



Article Using an Interval Type-2 Fuzzy AROMAN Decision-Making Method to Improve the Sustainability of the Postal Network in Rural Areas

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Abstract: One of the crucial pillars of each state's development strategy relates to service provision in rural areas. An adequate scope of these services is a prerequisite for uniform regional progress. Postal operators play a key role in supporting these development policies, by providing postal, financial and transportation services to each citizen in a state, regardless of place of residence. The postal network represents one of the biggest logistics networks worldwide. However, since it is not financially justified to provide services to all citizens, even to those that live in the most remote areas, the question of how to optimize the postal network is always topical. This problem is very complex because the postal units' existence in rural areas cannot be considered just from an economic standpoint; many other criteria should be considered. The model proposed in this paper can be considered a decision-making tool designed to support policymakers in planning the postal network. First, we identify the criteria that should be considered in decision-making by an extensive literature review. We then apply the FUCOM method to determine the importance of individual criteria. Finally, we propose an Interval Type-2 Fuzzy AROMAN approach to determine which postal unit should be reorganized.

Keywords: multi-criteria decision-making; AROMAN; FUCOM; postal services; last-mile delivery; service networks; rural areas

MSC: 03E72; 47S40; 90B50

1. Introduction

Most rural residents are engaged in food production, which is a crucial pillar of human civilization [1,2]. However, the quality of life in rural areas is an intricate issue in numerous countries [3,4]. The appropriate infrastructure and availability of services are mostly lacking in these areas [5,6]. Many factors may have caused this problem; however, one of the main policy pillars of each government should be to contribute to the development of rural areas [7–9].

Individual countries define "rural areas" differently, the scope ranging from definitions in terms of dispersed population, agricultural-based economy, distance from major urban centers, and, as a direct consequence, lack of access to major services [10,11]. At the international level, the most frequently used approach is that proposed by the Organisation for Economic Co-operation and Development—OECD. The OECD has established a regional typology according to which regions have been classified as predominantly rural (PR), intermediate (IR), and predominantly urban (PU). This typology is based on a combination of three criteria: the population density, the percentage of the population of a region living in rural communities, and the presence of large urban centers in such a



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Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). region. As shown in Figure 1, among the considered 1348 regions in the EU-28, some 367 were classified as predominantly urban, 553 as intermediate, and 428 as predominantly rural regions in the year 2016 [12].



(1) Estimates.

- (*) Intermediate regions and predominantly rural regions: not applicable.
- (*) Predominantly urban regions and predominantly rural regions: not applicable.
- (*) Predominantly urban regions: not applicable.
- (*) Intermediate regions: not applicable.
- (6) Predominantly rural regions: not applicable

Figure 1. Population structure, by urban–rural topology (% share of the total population) [12].

The integration of developing countries into the modern world economy was in the past associated with a population migration tendency reflected mainly in the direction of rural to urban areas [13]. Along with a process of de-agriculturalization of rural households, there is also increasing development of new non-agricultural activities in rural areas, such as manufacturing, tourism, housing expansion, and new consumption patterns, connected to leisure and recreation that have increased demand for labor [14–16]. The nature of rural areas includes more than the production of agricultural products and rural areas are diversifying. Earlier, rural communities were almost isolated from various influences, but there is now a large degree of variability among rural residents. New social groups are emerging, and the result is a change in demand for services because new groups have new habits. Rural development is less and less associated with underdevelopment and more and more with the lack of attractiveness of these areas. Rural households have difficulties accessing food stores, banking and postal services, health and education, and public transport [17].

Postal services are of particular interest for regional growth, and this is the reason why the states consider them as services of general economic interest [18]. As a consequence, at the global level, the concept of universal postal obligation is introduced in the sector, which implies the existence of certain scope of postal services in every community [19,20]. Such

an extensive network of postal units represents one of the biggest logistics infrastructures worldwide [21]. However, since it is not profitable to offer the services at each point of a state, the problem of optimization of the postal network is one the most crucial in this industry [22,23].

In this paper, we propose a methodological approach for the optimization of the postal network at a micro level, considering just a handful of postal units. We demonstrate the application of the model on a small number of units (6 in our case), but we can repeat this analysis many times (6 + 6 + ...) and analyze the whole postal network by this (more thousand units). The concept of analysis implies assessing several postal units in a certain rural region, where the result of the implemented methodology should give the rank of the considered postal units. The unit with the lowest rank should be somehow reorganized. While assessing the importance rank of each unit, multiple criteria should be included in the analysis. Accordingly, the considered problem is a typical multi-criteria decision-making (MCDM) problem. Various techniques could be used for solving this kind of problem while providing ways to deal with uncertainty in data, using the theory of fuzzy or rough sets [24,25]. In this paper, we decided to propose the implementation of an alternative ranking order method accounting for two-step normalization—AROMAN [26–28]—in a type-2 fuzzy environment. According to the authors' knowledge, this is the first time in the literature that an interval type-2 fuzzy AROMAN has been proposed and implemented. The most important contributions of this study are as follows: (i) based on an in-depth literature review, we discovered the attributes for optimization of the postal network; (ii) the set of twenty identified attributes is reduced to seven criteria by grouping them into seven clusters; (iii) we determined the relevance ranks of the criteria by interviewing the experts from the postal industry and by applying the FUCOM method; (iv) we implemented the AROMAN method in the type-2 fuzzy environment for the first time in the literature; (v) we offered the proposal for reorganization of the lowest ranked postal units.

2. Literature Review on the Criteria for the Postal Network Assessment

This section reveals the criteria that various authors used to evaluate the efficiency of the postal and related sectors. The summarized overview is shown in Table 1.

Closing a postal network unit (PNU) has the most detrimental effect on sensitive population groups. These categories are characterized by limited mobility due to illness or commuting difficulties. Every relocation has an impact on their inability to access postal services or their overall experience with the postal network. Such a change also creates additional expenses for people with low income or problems with daily schedules to organize the additional commute required for gaining access to postal services [22,29,30].

One of the important attributes is the number of legal entities in the territory covered by PNU. Closing a PNU has a significant effect on the local business community as well. This slows down the information flow, causes delays and additional expenses for gas to reach alternative PNU, and leads to negative ecological effects [31,32].

Ralevic et al. [33] used the Data Envelopment Analysis (DEA) approach for public postal operators' profit efficiency measurement. They used different approaches and different inputs and outputs. It is also interesting that this method may be applied at different levels—an individual postal operator and its network; city level; regional, national, or for example, European postal market. For example, Filippini and Zola [34] use the econometrical approach for determining the cost efficiency of the Swiss Post. The analysis was carried out in the Italian-speaking area of Switzerland and included 47 small local post offices. Most studies that measure the efficiency of the postal network take the number of employees as one of the main inputs [18]. Dobrodolac et al. [35] proposed a model for the comparison of business units in the postal industry based on the stress level of employees.

Klingenberg et al. [36] analyzed the United States Postal Service, which possesses the largest retail network in the United States with over 30,000 retail locations. The authors consider various factors, such as geographical diversity, population density, Internet broadband access, diversity of transportation modes, transit routes or parking regulations, quality

of retail counter service/employee helpfulness, constraints related to the existing retail network, changes in population and employment over time, changes in the use of postal services over time, changes in demographic profile over time, changes in transportation networks and transit routes over time and accuracy of input data. For customers from underdeveloped and remote areas, the postal infrastructure is the only medium to ensure access to information [37].

Mizutani and Uranishi [38] analyze whether and to what extent the competition affects a reduction in expenses and overall productivity. The sample refers to the organizations that deliver parcels in Japan—one of them is state-owned, and the others are private operators.

The quality of service is an intricate question in the postal sector. This is because postal services should be provided to every citizen in a state due to universal service obligations, which are very costly and demanding [39].

Human capital is of crucial importance for each company, particularly in the service sector. Speaking about the expertise of employees, experienced workers are an advantage in complex systems such as the postal system [40]. The companies also implement various programs to stimulate their satisfaction, expecting that this would lead to increased kindness toward customers [41].

The interior and exterior of the post office also have a significant impact on user satisfaction. Accordingly, postal operators invest significant funds in the repair and improvement of their facilities [42]. Further, opening hours significantly affect the accessibility of public service delivery [43]. The researchers, and customers as well, assess the efficiency of service by analyzing the average waiting times [44]. A range of postal services and corresponding quality issues offered to rural communities generate constant debate in the postal sector [19,23].

The proximity of an alternative post office is a valuable attribute when considering reducing the postal network. The study by Vaishar et al. [45] showed that postal branches in Europe should be accessible to users in rural areas within a shorter time than 30 min. Accordingly, customers in rural areas often use various transportation modes to reach the post office [46].

The number of delivery points, i.e., the number of households that are served by a PNU, is an important attribute that gives information about the significance of a particular postal unit [47]. A similar attribute relates to the covered area by a PNU [48]. Even though the goal of many studies is to minimize capital resources, such as vehicles [49], here, we should maximize this criterion because the PNU that covers a wider area can be considered more important for the fulfillment of universal service obligations.

All services offered in a PNU are normalized, which makes it possible to use the number of services by type to determine the overall realized norm minutes for a certain period, which represents a productivity measure of a PNU. The number of norm minutes per month is a measure directly associated with the costs of a PNU, as one of the most important criteria in decision-making [50]. The higher values of norm minutes bring lower costs per provided service.

Based on the literature review, twenty attributes are identified. The authors of the paper concluded that these attributes, also named potential criteria, can be grouped into clusters which would be the final criteria used in the decision-making process (Figure 2). The grouping is carried out as follows: the first cluster includes vulnerable groups and access for people with disabilities; the second legal entities, covered area, number of mailboxes, number of routes and number of norm minutes per month; the third efficiency, quality of postal services and waiting time in the line; the fourth employees, the expertise of employees and the kindness of employees; the fifth mobile and Internet network coverage; the sixth competition and the proximity of an alternative post office; and the seventh interior and exterior of the post office, appropriate working hours, range of services, and easiness of access.

Attributes or Potential Criteria (PC)	Type of Attribute	Definition	Authors
Vulnerable groups—PC1	maximization	The number of people from vulnerable groups (older people, people with a lack of mobility, low-income people, single parents, etc.)	Milutinović, Marković, Stanivuković, Švadlenka, Dobrodolac [22]; Hamilton [29]
Legal entities—PC2	maximization	Number of legal entities in the territory covered by the observed PNU	Cabras, Lau [31]; Christiaanse, Haartsen [32]
Efficiency—PC3	maximization	The efficiency of a PNU as a ratio of the average monthly PNU incomes and the average monthly PNU outcomes	Ralevic, Dobrodolac, Markovic, Mladenovic [33]; Filippini, Zola [34]
Employees—PC4	maximization	Number of employees in the observed PNU	Ralević, Dobrodolac, Marković [18] Dobrodolac, Švadlenka, Čubranić-Dobrodolac, Čičević, Stanivuković [35]
Mobile and Internet network coverage—PC5	minimization	Mobile and internet network coverage in the area of observed PNU	Klingenberg, Bzhilyanskaya, Ravnitzky [36] Budziewicz-Guźlecka, Drab-Kurowska [37]
Competition—PC6	minimization	The number of competing organizations providing similar services	Mizutani, Uranishi [38]
Quality of postal services—PC7	maximization	User assessment of the provided service quality	Klingenberg, Bzhilyanskaya, Ravnitzky [36] Matúšková, Madleňáková [39]
The expertise of employees—PC8	maximization	User assessment of the expertise of employees	Neupane, Kyrönlahti, Prakash, Siukola, Kosonen, Lumme-Sandt, Nikander, Nygård [40]
The kindness of employees—PC9	maximization	User assessment of the kindness of employees	Drašković, Průša, Čičević, Jovčić [41]
Interior and exterior of the post office—PC10	maximization	Interior and exterior attractiveness of the observed PNU	Minami [42]
Appropriate working hours—PC11	maximization	Availability of the system at the daily and weekly level	Neutens, Delafontaine, Schwanen, van de Weghe [43]
Range of services—PC12	maximization	The range of services should be adjusted to customer needs	Dobrodolac, Ralević, Švadlenka, Radojičić [19]
Waiting time in the line—PC13	minimization	User perception of waiting time get access to post office counter	Doble [44]
Easiness of access—PC14	maximization	Easy access to the observed PNU (parking, bus station,)	Mostarac, Kavran, Rakić [46]
Access for people with disabilities—PC15	maximization	Width of the entrance, step-free access, assistance, low-level counters, portable PIN pads, hearing loops, staff interaction	Shergold, Parkhurst [30]
The proximity of an alternative post office—PC16	minimization	The proximity of an alternative post office in case of shutting down the observed PNU	Vaishar, Šťastná, Ilaria, Kataishi, Akhavan, Senjyu [45]

 Table 1. Attributes of the postal network identified from the relevant literature.

Attributes or Potential Criteria (PC)	Type of Attribute	Definition	Authors
Covered area—PC17	maximization	Delivery area of the observed PNU	Çakır, Perçin, Min [48]
Number of mailboxes—PC18	maximization	Number of delivery points/number of households	Mostarac, Mostarac, Kavran, Šarac [47]
Number of routes—PC19	maximization	Number of routes in the delivery area of a PNU	Nebro, García-Nieto, Berlí, Warchulski, Kozdrowski [49]
Number of norm minutes per month—PC20	maximization	The overall realized norm minutes for a certain period, which represents a productivity measure of a PNU	de Araújo, Dos Reis, da Silva, Aktas [50]

Table 1. Cont.



Figure 2. Structure of decision-making problem of postal network optimization.

3. Methods

Based on an extensive literature review, we identified twenty potential criteria for postal network optimization. We then further structured them into seven clusters representing the final criteria in the decision-making process. The following research methodology can be structured into two parts: determination of criteria weights and alternative ranking. For the first part, we apply the FUCOM method, while for the second we propose an implementation of the type-2 fuzzy AROMAN method. The research configuration is shown in Figure 3.



Figure 3. Research configuration.

3.1. Determination of Criteria Weights by the FUCOM Method

In this paper, we use the Full Consistency Method (FUCOM) to determine the weights of identified criteria. FUCOM is a relatively new method proposed by Pamučar et al. [51] in 2018.

A typical MCMD model can be described by the equation $max[f_1(x), f_2(x), \ldots, f_n(x)]$, $n \ge 2$, where $x \in A = [a_1, a_2, \ldots, a_m]$; n is the number of the criteria, m is the number of the alternatives, f_j represents the criteria $(j = 1, 2, \ldots, n)$ and A represents the set of the alternatives a_i $(i = 1, 2, \ldots, m)$. The values f_{ij} of each considered criterion f_j for each considered alternative a_i are known, namely $f_{ij} = f_j(a_i)$, $\forall (i, j); i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$, where each value of the attribute depends on the *j*-th criterion and the *i*-th alternative.

Commonly, real-life problems are not described by criteria of the same degree of importance, and deciding the relative weights of criteria in MCDM models is a specific problem that includes subjectivity. This FUCOM method enables the calculation of the weight coefficients of all of the elements mutually compared at a certain level of the hierarchy, simultaneously satisfying the conditions of comparison consistency [51].

FUCOM reduces the possibility of errors in comparison to the least possible extent due to (1) a small number of comparisons (n - 1) and (2) the constraints defined when calculating the optimal values of criteria. FUCOM offers the possibility to validate the

model by calculating the error for the obtained weight vectors by determining DFC. In the following text, the procedure of the FUCOM method is explained in more detail.

Step 1: In the first step, the criteria from the predefined set of evaluation criteria $C = \{C_1, C_2, ..., C_n\}$ are ranked. The ranking is performed according to the significance of the criteria; i.e., starting from the criterion that is expected to have the highest weight coefficient to the criterion of the least significance. Thus, the criteria ranked according to the expected values of the weight coefficients are obtained [51]:

$$C_{j(1)} > C_{j(2)} > \ldots > C_{j(k)}$$
 (1)

where k represents the rank of the observed criterion. If there is a judgment of the existence of two or more criteria with the same significance, the sign of equality is placed instead of ">" between these criteria in the expression (1).

Step 2: Further comparison of the ranked criteria is carried out by determining the comparative priority ($\varphi_{k/(k+1)}$, k = 1, 2, ..., n, where k represents the rank of the criteria). The comparative priority of the evaluation criteria ($\varphi_{k/(k+1)}$) is an advantage of the criterion of the $C_{j(k)}$ rank compared to the criterion of the $C_{j(k+1)}$ rank. Thus, the vectors of the comparative priorities of the evaluation criteria are obtained, as in expression (2) [51]:

$$\Phi = (\varphi_{1/2}, \varphi_{2/3}, \dots, \varphi_{k/(k+1)})$$
(2)

where $\varphi_{k/(k+1)}$ represents the significance (priority) that the criterion of the $C_{j(k)}$ rank has been compared to the criterion of the $C_{j(k+1)}$ rank.

The comparative priority of the criteria is assessed in one of two ways: (a) according to their preferences, decision-makers define the comparative priority $\varphi_{k/k+1}$ among the observed criteria. When solving real problems, decision-makers compare the ranked criteria based on internal knowledge, so they determine the comparative priority $\varphi_{k/k+1}$ based on subjective preferences. If the decision-maker thinks that the criterion of the $C_{j(k)}$ rank has the same significance as the criterion of the $C_{j(k+1)}$ rank, then the comparative priority is $\varphi_{k/k+1} = 1$. (b) Based on a predefined scale for the comparison of criteria, decision-makers compare the criteria and thus determine the significance of each criterion in the expression (1). The comparison is made concerning the first-ranked (the most significant) criterion. Thus, the significance of the criteria $\omega_{C_{j(k)}}$ for all of the criteria ranked in Step 1 is obtained. Since the first-ranked criterion is compared with itself (its significance is $\omega_{C_{j(1)}} = 1$), it means that the n - 1 comparison of the criteria should be performed.

The FUCOM model allows the pairwise comparison of the criteria employing integer, decimal values, or the values from the predefined scale for the pairwise comparison of the criteria [52].

Step 3:The final values of the weight coefficients should be calculated in Step 3 (ω_1 , ω_2 ,..., ω_n)^{*T*}. These values should satisfy the two conditions:

(1) that the ratio of the weight coefficients is equal to the comparative priority among the observed criteria ($\varphi_{k/(k+1)}$) defined in Step 2; i.e., that the following condition is met [51]:

$$\frac{\omega_k}{\omega_{k+1}} = \varphi_{k/(k+1)} \tag{3}$$

(2) In addition to condition (3), the final values of the weight coefficients should satisfy the condition of mathematical transitivity; i.e., that $\varphi_{k/(k+1)} \otimes \varphi_{(k+1)/(k+2)} = \varphi_{k/(k+2)}$. Since $\varphi_{k/(k+1)} = \frac{\omega_k}{\omega_{k+1}}$ and $\varphi_{(k+1)/(k+2)} = \frac{\omega_{k+1}}{\omega_{k+2}}$, the condition that $\frac{\omega_k}{\omega_{k+1}} \otimes \frac{\omega_{k+1}}{\omega_{k+2}} = \frac{\omega_k}{\omega_{k+2}}$ is obtained. Thus, yet another condition that the final values of the weight coefficients of the evaluation criteria need to meet is obtained, namely [51]:

$$\frac{\omega_k}{\omega_{k+2}} = \varphi_{k/(k+1)} \otimes \varphi_{(k+1)/(k+2)} \tag{4}$$

Full consistency, i.e., minimum DFC (χ) is satisfied only if transitivity is fully respected, i.e., when the conditions of $\frac{\omega_k}{\omega_{k+1}} = \varphi_{k/(k+1)}$ and $\frac{\omega_k}{\omega_{k+2}} = \varphi_{k/(k+1)} \otimes \varphi_{(k+1)/(k+2)}$ are met. In that way, the requirement for maximum consistency is fulfilled, i.e., DFC is $\chi = 0$ for the obtained values of the weight coefficients. For the conditions to be met, it is necessary that the values of the weight coefficients ($\omega_1, \omega_2, \dots, \omega_n$)^{*T*} meet the condition of $\left| \frac{\omega_k}{\omega_{k+1}} - \varphi_{k/(k+1)} \right| \leq \chi$ and $\left| \frac{\omega_k}{\omega_{k+2}} - \varphi_{k/(k+1)} \otimes \varphi_{(k+1)/(k+2)} \right| \leq \chi$ with the minimization of the value χ . In that

manner, the requirement for maximum consistency is satisfied. Based on the defined settings, the final model for determining the final values of the

$$\min \chi s.t. \left| \frac{\omega_{j(k)}}{\omega_{j(k+1)}} - \varphi_{k/(k+1)} \right| \leq \chi, \forall j \left| \frac{\omega_{j(k)}}{\omega_{j(k+2)}} - \varphi_{k/(k+1)} \otimes \varphi_{(k+1)/(k+2)} \right| \leq \chi, \forall j \sum_{j=1}^{n} \omega_j = 1, \forall j \omega_i > 0, \forall j$$

$$(5)$$

By solving model (5), the final values of the evaluation criteria $(\omega_1, \omega_2, ..., \omega_n)^T$ and the degree of DFC (χ) are generated [53].

3.2. Ranking Alternatives Using a Type-2 Fuzzy AROMAN Method

weight coefficients of the evaluation criteria can be defined [51].

The type-2 fuzzy AROMAN method is, for the first time in the literature, implemented in this paper. First, we will provide some preliminaries on type-2 fuzzy arithmetic.

3.2.1. Preliminaries on Type-2 Fuzzy Arithmetic

In this section, we provide the definitions concerning the type-2 fuzzy sets and principles of type-2 fuzzy arithmetic that will be used in calculations related to the type-2 fuzzy AROMAN method.

Definition 1. A type-2 fuzzy set \widetilde{A} in the universe of discourse X can be represented by a type-2 membership function $\mu_{\widetilde{A}}$, shown as follows [54]:

$$A = \{ ((x, u), \mu_{A}(x, u)) | \forall x \in X, \forall u \in J_{X} \subseteq [0, 1], 0 \le \mu_{A}(x, u) \le 1 \}$$
(6)

where J_X denotes an interval in [0, 1]. Moreover, the type-2 fuzzy set A can also be represented as follows [54]:

$$\mathbf{A} = \int_{x \in X} \int_{u \in J_X} \mu_{\mathbf{A}}(x, u) / (x, u), \tag{7}$$

where $J_X \subseteq [0,1]$ and $\int \int denotes union over all admissible x and u.$

Definition 2. Let $\overset{\approx}{A}$ be a type-2 fuzzy set in the universe of discourse X represented by the type-2 membership function $\mu_{\overset{\approx}{A}}$. If all $\mu_{\overset{\approx}{A}}(x, u) = 1$, then $\overset{\approx}{A}$ is called an interval type-2 fuzzy set [54]. An interval type-2 fuzzy set $\overset{\approx}{A}$ can be regarded as a special case of a type-2 fuzzy set, represented as follows [54]:

$$\mathbf{A} = \int_{x \in X} \int_{u \in J_X} 1/(x, u), \tag{8}$$

where $J_X \subseteq [0,1]$.

Definition 3. *The upper membership function and the lower membership function of an interval type-2 fuzzy set are type-1 membership functions, respectively. In this paper, we propose the* application of interval type-2 fuzzy sets for solving fuzzy MCDM problems, where the points with maximum membership degrees of the upper and the lower membership functions of interval type-2 fuzzy sets are used to characterize interval type-2 fuzzy sets. Figure 4 illustrates a trapezoidal interval type-2 fuzzy set $\widetilde{A} = \left(\widetilde{A}_i^U, \widetilde{A}_i^L \right) = \left((a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U, H_1 \left(\widetilde{A}_i^U \right), H_2 \left(\widetilde{A}_i^U \right) \right), \left(a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L, H_1 \left(\widetilde{A}_i^L \right), H_2 \left(\widetilde{A}_i^L \right) \right) \right)$ where \widetilde{A}_i^U and \widetilde{A}_i^L are type-1 fuzzy sets, $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U, a_{i2}^L, a_{i3}^L, a_{i4}^L, H_1 \left(\widetilde{A}_i^L \right), H_2 \left(\widetilde{A}_i^L \right) \right)$ where \widetilde{A}_i^U and \widetilde{A}_i^L are type-1 fuzzy sets, $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U, a_{i2}^L, a_{i3}^L, a_{i4}^L, a_{i4}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L, a_{i4}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L, a_{i4}^L, a_{i4}^L, a_{i5}^L, a_{i5}^L, a_{i6}^L, a_{i4}^L, a_{i6}^L, a_{i6}^L, a_{i6}^L, a_{i6}^L, a_{i6}^L, a_{i6}^L, a_{i7}^L, a_{i6}^L, a_{i6}^L, a_{i6}^L, a_{i7}^L, a_{i6}^L, a_{$



Figure 4. The upper and lower trapezoidal membership functions of the interval type-2 fuzzy set.

Definition 4. The addition operation between the trapezoidal interval type-2 fuzzy sets $\stackrel{\approx}{A_1} = \stackrel{\sim U}{(A_1, A_1)} = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(\stackrel{\sim}{A_1}), H_2(\stackrel{\sim}{A_1})), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(\stackrel{\sim}{A_1}), H_2(\stackrel{\sim}{A_1})))$ and $\stackrel{\approx}{A_2} = (A_2, A_2) = ((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1(\stackrel{\sim}{A_2}), H_2(\stackrel{\sim}{A_2})), (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1(\stackrel{\sim}{A_2}), H_2(\stackrel{\sim}{A_2})))$ is defined as follows [54–56]:

$$\widetilde{\widetilde{A}}_{1} \oplus \widetilde{\widetilde{A}}_{2} = (\widetilde{A}_{1}^{U}, \widetilde{A}_{1}^{L}) \oplus (\widetilde{A}_{2}^{U}, \widetilde{A}_{2}^{L}) = ((a_{11}^{U} + a_{21}^{U}, a_{12}^{U} + a_{22}^{U}, a_{13}^{U} + a_{23}^{U}, a_{14}^{U} + a_{24}^{U};
\widetilde{M}_{1} = (A_{1}^{U}, A_{1}^{U}), M_{1} = (A_{2}^{U}), M_{2} = ((A_{1}^{U} + a_{21}^{U}, a_{12}^{U} + a_{22}^{U}, a_{13}^{U} + a_{24}^{U}; a_{14}^{U} + a_{24}^{U};
(a_{11}^{L} + a_{21}^{L}, a_{12}^{L} + a_{22}^{U}, a_{13}^{L} + a_{23}^{L}, a_{14}^{L} + a_{24}^{L};
(a_{11}^{L} + a_{21}^{L}, a_{12}^{L} + a_{22}^{U}, a_{13}^{L} + a_{23}^{U}, a_{14}^{L} + a_{24}^{L};
(M) = (A_{1}^{U}, A_{1}^{U}), M_{1} = (A_{2}^{U}), M_{1} = (A_{1}^{U}), M_{2} = (A_{1}^{U}, A_{2}^{U})).$$
(9)

Definition 5. The subtraction operation between the trapezoidal interval type-2 fuzzy sets $\widetilde{A}_1 = (\widetilde{A}_1, \widetilde{A}_1) = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1 (\widetilde{A}_1), H_2 (\widetilde{A}_1)), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1 (\widetilde{A}_1), H_2 (\widetilde{A}_1))$

Definition 6. The multiplication operation between the trapezoidal interval type-2 fuzzy sets $\stackrel{\sim}{A_1} = \stackrel{\sim}{(A_1, A_1)} = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1 (\stackrel{\sim}{A_1}), H_2 (\stackrel{\sim}{A_1})), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1 (\stackrel{\sim}{A_1}), H_2 (\stackrel{\sim}{A_1})))$ and $\stackrel{\sim}{A_2} = (\stackrel{\sim}{A_2}, \stackrel{\sim}{A_2}) = ((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1 (\stackrel{\sim}{A_2}), H_2 (\stackrel{\sim}{A_2})), (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1 (\stackrel{\sim}{A_2})))$ is defined as follows [54–56]:

$$\widetilde{\widetilde{A}}_{1} \otimes \widetilde{\widetilde{A}}_{2} = (\widetilde{A}_{1}^{U}, \widetilde{A}_{1}^{L}) \otimes (\widetilde{A}_{2}^{U}, \widetilde{A}_{2}^{L}) = ((a_{11}^{U} \times a_{21}^{U}, a_{12}^{U} \times a_{22}^{U}, a_{13}^{U} \times a_{23}^{U}, a_{14}^{U} \times a_{24}^{U}; \widetilde{A}_{1}^{U} \otimes (\widetilde{A}_{2}^{U}, \widetilde{A}_{2}^{U}) = ((a_{11}^{U} \times a_{21}^{U}, a_{12}^{U} \times a_{22}^{U}, a_{13}^{U} \times a_{23}^{U}, a_{14}^{U} \times a_{24}^{U}; \widetilde{A}_{11}^{U} \otimes (\widetilde{A}_{1}^{L}), H_{1}(\widetilde{A}_{2})), \min(H_{2}(\widetilde{A}_{1}), H_{2}(\widetilde{A}_{2}))), (a_{11}^{L} \times a_{21}^{L}, a_{12}^{L} \times a_{22}^{L}, a_{13}^{L} \times a_{23}^{L}, a_{14}^{L} \times a_{24}^{L}; \widetilde{A}_{11}^{U} \otimes (\widetilde{A}_{1}^{U}), \min(H_{2}(\widetilde{A}_{1}), H_{2}(\widetilde{A}_{2}))).$$

$$(11)$$

Definition 7. The division operation between the trapezoidal interval type-2 fuzzy sets $\stackrel{\approx}{A_1} = (\stackrel{\sim}{A_1}, \stackrel{\sim}{A_1}) = ((a_{11}^{U}, a_{12}^{U}, a_{13}^{U}, a_{14}^{U}; H_1 (\stackrel{\sim}{A_1}), H_2 (\stackrel{\sim}{A_1})), (a_{11}^{L}, a_{12}^{L}, a_{13}^{L}, a_{14}^{L}; H_1 (\stackrel{\sim}{A_1}), H_2 (\stackrel{\sim}{A_1})), (a_{11}^{L}, a_{12}^{L}, a_{13}^{L}, a_{14}^{L}; H_1 (\stackrel{\sim}{A_1}), H_2 (\stackrel{\sim}{A_1}))) and \stackrel{\approx}{A_2} = (\stackrel{\sim}{A_2}, \stackrel{\sim}{A_2}) = ((a_{21}^{U}, a_{22}^{U}, a_{23}^{U}, a_{24}^{U}; H_1 (\stackrel{\sim}{A_2}), H_2 (\stackrel{\sim}{A_2})), (a_{21}^{L}, a_{22}^{L}, a_{23}^{L}, a_{24}^{L}; H_1 (\stackrel{\sim}{A_2}), H_2 (\stackrel{\sim}{A_2}))) is defined as follows [54–56]:$

$$\overset{\approx}{A_{1}} \ominus \overset{\sim}{A_{2}} = (\overset{\sim}{A_{1}}, \overset{\sim}{A_{1}}) \ominus (\overset{\sim}{A_{2}}, \overset{\sim}{A_{2}}) = ((\overset{a_{11}^{U}}{a_{24}^{U}}, \overset{a_{12}^{U}}{a_{23}^{U}}, \overset{a_{13}^{U}}{a_{22}^{U}}, \overset{a_{14}^{U}}{a_{21}^{U}}; \overset{a_{14}^{U}}{a_{21}^{U}}, \overset{a_{14}^{U}}{a_{21}^{U}}; \overset{a_{14}^{U}}{a_{21}^{U}}; \overset{a_{14}^{U}}{a_{21}^{U}}; \overset{a_{14}^{U}}{a_{21}^{U}}; \overset{a_{14}^{U}}{a_{21}^{U}}; \overset{a_{14}^{U}}{a_{21}^{U}}; \overset{a_{14}^{U}}{a_{21}^{U}}; \overset{a_{14}^{U}}{a_{21}^{U}}; (\overset{\sim}{A_{14}^{U}}), H_{1}(\overset{\sim}{A_{2}^{U}})), \min(H_{2}(\overset{\sim}{A_{1}^{U}}), H_{2}(\overset{\sim}{A_{2}^{U}}))).$$

$$(12)$$

Definition 8. The arithmetic operations between the trapezoidal interval type-2 fuzzy sets $\widetilde{A}_1 = (\widetilde{A}_1^U, \widetilde{A}_1) = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(\widetilde{A}_1), H_2(\widetilde{A}_1)), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(\widetilde{A}_1), H_2(\widetilde{A}_1)))$ and the crisp value k is defined as follows [54–56]:

$$\widetilde{\overset{\alpha}{k}}_{1}^{\widetilde{\alpha}} = ((k \times a_{11}^{U}, k \times a_{12}^{U}, k \times a_{13}^{U}, k \times a_{14}^{U}; .H_{1}(\overset{\alpha}{A_{1}}), H_{2}(\overset{\alpha}{A_{1}})), (k \times a_{11}^{L}, k \times a_{12}^{L}, k \times a_{13}^{L}, k \times a_{14}^{L}; .H_{1}(\overset{\alpha}{A_{1}}), H_{2}(\overset{\alpha}{A_{1}})), (\overset{\alpha}{\overset{\beta}{A_{1}}}_{\frac{\widetilde{A_{1}}}{k}} = ((\frac{1}{k} \times a_{11}^{U}, \frac{1}{k} \times a_{12}^{U}, \frac{1}{k} \times a_{13}^{U}, \frac{1}{k} \times a_{14}^{U}; .H_{1}(\overset{\alpha}{A_{1}}), H_{2}(\overset{\alpha}{A_{1}})), (\overset{\alpha}{\overset{\alpha}{A_{1}}}), (13) (\overset{\alpha}{k} \times a_{11}^{L}, \frac{1}{k} \times a_{12}^{L}, \frac{1}{k} \times a_{13}^{L}, \frac{1}{k} \times a_{14}^{L}; .H_{1}(\overset{\alpha}{A_{1}}), H_{2}(\overset{\alpha}{A_{1}})), (13) (\overset{\alpha}{k} \times a_{11}^{L}, \frac{1}{k} \times a_{12}^{L}, \frac{1}{k} \times a_{13}^{L}, \frac{1}{k} \times a_{14}^{L}; .H_{1}(\overset{\alpha}{A_{1}}), H_{2}(\overset{\alpha}{A_{1}})), (14) (\overset{\alpha}{k} \times a_{11}), \overset{\alpha}{k} \times a_{12}), (15) (\overset{\alpha}{k} \times a_{11}), \overset{\alpha}{k} \times a_{12}), (15) (\overset{\alpha}{k} \times a_{11}), \overset{\alpha}{k} \times a_{12}), (15) (\overset{\alpha}{k} \times a_{11}), \overset{\alpha}{k} \times a_{12}), \overset{\alpha}{k} \times a_{13}), (15) (\overset{\alpha}{k} \times a_{11}), \overset{\alpha}{k} \times a_{12}), (15) (\overset{\alpha}{k} \times a_{11}), \overset{\alpha}{k} \times a_{12}), (15) (\overset{\alpha}{k} \times a_{11}), \overset{\alpha}{k} \times a_{12}), \overset{\alpha}{k} \times a_{12}), (15) (\overset{\alpha}{k} \times a_{12}), \overset{\alpha}{k} \times a_{12}), (15) (\overset{\alpha}{k} \times a_{11}), \overset{\alpha}{k} \times a_{12}), \overset{\alpha}{k} \times a_{12}), \overset{\alpha}{k} \times a_{12}), \overset{\alpha}{k} \times a_{12}), (15) (\overset{\alpha}{k} \times a_{12}), \overset{\alpha}{k} \times a_{12}), \overset{\alpha}{k}$$

3.2.2. Type-2 Fuzzy AROMAN Method

An extension of the AROMAN method [26–28] to the type-2 fuzzy environment is proposed in this section. The procedure is described in the following steps.

Step 1: Determine the initial decision-making matrix with the input data.

A type-2 fuzzy MCDM problem can be shown in the matrix format as:

$$\widetilde{\widetilde{D}} = \begin{bmatrix} \widetilde{\widetilde{x}}_{11} & \cdots & \widetilde{\widetilde{x}}_{1j} & \cdots & \widetilde{\widetilde{x}}_{1n} \\ \widetilde{\widetilde{x}}_{21} & \cdots & \widetilde{\widetilde{x}}_{2j} & \cdots & \widetilde{\widetilde{x}}_{2n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \widetilde{\widetilde{x}}_{m1} & \cdots & \widetilde{\widetilde{x}}_{mj} & \cdots & \widetilde{\widetilde{x}}_{mn} \end{bmatrix}, \ i = 1, 2, \dots, m, j = 1, 2, \dots, n.$$

where \tilde{x}_{ij} are linguistic variables.

To rate the qualitative criteria, the inputs are linguistic variables. These linguistic variables can be expressed as type-2 trapezoidal fuzzy numbers. The scale is offered in Table 2.

Table 2. Linguistic variables for the ratings of criteria.

Linguistic Variable	Type-2 Fuzzy Numbers
Very low (VL)	(0, 0, 0, 1; 1, 1), (0, 0, 0, 0.5; 0.9, 0.9)
Low (L)	(0, 1, 1, 3; 1, 1), (0.5, 1, 1, 2; 0.9, 0.9)
Medium-low (ML)	(1, 3, 3, 5; 1, 1), (2, 3, 3, 4; 0.9, 0.9)
Medium (M)	(3, 5, 5, 7; 1, 1), (4, 5, 5, 6; 0.9, 0.9)
Medium-high (MH)	(5, 7, 7, 9; 1, 1), (6, 7, 7, 8; 0.9, 0.9)
High (H)	(7, 9, 9, 10; 1, 1), (8, 9, 9, 9.5; 0.9, 0.9)
Very High (VH)	(9, 10, 10, 10; 1, 1), (0.95, 10, 10, 10; 0.9, 0.9)

If there are *K* experts that evaluate the alternatives based on set criteria, then the ratings can be calculated as:

$$\widetilde{\widetilde{x}}_{ij} = \frac{1}{K} \left[\widetilde{\widetilde{x}_{ij}^{*}} \left(+ \right) \widetilde{\widetilde{x}_{ij}^{2}} \left(+ \right) \dots \left(+ \right) \widetilde{\widetilde{x}_{ij}^{K}} \right].$$
(14)

Next, the normalization of data should be carried out. The AROMAN method implies two types of normalization, as explained in Steps 2 and 3.

Step 2: Normalization No. 1.

$$\widetilde{\widetilde{t}}_{ij} = \frac{\widetilde{\widetilde{x}}_{ij} - \min_{i} \widetilde{\widetilde{x}}_{ij}}{\max_{i} \widetilde{\widetilde{x}}_{ij} - \min_{i} \widetilde{\widetilde{x}}_{ij}}, i = 1, 2, \dots, m; j = 1, 2, \dots, n;$$
(15)

Step 3: Normalization No. 2.

$$\tilde{\tilde{x}}_{ij} = \frac{\tilde{\tilde{x}}_{ij}}{\sqrt{\sum_{i=1}^{m} \tilde{x}_{ij}^{2}}}; i = 1, 2, \dots, m; j = 1, 2, \dots, n;$$
(16)

The normalization procedure in Steps 2 and 3 should be applied for both criterion types (min and max).

Step 4: Aggregated normalization.

The aggregated normalization is obtained by Equation (17).

$$\widetilde{\widetilde{t}}_{ij}^{norm} = \frac{\beta \widetilde{\widetilde{t}}_{ij} + (1 - \beta) \widetilde{\widetilde{t}}_{ij}^{\ast}}{2}; \ i = 1, 2, \dots, m; j = 1, 2, \dots, n;$$
(17)

where \tilde{t}_{ij}^{norm} denotes the aggregated averaged normalization. β is a weighting factor for each type of normalization varying from 0 to 1.

Step 5: Weighted aggregated normalized decision-making matrix.

The aggregated normalized decision-making (DM) matrix should be multiplied by the criteria weights to obtain a weighted DM matrix.

$$\hat{\hat{t}}_{ij} = W_{ij} \cdot \hat{t}_{ij}^{norm}; \ i = 1, 2, \dots, m; j = 1, 2, \dots, n;$$
(18)

Step 6: Summation of weighted aggregated normalized DM per the criteria type.

Further procedure relates to a summation of the normalized weighted values separately for the criteria type min $(\tilde{\tilde{L}}_i)$ and the type max $(\tilde{\tilde{A}}_i)$.

$$\widetilde{\widetilde{L}}_{i} = \sum_{j=1}^{n} \widetilde{\widetilde{t}}_{ij}^{(min)}; i = 1, 2, \dots, m; j = 1, 2, \dots, n;$$
(19)

$$\widetilde{\widetilde{A}}_{i} = \sum_{j=1}^{n} \widetilde{\widetilde{t}}_{ij}^{(max)}; i = 1, 2, \dots, m; j = 1, 2, \dots, n;$$
(20)

Step 7: Raise the obtained \tilde{L}_i and \tilde{A}_i values to the degree of λ .

$$\widetilde{\widetilde{L}}_{i} = \widetilde{\widetilde{L}}_{i}^{\lambda} = \left(\sum_{j=1}^{n} \widetilde{\widetilde{t}}_{ij}^{(min)}\right)^{\lambda}; i = 1, 2, \dots, m; j = 1, 2, \dots, n;$$
(21)

$$\hat{\tilde{A}}_{i} = \hat{\tilde{A}}_{i}^{1-\lambda} = (\sum_{j=1}^{n} \hat{\tilde{t}}_{ij}^{(max)})^{1-\lambda}; \ i = 1, 2, \dots, m; j = 1, 2, \dots, n;$$
(22)

where λ represents the coefficient degree of the criterion type. Parameter λ can be set in different ways; however, here, we apply the weights obtained by the FUCOM method. If we mark the weights of the criteria of min type by w_j^{min} , then the parameter λ can be obtained by Equation (23).

$$\lambda = \sum_{j=1}^{n} w_{j}^{min}; \ j = 1, 2, \dots, n$$
(23)

Step 8: Calculate the final ranking.

To obtain the final ranking of alternatives (R_i) , the difference between the values $A_i \approx \hat{L}_i$ and \hat{L}_i should be calculated and the final ranking equation applied. To transform the type-2 fuzzy numbers to crisp values, we will apply the approach proposed by Lee and Chen [57] for ranking values of trapezoidal interval type-2 fuzzy sets.

Let A_i be an interval type-2 fuzzy set shown in Figure 4, where $A_i = (\tilde{A}_i^U, \tilde{A}_i^L) = (a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1(\tilde{A}_i^U), H_2(\tilde{A}_i^U))., (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L)))$. The ranking value $Rank(A_i)$ of the trapezoidal interval type-2 fuzzy set A_i is defined as follows [57]:

$$Rank \overset{\approx}{A}_{i} = M_{1}(\tilde{A}_{i}^{U}) + M_{1}(\tilde{A}_{i}^{L}) + M_{2}(\tilde{A}_{i}^{U}) + M_{2}(\tilde{A}_{i}^{L}) + M_{3}(\tilde{A}_{i}^{U}) + M_{3}(\tilde{A}_{i}^{L}) +$$

where $M_p(\widetilde{A}_i^j)$ denotes the average of the elements a_{ip}^j and $a_{i(p+1)}^j$, $M_p(\widetilde{A}_i^j) = \frac{a_{ip}^j + a_{i(p+1)}^j}{2}$, $1 \le p \le 3$, $S_q(\widetilde{A}_i^j)$ denotes the standard deviation of the elements a_{iq}^j and $a_{i(q+1)}^j$, $S_q(\widetilde{A}_i^j) = \sqrt{\frac{1}{2}\sum_{k=q}^{q+1} (a_{ik}^j - \frac{1}{2}\sum_{k=q}^{q+1} a_{ik}^j)^2}$, $1 \le q \le 3$, $S_4(\widetilde{A}_i^j)$ denotes the standard deviation of the elements a_{iq}^j and $a_{i(q+1)}^j$, $S_q(\widetilde{A}_i^j) = \sqrt{\frac{1}{2}\sum_{k=q}^{q+1} (a_{ik}^j - \frac{1}{2}\sum_{k=q}^{q+1} a_{ik}^j)^2}$, $1 \le q \le 3$, $S_4(\widetilde{A}_i^j)$ denotes the standard deviation of the elements a_{iq}^j and $a_{i(q+1)}^j$, $S_q(\widetilde{A}_i^j) = \sqrt{\frac{1}{2}\sum_{k=q}^{q+1} (a_{ik}^j - \frac{1}{2}\sum_{k=q}^{q+1} a_{ik}^j)^2}$, $1 \le q \le 3$, $S_4(\widetilde{A}_i^j)$ denotes the standard deviation of the elements a_{iq}^j and $a_{i(q+1)}^j$. ments $a_{i1}^j, a_{i2}^j, a_{i3}^j, a_{i4}^j, S_4(\widetilde{A}_i^j) = \sqrt{\frac{1}{4}\sum_{k=1}^4 (a_{ik}^j - \frac{1}{4}\sum_{k=1}^4 a_{ik}^j)^2}$, $H_p(\widetilde{A}_i^j)$ denotes the membership value of the element $a_{i(q+1)}^j$ in the trapezoidal membership function $\widetilde{A}_i^j, 1 \le p \le 2, j \in \{U, L\}$, and $1 \le i \le n$.

In Equation (24), the summation of $M_1(\overset{U}{A_i}), M_1(\overset{U}{A_i}), M_2(\overset{U}{A_i}), M_2(\overset{U}{A_i}), M_3(\overset{U}{A_i}), \overset{U}{A_i}), M_3(\overset{U}{A_i}), H_1(\overset{U}{A_i}), H_1(\overset{U}{A_i}), H_2(\overset{U}{A_i}), H_2(\overset{U}{A_i})$ and $H_2(\overset{U}{A_i})$ is called the basic ranking score, where we deduct the average of $S_1(\overset{U}{A_i}), S_1(\overset{U}{A_i}), S_2(\overset{U}{A_i}), S_2(\overset{U}{A_i}), S_3(\overset{U}{A_i}), S_3(\overset{U}{A_i}), S_4(\overset{U}{A_i})$ and $S_4(\overset{U}{A_i})$ from the basic ranking score to give the dispersive interval type-2 fuzzy set a penalty, where $1 \le i \le n$.

Accordingly, the final equation for the calculation of alternative ranks is as follows:

$$\mathbf{R}_{i} = e^{Rank(\widetilde{A}_{i} - \widetilde{L}_{i})}; \ i = 1, 2, \dots, m$$
(25)

4. Case Study—Optimization of the Rural Postal Network in the Region of Bajina Bašta, Serbia

To illustrate the applicability of the proposed methodology, we present a real-life case study in this section. The task will be to determine the importance ranks of six postal branches of the rural postal network in the region of Bajina Bašta, Serbia. In the considered region, there is just one post office in the urban area, 31250 Bajina Bašta, while the remaining six, which are the subjects of a case study and can be considered as alternatives, are in rural areas, and their names are:

- 31251 Mitrovac—alternative 1 (A1);
- 31253 Zlodol—alternative 2 (A2);
- 31254 Kostojevići—alternative 3 (A3);
- 31255 Rogačica—alternative 4 (A4);
- 31256 Perućac—alternative 5 (A5);
- 31258 Bačevci—alternative 6 (A6).

The number before the name of the location of the post office represents its postal code. The position of the Bajina Bašta region on the map of Europe is presented in Figure 5. A layout of post offices in the considered region is shown in Figure 6, where the red point is an urban post office and the remaining six in green are post offices in rural areas. The numbers correspond to the serial number of each alternative. The visual impression of the buildings where post offices are located is presented in Figure 7.

After we identified seven criteria, as previously explained in the section concerning the literature review, and six alternatives, we started the procedure by interviewing experts. The experts were interviewed twice: the first time to obtain the criteria weights by the FUCOM method, and the second time to implement the type-2 fuzzy AROMAN method. In this case study, we collected the answers from three experts from the postal industry. All experts have more than 20 years of professional experience. Moreover, two of them possess Ph.D. degrees, while the remaining one is a postal technology engineer.



Figure 5. Position of the Bajina Bašta region on the map of Europe.



Figure 6. Layout of post offices in the region of Bajina Bašta. 1—31251 Mitrovac; 2—31253 Zlodol; 3—31254 Kostojevići; 4—31255 Rogačica; 5—31256 Perućac; 6—31258 Bačevci. Red Circle is an urban post office.



Figure 7. The appearance of the considered post offices. 1—31251 Mitrovac; 2—31253 Zlodol; 3—31254 Kostojevići; 4—31255 Rogačica; 5—31256 Perućac; 6—31258 Bačevci.

4.1. The Results of the FUCOM Method

Step 1: The initial interrelation between the criteria is examined by using the numbers from 1 to 7. The criteria ranked 1 are the most important, while the criteria marked 7 are the least important. The results of evaluations by five experts are presented in Table 3.

Table 3. The answers of experts about the initial interrelation between the criteria.

	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6	Criterion 7
Expert 1	3	1	4	6	5	2	7
Expert 2	3	1	4	7	5	2	6
Expert 3	4	2	3	7	5	1	6

Step 2: Further comparison of the ranked criteria is carried out by determining the comparative priority, and the answers are in Table 4.

Table 4. The answers of experts about the comparative priority of the criteria.

	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6	Criterion 7
Expert 1	2.8	1	2.9	3.5	3.2	2.1	3.9
Expert 2	1.8	1	2.4	4	2.9	1.2	3.5
Expert 3	3	2	2.1	6	4	1	4.5

Step 3: According to the explanation of Step 3 in the methodological part, we calculated the criteria weights. The achieved results are presented in Table 5. These results will be further used in the procedure of type-2 fuzzy AROMAN implementation.

	Criterion 1	Criterion 2	Criterion 3	Criterion 4	Criterion 5	Criterion 6	Criterion 7
Expert 1	0.118	0.330	0.114	0.094	0.103	0.157	0.085
Expert 2	0.151	0.271	0.113	0.068	0.094	0.226	0.078
Expert 3	0.113	0.170	0.162	0.057	0.085	0.339	0.075
Average	0.127	0.257	0.129	0.073	0.094	0.241	0.079

Table 5. The final values of the weight coefficients.

The results of the FUCOM method implementation indicate that the most important criterion for the optimization of the postal network is Criterion 2—development potential (Figure 8). This can be explained by the significant benefits that the attributes relating to Criterion 2 might bring to the rural area. For example, the presence of legal entities in certain areas can generate new demand for services and products produced by local community members. The second-ranked criterion is C6—alternative services. It is described by the attributes competition and the proximity of an alternative post office. The high importance of this criterion is logical, keeping in mind that the potential closing of a post office is very difficult to overcome if the alternative solutions are not available.



Figure 8. The result of ranking the criteria importance.

The third-ranked criterion is C3—service quality. It involves attributes such as the efficiency, quality of the provided postal services or waiting time in the line. Unlike the previous two criteria, which can be considered as external or non-dependent from the efforts of managers in the postal branch, C3 is mostly dependent. Therefore, these managers should be aware of the relatively high impact of their business efforts on the long-term existence of their postal branch. The social value, Criterion 1, relates to the number of inhabitants that belong to vulnerable groups. This should remind the policymakers that post offices in rural areas are important for more than just providing postal services. Indeed, they can be considered as entities supporting vulnerable groups in different ways, from providing financial aid and representing a medium for supplying basic necessities to acting as the main pillar of social life and interconnection of these people with the community.

The fifth-ranked criterion is C5—infrastructure, in terms of mobile and Internet network coverage. Developed infrastructure can replace certain needs for postal services, however, not in total, especially as it cannot replace the post office as a unit of social value.

The sixth-ranked criterion is Criterion 7—the postal network unit arrangement. This criterion includes the attributes: interior and exterior of the post office, appropriate working hours, range of services, and easiness of access. The last-ranked criterion, C4, is closely related to the previous one, since both are internal criteria, depending mostly on the managers and employees in the postal branch. C4 involves the expertise and kindness of employees.

4.2. The Results of a Type-2 Fuzzy AROMAN Method

The procedure of type-2 fuzzy AROMAN starts by interviewing experts about the values considering each criterion, per each considered alternative. In this case study, we collected the answers as shown in Table 6, in this way forming the initial decision-making matrix. Then, the linguistic inputs are converted into type-2 fuzzy numbers following the rules presented in Table 2. The results are averaged for all three experts and presented in Table 7.

Table 6.	The	ratings	of al	ternatives.
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Criteria	Alternatives	Experts		
Criteria		E1	E2	E3
C1	A1	М	MH	MH
	A2	MH	Н	Н
	A3	М	ML	М
	A4	VH	Н	VH
	A5	L	ML	ML
	A6	VL	L	L
C2	A1	L	ML	ML
	A2	Μ	ML	М
	A3	MH	М	М
	A4	Н	Н	VH
	A5	ML	ML	ML
	A6	MH	М	MH
C3	A1	L	L	ML
	A2	VH	Н	Н
	A3	VH	Н	VH
	A4	Н	Н	MH
	A5	ML	Μ	М
	A6	MH	М	М
C4	A1	L	L	ML
	A2	Н	Н	MH
	A3	Н	Н	MH
	A4	VH	Н	Н
	A5	Н	Н	MH
	A6	L	L	ML
C5	A1	ML	М	М
	A2	MH	MH	М
	A3	ML	М	М
	A4	Н	MH	Н
	A5	ML	М	М
	A6	MH	М	М
C6	A1	MH	М	М
	A2	М	MH	М
	A3	ML	М	М
	A4	MH	MH	Н
	A5	ML	М	М
	A6	ML	М	ML
C7	A1	Μ	MH	MH
	A2	Μ	MH	Μ
	A3	ML	М	Μ
	A4	MH	Н	Н
	A5	Μ	ML	Μ
	A6	ML	М	М

Criteria	Alternatives	Experts (Average)
C1	A1	(4.33, 6.33, 6.33, 8.33; 1, 1), (5.33, 6.33, 6.33, 7.33; 0.9, 0.9)
	A2	(6.33, 8.33, 8.33, 9.67; 1, 1), (7.33, 8.33, 8.33, 9.00; 0.9, 0.9)
	A3	(2.33, 4.33, 4.33, 6.33; 1, 1), (3.33, 4.33, 4.33, 5.33; 0.9, 0.9)
	A4	(8.33, 9.67, 9.67, 10.00; 1, 1), (9.00, 9.67, 9.67, 9.83; 0.9, 0.9)
	A5	(0.67, 2.33, 2.33, 4.33; 1, 1), (1.50, 2.33, 2.33, 3.33; 0.9, 0.9)
	A6	(0.00, 0.67, 0.67, 2.33; 1, 1), (0.33, 0.67, 0.67, 1.50; 0.9, 0.9)
C2	A1	(0.67, 2.33, 2.33, 4.33; 1, 1), (1.50, 2.33, 2.33, 3.33; 0.9, 0.9)
	A2	(2.33, 4.33, 4.33, 6.33; 1, 1), (3.33, 4.33, 4.33, 5.33; 0.9, 0.9)
	A3	(3.67, 5.67, 5.67, 7.67; 1, 1), (4.67, 5.67, 5.67, 6.67; 0.9, 0.9)
	A4	(7.67, 9.33, 9.33, 10.00; 1, 1), (8.50, 9.33, 9.33, 9.67; 0.9, 0.9)
	A5	(1.00, 3.00, 3.00, 5.00; 1, 1), (2.00, 3.00, 3.00, 4.00; 0.9, 0.9)
	A6	(4.33, 6.33, 6.33, 8.33; 1, 1), (5.33, 6.33, 6.33, 7.33; 0.9, 0.9)
C3	A1	(0.33, 1.67, 1.67, 3.67; 1, 1), (1.00, 1.67, 1.67, 2.67; 0.9, 0.9)
	A2	(7.67, 9.33, 9.33, 10.00; 1, 1), (8.50, 9.33, 9.33, 9.67; 0.9, 0.9)
	A3	(8.33, 9.67, 9.67, 10.00; 1, 1), (9.00, 9.67, 9.67, 9.83; 0.9, 0.9)
	A4	(6.33, 8.33, 8.33, 9.67; 1, 1), (7.33, 8.33, 8.33, 9.00; 0.9, 0.9)
	A5	(2.33, 4.33, 4.33, 6.33; 1, 1), (3.33, 4.33, 4.33, 5.33; 0.9, 0.9)
	A6	(3.67, 5.67, 5.67, 7.67; 1, 1), (4.67, 5.67, 5.67, 6.67; 0.9, 0.9)
C4	A1	(0.33, 1.67, 1.67, 3.67; 1, 1), (1.00, 1.67, 1.67, 2.67; 0.9, 0.9)
	A2	(6.33, 8.33, 8.33, 9.67; 1, 1), (7.33, 8.33, 8.33, 9.00; 0.9, 0.9)
	A3	(6.33, 8.33, 8.33, 9.67; 1, 1), (7.33, 8.33, 8.33, 9.00; 0.9, 0.9)
	A4	(7.67, 9.33, 9.33, 10.00; 1, 1), (8.50, 9.33, 9.33, 9.67; 0.9, 0.9)
	A5	(6.33, 8.33, 8.33, 9.67; 1, 1), (7.33, 8.33, 8.33, 9.00; 0.9, 0.9)
6-	A6	(0.33, 1.67, 1.67, 3.67; 1, 1), (1.00, 1.67, 1.67, 2.67; 0.9, 0.9)
C5	Al	(2.33, 4.33, 4.33, 6.33; 1, 1), (3.33, 4.33, 4.33, 5.33; 0.9, 0.9)
	A2	(4.33, 6.33, 6.33, 8.33; 1, 1), (5.33, 6.33, 6.33, 7.33; 0.9, 0.9)
	A3	(2.33, 4.33, 4.33, 6.33; 1, 1), (3.33, 4.33, 4.33, 5.33; 0.9, 0.9)
	A4	(6.33, 8.33, 8.33, 9.67; 1, 1), (7.33, 8.33, 8.33, 9.00; 0.9, 0.9)
	A5	(2.53, 4.53, 4.53, 0.53; 1, 1), (3.53, 4.53, 4.53, 5.53; 0.9, 0.9)
<u>C</u> 6	A0 A1	(5.07, 5.07, 5.07, 7.07, 1, 1), (4.07, 5.07, 5.07, 0.07, 0.9, 0.9)
0	AI A2	(3.07, 5.07, 5.07, 7.07, 1, 1), (4.07, 5.07, 5.07, 0.07, 0.9, 0.9)
	AZ A 2	(5.07, 5.07, 5.07, 7.07, 1, 1), (4.07, 5.07, 5.07, 0.07, 0.9, 0.9)
	A3 A4	(2.33, 4.33, 4.33, 0.33, 1, 1), (3.33, 4.33, 4.33, 5.33, 0.9, 0.9)
	A4 A5	(3.07, 7.07, 7.07, 9.33, 1, 1), (0.07, 7.07, 7.07, 0.30, 0.9, 0.9)
	A5 A6	(2.55, 4.55, 4.55, 0.55, 1, 1), (3.55, 4.55, 4.55, 0.55, 0.5, 0.7, 0.7)
C7	Α0 Δ1	(1.07, 5.07, 5.07, 5.07, 1, 1), (2.07, 5.07, 5.07, 4.07, 0.9, 0.9)
C/	Δ2	(3.67 5.67 5.67 7.67 1 1) (4.67 5.67 5.67 6.70 0.9)
	A3	(2 33 4 33 4 33 6 33 1 1) (3 33 4 33 4 33 5 33 0 0 0)
	A4	(6.33, 8.33, 8.33, 9.67, 1, 1) $(7.33, 8.33, 8.33, 9.00, 0.9, 0.9)$
	A5	(2.33, 4.33, 4.33, 6.33; 1, 1), (3.33, 4.33, 4.33, 5.33; 0.9, 0.9)
	A6	(2.33, 4.33, 4.33, 6.33; 1, 1), (3.33, 4.33, 4.33, 5.33; 0.9, 0.9)

 Table 7. The type-2 fuzzy decision matrix.

The results of Steps 2, 3, and 4 will not be displayed here to keep the length of the article reasonable. However, the results of the weighted aggregated normalized decision-making matrix that need to be calculated in Step 5 are shown in Table 8.

Table 8. The weighted	aggregated normal	lized type-2 fuzzy	decision matrix.

Criteria	Alternatives	Type-2 Fuzzy Numbers—Average Experts' Answers
C1	A1	(0.03, 0.04, 0.04, 0.06; 1, 1), (0.03, 0.04, 0.04, 0.05; 0.9, 0.9)
	A2	(0.04, 0.06, 0.06, 0.07; 1, 1), (0.05, 0.06, 0.06, 0.06; 0.9, 0.9)
	A3	(0.02, 0.03, 0.03, 0.04; 1, 1), (0.02, 0.03, 0.03, 0.04; 0.9, 0.9)
	A4	(0.05, 0.06, 0.06, 0.07; 1, 1), (0.06, 0.07, 0.07, 0.07; 0.9, 0.9)
	A5	(0.00, 0.02, 0.02, 0.03; 1, 1), (0.01, 0.01, 0.01, 0.02; 0.9, 0.9)
	A6	(0.00, 0.00, 0.00, 0.02; 1, 1), (0.00, 0.00, 0.00, 0.01; 0.9, 0.9)

Criteria	Alternatives	Type-2 Fuzzy Numbers—Average Experts' Answers
C2	A1	(0.00, 0.02, 0.02, 0.06; 1, 1), (0.00, 0.01, 0.01, 0.03; 0.9, 0.9)
	A2	(0.02, 0.05, 0.05, 0.09; 1, 1), (0.03, 0.05, 0.05, 0.07; 0.9, 0.9)
	A3	(0.04, 0.07, 0.07, 0.11; 1, 1), (0.05, 0.07, 0.07, 0.09; 0.9, 0.9)
	A4	(0.10, 0.13, 0.13, 0.14; 1, 1), (0.11, 0.13, 0.13, 0.14; 0.9, 0.9)
	A5	(0.01, 0.03, 0.03, 0.07; 1, 1), (0.01, 0.03, 0.03, 0.04; 0.9, 0.9)
	A6	(0.05, 0.08, 0.08, 0.12; 1, 1), (0.06, 0.08, 0.08, 0.10; 0.9, 0.9)
C3	A1	(0.00, 0.01, 0.01, 0.02; 1, 1), (0.00, 0.01, 0.01, 0.01; 0.9, 0.9)
	A2	(0.05, 0.06, 0.06, 0.07; 1, 1), (0.06, 0.06, 0.06, 0.07; 0.9, 0.9)
	A3	(0.05, 0.06, 0.06, 0.07; 1, 1), (0.06, 0.07, 0.07, 0.07; 0.9, 0.9)
	A4	(0.04, 0.06, 0.06, 0.07; 1, 1), (0.05, 0.06, 0.06, 0.06; 0.9, 0.9)
	A5	(0.01, 0.03, 0.03, 0.04; 1, 1), (0.02, 0.03, 0.03, 0.03; 0.9, 0.9)
	A6	(0.02, 0.04, 0.04, 0.05; 1, 1), (0.03, 0.04, 0.04, 0.04; 0.9, 0.9)
C4	A1	(0.00, 0.01, 0.01, 0.01; 1, 1), (0.00, 0.00, 0.00, 0.01; 0.9, 0.9)
	A2	(0.02, 0.03, 0.03, 0.04; 1, 1), (0.03, 0.03, 0.03, 0.04; 0.9, 0.9)
	A3	(0.02, 0.03, 0.03, 0.04; 1, 1), (0.03, 0.03, 0.03, 0.04; 0.9, 0.9)
	A4	(0.03, 0.04, 0.04, 0.04; 1, 1), (0.03, 0.04, 0.04, 0.04; 0.9, 0.9)
	A5	(0.02, 0.03, 0.03, 0.04; 1, 1), (0.03, 0.03, 0.03, 0.04; 0.9, 0.9)
	A6	(0.00, 0.01, 0.01, 0.01; 1, 1), (0.00, 0.00, 0.00, 0.01; 0.9, 0.9)
C5	A1	(0.00, 0.01, 0.01, 0.03; 1, 1), (0.00, 0.01, 0.01, 0.02; 0.9, 0.9)
	A2	(0.01, 0.03, 0.03, 0.04; 1, 1), (0.02, 0.03, 0.03, 0.04; 0.9, 0.9)
	A3	(0.00, 0.01, 0.01, 0.03; 1, 1), (0.00, 0.01, 0.01, 0.02; 0.9, 0.9)
	A4	(0.03, 0.04, 0.04, 0.05; 1, 1), (0.03, 0.04, 0.04, 0.05; 0.9, 0.9)
	A5	(0.00, 0.01, 0.01, 0.03; 1, 1), (0.00, 0.01, 0.01, 0.02; 0.9, 0.9)
	A6	(0.01, 0.02, 0.02, 0.04; 1, 1), (0.01, 0.02, 0.02, 0.03; 0.9, 0.9)
C6	A1	(0.03, 0.07, 0.07, 0.11; 1, 1), (0.04, 0.07, 0.07, 0.09; 0.9, 0.9)
	A2	(0.03, 0.07, 0.07, 0.11; 1, 1), (0.04, 0.07, 0.07, 0.09; 0.9, 0.9)
	A3	(0.01, 0.04, 0.04, 0.08; 1, 1), (0.02, 0.04, 0.04, 0.06; 0.9, 0.9)
	A4	(0.07, 0.10, 0.10, 0.14; 1, 1), (0.09, 0.11, 0.11, 0.13; 0.9, 0.9)
	A5	(0.01, 0.04, 0.04, 0.08; 1, 1), (0.02, 0.04, 0.04, 0.06; 0.9, 0.9)
	A6	(0.00, 0.03, 0.03, 0.07; 1, 1), (0.00, 0.02, 0.02, 0.05; 0.9, 0.9)
C7	A1	(0.01, 0.02, 0.02, 0.04; 1, 1), (0.01, 0.02, 0.02, 0.03; 0.9, 0.9)
	A2	(0.01, 0.02, 0.02, 0.03; 1, 1), (0.01, 0.02, 0.02, 0.03; 0.9, 0.9)
	A3	(0.00, 0.01, 0.01, 0.02; 1, 1), (0.00, 0.01, 0.01, 0.02; 0.9, 0.9)
	A4	(0.02, 0.03, 0.03, 0.04; 1, 1), (0.03, 0.04, 0.04, 0.04; 0.9, 0.9)
	A5	(0.00, 0.01, 0.01, 0.02; 1, 1), (0.00, 0.01, 0.01, 0.02; 0.9, 0.9)
	A6	(0.00, 0.01, 0.01, 0.02; 1, 1), (0.00, 0.01, 0.01, 0.02; 0.9, 0.9)

Table 8. Cont.

In Step 6, the summation of the weighted aggregated normalized type-2 fuzzy decisionmaking matrix should be carried out per the criterion type. In our case, the min type criteria are C5 and C6, while the max type criteria are C1, C2, C3, C4, and C7. The results are shown in Table 9.

Table 9. Summation of weighted type-2 fuzzy decision matrix per the criteria type.

	$\widetilde{\widetilde{L}}_i$	$\widetilde{\widetilde{A}}_i$
A1	(0.03, 0.08, 0.08, 0.14; 1, 1), (0.04, 0.08, 0.08, 0.11; 0.9, 0.9)	(0.04, 0.10, 0.10, 0.19; 1, 1), (0.05, 0.09, 0.09, 0.13; 0.9, 0.9)
A2	(0.05, 0.09, 0.09, 0.15; 1, 1), (0.06, 0.09, 0.09, 0.13; 0.9, 0.9)	(0.15, 0.22, 0.22, 0.29; 1, 1), (0.17, 0.22, 0.22, 0.25; 0.9, 0.9)
A3	(0.01, 0.06, 0.06, 0.11; 1, 1), (0.02, 0.05, 0.05, 0.08; 0.9, 0.9)	(0.14, 0.21, 0.21, 0.28; 1, 1), (0.16, 0.20, 0.20, 0.24; 0.9, 0.9)
A4	(0.09, 0.14, 0.14, 0.19; 1, 1), (0.12, 0.15, 0.15, 0.18; 0.9, 0.9)	(0.25, 0.31, 0.31, 0.36; 1, 1), (0.28, 0.32, 0.32, 0.35; 0.9, 0.9)
A5	(0.01, 0.06, 0.06, 0.11; 1, 1), (0.02, 0.05, 0.05, 0.08; 0.9, 0.9)	(0.05, 0.12, 0.12, 0.20; 1, 1), (0.06, 0.10, 0.10, 0.15; 0.9, 0.9)
A6	(0.01, 0.06, 0.06, 0.11; 1, 1), (0.01, 0.04, 0.04, 0.08; 0.9, 0.9)	(0.08, 0.14, 0.14, 0.22; 1, 1), (0.09, 0.13, 0.13, 0.17; 0.9, 0.9)

In Step 7, we should raise the obtained values from Step 6 to the degree of λ . We calculated λ as explained in the methodological part, and in our case, $\lambda = 0.335$. The obtained results from Step 7 are in Table 10.

	$\widetilde{\widetilde{L}}_i$	$\stackrel{\approx}{A_i}^{\prime}$
A1	(0.32, 0.43, 0.43, 0.51; 1, 1), (0.35, 0.42, 0.42, 0.47; 0.9, 0.9)	(0.12, 0.22, 0.22, 0.33; 1, 1), (0.14, 0.20, 0.20, 0.26; 0.9, 0.9)
A2	(0.36, 0.45, 0.45, 0.53; 1, 1), (0.39, 0.45, 0.45, 0.50; 0.9, 0.9)	(0.28, 0.37, 0.37, 0.44; 1, 1), (0.31, 0.36, 0.36, 0.40; 0.9, 0.9)
A3	(0.23, 0.39, 0.39, 0.48; 1, 1), (0.25, 0.36, 0.36, 0.43; 0.9, 0.9)	(0.27, 0.35, 0.35, 0.43; 1, 1), (0.30, 0.35, 0.35, 0.39; 0.9, 0.9)
A4	(0.45, 0.52, 0.52, 0.57; 1, 1), (0.49, 0.53, 0.53, 0.56; 0.9, 0.9)	(0.39, 0.46, 0.46, 0.50; 1, 1), (0.43, 0.47, 0.47, 0.49; 0.9, 0.9)
A5	(0.23, 0.39, 0.39, 0.48; 1, 1), (0.25, 0.36, 0.36, 0.43; 0.9, 0.9)	(0.13, 0.24, 0.24, 0.34; 1, 1), (0.16, 0.22, 0.22, 0.28; 0.9, 0.9)
A6	(0.21, 0.38, 0.38, 0.48; 1, 1), (0.23, 0.35, 0.35, 0.42; 0.9, 0.9)	(0.18, 0.27, 0.27, 0.37; 1, 1), (0.20, 0.26, 0.26, 0.31; 0.9, 0.9)

Table 10. The results of Step 7 of the type-2 fuzzy AROMAN method.

To achieve the final result, we should first calculate the ranking value of trapezoidal interval type-2 fuzzy sets. The following is obtained:

$$Rank(\tilde{A}_{1} - \tilde{L}_{1}) = 244; Rank(\tilde{A}_{2} - \tilde{L}_{2}) = 317; Rank(\tilde{A}_{3} - \tilde{L}_{3}) = 336;$$
$$Rank(\tilde{A}_{4} - \tilde{L}_{4}) = 340; Rank(\tilde{A}_{5} - \tilde{L}_{5}) = 291; Rank(\tilde{A}_{6} - \tilde{L}_{6}) = 314$$

Finally, the ranking of considered alternatives is shown in Table 11. As can be noticed, the alternative A4 achieved the best score. On the other hand, A1 is the postal branch with the lowest rank, which means that it should be reorganized first to optimize the postal network.

Table	11.	Final	ranking	ς.
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Alternatives	R _i
A1	11.52
A2	23.74
A3	28.92
A4	29.87
A5	18.28
A6	23.18

5. Discussion

The discussion section is related to three phenomena. The first relates to the sensitivity analysis based on different defuzzification approaches in the interval type-2 fuzzy AROMAN, the second to the calculation of computational complexity of the proposed algorithm, and the final to the possible approaches for postal network reorganization.

5.1. Sensitivity Analysis Based on Different Defuzzification Approaches

In the proposed methodology, we used the defuzzification approach proposed by Lee and Chen [57]. However, to examine the stability of the obtained results, we implement a different defuzzification approach in this subsection. It is based on the calculation of the centroid of a type-2 fuzzy set, as proposed by Karnik and Mendel [58]. The newly obtained results are shown in Table 12. It can be concluded that the obtained results are stable, meaning that the order of alternatives is not changed by using different defuzzification approaches.

	$\stackrel{\approx}{A}_{i}^{h}-\stackrel{\approx}{L}_{i}^{h}$	The Centroid of Type-2 Fuzzy Number (µ _A ,x)	New Ranking Values	Previous Ranking Values
A1	(-0.39, -0.20, -0.20, 0.00; 1, 1), (-0.33, -0.22, -0.22, -0.09; 0.9, 0.9)	-0.18, 0.37	0.93	11.52
A2	(-0.25, -0.08, -0.08, 0.08; 1, 1), (-0.18, -0.08, -0.08, 0.00; 0.9, 0.9)	-0.08, 0.37	0.97	23.74
A3	(-0.21, -0.03, -0.03, 0.20; 1, 1), (-0.13, -0.01, -0.01, 0.13; 0.9, 0.9)	-0.03, 0.38	0.98	28.92
A4	(-0.18, -0.05, -0.05, 0.05; 1, 1), (-0.13, -0.06, -0.06, 0.00; 0.9, 0.9)	-0.05, 0.37	0.98	29.87
A5	(-0.35, -0.14, -0.14, 0.12; 1, 1), (-0.27, -0.13, -0.13, 0.03; 0.9, 0.9)	-0.13, 0.38	0.95	18.28
A6	(-0.30, -0.11, -0.11, 0.15; 1, 1), (-0.22, -0.09, -0.09, 0.08; 0.9, 0.9)	-0.09, 0.38	0.96	23.18

Table 12.	Final r	anking	based	on the	centroid	defuzz	ification	appi	coaches.

5.2. Computational Complexity

The computational complexity of the MCDM methods can be evaluated by considering the time complexity—T [59]. The parameter T should be determined inside the calculations by considering the number of augmentations. Several examples are offered here to compare the complexity of the *AROMAN* approach to some other MCDM methods. Considering that there are c criteria and p alternatives, the complexity of the *AHP* approach can be calculated as follows [59]:

$$T_{AHP} = c(c+1) + p(c+1) + pc$$
(26)

The time complexity of the *TOPSIS* technique is computed by the following equation [59]:

$$T_{TOPSIS} = pc + pc + p(c+1) + p(c+1) + p = 4pc + 3p$$
(27)

Further, when it comes to the *VIKOR* approach, the time complexity is expressed through the following equation [59]:

$$T_{VIKOR} = 3c + 3p + 4 \tag{28}$$

Finally, the *AROMAN* method requires *pc* operations to compute normalization No. 1, and an additional *pc* for normalization No. 2. Further, *pc* operations are needed for aggregated normalization and the same number for weighted normalized matrix. For a summation of a weighted aggregated normalized matrix per the criteria type, there are *c* operations, while for raising the obtained values to the degree of λ , there are 2*p* more. To calculate the final ranking, there are additional *p* operations. The explained calculations can be expressed by Equation (29).

$$T_{AROMAN} = 4pc + c + 2p + p = 4pc + 3p + c$$
 (29)

In the concrete case, we considered seven criteria and six alternatives; thus, the computational complexity per selected MCDM methods are the following: $T_{AHP} = 146$, $T_{TOPSIS} = 186$, $T_{VIKOR} = 43$ and $T_{AROMAN} = 193$. Figure 9 shows the computational complexity of four considered approaches.



Figure 9. The computational complexity of selected MCDM techniques.

5.3. Possible Directions for Postal Network Reorganization

The options for the reorganization of the inefficient postal branches can be different. First, since the post office in a certain place can be considered to be of general interest to many stakeholders, people living in the community, business entities and state administration, then a possible solution to keep the postal branch is to provide additional funds for its functioning. These funds can be collected from postal service users or municipalities. Another option is to change the form of the postal branch. For example, it is possible to introduce a mobile post office (Figure 10), a vehicle transformed into a post office that spends a certain time parked in predefined points of interest. As can be concluded, the question of the reorganization of the inefficient postal branches can also be considered as an MCDM problem, and solving it may be a good possible direction for future research.



Figure 10. An example of a mobile post office [60].

6. Conclusions

The problem of the optimization of the postal network is very complex, keeping in mind that the interests of many stakeholders should be satisfied. The process of decision-making considering locations and the existence of the postal branch should be considered from multiple aspects, not just from the standpoint of achieved profit. Various criteria that are considered in this field are often described by uncertain or imprecise data. As a

solution, we proposed an interval type-2 fuzzy AROMAN method to be implemented by policymakers and industry managers to optimize the postal network.

This study provides several contributions. Through an extensive literature review, we recognized the attributes that are important for the optimization of the postal network. Then, we grouped them into seven clusters by their similarity. The next step was to interview the experts to determine the relevance ranks of the set criteria, which was achieved using the FUCOM method. In the end, for the first time in the literature, we implemented the interval type-2 fuzzy AROMAN method. By solving a real-life numerical example, we confirmed the applicability of the proposed model. We demonstrated its implementation in the postal industry; however, it should be noted that the proposed model is general and can be used for various types of MCDM problems.

However, the model also has certain limitations. First, the procedure of the AROMAN model implies the existence of both criteria to be minimized and others to be maximized. This limitation is relatively easy to overcome in real-life problems, keeping in mind that complex phenomena are almost always described by both types of criteria. Further, the limitation of this case study can be related to a restricted group of interviewed experts that are from the postal industry. Further research can be directed toward interviewing different stakeholders, for example, people living in the community or representatives of local authorities. In addition, the proposed methodology can be expanded by using discrete type-2 fuzzy numbers or intuitionistic, Pythagorean or picture fuzzy numbers.

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