Modeling Retail Supply Chain Efficiency: Exploration and Comparative Analysis of Different Approaches

Milan Andrejić

Faculty of Transport and Traffic Engineering, University of Belgrade, Vojvode Stepe 305, 11000 Belgrade, Serbia; m.andrejic@sf.bg.ac.rs

Abstract: Retail supply chains are key on any market. Their significance has long been recognized in the literature and in practice. Various factors such as pandemics, economic crises, wars, and natural disasters have further increased interest in this area. However, the most recent research has focused more on resilience, sustainability, energy consumption, and a circular economy, while the efficiency of logistics processes has been almost completely overlooked. Logistics process efficiency in retail supply chain is a fundamental principle without which all mentioned performances cannot have desired values. This gap is precisely the main motivation of this research. In this paper, research models in literature are presented which can be used, with some modifications, to measure the efficiency of the retail supply chain. The models were based on the data envelopment analysis (DEA) approach. Four main groups were identified: standard DEA models, efficiency decomposition models, network models, and game-theory-based models. In the second part of the paper, various approaches were tested on a real example of a trading company operating in Serbia. Seven supply chains were observed, each consisting of a distribution center (DC) and retail store (RS). Variables used were the number of pallet places, logistics costs, number of deliveries, accuracy of deliveries, and turnover. The results showed the advantages and disadvantages of different approaches in real examples. The main contributions of this paper lie in unique approaches to measuring the efficiency of the retail supply chain. The paper creates an excellent foundation for future research and measurements on real systems, which is equally useful for researchers and industry experts.

Keywords: efficiency; retail; supply chain; DEA; game theory; logistics

MSC: 90B06

1. Introduction

Supply chains represent an important subject of research in theory and practice. In recent years, due to the pandemic, economic crises, wars, and other factors, increasing attention has been devoted to them. A lot of papers deal with resilience, sustainability and the circular economy ignoring the operational efficiency of supply chain [1–4]. Supply chain efficiency is also important. This field is in a phase of expansion, and the number of research studies is constantly increasing. Optimization models in supply chains are based on various criteria such as costs, inventory levels, profit, order fulfillment, probability of stockouts, variations in demand, and system capacity. Most deterministic and stochastic problems focus on isolated parts of the supply chain, such as supplier–manufacturer, manufacturer–distribution center, etc., [5–8]. Some models address strategic tasks in supply chains (such as warehouses, factories, distribution centers, etc.), while others address problem-solving that is more operational in nature (order size, inventory levels, order fulfillment rate, etc.).

Supply chains represent complex systems composed of a large number of interconnected and interdependent processes [9–14]. For a long time, supply chains and similar complex systems and processes have been viewed as “black boxes”, meaning that their structure and functioning have not been considered [15]. To assess the efficiency of supply
chains, it is necessary to measure the performance of all participants in the chain, including suppliers, manufacturers, retailers, as well as end-users. Retail supply chain has been recognized in the literature as a very important and actual topic. However, there is a lack of papers proposing concrete models for measuring the efficiency of participants [16–20].

There are a number of studies in the literature that deal with measuring the efficiency of independent participants in supply chains using the DEA method. However, the number of studies that deal with measuring the efficiency of retail supply chains is negligible. The problem of measuring the efficiency of multiphase processes is often referred to in the literature. This is most commonly the case with two-phase production systems. The aim of this paper is to propose models for measuring the efficiency of retail supply chains and to test their applicability. As there are not enough studies in the literature that deal with measuring the efficiency of retail supply chains, this paper tests the possibilities of applying existing models.

The contributions of this work are reflected in the adaptation of theoretical models from the literature that can be used to measure the efficiency of retail supply chain management and supply chains in general. The results show the advantages and disadvantages of different approaches, as well as opportunities for further development in future research. The practical contributions are reflected in the fact that practitioners can easily select and apply the proposed models depending on the size, organization, and indicators they track.

The first part of this paper describes the problem with a literature review. After that, mathematical formulations of the most significant approaches are presented: efficiency decomposition models, network models, and game-theory-based models. In the fourth chapter, the presented models are applied to a real example of a retail chain operating in Serbia. The obtained results are analyzed in detail. The concluding remarks and directions for future research are given at the end of the paper.

2. Problem Description and Literature Review

The majority of the proposed models are based on the DEA method. In that manner, in this paper, a special emphasis is placed on DEA models. Other efficiency measurement models are also presented, both for participants in supply chains and for entire supply chains, as well as those models that were originally not intended for measuring efficiency in supply chains but that can be successfully applied with appropriate adaptations. The universality of supply chain efficiency measurement models is also explored, as well as the possibility of applying them in real-world conditions.

The problem of measuring the performance of supply chains has been widely discussed in the literature but is rarely solved. Beamon emphasizes the importance of qualitative assessments such as good, bad, suitable, weak, etc., [21]. The author points out that qualitative indicators cannot describe the performance of a system in the best way. There are many studies in the literature that deal with analyzing performance indicators in supply chains. It is clear that there are no universal measures for all supply chains. It is necessary to consider each supply chain as a separate unit characterized by a unique set of performance measures. The main groups of performances that dominate the literature and can be analyzed in more detail are delivery performance, chain reliability, profit, inventory, and costs. Recently, efficiency and effectiveness have increasingly been used. It is difficult to classify these measures into any of the previous categories. The authors in [22] shared a similar view, emphasizing that differently formed units in different companies required slightly different measurement systems. They also noted that for effective performance measurement and improvement, measurement goals must represent organizational objectives and measures that describe a balance between financial and non-financial measures related to strategic, tactical, and operational levels of decision-making and control.

In the era of globalization and outsourcing, most companies see logistics and supply chains as a chance for competitiveness on the market. Companies realize that in order to achieve efficient and effective supply chains, they must first have a well-developed measurement system. The authors developed a framework for measuring performance at
the strategic, tactical, and operational levels. In [23], the authors analyzed the performance of 115 manufacturing companies, differentiating between four basic dimensions: cost, time, flexibility, and quality.

Despite the importance of measuring and improving performance, there are still not enough works in the literature that analyze the efficiency measurement in supply chains, with all important and connected entities, such as suppliers, warehouses and distribution centers, logistics providers, etc. The profitability and survival of companies largely depend on the performance of supply chains. Most works in the literature focus on internal efficiency performance without analyzing the mutual influence of other participants in the chain on performance.

The existence of adequate performance measurement systems is a prerequisite for efficient and successful supply chains. However, there is no relevant research in the literature that deals with this issue. Supply chains are complex systems composed of a large number of interconnected and conditioned processes. For a proper evaluation of supply chain efficiency, it is necessary to measure the performance of all participants in the chain, including suppliers, manufacturers, retailers, and end-users.

There are significant works in the literature that focus on measuring the efficiency of independent participants in supply chains using the DEA method. Collin [24] emphasized that inventory turnover was one of the most commonly used indicators of efficiency, as it describes the speed of material movement in the supply chain. The same author pointed out the importance of aligning internal processes and activities with environmental characteristics in order to achieve efficient supply chains. Requirements significantly affect the structure of chains. Companies must provide much more than competitive products. In the process of reducing costs and improving service levels, companies must take into account the interaction at different levels in the supply chain [25]. Supply chains can also be viewed as complex logistics networks consisting of suppliers, manufacturers, warehouses, distribution centers, stores, etc. When improving efficiency in supply chains, it is necessary to carry out actions that improve the efficiency of the entire chain, not just some of its parts.

In [26], the authors analyzed energy efficiency indicators in supply chains. The authors emphasized the importance of fuel, water, and electricity consumption, and highlighted the relationship between supply chain efficiency and the requirements of the ISO 14301 standard. Neto emphasized the problem of balancing environmental requirements and company operations [27]. Hervani provided a detailed overview of indicators for measuring supply chains from an environmental perspective [28].

The problem of measuring the efficiency of supply chains in the literature is highlighted as a problem of measuring the efficiency of multiphase processes. Most often, this concerns two-phase production systems. From the perspective of the DEA method, which is the most commonly used method for measuring the efficiency of multiphase processes, there are a large number of models and methods that can be applied directly or with some modifications to supply chains [29]. All models can be classified into four groups as shown in Table 1 [7,12,16,23,29]:

- Standard DEA models;
- Efficiency decomposition models;
- Network models;
- Game-theory-based models.
Table 1. Advantages and disadvantages of existing model in the literature.

<table>
<thead>
<tr>
<th>Model</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard DEA models</td>
<td>Suitable for independent participants;</td>
<td>Unsuitable for complex systems;</td>
</tr>
<tr>
<td></td>
<td>They are used in theory and practice;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Many easy-to-use ready-made software;</td>
<td></td>
</tr>
<tr>
<td>Efficiency decomposition models</td>
<td>Suitable for decomposition of two-phase systems;</td>
<td>Limited number of inputs, outputs, and intermediate products;</td>
</tr>
<tr>
<td>Network models</td>
<td>In a theoretical formulation, they can be used for more complex systems;</td>
<td>Limited application in practice due to complexity;</td>
</tr>
<tr>
<td>Game-theory-based models</td>
<td>In a theoretical formulation, they can be used for more complex systems;</td>
<td>Limited practical application; Limited number of parameters (inputs and outputs) that can be applied;</td>
</tr>
</tbody>
</table>

3. Mathematical Formulations

3.1. Standard DEA Model

Standard DEA approaches determine the efficiency of DMUs using the CCR DEA model [30]. In this paper, the dual formulation of this model was used. The mentioned model has the following form:

\[
\text{M I} \quad \text{Min} \ Z_k - \varepsilon \left( \sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^- \right) \\
\sum_{j=1}^{n} \lambda_j y_{rj} - y_{rk} - s_r^+ = 0, \ r = 1, 2, \ldots, s \\
Z_k x_{ik} - \sum_{j=1}^{n} \lambda_j x_{ij} - s_i^- = 0, \ i = 1, 2, \ldots, m \\
\lambda_j, s_r^+, s_i^- \geq 0, \ i = 1, 2, \ldots, m, \ r = 1, 2, \ldots, s, j = 1, 2, \ldots, n \\
Z_k - \text{unlimited}
\]

The set of decision-making units (DMUs) consists of \( n \) units (\( j = 1, 2, \ldots, n \)), where each is characterized by \( m \) input (\( i = 1, 2, \ldots, m \)) and \( s \) output variables (\( r = 1, 2, \ldots, s \)). The value of the \( i \)th input is denoted by \( x_{ij} \), while the value of the \( r \)th output variable of DMU\(_j\) is denoted by \( y_{rj} \). The variable \( \varepsilon \) represents a small positive value. The variable \( Z_k \) is called the intensity factor and shows how much DMU\(_k\) can proportionally reduce all input variables in the case of an input orientation or increase the outputs in the case of an output orientation. The target values for inefficient DMUs are defined using weight coefficients \( \lambda_j \).

The presented model assumes constant returns to scale. The variables \( s_r^- \) and \( s_i^+ \) represent deficiencies in input and excesses in output variables, respectively.

The presented model can be solved in two phases. In the first phase, i.e., in the first linear programming task, the value of \( Z_k \) is optimized, while in the second phase, the slack values are optimized. Thus, the previously described model has the following form:

\[
\text{M II} \quad \text{Min} \ Z_k \\
\sum_{j=1}^{n} \lambda_j y_{rj} - y_{rk} \geq 0, \ r = 1, 2, \ldots, s \\
Z_k x_{ik} - \sum_{j=1}^{n} \lambda_j x_{ij} \geq 0, \ i = 1, 2, \ldots, m \\
\lambda_j \geq 0, j = 1, 2, \ldots, n
\]
II phase

\[
\begin{align*}
\text{Max} \ e & \left( \sum_{r=1}^{s} s_{r}^{+} + \sum_{i=1}^{n} s_{i}^{-} \right) \\
\sum_{j=1}^{n} \lambda_{j} y_{rj} - y_{rk} - s_{r}^{+} &= 0, r = 1, 2, \ldots, s \\
Z_{k} x_{rk} - \sum_{j=1}^{n} \lambda_{j} x_{ij} - s_{i}^{-} &= 0, i = 1, 2, \ldots, m \\
\lambda_{j} s_{r}^{+}, s_{i}^{-} &\geq 0, i = 1, 2, \ldots, m, r = 1, 2, \ldots, s, j = 1, 2, \ldots, n \\
Z_{k} &\text{— unlimited}
\end{align*}
\]

where:
- \(s\)—number of inputs;
- \(m\)—number of outputs;
- \(n\)—number DMU;
- \(e\)—unity vector;
- \(Z_{k}^{*}\)—optimal value in phase I.

3.2. Supply Chain Efficiency Decomposition Models

One of the basic approaches for measuring the efficiency of supply chains is the approach of efficiency decomposition. Two basic approaches are distinguished in the literature. According to the first approach, the efficiency of the chain is defined as the arithmetic mean \([31]\) of its components (participants, processes, etc.), while according to the second approach, it is defined as a product \([32]\). The following provides a more detailed explanation of these approaches.

The “average” efficiency approach

Unlike the standard approach of measuring efficiency where systems are treated as “black boxes”, in the approach of average efficiency, the efficiency of each component is independently estimated first, after which the overall efficiency is obtained as the arithmetic mean \([31]\) of its components. According to the second approach, it is defined as a product \([32]\). The following provides a more detailed explanation of these approaches.

Figure 1. Supply chain with two components \([31]\).

Figure 1. Supply chain with two components \([31]\).

\(X_{A}\) is the input vector of component 1, and \(Y_{A}\) is component 1’s output vector. \(Y_{A}\) is also an input vector of component 2, along with \(X_{B}\), with \(Y_{B}\) being component 2’s output vector. \(U_{T}, I, V_{T}\), represent weight coefficient for inputs and outputs, while \(U_{T}, I, V_{T}\) represents transpose vectors.

M II

\[
\begin{align*}
\text{Max} & \ 1/2 \left[ U_{A}^{T} Y_{AO} + U_{B}^{T} Y_{BO} \right] \left[ V_{A}^{T} X_{AO} + V_{B}^{T} (X_{BO}, Y_{AO}) \right] \\
U_{A}^{T} Y_{AI} &\leq V_{A}^{T} X_{AI} \leq 1, j = 1, 2, \ldots, n \\
U_{B}^{T} Y_{BI} &\leq V_{B}^{T} (X_{Bj}, Y_{A}) \leq 1, j = 1, 2, \ldots, n \\
U_{A}^{T}, V_{A}^{T}, U_{B}^{T}, V_{B}^{T} &\geq 0
\end{align*}
\]

In the considered example, \(Y_{A}\) was used as the output variable for assessing the efficiency of component 1 and as the input variable for assessing the efficiency of component 2. The approach of average efficiency, the efficiency of each component is independently estimated first, after which the overall efficiency is obtained as the arithmetic mean of its components.
2. However, the drawback of this model is that the $Y_A$ vector does not show the relationship between component 1 and component 2. The weight coefficients of the observed vector may be different when assessing the efficiency of the first and second components. The model considers components as independent participants and does not describe the ideal functioning of the complete system. The authors believe that without considering the relationships between participants in the chain, there can be no successful assessment of the efficiency. Therefore, the average efficiency approach represents one of the basic models of efficiency decomposition, i.e., an aggregated efficiency.

The “product” efficiency approach

As previously mentioned, one of the basic approaches for the efficiency decomposition of two-stage systems is the application of standard DEA models to independently measure the efficiency of the entire process or some of its components. Kao and Hwang (2008) introduced a slightly different approach that considered intermediate products. The subject of their consideration was systems whose activities were carried out in two stages. These stages were sequentially linked, so the product of their efficiency represented the efficiency of the entire system or decision-making unit (DMU). $X_{ij}, i = 1, \ldots, m, Y_{rk}, r = 1, \ldots, s$ represent the ith input or rth output input, and DMU$_{ij}, j = 1, \ldots, n$. The standard CCR DEA model for measuring the efficiency of DMU$_k$ has the following form [30]:

**M IV**

$$E_k = \max \sum_{r=1}^{s} u_r Y_{rk} / \sum_{i=1}^{m} v_i X_{ik}$$

$$\sum_{r=1}^{s} u_r Y_{rk} / \sum_{i=1}^{m} v_i X_{ik} \leq 1, j = 1, \ldots, n$$

$u_r, v_j \geq \varepsilon, r = 1, \ldots, s; i = 1, \ldots, m$, where $\varepsilon$ (about 0.000001) represents a small number.

A two-stage production process can be represented as shown in Figure 2. The system consists of two sequentially linked subsystems. The entire process uses $m$ inputs $X_{ik}$, $i = 1, \ldots, m$ to produce $s$ outputs $Y_{rk}$, $r = 1, \ldots, s$. Unlike conventional DEA approaches for measuring the efficiency of two-stage processes, this approach distinguishes $q$ intermediate products $Z_{pk}, p = 1, \ldots, q$. The intermediate products represent the outputs of the first stage and the inputs to the second stage, as can be seen in Figure 2. To measure the efficiency of the first and second stage, the next models are used:

**M V**

$$E_k^1 = \max \sum_{p=1}^{q} w_p Z_{pk} / \sum_{i=1}^{m} v_i X_{ik}$$

$$\sum_{p=1}^{q} w_p Z_{pk} / \sum_{i=1}^{m} v_i X_{ik} \leq 1, j = 1, \ldots, n, q$$

$w_p, v_i \geq \varepsilon, p = 1, \ldots, q, i = 1, \ldots, m$.

$$E_k^2 = \max \sum_{r=1}^{s} u_r Y_{rk} / \sum_{p=1}^{q} w_p Z_{pk}$$

Figure 2. Network production system with intermediate products [33].
\[ \sum_{r=1}^{s} u_r Y_{rf} / \sum_{p=1}^{q} w_p Z_{pjk} \leq 1, j = 1, \ldots, n, \]

\[ u_r, w_p \geq \epsilon, r = 1, \ldots, s, p = 1, \ldots, q. \]

Unlike the previous approach in which intermediate products were completely neglected and the efficiency of individual stages and the entire process was independently determined, Kao and Hwang overcame the described problem with their approach [32]. Let \( u^*_r, v^*_i, \) and \( w^*_i \) represent the coefficients of output, input variables, and intermediate products that DMU_k has selected in order to calculate the overall efficiency \( E_k \) and the efficiency of the subprocesses \( E^1_k \) and \( E^2_k \):

M VI

\[
E_k = \sum_{r=1}^{s} u^*_r Y_{rk} / \sum_{i=1}^{m} v^*_i X_{ik} \leq 1
\]

\[
E^1_k = \sum_{p=1}^{q} w^*_p Z_{pk} / \sum_{i=1}^{m} v^*_i X_{ik} \leq 1
\]

\[
E^2_k = \sum_{r=1}^{s} u^*_r Y_{rk} / \sum_{p=1}^{q} w^*_p Z_{pk} \leq 1
\]

The overall efficiency of the system is the product of the efficiency of its subprocesses, i.e., \( E_k = E^1_k E^2_k \). By introducing additional constraints for the subprocesses, taking into account their sequential linkage, the following model is obtained.

M VII

\[
E_k = \max \sum_{r=1}^{s} u_r Y_{rk} / \sum_{i=1}^{m} v_i X_{ik}
\]

\[
\sum_{r=1}^{s} u_r Y_{rf} / \sum_{i=1}^{m} v_i X_{ij} \leq 1, j = 1, \ldots, n
\]

\[
\sum_{p=1}^{q} w_p Z_{pk} / \sum_{i=1}^{m} v_i X_{ik} \leq 1, j = 1, \ldots, n
\]

\[
\sum_{r=1}^{s} u_r Y_{rk} / \sum_{p=1}^{q} w_p Z_{pk} \leq 1, j = 1, \ldots, n
\]

The model can be reduced to a linear programming model with minor transformations.

M VIII

\[
E_k = \max \sum_{r=1}^{s} u_r Y_{rk}
\]

\[
\sum_{i=1}^{m} v_i X_{ik} = 1
\]

\[
\sum_{r=1}^{s} u_r Y_{rf} - \sum_{i=1}^{m} v_i X_{ij} \leq 0, j = 1, \ldots, n
\]

\[
\sum_{p=1}^{q} w_p Z_{pk} - \sum_{i=1}^{m} v_i X_{ik} \leq 0, j = 1, \ldots, n
\]

\[
\sum_{r=1}^{s} u_r Y_{rk} - \sum_{p=1}^{q} w_p Z_{pk} \leq 0, j = 1, \ldots, n
\]

\[ u_r, v_i, w_p \geq \epsilon, r = 1, \ldots, s; i = 1, \ldots, m, p = 1, \ldots, q \]
After determining the optimal values of $u^*_r, v^*_i, w^*_p$, as well as the efficiency of the first and second stages, the overall efficiency of the DMU can be easily determined using the following formulas:

\[ E_k = \sum_{r=1}^{s} u^*_r Y_{rk}, \]

\[ E^1_k = \sum_{r=1}^{s} w^*_r Z_{pk} / \sum_{i=1}^{m} v^*_i X_{ik} \]

\[ E^2_k = \sum_{r=1}^{s} u^*_r Y_{rk} / \sum_{p=1}^{q} w^*_p Z_{pk} \]

\[ E_k = E^1_k \times E^2_k \]

However, solving the previous model does not necessarily lead to unique solutions $u^*_r, v^*_i, w^*_p$ (i.e., $E_k = E^1_k \times E^2_k$ can be decomposed in different ways). This problem is present for all DMUs and can be addressed in multiple ways. Kao and Hwang [32] suggested one solution, which involved finding a set of weights that maximized the value of the first stage efficiency $E^1_k$ while keeping the overall efficiency $E_k$. This idea can be formulated as follows:

\[ E^1_k = \max \sum_{p=1}^{q} w^*_p Z_{pk} \]

\[ \sum_{i=1}^{m} v^*_i X_{ik} = 1 \]

\[ \sum_{r=1}^{s} u^*_r Y_{rk} - E_k \sum_{i=1}^{m} v^*_i X_{ik} = 0 \]

\[ \sum_{r=1}^{s} u^*_r Y_{rk} - \sum_{i=1}^{m} v^*_i X_{ij} \leq 0, \quad j = 1, \ldots, n \]

\[ \sum_{p=1}^{q} w^*_p Z_{pk} - \sum_{i=1}^{m} v^*_i X_{ik} \leq 0, \quad j = 1, \ldots, n \]

\[ \sum_{r=1}^{s} u^*_r Y_{rk} - \sum_{p=1}^{q} w^*_p Z_{pk} \leq 0, \quad j = 1, \ldots, n \]

\[ u_r, v_i, w_p \geq \epsilon, \quad r = 1, \ldots, s; \quad i = 1, \ldots, m; \quad p = 1, \ldots, q \]

Depending on the situation, it is possible to apply the opposite strategy, i.e., to maximize the second stage, under the same conditions. It is clear that in the first case, the efficiency of the first stage will be maximal, and the efficiency of the second stage minimal, and vice versa.

The described model was originally developed and tested on the example of insurance companies. However, the model can also be applied in the case of logistics processes. The methodology and the condition of equal weights of the intermediate products of the first and second stages represent a significant advancement and a good basis for the development of future approaches. However, the presented model is based on a simplified example. The problem is significantly complicated in cases where there are additional inputs in the second stage, or when there are outputs from the first stage that are not inputs to the second stage.
3.3. Network Supply Chain Efficiency Models

One of the basic models on which modern network models are based is described in [33], where the authors presented a model for measuring efficiency that included intermediate products and did not require the allocation of resources within subsystems.

Certain assumptions were made for the development of the model. The structure of the DMU consisted of two interconnected nodes that together made up the production technology. The first node generated outputs, some of which were used as inputs in the second node, while the rest were freely “sold” (external output). The definition of the production technology was based on \( k = 1, \ldots, K \) observations (number of DMUs) of external inputs \( x^k \) and final outputs \( y^k \). The total external input was allocated to the inputs used in the first and second nodes and could be represented as \( x^k = x + \tilde{x} \) (Figure 3).

![Figure 3. Two-stage process [34].](image)

In a similar way, it is possible to define the total output of final products (products that are sold on the external market) \( y^k = \frac{1}{2}y + \frac{1}{2}y \). However, a certain number of products from node one are used as intermediate inputs in node two. If this number is denoted as \( \frac{1}{2}y \), the total output from node one represents the sum of the sold products and the products used by node two, i.e., \( \frac{1}{2}y + \frac{1}{2}y \).

The DEA production technology of node two can be represented in the following way:

\[
\begin{align*}
M \ XI & \quad \sum_{k=1}^{K} \lambda_k \tilde{y}_{km} \geq \frac{1}{2}y_m, \quad m = 1, \ldots, M, \\
M \ XII & \quad \sum_{k=1}^{K} \lambda_k \tilde{y}_{km} \geq \frac{1}{2}y_m, \quad m = 1, \ldots, M, \\
& \quad \sum_{k=1}^{K} \lambda_k x_{kn} \leq \frac{1}{2}x_n, \quad n = 1, \ldots, N, \\
& \quad \lambda_k \geq 0, \quad k = 1, \ldots, K,
\end{align*}
\]

where \( \lambda_k \) denotes the intensity of the \( k \)th variable of the second node. The production technology of the first node is:

\[
\begin{align*}
M \ XI & \quad \sum_{k=1}^{K} \mu_k \left( \frac{1}{2}y_{km} + \frac{1}{2}y_{km} \right) \geq \left( \frac{1}{2}y_m + \frac{1}{2}y_m \right), \quad m = 1, \ldots, M, \\
M \ XII & \quad \sum_{k=1}^{K} \mu_k x_{kn} \leq \frac{1}{2}x_n, \quad n = 1, \ldots, N, \\
& \quad \mu_k \geq 0, \quad k = 1, \ldots, K,
\end{align*}
\]

where \( \mu_k \) represents the variables assigned to the inputs/outputs of the first node.

3.4. Model for Measuring Supply Chain Efficiency Based on Game Theory

The subject of this approach also includes two-stage systems. For example, the manufacturer, as the leader, has an advantage in the process of achieving efficiency, while the trader, as a follower, must adapt to a noncooperative strategy with the seller as the leader. In a cooperative strategy, the seller and the buyer want to maximize the efficiency
of the entire supply chain, not just some of its parts. The proposed model is based on the simultaneous maximization of the efficiency of both participants. The authors emphasize that their approach is not entirely consistent with the well-known theory of cooperative games, in which players can make joint decisions in the space of multiple players, and that it is more appropriate to consider it as a centralized approach to measuring efficiency.

3.4.1. Game-Theory Models

Unlike standard DEA approaches [34–36], noncooperative and centralized systems provide the possibility of decomposing efficiency into basic components. In their work, Liang developed an approach on a simpler example whose schematic representation is given in Figure 4 [34]. The figure shows the input and output variables, as well as intermediate products.

For DMU, the first phase is denoted as with the second phase is . The CCR DEA model is:

\[ e^1_j = \sum_{d=1}^{D} w_d z_{d,j} / \sum_{i=1}^{m} v_i x_{i,j} \]

\[ e^2_j = \sum_{r=1}^{s} u_r y_{r,j} / \sum_{d=1}^{D} w_d z_{d,j} \]

The efficiency of the complete process can be defined as a product or as an arithmetic mean of the subsystem efficiencies.

**Noncooperative model**

As in [31], one of the participants represents the leader in achieving efficiency, while the other represents the follower. Assuming that the first stage represents the leader and the second stage the follower, the efficiency of the first stage of the observed DMU is obtained by solving the CCR DEA linear programming model:

\[ e^{1*}_j = \text{Max} \sum_{d=1}^{D} w_d z_{d,0} \]

\[ \sum_{i=1}^{m} v_i x_{i,0} = 1 \]

\[ \sum_{d=1}^{D} w_d z_{d,j} - \sum_{i=1}^{m} v_i x_{i,j} \leq 0, j = 1, \ldots, n \]

\[ v_r, w_d \geq 0, d = 1, \ldots, D; i = 1, \ldots, m \]

After determining the efficiency of the first stage, the efficiency of the second stage is calculated while keeping the efficiency of the first stage constant. In fact, in the second stage, \( \sum_{d=1}^{D} w_d z_{d,j} \) is considered as an input. The model for calculating the efficiency of the second stage has the following form:

\[ e^{2*}_j = \text{Max} \sum_{r=1}^{s} U_r y_{r,j} / Q \sum_{d=1}^{D} w_d z_{d,0} \]
\[
\sum_{r=1}^{s} U_r y_{rj} / Q \sum_{d=1}^{D} w_d z_{dj} \leq 1, j = 1, \ldots, n
\]
\[
\sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, \ldots, n
\]
\[
\sum_{i=1}^{m} v_i x_{i0} = 1
\]
\[
\sum_{p=1}^{q} w_d z_{d0} = e_0^{1s}
\]
\[
U_r, Q, v_i, w_d \geq 0, d = 1, \ldots, D; i = 1, \ldots, m, r = 1, 2, \ldots s
\]

In the last model, one of the conditions is that the efficiency of the second stage remains unchanged \(e_1\). If \(u_r = U_r / Q, r = 1, 2, \ldots, s\), then the model has the following form:

M XVI
\[
e_0^{1s} = \text{Max} \sum_{r=1}^{s} u_r y_{r0} / e_0^{1s}
\]
\[
\sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leq 0, j = 1, \ldots, n
\]
\[
\sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, \ldots, n
\]
\[
\sum_{i=1}^{m} v_i x_{i0} = 1
\]
\[
\sum_{d=1}^{D} w_d z_{d0} = e_0^{1s}
\]
\[
u_r, v_i, w_d \geq 0, d = 1, \ldots, D; i = 1, \ldots, m, r = 1, 2, \ldots s
\]

In the same way, the efficiency can be calculated in the case where the second phase is the leader, and the first phase is the follower. According to the authors, the efficiency can be shown as:

M XVII
\[
e_0^{1s} \cdot e_0^{2s} = \sum_{r=1}^{s} u_r y_{r0}
\]
subject to:
\[
\sum_{i=1}^{m} v_i x_{i0} = 1
\]

Then,
\[
e_0^{1s} \cdot e_0^{2s} = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}}
\]

Based on all of the above, it can be concluded that the efficiency decomposition in this case has a unique solution.

Centralized model

The centralized approach involves determining the optimal weights of intermediate products that maximize the aggregated efficiency of the entire DMU. The centralized approach is characterized by the equality constraint \(w_d = \tilde{w}_d\). By introducing this constraint, the centralized model has the following form:

M XVIII
\[
e_0^{\text{centr}} = \text{Max} e_0^{1s} \cdot e_0^{2s} = \sum_{r=1}^{s} u_r y_{r0} / \sum_{i=1}^{m} v_i x_{i0}
\]
The model can be further transformed into the following linear programming model:

\[ e_1^0 \leq 1, \quad e_2^0 \leq 1, \quad w_d = \tilde{w}_d \]

The model provides the overall efficiency of the two-phase process. Assuming the model has a unique solution, the efficiency of the first and second phases can be obtained as follows:

\[ e_{1,\text{centr}} = \max \sum_{r=1}^s u_r y_{r0} \]

\[ \sum_{r=1}^s u_r y_{rj} - \sum_{d=1}^D w_d z_{dj} \leq 0, \quad j = 1, \ldots, n \]

\[ \sum_{d=1}^D w_d z_{dj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \ldots, n \]

\[ \sum_{i=1}^m v_i x_{i0} = 1 \]

\[ u_r, v_i, w_d \geq 0, \quad d = 1, \ldots, D; \quad i = 1, \ldots, m, \quad r = 1, 2, \ldots, s \]

The game theory is based on the fact that increasing the efficiency of one player directly affects the decrease of the efficiency of the other player. In this sense, maximizing the efficiency of the first phase leads to minimizing the efficiency of the second phase because

\[ e_2^{\text{centr}} = e_1^{\text{centr}} \cdot e_0^{\text{centr}} \]

Given that \( e_{0,\text{centr}} = e_0^{1,\text{centr}} \cdot e_0^{2,\text{centr}} \), the optimal solution may not be unique. Different combinations of subprocess efficiencies can yield the same overall efficiency. One way to solve this problem is by maximizing the efficiency of the first stage, as shown below:

\[ e_0^{1,\text{centr}} = \sum_{d=1}^D \omega_d z_{d0} / \sum_{i=1}^m v_i x_{i0} = \sum_{d=1}^D \omega_d z_{d0} \]

\[ e_0^{2,\text{centr}} = \sum_{r=1}^s u_r y_{r0} / \sum_{d=1}^D \omega_d z_{d0} \]

The game theory is based on the fact that increasing the efficiency of one player directly affects the decrease of the efficiency of the other player. In this sense, maximizing the efficiency of the first phase leads to minimizing the efficiency of the second phase because

\[ e_0^{2,\text{centr}} = e_0^{\text{centr}} / e_0^{1,\text{centr}} \]

Similarly, the efficiency of the second phase can be maximized, while the efficiency of the first phase will be minimized. Unlike the noncooperative approach, the centralized approach does not provide unique solutions when decomposing the overall efficiency.

Regardless of certain assumptions, this approach provides a convenient way to measure efficiency in supply chains. Its basic applicability lies in resolving conflicts that exist among participants in supply chains. Its application provides both the efficiency of the entire supply chain, as well as all that of its components. The described approach can be used to model supply chains, i.e., multistage systems in general.
3.4.2. A “Bargaining” Model for Measuring Efficiency in Supply Chains Based on Game Theory

As in previous models, this approach considers a chain consisting of two participants, a supplier and a manufacturer. The “efficiency game” between the two participants is explored. A greater number of Nash equilibria plans of efficiency are observed for the supplier or the manufacturer. A bargaining model is then proposed as a decision-making process or the adoption of the best decision-making strategy [37,38].

At the outset, the efficiency functions of the supplier and the manufacturer are defined as the inverse of each other. The conflict between the participants in the chain is related to the interdependent inputs/outputs. For example, if $Y_S$ represents the supplier’s profit and the manufacturer’s cost, the supplier seeks to maximize profit while the manufacturer seeks to minimize production costs. This is precisely why it is difficult to define the efficiency of the entire supply chain. The problem can be overcome by assuming that there is one decision-maker who has all the necessary information about the participants in the chain. This approach is called a centralized decision-making (control) system. However, in reality, neither the supplier nor the manufacturer controls the entire chain, and each of them has their own initiatives and strategies. This case is known as a decentralized control system.

The manufacturer will not do business with the supplier if it cannot achieve a minimum level of efficiency $\theta_M$. Accordingly, the supplier tries to achieve the maximum conditional efficiency $E_S(\theta_M)$. Similarly, the supplier can define a minimum level of efficiency $\theta_S$. In this case, the maximum dependent efficiency of the manufacturer would be $E_M(\theta_S)$. The following nonlinear programming models can be set up:

$$E_{S0}(\theta_M) = \max \{ \theta_{S0} : \theta_{Sj} \leq 1, \theta_{Mj} \leq 1, \theta_{M0} \geq \theta_M, j = 1,2, \ldots N \}$$

$$E_{M0}(\theta_S) = \max \{ \theta_{M0} : \theta_{Sj} \leq 1, \theta_{Mj} \leq 1, \theta_{S0} \geq \theta_S, j = 1,2, \ldots N \}$$

It is clear that there is a game between the supplier and the manufacturer. Based on the presented models, it can be concluded that $E_S(\theta_M)$ is a function of $\theta_M$, and $E_M(\theta_S)$, is a function of $\theta_S$. If the manufacturer’s minimum acceptable efficiency is $\theta_M$, then the supplier’s maximum conditional efficiency is $E_S(\theta_M)$. If the supplier’s minimum acceptable efficiency is $E_S(\theta_M)$, then the maximum conditional efficiency of the manufacturer is $E_M(\theta_M)$. The pair $(\theta_M, \theta_S)$ represents a Nash equilibrium if the equality $\theta_M = E_M(E_S(\theta_M))$ is satisfied. An equilibrium also exists in the case of $\theta_S = E_S(E_M(\theta_S))$.

Centralized and decentralized control

In the case of centralized control, there is one decision-maker who controls the efficiency of both the supplier and the manufacturer in order to maximize the efficiency of the entire supply chain. The efficiency measurement model takes the following form:

$$\theta_{DEACCS} = \max \{ \theta_T : \theta_T = \theta_{S0} \cdot \theta_{M0}, \theta_{Sj} \leq 1, \theta_{Mj} \leq 1, (\theta_{S0}, \theta_{M0}) \in T_E \}$$

However, in the case of decentralized control, the supplier and the manufacturer control their own efficiencies. Both participants strive to minimize the efficiency of the other and maximize their own efficiency. Figures 4 and 5 explain the bargaining process.

Figure 5. Three-stage bargaining model [37].
Figure 4 shows the bargaining model with an unlimited number of offers. The game starts when the supplier proposes a strategy $S_1$. The manufacturer either accepts or rejects the strategy. If the manufacturer accepts the proposed strategy, the game ends with efficiencies $(\theta_1, \theta_2)$. In the case where strategy $S_1$ is rejected, the manufacturer proposes strategy $S_2$. The game does not end until one member accepts the offer of the other member of the supply chain. It is necessary to note that participants are exposed to certain costs during the bargaining process.

The model also includes a time component. It is assumed that the rate of efficiency reduction for the supplier is $\delta_1$, and $\delta_2$ for the manufacturer, where $0 \leq \delta_1 \leq 1$, $0 \leq \delta_2 \leq 1$. For $k$ possible offers, if one player accepts the strategy of the other player $S_k: (\theta_M^k, \theta_S^k)$, the actual efficiency of the participants is $(\delta_1^{k-1}\theta_S^k, \delta_2^{k-1}\theta_M^k)$. The efficiencies of the manufacturer and the supplier will be zero if the number of offers tends to infinity. Games with an infinite number of offers are reduced to games with three offers.

In the second offer, the supplier knows that if their offer is not accepted, they must unconditionally accept the offer in the third step. If the supplier’s strategy satisfies $\theta_1\theta_{S2} = \delta_1^{2}\theta^*_S$ and $\theta_2\theta_{M2} = \delta_2^{2}\theta^*_M$, the producer will not reject the supplier’s strategy, so their efficiency will be improved.

If $\delta_1\theta_{S2} = \delta_1^{2}\theta^*_S$ and $\delta_2\theta_{M2} = \delta_2^{2}\theta^*_M$, then $\theta_{M2} \geq \theta^*_M$ and $\theta_{S2} \geq \theta^*_S$. Ideally, if the supplier’s offer in the second step satisfies the condition $\delta_1\theta_{S2} = \delta_1^{2}\theta^*_S$, that would be optimal. Moreover, in the first step, the supplier can devise a strategy that satisfies the condition $\delta_1\theta_{S2} = \delta_1^{2}\theta^*_S$. Then, it holds that:

$$(\theta^*_S, \theta^*_M) = (\theta_{S1}, \theta_{M1}),$$ since that $(\theta^*_S, \theta^*_M)$ is an equilibrium.

If $\theta^*_S = \delta_2\theta_{M2}$ and $\theta^*_S = \delta_1\theta_{S2}$, then $(\theta^*_S, \theta^*_M)$ is a Nash equilibrium. It must satisfy the conditions $\theta^*_M = E_M(\theta^*_S)$ and $\theta^*_S = E_M(\theta^*_S)$. For this reason, the authors define the DEA supply chain efficiency:

$$\theta^*_{DEACDS} = \text{Max} \left\{ \theta_T; \theta_T = \theta_{S0} \cdot \theta_{M0} = E_M(\theta_{S0}), \frac{1}{\delta_2}\theta_{M0} = E_M(\delta_1\theta_{S0}) \right\}$$

This approach corresponds to a certain extent to the real situation. However, the application of the model in larger chains is not entirely possible.

4. Measuring Efficiency of Retail Supply Chain—Case Study

The previously described models, with minor or major modifications, can be applied to supply chain management. What all models have in common is that they were primarily developed on simpler examples, usually with two participants in the chain. A practical application in cases of complex chains with dozens of horizontally and vertically integrated participants (serial and parallel) represents a special problem. In order to approach the DEA models of efficiency measurement in supply chains and logistics in general, this paper proposes and, with certain adaptation, tests certain models. The models are tested on a real-life example. The first part of this section provides a detailed description of the discussed chain. After that, various efficiency measurement models for logistics systems are tested in sequence. The obtained results are compared in the last part of the chapter.

4.1. System Description

The models were tested at one retail chain operating in Serbia. Due to limitations in the data collection process, five indicators were used in this case (Figure 6).

![Figure 6. Two-component retail supply chain.](image)
The supply chain shown in Figure 6 can be viewed as a two-phase production system, where the first phase is represented by a distribution center that uses resources expressed in monetary units and the number of pallet spaces to generate a certain number of deliveries, as well as the delivery accuracy. The retail store represents the second phase, in which the inputs from the first phase (number of deliveries and delivery accuracy) are used to realize the output (sales). In compliance with the recommendations on the relationship between the number of decision-making units and the number of variables, the variables that best described supply chains of commercial companies were selected. The values of the described variables are shown in Table 2. Each DMU represent independent SC which consist of a distribution center (DC) and a retail store (RS).

Table 2. Input and output variables for observed SC.

<table>
<thead>
<tr>
<th>DMU</th>
<th>Pallet Places</th>
<th>Logistics Costs (m.u.)</th>
<th>Deliveries</th>
<th>Accuracy of Deliveries (%)</th>
<th>Turnover (m.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU 1</td>
<td>6775</td>
<td>27</td>
<td>10,377</td>
<td>0.75</td>
<td>483</td>
</tr>
<tr>
<td>DMU 2</td>
<td>548</td>
<td>2</td>
<td>3057</td>
<td>0.95</td>
<td>52</td>
</tr>
<tr>
<td>DMU 3</td>
<td>4486</td>
<td>36</td>
<td>10,825</td>
<td>0.99</td>
<td>523</td>
</tr>
<tr>
<td>DMU 4</td>
<td>6286</td>
<td>22</td>
<td>6931</td>
<td>1.00</td>
<td>334</td>
</tr>
<tr>
<td>DMU 5</td>
<td>3234</td>
<td>13</td>
<td>5576</td>
<td>0.97</td>
<td>146</td>
</tr>
<tr>
<td>DMU 6</td>
<td>5241</td>
<td>17</td>
<td>3828</td>
<td>0.53</td>
<td>217</td>
</tr>
<tr>
<td>DMU 7</td>
<td>4824</td>
<td>9</td>
<td>3614</td>
<td>0.95</td>
<td>90</td>
</tr>
</tbody>
</table>

The proposed model includes variables that are more operational than strategic in nature. The number of pallet spaces provides more information about the capacity of the facility than the warehouse area, which is more commonly used, as it takes into account the height of the facility. The logistics costs of distribution centers represent the sum of transport and storage costs, expressed in millions of monetary units. Delivery accuracy is a variable that rarely appears in DEA models. This variable represents the ratio of the number of successfully delivered shipments to the total number of shipments and is expressed as a percentage. Generally, delivery accuracy is a variable that both the distribution center and the retail store strive to maximize. The main reason is to reduce the cost of returning goods and reduce the additional time required for redelivery. However, unlike the retail sector, which aims for 100% accurate deliveries, such deliveries for the distribution center require certain investments, i.e., they cause certain costs. In real systems, it is usually necessary to invest in information systems, equipment, and employees. Considering the situation in the Serbian market, where the observed companies operate, a conflict in achieving appropriate values of delivery accuracy for the distribution center and the retail store can easily be observed. The only output variable from the retail system is sales. Sales are one of the universal financial indicators for which there are conflicting opinions in the literature. In this case, sales were expressed in millions of monetary units. Financial indicators describe retail and trade very well in general.

At the beginning, the simplest models were tested and therefore, the results of the standard approach were first analyzed. After that, more complex models based on game theory were analyzed.

4.2. Standard Approach to Measuring the Efficiency of Retail Supply Chains

As already mentioned, according to this approach, the supply chain is viewed as a “black box” where the mutual influences of individual phases are not considered. In the analyzed case, an independent assessment of the efficiency of the distribution center and the retail store, or that of the entire chain without intermediates, was performed. The distribution center was viewed as a production system with two inputs (logistics costs and number of pallet places) and two outputs (number of deliveries and delivery accuracy), while the retail store was viewed as a system with two inputs (number of deliveries and
delivery accuracy) and one output (sales). On the other hand, the supply chain used two inputs (logistics costs and number of pallet places) to achieve sales.

The linear programming task for determining the efficiency of the DC in the first supply chain was as follows:

**Phase II**

\[
\text{Min } Z_k = 6775Z_k - 6775\lambda_1 - 548\lambda_2 - 4486\lambda_3 - 6286\lambda_4 - 3234\lambda_5 - 5241\lambda_6 - 4824\lambda_7 \geq 0
\]

\[
+ 27Z_k - 27\lambda_1 - 2\lambda_2 - 36\lambda_3 - 22\lambda_4 - 13\lambda_5 - 17\lambda_6 - 9\lambda_7 \geq 0
\]

\[
+ 0.25\lambda_1 - 0.95\lambda_2 - 0.99\lambda_3 - 1.00\lambda_4 - 0.97\lambda_5 - 0.53\lambda_6 - 0.095\lambda_7 - 0.25 \geq 0
\]

\[
+ 10,377\lambda_1 - 3057\lambda_2 - 10,825\lambda_3 - 6931\lambda_4 - 5576\lambda_5 - 3828\lambda_6 - 3614\lambda_7 - 10,377 \geq 0
\]

\[
\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7 \geq 0
\]

**Phase II**

\[
\text{Max } s_1^+ + s_2^+ + s_3^+ \geq 0
\]

\[
6775 \times 0.2746 - 6775\lambda_1 - 548\lambda_2 - 4486\lambda_3 - 6286\lambda_4 - 3234\lambda_5 - 5241\lambda_6 - 4824\lambda_7 - s_1^+ \geq 0
\]

\[
+ 27 \times 0.2746 - 27\lambda_1 - 2\lambda_2 - 36\lambda_3 - 22\lambda_4 - 13\lambda_5 - 17\lambda_6 - 9\lambda_7 - s_2^+ \geq 0
\]

\[
+ 0.25\lambda_1 - 0.95\lambda_2 - 0.99\lambda_3 - 1.00\lambda_4 - 0.97\lambda_5 - 0.53\lambda_6 - 0.095\lambda_7 - 0.25 - s_3^+ \geq 0
\]

\[
+ 10,377\lambda_1 - 3057\lambda_2 - 10,825\lambda_3 - 6931\lambda_4 - 5576\lambda_5 - 3828\lambda_6 - 3614\lambda_7 - 10,377 - s_1^+ \geq 0
\]

\[
\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, s_1^+, s_2^+, s_3^+ \geq 0
\]

where:

\[ s = 2; \]

\[ m = 1; \]

\[ n = 1; \]

\[ e—unity \text{ vector; } \]

\[ Z^*_k = 0.2746. \]

After solving the given tasks, the efficiency of the first supply chain was obtained. In the same way, the efficiencies of the other six DMUs were determined. By solving the model, in addition to efficiency values, reference values as well as values of input/output shortages and surpluses were obtained. Due to the limitations of the standard approach, corrective actions were not thoroughly considered but only the efficiencies were compared to the values of other models. In the same way, by respecting the appropriate input and output variables, the efficiencies of retail stores and entire supply chains were calculated as “black boxes”.

The results are shown in Table 2. By examining the obtained results in more detail, the following conclusions can be drawn. Only one distribution center, DMU 2, was efficient. Of all retail stores, only DMU 1 and DMU 6 could be considered efficient. The efficiencies of the supply chains showed that DMU 2 and DMU 3 were efficient units. It is clear that there was no DMU whose both subsystems, as well as the entire chain, were efficient. DMU 2 represented the decision-making unit with the highest efficiency values, since both the DC and the supply chain of this unit were efficient. Generally speaking, the obtained results could not be considered reliable due to the limitations of the standard approach.

4.3. Approaches of “Average” Efficiency and “Product” Efficiency

According to these approaches, the efficiency of the entire chain is defined by the efficiency of its individual components. Due to their frequent use, two approaches are distinguished, according to which the efficiency of the chain is defined as the arithmetic mean [31] or as the product [32] of its subprocesses.
“Average” efficiency

Unlike the standard approach, in the average efficiency approach, the authors first independently assessed the efficiency of all participants in the chain and then calculated the arithmetic mean of the previously obtained efficiencies. In this particular case, it was necessary to calculate the arithmetic mean of the efficiencies obtained by the previous approach, i.e., the average efficiency of the distribution center and retail outlet. The results are shown in Table 2.

Based on the obtained results, it can be concluded that there were no efficient supply chains among the seven observed ones. This phenomenon could be explained by the fundamental idea of decomposition, that a supply chain cannot be efficient if all of its participants are not efficient, i.e., in this case, both the distribution center and the retail needed to be efficient.

“Product” efficiency

A somewhat different approach, respecting the intermediate products as variables that affect the efficiency in both first and second stages, was proposed in [32]. According to that model, the efficiency of DMU 1 could be determined by solving the following linear programming problem:

\[
E_i = \max_{u_1, v_1, v_2, w_1, w_2} 483u_1 \\
6775v_1 + 27v_2 = 1 \\
483u_1 - 775v_1 - 27v_2 \leq 0 \\
52u_1 - 548v_1 - 2v_2 \leq 0 \\
523u_1 - 4486v_1 - 36v_2 \leq 0 \\
334u_1 - 6286v_1 - 22v_2 \leq 0 \\
146u_1 - 3234v_1 - 13v_2 \leq 0 \\
217u_1 - 5241v_1 - 17v_2 \leq 0 \\
90u_1 - 4824v_1 - 9v_2 \leq 0 \\
0.25w_1 - 10.377w_2 - 775v_1 - 27v_2 \leq 0 \\
0.95w_1 - 3057w_2 - 548v_1 - 2v_2 \leq 0 \\
0.99w_1 - 10.825w_2 - 4486v_1 - 36v_2 \leq 0 \\
1.00w_1 - 6931w_2 - 6286v_1 - 22v_2 \leq 0 \\
0.97w_1 - 5576w_2 - 3234v_1 - 13v_2 \leq 0 \\
0.53w_1 - 3828w_2 - 5241v_1 - 17v_2 \leq 0 \\
0.95w_1 - 3614w_2 - 4824v_1 - 9v_2 \leq 0 \\
483u_1 - 0.25w_1 - 10.377w_2 \leq 0 \\
52u_1 - 0.95w_1 - 3057w_2 \leq 0 \\
523u_1 - 0.99w_1 - 10.825w_2 \leq 0 \\
334u_1 - 1.00w_1 - 6931w_2 \leq 0 \\
146u_1 - 0.97w_1 - 5576w_2 \leq 0 \\
217u_1 - 0.53w_1 - 3828w_2 \leq 0 \\
90u_1 - 0.95w_1 - 3614w_2 \leq 0 \\
\lambda, v_1, v_2, w_1, w_2 \geq 0.000001
By solving the previous problem, the objective function value of 0.2259 was obtained. The obtained value represented the efficiency of the supply chain. However, since \( E_k = E_1^k \times E_2^k \), the model does not guarantee unique solutions for the weights, and solving the formulated model may not result in a unique solution for the weights. The authors of this approach proposed determining the weights that maximized the efficiency of the first stage, in this case, the distribution center, while maintaining the value of the chain efficiency. Accordingly, it is necessary to solve the following linear programming problem:

\[
E_k = \max 0.25w_1 + 10,377w_2 \\
6775v_1 + 27v_2 = 1 \\
483u_1 - 0.2259(6775v_1 + 27v_2) = 0 \\
483u_1 - 775v_1 - 27v_2 \leq 0 \\
52u_1 - 548v_1 - 2v_2 \leq 0 \\
523u_1 - 4486v_1 - 36v_2 \leq 0 \\
334u_1 - 6286v_1 - 22v_2 \leq 0 \\
146u_1 - 3234v_1 - 13v_2 \leq 0 \\
217u_1 - 5241v_1 - 17v_2 \leq 0 \\
90u_1 - 4824v_1 - 9v_2 \leq 0 \\
0.25w_1 - 10,377w_2 - 775v_1 - 27v_2 \leq 0 \\
0.95w_1 - 3057w_2 - 548v_1 - 2v_2 \leq 0 \\
0.99w_1 - 10,825w_2 - 4486v_1 - 36v_2 \leq 0 \\
1.00w_1 - 6931w_2 - 6286v_1 - 22v_2 \leq 0 \\
0.97w_1 - 5576w_2 - 3234v_1 - 13v_2 \leq 0 \\
0.53w_1 - 3828w_2 - 5241v_1 - 17v_2 \leq 0 \\
0.95w_1 - 3614w_2 - 4824v_1 - 9v_2 \leq 0 \\
483u_1 - 0.25w_1 - 10,377w_2 \leq 0 \\
52u_1 - 0.95w_1 - 3057w_2 \leq 0 \\
523u_1 - 0.99w_1 - 10,825w_2 \leq 0 \\
334u_1 - 1.00w_1 - 6931w_2 \leq 0 \\
146u_1 - 0.97w_1 - 5576w_2 \leq 0 \\
217u_1 - 0.53w_1 - 3828w_2 \leq 0 \\
90u_1 - 0.95w_1 - 3614w_2 \leq 0 \\
w_1, v_1, v_2, w_1, w_2 \geq 0.000001
\]

After solving the presented model, the efficiency of the distribution center was 0.2746. Considering that the efficiency of the supply chain was 0.2259, the efficiency of the retail store DMU 1 was 0.8228. It should be noted that the procedure for determining a unique solution could have been reversed, i.e., one could aim to maximize the efficiency of the second phase. The efficiencies of other decision-making units were determined in the same way. The results of the model are given in Table 2.

4.4. Game-Theory Approach

Concrete models based on game theory were defined for the supply chains of the observed trading company. The distribution center and retail store represented players with certain strategies aimed at maximizing efficiency. As previously mentioned, cooperative and noncooperative games differ. In the following section, the models proposed in [38] are tested.

Noncooperative approach

In the noncooperative approach, one participant represents the leader while the other represents the follower. Despite recommendations in the literature that retail stores, or trade in general, become the most influential participants in supply chains, in this case, it was assumed that the distribution center was the leader, while retail store represented the follower. Putting the emphasis on distribution centers was a result of the primary goal, which was to investigate the impact of logistics centers on the efficiency of the entire chain.
The leadership position of the distribution center provided the opportunity for an independent maximization of the efficiency. Practically, this meant that the efficiency of distribution centers was calculated as in the standard approach. The values of the thus obtained efficiencies are shown in Table 2. In the first chain, the efficiency of the distribution center was 0.2746. The retail store, as a follower, maximized the efficiency while respecting the values of the interproduct weights (number and accuracy of deliveries). The problem was reduced to the following linear programming task:

\[
\begin{align*}
w^3_0 &= \max 483u_1 \\
6775v_1 + 27v_2 &= 1 \\
0.25w_1 + 10, 377w_2 &= 0.2746 \\
0.25w_1 - 10, 377w_2 - 775v_1 - 27v_2 &\leq 0 \\
0.95w_1 - 3057w_2 - 548v_1 - 2v_2 &\leq 0 \\
0.99w_1 - 10, 825w_2 - 4486v_1 - 36v_2 &\leq 0 \\
1.00w_1 - 6931w_2 - 6286v_1 - 22v_2 &\leq 0 \\
0.97w_1 - 5576w_2 - 3234v_1 - 13v_2 &\leq 0 \\
0.53w_1 - 3828w_2 - 5241v_1 - 17v_2 &\leq 0 \\
0.95w_1 - 3614w_2 - 4824v_1 - 9v_2 &\leq 0 \\
483u_1 - 0.25w_1 - 10, 377w_2 &\leq 0 \\
52u_1 - 0.95w_1 - 3057w_2 &\leq 0 \\
523u_1 - 0.99w_1 - 10, 825w_2 &\leq 0 \\
334u_1 - 1.00w_1 - 6931w_2 &\leq 0 \\
146u_1 - 0.97w_1 - 5576w_2 &\leq 0 \\
217u_1 - 0.53w_1 - 3828w_2 &\leq 0 \\
90u_1 - 0.95w_1 - 3614w_2 &\leq 0 \\
u_1, v_1, v_2, w_1, w_2 &\geq 0.00001
\end{align*}
\]

The efficiency of the first retail store was 0.0018, while the efficiency of the supply chain was equal to the product of the efficiency of the distribution center and the retail store, and it was 0.0005. The efficiencies of other supply chains were also determined using the same approach (Table 2).

The centrally low efficiencies of the supply chains and retail stores obtained by this approach indicated that the distribution center as a leader in this specific case was not a good strategy. In this way, they led to a reduction of the efficiency of retail stores and therefore of the supply chains.

**Centralized approach**

The centralized approach is based on cooperative games in which players make joint strategies in order to maximize the overall chain. According to this approach, determining the optimal intermediate product weights is done under the condition of maximizing the aggregated efficiency of the entire supply chain. The centralized approach is characterized by the condition of equal intermediate product weights \(w_d = \bar{w}_d\). The efficiency of the first supply chain is obtained by solving the following linear programming problem:

\[
\begin{align*}
c^\text{centr}_0 &= \max 483u_1 \\
6775v_1 + 27v_2 &= 1 \\
0.25w_1 - 10, 377w_2 - 775v_1 - 27v_2 &\leq 0 \\
0.95w_1 - 3057w_2 - 548v_1 - 2v_2 &\leq 0 \\
0.99w_1 - 10, 825w_2 - 4486v_1 - 36v_2 &\leq 0 \\
1.00w_1 - 6931w_2 - 6286v_1 - 22v_2 &\leq 0 \\
0.97w_1 - 5576w_2 - 3234v_1 - 13v_2 &\leq 0 \\
0.53w_1 - 3828w_2 - 5241v_1 - 17v_2 &\leq 0 \\
0.95w_1 - 3614w_2 - 4824v_1 - 9v_2 &\leq 0
\end{align*}
\]
The efficiency of the first supply chain was 0.2259. Since \( e_{0}^{\text{centr}} = e_{0}^{1,\text{centr}} \cdot e_{0}^{2,\text{centr}} \), the optimal solution does not have to be unique. Different combinations of subprocess efficiencies can yield the same overall efficiency. One way to solve this problem is to maximize the efficiency of the first phase \( e_{0}^{1,\text{centr}} \), as shown below:

\[
\begin{align*}
483u_1 - 0.25w_1 - 10,377w_2 & \leq 0 \\
52u_1 - 0.95w_1 - 3057w_2 & \leq 0 \\
523u_1 - 0.99w_1 - 10,825w_2 & \leq 0 \\
334w_1 - 1.00w_1 - 6931w_2 & \leq 0 \\
146u_1 - 0.97w_1 - 5576w_2 & \leq 0 \\
217u_1 - 0.53w_1 - 3828w_2 & \leq 0 \\
90u_1 - 0.95w_1 - 3614w_2 & \leq 0 \\
u_1, v_1, v_2, w_1, w_2 & \geq 0.000001
\end{align*}
\]

After determining the maximum efficiency of the distribution center, which was 0.2746, the efficiency of the retail store was simply determined, and it was 0.8224. Determining the efficiency of other supply chains, i.e., its elements, was done in the same way (Table 2).

4.5. Results and Discussion

The results of the tested models largely confirmed the previously stated claims. The efficiency of the considered supply chain was also examined, as well as the influence of the distribution center and retail outlet on the overall efficiency. In order to simplify the analysis, the combined results are presented in Table 3.

By applying the standard (independent) approach, relatively low values of the distribution center’s efficiency can be observed, with only DMU 2 being fully efficient while the efficiencies of other units were less than 0.5. On the other hand, the efficiencies of retail outlets were significantly higher, with two units forming the efficient frontier. The supply chain efficiencies, ignoring intermedia products, were relatively high, with DMU 2 and DMU 3 being efficient. By comparing the first three columns of Table 2, it can be easily concluded that no DMU was efficient according to all approaches, i.e., no DMU simultaneously had an efficient distribution center, retail outlet, and supply chain. The distribution center and the supply chain of the first decision unit were efficient while the retail outlet was efficient only at 0.30. On the other hand, the supply chain and retail outlet of DMU 3 were almost fully efficient, while the distribution center was efficient at 0.43.
Table 3. Efficiency scores form different approaches.

<table>
<thead>
<tr>
<th>Component</th>
<th>Standard Approach</th>
<th>Kao and Hwang’s Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DC</td>
<td>RS</td>
</tr>
<tr>
<td>DMU 1</td>
<td>0.275</td>
<td>1.000</td>
</tr>
<tr>
<td>DMU 2</td>
<td>1.000</td>
<td>0.300</td>
</tr>
<tr>
<td>DMU 3</td>
<td>0.433</td>
<td>0.920</td>
</tr>
<tr>
<td>DMU 4</td>
<td>0.206</td>
<td>0.850</td>
</tr>
<tr>
<td>DMU 5</td>
<td>0.309</td>
<td>0.462</td>
</tr>
<tr>
<td>DMU 6</td>
<td>0.147</td>
<td>1.000</td>
</tr>
<tr>
<td>DMU 7</td>
<td>0.263</td>
<td>0.439</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Component</th>
<th>Noncooperative Approach</th>
<th>Centralized Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DC</td>
<td>RS</td>
</tr>
<tr>
<td>DMU 1</td>
<td>0.275</td>
<td>0.002</td>
</tr>
<tr>
<td>DMU 2</td>
<td>1.000</td>
<td>0.303</td>
</tr>
<tr>
<td>DMU 3</td>
<td>0.433</td>
<td>0.001</td>
</tr>
<tr>
<td>DMU 4</td>
<td>0.206</td>
<td>0.933</td>
</tr>
<tr>
<td>DMU 5</td>
<td>0.309</td>
<td>0.463</td>
</tr>
<tr>
<td>DMU 6</td>
<td>0.147</td>
<td>1.000</td>
</tr>
<tr>
<td>DMU 7</td>
<td>0.263</td>
<td>0.000</td>
</tr>
</tbody>
</table>

By analyzing the average efficiency, interesting observations can be made. None of the supply chains were efficient since none of them had efficient distribution centers and retail stores. Kao and Hwang’s approach in this specific example provided relatively small efficiency values for the chain [32]. Similar to the standard approach, it was also shown that the efficiency of retail stores was higher than that of the efficiency of distribution centers. Models based on game theory provided interesting results. By applying the noncooperative model, extremely low efficiency values were obtained for the chains. This can be explained by the fact that the distribution center as a leader with very low efficiency values could not positively influence the efficiency of the chain enough. Moreover, the retail store, as a follower, by retaining the coefficient values defined by the distribution center, could not increase the efficiency. On the other hand, centralized approaches achieved significantly higher efficiency values. This confirmed the suitability of applying that approach, as well as the fact that cooperative games, i.e., a centralized approach, provided no smaller efficiency values than noncooperative approach. However, in more complex real systems that are not owned by a single company, centralized management is very difficult.

The results of the centralized approach were the same as the results obtained by applying Kao and Hwang’s approach [32]. This can be explained by the fact that the models were relatively similar, and the only difference was in the additional constraints of the second model.

The decision on whether to use centralized or decentralized management in retail supply chains depends on several factors, such as the size of the chain, the number of participants, whether it is domestic or international, the ownership structure of the participants, and so on. In practice, decentralized management is more common. Centralized management may be present in smaller national retail chains or in strong large international chains that have a dominant position and significant influence over other participants. They practically dictate the rules and influence the decision-making of other links. None of the approaches tested (except the standard approach) provided efficient DMUs, i.e., none of the approaches found efficient supply chains. The suitability of the considered model was primarily reflected in the inclusion of a quality measure in the tested models. The results of the model were mainly influenced by the choice of input and output variables. Different combinations of inputs and outputs gave different results. By comparing the obtained results with the knowledge of the functioning of certain chains of the considered
company, it can be concluded that the results of the centralized model best corresponded to
the real situation.

This paper made significant contributions by adapting existing theoretical models
from the literature to evaluate the effectiveness of retail supply chain management and
supply chains overall. The research conducted identified and analyzed the pros and cons
of various approaches, while also highlighting areas for further study in the future. Our
practical contributions are particularly noteworthy, as we provided practitioners with easily
implementable models that can be tailored to suit the needs of their organization, based on
their size, structure, and performance metrics.

5. Conclusions

The modern problems of supply chains have been increasingly evident in recent years.
Factors such as pandemics, recessions, wars, and natural disasters have been some of the
main obstacles to the smooth operation of global and national supply chains. As a result,
companies have become increasingly interested in improving the efficiency of their supply
chain processes. This paper presented the most commonly used models in the literature,
which were tested and found to be applicable to simpler and less complex supply chains.

The example discussed in this paper was a real case of a trading company operating
in Serbia. The main entities in these simpler (national) supply chains were identified as
the distribution center and retail outlet. The limitation of this study lay in neglecting
suppliers as important links in the chain, due to the inability to apply the described models
to multiphase systems. Most models were limited to two-phase systems. Additional
limitation of this research were the input and output variables. This paper used the
most important variables, but future research should also include additional variables
(energy, environmental, operational, etc.). The results showed that none of the tested
approaches (except for the standard one) yielded efficient DMUs, or in other words, none
of the approaches found efficient supply chains, which to some extent confirmed the
discriminatory power of these approaches.

For an initial application and measurement of retail supply chain efficiency, standard
and simpler models can be used. This is indeed the predominant case in practice. It most
often refers to independent links. However, retail supply chains are very labor-intensive
from a logistics perspective, with large assortments, quantities of goods, and limited time.
This motivates them to carry out the most efficient logistics activities possible. However,
as noted, it is not enough to track the efficiency of just one partial participant but of all
participants (or as many participants as possible). Network models and more complex
models based on game theory are suitable for this. As described, each of the models can
be applied to specific examples and yield certain results. However, the decision on which
model to apply depends on the size of the chain, the number of participants, whether it is
domestic or international, the ownership structure of the participants, and so on. In that
sense, there is no one best model for all cases.

However, to measure the efficiency of more complex supply chains, it is necessary to
develop new models and improve existing approaches. It is essential to respect the complex
structure and organization of global supply chains. Another direction for future research
is to focus on selecting performance indicators, relevant input, and output variables. To
solve this problem, numerous MCDM approaches can be used, as well as information
from real systems. Companies face the problem of tracking too many indicators, and
decision-making is often based on only a fraction of the available data.

The third direction for future research is the application of simulation models, which
would enable a faster testing of new models and the exploration of different scenarios.
Simulations can also examine certain influential factors. The final direction for future
research identified in this paper is the application of models to real systems. Only real
systems can provide reliable results and highlight the advantages and disadvantages of
developed approaches. Testing needs to be done on supply chains from different industries
and markets to identify all influential factors of activities, markets, and other elements.


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