Large-Signal Stability of the Quadratic Boost Converter Using a Disturbance Observer-Based Sliding-Mode Control

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Abstract: The quadratic boost (QB) converter is a fourth-order system with a dc gain that is higher than the traditional second-order step-up configuration. The modern controllers that control these high-order dc–dc converters often only guarantee local stability around a steady-state equilibrium point, which is one of their primary drawbacks. In this article, a non-linear robust control law design to attain large-signal stability in this single switch QB converter is presented. In the presence of an unpredictable load, the control objective is to maintain the regulation of an output voltage. The Brunovsky canonical model of the converter was derived first, and the non-linear disturbance observer-based sliding-mode (SM) control law is designed based on it. An observer variable precisely estimates the output disturbances. The detailed process for deriving the control signal is described in this paper and the large-signal stability of the closed-loop converter system is ensured via the Lyapunov function. Finally, some simulation results are shown to validate the usefulness of the given controller.

Keywords: quadratic converter; sliding-mode control; observer

MSC: 93D05

1. Introduction

The dc–dc boost converter is employed in several fields, including electric vehicles, telecommunication equipment, energy systems based on non-conventional resources, and so on [1–3]. For instance, the voltage at the output of a single fuel cell is very low, of the order of 1.1 V, and its stacked version could produce around from 24 V to 60 V. However, this voltage is not enough at the input of an inverter for applications in the power range from 1 kW to 5 kW. Thus, a dc–dc converter can be employed between the nonconventional energy resource and inverter and its gain should be high enough to make up for the differences [1]. The second-order classical boost converter can step up the output voltage, but its gain is limited because it needs to operate at a considerably high duty ratio to produce high gain, and switching devices have limited finite switching durations. It may also incur EMI and reverse recovery issues of the diode. Lastly, working at high duty ratio values could affect the system’s dynamic response to parameter variations [4]. One of the solutions to address this problem is using transformer-based dc–dc power converters to provide high gain before interfacing with inverters. However, if a particular utilization area does not need any isolation, the usage transformer becomes redundant, and it ultimately increases the system’s size and pricing. The high-order transformer-less power
converters are thus receiving attention because they do not only eliminate the use of transformers but also avoid high values of the duty ratio to offer a large conversion ratio [5–8]. Moreover, the voltage stress is also limited in their cases. Among them, the quadratic boost (QB) converter is a popular candidate due to its high efficiency, smaller size, and ease of control due to having a single active switch [6].

The control aspect of high-step-up converters like the QB converter has recently been the interest of some research [6–12]. However, their control is not very straightforward because right half-plane zeroes are present in the control to load voltage transfer function of these converters [13]. Because of this, the closed-loop system could lose its stability. To address the first concern, one of the widely used regulation methods for the higher order power converters is employing the current through the inductor for feedback purposes [14]. The additional current-loop, apart from the basic voltage-loop, provides stability to the system and gives inbuilt overcurrent protection. In [13], the application of the current-based control scheme for the quadratic boost converter was investigated. The more advanced current-mode control, based on an adaptation algorithm for the sixth-order boost converter, was discussed in [15]. Even though all of these current-mode controllers are shown to provide a satisfactory response over a large range of parameter changes, they are based on the small-signal averaged model of the converter, which can only ensure stability in the vicinity of a steady-state operating point.

A sliding-mode (SM) scheme is another well-employed scheme that is suitable for dc–dc converters [16–24]. Traditionally, hysteresis-modulation is used for the implementation of the SM controller for dc–dc converters [6,20–22]. This method has recently been used to regulate the output voltage of several high-gain converters like the quadratic boost converter [6,20], the hybrid boost converter [21], and the zeta converter [22]. Although this method is easy to implement, its main drawback is that it may lead to chattering in the response. Also, since the switching frequency is not fixed, there could be large variations in the switching frequency in the presence of load and line variations. This may lead to increased switching losses and electromagnetic interference (EMI) issues. To address these concerns, recently, the constant-frequency SM scheme has been employed for high-order boost converters like the quadratic boost converter in [16]. In this method, the pulse-width-modulation (PWM) technique is used to generate the control signal. The various advantages offered by this method are ease of implementation, reduced chattering, and lower electromagnetic interference (EMI) issues. Some of the other state-of-the-art SM controllers for dc–dc converters based on the PWM approach are discussed in [17,18]. As can be seen, there has been considerable efforts made towards the implementation of several non-linear controllers for high-order dc–dc converters. However, the main drawback of most of these controllers is that their stability is guaranteed only in the neighborhood of the equilibrium point. In other words, they guarantee only small-signal stability and none of the works discussed so far address the large-signal stability of the controlled high-step-up power converters. Thus, to ensure smooth tracking in the presence of large and fast variations in the system parameters, the problem of the design of a robust and globally stable controller for high-step-up dc–dc converters still needs to be addressed.

To address this, a new SM controller design based on disturbance observer (DO) for the QB converter’s output voltage regulation is presented. The main contributions of the paper are—(i) first, as opposed to the existing methods discussed above, the proposed controller ensures global stability, which has been proved using the Lyapunov function. To this end, the sliding surface and corresponding observer variables are selected such that the suitable Lyapunov function can be selected for the global stability analysis; (ii) secondly, instead of using the conventional averaged state-space model for the controller design and analysis, the Brunovsky canonical form of the model for the quadratic converter is derived and used for the controller design. This model accommodates the disturbance variables and aids in the derivation of the control law based on the proposed DO-based sliding surface; (iii) lastly, in order to avoid the chattering and EMI concerns, the PWM method of implementation is used for the implementation of the globally stable
SM controller. The main control objective is to regulate the output voltage in the presence of parameter variations such as load changes. An in-depth derivation of the equivalent control law and a thorough stability analysis are presented. The suitability of the proposed control scheme has been authenticated by simulation results performed in MATLAB Simulink. It is important to mention that the design methodology of the given control law is such that it can be applied for the control of other types of high-step-up converters as well.

The manuscript is structured as follows. In Section 2, the circuit diagram along with an averaged model of the QB converter is given. Next, Section 3 discusses the detailed control law design and the global stability analysis of the system. Finally, in Section 4, some simulation results are given to establish the ability of the derived control law to handle large signal disturbances, followed by the conclusion in the last section.

2. State-Space Modeling for Quadratic-Ratio Converter

The quadratic boost topology’s circuit schematic is depicted in Figure 1a. It has an extra step-up arrangement compared to the second-order conventional boost topology. This additional arrangement primarily consists of an additional boost stage but without an additional active switch. The use of a single active switch reduces the converter switching losses. In summary, to increase the gain of the orthodox step-up topology, two boost converters are combined using one active switch to create this converter [25].

![Circuit Diagram](image1.png)

(a) Circuit schematic; (b) switch ON; (c) switch OFF.

Figure 1. Circuit diagram and operational modes of the quadratic converter: (a) Circuit schematic; (b) switch ON; (c) switch OFF.
The following presumptions are made in order to streamline the modeling and create the topology’s averaged modeling equations: (a) The MOSFET ‘S’ switches on and off in synchrony with all of the diodes; (b) the dc–dc system works in a continuous mode of conduction; (c) all of the diodes and the semiconductor switches are viewed as perfect components with very low parasitic resistance.

The following describes the system’s two operational modes.

‘Mode 1’: In this mode, diodes $D_2$ and $D_3$ are biased in the reverse direction while $D_1$ is forward biased. Also, the semiconductor device ‘S’ is closed while the device is working in this first condition. Energy is stored in the two inductors, $L_1$ and $L_2$, by the input voltage sources $E$ and $C_1$, respectively. The derivative expressions for this mode of operation can be obtained by employing Kirchhoff’s laws of voltage and current (KVL and KCL) in Figure 1b, and as a result we obtain (see Appendix A for detailed derivation):

$$\frac{dx_1}{dt} = \frac{E}{L_1}$$

$$\frac{dx_2}{dt} = \frac{x_3}{L_2}$$

$$\frac{dx_3}{dt} = -\frac{x_2}{C_1}$$

$$\frac{dx_4}{dt} = -\frac{x_4}{RC_2}$$

where $x_1 = i_{L_1}$ and $x_2 = i_{L_2}$ are the currents through $L_1$ and $L_2$, respectively, and $x_3 = v_{C_1}$ and $x_4 = v_{C_2}$ are the voltages across $C_1$ and $C_2$, respectively. Also, $E$ and $R$ are the source voltage and actual load value, respectively.

‘Mode 2’: In this operational mode, the semiconductor MOSFET ‘S’ is in an OFF state and $D_2$ and $D_3$ are biased in the forward direction while $D_1$ is also forward biased. This ensures a way for the flow of the inductor current $x_1$ and $x_2$ to the load, and the energy from the input and these two inductors is transferred to the load. The derivative expressions for this mode of operation can be obtained by employing Kirchhoff’s laws of voltage and current (KVL and KCL) to Figure 1c, for which we obtain (see Appendix B for detailed derivation):

$$\frac{dx_1}{dt} = \frac{E - x_3}{L_1}$$

$$\frac{dx_2}{dt} = \frac{x_3 - x_4}{L_2}$$

$$\frac{dx_3}{dt} = \frac{x_1 - x_2}{C_1}$$

$$\frac{dx_4}{dt} = \frac{x_2}{C_2} - \frac{x_4}{RC_2}$$

Next, the averaged state-space model of the QB converter is obtained. In this technique, the differential equations for the ‘ON’ state and the ‘OFF’ state are averaged over one switching period, $T$. Basically, the ‘ON’ state equations given by (1)–(4) are multiplied by $kT$, the ‘OFF’ state equations given by (5)–(8) are multiplied by $(1-k)T$, and then these equations are added with each other and divided by the total time period, $T$. Here, $k$ is the duty ratio which is also the control signal of the converter such that $0 < k < 1$. Using (1)–(8), one can obtain the averaged state-space expression of the system, given by:
\[
\frac{dx_1}{dt} = -\frac{1}{L_1} (1-k)x_3 + \frac{1}{L_1} E \quad (9)
\]
\[
\frac{dx_2}{dt} = -\frac{1}{L_2} (1-k)x_4 + \frac{1}{E} x_3 \quad (10)
\]
\[
\frac{dx_3}{dt} = \frac{1}{C_1} (1-k)x_1 - \frac{1}{C_1} x_2 \quad (11)
\]
\[
\frac{dx_4}{dt} = \frac{1}{C_2} (1-k)x_2 - \frac{1}{RC_2} x_4 \quad (12)
\]

Now, one can determine the equilibrium values of the converter by equating (9)–(12) with zero as follows:

\[
X_{ref}^1 = \frac{v_1^2}{2R_0}, \quad X_{ref}^2 = \frac{v_d^2}{2}, \quad X_{ref}^3 = \frac{v_x E}{2}, \quad X_{ref}^4 = V_d \quad (13)
\]

where \(X_{ref}^1, X_{ref}^2, X_{ref}^3,\) and \(X_{ref}^4\) signify the reference values of \(x_1, x_2, x_3,\) and \(x_4,\) respectively, and \(V_d\) is the reference output voltage.

The aim is to design a suitable non-linear control law to ensure the global stability of this converter when an uncertain load occurs.

3. Controller Design

In this section, on the basis of the averaged model given by (9)–(12), a non-linear SM is designed. The goal is to track the output \(x_4\) to its reference \(X_{ref}^4\).

3.1. Transformation into Canonical Form

The widely used averaged state-space model of the dc–dc converter is not suitable to design the proposed controller in order to achieve a large signal stability. To this end, the averaged state-space model given by (9)–(12) is converted into the canonical form such that the first state variable, \(p_1\), is the overall system’s energy and the second state variable, \(p_2\), is the difference between the input and output power [26]. This allows the disturbance variables \(\delta_1\) and \(\delta_2\) to be included in the model and, later, their observers can be used for the design of the SM controller. The model in the revised form is shown below (see Appendix C for detailed derivation):

\[
p_1 = p_2 + \delta_1 \quad (14)
\]
\[
p_2 = m + \delta_2 \quad (15)
\]

Here, \(p_1 = 0.5(L_1x_1^2 + L_2x_2^2 + C_1x_2^2 + C_2x_4^2)\) and \(p_2 = Ex_1 - x_4^2/R_o\), such that \(R_o\) is the system’s nominal resistance. Also, the mis-matched disturbance is given by \(\delta_1 = x_4^2/R_o - x_4^2/R\) and the matched disturbance is given by \(\delta_2 = (2/R_oC_2)(-x_4^2/R_o + x_4^2/R)\). The virtual control law is \(m = E^2/L_1 + 2x_4^2/R_o C_2 - (Ex_3/L_1 + x_2 x_4/R_o C_2)(1-k)\).

Next, this model is used for the controller design. From the expression of \(m\), the value of the control signal is obtained as:

\[
k = 1 - \frac{C_2 R_o^2 E^2 + 2L_1v_c^2 - L_1 C_2 R_o^2 m}{C_2 R_o^2 E v_c + 2L_1 R_0 i_{L_2} v_c} \quad (16)
\]

The original control objective that the actual voltage \(x_4\) follows the reference voltage, \(x_{ref}^4\), trajectory is now changed as the state-variables are modified. The new objective is that the state variables \(p_1\) and \(p_2\) follow their reference paths of \(p_{1ref}\) and \(p_{2ref}\), respectively, such that:
\[ p_{1\text{ref}} = 0.5\left( L_1 X_{1\text{ref}}^2 + L_2 X_{2\text{ref}}^2 + C_1 X_{3\text{ref}}^2 + C_2 X_{4\text{ref}}^2 \right) \] (17)

\[ p_{2\text{ref}} = E X_{1\text{ref}} - X_{4\text{ref}}^2 / R_o \] (18)

where \( X_{1\text{ref}}, X_{2\text{ref}}, X_{3\text{ref}}, \) and \( X_{4\text{ref}} \) are the reference values of \( x_1, x_2, x_3, \) and \( x_4, \) respectively. Since \( EX_{1\text{ref}} \) is the steady-state input power of the converter, and assuming a lossless converter, we can write:

\[ p_{2\text{ref}} = P_{ss} - X_{4\text{ref}}^2 / R_o \] (19)

\[ P_{ss} = X_{4\text{ref}}^2 / R \] (20)

\[ X_{1\text{ref}} = P_{ss} / E \] (21)

where \( P_{ss} \) is the actual steady-state output power of the converter. Also, it is to be noted that the disturbances \( \delta_1 \) and \( \delta_2 \) are assumed to be bounded such that:

\[ |\delta_i(t)| \leq \delta_i \] and \[ |\delta_i^*(t)| \leq \delta_i^* \] (22)

where \( \delta_i \) and \( \delta_i^* \) are positive constants.

3.2. SM Scheme Based on Current through Input Inductor

Initially, an observer to estimate the disturbances in \( \delta_1 \) and \( \delta_2 \) is written as [27]:

\[ \delta_1 = G_{d1} p_1 + \alpha_1 \] (23)

\[ \dot{\delta}_1 = -G_{d1} (p_2 + \delta_1) \] (24)

and

\[ \delta_2 = G_{d2} p_2 + \alpha_2 \] (25)

\[ \dot{\delta}_2 = -G_{d2} (m + \delta_2) \] (26)

where \( G_{d1} \) and \( G_{d2} \) are the constant gains of an observer and \( \alpha_1 \) and \( \alpha_2 \) are the auxiliary gains of an observer. Using (23)–(26), we obtain:

\[ \dot{\varepsilon}_{\delta_1} = -G_{d1} \varepsilon_{\delta_1} + \delta_1 \] (27)

Also:

\[ \dot{\varepsilon}_{\delta_2} = -G_{d2} \varepsilon_{\delta_2} + \delta_2 \] (28)

where \( \varepsilon_{\delta_1} = \delta_1 - \hat{\delta}_1 \) and \( \varepsilon_{\delta_2} = \delta_2 - \hat{\delta}_2 \) are the errors in the estimation of two disturbances. Next, considering the difficulty of measuring the actual output power, its estimation is obtained as:

\[ \hat{P}_{ss} = X_{4\text{ref}}^2 / R + \delta_{1\text{ref}} - \hat{\delta}_1 \] (29)

Now, \( \delta_{1\text{ref}} = X_{4\text{ref}}^2 / R_o - X_{4\text{ref}}^2 / R. \) Thus,

\[ \hat{P}_{ss} = X_{4\text{ref}}^2 / R_o - \hat{\delta}_1 \] (30)

Substituting (30) in (17) and (19), and using (21), the estimation of the reference values of new state variables is obtained using:
\( \hat{p}_{1\text{ref}} = \frac{1}{2} \frac{L_1}{E} \left( \frac{X_{4\text{ref}}^2}{R_o} - \delta_1 \right)^2 \right) + \frac{1}{2} C_{1} X_{3\text{ref}}^2 + \frac{1}{2} C_{2} X_{2\text{ref}}^2 + \frac{1}{2} C_{2} X_{4\text{ref}}^2 \) (31)

\( \hat{p}_{2\text{ref}} = -\delta_1 \) (32)

where \( \hat{p}_{1\text{ref}} \) is the estimation of \( p_{1\text{ref}} \) and \( \hat{p}_{2\text{ref}} \) is the estimation of \( p_{2\text{ref}} \).

Next, the form of the proposed SM controller for the regulation of the QB converter is proposed as:

\[ s = c e_{p_1} + e_{p_2} - \hat{p}_{1\text{ref}} \] (33)

where \( e_{p_1} = p_1 - \hat{p}_{1\text{ref}} \) and \( e_{p_2} = p_2 - \hat{p}_{2\text{ref}} \) are the errors in the new state variables \( p_1 \) and \( p_2 \), respectively, and \( c \) is the constant gain. In order to satisfy the condition that ‘s’ converges to zero, we obtain:

\[ m = -c (e_{p_1} - \hat{p}_{1\text{ref}}) + \hat{p}_{1\text{ref}} - \delta_1 - \delta_2 - K_{b_1} \text{sgn}(s) - K_{b_2} s \] (34)

where \( K_{b_1} \) and \( K_{b_2} \) are user-defined controller gains.

### 3.3. Global stability Analysis

Next, the Lyapunov function is selected such that \( V(s) \) is positive definite and the condition \( \dot{V}(s) \leq 0 \) can be satisfied for certain values of the controller gains. We need \( \dot{V}(s) \) to be negative definite to prove the asymptotic stability. Thus, let us define the Lyapunov function as given by (35).

\[ V(s) = \frac{1}{2} s^2 \] (35)

Using (14) and (15), (27) and (28), and (32)–(34), we obtain:

\[ \dot{s} = c e_{\delta_1} + e_{\delta_2} - K_{b_1} \text{sgn}(s) - K_{b_2} s \] (36)

Using (35) and (36), we obtain:

\[ \dot{V}(s) = s \dot{s} = s(c e_{\delta_1} + e_{\delta_2} - K_{b_1} \text{sgn}(s) - K_{b_2} s) \] (37)

Since \( s \cdot \text{sgn}(s) = |s| \), we obtain:

\[ \dot{V}(s) = -K_{b_1} |s| - K_{b_2} s^2 + s(c e_{\delta_1} + e_{\delta_2}) \] (38)

Thus:

\[ \dot{V}(s) \leq -K_{b_1} |s| - K_{b_2} s^2 + |s|(c e_{\delta_{1\text{max}}} + e_{\delta_{2\text{max}}}) \] (39)

where \( e_{\delta_{i\text{max}}} = \sup |e_{\delta_i}(t)|, t > 0 \) and \( i \in \{1, 2\} \). From (35), \( |s| = \sqrt{2V} \). Thus, (39) becomes:

\[ \dot{V}(s) \leq -\sqrt{2V} \left[ K_{b_1} + K_{b_2} |s| - (c e_{\delta_{1\text{max}}} + e_{\delta_{2\text{max}}}) \right] \] (40)

Now, \( \dot{V}(s) \leq 0 \), as long as \( (K_{b_1} + K_{b_2} |s|) \geq (c e_{\delta_{1\text{max}}} + e_{\delta_{2\text{max}}}) \).

Thus, the QB converter controlled by the proposed SM controller is asymptotically stable even when large parameter variations occur, if the controller gains are selected such that \( (K_{b_1} + K_{b_2} |s|) \geq (c e_{\delta_{1\text{max}}} + e_{\delta_{2\text{max}}}) \). Next, using \( s = 0 \) in (33), we obtain:

\[ e_{p_2} = -c e_{p_1} + \hat{p}_{1\text{ref}} \] (41)

Thus, using the definitions of \( e_{p_1} \), \( e_{p_2} \), and \( e_{\delta_1} \), and using (14), (32), and (41), we obtain:

\[ \dot{e}_{p_1} = -c e_{p_1} + e_{\delta_1} \] (42)

Finally, the dynamics of the controlled system are stated by:
\[
\dot{e} = Me + N\delta
\]  
(43)

where \( \dot{e} = [\dot{e}_{p_1}, \dot{e}_{\delta_1}, \dot{e}_{\delta_2}] \) and \( \delta = [\delta_1, \delta_2] \), and the matrices \( M \) and \( N \), are given by:

\[
M = \begin{bmatrix}
-c & 1 & 0 \\
0 & -G_{d_1} & 0 \\
0 & 0 & -G_{d_2}
\end{bmatrix}
\quad \text{and} \quad
N = \begin{bmatrix}
0 & 0 \\
1 & 0 \\
0 & 1
\end{bmatrix}
\]

Here, \( M \) is the stability matrix of the system. A square matrix, \( M \), is called a Hurwitz matrix if all eigenvalues of \( M \) have strictly negative real parts, i.e., \( \text{Re}[\lambda_i] < 0 \), where \( \lambda_i \) is the \( i \)th eigen value. Since all of the constants in the matrix \( M \) viz. \( c, G_{d_1}, \) and \( G_{d_2} \) are positive constants and their coefficients are negative, the matrix \( M \) is a Hurwitz matrix. The system is globally stable [28]. Also, since there exists an input-to-state system of the form \( \dot{p} = f(p, \delta) \) which is globally stable and also the input of which satisfies the condition \( \lim_{t \to \infty} \delta(t) = 0 \), the system states satisfy the condition \( \lim_{t \to \infty} p(t) = 0 \). Thus, the system errors asymptotically converge such that \( \lim_{t \to \infty} e_{p_1}(t) = 0, \lim_{t \to \infty} e_{\delta_1}(t) = 0, \) and \( \lim_{t \to \infty} e_{\delta_2}(t) = 0 \).

Discussions: The key element in addressing the control of the proposed system is the eigenproblem. All negative coefficients in the stability matrix of the system dynamics indicate that all of the real parts of the eigen values are in the left half of the plane. This proves the bounded nature of the system as time approaches infinity. Such an eigenproblem has been previously used to validate the stability of several control systems, as described in [29–32].

4. Simulation Outcomes

In this section, some simulation outcomes are presented to validate the use of the proposed control scheme to regulate the QB converter. The control scheme was realized in MATLAB Simulink version 2022b. Figure 2 shows the block diagram of the control scheme’s realization. The converter parameters used were: \( E = 10 \), \( L_1 = 180 \) \( \mu \)H, \( L_2 = 180 \) \( \mu \)H, \( C_1 = 930 \) \( \mu \)F, \( C_2 = 930 \) \( \mu \)F, \( R = 100 \) \( \Omega \), and \( X_{\text{ref}} = 40 \) V. Also, the controller gains used were: \( G_{d_1} = 100, \ G_{d_2} = 100, \ c = 8000, \ K_{b_1} = 2000, \) and \( K_{p_2} = 500 \). Next, it is worth mentioning that the proposed controller is based on the pulse width modulation-based approach in which the control signal \( k \) is compared with a carrier sawtooth signal of fixed frequency and the resulting PWM signal is generated using a comparator. The switching frequency used was 100 KHz. The output PWM signal of a comparator then drives the switch of the quadratic boost converter. The control input is the duty signal given by (16).

Figure 2. Block diagram of the control scheme’s realization.

Initially, the effect of load disturbances on the output response was investigated. Figure 3a shows the system’s response when the load was varied by 50% from \( R = 100 \) \( \Omega \) to \( R = 150 \) \( \Omega \) at time \( t = 0.4 \) s, and then back to \( R = 100 \) \( \Omega \) at \( t = 0.6 \) s. The response in Figure 3a includes the response of the output voltage, the load current change, and a zoomed version of the control signal. It can be observed that the output quickly reached the reference voltage. The response has an overshoot of ~1% and settling time of ~0.01 s. The ability
of the converter to handle heavy load disturbances was also investigated and, to this end, the load was changed from 100% of its nominal value. Figure 3b shows the system’s response when the load was varied by 100% from $R = 100 \, \Omega$ to $R = 200 \, \Omega$ at time $t = 0.8 \, s$, and then back to $R = 100 \, \Omega$ at $t = 1 \, s$. Again, the output settled to its reference value with an overshoot of only $\sim 1.2\%$ and a settling time of $\sim 0.02 \, s$.

Figure 3. Load-change response (output voltage response: middle; output current response: center; zoomed control signal: bottom). (a) 50 % variation in load from $R = 100 \, \Omega$ to $R = 150 \, \Omega$ at time $t = 0.4 \, s$, and then back to $R = 100 \, \Omega$ at $t = 0.6 \, s$; (b) 100 % variation in load from $R = 100 \, \Omega$ to $R = 200 \, \Omega$ at time $t = 0.8 \, s$, and then back to $R = 100 \, \Omega$ at $t = 1 \, s$. 
Next, the ability of the closed-loop system to handle the input voltage variations was verified. First, the input battery voltage was changed by 20% from $E = 10 \, V$ to $E = 12 \, V$ at $t = 1.2 \, s$, and then back to $E = 10 \, V$ at $t = 1.4 \, s$. Figure 4a shows the converter’s response and the corresponding input voltage and control signal variables. Next, the supply was changed by 50% from $10 \, V$ to $14 \, V$ at $t = 1.6 \, s$ and then back to $10 \, V$ at $t = 1.8 \, s$. The response is depicted in Figure 4b. As can be observed, the response settles to its nominal value in $\approx 0.02 \, s$. 
Figure 4. Line change response (output voltage response: middle; input voltage response: center; zoomed control signal: bottom). (a) 20% change in input from $E = 10 \, \text{V}$ to $E = 12 \, \text{V}$ at $t = 1.2 \, \text{s}$ and then back to $E = 10 \, \text{V}$ at $t = 1.4 \, \text{s}$; (b) 50% change in input from $10 \, \text{V}$ to $14 \, \text{V}$ at $t = 1.6 \, \text{s}$ and then back to $10 \, \text{V}$ at $t = 1.8 \, \text{s}$.

Lastly, the capacity of the proposed SM-controlled system to handle reference voltage variations was investigated. Figure 5 shows the converter’s response, including the output voltage, disturbance variable, and control signal variables when the reference voltage was changed from $X_{\text{ref}} \rightarrow 40 \, \text{V}$ to $X_{\text{ref}} \rightarrow 45 \, \text{V}$ at $t = 2 \, \text{s}$ and then $X_{\text{ref}} \rightarrow 50 \, \text{V}$ at $t = 2.2 \, \text{s}$. All of these results verify the converter’s ability to tightly regulate the output voltage to its reference value.

Figure 5. Reference voltage change response of the system when reference voltage was changed from $X_{\text{ref}} \rightarrow 40 \, \text{V}$ to $X_{\text{ref}} \rightarrow 45 \, \text{V}$ at $t = 2 \, \text{s}$ and then $X_{\text{ref}} \rightarrow 50 \, \text{V}$ at $t = 2.2 \, \text{s}$.

5. Conclusions

In this article, a detailed design and analysis of a globally stable SM controller for the high-step-up quadratic boost converter is presented. The detailed controller design, including the derivation of the control signal, is presented. The proposed controller employs observer variables of the disturbances which estimate the changes in the system’s power and capacitor current. The main contribution of this paper is that the Lyapunov stability criterion is employed to validate the large signal stability of the SM-controlled QB converter. Also, the PWM-based SM control scheme has been employed to avoid the chattering effect. Finally, some simulation outcomes are shown to support the theoretical results. They validate the ability of the proposed controller to handle the load, line, and reference voltage variations. It is important to note that, while the suggested controller is used to control a quadratic boost converter, it can also easily be implanted in other high-order dc–dc converters to control their output voltage.
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Appendix A

Here, the proofs of Equations (1)–(4) are presented for the ‘ON’ state of the QB converter. In Figure 1b, applying KVL in the loop, including $E$, $L_1$, $D_1$, and closed switch $S$, we obtain:

$$ E - v_{L_1} = E - L_1 \frac{dx_1}{dt} = 0 \Rightarrow \frac{dx_1}{dt} = \frac{E}{L_1}. \quad (A1) $$

where $v_{L_1}$ is the voltage across inductor $L_1$, which is equal to $L_1 \frac{dx_1}{dt}$.

Similarly, applying KVL in the loop, including $C_1$, $L_1$, and closed switch $S$, we obtain:

$$ x_3 - v_{L_1} = x_3 - L_2 \frac{dx_2}{dt} = 0 \Rightarrow \frac{dx_2}{dt} = \frac{x_3}{L_2}. \quad (A2) $$

where $v_{L_2}$ is the voltage across inductor $L_2$, which is equal to $L_2 \frac{dx_2}{dt}$.

Next, if the current through capacitor $C_1$ is assumed as $i_{C_1}$, which is equal to $C_1 \frac{dx_1}{dt}$, then:

$$ i_{C_1} = -x_2 \Rightarrow C_1 \frac{dx_1}{dt} = -x_2 \Rightarrow \frac{dx_1}{dt} = -\frac{x_2}{C_1}. \quad (A3) $$

Similarly, if the current through capacitor $C_2$ is assumed as $i_{C_2}$, which is equal to $C_2 \frac{dx_2}{dt}$, then:

$$ i_{C_2} = -\frac{x_4}{R} \Rightarrow C_2 \frac{dx_2}{dt} = -\frac{x_4}{R} \Rightarrow \frac{dx_2}{dt} = -\frac{x_4}{RC_2}. \quad (A4) $$

Equations (A1)–(A4) prove Equations (1)–(4), respectively.

Appendix B

Here, the proofs of Equations (5)–(8) are presented for the ‘OFF’ state of the QB converter. In Figure 1c, applying KVL in the loop, including $E$, $L_1$, and $C_1$, we obtain:

$$ E - v_{L_1} - x_3 = 0 \Rightarrow E - L_1 \frac{dx_1}{dt} - x_3 = 0 \Rightarrow \frac{dx_1}{dt} = \frac{E-x_3}{L_1}. \quad (A5) $$

Also, applying KVL in the loop including $C_1$, $L_2$, and $C_2$, we obtain:

$$ x_3 - v_{L_2} - x_4 = x_3 - L_2 \frac{dx_3}{dt} - x_4 = 0 \Rightarrow \frac{dx_3}{dt} = \frac{x_3-x_4}{L_2}. \quad (A6) $$

Next, if the current through capacitor $C_1$ is assumed as $i_{C_1}$, which is equal to $C_1 \frac{dx_1}{dt}$, then:

$$ i_{C_1} = x_1 - x_2 \Rightarrow C_1 \frac{dx_3}{dt} = x_1 - x_2 \Rightarrow \frac{dx_3}{dt} = \frac{x_1-x_2}{C_1}. \quad (A7) $$

Similarly, if the current through capacitor $C_2$ is assumed as $i_{C_2}$, which is equal to $C_2 \frac{dx_3}{dt}$, then:
\[ i_{C_2} = x_2 - \frac{x_4}{R} \Rightarrow C_2 \frac{dx_4}{dt} = x_2 - \frac{x_4}{R} \Rightarrow \frac{dx_4}{dt} = \frac{x_2 - x_4}{RC_2} \]  
(A8)

Equations (A5)–(A8) prove Equations (5)–(8), respectively.

Appendix C

The transformed system’s first state variable is given as:

\[ p_1 = 0.5(L_1 x_1^2 + L_2 x_2^2 + C_1 x_3^2 + C_2 x_4^2) \]  
(A9)

Taking first derivative of (A9), we obtain:

\[ \dot{p}_1 = L_1 x_1 \dot{x}_1 + L_2 x_2 \dot{x}_2 + C_1 \dot{x}_3 + C_2 \dot{x}_4 \]  
(A10)

Substituting (1)–(4) in (A10), we obtain:

\[ \dot{p}_1 = x_1(-(1-k)x_3 + E) + x_2(-(1-k)x_4 + x_3) + x_3((1-k)x_1 - x_2) + x_4((1-k)x_2 - \frac{1}{R}x_4) \]  
(A11)

Simplifying (A11), we obtain:

\[ \dot{p}_1 = E x_1 - \frac{x_4^2}{R} \]  
(A12)

Using the transformed system’s second state variable as \( p_2 = E x_1 - x_4^2/R_o \), we obtain:

\[ \dot{p}_2 = \frac{x_4^2}{R_o} - \frac{x_4^2}{R} \]  
(A13)

Now, if disturbance variable \( \delta_1 = \frac{x_4^2}{R_o} - \frac{x_4^2}{R} \) we obtain:

\[ \bar{p}_1 = p_2 + \delta_1 \]  
(A14)

Next, from \( p_2 = E x_1 - x_4^2/R_o \) as defined in (A12), we obtain:

\[ \dot{p}_2 = E \dot{x}_1 - 2 \frac{x_4}{R_o} \dot{x}_4 \]  
(A15)

Substituting (1) and (4) in (A15) we obtain:

\[ \dot{p}_2 = E \left(- \frac{1}{L_1} (1-k) x_3 + \frac{1}{L_1} E \right) - 2 \frac{x_4}{R_o} \left( \frac{1}{C_2} (1-k) x_2 - \frac{1}{RC_2} x_4 \right) \]  
(A16)

Solving (A16), we obtain:

\[ \dot{p}_2 = m + \delta_2 \]  
(A17)

where \( m = E^2/L_1 + 2x_4^2/R_o C_2 - (E x_4/L_1 + 2x_2 x_4/R_o C_2) (1-k) \) and \( \delta_2 = (2/R_o C_2)(-x_4^2/R_o + x_4^2/R) \).

References


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