Nonlinear Functional Observer Design for Robot Manipulators

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Abstract: In this paper, a nonlinear functional observer (NFO) is first proposed for the control design of robot manipulators under model uncertainties, external disturbances, and a lack of joint velocity information. In principle, the proposed NFO can estimate not only lumped disturbances and uncertainties but also unmeasurable joint velocities, which are then fed back into the main controller. Compared to the well-known ESO design, the proposed NFO has a simpler structure, more accurate estimations, and less computational effort, and consequently, it is easier for practical implementation. Moreover, unnecessary observations of joint displacements are avoided when compared to the well-known extended state observer (ESO). Based on the Lyapunov theory, globally uniformly ultimately bounded estimation performance is guaranteed by the proposed NFO. Consequently, it is theoretically proven that the estimation performances of the NFO are better than those of the ESO. Simulations with a two-degree-of-freedom (2-DOF) robot manipulator are conducted to verify the effectiveness of the proposed algorithm in terms of not only the estimation performance but also the closed-loop control performance.

Keywords: robot manipulators; functional observer; disturbance observer; state observer

MSC: 93B35

1. Introduction

Due to their capability of performing repetitive tasks with high speeds and accuracies [1], robot manipulators have been developed and used in many fields, such as industrial automation [2], robotic surgery [3,4], and in-space missions [5]. Besides the development of hardware components, such as sophisticated mechanical mechanisms and electrical devices, control design plays an important role in ensuring that the robot manipulators operate precisely and achieve the desired performance. Recently, advanced control techniques have been proposed to deal with practical problems such as model uncertainties and external disturbances [6–15], output and state constraints [16–19], control input saturation [20–22], convergence time [23–27], and their combinations. Among them, studies focusing on uncertainty and disturbance attenuation are essential to guarantee nominal control performance before considering any further problems.

For a long time, disturbance/uncertainty estimation and attenuation for robot manipulators have been widely investigated. Originally, adaptive control was developed to deal with unknown kinematic and dynamic model parameters [28–30]. These parameters are estimated by adaptive laws and then fed back to the control law to achieve high-accuracy tracking performance. However, this performance is guaranteed by assuming that the structure of the model uncertainties is known and presented in linear-in-parameter forms with constant or slow-varying model parameters. To cope with these drawbacks, intelligent approximators such as neural networks (NNs) [6–8,31] and fuzzy logic systems (FLSs) [9,10,32] were adopted to handle unstructured uncertainties caused by uncertain parameters and unknown nonlinearities, such as nonlinear frictions. Basically, instead of
being described through regression functions, these uncertainties are approximated by linear combinations of weighted activation functions, and weighting factors are updated by adaptive laws. However, since the structure of these approximators is complicated with a lot of parameters to be selected, experts’ knowledge is required with heavy computational effort for the implementation. Furthermore, these intelligent approximators receive inputs as system states; therefore, they are no longer available to cope with state-independent terms, i.e., external disturbances.

Overcoming the above-mentioned issues, DOBs have been developed and widely applied to not only position but also force control for robot manipulators due to their ability to estimate lumped disturbances and uncertainties whose derivatives are assumed to be bounded [14,16,33–36]. For example, a generalized-momentum observer (GMO) was previously proposed to estimate the lumped force/torque disturbances based on the difference between the real robot momentum and its estimate [37]. However, since joint velocity information is necessary, velocity sensor installment is required, which is not cost-effective and complicates the hardware design. The same problem occurs when a nonlinear disturbance observer (NDOB) is applied since both joint displacements and velocities are demanded to achieve asymptotic disturbance estimation performance with the assumption that the disturbance is constant [38]. In [39,40], time delay estimation (TDE) has been designed for robot manipulators based on the assumption that the lumped disturbances in the current time step can be approximated by those in the previous time step. However, since the acceleration information is used in the calculation, further acceleration measurements or observations are required. To reduce the system cost and the complexity of the overall control system, the ESO has been proposed to estimate both lumped disturbances and uncertainties and joint velocities simultaneously [16,41]. However, the order of the ESO, i.e., the order of the set of differential equations, is higher than that of the original system model, which requires high computational costs. Furthermore, the estimations of joint displacements are unnecessary since their measurements are available. In recent years, there have been several new DOBs [34,42] developed for robotic applications that partially rely on the above-mentioned DOB techniques. However, the designs are quite conservative and coupled with the designed control laws; therefore, they are difficult to inherit and apply for a general control structure. Some recent studies have utilized the frequency-domain DOB design for robot manipulators, which is not suitable due to the highly nonlinear dynamics of the system [14,43]. Overall, the problem of developing an observer to efficiently estimate both the state and disturbances of robot manipulators is still challenging.

A similar problem was considered when the concepts of reduced-order observers or functional observers (FOs) were introduced to deal with system state observation [44–46]. In principle, FOs are designed to estimate a linear combination of system states with the minimum effort, i.e., estimations of excessive states as the system outputs are avoided and the order of the observer is chosen to be as low as possible. In the beginning, FOs were proposed for linear time-invariant systems [47,48]. Many studies have been conducted to find the existent conditions [44,49] and the minimum order of FOs [45]. After that, the applications of FOs have been extended to linear systems with time-varying parameters [50], time delays [51,52], and unknown inputs [53,54]. For nonlinear systems, it is assumed that Lipschitz conditions hold for nonlinear dynamic functions to guarantee the estimation performance of FOs [55–57]. Some studies considered the FO design for discrete-time systems [58–61]. However, from the authors’ point of view, only a few studies extended the application of the FOs to estimating lumped disturbances and uncertainties [60,62]. Furthermore, these DOBs are limited to disturbance estimation in linear systems, not both state and disturbance estimation in nonlinear systems as in robot manipulators.

Based on the above-mentioned analysis, in this paper, an NFO is first proposed for robot manipulators to estimate both joint velocities and lumped uncertainties and disturbances. Technically, compared to the GMO, NDOB, and TDE, the proposed observer not only does not require further state estimation but also provides it to the main controller.
Furthermore, the NFO requires less computational effort than the ESO since the estimations of joint angles are eliminated and the order of the observer is reduced. The stability of the proposed observer is proven based on the Lyapunov theorem. Finally, comparisons by both proof theory and simulation with a 2-DOF manipulator are conducted to validate not only the estimation performance but also its effects on the closed-loop control performance of the proposed observer compared to those of the ESO.

The rest of the paper is organized as follows: The system modeling is presented in Section 2. Nonlinear functional observer design and its stability analysis are given in Section 3. The theoretical comparison with the ESO is conducted in Section 4. Numerical simulations are described in Section 5. Finally, Section 6 concludes this work.

2. System Modeling

The dynamics of an n-DOF robot manipulator are given by:

\[ M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = u + \tau_d \]  

where \( q \in \mathbb{R}^n \), \( M(q) \in \mathbb{R}^{n \times n} \), \( C(q, \dot{q}) \in \mathbb{R}^{n \times n} \), and \( G(q) \in \mathbb{R}^n \) are the vector of joint displacements, inertia matrix, Coriolis and centrifugal vector, and gravity vector, respectively. \( u \in \mathbb{R}^n \) is the control force/torque vector and \( \tau_d \in \mathbb{R}^n \) is the vector of lumped disturbances and uncertainties existing in the system.

To design the proposed observer more conveniently, the manipulator dynamics are rewritten in the following form:

\[ \begin{align*}
    \dot{x}_1 &= x_2 \\
    \dot{x}_2 &= f_1(x_1)u + f_2(x_1, x_2) + d
\end{align*} \]  

where \( x_1 = q, x_2 = \dot{q}, d = M(x_1)^{-1}\tau_d, f_1 = M(x_1)^{-1}, \text{and } f_2 = -M(x_1)^{-1}(C(x_1, x_2)x_2 + G(x_1)) \).

Assumption 1. There exists a constant \( \alpha > 0 \) so that the following Lipchitz condition holds [16,34]:

\[ \|f_2(x_1, x_2 + \Delta x_2) - f_2(x_1, x_2)\| \leq \alpha \|\Delta x_2\| \]  

Assumption 2. The derivative of the lumped disturbances and uncertainties is bounded, i.e., \( \|d\| \leq \delta_h \) where \( \delta_h > 0 \) [16,33–35].

Remark 1. Since velocity information is unavailable, Assumption 1 is added to realize the implementation of state and disturbance observations. Besides, Assumption 2 is reasonable since it is impossible for the observer and main controller to perfectly react to step disturbances when they occur.

Remark 2. The investigated problem is how to estimate both joint velocities and lumped disturbances and uncertainties with only joint displacement measurements.

3. Nonlinear Functional Observer Design

Inspired by previous works [44,60], the NFO structure is selected for robotic manipulators as follows:

\[ \begin{align*}
    \dot{z} &= Nz + \Psi + Jy \\
    \hat{\nu} &= z + Ey
\end{align*} \]  

where \( z \) is the state vector of the observer; \( \hat{\nu} = [\hat{x}_2^T, \hat{d}^T]^T \) is the estimate of joint velocities and lumped disturbances \( \nu = [x_2^T, d^T]^T \); \( y \) is the output of the system, i.e., the joint angle measurement \( x_1 \); \( N, J, \) and \( E \) are matrices to be designed with appropriate dimensions; and \( \Psi \) is introduced to cover nonlinearities in the manipulator dynamics.
Define the error $e = \hat{v} - v$. From (2) and (4), the error dynamics are derived as

$$
\dot{e} = Nz + \Psi f y + Ex_2 - \left[ \frac{f_1(x_1)u + f_2(x_1, x_2) + d}{d} \right]$$

$$
= N(z + Ey) + (J - NE)y + Ex_2 - \left[ \frac{d}{0_{n \times 1}} \right] + \Psi - \left[ \frac{f_1(x_1)u + f_2(x_1, x_2)}{0_{n \times 1}} \right] - \left[ \frac{0_{n \times 1}}{d} \right]
$$

(5)

To obtain the desired performance $\dot{e} = Ne$, the following equation needs to be solved:

$$
J = NE
$$

(6)

$$
N \eta = -Ex_2 + \left[ \frac{d}{0_{n \times 1}} \right]
$$

(7)

$$
\Psi = \left[ \begin{array}{c} f_1(x_1)u + f_2(x_1, \hat{x}_2) \\ 0_{n \times 1} \end{array} \right]
$$

(8)

Define $E = [E_1^T, E_2^T]^T$. From (7), the formula of $N$ is derived as

$$
N = \left[ \begin{array}{c} -E_1 \\ -E_2 \\ 0_{n \times n} \end{array} \right]
$$

(9)

Substituting this into (6), matrix $J$ is computed as

$$
J = NE = \left[ \begin{array}{c} -3\omega^2 I_n \\ -2\omega^3 I_n \end{array} \right]
$$

(11)

**Theorem 1.** Based on Assumptions 1 and 2, the proposed NFO (4) with matrices constructed by (8), (10), and (11) guarantees globally uniformly ultimately bounded estimation performances of both unmeasurable velocities and lumped disturbances and uncertainties for the robot manipulator (2).

**Proof of Theorem 1.** Substituting (8), (10), and (11) into (5), the error dynamics become

$$
\dot{e} = Ne + D_1 \tilde{f}_2 - D_2 \frac{d}{d} \Leftrightarrow \left\{ \begin{array}{l}
\dot{e}_1 = -2\omega e_1 + e_2 + \tilde{f}_2 \\
\dot{e}_2 = -\omega^2 e_1 - d
\end{array} \right.
$$

(12)

where $\tilde{f}_2 = f_2(x_1, \hat{x}_2) - f_2(x_1, x_2)$, $D_1 = [I_n, 0_{n \times n}]^T$, and $D_2 = [0_{n \times n, I_n}]^T$. Define a scaled estimation error $\eta = [e_1^T, \omega^{-1} e_2^T]^T$. The scaled error dynamics are expressed as

$$
\left\{ \begin{array}{l}
\dot{\eta}_1 = -2\omega \eta_1 + \omega \eta_2 + \tilde{f}_2 \\
\dot{\eta}_2 = -\omega \eta_1 - \omega^{-1} d \\
\end{array} \right. \Leftrightarrow \eta = \omega A_\eta \eta + D_1 y \tilde{f}_2 - D_2 y \frac{d}{d}
$$

(13)

where $A_\eta = \left[ \begin{array}{c} -2I_n \\ -I_n \\ 0_{n \times n} \end{array} \right]$, $D_1 y = \left[ \begin{array}{c} I_n \\ 0_{n \times n} \end{array} \right]$, and $D_2 y = \left[ \begin{array}{c} 0_{n \times n} \\ I_n \end{array} \right]$. 
Since all eigenvalues of $A_\eta$ are $-1$, the following Lyapunov equation has a solution:

$$A_\eta^T P_\eta + P_\eta A_\eta = -2I_{2n}$$  (14)

where $P_\eta$ is a positive definite matrix.

A Lyapunov function is considered as $V_\eta = 0.5\eta^T P_\eta \eta$. Taking the derivative of it, one obtains:

$$\dot{V}_\eta = -\omega \eta^T \eta + \eta^T P_\eta D_{1\eta} \tilde{f}_2 - \eta^T P_\eta D_{2\eta} \frac{d}{\omega}$$  (15)

Based on Young’s inequality, the following inequalities hold:

$$\eta^T P_\eta D_{1\eta} \tilde{f}_2 \leq \frac{1}{2} \eta^T \eta + \frac{1}{2} \tilde{f}_2^T (P_\eta D_{1\eta})^T P_\eta D_{1\eta} \tilde{f}_2$$

$$\eta^T P_\eta D_{2\eta} \frac{d}{\omega} \leq \frac{1}{2} \eta^T \eta + \frac{1}{2\omega^2} d \left( P_\eta D_{2\eta} \right)^T P_\eta D_{2\eta} d$$  (16)

Set $\lambda_{1\eta} = \lambda_{\text{min}}((P_\eta D_{1\eta})^T P_\eta D_{1\eta})$ and $\lambda_{2\eta} = \lambda_{\text{min}}((P_\eta D_{2\eta})^T P_\eta D_{2\eta})$ with $\lambda_{\text{min}}(X)$ and $\lambda_{\text{max}}(X)$ as the minimum and maximum eigenvalues of the matrix $X$, respectively. Substituting (16) into (15), one obtains:

$$\dot{V}_\eta \leq - (\omega - 1)\eta^T \eta + \frac{1}{2} \lambda_{1\eta} \tilde{f}_2^T \tilde{f}_2 + \frac{1}{2\omega^2} \lambda_{2\eta} \frac{d^T d}{\omega}$$  (17)

From Assumptions 1 and 2, the following inequalities hold

$$\tilde{f}_2^T \tilde{f}_2 \leq a^2 \left\| \tilde{x}_2 - x_2 \right\|^2 = a^2 \eta_1^T \eta_1$$  (18)

$$\frac{d^T d}{\omega} \leq \delta_n^2$$  (19)

Substituting these into (17), the derivative of the Lyapunov function $V_\eta$ satisfies

$$\dot{V}_\eta \leq - (\omega - 1)\eta^T \eta + \frac{1}{2} \lambda_{1\eta} a^2 \eta_1^T \eta_1 + \frac{1}{2\omega^2} \lambda_{2\eta} \delta_n^2$$

$$\leq - (\omega - 1 - \frac{\lambda_{1\eta} a^2}{2}) \eta^T \eta + \frac{1}{2\omega^2} \lambda_{2\eta} \delta_n^2$$

$$\leq - (\omega - 1 - \frac{\lambda_{1\eta} a^2}{2}) \eta_1^T V_\eta + \frac{1}{2\omega^2} \lambda_{2\eta} \delta_n^2$$

$$= - a_\eta V_\eta + b_\eta$$

where $a_\eta = \left( \omega - 1 - \frac{\lambda_{1\eta} a^2}{2} \right) \frac{2}{\omega^2}$, $b_\eta = \frac{1}{2\omega^2} \lambda_{2\eta} \delta_n^2$, and $\lambda_{\text{max}}(P_\eta)$. After some mathematical transformations [16], one obtains

$$V_\eta(t) \leq e^{-a_\eta(t-t_0)} \left( V_\eta(t_0) - \frac{b_\eta}{a_\eta} \right) + \frac{b_\eta}{a_\eta}$$  (21)

To analyze the stability of the proposed observer [63], the following function is introduced:

$$T = \begin{cases} \frac{1}{a_\eta} \ln \left( \frac{c_\eta - \frac{b_\eta}{a_\eta}}{\frac{b_\eta}{a_\eta}} \right), & \text{if } \left( c_\eta - \frac{b_\eta}{a_\eta} \right) \frac{1}{a_\eta} > 1 \\ 0, & \text{otherwise} \end{cases}$$  (22)

where $c_\eta$ and $d_\eta$ are positive constants.

From (21) and (22), it is observed that for an arbitrarily large constant $c_\eta$, there exists a positive constant $d_\eta$ and an interval $T(c_\eta,d_\eta)$ that are independent of $t_0 \geq 0$ such that

$$V_\eta(t_0) \leq c_\eta \Rightarrow V_\eta(t) \leq d_\eta + \frac{b_\eta}{a_\eta}, \forall t \geq t_0 + T$$  (23)

Hence, the proof of Theorem 1 is completed. □
In the steady state, i.e., when time goes to infinity, the Lyapunov function $V_\eta$ converges to a region that

$$V_\eta \leq \frac{b_\eta}{a_\eta} = \frac{\lambda_{2\eta} \lambda_{P\eta} \delta_\eta^2}{4 \omega^2 \left( \omega - 1 - \frac{\lambda_{P\eta}^2}{2} \right)}$$  (24)$$

From the definition of $V_\eta$ and the scaled error $\eta$, one obtains

$$\| \hat{x}_2 - x_2 \| \leq \frac{\delta_\eta}{\omega^{3/2}} \sqrt{\frac{\lambda_{2\eta} \lambda_{P\eta}}{2 \lambda_{P\eta} \left( 1 - \frac{1}{2} - \frac{\lambda_{P\eta}^2}{2 \omega^2} \right)}}$$  (25)$$

where $\lambda_{P\eta} = \lambda_{\min}(P_\eta)$.

**Remark 3.** From (21) and (25), it is observed that not only the convergence speed but also the steady-state estimation performance of the proposed observer is improved when the observer gain increases. However, in real applications, since the observer is implemented in hardware with nonzero sample time, the observer gain cannot be selected at a too large value to avoid instability in the observation performance.

4. Theoretical Comparison with an Extended State Observer

Since both the NFO and the well-known ESO can estimate both unmeasurable velocities and lumped disturbances and uncertainties in robot manipulators, it is reasonable to determine which one is better in terms of estimation accuracy and computational efforts. The ESO design for robot manipulators is briefly presented as follows:

\[
\begin{align*}
\dot{x}_1 &= \hat{x}_2 + 3\omega(x_1 - \hat{x}_1) \\
\dot{x}_2 &= f_1(x_1) + f_2(x_1, \hat{x}_2) + 3\omega^2(x_1 - \hat{x}_1) \\
\dot{x}_3 &= \omega^3(x_1 - \hat{x}_1)
\end{align*}
\]  (26)$$

where $\hat{x}_1$, $\hat{x}_2$, and $\hat{x}_3$ are the estimates of $x_1$, $x_2$, and $x_3 = d$, respectively. Define the scaled estimation error vector $\varepsilon = [\hat{x}_1^T, \omega^{-1} \hat{x}_2^T, \omega^{-2} \hat{x}_3^T]^T$. The error dynamics are given by

\[
\dot{\varepsilon} = \omega A_\varepsilon \varepsilon + D_{\varepsilon 1} \frac{\hat{f}_2}{\omega} - D_{\varepsilon 2} \frac{d}{\omega^2}
\]  (27)$$

where $A_\varepsilon = \begin{bmatrix} -3I_n & I_n & 0_{n \times n} \\ -3I_n & 0_{n \times n} & I_n \\ -I_n & 0_{n \times n} & 0_{n \times n} \end{bmatrix}$, $D_{\varepsilon 1} = \begin{bmatrix} 0_{n \times 1} \\ I_n \\ 0_{n \times 1} \end{bmatrix}$, and $D_{\varepsilon 2} = \begin{bmatrix} 0_{n \times 1} \\ I_n \end{bmatrix}$.

Matrix $A_\varepsilon$ is negative-definite; therefore, there exists a positive definite matrix $P_\varepsilon$ satisfying the following Lyapunov equation:

\[
A_\varepsilon^T P_\varepsilon + P_\varepsilon A_\varepsilon = -2I_{2n}
\]  (28)$$

Considering a Lyapunov function $V_\varepsilon = 0.5 \varepsilon^T P_\varepsilon \varepsilon$, the derivative of it is expressed as

\[
\dot{V}_\varepsilon = -\omega \varepsilon^T \varepsilon + \varepsilon^T P_\varepsilon D_{\varepsilon 1} \frac{\hat{f}_2}{\omega} - \varepsilon^T P_\varepsilon D_{\varepsilon 2} \frac{d}{\omega^2}
\]  (29)$$
Setting $\lambda_{1e} = \lambda_{\min}((P_1D_{1e})^T P_1D_{1e})$ and $\lambda_{2e} = \lambda_{\min}((P_2D_{2e})^T P_2D_{2e})$, the following inequality holds:

$$\dot{V}_e \leq -(\omega - 1)\varepsilon^T e + \frac{1}{2\omega^2} \lambda_{1e} \varepsilon_e^T \varepsilon_e + \frac{1}{2\omega^4} \lambda_{2e} d^T d$$

(30)

Similar to the previous section, based on Assumptions 1 and 2, inequality (30) becomes

$$\dot{V}_e \leq -(\omega - 1)\varepsilon^T e + \frac{1}{2\omega^2} \lambda_{1e} \varepsilon^T e + \frac{1}{2\omega^4} \lambda_{2e} d^T d$$

(31)

where $\lambda_{p_1} = \lambda_{\max}(P_1)$.

From this, the globally uniformly ultimately bounded convergence of the Lyapunov function $V_e$ is guaranteed, which leads to the following steady-state performances:

$$\| \mathbf{x}_2 - x_2 \| \leq \frac{\delta_h}{\omega^{\frac{3}{2}}} \sqrt{\frac{\lambda_{2i} \lambda_{p_1}}{\lambda_{p_2}}}$$

(32)

$$\| \mathbf{d} - d \| \leq \frac{\delta_h}{\omega^{\frac{3}{2}}} \sqrt{\frac{\lambda_{2i} \lambda_{p_1}}{\lambda_{p_2}}}$$

where $\lambda_{p_2} = \lambda_{\min}(P_2)$.

Remark 4. Inequalities (25) and (32) indicate the estimation performance of the proposed NFO and the ESO, respectively. Comparing them, it is observed that the criterion to determine which observer is better is given by

$$R_i = \sqrt{\frac{\lambda_{2i} \lambda_{p_1}}{\lambda_{p_2}}} \left( i = 1, 2 \right)$$

(33)

After computing matrices and parameters by MATLAB, the following result is obtained:

$$R_1 \approx \frac{10.34 \times 3.41}{2 \times 0.586 \times (1 - \frac{1}{\omega} - \frac{n_i}{\omega})} = \frac{5.39}{1 - \frac{2}{\omega}}$$

$$R_2 \approx \frac{69 \times 8.67}{2 \times 0.391 \times (1 - \frac{1}{\omega} - \frac{n_i}{\omega})} = \frac{27.61}{1 - \frac{2}{\omega}}$$

(34)

From this, one concludes that in the worst situation, i.e., when the estimation errors reach their boundary, the proposed NFO dominates the ESO. Moreover, since the order of the proposed NFO, i.e., $2n$, is smaller than the order of the ESO, i.e., $3n$, it is observed that less computational effort is required for the implementation of the proposed NFO.

5. Numerical Simulation

5.1. Simulation Setup

In this section, a 2-DOF robot manipulator with revolute joints is utilized to verify the effectiveness of the proposed NFO. The robot configuration is shown in Figure 1. Matrices in the manipulator dynamics are given by

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}; \ C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}; \ G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$$

(35)

where $M_{11} = I_1 + m_1l_1^2 + I_2 + m_2(l_1^2 + l_2^2 + 2l_1l_2c_2)$, $M_{12} = M_{21} = I_2 + m_2(l_2^2 + l_1l_2c_2)$, $M_{22} = I_2 + m_2l_2^2$, $C_{11} = -m_2l_1l_2s_2\dot{q}_2$, $C_{12} = -m_2l_1l_2s_2(\dot{q}_1 + \dot{q}_2)$,
C_{21} = m_2 l_1 l_2 s_2 \dot{q}_1, \quad C_{22} = 0, \quad G_1 = m_2 g (l_2 c_{12} + l_1 c_1) + m_1 g l_1 c_1, \quad \text{and} \quad G_2 = m_2 g l_2 c_{12}.

Model parameters are given in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>l_1</td>
<td>inertia of link 1</td>
<td>61.25 \times 10^{-3} \text{kg.m}^2</td>
</tr>
<tr>
<td>l_2</td>
<td>inertia of link 2</td>
<td>20.42 \times 10^{-3} \text{kg.m}^2</td>
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<td>m_1</td>
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</tr>
<tr>
<td>m_2</td>
<td>mass of link 2</td>
<td>0.85 \text{kg}</td>
</tr>
<tr>
<td>l_1</td>
<td>length of link 1</td>
<td>0.35 \text{m}</td>
</tr>
<tr>
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<tr>
<td>g</td>
<td>gravitational acceleration</td>
<td>9.81 \text{m/s}^2</td>
</tr>
</tbody>
</table>

To illustrate uncertainties in the system, all the nominal model parameters are chosen with 90% of their true values. Disturbances are injected into the system by

\[
d = \begin{bmatrix}
10 + 10 \sin(2\pi t) \\
10 + 10 \cos(2\pi t)
\end{bmatrix} \text{ (ms}^{-2})
\] (36)

For fair and comprehensive comparisons, different observer gains are selected as \(\omega_1 = 20\), \(\omega_2 = 50\), and \(\omega_3 = 80\) for both the NFO and ESO with the same initial conditions. Besides, a computed torque controller (CTC) is adopted as the main controller with control gains \(k_D = 50\) and \(k_P = 1000\). The design of the CTC is presented in Appendix A.

The reference trajectory is selected as follows:

\[
q_d = \begin{bmatrix}
0.4 + 0.2 \sin(2\pi t) \\
0.4 + 0.2 \sin(2\pi t)
\end{bmatrix} \text{ (rad)}
\] (37)

5.2. Simulation Results

5.2.1. Case 1: Estimation Performance Comparison

In this case, to compare the estimation performances without considering their effects on the final control performance, the same CTC is used to test both the NFO and ESO without utilizing velocity and disturbance estimations from DOBs to design the controllers.

Figure 2 shows that both compared observers guarantee that the estimated disturbances track the real ones. Larger observer gains achieve better disturbance estimation performances in both the ESO and NFO. However, it is observed that the proposed NFO provides better disturbance estimation accuracy in not only the steady-state but also the transient time compared to the ESO with the same observer gain as shown in Figure 3.
Especially, the transient response of the ESO dramatically increases when the observer gain increases, which does not happen in the case of the NFO. To precisely evaluate the estimation results, performance indices including the maximum $M_e$, average $\mu_e$, and standard deviation $\sigma_e$, of the disturbance estimation errors at the steady state are calculated and presented in Table 2, which once again proves the statement.

![Figure 2](image1.png)

**Figure 2.** Estimation performances of lumped uncertainties and disturbances.

![Figure 3](image2.png)

**Figure 3.** Estimation errors of lumped uncertainties and disturbances.
Table 2. Disturbance estimation performance indices.

<table>
<thead>
<tr>
<th>Observer</th>
<th>$M_e$ (rad/s$^2$)</th>
<th>$\mu_e$ (rad/s$^2$)</th>
<th>$\sigma_e$ (rad/s$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d_1$</td>
<td>$d_2$</td>
<td>$d_3$</td>
</tr>
<tr>
<td>ESO, $\omega_1$</td>
<td>27.82</td>
<td>20.22</td>
<td>17.42</td>
</tr>
<tr>
<td>NFO, $\omega_1$</td>
<td>19.56</td>
<td>14.78</td>
<td>12.20</td>
</tr>
<tr>
<td>ESO, $\omega_2$</td>
<td>12.62</td>
<td>9.958</td>
<td>7.810</td>
</tr>
<tr>
<td>NFO, $\omega_2$</td>
<td>8.566</td>
<td>6.785</td>
<td>5.293</td>
</tr>
<tr>
<td>ESO, $\omega_3$</td>
<td>8.072</td>
<td>6.433</td>
<td>4.985</td>
</tr>
<tr>
<td>NFO, $\omega_3$</td>
<td>5.449</td>
<td>4.340</td>
<td>3.362</td>
</tr>
</tbody>
</table>

Similar results are realized in the estimation performances, estimation errors, and performance indices of the joint velocities as shown in Figures 4 and 5, and Table 3, respectively. Even though both the NFO and ESO provide good velocity estimation performances when the observer gain is high, the proposed NFO still guarantees better velocity estimation accuracy. These results fit with the theoretical comparison between two observers as presented in Section 4.

Table 3. Velocity estimation performance indices.

<table>
<thead>
<tr>
<th>Observer</th>
<th>$M_e$ (rad/s$^2$)</th>
<th>$\mu_e$ (rad/s$^2$)</th>
<th>$\sigma_e$ (rad/s$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\dot{q}_1$</td>
<td>$\dot{q}_2$</td>
<td>$\ddot{q}_1$</td>
</tr>
<tr>
<td>ESO, $\omega_1$</td>
<td>1.395</td>
<td>0.980</td>
<td>0.875</td>
</tr>
<tr>
<td>NFO, $\omega_1$</td>
<td>0.487</td>
<td>0.359</td>
<td>0.304</td>
</tr>
<tr>
<td>ESO, $\omega_2$</td>
<td>0.254</td>
<td>0.198</td>
<td>0.157</td>
</tr>
<tr>
<td>NFO, $\omega_2$</td>
<td>0.085</td>
<td>0.067</td>
<td>0.053</td>
</tr>
<tr>
<td>ESO, $\omega_3$</td>
<td>0.100</td>
<td>0.080</td>
<td>0.062</td>
</tr>
<tr>
<td>NFO, $\omega_3$</td>
<td>0.034</td>
<td>0.027</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Figure 4. Estimation performances of unmeasurable velocities.
5.2.2. Case 2: Closed-Loop Control Performance Comparison

In this case, the estimates of unmeasurable velocities and lumped uncertainties and disturbances are fed back to the main controller. Based on this, the effectiveness of the proposed NFO and ESO on closed-loop control performances can be compared.

Figure 6 presents the tracking performances of both controllers with different observer gains. It is observed that both the NFO-based CTC and the ESO-based CTC guarantee position-tracking performances for the control system suffering from unmeasurable velocities, external disturbances, and uncertain model parameters. Nevertheless, in Figure 7 and Table 4, the tracking errors and their performance indices at the steady state of both controllers are given, respectively, which indicates the superiority of the proposed NFO in closed-loop performances. These results are obvious since the estimated velocities and lumped disturbances of the proposed NFO are better than those of the ESO, as shown in Case 1, which directly contributes to the final control performances. Furthermore, the transient performance of the ESO-based CTC is quite sensitive to the increase in observer gain compared to those of the ESO-based CTC, which is similar to the velocity and disturbance estimation performances mentioned above.

Table 4. Position tracking performance indices.

<table>
<thead>
<tr>
<th>Observer</th>
<th>( \dot{q}_1 ) (rad/s)</th>
<th>( \dot{q}_2 ) (rad/s)</th>
<th>( \mu_e ) (rad)</th>
<th>( \sigma_e ) (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESO, ( \omega_1 )</td>
<td>17.02</td>
<td>6.894</td>
<td>10.69</td>
<td>2.788</td>
</tr>
<tr>
<td>NFO, ( \omega_1 )</td>
<td>6.314</td>
<td>3.934</td>
<td>4.037</td>
<td>2.040</td>
</tr>
<tr>
<td>ESO, ( \omega_2 )</td>
<td>3.418</td>
<td>2.405</td>
<td>2.212</td>
<td>1.271</td>
</tr>
<tr>
<td>NFO, ( \omega_2 )</td>
<td>1.708</td>
<td>1.248</td>
<td>1.097</td>
<td>0.676</td>
</tr>
<tr>
<td>ESO, ( \omega_3 )</td>
<td>1.726</td>
<td>1.251</td>
<td>1.112</td>
<td>0.685</td>
</tr>
<tr>
<td>NFO, ( \omega_3 )</td>
<td>0.965</td>
<td>0.725</td>
<td>0.616</td>
<td>0.394</td>
</tr>
</tbody>
</table>
at the steady-state, similar shapes are realized since both controllers guarantee similar position-tracking performances. However, responses of the ESO are worse than those of the NFO, the control signals of the ESO-based controller are consequently larger than those of the NFO. Nevertheless, in the transient regime, since the transient performance of the ESO-based CTC is quite sensitive to the increase in observer gains. It is observed that both the NFO and ESO on closed-loop control performances can be compared in performance comparison.

Table 3. Velocity estimation performance indices.

Table 4. Position tracking performance indices.

Figure 6. Position tracking performances.

Figure 7. Position tracking errors.

Control signals of all controllers are plotted in Figure 8 with smooth shapes. It is observed that controllers with lower gains generate control signals with higher magnitudes, but lower tracking accuracies as shown above. In the transient regime, since the transient responses of the ESO are worse than those of the NFO, the control signals of the ESO-based controller are consequently larger than those of the NFO-based controller. However, at the steady-state, similar shapes are realized since both controllers guarantee similar position-tracking performances.
5.3. Discussion of the Results

Based on the simulation results, it is observed that compared to the well-known ESO, the proposed NFO provides better estimation performance in terms of both unknown joint velocities and lumped uncertainties/disturbances for the control tasks of robot manipulators.

However, practical problems such as measurement noise and sampling time for digital control structures have not been considered in the validation of the proposed NFO and the comparison between it and the ESO. Furthermore, experiment results are not yet available, which requires further studies in the future.

6. Conclusions

This paper proposed a novel NFO to estimate both unmeasurable velocities and lumped uncertainties and disturbances of robot manipulators. Compared to the well-known ESO, the proposed observer provides better estimation performances, as proved by theoretical analysis and simulation results with a 2-DOF manipulator. The effects of the estimation performances of the proposed NFO on the closed-loop control system are also validated with high-accuracy tracking performances. Future works will be conducted to investigate further applications of the NFO in the practical control problems of robot manipulators.

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Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

The computed torque control signal is designed as follows:

$$ u = M(q) \left( \ddot{q}_d + k_D (\ddot{q}_d - \ddot{q}) + k_p (q_d - q) - \ddot{d} \right) + C(q, \dot{q}) \dot{q} + G(q) $$  \hspace{1cm} (A1)

where \( q_d \) is the reference trajectory, \( k_D \) and \( k_p \) are the control parameters, and \( \ddot{q} \) is the vector of joint velocity estimates.

Substitute this into the manipulator dynamics (1), one obtains the tracking error dynamics as

$$ \dot{e} + k_D \dot{e} + k_p e = k_D (\ddot{q} - \ddot{d}) - M(q)^{-1} C \dot{q} $$  \hspace{1cm} (A2)

where \( e = q_d - q, \ddot{q} = \ddot{q} - \ddot{q}, \ddot{d} = \ddot{d} - d, \) and \( C \ddot{q} = C(q, \dot{q}) \dot{q} - C(q, \dot{q}) q \).

From this, system stability is guaranteed based on the arbitrarily bounded estimation performances of both the NFO and ESO.

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