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Vibration Suppression of an Input-Constrained Wind Turbine Blade System

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Abstract: During the actual wind power generation process, wind turbines are often affected by side effects such as blade vibrations, input constraints, and actuator faults. This can lead to a reduction in power generation efficiency and even result in unforeseen losses. This study discusses a robust adaptive fault-tolerant boundary control approach to address the issues of input-constrained and actuator-fault problems in wind turbine blade vibration control. By employing projection mapping techniques and hyperbolic tangent functions, a novel robust adaptive controller based on online dynamic updates is constructed to constrain vibrations, compensate for unknown disturbance upper bounds, and ensure the robustness of the coupled system. Additionally, considering the possibility of actuator faults during the control process, a fault-tolerant controller is proposed to effectively suppress elastic vibrations in the wind turbine blade system even in the presence of actuator faults. The effectiveness of the proposed controller is validated through numerical simulations.

Keywords: active vibration control; wind turbine blades; fault-tolerant control; input constraints

MSC: 93C40

1. Introduction

In recent years, the wind power industry has experienced rapid growth [1–3]. Scholars have been actively investigating the appropriate control of wind turbines to optimize wind energy capture during power generation. The blades play a crucial role in absorbing energy from the wind. However, the presence of aeroelastic flutter in the generator can lead to blade vibrations during rotation [4,5]. Severe flutter poses a significant risk of turbine damage. To address this issue, it is essential to conduct an effective dynamic analysis of wind turbine blades and establish efficient control strategies for suppressing blade flutter. Various methods have been proposed to mitigate blade vibrations, including tuned liquid column dampers, tuned mass dampers, and vibration controllers based on active equipment. Compared to the first two methods, active control offers higher flexibility and robustness. For example, in [6], a novel bandwidth selection methodology for the Vold–Kalman filter was proposed to address the issue of unreliable monitoring in wind turbine systems. A significant scheme was proposed for Vold–Kalman filter bandwidth selection to guarantee the consistency and accuracy of the offshore wind turbine condition monitoring process, ensuring reliable fault diagnosis in [7]. Both of the above-mentioned methods can effectively address the vibration issues in wind turbine systems. However, the modes considered are limited. In order to consider all modes of the system, we model the wind turbine blade as a distributed parameter system (DPS) [8–10] in this study and adopt a boundary control (BC) scheme [11,12] to consider all system modes, effectively avoiding issues related to mode truncation and overflow.

Compared to other active control strategies, boundary control (BC) offers the advantage of requiring fewer sensors and actuators during its implementation. As a result, it has drawn significant interest from researchers in recent years. For instance, in [13], BC...
technology was applied to a tall building incorporating an active vibration control system, effectively dealing with input and output constraints. Similarly, in [14], a single limit controller was utilized for the flexible tail of an autonomous robotic fish to achieve complex oscillating bodily actions. In [15], an innovative adaptive offset approach was proposed for a flexible manipulator to handle unpredicted system dynamics and a countless number of time-varying actuator faults. Moreover, in [16], a unique out-of-sync boundary control design was introduced for Markov jump reaction-diffusion neural networks. Despite these impressive developments, the research on BC methods specifically aimed at suppressing wind turbine blade vibrations is still limited, which serves as the driving force behind the motivation for the present study.

Numerous techniques have been developed to address the issue of actuator input constraints, catering to both linear and nonlinear systems. For instance, in [17], a fresh formulation was introduced to enhance tracking performance while ensuring stability, even when dealing with various constraints and unaccounted dynamics. In [18], a distributed cooperative control method for multiple unmanned aerial vehicles (UAVs) with directed topology was studied. Similarly, in [19], the obstacle of automatic carrier landing of a UAV was investigated, taking into account external disturbance and input constraints. Moreover, the distributed formation tracking control problem with multiple leaders under actuator faults and constraints was addressed in [20]. However, despite these developments, there has been limited exploration in addressing the collective influence of input constraints in flexible wind turbine blade systems. This research gap has motivated our study, driving us to seek a solution to this specific problem.

During practical production and control processes, actuators may encounter unknown faults, such as partial failures or getting stuck. These faults can lead to changes in system parameters, causing the actual actuator output to deviate from the desired set value and adversely affecting the overall control performance. To address these issues, a well-designed fault-tolerant control (FTC) scheme becomes essential. In recent research, various adaptive FTC approaches have been presented for different systems. In [21], an adaptive quantized FTC was proposed for a 2-DOF helicopter system with actuator failures and dead-zone nonlinearity. Additionally, an innovative adaptive fuzzy FTC was proposed for a three-dimensional riser-vessel system under input backlash nonlinearity [22]. These advancements represent significant progress in the field of FTC. However, the control design for a wind turbine blade system with input constraints, actuator faults, and unknown disturbances remains an unresolved issue, which motivates further investigation efforts to develop effective solutions in this area.

This paper aims at suppressing the instability of a wind turbine blade system under the input constraint and actuator faults. The following presents a concise summary of the primary contributions: (i) Compared to reference [23], this paper takes into consideration the distributed disturbances affecting the blades and updates the model accordingly. This update improves the fidelity of the system’s dynamics, making it more in line with the actual real-world model. (ii) A novel approach by utilizing the hyperbolic tangent function (HTF) is introduced for constructing adaptive controllers. Additionally, the controllers are coupled with dynamically updated adaptive laws based on the projection mapping operator (PMO), effectively suppressing vibrations and ensuring the robustness of the system. (iii) This study effectively addresses the inherent vibration problem in wind turbine blades, and the proposed approach ensures that the system control does not exhibit any spillover effect.

2. Problem Statement

2.1. System Description

Figure 1 depicts a wind turbine system subjected to unknown external disturbances. In this paper, we consider the dynamic model of the flexible wind turbine blade whose governing equations are as follows [23]
\[ \xi y_b \varepsilon \ddot{A}(\tau, t) - \varphi \dddot{Z}(\tau, t) - \Pi B R \dddot{Z}(\tau, t) + f_b(\tau, t) = 0, \quad (1) \]

\[ T R A''(\tau, t) + \xi y_b \varepsilon \ddot{Z}(\tau, t) + \Pi T R A''(\tau, t) - Y \ddot{A}(\tau, t) - \psi f_b(\tau, t) = 0. \quad (2) \]

The boundary conditions governing the tip vibration as presented:

\[ B R \dddot{Z}(\tau, t) + \Pi B R \dddot{Z}(\tau, t) - f_1(t) - f_1(t) = 0, \quad (3) \]

\[ T R A'(\tau, t) + \Pi T R A'(\tau, t) - f_2(t) - f_2(t) = 0, \quad (4) \]

\[ Z(0, t) = \dot{Z}(0, t) = \ddot{Z}(0, t) = \Lambda(0, t) = 0, \quad (5) \]

where \( y_b \) denotes the distance from the blade’s center of mass to its shear center. The aerodynamic center offset from the shear center of the blade is represented by \( \psi \). \( \varphi \) signifies the density of the blade, \( \xi \) represents the bending rigidity, \( Y \) denotes the axial moment of inertia of the flexible blade cross-section, \( B R \) represents the torsion rigidity, \( T R \) is the torsion rigidity, \( f_b(\tau, t) \) denotes the distributed disturbance, \( \mathbb{R} \) denotes the set of real numbers, \( f_1(t) \) and \( f_2(t) \) symbolize the control inputs, and \( \Pi \) indicates the damping coefficient. Additionally, \( f_1(t) \) and \( f_2(t) \) are the unidentified disturbances at the blade tip \( \tau = \tau \).

![Figure 1. The wind turbine system.](image)

**Remark 1.** To enhance clarity, the following symbol substitutions are used: \( \dot{\cdot} = \partial(\cdot)/\partial t \), \( \cdot' = \partial(\cdot)/\partial \tau \), \( \cdot'' = \partial^2(\cdot)/\partial \tau^2 \), and \( \cdot'' = \partial^2(\cdot)/\partial \tau^2 \).

### 2.2. Actuator Fault

This paper considers the form of the actuator faults as described

\[ F_j(t) = \mu_j F_{d_j}(t) + F_{f_j}(t), \quad (6) \]

where \( 0 < \mu_i \leq 1, j = 1, 2 \), represent the extent of actuator faults, \( F_{d_j}, j = 1, 2 \), denote the ideal inputs of the actuator, and \( F_{f_j}, j = 1, 2 \) are bounded and unknown functions, which represent the floating faults. Thus, combining (6), the boundary conditions (3) and (4) can be rewritten as

\[ B R \dddot{Z}(\tau, t) + \Pi B R \dddot{Z}(\tau, t) - \mu_1 F_{d_1}(t) - F_{f_1}(t) - f_1(t) = 0, \quad (7) \]

\[ T R A'(\tau, t) + \Pi T R A'(\tau, t) - \mu_2 F_{d_2}(t) - F_{f_2}(t) - f_2(t) = 0. \quad (8) \]
We define the “disturbance-like” terms $v_j(t), j = 1, 2$, including external disturbances $f_j(t), j = 1, 2$, and the floating faults $F_{f_j}(t), j = 1, 2$.

**Remark 2.** Equation (6) represents a conventional actuator fault expression. The parameters $\mu_j$, where $j = 1, 2$, determine the efficiency of the actuators. When $\mu_j = 1$, it explains that the actuators are under normal operation. In contrast, if $\mu_j = 0$, it illustrates the actuators have lost all power.

2.3. Preliminaries

**Assumption 1.** We make the assumption that there are positive constants $\Delta_1, \Delta_2, \text{ and } f_{\text{limax}}$ such that $|v_1(t)| \leq \Delta_1, |v_2(t)| \leq \Delta_2, \text{ and } |f_j(\tau, t)| \leq f_{\text{limax}}$, for all $(\tau, t) \in [0, \tau] \times [0, +\infty)$.

**Lemma 1** ([24]). By assuming the existence of $\alpha_1(\tau, t)$ and $\alpha_2(\tau, t) \in \mathbb{R}$, and $(\tau, t) \in [0, \tau] \times [0, +\infty)$, and considering $m > 0$, the following expression can be derived

\[
\alpha_1\alpha_2 \leq |\alpha_1\alpha_2| \leq \alpha_1^2 + \alpha_2^2, \tag{9}
\]

\[
\alpha_1\alpha_2 \leq \frac{1}{m} \alpha_1^2 + m\alpha_2^2. \tag{10}
\]

**Lemma 2** ([24]). Assuming the existence of $\alpha(\tau, t) \in \mathbb{R}$, and $(\tau, t) \in [0, \tau] \times [0, +\infty)$, which satisfying the condition $\alpha(0, t) = 0$, we can express it as follows

\[
\alpha^2 \leq \tau \int_0^\tau \alpha^2 d\tau, \tag{11}
\]

where $(\tau, t) \in [0, \tau] \times [0, +\infty)$.

**Lemma 3** ([24]). For every $\alpha(t) \in \mathbb{R}$, the subsequent inequality holds

\[
0 < |\alpha| - \alpha \tanh(\alpha) \leq \beta, \tag{12}
\]

where $\beta = 0.2785$.

3. Control Design

First, we establish the “disturbance-like” components, encompassing external disturbances and the floating faults. Subsequently, an adaptive FTC strategy is formulated to compensate for the actuator fault, handle the composite disturbance, dampen vibration, and maintain displacements within a predefined range.

The control inputs are designed as

\[
F_{d1}(t) = \rho_1\hat{p}_1[v_1\Xi(\tau, t) + v_2\hat{\Xi}(\tau, t)] - \hat{p}_1\tanh[v_1\Xi(\tau, t) + v_2\hat{\Xi}(\tau, t)]\hat{\Delta}_1, \tag{13}
\]

\[
F_{d2}(t) = -\rho_2\hat{p}_2[v_1\Lambda(\tau, t) + v_2\hat{\Lambda}(\tau, t)] - \hat{p}_2\tanh[v_1\Lambda(\tau, t) + v_2\hat{\Lambda}(\tau, t)]\hat{\Delta}_2, \tag{14}
\]

where $v_1, v_2, \rho_1, \text{ and } \rho_2$ are positive numbers. Let $p_j = \frac{1}{\hat{p}_j}, j = 1, 2$, and $\hat{p}_j, j = 1, 2$, denote the estimated values of $p_j$, while the estimated errors are identified as $\hat{p}_j = p_j - \hat{p}_j, j = 1, 2$. $\hat{\Delta}_j, j = 1, 2$, denote the estimated values of $\Delta_j, j = 1, 2$, while the estimated errors are identified as $\hat{\Delta}_j = \Delta_j - \hat{\Delta}_j, j = 1, 2$.

**Step 1.** A novel robust adaptive control using PMO and HTF is employed to guarantee the stabilization of the wind turbine blade system.

\[
\hat{\Delta}_1(t) = -\gamma_1[v_1\Xi(\tau, t) + v_2\hat{\Xi}(l, t)]\tanh[v_1\Xi(\tau, t) + v_2\hat{\Xi}(\tau, t)] - \gamma_1\hat{\Delta}_1(t), \tag{15}
\]
where we can arrive at
\[
\dot{\Lambda}_2(t) = -\gamma_2[v_1 \Lambda(t, t) + v_2 \dot{\Lambda}(t, t)] \tanh [v_1 \Lambda(t, t) + v_2 \dot{\Lambda}(t, t)] - \gamma_2 \dot{\Lambda}_2(t),
\] (16)

\[
\dot{p}_1(t) = \text{Proj}_{\dot{p}_1} \{ \gamma_3[v_1 \Xi(t, t) + v_2 \dot{\Xi}(t, t)] \times \{ p_1 \text{tanh}[v_1 \Xi(t, t) + v_2 \dot{\Xi}(t, t)] \} \} - \dot{p}_1,
\] (17)

\[
\dot{p}_2(t) = \text{Proj}_{\dot{p}_2} \{ \gamma_4[v_1 \Lambda(t, t) + v_2 \dot{\Lambda}(t, t)] \times \{ p_2 \text{tanh}[v_1 \Lambda(t, t) + v_2 \dot{\Lambda}(t, t)] \} \} - \dot{p}_2,
\] (18)

where \( \gamma_i, i = 1, \ldots, 4 \) are positive numbers, and \( \text{Proj}_{\dot{p}_j}, j = 1, 2 \), denote the PMO, which restricts the parameters \( \dot{p}_j, j = 1, 2 \), within the interval \([p_j, \bar{p}_j], j = 1, 2 \) [22].

**Step 2.** A Lyapunov function \( \Omega(t) \) is selected as
\[
\Omega(t) = \Omega_1(t) + \Omega_2(t) + \Omega_3(t),
\] (19)

where
\[
\Omega_1(t) = \frac{v_2}{2} \int_0^T [\dot{\Xi}(\tau, t)]^2 \, d\tau + \frac{v_2}{2} BR \int_0^T [\Xi''(\tau, t)]^2 \, d\tau + \frac{v_2}{2} Y \int_0^T [\dot{\Lambda}(\tau, t)]^2 \, d\tau
\]
\[+ \frac{v_2}{2} TR \int_0^T [\Lambda'(\tau, t)]^2 \, d\tau,
\] (20)

\[
\Omega_2(t) = v_1 \gamma_3 \int_0^T A(\xi, t) \Xi(\tau, t) \, d\tau + v_1 Y \int_0^T \Lambda(\xi, t) \Lambda(\xi, t) \, d\tau
\]
\[+ v_1 \gamma_3 \int_0^T \Lambda(\xi, t) \Xi(\tau, t) + \Lambda(\xi, t) \Xi(\tau, t) \, d\tau
\]
\[+ v_1 \gamma_4 \int_0^T \Xi'(\xi, t) \Lambda'(\xi, t) \, d\tau,
\] (21)

\[
\Omega_3(t) = \frac{1}{2\gamma_1} \dot{\Lambda}_1^2 + \frac{1}{2\gamma_2} \dot{\Lambda}_2^2 + \frac{\mu_1}{2\gamma_3} \rho_1^2 + \frac{\mu_2}{2\gamma_4} \rho_2^2
\] (22)

**Remark 3.** In this context, \( \Omega_1(t) \) is derived from the system’s kinetic energy \( E_k(t) \) and potential energy \( E_p(t) \) and is also referred to as the energy term. The term \( \Omega_2(t) \) arises from the coupling of various state variables within the system, manifesting as a cross-term. Additionally, \( \Omega_3(t) \) is associated with auxiliary states, denoted as the auxiliary term, representing auxiliary components within the system designed to address disturbance errors.

**Theorem 1.** The function (19) is verified to be positive
\[
0 \leq \varrho_2[\zeta(t) + \Omega_3(t)] \leq \varrho_1[\zeta(t) + \Omega_3(t)],
\] (23)

where \( \varrho_1 \) and \( \varrho_2 > 0 \).

**Proof.** An auxiliary function is presented as
\[
\zeta(t) = \int_0^T \left[ (\dot{\Xi}(\tau, t))^2 + (\Xi''(\tau, t))^2 + (\dot{\Lambda}(\tau, t))^2 + (\Lambda'(\tau, t))^2 \right] \, d\tau.
\] (24)

Thus, we have
\[
\varrho_2 \zeta(t) \leq \Omega_1(t) \leq \varrho_1 \zeta(t),
\] (25)

where \( \varrho_1, \varrho_2 > 0 \), and \( \varrho_1 = \frac{\varphi}{\mu_1} \max \{ TR, Y, \zeta, BR \} \), and \( \varrho_2 = \frac{\varphi}{\mu_2} \min \{ TR, BR, \zeta, Y \} \). For \( \Omega_2(t) \), we can arrive at
\[ |\Omega_2(t)| \leq v_1 \left\{ \int_0^T [\Xi(\alpha, t)]^2 d\alpha + \tau^4 \int_0^T [\Xi''(\alpha, t)]^2 d\alpha \right\} \\
+ v_1 Y \left\{ \int_0^T [\Lambda(\alpha, t)]^2 d\alpha + \tau^2 \int_0^T [\Lambda'(\alpha, t)]^2 d\alpha \right\} \\
+ v_1 \xi_h c \left\{ \int_0^T [\Xi(\alpha, t)]^2 d\alpha + \int_0^T [\Lambda(\alpha, t)]^2 d\alpha \right\} \\
+ v_1 \xi_h c \left\{ \tau^4 \int_0^T [\Xi''(\alpha, t)]^2 d\alpha + \tau^2 \int_0^T [\Lambda'(\alpha, t)]^2 d\alpha \right\} \\
+ v_2 \xi_h c \left\{ \int_0^T [\Xi(\alpha, t)]^2 d\alpha + \int_0^T [\Lambda(\alpha, t)]^2 d\alpha \right\} \\
= (v_1 + v_1 \xi_h c + v_2 \xi_h c) \int_0^T [\Xi(\alpha, t)]^2 d\alpha + (v_1 + v_1 \xi_h c) \tau^4 \int_0^T [\Xi''(\alpha, t)]^2 d\alpha \\
+ (v_1 + v_1 \xi_h c + v_2 \xi_h c) \int_0^T [\Lambda(\alpha, t)]^2 d\alpha + (v_1 + v_1 \xi_h c) \tau^2 \int_0^T [\Lambda'(\alpha, t)]^2 d\alpha \\
\leq \varrho_3 \zeta(t), \tag{26} \]

where \( \varrho_3 = \max\{v_1 + v_1 \xi_h c + v_2 \xi_h c, (v_1 + v_1 \xi_h c) \tau^4, v_1 Y + v_1 \xi_h c + v_2 \xi_h c, (v_1 + v_1 \xi_h c) \tau^2\} \) and \( v_2 > \frac{2 \varrho_3}{\min\{BR, Y, TR, \xi\}} \).

Then, (23) can be proved:

\[ 0 \leq \varrho_2 [\zeta(t) + \Omega_3(t)] \leq \Omega(t) \leq \varrho_1 [\zeta(t) + \Omega_3(t)], \tag{27} \]

where \( \varrho_2 = \varrho_2 - \varrho_3 \) and \( \varrho_1 = \varrho_1 + \varrho_3 \).

**Step 3.** Moreover, the \( \Omega_1(t) \) is given as

\[ \Omega_1(t) \leq -v_2 \Xi(\alpha, t)[BR \Xi''(\alpha, t) + PIBR \Xi''(\alpha, t)] + v_2 \Lambda(\alpha, t)[TRA' (\alpha, t) + PIBR \Xi''(\alpha, t)] \\
+ v_2 \xi_h c \int_0^T [\Lambda(\alpha, t) \Xi(\alpha, t)] d\alpha - \frac{v_2 PIBR}{2} \int_0^T [\Lambda''(\alpha, t)]^2 d\alpha \\
- \frac{v_2 PIBR}{2 \tau^4} - \frac{v_2 \Xi_h c}{2 \tau^2} - \frac{v_2 \Xi_h c}{2} \int_0^T [\Lambda''(\alpha, t)]^2 d\alpha \\
+ \frac{v_2 \Xi_h c}{2} \tau f_{\text{max}} - \frac{v_2 PIBR}{2} \int_0^T [\Xi''(\alpha, t)]^2 d\alpha. \tag{28} \]

Next, we differentiate \( \Omega_2(t) \) and get

\[ \Omega_2(t) = v_1 Y \int_0^T \hat{\Lambda}(\alpha, t) \Lambda(\alpha, t) d\alpha + v_1 \xi_h c \int_0^T \hat{\Xi}(\alpha, t) \Xi(\alpha, t) d\alpha \\
- v_2 \xi_h c \int_0^T \hat{\Lambda}(\alpha, t) \Xi(\alpha, t) d\alpha - v_1 \xi_h c \int_0^T \hat{\Xi}(\alpha, t) \Lambda(\alpha, t) d\alpha \\
- v_1 \xi_h c \int_0^T \Xi(\alpha, t) \hat{\Lambda}(\alpha, t) d\alpha + v_1 Y \int_0^T \hat{\Xi}(\alpha, t) \Xi(\alpha, t) d\alpha \\
- 2 v_1 \xi_h c \int_0^T \hat{\Xi}(\alpha, t) \Xi(\alpha, t) d\alpha - v_2 \xi_h c \int_0^T \hat{\Lambda}(\alpha, t) \Xi(\alpha, t) d\alpha. \tag{29} \]

Similarly, we can obtain the \( \Omega_3(t) \)

\[ \Omega_3(t) = -\frac{1}{\gamma_1} \hat{\Delta}_1 \beta_1 - \frac{1}{\gamma_2} \hat{\Delta}_2 \beta_2 - \frac{\mu_1}{\gamma_3} \tilde{p}_1 - \frac{\mu_2}{\gamma_4} \tilde{p}_2. \tag{30} \]

Considering \( \hat{p}_j(\alpha) - \tilde{p}_j \text{Proj}_{\tilde{p}_j}(\alpha) \leq 0, j = 1, 2 \), we derive
\[ \dot{\Omega}(t) \leq -\rho_2|v_1\lambda(t) + v_2\zeta(t)|^2 - \rho_1|v_1\Xi(t) + v_2\zeta(t)|^2 - (v_1BR - \frac{v_1\Pi BR}{\eta_3}) - \eta_6 v_1^4 \int_0^T [\Xi''(\lambda(t))]^2 d\lambda - (v_1TR - \frac{v_1\Pi TR}{\eta_4} - \eta_7 \tau^2 \psi) \int_0^T [\lambda'(\lambda(t))]^2 d\lambda \\
- \frac{(v_2\Pi BR)}{2} - \frac{(v_1\Pi BR)}{2} \int_0^T [\Xi''(\lambda(t))]^2 d\lambda - \frac{(v_2\Pi TR)}{2} \int_0^T [\lambda'(\lambda(t))]^2 d\lambda \\
- \frac{(v_1\Pi BR)}{2t^4} - \eta_1v_2 - v_1\zeta - 2v_1\zeta y_5 \int_0^T [\Xi(\lambda(t))]^2 d\lambda - \frac{\gamma_1}{2} \bar{\lambda}_1 - \frac{\gamma_2}{2} \bar{\lambda}_2 - \frac{\mu_1}{2} \bar{\rho}_2^2 \\
+ \left( \frac{\nu_1}{\eta_3} + \frac{\nu_2}{\eta_2} + \frac{\nu_1}{\eta_6} + \frac{\nu_2}{\eta_7} + \frac{\nu_3}{\eta_5} \right) \tau f_2^{h_{\text{max}}} + m(\Delta_1 + \Delta_2) + \frac{\mu_1}{2} p_2^2 + \frac{\mu_2}{2} p_2^2 \leq -\sigma_3\zeta(t) + \omega, \quad (31) \]

where \( \eta_i > 0, i = 1 \ldots 7. \)

The intermediate parameters are chosen to satisfy

\[ \theta_1 = \frac{v_2\Pi BR}{2t^4} - \eta_1v_2 - v_1\zeta - 2v_1\zeta y_5 \zeta y_5 > 0, \quad (32) \]
\[ \theta_2 = \frac{v_2\Pi TR}{2t^2} - \eta_2v_2\psi - v_1\zeta - 2v_1\zeta y_5 \frac{\eta_5}{\eta_2} > 0, \quad (33) \]
\[ \theta_3 = v_1BR - \frac{v_1\Pi BR}{\eta_3} - \eta_6 v_1^4 > 0, \quad (34) \]
\[ \theta_4 = v_1TR - \frac{v_1\Pi TR}{\eta_4} - \eta_7 \tau^2 \psi > 0, \quad (35) \]

\[ \omega = \omega_0 \tau f_2^{h_{\text{max}}} + \frac{\gamma_2}{2} \bar{\lambda}_2^2 + \frac{\gamma_1}{2} \bar{\lambda}_1^2 + \frac{\mu_1}{2} p_2^2 + \frac{\mu_2}{2} p_2^2 + m(\Delta_1 + \Delta_2) < +\infty, \quad (36) \]

Therefore, we can obtain

\[ \dot{\Omega}(t) \leq -\sigma\Omega(t) + \omega, \quad (37) \]

where \( \sigma = \sigma_3/\theta_1, \sigma_3 = \min\{\theta_1, \theta_2, \theta_3, \theta_4\} > 0. \)

**Theorem 2.** For the wind turbine blade system represented by (1)–(5) under the influence of the implemented adaptive controllers (13) and (14) and adaptive updating rules (15)–(18), assuming the designed parameters \( v_1, v_2, p_1, p_2, \) and \( \gamma_i, \) for \( i = 1, \ldots, 4, \) meeting the constraint conditions (32)–(36) and the initial conditions are bounded.

**Proof.** By using Equation (37) and multiplying both sides by \( e^{\theta t}, \) we obtain

\[ \dot{\Omega}(t)e^{\theta t} \leq -\theta\Omega(t)e^{\theta t} + \omega e^{\theta t}. \quad (38) \]

By integrating Equation (38), we get

\[ \Omega(t) \leq \left( \Omega(0) - \frac{\omega}{\theta} \right)e^{-\theta t} + \frac{\omega}{\theta} \leq \Omega(0)e^{-\theta t} + \frac{\omega}{\theta}. \quad (39) \]

Furthermore, by utilizing Equation (23) and applying Lemma 1, we obtain

\[ \frac{1}{\tau^3} |\Xi(\lambda(t))|^2 \leq \frac{1}{\tau^3} \int_0^T |\Xi'(\lambda(t))|^2 d\lambda \leq \int_0^T |\Xi''(\lambda(t))|^2 d\lambda \leq \zeta(t) \leq \frac{1}{\theta} \Omega(t). \quad (40) \]
Finally, we obtain
\[
| \Xi(\mathcal{C}, t) | \leq \sqrt{\frac{T^2}{\sigma_1^2} (\Omega(0) e^{-\theta t} + \Theta_1^2)} \forall (\mathcal{C}, t) \in [0, \tau] \times [0, +\infty), \quad (41)
\]
\[
| \Lambda(\mathcal{C}, t) | \leq \sqrt{\frac{T^2}{\sigma_2^2} (\Omega(0) e^{-\theta t} + \Theta_2^2)} \forall (\mathcal{C}, t) \in [0, \tau] \times [0, +\infty). \quad (42)
\]
\[
\blacksquare
\]

4. Simulation

Some numerical methods such as the finite difference method, the finite element method, and the finite volume method can be applied to get the approximate solution of the system (1)–(5) when there is no obtainable analytical solution. Among these methods, the finite difference method is relatively simple and well-established, and its advantages are as described in Remark 4. Therefore, in this paper, the finite difference method is chosen for conducting simulation experiments. The system parameters as listed below:
\[
BR = 3 \text{Nm}^2, \quad \psi = 0.05 \text{N}, \quad \zeta = 4 \text{kg/m}, \quad \tau = 2.0 \text{m}, \quad \gamma_{hc} = 0.25 \text{m}, \quad Y = 10 \text{kNm}, \quad \Pi = 0.6, \quad \text{and} \quad TR = 20 \text{Nm}^2.
\]
The initial states of the system are set to $\Xi(\mathcal{C}, 0) = \mathcal{C}/\tau$ and $\Lambda(\mathcal{C}, 0) = \frac{\pi}{2\tau}$, along with $\Xi(\mathcal{C}, 0) = \Lambda(\mathcal{C}, 0) = 0$. The external disturbances of the system are detailed as follows [23].
\[
f_1(t) = 0.02 + 0.06\sin(0.05t) \quad f_2(t) = 0.04 + 0.02\sin(0.1t).
\]

For simulating the actuator efficacy fault, we configure the efficiency factors as $\mu_1 = \mu_2 = 0.5$ at $t = 2.5$ s, and $F f_1(t) = F f_2(t) = \sin(0.1t)$.

Figures 2 and 3 depict the deformation of the flexible wind turbine blade without control, i.e., when $F_1(t) = 0$ and $F_2(t) = 0$. Figures 4 and 5 illustrate the stereoscopic representation of the flexible wind turbine blade with the proposed control. The controllers are parameterized with $\rho_1 = 2, \rho_2 = 1, \gamma_1 = 0.02, \gamma_2 = 5, \gamma_3 = 0.1,$ and $\gamma_4 = 3$. The input signal constraints in this paper are set as $F_1 = 3$ and $F_2 = 3.5$. We conducted simulations for the following two cases: (1) In case 1, we adopted the control scheme proposed as (44). (2) In case 2, we used the control laws (13)–(18) proposed in this paper for the simulation.

\[
F_{d1}(t) = \rho_1 [v_1 \Xi(\tau, t) + v_2 \Xi(\tau, t)] \quad F_{d2}(t) = -\rho_2 [v_1 \Lambda(\tau, t) + v_2 \Lambda(\tau, t)] \quad (44)
\]

When employing the control scheme proposed by [25], it becomes evident from Figures 6 and 7 that the control inputs required to achieve the same control objectives exceed the input constraints significantly. The actual inputs’ maximum value even reaches six times the input upper limit, which poses serious risks in practical engineering scenarios with input signal constraints. In contrast, the controllers proposed in this paper with input constraints, as shown in Figures 8 and 9, ensure that the tip forces $F_1$ and $F_2$ strictly remain within the specified constraints.

As observed in Figures 8 and 9, there is a noticeable jump in the control input at 2.5 s, which corresponds to the moment of the actuator fault. However, despite the actuator fault, the preset control targets can still be achieved, demonstrating the strong fault tolerance capability of the loop system.

Remark 4. The finite difference method is an approximate numerical solution that directly turns a differential problem into an algebraic problem. The mathematical concept is intuitive and the expression is simple. It is an early and mature numerical method. The finite difference method can solve many kinds of partial differential equations systems easily [26]. So in this manuscript, we use the finite difference method to finish the simulations.
Figure 2. Bending deformations without control.

Figure 3. Twist deformation without control.

Figure 4. Bending deformation with control.

Figure 5. Twist deformation with control.
5. Conclusions

In this study, an innovative and robust adaptive FTC strategy is proposed for a wind turbine blade system with constrained inputs, which is subjected to uncertainties in the upper bounds of external disturbances. The proposed adaptive FTC approach is specifically designed to attenuate vibrations and rectify actuator faults effectively. By incorporating PMO and HTF into the control method, the system’s robustness is assured,
and the inputs are strictly maintained within the imposed constraints. Through rigorous Lyapunov analysis, the controlled system’s uniformly bounded stability is ensured. The effectiveness of the control scheme is validated via comprehensive simulations, where the results are compared to assess the control performance. In the future, we intend to develop a neural network control or reinforcement learning control strategy to handle system uncertainties and external disturbances [27,28].


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