}||), p_k(x)] \).

Combining Equations (9) and (10), \( \Psi(x) \) is found as

\[
\Psi(x) u'(x) = \Phi(x)(A_{new})^{-1} u'(x) \implies \Psi(x) = \Phi(x)(A_{new})^{-1},
\]

where \( \Psi \) is a vector of size \((ns + m) \times m\), but the last \( m \) terms are not used. Derivatives of this \( \Psi \) allow us to obtain differentiation matrices.

The steps of the current in-house implementation may be given as

1. Nodes lying in a stencil centered at a node \( x_i \) are determined. Let these nodes be denoted by \( x_i^{loc} \).
2. \( x_i^{loc} \) are shifted and centered as described in [37]. Then, a scaling in \( x_i^{loc} \) is performed with \( scale = 1/\max(abs(x_i^{loc})) \) as introduced by Tominec [38]. Another useful scaling is also mentioned in [39].
3. In stencil, using a polyharmonic spline RBF, \( f = r^p \), and cubic augmented polynomial terms matrix, \( A_{new} \) is built.
4. \( \Psi \) or \( \Psi_x \) (for \( x \)-derivatives) or \( \Psi_y \) (for \( y \)-derivatives) or \( \Psi_z \) (for Laplacian) is found, and its first \( ns \) terms are saved. Note that scaling also affects \( \Psi_x \), \( \Psi_y \) and \( \Psi_z \). Therefore, the differentiation matrices \( D_x, D_y \) and \( D_z \) are constructed by the first \( ns \) terms of \( \Psi_x, \Psi_y, \Psi_z \).

Thus, the iterative solution of the dimensionless nonlinear governing equations is performed as follows:

\[
D_2 \psi^{n+1} = -w^n, \tag{12a}
\]

\[
u = u^{n+1} = D_y \psi^{n+1}, v = v^{n+1} = -D_x \psi^{n+1}, \tag{12b}
\]

\[
\left( \frac{\rho_{hnf}}{\mu} D_2 - M \right) T^{n+1} = 0, \tag{12c}
\]

\[
D_2 T_s^{n+1} = 0, \tag{12d}
\]

\[
\left( \frac{\rho_{hnf}}{\mu} PrD_2 - \frac{\rho_{hnf}}{\mu_f} M \right) w^{n+1} = -Ra Pr \left( \frac{\rho}{\rho_f} \right)_{hnf} D_x T^{n+1}, \tag{12e}
\]

where \( M \) is the matrix equal to \([u]_d D_x + [v]_d D_y\), \( d \) stands for diagonal, and \( n \) is the iteration level.

For stream function and vorticity equations, only the matrices \( D_x, D_y \), and \( D_z \) constructed in the fluid part are used. For temperature equations, these matrices are constructed in fluid and solid regions separately and are combined, keeping in mind the flux conditions at the interface.

Vorticity boundary conditions are achieved by using the definition of vorticity:

\[
\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = D_x v^{n+1} - D_y u^{n+1}. \tag{13}
\]

The iteration is terminated if

\[
\|\psi^{n+1} - \psi^n\|_\infty + \|T^{n+1} - T^n\|_\infty + \|\omega^{n+1} - \omega^n\|_\infty < \varepsilon, \tag{14}
\]

in which \( \varepsilon = 10^{-5} \) is the tolerance.

A parameter \( \tau \) in interval \((0,1)\) eases the vorticity equation once it is solved as

\[
\omega^{n+1} = \tau \omega^{n+1} + (1 - \tau) \omega^n. \tag{15}
\]

Average Nu number along the interface separating the solid and the fluid part is calculated as

\[
\overline{Nu_i} = \frac{k_{hnf}}{k_f} \int_0^s \frac{\partial T}{\partial n} ds, \tag{16}
\]

where \( s \) is the length of the interface.

In the fluid part, local HT irreversibility and local fluid friction irreversibility are adopted as \([40]\)

\[
HTI_{loc} = \frac{k_{hnf}}{k_f} \left( \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right), \tag{17a}
\]

\[
FFI_{loc} = \frac{\rho_{hnf}}{\mu_{hnf}} \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] + \frac{\rho_{hnf}}{\rho_{hnf}} \frac{\mu_f}{\mu_{hnf}} Ha^2 \left( u \sin \gamma - v \cos \gamma \right)^2, \tag{17b}
\]

\[
S_{tot} = HTI_{loc} + FFI_{loc}, \tag{17c}
\]

\[
Be_{loc} = \frac{HTI_{loc}}{FFI_{loc}}, \tag{17d}
\]
endroup where $\varphi$ is the irreversibility ratio held as 0.01 and $Be_{loc}$ is the local Bejan number. Note that $S_{loc}^{tot}$ and in turn $Be$ are obtained as vectors in the entire domain. In other words, we have discrete data for $S_{loc}^{tot}$ and $Be_{loc}$. Average entropy over the entire two-dimensional domain is computed by using the double integral as

$$\text{avg}(S_{loc}^{tot}) = \iint_{\Omega} S_{loc}^{tot} dA, \quad \text{avg}(Be) = \frac{\iint_{\Omega} HTI_{loc} dA}{\iint_{\Omega} S_{loc}^{tot} dA}.$$  \hspace{1cm} (18)

This double integral is calculated by using these discrete data cumulatively as introduced in ‘cumtrapz’ in Matlab’s library. In this study, composite Simpson’s rule is applied based on the idea expressed in ‘cumtrapz’. As a note, statistical mean for $Be$ number defined by $\text{mean}(HTI_{loc})/\text{mean}(S_{loc}^{tot})$ also gives very close results to this double integral result for $Be$. $Be > 0.5$ corresponds to the domination by HTI.

4. Numerical Outputs
4.1. Validation

In order to validate the current implemented code, some published problems are solved. Firstly, the basic unsteady benchmark problem as natural convection flow of air in a square cavity is considered using $51 \times 51$ uniform nodes with RBF-FD in space derivatives and the Backward-Euler method in time derivatives. The obtained average Nu numbers along the heated left vertical wall are compared with the results in Reference [41]. As seen in Table 2, our results are in good agreement.

Table 2. Comparison of average Nu numbers in a NC flow problem in a square cavity.

<table>
<thead>
<tr>
<th>$Ra$</th>
<th>Present</th>
<th>Ref. [41]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>1.1181</td>
<td>1.118</td>
</tr>
<tr>
<td>$10^4$</td>
<td>2.2440</td>
<td>2.243</td>
</tr>
<tr>
<td>$10^5$</td>
<td>4.5123</td>
<td>4.519</td>
</tr>
</tbody>
</table>

Secondly, the steady NC flow of a NF in a wavy cavity is considered, and average Nu values are compared in Table 3 in the case of a zero inclination angle of cavity, 0.05 solid volume fraction, and 0.05 amplitude of waviness in wavy walls. The $51 \times 51$ grid points are arranged for that geometry. In the reference study, Ansys is utilized.

Table 3. Comparison of average Nu values in the case of NC flow of an NF in a wavy cavity.

<table>
<thead>
<tr>
<th>$n = 1$</th>
<th>$n = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ra$</td>
<td>Present</td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>$10^4$</td>
<td>2.5379</td>
</tr>
<tr>
<td>$10^5$</td>
<td>5.1548</td>
</tr>
</tbody>
</table>

The third validation is to compare total EG contours of $S_{loc}^{tot}$ in different Rayleigh numbers when the irreversibility ratio is $\varphi = 10^{-4}$. As is seen in Figure 2, our results are in good agreement with Reference [26].
5. Conclusion

Entropy generation in transient state for natural convection was calculated numerically by using a control volume finite-element method. The influence of the Rayleigh number and the irreversibility distribution ratio on the total entropy generation and the Bejan number.

Figure 11. Entropy generation maps due to heat transfer: (a) $Ra = 10^3$, (b) $Ra = 10^4$ and (c) $Ra = 10^5$.

Figure 12. Entropy generation maps due to heat transfer and fluid friction for irreversibility distribution ratio $u = \frac{10^4}{C0}$: (a) $Ra = 10^3$, (b) $Ra = 10^4$ and (c) $Ra = 10^5$.

4.2. Grid Independency

Grid distributions for each case are designed as shown in Figure 3.

Figure 2. Comparison of total entropy contours in a NC flow problem. (a–c) Reference [26] (the left); (d–f) Present (the right with $N = 51$).

Figure 3. Design of grid distribution. The (left) is for Case 1, the (middle) is for Case 2 and the (right) is for Case 3.
Grid independence is checked in Table 4. In all computations, \( N = 57 \), which means \( 57 \times 57 \) nodes, are used in all computations.

**Table 4.** Grid independence analysis with \( Ra = 10^5, Ha = 10, kr = 1, A = 0.05, n = 3, \gamma = 0^\circ \).

<table>
<thead>
<tr>
<th>( N )</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>2.5238</td>
<td>1.7815</td>
<td>2.0733</td>
</tr>
<tr>
<td>33</td>
<td>2.5081</td>
<td>1.7842</td>
<td>2.0846</td>
</tr>
<tr>
<td>37</td>
<td>2.4960</td>
<td>1.7842</td>
<td>2.0950</td>
</tr>
<tr>
<td>41</td>
<td>2.4919</td>
<td>1.7866</td>
<td>2.0945</td>
</tr>
<tr>
<td>45</td>
<td>2.4855</td>
<td>1.7925</td>
<td>2.1014</td>
</tr>
<tr>
<td>49</td>
<td>2.4845</td>
<td>1.7900</td>
<td>2.1065</td>
</tr>
<tr>
<td>53</td>
<td>2.4800</td>
<td>1.7874</td>
<td>2.1050</td>
</tr>
<tr>
<td>57</td>
<td>2.4792</td>
<td>1.7894</td>
<td>2.1039</td>
</tr>
</tbody>
</table>

### 4.3. Discussion on the Current Problem

In this part, numerical results are visualized as streamlines (\( \psi \)) and temperature (\( T \)) contours, as well as some graphs involving \( Nu \) and EG parameters. The fixed parameters at all calculations are \( Pr = 6.2, \phi_1 = \phi_2 = 0.02, d_s = 0.25 \). The other pertinent parameter ranges are \( Ha = 10 - 100, Ra = 10^3 - 10^5, kr = 0.1 - 10, A = 0.01 - 0.1, n = 1 - 4, \gamma = 0^\circ - 90^\circ \).

In contours of \( \psi \), the given numbers almost at the center stand for \( |\psi|_{\text{max}} \) values.

Figure 4 shows the behavior of FF and HT in various values of \( Ha \). The dampening effect of Lorentz force at large \( Ha \) numbers is expected. This is verified from the decreasing values of \( |\psi|_{\text{max}} \). Further, MF comes along the hot wall horizontally at an angle \( \gamma = 0^\circ \). Therefore, the primary cell, particularly in Case 2 and 3, tends to be separated into new cells in large \( Ha \) numbers. Isotherms almost exhibit conductive behavior at \( Ha = 100 \). That is, convection is suppressed by large Lorentz force.

![Figure 4. Cont.](image-url)
Figure 4. Variation in $Ha$ when $Ra = 10^5$, $kr = 1$, $\gamma = 0^\circ$, $A = 0.05$, $n = 3$. Streamlines and isotherms at the (top) is for case 1, at the (middle) is for case 2 and the at (bottom) is for case 3.

Figure 5 illustrates the variation in $Ra$ number in three cases. In each cases, maximum absolute stream function values increases with the rise in $Ra$ number due to the increase in buoyancy force inside the fluid part. The core vortex in the primary cell in streamlines becomes smaller at $Ra = 10^5$, pointing to a faster fluid flow. The smallest values of $|\psi|_{\text{max}}$ when comparing each case are noted in Case 2. This may be due to the inhibition as a result of the larger area of the conductive solid block in Case 2 than in other cases. While isotherms exhibit a conductive behavior at $Ra = 10^3$, free convection behavior at $Ra = 10^5$ is noted with a pronounced thermal gradient on the right vertical wall. On the contrary, increasing Rayleigh number does not make an important change in the solid wall.

Figure 5. Cont.
Figure 5. Variation in $Ra$ when $Ha = 25$, $kr = 1$, $\gamma = 0^\circ$, $A = 0.05$, $n = 3$. Streamlines and isotherms at the (top) is for case 1, at the (middle) is for case 2 and the at (bottom) is for case 3.

The change in conductivity ratio is given in Figure 6. In all cases, the rise in $kr$ causes fluid to flow faster, which is noted from the absolute maximum stream function values. Once again, the smallest $|\psi|_{\text{max}}$ values are obtained in Case 2. Since $kr$ is directly proportional to $k_s$, it is expected that conduction in the solid part should increase at $kr = 10$. It is seen in isotherms at $kr = 10$, in which no convective behavior occurs inside the solid part, while convection is clearly noted at $kr = 1$. At $kr = 0.1$, isotherms significantly cover the solid block, and these become rare in the fluid part. Furthermore, fluid flows faster as $kr$ rises since the convective behavior in the fluid part becomes prominent due to the increase in $k_f$.

The influence of amplitude of waviness is examined in Figure 7. In Case 1, $|\psi|_{\text{max}}$ increases with the increase in $A$. This may be due to the fact that more heat along the wavy wall in Case 1 is transferred to the fluid part, which makes the fluid flow faster. However, in Cases 2 and 3, a decrease in fluid velocity is noticed with the rise in $A$. This may be due to the smaller area of fluid part as a result of larger $A$.

Figure 6. Cont.
Figure 6. Variation in $kr$ when $Ra = 10^5$, $Ha = 25$, $\gamma = 0^\circ$, $A = 0.05$, $n = 3$. Streamlines and isotherms at the (top) is for case 1, at the (middle) is for case 2 and the at (bottom) is for case 3.

Figure 7. Variation in $A$ when $Ra = 10^5$, $Ha = 25$, $kr = 1$, $\gamma = 0^\circ$, $n = 3$. Streamlines and isotherms at the (top) is for case 1, at the (middle) is for case 2 and the at (bottom) is for case 3.

The impact of the number of waves in wavy wall is checked in Figure 8. In Case 1, $|\psi|_{\text{max}}$ stays almost the same in each $n$. In Cases 2 and 3, as $n$ changes from 1 to 2, faster
fluid flow is found, and then $|\psi|_{\text{max}}$ declines from $n = 2$ to $n = 3$. However, compared to $n = 1$, fluid flows more rapidly when $n \geq 2$. Further, the smallest values of $|\psi|_{\text{max}}$ at any $n$ are noted in Case 2 due to the larger area of conducting solid block than the other cases. If $|\psi|_{\text{max}}$ values in Case 1 and 3 are compared, not only the left wavy wall but also the waviness in the interface reduce the fluid velocity. That is, the hinderance in fluid flow increases with the rise in waviness. Convective behavior in isotherms in these two cases also become a bit more remarkable at $n = 3$ than $n = 1$.

![Figure 8](image-url)

**Figure 8.** Variation in $n$ when $Ra = 10^5$, $Ha = 25$, $kr = 1$, $\gamma = 0^\circ$, $A = 0.05$. Streamlines and isotherms at the (top) is for case 1, at the (middle) is for case 2 and the at (bottom) is for case 3.

Figure 9 presents the variation in inclination angle $\gamma$ of uniform MF. In Case 1, the angle $45^\circ$ has a significant effect on $|\psi|_{\text{max}}$, which means that a quicker flow is noted in the flow part. Furthermore, the core vortex tends to obey the direction of the magnetic field. In Cases 2 and 3, $|\psi|_{\text{max}}$ ascends with the augmentation in the inclination angle of the MF, and
isotherms are also more perturbed with this increment in angle $\gamma$. In other words, the flow can be controlled via a inclination angle of the MF.

**Figure 9.** Variation in $\gamma$ when $Ra = 10^5$, $Ha = 25$, $kr = 1$, $A = 0.05$, $n = 3$. Streamlines and isotherms at the (top) is for case 1, at the (middle) is for case 2 and the at (bottom) is for case 3.

Figure 10 shows the average Nu number along the interface, general EG, and average Bejan number in the variation in pertinent parameters. These outcomes are interpreted on the fluid part. In (a), $Nu_i$ escalates with the rise in $Ra$ number. That is, the convective heat transfer (CHT) rises in the fluid part. $avg(S_{totloc})$ also rises with this increment in $Ra$ due to the rise in both HT and fluid friction irreversibilities. A reverse behavior is noted in the Be number.

In (b), the rise in $Ha$ number weakens the CHT due to the dampening effect of the large Lorentz force. $avg(S_{totloc})$ is also significantly reduced while the Bejan number rises. The reduction occurs the most in Case 3.

In (c), the augmentation in amplitude of the wavy wall affects the CHT in an increasing trend in Case 1 and 2 and a decreasing trend in Case 3. The same trend is exhibited in $avg(S_{totloc})$. Remember that Case 1 and 2 only had one wavy wall on the conducting solid,
while the other vertical wall on the solid was straight. Therefore, it may be concluded that the wavy conducting block with a large amplitude has a reducing impact on CHT in FF.

In (d), the increment in the number of undulations has a rising influence on CHT and $\text{avg}(S_{\text{loc}}^{\text{tot}})$ in Case 1. In Case 2 and Case 3, there is a rise and reduction behavior with the change in $n$. However, looking at the discrete data, CHT is efficient at $n = 3$ in Case 3 and at $n = 1$ in Case 2. In Case 2, $\text{avg}(S_{\text{loc}}^{\text{tot}})$ decreases with the rise in $n$.

In (e), diverse values of $kr$ are observed. CHT, $\text{avg}(S_{\text{loc}}^{\text{tot}})$ and $Be$ are obviously increasing with the rise in $kr$. With a large $kr$, conduction in the solid part rises while convection is boosted in the fluid part. Therefore, the current rising behavior is expected.

In (f), the change in the inclination angle $\gamma$ of uniform MF from the horizontal MF to vertical MF is examined. Vertical MF has a greater improving influence on CHT in each case. As seen from the figure, $\text{avg}(S_{\text{loc}}^{\text{tot}})$ reaches a peak and Be number has a minimum value at an angle of $45^\circ$. This may be due to the last term in $FFI_{\text{loc}}$ in which $\gamma = 45^\circ$ seems to have additional effect than other angles, and therefore $S_{\text{loc}}^{\text{tot}}$ becomes larger at $45^\circ$ than other angles. The reverse case, as the minimum, occurs at Be. This means that there is an optimum value for entropy generation.

Figure 10. Cont.
5. Conclusions

In this study, steady MHD free convection flow of a hybrid nanofluid is investigated in a cavity with a wavy, thermally conducting solid block attached to the left wall. RBF-FD is employed to examine the pertinent parameters numerically. Three different designs of waviness in the solid part of the enclosure are also examined. Some of the basic results may be listed as follows:

- The rise in the Lorentz force results in a reduction in the fluid velocity, CHT, and total entropy. If $Ha$ is changed from $10$ to $100$, $71.58\%$ reduction in total EG, and $49.16\%$ reduction in $Nu$, are found, while $Be$ number increases by $146.3\%$ in Case 1.
- The more the buoyancy force exists, the faster the fluid flows and the more CHT improves. From $Ra = 10^3$ to $Ra = 10^5$, the greatest increase in $Nu$ occurs in Case 1 as $74.5\%$, and an almost $100\%$ reduction in $Be$ in each cases is observed.
- With large values of $kr$, CHT is more pronounced in the fluid part. Total EG and $Be$ number also ascend with the rise in $kr$.
- The amplitude of waviness has a reducing effect on $Nu$ and total EG in Case 2 and an increasing impact in Cases 1 and 3.
- In Case 1 and 3, the increment in the number of undulations is directly proportional to $Nu$.
- If the angle of uniform MF is changed from $0^\circ$ to $90^\circ$, $Nu$ rises $8.79\%$ in Case 1, $7.73\%$ in Case 2, and $7.58\%$ in Case 3. Total EG initially increases from angle $0^\circ$ to $45^\circ$, and then it decreases from $45^\circ$ to $90^\circ$. $Be$ number is not significantly affected by this angle.

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