Analysis of a Collision-Affected M/GI/1/N Retrial Queuing System Considering Negative Customers and Transmission Errors

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Abstract: This paper considers a retrial G-queue with collisions, transmission errors, and a finite number of sources, where service and repair time are both general distributions. The number of sources (terminals) is finite and a source cannot generate new requests until the channel (server) finishes its work, i.e., the rate at which new primary requests are generated varies inversely with the number of data frame (customer) in the system. A collision occurs when service requests arrive at a busy channel, and transmission errors prevent data frames from leaving the system after completing service. Two types of arrivals are considered. Negative customers will break down the system in the busy state and remove the customer under service. The application of our model is indicated, with a particular emphasis on communication networks such as the local-area networks (LAN) with CSMA/CD protocol. Recursive formulas have been derived to calculate the stationary joint distributions and the Laplace transform of reliability function by applying the discrete transformations method along with the supplementary variables technique (SVT). Furthermore, the comparative performance and reliability analysis have been conducted numerically. Numerical examples are provided to investigate the sensitivity of different parameters on performance measures and reliability indicators.

Keywords: retrial G-queue; finite source; collisions; transmission errors; reliability indices

MSC: 60K25; 68M20; 90B22

1. Introduction

Retrial queues, with the characteristic feature that concerns the behavior of data retransmission, are competent for modeling many computer systems, where computers compete for service from a central processing server, etc. And also for many telecommunication systems, including telephone switching systems, telecommunication networks, cellular and local area networks, call centers, wireless communication systems, etc. When the node fails to transmit the data frame, it will wait for a random length of time and try to transmit again. The failed attempts will enter a waiting pool, also known as the orbit. In order to systematically give an overview of the basic techniques and the results of this topic up to date, we refer the interested reader to, for instance [1–4].

In recent years, with the rapid development of information technology, the research of retrial systems in computer networks and communication systems has grown gradually. These systems may encounter several challenges like collisions, data loss, retransmission, transmission errors, and server failures. For example, in computer networks where processors are connected to a central transmission unit. If the medium is available, the central
unit immediately transmits the message. Otherwise, messages are stored in the buffer for retry. Such computer systems may suffer from viruses, which will destroy the data being transmitted (negative arrivals), and also the mechanical parts may be crashed, which must be repaired. Another example exists in wireless data networks with the CSMA/CA (Carrier Sense Multiple Access With Collision-Avoidance) protocol. Before sending packets, the station is asked to sense the state of the channel. If the channel is free and the station has received the CTS (Clean-To-Send) message, it starts to transmit packets; otherwise, it generates repeated attempts for retrying. The transmitting station initiates communication by sending an RTS (Request-to-Send) signal to the receiving station. In response, the receiving station acknowledges the request by emitting a CTS signal. The CTS is not received due to either the occupied channel or a transmission error. The transmission of the packet is contingent upon the successful completion of the RTS/CTS exchange. The impact of these factors on the performance, reliability, and availability of the system cannot be ignored. This motivates us to propose a stochastic model that is compatible with realistic applications and takes into account all the above aspects. Considering that the number of terminals in a data network is often limited, we propose a finite source retrial queuing system to analyze the random access to the channel in wireless communication networks and computer systems. Utilizing the aforementioned model, an analytical investigation has been conducted to evaluate the performance and reliability of the system. Furthermore, it is worth noting that this model has not been investigated in previous research, thus rendering it a topic of theoretical significance.

In numerous cases involving data transmission, the uncoordinated transmission usually happens; data from diverse sources simultaneously send transmission requests to a channel or a service facility. Due to the limited service capability of the channel, this situation results in conflicts, leading to message delays. Nowadays, data collision is still a significant concern for communication networks. Nevertheless, although many protocols have been proposed, collisions cannot be wholly avoided; see [5–7]. Within queuing theory, retrial queues with collisions are often applied to model such a phenomenon. Furthermore, retrial queuing systems with collisions are proposed and generally utilized in modeling many practical problems arising in magnetic disk memory systems, cellular mobile networks, and wireless local-area networks with the CSMA/CA (Carrier Sense Multiple Access With Collision-Avoidance) protocol. The CSMA/CA protocol requires stations to monitor the channel before sending data. The station only sends data when it detects that the channel is free. But if two stations simultaneously find that the channel is free and both start to transmit data, it will almost immediately cause a conflict. Falin [8] investigated a retrial queue with collisions called double connections. Choi [9] extended this model to a more specific situation, i.e., unslotted CSMA/CD protocol. Wu [10] considered this model with preemptive priority and collisions under the discrete-time condition. Jailaxmi [11] assumed a general distributed service time and introduced collisions and modified M vacations in the model. Phung-Duc [12] studied a retrial queue with collisions where the transmission of packets is divided into two phases. Transmission errors exist in many communication systems. It may occur when a wrong packet is transmitted to the channel, or the conditions of the channel are not up to standard. This situation has been taken into account in [13].

The multiple access system is divided into two layers according to operation and source: the access and communication layer. During data transmission, attempts may fail in the access layer due to collisions and cause collision loss. However, data that successfully pass through the access layer may still be retransmitted or lost due to the unavailability of channels or service facilities, see [14]. The service is interrupted by the server breakdown, which can be modeled by retrial queues with server breakdowns. Rajadura [15] investigated an unreliable M/G/1 retrial queue with feedback and vacations. Melikov [16] developed an approximate method based on the space merging approach to examine unreliable multi-server retrial queues with delayed feedback. Lakaour [17] considered the transmission errors and collisions in an unreliable retrial queuing system. Yiming [18] studied the
asymptotic behavior of an M/G/1 retrial queuing system with server breakdowns. Most related papers on unreliable retrial queues with collisions barely take finite sources into consideration. However, the assumption of unlimited sources in these articles is not quite realistic. For instance, the number of terminals is finite in wireless data networks in which the CSMA/CA protocol is used logically. Meanwhile, queues with finite sources can also reflect the characteristic, i.e., the rate that new primary requests are generated varies inversely with the number of customers within the queuing system. Dragieva [19] have considered two types of calls: incoming calls and outgoing calls. Following the discrete transformations method, formulas for the stationary probabilities were derived. Bérczes [20] investigate the performance of an unreliable finite source retrial queue by software MOSEL-2. Nazarov [21–23] conduct an asymptotic analysis of unreliable finite source retrial queue with collisions. Although the introduction of finite sources brings the proposed model closer to reality, it also increases the complexity of the study. So there are still very few related studies, and most of them used software, see [20] or other approximate methods, see [21–23] to study this kind of queuing model.

The concept ‘server breakdowns’ in the above research represents that the server reaches its lifetime, but in fact, the server may suffer many accidents, which will cause it to break down before it reaches its lifetime. For example, viruses or commands from the outside will also interrupt the transmission, and random loss of data will also occur simultaneously. This kind of behavior can be described by retrial G-queues, first introduced by [24], where the arrival of negative customers (packets) will result in an unavailable server and the loss of customers (packets). Retrial queues with negative customers are widely used in modeling the design and control of packet-switching networks. Peng [25] studied an unreliable M/G/1 retrial queue under collisions, preemptive priority, and delayed repairs. Nesrine [26] investigated negative arrivals on multi-server retrial queues. Then, they extend their work into finite service capacity and exponential abandonment in [27]. Upadhyaya [28] investigated a general service retrial queue with negative arrivals and declared its applications in cognitive radio (CR) networks. Rajadurai [29] investigated the performance of a repairable retrial G-queue with feedback and general distributed service times. Singh [30] took optional service and delayed repair into consideration. Lisovskaya [31] explored negative customers in a retrial queue with two orbits.

Systems that suffer from random breakdowns will result in a serious negative effect on the service quality in almost all fields including computer and communication systems, manufacturing, and production processes. In today’s era of information explosions, communication systems depend on correct and timely information transmission and specific quality of service, even if failures occur from time to time. In the retrial queuing system with an unreliable server, the reliability measures provide the information, which is required for the improvement of the system. Therefore, it is imperatively vital to investigate the reliability of the finite source retrial queue with unreliable servers. Wang [32] gave a detailed reliability analysis of unreliable retrial queuing systems. In the retrial queuing system with an unreliable server, the reliability measures provide the information, which is required for the improvement of the system. Zirem [33] dealt with a batch arrivals queue with general retrial time, breakdowns, repairs, and reserved time. They calculated the availability (AV) and the failure frequency of the server (Wf). However, a detailed numerical analysis of the reliability metrics is missing. Gao [34] investigated an M/G/1 retrial queue with two types of breakdowns. A reliability analysis was conducted. Abdollahi [35] studied reliability and sensitivity analysis of retrial queue with optional k-phases services, vacation, and feedback. They only examined one of the reliability indices, i.e., the steady-state availability (AV). In order to study the unreliable retrial systems with a finite number of sources and multiple servers, Gharbi [36] applied the generalized stochastic petri nets model. Multi-server queuing systems have a background in many fields of applications, but the inclusion of collisions and feedback makes the study very complex and requires more settings to be considered. For example, which server to go to for newly arrived requests and how collisions occur, etc. We will do further study on multi-server retrial systems with collisions in the future,
whereas, in this article, we will only consider a single server. For all we know, not much work is found in analyzing the reliability of retrial queues with collisions and finite sources in the literature. Although [20–23] have conducted an analysis of unreliable finite source retrial queues with collisions by software and the asymptotic method, the reliability and availability have not been derived. This motivates us to deal with such a queuing model in this paper.

The summarized review has been presented in Table 1; hence, the contribution of this study can be illustrated clearly. The notation * in this table indicates the corresponding models have considered collisions. P.G.F. and S.V.T. mean the probability-generating function and the supplementary technique.

The contributions of this work can be summarized as follows:

- To conclude our main contributions made in this paper, we emphasize that to the best of our knowledge, this is the first study to combine the above characteristics and conduct a reliability and availability analysis. Unlike previous studies that used software simulations or approximation methods, we employed the discrete transformation method to deal with Kolmogorov equations and derived recursive formulas for steady-state probabilities, providing another more intuitive form of solution for such queuing models.
- To better reflect practical scenarios in communication networks, this paper proposes a finite retrial G-queue with collisions and transmission errors, which takes into account the limited number of terminals, potential service interruptions (such as packet collisions and unavailable channels), packet loss, and the need for retransmission caused by erroneous data or unfavorable channel conditions. These features make the queuing model more complex but more suitable for practical application scenarios, such as wireless local area network (WLAN) systems using CSMA/CA protocols.
- The reliability indicators are derived by taking a new model into account and using the Laplace transforms. Reliability analysis of unreliable retrial queuing systems with collisions and finite sources is seldom found in existing research. Modern communication systems rely on accurate and timely information transmission, and service interruptions and failures can affect system performance and service quality. Therefore, the reliability analysis of the proposed model is also crucial.
- The sensitivities of different parameters on main performance and reliability indicators are investigated in the numerical section. We also compared the results with those of previous literature.

The structure of the subsequent content of this paper will be given in the following. Section 2 states the proposed model in a succinct expression. Section 3 regards the elapsed service and repair time as the supplementary variables while applying the supplementary variables technique (SVT). A series of Kolmogorov equations are derived, and we use the discrete-transformation method to obtain the stationary characteristics. In Sections 4 and 5, we calculate several main reliability and performance indices. In the end, some detailed numerical illustrations are carried out in the form of figures and tables. Furthermore, we compare our results with previous research and find some interesting phenomena. We first summarize the definitions of all parameters and variables covered in this paper in Table 2 for the reader to keep track.
Table 1. Comparison between the published retrial queuing models.

<table>
<thead>
<tr>
<th>References</th>
<th>Sources</th>
<th>Arrival</th>
<th>Service</th>
<th>Repair</th>
<th>Negative arr.</th>
<th>Feedback</th>
<th>Method</th>
<th>Reliability Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>* [9]</td>
<td>infinite</td>
<td>Poi</td>
<td>constant + general</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>PG.F</td>
<td>-</td>
</tr>
<tr>
<td>* [10]</td>
<td>infinite</td>
<td>Geo</td>
<td>General</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>PG.F</td>
<td>-</td>
</tr>
<tr>
<td>* [11]</td>
<td>infinite</td>
<td>Poi</td>
<td>General</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>PG.F</td>
<td>-</td>
</tr>
<tr>
<td>* [12]</td>
<td>infinite</td>
<td>Poi</td>
<td>Exp.</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>PG.F</td>
<td>-</td>
</tr>
<tr>
<td>* [13]</td>
<td>infinite</td>
<td>Poi</td>
<td>Exp.</td>
<td>-</td>
<td>✓</td>
<td>✓</td>
<td>PG.F</td>
<td>-</td>
</tr>
<tr>
<td>[15]</td>
<td>infinite</td>
<td>Poi</td>
<td>General</td>
<td>Exp.</td>
<td>-</td>
<td>-</td>
<td>S.V.T.</td>
<td>-</td>
</tr>
<tr>
<td>[16]</td>
<td>infinite</td>
<td>Poi</td>
<td>Exp.</td>
<td>Exp.</td>
<td>✓</td>
<td>✓</td>
<td>Approximation</td>
<td>-</td>
</tr>
<tr>
<td>* [17]</td>
<td>infinite</td>
<td>Poi</td>
<td>Exp.</td>
<td>Exp.</td>
<td>✓</td>
<td>✓</td>
<td>PG.F</td>
<td>-</td>
</tr>
<tr>
<td>[18]</td>
<td>infinite</td>
<td>Poi</td>
<td>General</td>
<td>Exp.</td>
<td>-</td>
<td>-</td>
<td>Asymptotic method</td>
<td>-</td>
</tr>
<tr>
<td>[19]</td>
<td>finite</td>
<td>Poi</td>
<td>General</td>
<td>Exp.</td>
<td>-</td>
<td>-</td>
<td>Discrete transformations</td>
<td>-</td>
</tr>
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<td>* [20]</td>
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<td>Poi</td>
<td>Exp.</td>
<td>Exp.</td>
<td>-</td>
<td>-</td>
<td>MOSEL</td>
<td>-</td>
</tr>
<tr>
<td>* [21]</td>
<td>finite</td>
<td>Poi</td>
<td>General</td>
<td>Exp.</td>
<td>-</td>
<td>-</td>
<td>S.V.T.</td>
<td>-</td>
</tr>
<tr>
<td>* [22]</td>
<td>finite</td>
<td>Poi</td>
<td>Exp.</td>
<td>Exp.</td>
<td>-</td>
<td>-</td>
<td>Asymptotic method</td>
<td>-</td>
</tr>
<tr>
<td>* [23]</td>
<td>finite</td>
<td>Poi</td>
<td>General</td>
<td>Exp.</td>
<td>-</td>
<td>-</td>
<td>Asymptotic method</td>
<td>-</td>
</tr>
<tr>
<td>[24]</td>
<td>infinite</td>
<td>Poi</td>
<td>Exp.</td>
<td>Exp.</td>
<td>-</td>
<td>-</td>
<td>Analytic method</td>
<td>-</td>
</tr>
<tr>
<td>* [25]</td>
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<td>Poi</td>
<td>General</td>
<td>Exp.</td>
<td>-</td>
<td>-</td>
<td>S.V.T.</td>
<td>-</td>
</tr>
<tr>
<td>* [26]</td>
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<td>Poi</td>
<td>Exp.</td>
<td>Exp.</td>
<td>✓</td>
<td>✓</td>
<td>Truncation</td>
<td>-</td>
</tr>
<tr>
<td>[27]</td>
<td>infinite</td>
<td>Poi</td>
<td>Exp.</td>
<td>Exp.</td>
<td>✓</td>
<td>✓</td>
<td>S.V.T.</td>
<td>AV, Wf</td>
</tr>
<tr>
<td>[28]</td>
<td>infinite</td>
<td>Poi</td>
<td>General</td>
<td>General</td>
<td>-</td>
<td>-</td>
<td>S.V.T.</td>
<td>AV, Wf</td>
</tr>
<tr>
<td>[29]</td>
<td>infinite</td>
<td>Poi</td>
<td>General</td>
<td>Exp.</td>
<td>✓</td>
<td>✓</td>
<td>S.V.T.</td>
<td>-</td>
</tr>
<tr>
<td>[31]</td>
<td>infinite</td>
<td>Poi</td>
<td>Exp.</td>
<td>Exp.</td>
<td>✓</td>
<td>✓</td>
<td>S.V.T.</td>
<td>Asymptotic method</td>
</tr>
<tr>
<td>[32]</td>
<td>infinite</td>
<td>Poi</td>
<td>General</td>
<td>General</td>
<td>-</td>
<td>-</td>
<td>S.V.T.</td>
<td>No numerical results</td>
</tr>
<tr>
<td>[33]</td>
<td>Compound</td>
<td>Poi</td>
<td>General</td>
<td>General</td>
<td>-</td>
<td>-</td>
<td>S.V.T.</td>
<td>No numerical results</td>
</tr>
<tr>
<td>[34]</td>
<td>infinite</td>
<td>Exp.</td>
<td>General</td>
<td>General</td>
<td>-</td>
<td>-</td>
<td>S.V.T.</td>
<td>AV, Wf, MTTF</td>
</tr>
<tr>
<td>[35]</td>
<td>infinite</td>
<td>Exp.</td>
<td>General</td>
<td>Exp.</td>
<td>✓</td>
<td>✓</td>
<td>S.V.T.</td>
<td>-</td>
</tr>
<tr>
<td>[36]</td>
<td>infinite</td>
<td>Exp.</td>
<td>Exp.</td>
<td>Exp.</td>
<td>-</td>
<td>-</td>
<td>Generalized stochastic petri nets</td>
<td>-</td>
</tr>
</tbody>
</table>

* our study finite Poi General General ✓ ✓ Discrete transformations AV, Wf, MTTF

* this indicates that the corresponding models have considered collisions.
Table 2. The parameters and variables used in this paper.

<table>
<thead>
<tr>
<th>Parameters and Variables</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>The number of sources</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>The total arrival rate of the primary requests</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>The total retrial rate</td>
</tr>
<tr>
<td>$p$</td>
<td>The probability of generating a positive request.</td>
</tr>
<tr>
<td>$\theta$</td>
<td>The probability of an arriving request which join the orbit alone after collisions</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>The probability of no transmission errors</td>
</tr>
<tr>
<td>$\mu^*$</td>
<td>The service rate of the server</td>
</tr>
<tr>
<td>$\beta^*$</td>
<td>The rate of repairing the server</td>
</tr>
<tr>
<td>$b(x)$</td>
<td>The hazard rate function of the service time</td>
</tr>
<tr>
<td>$B(x)$</td>
<td>The probability distribution function of the service time</td>
</tr>
<tr>
<td>$B(s)$</td>
<td>The Laplace–Stieltjes transform of the service time</td>
</tr>
<tr>
<td>$a(x)$</td>
<td>The hazard rate function of the repair time</td>
</tr>
<tr>
<td>$A(x)$</td>
<td>The probability distribution function of the repair time</td>
</tr>
<tr>
<td>$\tilde{A}(s)$</td>
<td>The Laplace–Stieltjes transform of the repair time</td>
</tr>
<tr>
<td>$\bar{\lambda}$</td>
<td>The effective arrival rate</td>
</tr>
<tr>
<td>$P_0, P_1, P_2$</td>
<td>The probability of idle state, busy state, repair state</td>
</tr>
<tr>
<td>$E(L), E(O)$</td>
<td>The expected number of packets in the system, in the orbit</td>
</tr>
<tr>
<td>$E(T), E(W)$</td>
<td>The mean response time of the system, the mean waiting time in the orbit</td>
</tr>
<tr>
<td>$E(S), E(\tau)$</td>
<td>The mean total service time, sojourn time in the source</td>
</tr>
<tr>
<td>$P_B$</td>
<td>The blocking probability</td>
</tr>
<tr>
<td>$AV$</td>
<td>The steady-state availability</td>
</tr>
<tr>
<td>$W_f$</td>
<td>The failure frequency</td>
</tr>
<tr>
<td>$R_Y(t)$</td>
<td>The reliability function of the server</td>
</tr>
<tr>
<td>MTTF</td>
<td>The mean time to the failure of the system</td>
</tr>
</tbody>
</table>

*This indicates that the definitions of the corresponding parameters are based on the premise of the exponential distribution.

2. Model Description

We present an M/G/1/N type queuing system with collisions, G-queue, and transmission errors. Two types of customers, positive and negative, are considered. Negative customers will affect the system only if the server is in a busy state. In other words, negative customers will disappear naturally if they arrive at the system in idle states or repair states. When a busy server is accessed by a negative customer, it will stop working and will be sent to repair immediately. At the same time, the customer being served will be taken away from the system by the negative customer. When the server (channel) is in an idle state, the arriving primary customer will receive service immediately. If a primary or retrial customer arrives at a busy server, a collision occurs under this circumstance. In this work, the transmission error is also under consideration, i.e., when the customer finishes his/her service, it may not be possible for the customer to leave the system. The following are the assumptions made for the mathematical formulation of the proposed retrial queuing system:

(i) There are $N$ sources generating the primary requests with rate $\lambda/N$, and $N$ is a finite number. A new request can only be generated after the customer under service completes the required service and exits the system. The probability of generating a positive and negative customer is $p$ and $1-p$, respectively.

(ii) The service time follows the general probability distribution function $B(x)$, Laplace–Stieltjes transform $\tilde{B}(s)$, and hazard rate function $b(x) = \frac{B'(x)}{1-B(x)}$.

(iii) The repair time follows the general distribution with probability distribution function $A(x)$, Laplace–Stieltjes transform $\tilde{A}(s)$, and hazard rate function $a(x) = \frac{A'(x)}{1-A(x)}$.

(iv) Two situations after collisions are considered here. When collisions happen, the transmission is interrupted. The arriving customer will join the orbit together with the
customer under service with probability $1 - \theta$ or enter into the orbit alone with probability $\theta$. After a time of random length, customers in orbit will generate a new service request. We assume the exponential distribution of retrial time. Considering the time interval $(t, t + dt)$, the probability of retrial is $\frac{\theta}{N} dt + o(dt)$ as $dt \to 0$. The classical retrial policy is considered here, in which all customers in orbit can generate service requests.

(v) Transmission errors can be caused by wrong packets or inappropriate channel conditions. The customer may be sent to the orbit due to transmission errors with probability $1 - \gamma$, even if the service is completed.

For the purpose of a better interpretation of the proposed model, the state transition diagram of the corresponding model under exponential distribution has been given in Figure 1.

We assume the random processes involved above are all independent of each other. The concept ‘customer’ mentioned can sometimes represent nodes, data, packets or information, etc.

Figure 1. The state transition diagram of the corresponding model under exponential assumption.

3. Analysis of Stationary Probability Distribution

For the purpose of deriving the equilibrium probabilities, the first step is to construct a Markov chain $\{X(t), t \geq 0\} = \{C(t), N(t), Z(t), t \geq 0\}$ by treating the elapsed service and repair time as the supplementary variables. Consider $C(t)$, represent the state of the server (channel) as follows

$$C(t) = \begin{cases} 
0, & \text{when the server is idle;} \\
1, & \text{when the server is busy;} \\
2, & \text{when the server is under repair.}
\end{cases}$$

Let $N(t)$ be the number of customers in the system at time $t$, and $Z(t)$ represent different time periods when the system is at different states, which is described more concisely as follows

$$Z(t) = \begin{cases} 
\text{elapsed service time, } C(t) = 1; \\
\text{elapsed repair time, } C(t) = 2.
\end{cases}$$

Now, we define the following probabilities:
\[ P_{0,i} = P\{C(t) = 0, N(t) = i\}, \quad 0 \leq i \leq N, \]
\[ P_{1,i}(x)dx = \lim_{t \to \infty} P\{C(t) = 1, N(t) = i, x < Z(t) < x + dx\}, \quad 1 \leq i \leq N, \]
\[ P_{2,i}(x)dx = \lim_{t \to \infty} P\{C(t) = 2, N(t) = i, x < Z(t) < x + dx\}, \quad 0 \leq i \leq N, \]
\[ P_{k,j} = \int_0^\infty P_{k,j}(x)dx, \quad 1 - \delta_{2,k} \leq i \leq N, k = 1,2 \]

As is stated in Section 2, consider there are \( N \) sources, which means the number of customers is up to \( N \) at any given time in the system. Therefore, the stationary regime always exists. By adopting the method of supplementary variables, equilibrium equations can be obtained as follows:

\[
\frac{dP_{1,i}(x)}{dx} = -\left(\frac{N - i}{N}\right)\lambda + b(x) + \frac{i - 1}{N} + \frac{N - i + 1}{N} p\lambda \theta P_{1,i-1}(x), \quad 1 \leq i \leq N, \tag{1}
\]
\[
\frac{dP_{2,i}(x)}{dx} = -\left(\frac{N - i}{N}\right)\lambda p + a(x)P_{2,i}(x) + \frac{N - i + 1}{N} \lambda p P_{2,i-1}(x), \quad 0 \leq i \leq N, \tag{2}
\]
\[
(\lambda p + (\sigma - \lambda p)\frac{i}{N})P_{0,i} = \int_0^\infty a(x)P_{2,i}(x)dx + \int_0^\infty \frac{N - i + 1}{N} p\lambda (1 - \theta) P_{1,i-1}(x)dx \\
+ \int_0^\infty \left(\frac{i - 1}{N} \sigma + b(x)(1 - \gamma)\right) P_{1,i}(x)dx + \int_0^\infty \gamma b(x) P_{1,i+1}(x)dx, \quad 0 \leq i \leq N, \tag{3}
\]

where \( P_{1,N-1}(x) = 0, P_{1,0}(x) = 0, P_{2,N-1}(x) = 0 \) and \( P_{1,N+1}(x) = 0 \). We can obtain the following boundary conditions of steady states:

\[
P_{1,i}(0) = \frac{i}{N}\sigma P_{0,i} + (1 - \frac{i - 1}{N})\lambda p P_{0,i-1}, 1 \leq i \leq N, \tag{4}
\]
\[
P_{2,i}(0) = \int_0^\infty \frac{N - i - 1}{N} (1 - p) \lambda P_{1,i+1}(x)dx, 0 \leq i \leq N - 1. \tag{5}
\]

Also, the normalization condition is given by

\[
\sum_{i=0}^N P_{0,i} + \sum_{i=1}^N \int_0^\infty P_{1,i}(x)dx + \sum_{i=0}^N \int_0^\infty P_{2,i}(x)dx = 1. \tag{6}
\]

It is a well-known conclusion in [37] that the method of discrete transformations is convenient in solving such differential equations. Discrete transformations is a technique with the feature replacing a set of unknown variables \( p = (p_0, p_1, \ldots, p_n) \) with a specific (linear) transformation of them. The most general form is the linear replacement \( q^T = Ap^T \), where \( A \) is the non-singular matrix, and \( q^T \) is called the image of \( p \). Equation (1) can be written as \( (A_1 I - B_1)P(x) = 0 \), where \( P(x) = (P_{1,1}(x), P_{1,2}(x), \ldots, P_{1,N}(x))^T \), and \( A_1 = N \frac{d}{dx} + N b(x) \). Now, we can construct \( B_1 \) from (1):

\[
B_1^{N \times N} = \begin{pmatrix}
-(N-1)\lambda & 0 & \cdots & 0 & 0 & 0 \\
(N-1)\lambda p\theta -(N-2)\lambda - \sigma & 0 & 0 & 0 & 0 & 0 \\
0 & (N-2)\lambda p\theta & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & 2\lambda p\theta \lambda - (N-2)\sigma & 0 & 0 \\
0 & 0 & \cdots & 0 & \lambda p\theta & -(N-1)\sigma
\end{pmatrix}
\]

We need to find matrix \( V_1 \), s.t. \( V_1^{-1}B_1V_1 = \Lambda_1 \), where \( \Lambda_1 \) is a diagonal matrix whose elements are the eigenvalues of matrix \( B_1 \). After some calculations, it is not difficult to obtain \( \Lambda_1 = \text{diag} \{- (N-1)\sigma, -\lambda - (N-2)\sigma, \cdots, -(N-2)\lambda - \sigma, -(N-1)\lambda\} \) and
\[
V_1^{N \times N} = \begin{pmatrix}
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 1 \\
0 & 0 & \cdots & (N-2) \frac{\lambda p \theta}{\sigma - \lambda} \\
\vdots & \vdots & \ddots & \vdots \\
1 & \frac{\lambda p \theta}{\sigma - \lambda} & \cdots & (N-3) \frac{\lambda p \theta}{\sigma - \lambda} \\
0 & 1 & \cdots & (N-2) \frac{\lambda p \theta}{\sigma - \lambda} \\
\end{pmatrix} (N-1) \frac{\lambda p \theta}{\sigma - \lambda}, \quad (N-2) \frac{\lambda p \theta}{\sigma - \lambda}, \quad (N-3) \frac{\lambda p \theta}{\sigma - \lambda}, \quad (N-2) \frac{\lambda p \theta}{\sigma - \lambda}, \quad (N-1) \frac{\lambda p \theta}{\sigma - \lambda}, \quad 1
\]

Thus, \( P_{1,j}(x) \) can be expressed by \( q_k^1(x) \):
\[
P_{1,j}(x) = \sum_{k=N-i+1}^{N} v_{j,k} q_k^1(x) = \sum_{k=N-i+1}^{N} \left( \frac{k-1}{N-i} \right) \left( \frac{\lambda p \theta}{\sigma - \lambda} \right)^{i+k-N-1} q_k^1(x).
\]

The method of discrete transformations is still effective in solving (2). (2) can be shortly written as \( (A_2 I - B_2) P_2(x) = 0 \), where \( P_2(x) = (P_{2,0}(x), P_{2,1}(x), \ldots, P_{2,N}(x))^T \), \( A_2 = N \frac{1}{\sigma} + N a(x) \) and
\[
B_2^{(N+1) \times (N+1)} = \begin{pmatrix}
-N \lambda p & 0 & \cdots & 0 & 0 & 0 \\
-N \lambda p & -(N-1) \lambda p & \cdots & 0 & 0 & 0 \\
0 & (N-1) \lambda p & -(N-2) \lambda p & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 2 \lambda p & -\lambda p & 0 \\
0 & 0 & \cdots & 0 & \lambda p & 0 \\
\end{pmatrix}
\]

After some regular calculations, matrix \( V_2 \) can be obtained, and its elements are as follows:
\[
v_{i,j}^2 = \begin{cases}
(-1)^{j-N+i} i^{-N-i}, & i + j \geq N, \\
0, & \text{otherwise,}
\end{cases}
\]

Thus, \( P_{2,j}(x) \) can be expressed by \( q_k^2(x) \):
\[
P_{2,j}(x) = \sum_{k=0}^{N} v_{j,k}^2 q_k^2(x) = \sum_{k=N-i}^{N} (-1)^{k-N+i} \left( \frac{k}{N-i} \right) q_k^2(x).
\]

**Proposition 1.** The stationary joint distribution of the server state and the system size can be shown according to the following formulas:
\[
P_{1,i}(x) = \sum_{k=N+1-i}^{N} \left( \frac{k-1}{N-i} \right) \left( \frac{\lambda p \theta}{\sigma - \lambda} \right)^{i+k-N-1} q_k^1(0)(1 - B(x)) e^{-\lambda x}, \quad 1 \geq i \geq N,
\]
\[
P_{2,i}(x) = \sum_{k=N-i}^{N} (-1)^{i+k-N} \left( \frac{k}{N-i} \right) q_k^2(0)(1 - A(x)) e^{-\frac{k}{\sigma} \lambda x}, \quad 0 \geq i \geq N,
\]

and
\[ P_{0,i} = \frac{N}{N\lambda p + (\sigma - \lambda p)i} \left[ \sum_{k=N-i}^{N} (-1)^{k-N+i} \binom{k}{N-i} q_{k}^2(0) A_{i}\frac{k}{N} \lambda p + (1 - \gamma) Q_{i-1} + \gamma Q_{i} \right. \]
\[ + \left. \frac{(N - i + 1)p\lambda(1 - \theta)}{N} (M_{i-2} - Q_{i-2}) + \frac{(i - 1)\sigma}{N} (M_{i-1} - Q_{i-1}) \right]. \]  
(12)

The elements \( q_{k}^1(0) \), are connected by the following linear formula:
\[ M_{i-1} = \frac{ie^\sigma}{N\lambda p + (\sigma - \lambda p)i} \left[ S_{i} + (1 - \gamma) Q_{i-1} + \gamma Q_{i} + \frac{\sigma}{N} T_{i-1} \right] \]
\[ + \frac{N - i + 1}{N\lambda p + (\sigma - \lambda p)(i - 1)} \left[ S_{i-1} + (1 - \gamma) Q_{i-2} + \gamma Q_{i-1} + \frac{\sigma}{N} T_{i-2} \right]. \]  
(13)

The elements \( q_{k}^2(0) \) satisfy the following equations:
\[ q_{N-i}(0) = \sum_{m=0}^{i} \binom{N - i + m}{N - i} \frac{N - i + m - 1}{N} (1 - p)\lambda(M_{i-m} - Q_{i-m}), \quad 0 \leq i \leq N - 1, \]  
(14)
\[ q_{0}(0) = \sum_{k=1}^{N} \frac{k\lambda p\sigma}{\lambda p + (N - 1)\sigma} - 1\binom{-1}{k} q_{k}^2(0) A_{k}\frac{k}{N} \lambda p - \gamma Q_{N} - \frac{\lambda p(1 - \gamma)\sigma}{\lambda p + (N - 1)\sigma} Q_{N-2} \]
\[ - (1 - \gamma - \sigma + \frac{\lambda p\gamma\sigma}{\lambda p + (N - 1)\sigma}) Q_{N-1} + \frac{\sigma}{N} (M_{N-1} - Q_{N-1}) \]
\[ - \frac{\lambda p(1 - \theta)}{N} + \frac{(N - 2)\sigma^2}{N\lambda p + (N - 1)\sigma} \right] (M_{N-2} - Q_{N-2}) \]
\[ - \frac{2\sigma(1 - \theta)(\lambda p)^2}{N\lambda p + (N - 1)\sigma} (M_{N-3} - Q_{N-3}). \]  
(15)

where
\[ c_{k} = \frac{k - 1}{N} \lambda + \frac{N - k}{N} \sigma, \]
\[ M_{i} = \sum_{k=N-i}^{N} \binom{k - 1}{N - i - 1} \frac{\lambda p\theta_{i+k-N}}{\sigma - \lambda} q_{k}^1(0), \]
\[ Q_{i} = \sum_{k=N-i}^{N} \binom{k - 1}{N - i - 1} \frac{\lambda p\theta_{i+k-N}}{\sigma - \lambda} q_{k}^1(0) B_{N-1} \]
\[ S_{i} = \frac{N - i - 1}{N} (1 - p)\lambda(M_{i} - Q_{i}) + \frac{(N - i + 1)p\lambda(1 - \theta)}{N} (M_{i-2} - Q_{i-2}), \]
\[ T_{i} = i(M_{i} - Q_{i}). \]

Proof. We can simply rewrite the above-mentioned differential equations as follows by substituting (8) and (9) into (1) and (2), respectively.
\[ N d\frac{d}{dx} q_{k}^1(x) = -[(k - 1)\lambda + (N - k)\sigma + Nb(x)] q_{k}^1(x), \]  
(16)
\[ N d\frac{d}{dx} q_{k}^2(x) = -[k\lambda p + a(x)] q_{k}^2(x). \]  
(17)

Therefore, solving the above differential equations can obtain \( q_{k}^1(x) \) and \( q_{k}^2(x) \).
\[ q_{k}^1(x) = q_{k}^1(0) \exp \left\{ - \int_{0}^{x} \frac{k - 1}{N} \lambda + \frac{N - k}{N} \sigma + b(y)dy \right\} \]
\[ = q_{k}^1(0)(1 - B(x))e^{-\left(\frac{N - k}{N} \lambda + \frac{N - k}{N} \sigma\right)x}, \text{ for all } k = 1, \cdots, N, \]  
(18)
\[ q_{k}^2(x) = q_{k}^2(0)(1 - A(x))e^{-\frac{k}{N} \lambda p x}, \text{ for all } k = 0, \cdots, N. \]  
(19)
Substituting the above two equations into (8) and (9), respectively, we can obtain (10) and (11). As $P_{1,i}(x)$ and $P_{2,i}(x)$ have been represented by $q^1_k(0)$ and $q^2_k(0)$, respectively, then $P_{0,i}$ can also be represented by $q^1_k(0)$ and $q^2_k(0)$ according to (3). After some regular calculations, (12) can be obtained.

The probabilities $P_{0,i}$, $P_{1,i}(x)$, and $P_{2,i}(x)$ are also known, once we obtain $q^1_k(0)$ and $q^2_k(0)$. We will explain how to obtain $q^1_k(0)$ and $q^2_k(0)$ in the rest of the proof.

Replace $P_{1,i+1}(x)$ in (5) by (10) and let $x = 0$ in (11), we can obtain the following equation according to (5):

$$
\sum_{k=N-i}^{N} (-1)^{k-N+i} \binom{k}{N-i} q^2_k(0) = \frac{N - i - 1}{N}(1 - p)\lambda(M_l - Q_i), \tag{20}
$$

where $i = 0, 1, \ldots, N - 1$. Through (20), we can conclude (14).

The next step will show that $q^2_0(0)$ can also be represented by $q^1_1(0), q^1_2(0), \ldots, q^1_N(0)$. Let $i = N$, $x = 0$ and substitute (10) in (4), (14) can be obtained directly. So far, $q^2_0(0), k = 0, \ldots, N$ can be represented by the formula containing $q^1_{i}(0), q^1_{i+1}(0), \ldots, q^1_{N}(0)$.

The third step is to show that $q^1_1(0), q^1_2(0), \ldots, q^1_{N-1}(0)$ can be represented by $q^1_N(0)$. Take $x = 0$ in (10) and substitute it with (12) in (4), (13) can be obtained.

When $i$ takes different values, we can iteratively obtain the relationship between $q^1_N(0)$ and $q^1_k(0)$, $k = 1, \ldots, N - 1$. Therefore, $\{q^1_k(0), 1 \leq N - 1\}$ and $\{q^2_k(0), 1 \leq N\}$ are all proportional to $q^1_N(0)$ according to (20) and (15). We can recursively calculate the coefficient of proportionality by setting $q^1_N(0) = 1$.

At the end of the calculation, the normalization condition is used to calculate $q^1_N(0)$ so that $q^1_k, k = 1, \ldots, N$ and $q^2_k(0), k = 0, \ldots, N$ can be obtained immediately. So far, we can obtain $P_{0,i}(i = 0, \ldots, N), P_{1,i}(x)(i = 1, \ldots, N)$, and $P_{2,i}(x)(i = 0, \ldots, N)$ directly through the above iteration.

4. Performance Measures

Several main performance indices are given in this section, such as the effective arrival rate, state probabilities, mean system content, mean response and waiting time, blocking probability, etc.

1. The effective arrival rate

The proposed model has two kinds of arrivals, i.e., positive arrivals and negative arrivals. The access of the negative customer to a busy server will break down the system and remove the customer receiving service. In other states, negative customers do not affect the system, which is called ineffective arrivals.

$$
X = \sum_{i=1}^{N} \frac{(N - i)\lambda p}{N} P_{0,i} + \int_{0}^{\infty} P_{1,i}(x)dx + \int_{0}^{\infty} P_{2,i}(x)dx + \lambda p P_{0,0} + \lambda p P_{2,0}
$$

$$
= \lambda p \sum_{i=1}^{N} (P_{0,i} + P_{1,i} + P_{2,i}) - \sum_{i=1}^{N} \frac{i}{N} \lambda p (P_{0,i} + P_{1,i} + P_{2,i}) + \lambda p P_{0,0} + \lambda p P_{2,0}
$$

$$
= \frac{\lambda p}{N} (N - E(L)).
$$

2. The state probabilities

$$
P_0 = \sum_{i=0}^{N} P_{0,i}, P_1 = \sum_{i=1}^{N} \int_{0}^{\infty} P_{1,i}(x)dx, P_2 = \sum_{i=0}^{N} \int_{0}^{\infty} P_{2,i}(x)dx.
$$

3. Define $E(L)$, $E(O)$ as the expected number of packets in the system (mean system content) and orbit, respectively.
\[ E(L) = \sum_{i=1}^{N} i(P_{0,i} + \int_0^\infty P_{1,i}(x)dx + \int_0^\infty P_{2,i}(x)dx), \]
\[ E(O) = \sum_{i=0}^{N} iP_{0,i} + \sum_{i=1}^{N} (i - 1) \int_0^\infty P_{1,i}(x)dx + \sum_{i=0}^{N} i \int_0^\infty P_{2,i}(x)dx = E(L) - P_1. \]

4. As a basic conclusion in queuing theory, describing the relations among the mean arrival rate, the mean system content, and the mean response time, Little's law is adopted here to calculate the mean response time and the mean waiting time in the orbit. The mean response time: \( E(T) = E(L)/\lambda \).

The mean waiting time in the orbit: \( E(W) = E(O)/\lambda \).

5. The mean total service time: \( E(S) = E(T) - E(W) \).

6. The blocking probability that a new arriving customer will not receive service directly, but will be trapped in the orbit.

\[ P_B = \frac{\sum_{i=1}^{N} \frac{N-i}{N} \lambda pP_{1,i} + \sum_{i=0}^{N} \frac{N-i}{N} \lambda pP_{2,i}}{\sum_{i=0}^{N} \frac{N-i}{N} \lambda pP_{0,i} + \sum_{i=1}^{N} \frac{N-i}{N} \lambda pP_{1,i} + \sum_{i=0}^{N} \frac{N-i}{N} \lambda pP_{2,i}} = \frac{N(P_1 + P_2) - N_1 - N_2}{N - E(L)} = \frac{\lambda p N(P_1 + P_2) - N_1 - N_2}{\lambda}. \]

5. Reliability and Availability Analysis

To many practical systems, it is often integral to maintain a required or relatively high level of availability and reliability. The reliability indicators are of considerable concern for queuing systems suffering random breakdowns. In this section, we calculate the mean time to the first failure, failure frequency, reliability function, and availability of the server. Meanwhile, some figures and tables are also presented in Section 5 to show how these reliability indices change with different parameters.

1. The steady-state availability \( AV \)

\[ AV = \lim_{t \to \infty} AV(t) = \sum_{i=0}^{N} P_{0,i} + \sum_{i=1}^{N} \int_0^\infty P_{1,i}(x)dx = 1 - \sum_{i=0}^{N} \int_0^\infty P_{2,i}(x)dx \]

2. The failure frequency \( W_f \)

\[ W_f = \lambda^- P_1 = (\sum_{i=1}^{N} \frac{N-i}{N} \lambda (1-p) \int_0^\infty P_{1,i}(x)dx)P_1, \]

where \( \lambda^- \) denotes the mean arrivals of negative customers(packets).

3. The reliability function of the server \( R_Y(t) \)

An iterative formula is derived in the subsequent part of this section in order to compute the reliability function of the server \( R_Y(t) = P(\tau > t) \), where the first failure time of the server is represented by \( \tau \). To calculate \( R_Y(t) \), a new model needs to be constructed in terms of making the failure states become absorbing states. In the entirely new system, the notations and descriptions are the same as in Section 2, but the absorbing states are no longer able to transfer to other states. Therefore, the Kolmogorov equations can be derived as follows:

\[
\left( \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \right) P_{1,i}(x,t) = -\left( \frac{N-i}{N} \lambda + b(x) + \frac{i-1}{N} \sigma \right) P_{1,i}(x,t) \\
+ \frac{N-i+1}{N} p \lambda \theta P_{1,i-1}(x,t), \quad 1 \leq i \leq N,
\]

(21)
\[
\frac{\partial}{\partial t} + \lambda p + (\sigma - \lambda p) \frac{i}{N} P_{0,i}(t) = \int_0^\infty \frac{N-i+1}{N} p\lambda(1-\theta)P_{1,i-1}(x,t)dx \\
+ \int_0^\infty \left[ \frac{i-1}{N} \sigma + b(x)(1-\gamma) \right] P_{i,i}(x,t)dx \\
+ \int_0^\infty \gamma b(x) P_{1,i+1}(x,t)dx, \quad 0 \leq i \leq N, \tag{22}
\]

with the initial condition: \( P_{0,0}(0) = 1 \) and the steady-state boundary condition:

\[
P_{1,i}(0,t) = \frac{i}{N} \sigma P_{0,i}(t) + (1 - \frac{i-1}{N}) \lambda p P_{0,i-1}(t), \quad 1 \leq i \leq N. \tag{23}
\]

For the purpose of calculating \( R_Y(t) \), the first step to carry out is finding the transient probabilities of the new system by taking the Laplace transform of (21)–(23).

\[
sP_{i,i}^*(x,s) + \frac{\partial}{\partial x} P_{i,i}^*(x,s) = -\left( \frac{N-i+1}{N} \lambda + b(x) + \frac{i-1}{N} \sigma \right) P_{i,i-1}^*(x,s), \quad 1 \leq i \leq N, \tag{24}
\]

\[
(s + \lambda p + (\sigma - \lambda p) \frac{i}{N}) P_{0,i}^*(s) - \delta_{0,i} = \frac{N-i+1}{N} \lambda(1-\theta) \int_0^\infty P_{i,i-1}^*(x,s)dx \\
+ \int_0^\infty \left[ \frac{i-1}{N} \sigma + b(x)(1-\gamma) \right] P_{i,i}^*(x,s)dx \\
+ \int_0^\infty \gamma b(x) P_{i,i+1}^*(x,s)dx, \quad 0 \leq i \leq N, \tag{25}
\]

\[
P_{i,i}^*(0,s) = \frac{i}{N} \sigma P_{0,i}^*(s) + (1 - \frac{i-1}{N}) \lambda p P_{0,i-1}^*(s), \quad 1 \leq i \leq N \tag{26}
\]

**Proposition 2.** The Laplace transforms of \( P_{0,i}(x,t) \), \( P_{1,i}(x,t) \) and \( R_Y(t) \) can be expressed as follows:

\[
P_{i,i}^*(s) = \frac{1}{s + \lambda p + (\sigma - \lambda p) \frac{i}{N}} \left[ \frac{N-i+1}{N} \lambda(1-\theta)(\tilde{M}_{i-2} - \tilde{Q}_{i-2}) \\
+ \frac{i-1}{N} \sigma(\tilde{M}_{i-1} - \tilde{Q}_{i-1}) + (1-\gamma)\tilde{Q}_{i-1} + \gamma \tilde{Q}_i + \delta_{0,i}) \right], \quad 0 \leq i \leq N. \tag{27}
\]

\[
P_{i,i}^*(x,s) = \sum_{k=N-i+1}^{N} \left( \frac{k-1}{N-i} \right) \left( \frac{\lambda p}{\sigma - \lambda} \right)^{i+k-N-1} q_k^{3*}(0,s)(1-B(x)e^{-(\tilde{c}_k + \tilde{d})x}, \quad 1 \leq i \leq N. \tag{28}
\]

The element \( q_k^{3*}(0,s) \) satisfies the following equation:

\[
\tilde{M}_{i-1} = \frac{is}{N(\lambda p + s + (\sigma - \lambda p))} \left[ \frac{\lambda p}{N} \tilde{S}_{i-2} + \frac{\sigma}{N} \tilde{T}_{i-1} + (1-\gamma)\tilde{Q}_{i-1} + \gamma \tilde{Q}_i + \delta_{0,i} \right] \\
+ \frac{(N-i+1)\lambda p}{N(\lambda p + s + (\sigma - \lambda p)(i-1))} \left[ \frac{\lambda p}{N} \tilde{S}_{i-3} + \frac{\sigma}{N} \tilde{T}_{i-2} + (1-\gamma)\tilde{Q}_{i-2} + \gamma \tilde{Q}_{i-1} + \delta_{0,i-1} \right] \tag{29}
\]

where
The mean time to failure (MTTF) of the system presented in the following.

6. Numerical Results and Comparative Discussions

conclusions about the characteristics of the system have also been summarized, which allow
sition 1, Proposition 2, and Sections 4 and 5. By analyzing these figures, some important
formance measures are presented in this section. All numerical results are obtained by
obtaining the inverse Laplace transforms of the above variables. This procedure
solution for $P$

The Laplace transform of the reliability function of the server is

$$ R^*_V(s) = \sum_{i=0}^{N} P^*_0(s) + \sum_{i=1}^{N} \int_{0}^{+\infty} P^*_V(x,s)dx. \quad (30) $$

Proof. It is worth noting that the partial differential Equation (24) has similar features
as (1), which indicates the discrete transformation can also be used here. Equation (24)
can be represented by $(A_3I - B_3)\tilde{P}^*_V(x,s) = 0$, where $\tilde{P}^*_V(x,s) = (\tilde{P}^*_{1,1}(x,s), \tilde{P}^*_{1,2}(x,s), \cdots, \tilde{P}^*_{1,N}(x,s))$ and $A_3 = N\frac{d}{dx} + Nb(x)$. It is not difficult to see that $B_3 = B_1 - sI$, then the
diagonal matrix of corresponding eigenvalues $\Lambda_3 = \Lambda - sI$ and the matrix of eigenvectors
$V_3 = V_1$.

Throughout the analysis above, we conclude that $P^*_{1,j}(x,s)$ has a similar solution form
presented in the following.

$$ P^*_{1,j}(x,s) = \sum_{k=0}^{N-i-1} \left( \begin{array}{c} k-1 \\ N-i-1 \end{array} \right) \frac{\lambda p^\theta}{\sigma^\lambda}^{i+k-N}{q^*_k}(0,s). \quad (31) $$

Solving (24) by plugging (31), we have

$$ q^*_k(x,s) = q^*_k(0,s)(1 - B(x))e^{-(c_k + \frac{N}{\lambda})x}, k = 1, \cdots, N \quad (32) $$

Therefore, (28) can be obtained. Substitute (28) in (25), the Laplace transform of $P^*_0(s)$ can be derived as (27). A recursive formula (29) for $\{q^*_k(0,s), k = 1, \cdots, N\}$ can be constructed according to (26).

It is quite clear that $q^*_0(0,s), \cdots, q^*_N(0,s)$ are all proportional to $q^*_N(0,s)$. The corre-
spoking coefficient of the proportionality can be calculated by setting $q^*_N(0,s) = 1$.
The unknown quantity $q^*_N(0,s)$ can be determined by the normalization condition. Then,
the Laplace transformations $P^*_{1,j}(x,s), j = 1, \cdots, N$ and $P^*_0(s), i = 0, \cdots, N$ can be derived.
However, it is difficult to obtain analytical solutions. Hence, we aim to obtain the numerical
solution for $P^*_{1,j}(x,s)$ and $P^*_0(s)$. In the sequel, the transient probabilities can finally be obtained by taking the inverse Laplace transforms of the above variables. This procedure can be implemented by MATLAB (R2017b) software.

4) The mean time to failure (MTTF) of the system

$$ MTTF = \lim_{s \to 0} R^*(s) = \sum_{i=0}^{N} P^*_0(0) + \sum_{i=1}^{N} \int_{0}^{+\infty} P^*_V(x,0)dx. $$

6. Numerical Results and Comparative Discussions

Numerical examples related to the impact on main reliability indicators and per-
formance measures are presented in this section. All numerical results are obtained by
programming using the software MATLAB (R2017b) according to the formulas in Proposition 1, Proposition 2, and Sections 4 and 5. By analyzing these figures, some important
conclusions about the characteristics of the system have also been summarized, which allow
us to have a deeper understanding of the proposed model. It is worth mentioning that we
did not analyze the impact of every parameter on all performance indicators, which would make the content too cumbersome. On the contrary, based on the preliminary analysis of each indicator, we selected a few of the most concerned indicators and the corresponding numerical conclusions are given in Figures 2–6.

![Graphs showing main performance indicators vs. λ and N.](image-url)
More specifically, we selected eight performance measures ($\bar{\lambda}$, $P_0$, $P_1$, $P_2$, $E(L)$, $E(W)$, $E(\tau)$, and $P_b$) out of 11 performance measures, and $E(O)$, $E(T)$ and $E(S)$ are not in consideration. For reliability and availability indices, we selected three ($AV$, $W_f$, and $MTTF$) out of four indices to conduct numerical studies, and $R_Y(t)$ is not in consideration. Some metrics reflect similar system performance, such as $E(L)$ (the expected number of packets in the system) and $E(O)$ (the expected number of packets in the orbit). And with the help of the relationship equation between the two in Section 4, it can be seen that their trends with parameters are similar, so we choose only one of them to study and analyze. The other indices are selected based on a similar thought.

Figure 4. The steady-state availability vs. $\lambda$.

Figure 5. The failure frequency vs. $\lambda$. 
Figure 6. The failure frequency and steady-state availability vs. \( N \).

We preliminarily conducted a brief study of all eight parameters (\( \lambda, \mu, \sigma, p, \theta, \gamma, \beta, \) and \( N \)) and found that \( \lambda, \mu, \theta, \) and \( N \) had a relatively large impact on the performance metrics. Furthermore, some of the four parameters brought about changes that were not in line with our expectations, see Figures 2 and 3. We follow the same considerations in selecting the parameters of \( AV \) and \( W_f \), see Figures 4–6. In the end, some supplementary results are summarized in Tables 3–9.

Table 3. Performance and reliability indicators versus \( \lambda \).

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( P_0 )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( E(\tau) )</th>
<th>MTTF</th>
</tr>
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<td>0.334484</td>
<td>0.15885</td>
<td>66.66667</td>
<td>18.16401</td>
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</tbody>
</table>

Table 4. Performance and reliability indicators versus \( \beta \).

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( P_0 )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( E(\tau) )</th>
<th>MTTF</th>
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<td>0.334484</td>
<td>0.15885</td>
<td>66.66667</td>
<td>18.16401</td>
</tr>
</tbody>
</table>

Table 5. Performance and reliability indicators versus \( \mu \).

<table>
<thead>
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<th>( \mu )</th>
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Table 6. Performance and reliability indicators versus \( \theta \).

<table>
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<th>( \theta )</th>
<th>( P_0 )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( E(\tau) )</th>
<th>( W_f )</th>
<th>MTTF</th>
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<td>18.16401</td>
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Table 7. Performance and reliability indicators versus $\gamma$.

<table>
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<th>$\gamma$</th>
<th>$P_0$</th>
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<th>$P_2$</th>
<th>$E(\tau)$</th>
<th>$W_f$</th>
<th>MTTF</th>
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<td>0.037845</td>
<td>15.74974</td>
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Table 8. Performance and reliability indicators versus $\sigma$.

<table>
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<th>$\sigma$</th>
<th>$P_0$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$E(\tau)$</th>
<th>$W_f$</th>
<th>MTTF</th>
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<td>21.73683</td>
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Table 9. Performance and reliability indicators versus $N$.

<table>
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<tr>
<th>$N$</th>
<th>$P_0$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$E(\tau)$</th>
<th>MTTF</th>
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<td>2</td>
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<td>0.061084</td>
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<tr>
<td>5</td>
<td>0.345021</td>
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<td>0.096695</td>
<td>83.33333</td>
<td>32.70835</td>
</tr>
</tbody>
</table>

For more clearly and comprehensively illustrating the numerical results, we assume the service time (with rate $\mu$) and repair time (with rate $\beta$) both follow the exponential distributions. We set $N = 8, \lambda = 0.2, \mu = 0.2, \beta = 0.5, \theta = 0.4, p = 0.3, \gamma = 0.3, \sigma = 2$ and change different parameter values in different figures.

Figures 2 and 3 depict the corresponding numerical results of the main performance measures. We plot the mean waiting time $E(W)$, effective arrival rate $\lambda$, blocking probability $P_B$ and mean system content $E(L)$ against parameter $\lambda$ and $\theta$ with three different values of $\mu = 0.3, 0.1$ and $0.05$ and $N = 12, 9, 7$. Each figure contains three curves with different colors to simultaneously describe the changes of each indicator with parameters.

As can be seen from the picture in the upper-left corner of Figure 2, $E(W)$ has a point of minimum value as a function of $\lambda$. This result is similar to a retrial queue with collisions and feedback in [38]. Here, we need to notice that there is a common phenomenon of most retrial queues with finite sources and a single server that both $E(W)$ and $P_B$ have a maximum point as a function of $\lambda$, see in [39–42]. The same feature is also reflected in the blocking probability $P_B$ and effective arrival rate in Figure 2. However, it is inconsistent with $E(W)$ in Figure 2. To sum up, this shows that transmission errors can be regarded as a kind of feedback behavior, and it has a significant influence on $E(W)$. This critical conclusion also reveals the importance and necessity of controlling transmission errors. The figure in the upper right corner of Figure 2 shows that the effective arrival rate as a function of $\lambda$ has a maximum value. This shows that for computers and communication systems with a limited number of terminals, increasing the packet arrival rate does not necessarily increase the effective arrival rate if it is affected by data collisions, server failures, and transmission errors. Instead, if the arrival rate exceeds the maximum point, the effective arrival rate will decrease. This is because an increase in the arrival rate increases the mean system content, and the data source will not make any more requests until the service is completed.

Figure 3 shows the effect of the probability of no collision $\theta$ on different performance indicators with three values of $N = 7, 9,$ and $12$. We can easily see that when the probability of no collision increases, the mean waiting time and the mean system content decrease which is in line with our intuition. Meanwhile, the effective arrival rate and the blocking
we hypothesize that this may be because as the repair rate increases, the probability that
with our expectations, i.e., as the repair rate increases, the failure frequency also increases.
Furthermore, the system becomes more
A totally different conclusion may possibly be reached when instinct is used to deduce the
combined with the results of Table 9 probability
We are also very concerned about the effect of the number of finite sources on our pro-
posed finite source retrial system. From the Figure 3, we can see that as the number of finite
sources \( N \) increases, the retrial rate of individual customers in the orbit decreases, while
the arrival rate of requests generated by individual data sources increases, and therefore,
the arrival rate of negative customers has increased, and the system is more likely to crash in a busy state
(this conclusion has also been verified by the results of Table 9 probability \( P_2 \).
Figure 4 gives numerical examples of the stationary system availability. As depicted
in the lower-left corner, \( AV \) has a minimum point as a function of \( \lambda \), which
indicates that our model guarantees minimal availability for the system, at least at some
value of parameters. Also, the availability decreases when \( \sigma \) or \( \mu \) increases. Due to the
many parameters involved here, it is quite complicated to analyze the specific changes.
A totally different conclusion may possibly be reached when instinct is used to deduce the
relations. As we can see in Tables 5 and 8, probability \( P_2 \) increases when \( \sigma \) or \( \mu \) increases,
which can offer a full explanation for this result. Furthermore, the system becomes more
available when there are fewer negative arrivals and a larger repair speed, which agrees
with intuitive expectations.
Figure 5 shows the effect of the parameters \( \lambda, \sigma, p, \) and \( \mu \) on \( W_f \). It is worth noting
that we have made some adjustments in the bottom left image, i.e., using \( \mu \) as the x-axis,
in order to better present the effect of \( \mu \) on \( W_f \). Through Figure 5 we can see that \( W_f \) as a
function of the arrival rate has a maximum value. This inspires us to take this property into
account when controlling the failure frequency of the corresponding system to avoid the
arrival rate falling near the point of maximum value. The upper-right image shows the
effect of the probability of generating a positive customer on the \( W_f \). As can be seen from
the figure, when \( p \) is very small, such as \( p = 0.2, W_f \) is relatively sensitive to changes in \( \lambda \),
i.e., when \( \lambda \) is changed, the change in \( W_f \) is relatively large; whereas, when \( p \) is gradually
increased, such as \( p = 0.5 \) and \( 0.7, W_f \) is relatively insensitive to changes in \( \lambda \), i.e., when \( \lambda \)
behaves near the maximum, which makes \( P_B \) increase as well. To conclude, at the expense of a portion of the
blocking probability, if the probability of collisions is reduced, the effective arrival rate can
be effectively improved, and it also reduces the mean system delay and system content at
the same time.

The numerical results of the effect of the number of finite sources on the indicators
\( AV \) and \( W_f \) are summarized in Figure 6. This figure depicts the property that as a function
of the number of finite sources \( N \), \( W_f \) has a maximum value, while \( AV \) has a minimum value.
This conclusion is clearly not in line with our expectations. As seen in the figure,
the maximum value of \( W_f \) increases with the arrival rate, while the minimum value of \( AV \)
decreases with the arrival rate. This inspires us that we can effectively control the extremes of $W_f$ and $AV$ by controlling the arrival rate of the packets.

Tables 3–9 show the influence of different parameters on some indicators, including two reliability indicators. In Table 3, when $\lambda$ increases, more customers enter the system no matter what state the system is in. We can only infer that $P_0$ will decrease as $\lambda$ increases because an arrival in the idle state will start its service at the server, which turns the system into a busy state. However, we cannot directly go to the conclusion that $P_1$ will increase as $\lambda$ increases. As we described in the proposed model in the previous section, two distinct types of customers are presented: positive and negative. An increase in $\lambda$ not only increases the rate that positive customers arrive but also increases the arrival rate of negative customers, which can lead to breakdowns of the server, then the system turns to a repair state ($P_2$ increases). Moreover, the arrival in the busy state also causes collisions, so $P_1$ decreases. It is obvious that the mean total sojourn time in the source only relates to the arrival rate $\lambda$ and the number of sources $N$. When $\lambda$ increases, the generating rate of requests increases, so $E(\tau)$ decreases. When $N$ increases, the generating rate of requests decreases, so $E(\tau)$ increases, see Table 9.

Table 4 illustrates the effect of repair rate on each of the indices. When the $\beta$ is increased, the failed server will be restored to an available state faster, so $P_2$ decreases; the status of the server immediately changes to idle after the repair is completed, so $P_0$ increases; and the sensitivity of $P_2$ to $\beta$ changes is greater than that of $P_0$, so according to the normalization condition, $P_1$ decreases.

From Table 5, we can observe the variation of each metric about the service rate $\mu$. When the service rate becomes large, the service is completed faster, so $P_1$ becomes smaller; and the probability that the system is in maintenance, $P_2$, increases and then decreases, from which it can be seen that the sensitivity of $P_1$ and $P_2$ to changes in the service rate also changes with the service rate. When the service rate is very small, $P_1$ is more sensitive than $P_0$, so that $P_1$ decreases more than $P_0$ increases, and $P_2$ increases; and as the service rate increases, $P_0$’s sensitivity to $\mu$ changes is greater than that of $P_1$, so that $P_1$ decreases by a smaller amount than $P_0$ and $P_2$ decreases.

We are also interested in the impact of the collision phenomenon on other metrics of the system, and so we give Table 6. As the probability of a non-collision occurring $\theta$ increases, the probability of an arriving new request interrupting the service decreases, which means that the probability of the server switching to an idle state after the service has been interrupted also decreases, and thus $P_0$ decreases while $P_1$ increases. In addition, since the probability that the server is busy becomes larger, then it will crash more frequently during this period ($W_f$ becomes larger), and so the probability that the server is in a maintenance state, $P_2$, also becomes larger.

Table 7 presents the impact of the probability of transmission without error $\gamma$ on the other metrics of the system. The higher the probability of error-free transmission, the earlier the system crashes (MTTF becomes small) and the frequency of crashes increases ($W_f$ increases). At the same time, the probability that the system is in idle and busy states decreases, while the probability that it is in a maintenance state increases.

Table 8 gives the effect of retrial rate $\sigma$ on other metrics of the system. Requests that are retried successfully go to the server to be served, so as the retrial rate of requests in the orbit increases, $P_0$ decreases and $P_1$ increases. And for $\sigma$, since $P_0$ is more sensitive than $P_1$, $P_2$ increases according to the normalization condition. As the probability of a busy state increases, the failure frequency of the server increases and also crashes faster.

Finally, for the finite source retrial system, we are also very concerned about the impact of the number of finite sources on the other metrics of the system, which are summarized in Table 9. It can be seen that when the number of sources increases, the rate at which each source generates positive and negative requests also increases, which makes the system more susceptible to collisions and crashes when it is busy ($P_1$ decreases), and thus shifts to the idle and repair states ($P_0$ and $P_2$ increase). In addition, the sensitivity of $P_0$, $P_1$, and $P_2$ with respect to changes in $N$ is somewhat reacted to. When $N$ increases by the same
magnitude, the change in $P_1 > P_0 > P_2$, i.e., the degree of sensitivity of probabilities to $N$: $P_1 > P_0 > P_2$.

7. Conclusions

This paper considers a retrial G-queue with collisions, finite sources, and transmission errors, in which both service and repair time are considered as general distributions. The proposed model can be applied to many practical problems arising in many communication systems. The supplementary variables and discrete transformation methods are adopted to derive the recursive formulas in order to calculate the joint stationary probabilities of the system state and system content. Numerical illustrations of the effect caused by parameters on performance and reliability indices are presented. We also compared the results with previous research and obtained some interesting characteristics. The average waiting time $E(W)$, a function of $\lambda$, exists a minimum point, which does not satisfy the common feature of most of the retrial queues with the single server and finite sources. However, this characteristic is in accordance with the result obtained from a feedback retrial queuing system with collisions. This intriguing finding could be explained by the fact that the transmission errors can be regarded as a feedback behavior, and it does have a great influence on $E(W)$. Furthermore, from the illustrations, the steady-state availability also has a minimum point as a function of $\lambda$. This conclusion shows the proposed model guarantees the minimal availability of the system.

For further research, we plan to study a multi-server retrial queuing system and also take more types of customers into consideration.

Author Contributions: Conceptualization, W.X.; methodology, W.X.; software, W.X., L.L. (Linhong Li) and Z.W.; formal analysis, W.X., L.L. (Linhong Li) and S.W.; investigation, W.X.; data curation, W.X.; writing—original draft preparation, W.X., L.L. (Liwei Liu), L.L. (Linhong Li) and S.W.; writing—review and editing, W.X., L.L. (Liwei Liu) and L.L. (Linhong Li) and S.W.; visualization, W.X.; supervision, L.L. (Liwei Liu) and S.W. All authors have read and agreed to the published version of the manuscript.

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