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Hyperconnectedness and Resolvability of Soft Ideal Topological Spaces

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Abstract: This paper introduces and explores the concept of soft ideal dense sets, utilizing soft open sets and soft local functions, to examine their fundamental characteristics under some conditions for the following notions: soft ideal hyperconnectedness, soft ideal resolvability, soft ideal irresolvability, and soft ideal semi-irresolvability in soft ideal topological spaces. Moreover, it explores the relationship between these notions if \( \tau \cap \overline{E} = \phi \) is obtained in the soft set environment.

Keywords: soft open set; soft dense; soft ideal; soft ideal hyperconnected; soft ideal resolvable; soft ideal irresolvable; soft ideal semi-irresolvable

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1. Introduction

In 1999, Molodtsov [1] initially suggested the idea of soft sets as a broad mathematical tool for handling uncertain situations. Molodtsov effectively utilized soft theory in some areas, including probability, theory of measurement, smoothness of functions, Perron integration, operations research, Riemann integration, and so on, in [2].

Shabir and Naz [3] started researching soft topological spaces in 2011. They defined the topology on the collection \( \tau \) of soft sets over \( X \). Thus, they developed many features of soft regular spaces, soft normal spaces, soft separation axioms, soft open and soft closed sets, soft subspace, soft closure, and soft nbd of a point. They also defined the fundamental concepts of soft topological spaces.

Kandil and colleagues introduced the concept of the soft ideal for the first time [4]. Additionally, they presented the idea of soft local functions. These ideas are presented with the goal of identifying new soft topologies, termed soft topological spaces with soft ideal \((X_E, \tau, \mathcal{I})\), from the original one. Numerous mathematical structures, such as soft group theory [5], soft ring theory [6], soft primalns [7], soft algebras [8,9], soft category theory [10], ideal spaces [11], ideal resolvability [12], and so on, have been addressed by soft set theory. Similarly, the notion of soft topology through soft grills was introduced in [13]. Additionally, a large number of academics and researchers developed gentle versions of the traditional topological ideas, such as soft resolvable spaces [14], soft hyperconnected spaces [15], suitable soft spaces [7], soft ideal spaces [4,16,17], soft extremally disconnected spaces [18], soft Menger spaces [19], soft countable chain condition, and soft caliber [20].

From here on, we shall refer to a soft ideal topological space \((X_E, \tau, \mathcal{I})\), a soft ideal space. The way this work is set out is as follows: Following the introduction, we discuss the definitions and findings that are necessary to understand the data in Section 2. Next, we recall the notion of soft local functions in Section 3. We study the fundamental operations on soft local functions. The definitions of soft hyperconnected and soft hyperconnected modulo ideal spaces, as well as a soft ideal topological space, are provided in Section 4.
We look at the basic characteristics and connections between soft hyperconnected and soft hyperconnected modulo ideals. A soft ideal resolvable space is defined in Section 5 and it is demonstrated that soft ideal resolvable topologies over soft ideal resolvable subspace are also soft ideal resolvable. The concept of soft ideal semi-irresolvable space and an overview of its properties are provided in Section 6. In Section 7, we finish off by providing an overview of the major contributions and some recommendations for the future.

2. Preliminary

Here, we provide the fundamental concepts and the outcomes of soft set theory that are required for the follow-up.

Definition 1 ([1]). Let X be an initial universe and E be a set of parameters. Let \( P(X) \) denote the power set of X and A be a non-null subset of parameters E. A pair \((F, A)\) symbolized by \( F_A \) is a soft set over \( X_E \), where \( F \) is a mapping given by \( F : A \rightarrow P(X) \). Otherwise put, a soft set over \( X_E \) is a parameterized family of subsets of the universe \( X_E \). For a particular \( e \in E \), \( F(e) \) might be regarded as the set of e-approximate elements of the soft set \((F, E) = F_E\) and, if \( e \notin E \), then \( F(e) = \phi \), i.e., \( F_E = \{F(e) : e \in E, F : E \rightarrow P(X)\} \). The collection of all these soft sets is symbolized by \( SS(X)_E \).

Definition 2 ([21]). Let \( F_E, G_E \in SS(X)_E \). Then
1. \( F_E \) is called a soft subset of \( G_E \), denoted by \( F_E \subseteq G_E \), if \( F(e) \subseteq G(e) \), for all \( e \in E \).
2. \( F_E \) is called absolute, symbolized by \( X_E \), if \( F(e) = X \) for all \( e \in E \).
3. \( F_E \) is called null, symbolized by \( \phi_E \), if \( F(e) = \phi \) for all \( e \in E \).

In this case \( F_E \) is said to be a soft subset of \( G_E \) and \( G_E \) is said to be a soft superset of \( F_E \), \( F_E \subseteq G_E \).

Definition 3 ([22]). 1. A soft set \( F_E \in SS(X)_E \) is called a soft point in \( X_E \) if there exist \( x \in X \) and \( e \in E \) such that \( F(e) = \{x\} \) and \( F(e') = \phi \) for each \( e' \in E \) - \( \{e\} \). This soft point \( F_E \) is denoted by \( x_e \).
2. Let \( \Delta \) be an arbitrary index set and \( \Omega = \{(F_\alpha)_E : \alpha \in \Delta\} \) be a subfamily of \( SS(X)_E \). Then:
   (a) The union of all \( F_\alpha \) is the soft set \( H_E \), where \( H(e) = \cup_{\alpha \in \Delta}(F_\alpha)_E(e) \) for each \( e \in E \). We write \( \cup_{\alpha \in \Delta}(F_\alpha)_E = H_E \).
   (b) The intersection of all \( F_\alpha \) is the soft set \( M_E \), where \( M(e) = \cap_{\alpha \in \Delta}(F_\alpha)_E(e) \) for each \( e \in E \). We write \( \cap_{\alpha \in \Delta}(F_\alpha)_E = M_E \).
3. A soft set \( G_E \) in a soft topological space \( (X_E, \tau) \) is called a soft neighborhood of the soft point \( x_e \in X_E \) if there exists a soft open set \( H_E \) such that \( x_e \in H_E \subseteq G_E \).

Definition 4 ([3]). Let \((X_E, \tau)\) be a soft topological space and \( F_E \in SS(X)_E \).
1. The soft closure of \( F_E \), symbolized by \( \overline{c}(F_E) \), is the intersection of all soft closed superset of \( F_E \), i.e., \( \overline{c}(F_E) = \cap\{H_E : H_E \subseteq F_E \} \).
2. The soft interior of \( F_E \) is the set \( \text{Int}(F_E) = \cup\{H_E : H_E \subseteq F_E \} \).
3. A difference of two soft sets \( F_E \) and \( G_E \) over the common universe \( X_E \), symbolized by \( F_E - G_E \), is the soft set \( H_E \) for all \( e \in E \), \( H(e) = F(e) - G(e) \).
4. A complement of a soft set \( F_E \), symbolized by \( F^c_E \), is defined as follows. \( F^c_E : E \rightarrow P(X) \) is a mapping given by \( F^c(e) = X_E(e) - F(e) \), for all \( e \in E \), and \( F^c \) is called a soft complement function of \( F_E \).
5. Let \( F_E \) be a soft set over \( X_E \) and \( x_e \in X_E \). We say that \( x_e \in F_E \) denotes that \( x_e \) belongs to the soft set \( F_E \) whenever \( x_e \in F(e) \), for all \( e \in E \).

For more details of soft set theory and its applications in a variety of mathematical structures, see [18,23–27].

3. Soft Local Functions

Definition 5 ([4]). The non-null collection of soft subsets \( \mathcal{T} \) of \( SS(X)_E \) is called a soft ideal on \( X_E \) if
(a) \( F_E \in \mathcal{I} \) and \( G_E \subseteq F_E \), then \( G_E \in \mathcal{I} \).
(b) \( F_E \in \mathcal{I} \) and \( G_E \in \mathcal{I} \), then \( F_E \cup G_E \in \mathcal{I} \).

**Definition 6** ([4]). Let \((X_E, \tau, \mathcal{I})\) be a SITS. Then, \( F_E' \mathcal{I} (\tau, \mathcal{I}) \) (or \( F_E^* \mathcal{I} \)) is called a soft local function of \( F_E \) with respect to \( \mathcal{I} \) and soft topology \( \tau \), where \( O_{x_e} \) is a soft open set containing \( x_e \).

A soft subset \( A_E \) of a soft ideal topological space “symbolized SITS” \((X_E, \tau, \mathcal{I})\) is said to be soft ideal dense if every soft point of \( X_E \) is in \( A_E \), i.e., if \( \overline{A_E} = X_E \).

**Remark 1.** For a SITS \((X_E, \tau, \mathcal{I})\), if \( D_E \subseteq X_E \) is soft ideal dense, then \( X_E \) is also soft ideal dense, i.e., \( \overline{X_E} = X_E \).

A soft set \( S_E \in SS(X_E) \) is called soft co-dense [28] if \( Int(S_E) = \phi_E \).

**Theorem 1.** Let \((X_E, \tau, \mathcal{I})\) be a SITS. Then, the next characteristics are interchangeable:
(a) \( \tau \cap \mathcal{I} = \phi_E \), where \( \phi_E \) is a null soft set;
(b) If \( S_E \in \mathcal{I} \), then \( Int(S_E) = \phi_E \);
(c) For any soft open \( F_E \), we have \( F_E \subseteq \mathcal{I} \);
(d) \( X_E = \overline{X_E} \).

**Proof.** (a) \( \rightarrow \) (b): Assume that \( \tau \cap \mathcal{I} = \phi_E \) and \( S_E \in \mathcal{I} \). Suppose that \( x_e \in Int(S_E) \).

Then, there exists a soft open set \( U_E \) such that \( x_e \in U_E \subseteq S_E \). Since \( S_E \in \mathcal{I} \), \( U_E \in \mathcal{I} \). This is contrary to \( \tau \cap \mathcal{I} = \phi_E \). Therefore, \( Int(S_E) = \phi_E \).

(b) \( \rightarrow \) (c): Assume that \( x_e \in F_E \). Let \( x_e \in \mathcal{I} \); then, there exists soft open set \( U_{x_e} \) containing \( x_e \) such that \( F_E \cap U_{x_e} \subseteq \mathcal{I} \). Since \( F_E \) is a soft open set, by (b) \( x_e \in F_E \cap U_{x_e} = Int(F_E \cap U_{x_e}) = \phi_E \).

This is incoherent, and so \( x_e \in F_E \cap \mathcal{I} \). Therefore, \( \mathcal{I} \).

(c) \( \rightarrow \) (d): Suppose that \( X_E = \overline{X_E} \) and \( X_E = \overline{X_E} \) are non-null soft open sets with \( X_E \cap X_E \).

Then, \( \tau \cap \mathcal{I} = \phi_E \).

4. Soft Hyperconnected Spaces

**Definition 7.** Let \((X_E, \tau, \mathcal{I})\) be a SITS. We say that this space is:
1. Soft hyperconnected “symbolized \( HC \)" [17] if every pair of non-null soft open sets of \( X_E \) has non-null intersection.
2. Soft \( HC \) modulo \( \mathcal{I} \) if the intersection of every two non-null soft open sets is not in \( \mathcal{I} \).
3. Soft ideal \( HC \) if every non-null soft open set is soft ideal dense in \( \mathcal{I} \).

**Lemma 1.** A SITS \((X_E, \tau, \mathcal{I})\) is soft \( HC \) modulo \( \mathcal{I} \) if there are no proper soft closed sets \( G_E \) and \( H_E \) such that \( X_E = (G_E \cup H_E) \in \mathcal{I} \).

**Proof.** If there are proper soft closed sets \( G_E \) and \( H_E \) such that \( X_E = [G_E \cup H_E] \in \mathcal{I} \).

If \( H_E = \phi_E \), then \( X_E = G_E \in \mathcal{I} \), \( X_E \subseteq G_E \) and \( X_E \subseteq H_E \). Then, \( X_E \cap G_E \neq X_E \cap H_E \in \mathcal{I} \). This is incoherent. Hence, \( G_E \neq \phi_E \) and \( H_E \neq \phi_E \).

Conversely, assume that \( A_E \neq \phi_E \) and \( B_E \neq \phi_E \) are soft open sets in \( X_E \). So, \( X_E - A_E \) and \( X_E - B_E \) are proper soft closed sets in \( X_E \) and \( X_E - [(X_E - A_E) \cup (X_E - B_E)] \neq \mathcal{I} \). This implies that \( X_E - [X_E - (A_E \cup B_E)] \neq \mathcal{I} \). Hence, \( (X_E, \tau, \mathcal{I}) \) is soft \( HC \) modulo \( \mathcal{I} \).

**Theorem 2.** Let \((X_E, \tau, \mathcal{I})\) be a SITS and \( \tau \cap \mathcal{I} = \phi_E \). Then, \((X_E, \tau, \mathcal{I})\) is soft \( HC \) modulo \( \mathcal{I} \) if and only if \((X_E, \tau, \mathcal{I})\) is soft \( HC \).
Mathematics 2023, 4697

A soft topological space

Theorem 5. Let $(X_E, \tau, \mathcal{I})$ be a soft $\mathcal{H}C$ modulo $\mathcal{T}$, $(X_E, \tau)$ is soft $\mathcal{H}C$.

Proof. Assume that $(X_E, \tau, \mathcal{I})$ is a soft $\mathcal{H}C$ modulo $\mathcal{T}$. So, since $\phi_E \in \mathcal{I}$, $(X_E, \tau)$ is soft $\mathcal{H}C$.

Conversely, let $(X_E, \tau)$ be a soft $\mathcal{H}C$ and $A_E, B_E$ be non-null soft open sets. Then, $A_E \cap B_E$ is a non-null soft open set in $(X_E, \tau)$. Since $\tau \cap \mathcal{I} = \phi_E$, $A_E \cap B_E \notin \mathcal{I}$. Thus, $(X_E, \tau, \mathcal{I})$ is soft $\mathcal{H}C$ modulo $\mathcal{T}$. □

The following example show that, if $\tau \cap \mathcal{I} \neq \phi_E$, Theorem 2 is not true.

Example 1. Let $(X_E, \tau, \mathcal{I})$ be a SITS, where $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ $\tau = \{(X_E, \phi_E), \{(e_1, \{h_1\}), (e_2, \{h_2\})\}, \{\{e_1, \{h_1\}\}, (e_2, \{X_E\})\}\}$, and $\mathcal{I} = \{\phi_E, \{(e_1, \{h_1\})\}, \{(e_2, \{h_2\})\}, \{\{e_1, \{h_1\}, \{e_2, \{h_2\}\}\}}\}$. Then, $\tau \cap \mathcal{I} \neq \phi_E$.

Since every pair of non-null soft open sets of $X_E$ has non-null soft intersection, $(X_E, \tau, \mathcal{I})$ is soft $\mathcal{H}C$. But it is clear that it is not soft $\mathcal{H}C$ modulo $\mathcal{T}$.

Theorem 3. A soft topological space $(X_E, \tau)$ is soft $\mathcal{H}C$ iff the union of two not soft dense sets is a not soft dense set.

Proof. Assume that $(X_E, \tau)$ is soft $\mathcal{H}C$ and $G_E, F_E$ are two not soft dense sets in $(X_E, \tau)$. Then there exist two non-null soft open sets $U_E$ and $V_E$ such that $U_E \cap G_E = \phi_E$ and $V_E \cap F_E = \phi_E$. Since $(X_E, \tau)$ is soft $\mathcal{H}C$, $U_E \cap V_E \neq \phi_E$. But $(U_E \cap V_E) \cap (G_E \cup F_E) = \phi_E$ and, hence, $G_E \cup F_E$ is not soft dense in $(X_E, \tau)$.

Conversely, if the condition is true in $(X_E, \tau)$ but $(X_E, \tau)$ is not soft $\mathcal{H}C$, then there exist two non-null soft open sets $U_E$ and $V_E$ such that $U_E \cap V_E = \phi_E$. Hence, $U_E \subseteq X_E - V_E$ and $V_E \subseteq X_E - U_E$. Then, $X_E - U_E$ and $X_E - V_E$ are not soft dense in $(X_E, \tau)$. But $(X_E - U_E) \cup (X_E - V_E) = X_E$. This contradicts the assertion that a union of two non-soft dense sets is also not a soft dense set. The theorem is therefore true. □

Lemma 2. Let $(X_E, \tau, \mathcal{I})$ be a SITS. Then, $(X_E, \tau, \mathcal{I})$ is soft ideal $\mathcal{H}C$ if and only if $(X_E, \tau)$ is soft $\mathcal{H}C$ and $\tau \cap \mathcal{I} = \phi_E$.

Proof. Clearly, every soft ideal $\mathcal{H}C$ space is soft $\mathcal{H}C$. Let $U_E$ be a non-null soft open set in the soft ideal. Then, $\mathcal{U}_E = X_E$. Conversely, yet, since $U_E \in \mathcal{I}$, $\mathcal{U}_E = \phi_E$. Hence, $X_E = \phi_E$. There is inconsistency here. Consequently, $\tau \cap \mathcal{I} = \phi_E$.

Conversely, let $U_E$ be a non-null soft open set. Let $x_E \in X_E$. Due to the soft $\mathcal{H}C$ property of $(X_E, \tau)$, every soft open set $V_E$ containing $x_E$ meets $U_E$. Moreover, $U_E \cap V_E$ is a soft open set and $U_E \cap V_E \notin \mathcal{I}$ because $\tau \cap \mathcal{I} = \phi_E$. Thus, $x_E \in \mathcal{U}_E$. This shows that $U_E$ is soft ideal dense. □

Theorem 4. Let $(X_E, \tau, \mathcal{I})$ be a SITS, where $\tau \cap \mathcal{I} = \phi_E$. Then, a set $D_E$ is soft ideal dense if and only if $(U_E - A_E) \cap D_E \neq \phi_E$ whenever $U_E$ is non-null soft open and $A_E \in \mathcal{I}$.

Proof. Let $D_E$ be soft ideal dense. So, $U_E \cap D_E \notin \mathcal{I}$ for all non-null soft open sets $U_E$. Hence, for all $A_E \in \mathcal{I}$, $(U_E - A_E) \cap D_E \neq \phi_E$, for, otherwise, $(U_E - A_E) \cap D_E = \phi_E$ and, hence, $\phi_E = U_E \cap (X_E - A_E) \cap D_E = (U_E \cap D_E) \cap (X_E - A_E)$. Therefore, $U_E \cap D_E \subseteq A_E$. Since $A_E \in \mathcal{I}$, $U_E \cap D_E \in \mathcal{I}$, which is contrary to $U_E \cap D_E \notin \mathcal{I}$. Therefore, $(U_E - A_E) \cap D_E \neq \phi_E$.

Conversely, let $(U_E - A_E) \cap D_E \neq \phi_E$ whenever $U_E$ is a non-null soft open set and $A_E \in \mathcal{I}$. Next, we assert that $D_E$ is soft ideal dense. Let $D_E$ be not soft ideal dense. Then, there exists some non-null soft open set $U_E$ such that $U_E \cap D_E \in \mathcal{I}$. Let $U_E \cap D_E = A_E$. So, since $\tau \cap \mathcal{I} = \phi_E$, $U_E - A_E$ is non-null but $(U_E - A_E) \cap D_E = \phi_E$. This defies everything we had assumed. □

Theorem 5. Let $(X_E, \tau, \mathcal{I})$ be a SITS, where $\tau \cap \mathcal{I} = \phi_E$. Then, $(X_E, \tau, \mathcal{I})$ is soft $\mathcal{H}C$ modulo $\mathcal{T}$ if and only if $(U_E - A_E) \cap D_E \neq \phi_E$ whenever $U_E$ and $D_E$ are non-null soft open sets and $A_E \in \mathcal{I}$.

Proof. From Lemma 2 and Theorem 4, the proof follows. □
5. Soft Ideal Resolvable Spaces

A soft space \((X_E, \tau)\) is soft resolvable [14], symbolized \((RS)\), if \(X_E\) is the union of two soft dense subsets which are disjoint.

A \(SITS\) \((X_E, \tau, \mathcal{I})\) is soft ideal \(RS\) if it has two disjoint soft ideal dense sets; alternatively, it is claimed to be soft ideal irresolvable, symbolized \((IRS)\).

Lemma 3. Let \((X_E, \tau, \mathcal{I})\) be a \(SITS\).

(1) \((X_E, \tau, \mathcal{I})\) is soft ideal \(RS\) iff \(X_E\) is the union of two disjoint soft ideal dense sets.

(2) If \((X_E, \tau, \mathcal{I})\) is soft ideal \(RS\), then \(\tau \cap \mathcal{I} = \phi_E\).

Proof. (1) Let \(A_E\) and \(B_E\) be disjoint soft ideal dense sets. Then, \(\overline{A_E} = X_E\) and \(X_E = B_E \subseteq (X_E - A_E)\), and, hence, \(X_E = (X_E - A_E)\). Therefore, \(X_E\) is the union of soft ideal dense sets \(A_E\) and \(X_E - A_E\). The opposite is evident.

(2) Let \(A_E\) and \(B_E\) be disjoint soft ideal dense sets. So, by Theorem 3.2 of [4], we have \(X_E = \overline{A_E} \subseteq X_E^\star\). Therefore, \(X_E\) is soft ideal dense. Thus, by Theorem 1, \(\tau \cap \mathcal{I} = \phi_E\). □

Remark 2. In cite kandil it was obtained that \(\overline{Cl}^\star (A_E) = A_E \cup A_E^\star\) is a soft Kuratowski closure operator. We will denote by \((X_E, \tau^\star, \mathcal{I})\) the soft topology generated by \(\overline{Cl}^\star\), that is, \(\tau^\star = \{U_E \subseteq X_E : \overline{Cl}^\star (X_E - U_E) = X_E - U_E\}\).

Theorem 6 ([29]). Let \((X_E, \tau, \mathcal{I})\) be a \(SITS\). Then \(\beta(\tau^\star, \mathcal{I}) = \{V_E - I : V_E\) is soft open set of \((X_E, \tau), I \in \mathcal{I}\}\) is a basis for \((X_E, \tau^\star)\).

Theorem 7. A \(SITS\) \((X_E, \tau, \mathcal{I})\) is soft ideal \(RS\) if and only if \((X_E, \tau^\star)\) is soft \(RS\) and \(\tau \cap \mathcal{I} = \phi_E\).

Proof. Let \((X_E, \tau, \mathcal{I})\) be soft ideal \(RS\). So, by Lemma 3 (1), \(X_E = A_E \cup B_E\), where \(A_E\) and \(B_E\) are disjoint soft ideal dense sets of \(X_E\). Note that \(\overline{Cl}^\star (A_E) = A_E \cup A_E^\star = A_E \cup X_E = X_E\). Hence, \(A_E\) and \(B_E\) are soft dense in \((X_E, \tau^\star)\). Thus, \((X_E, \tau^\star)\) is soft \(RS\). By Lemma 3 (2), \(\tau \cap \mathcal{I} = \phi_E\).

Conversely, let \((X_E, \tau^\star)\) be soft \(RS\) and \(\tau \cap \mathcal{I} = \phi_E\). Suppose that \(X_E = A_E \cup B_E\), \(A_E \cap B_E = \phi_E\), and both \(A_E\) and \(B_E\) are soft dense in \((X_E, \tau^\star)\). Let \(x_E \in X_E\) and \(x_E \notin \overline{A_E}\); then, there exists a soft open set \(U_E\) containing \(x_E\) such that \(V_E = U_E \cap \mathcal{I} \subseteq \mathcal{I}\). Since \(B_E\) is soft dense in \((X_E, \tau^\star)\) and \(\tau \cap \mathcal{I} = \phi_E\), \(V_E\) is non-null and also \(U_E \notin \mathcal{I}\). Hence, by Theorem 6, \(W_E = U_E - V_E \in (X_E, \tau^\star)\) is a non-null set and \(W_E \cap A_E = \phi_E\). This contradicts the fact that \(A_E\) is soft dense in \((X_E, \tau^\star)\). Thus, \(x_E \notin \overline{A_E}\) and, hence, \(A_E\) is soft ideal dense. A related argument demonstrates that \(B_E\) is soft ideal dense. Thus, \((X_E, \tau, \mathcal{I})\) is soft ideal \(RS\). □

Definition 8 ([3]). Let \(Y_E \neq \phi_E\) be a soft subset of \((X_E, \tau, E)\); then, \(\tau_{Y_E} = \{G_E \cap Y_E : G_E \in \tau\}\) is called a relative soft topology over \(Y\) and \((Y_E, \tau_{Y_E}, E)\) is a soft subspace of \((X_E, \tau, E)\).

Lemma 4. Let \(Y_E \subseteq X_E\) and \(\mathcal{I}\) be soft ideal in \(X_E\). Then, \(\mathcal{I}_{Y_E} = \{I \subseteq \mathcal{I} : I \subseteq Y_E\}\) is soft ideal in \(Y_E\).

Lemma 5. Let \((X_E, \tau, \mathcal{I})\) be a \(SITS\). The non-null soft \(\tau^\star\)-open subspace of a soft ideal \(RS\) space is a soft ideal \(RS\) space.

Proof. First, we know that the intersection of a soft dense and a soft open set is soft dense, so the soft resolvability is a soft open hereditary. Also, for all \(A_E \in \tau^\star\) we have \(\tau_{A_E} = (\tau_{A_E})^\star\). Thus, by Theorem 7, if \((X_E, \tau, \mathcal{I})\) is soft ideal \(RS\) and \(A\) is \(\tau^\star\)-open, then \((X_E, \tau^\star)\) is soft \(RS\); hence, \((A_E, \tau_{A_E}) = (A_E, (\tau_{A_E})^\star)\) is soft \(RS\) and, thus, \((A_E, \tau_{A_E}, \mathcal{I}_{A_E})\) is soft ideal \(RS\). □
Theorem 8. Let \((X_E, \tau, \mathcal{T})\) be a \(\mathcal{SITSS}\). Simple expansion of soft ideal \(\mathcal{RS}\) topologies over soft ideal \(\mathcal{RS}\) subspace are soft ideal \(\mathcal{RS}\).

Proof. Let \((X_E, \tau, \mathcal{T})\) be soft ideal \(\mathcal{RS}\) and \(S_E \subseteq X_E\) be a soft ideal \(\mathcal{RS}\) subspace. Let \((D_E, D'_E)\) be the soft ideal resolution of \((S_E, \tau|_{S_E}, \mathcal{T}|_{S_E})\). We examine the next two instances:

Case (1): \(S_E\) is soft \(\tau^*\)-dense in \((X_E, \tau, \mathcal{T})\); that is, \(X_E = S_E \cup S'_E\). We first establish that \(D_E\) is soft ideal dense in \((X_E, \tau, \mathcal{T})\). Let \(x_e \in X_E\). Suppose that for some soft open set \(U_E\) with \(x_e \in U_E\) we have \(U_E \cap \mathcal{D}_E \in \mathcal{T}\). The two subcases that follow are ours.

Subcase (a): \(x_e \in S_E\). Then, \(V_E = U_E \cap S_E \in \tau|_{S_E}\) is a soft open set of \(x_e\) in \((S_E, \tau|_{S_E}, \mathcal{T}|_{S_E})\) such that \(V_E \cap \mathcal{D}_E = U_E \cap S_E \cap D_E \in \mathcal{T}\) due to the heredity of \(\mathcal{T}\). This defies the assertion that \(D_E\) is soft ideal dense in \((S_E, \tau|_{S_E}, \mathcal{T}|_{S_E})\). So, \(D_E\) is soft ideal dense in \((X_E, \tau, \mathcal{T})\).

Subcase (b): \(x_e \notin S_E\). Since \(X_E = S_E \cup S'_E\), \(x_e \in S_E\). To demonstrate that \(x_e \in D'_E\), we believe the opposite, i.e., there exists a soft open set \(U_E\) with \(x_e \in U_E\) such that \(U_E \cap \mathcal{D}_E \in \mathcal{T}\). Note that \(U_E \cap S_E \neq \phi_E\); otherwise, \(x_e \notin S'_E\). Pick \(y_e \in U_E \cap S_E \in \tau|_{S_E}\). Since \(U_E \cap D_E \in \mathcal{T}\), then, by heredity of \(\mathcal{T}\), \(U_E \cap S_E \cap D_E \in \mathcal{T}\). So, \(D_E\) is not soft ideal dense in \((S_E, \tau|_{S_E}, \mathcal{T}|_{S_E})\). By contradiction \(x_e \in D'_E\), i.e., \(D_E\) is soft ideal dense in \((X_E, \tau, \mathcal{T})\). So, we have demonstrated that \(D'_E = X_E\). Using a comparable defense, \(D''_E = X_E\). Let \(x_e \in X_E\) and let \(U_E \cup (V_E \cap S_E)\) be a soft open set of \(x_e\) in \((X_E, \tau(S_E), \mathcal{T})\), where \(\tau(S_E)\) is the simple expansion of \(\tau\) over \(S_E\). If \((U_E \cup (V_E \cap S_E)) \cap D_E \in \mathcal{T}\), then, by heredity of \(\mathcal{T}\), \((V_E \cap S_E) \cap D_E\) is a member of \(\mathcal{T}\) so that \(V_E\) is a null set. Of course, \((V_E \cap S_E) \cap D_E\) cannot be a member of \(\mathcal{T}\) if \(V_E\) is non-null since then \(V_E\) must contain an element of \(S_E\). So, \(x_e\) belongs to \(U_E \cap D_E\) which is also not eligible to join with \(\mathcal{T}\) since \(D'_E = X_E\). This contradiction shows that \(D_E\) is soft \(\tau(S_E)\)-dense. Using a comparable defense of \(D'_E\), we determine that \((X_E, \tau(S_E), \mathcal{T})\) is soft ideal \(\mathcal{RS}\).

Case (2): \(S_E\) is not soft \(\tau^*\)-dense in \((X_E, \tau, \mathcal{T})\). Then, \(S'_E = X_E \setminus \text{Cl}^* (S_E)\), so it is \(\tau^*\)-open and non-null. By Lemma 5, \(S'_E\) is soft ideal \(\mathcal{RS}\) (more precisely said soft ideal \(\mathcal{RS}\) with respect to \(S_E\)). Let \((A_E, B_E)\) be the soft ideal resolution of \(S'_E\). By using reasoning akin to that of Case (1), we can prove that \((D_E \cap A_E, D_E \cap B_E)\) is a soft ideal resolution of \((X_E, \tau, \mathcal{T})\). Additionally, employing the same method as at the conclusion of Case (1), we find that \((X_E \tau(S_E); \mathcal{T})\) is soft ideal \(\mathcal{RS}\).

\(\square\)

Theorem 9. A \(\mathcal{SITSS}(X_E, \tau, \mathcal{T})\) is soft ideal \(\mathcal{RS}\) if there exists a soft ideal dense set \(D_E\) such that, for all non-null soft open sets \(U_E\) and all \(A_E \in \mathcal{T}\), \(U_E \setminus A_E \neq \phi_E\) implies \(U_E \setminus A_E \not\subseteq \mathcal{D}_E\).

Proof. Let \((X, \tau, \mathcal{T})\) be soft ideal \(\mathcal{RS}\). So, by Remark 1 and Theorem 1, \(\tau \cap \mathcal{T} = \phi_E\). Now, there exist two disjoint soft ideal dense sets, say \(D'_E\) and \(D''_E\). We demonstrate that \((U_E - A_E) \not\subseteq \mathcal{D}_E\) whenever \(U_E - A_E \neq \phi_E\) for all non-null soft open sets \(U_E\) and \(A_E \in \mathcal{T}\). If possible, let \((U_E - A_E) \not\subseteq \mathcal{D}_E\) for some non-null soft open set \(U_E\) and \(A_E \in \mathcal{T}\). Then, \((U_E - A_E) \not\subseteq \mathcal{D}_E\) for all non-null soft open sets \(U_E\) and \(A_E \in \mathcal{T}\). Clearly, \(U_E \cap (X_E - D_E) \neq \phi_E\), for otherwise \(V_E \not\subseteq \mathcal{D}_E\), which is contrary to our assumption. Let \(A_E = V_E \cap (X_E - D_E)\). Then, \(V_E - A_E \neq \phi_E\). For if \(V_E - A_E = \phi_E\) then \(V_E \not\subseteq A_E\) and,
hence, $V_E \in \mathcal{I}$, which suggests $V_E \cap D_E \in \mathcal{I}$. Contrary to that, this $D_E$ is soft ideal dense. Therefore, $V_E - A_E \subseteq D_E$. It goes against our presumption once more. Thus, $X_E - D_E$ is soft ideal dense and so $(X_E, \tau, \mathcal{I})$ is soft ideal RS. \qed

**Corollary 1.** A SITSS $(X_E, \tau, \mathcal{I})$ is soft ideal IRS iff, for each soft ideal dense set $D_E$, there exist a soft open set $U_E$ and $A \in \mathcal{I}$ such that $\phi_E \neq (U_E - A_E) \subseteq D_E$.

**Theorem 10.** If $(X_E, \tau, \mathcal{I})$ is a SITSS such that $\tau \cap \mathcal{I} = \phi_E$ and if $D_E$ is soft ideal dense in $(X_E, \tau, \mathcal{I})$, then, for all $Y_E = U_E - A_E$, where $U_E$ is non-null soft open and $A_E \in \mathcal{I}$, $Y_E \cap D_E$ is soft ideal dense in $(Y_E, \tau_{Y_E}, \mathcal{I}_{Y_E})$.

**Proof.** Clearly, we suppose that $\tau \cap \mathcal{I} = \phi_E$. Then, by Proposition 11 of [3], a soft open set in $Y_E$ is of the form $Y_E \cap O_E = (U_E - A_E) \cap O_E = (U_E \cap O_E) - A_E$, where $O_E$ is a soft open set in $(X_E, \tau)$. Let $\phi_E \neq U_E \cap O_E - A_E$. Consider $\phi_E \neq \left( (U_E \cap O_E) - A_E \right) - B_E$, $B_E \in \mathcal{I}_{Y_E}$. Then, since $D_E$ is soft ideal dense and $U_E \cap O_E$ is a soft open set in $(X_E, \tau)$, by Theorem 4, $\left( (U_E \cap O_E) - (A_E \cup B_E) \right) \cap D_E \neq \phi_E$. Hence, $\left( (U_E \cap O_E) - A_E \right) - B_E \cap D_E \neq \phi_E$. Therefore, again by Theorem 4, $Y_E \cap D_E$ is soft ideal dense in $(Y_E, \tau_{Y_E}, \mathcal{I}_{Y_E})$. \qed

**Theorem 11.** Let $(X_E, \tau, \mathcal{I})$ be a SITSS such that $\tau \cap \mathcal{I} = \phi_E$ and $P_E \subseteq Y_E = U_E - A_E$, where $U_E$ is a non-null soft open set, $A_E \in \mathcal{I}$. Then, $P_E$ is soft ideal dense in $(Y_E, \tau_{Y_E}, \mathcal{I}_{Y_E})$ if and only if $P_E = Y_E - D_E$, where $D_E$ is soft ideal dense in $(X_E, \tau, \mathcal{I})$.

**Proof.** Assume that $P_E$ is soft ideal dense in $(Y_E, \tau_{Y_E}, \mathcal{I}_{Y_E})$. Consider the set $P_E \cup (X_E - Y_E)$. Then, $\phi_E = (P_E \cup (X_E - Y_E)) \cap O_E = (P_E \cap O_E) \cup (X_E - Y_E) \cap O_E$, where $O_E$ is a non-null soft open set. Now, if $O_E \subseteq X_E - Y_E$, then $P_E \subseteq Y_E$ and $P_E \cap O_E = \phi_E$, and we have $(P_E \cup (X_E - Y_E)) \cap O_E = O_E$ which is not in $\mathcal{I}$ because $\tau \cap \mathcal{I} = \phi_E$. Moreover, if $O_E \cap \phi_E \neq \phi_E$, then, since $P_E$ is soft ideal dense in $(Y_E, \tau_{Y_E}, \mathcal{I}_{Y_E})$, $P_E \cap (O_E \cap Y_E) \notin \mathcal{I}_{Y_E}$ and so $P_E \cap O_E \notin \mathcal{I}$. Therefore, $(P_E \cup (X_E - Y_E)) \cap O_E \notin \mathcal{I}$. Thus, $(P_E \cup (X_E - Y_E)) = D_E$, say, is soft ideal dense in $(X_E, \tau, \mathcal{I})$ and, hence, $P_E = Y_E - D_E$. Next, let $P_E = Y_E - D_E$, where $D_E$ is soft ideal dense in $(X_E, \tau, \mathcal{I})$. Hence, by Theorem 10, $P_E$ is soft ideal dense in $(Y_E, \tau_{Y_E}, \mathcal{I}_{Y_E})$. This completes the proof of the theorem. \qed

Note that, as per the condition in Theorem 11, for $D_E$ soft ideal dense is necessary because if $D_E$ is not soft ideal dense then $P_E = \phi_E$ for some non-null soft open set $U_E$, $A_E \in \mathcal{I}$ and, hence, $P_E$ is not soft ideal dense in $(Y_E, \tau_{Y_E}, \mathcal{I}_{Y_E})$.

**6. Soft Ideal Semi-Irresolvable Spaces**

Next, we will define and go over the characteristics of a soft ideal semi-IRS space.

**Definition 9.** A SITSS $(X_E, \tau, \mathcal{I})$ is said to be soft ideal semi-IRS if for each soft ideal dense set $D_E$ and each non-null soft open set $U_E$ and $A_E \in \mathcal{I}$ such that $U_E - A_E$ is non-null set, there exists a non-null soft open set $V_E$ and $B_E \in \mathcal{I}$ such that $\phi_E \neq (V_E - B_E) \subseteq (U_E - A_E) \cap D_E$.

**Theorem 12.** A SITSS $(X_E, \tau, \mathcal{I})$ is a soft ideal semi-IRS iff the intersection of soft ideal dense sets is a soft ideal dense set, where $\tau \cap \mathcal{I} = \phi_E$.

**Proof.** Assume that $(X_E, \tau, \mathcal{I})$ is a soft ideal semi-IRS and $\tau \cap \mathcal{I} = \phi_E$. Let $D'_E$ and $D''_E$ be two soft ideal dense sets in $(X_E, \tau, \mathcal{I})$. We demonstrate that $D'_E \cap D''_E$ is soft ideal dense. Consider $U_E - A_E$, where $U$ is a non-null soft open set and $A_E \in \mathcal{I}$. As we demonstrate, $(U_E - A_E) \cap D'_E \cap D''_E = \phi_E$. Since $D'_E$ is soft ideal dense, by Theorem 4, $(U_E - A_E) \cap D'_E = \phi_E$. Since $(X_E, \tau, \mathcal{I})$ is soft ideal semi-IRS, there exists a non-null soft open set $V'_E$ and $B'_E \in \mathcal{I}$ such that $\phi_E \neq (V'_E - B'_E) \subseteq (U_E - A_E) \cap D'_E$. Again, since $D''_E$ is soft ideal dense, there exists a non-null soft open set $V''_E$ and $B''_E \in \mathcal{I}$ such
that \( \phi_E \neq (V_E' - B_E') \subseteq (V_E' - B_E') \cap D_E'. \) Hence, \( \phi_E \neq V'_E - B'_E \subseteq (U_E - A_E) \cap D'_E \cap D''_E. \) Therefore, \( (U_E - A_E) \cap (D'_E \cap D''_E) \neq \phi_E \) and, by Theorem 4, \( \text{IRS} \cap D''_E \) is soft ideal dense.

Conversely, assume that the intersection of soft ideal dense sets is soft ideal dense. Assume that \((X_E, \tau, \mathcal{I})\) is not soft ideal semi-IRS. Then, there exists a soft ideal dense set \( D'_E \), and a non-null soft open set \( U_E \) and \( A_E \in \mathcal{I} \), where \( \phi_E \neq U_E - A_E \), such that \((U_E - A_E) \cap D'_E \) does not contain any non-null soft open set \( V_E \) and \( B_E \in \mathcal{I} \).

Consider the set \( D''_E = (X_E - (U_E - A_E)) \cap ((U_E - A_E) - (U_E - A_E) \cap D'_E) \). By Theorem 4, \( D''_E \) is soft ideal dense since \( (V_E - B_E) \cap D''_E \neq \phi_E \). But \((U_E - A_E) \cap D''_E \cap D''_E = \phi_E \). This contradicts the reality that the intersection of two soft ideal dense sets is a soft ideal dense set. Hence, \((X_E, \tau, \mathcal{I})\) must be soft ideal semi-IRS. This concludes the theorem’s proof. □

**Example 2.** Let \((X_E, \tau, \mathcal{I})\) be a SITS, where \( X = \{h_1, h_2, h_3\} \), \( E = \{e\} \). Consider \( \tau = \{X_E, \phi_E, \{(e, \{h_1, h_2\})\} \) and \( \mathcal{I} = \{\phi_E, \{(e, \{h_2, h_3\})\}, \{(e, \{h_1, h_3\})\} \). Then, we have the following.

1. \( \tau \cap \mathcal{I} = \phi_E \).
2. The collection of all soft ideal dense sets is \( X_E, \{(e, \{h_1\})\}, \{(e, \{h_2, h_3\})\} \) and \( \{(e, \{h_1, h_3\})\} \).
3. The soft intersection of any soft ideal dense sets is soft ideal dense.

Hence, by Theorem 12, \((X_E, \tau, \mathcal{I})\) is soft ideal semi-IRS.

**Theorem 13.** Let \((X_E, \tau, \mathcal{I})\) be a SITS and \( \tau \cap \mathcal{I} = \phi_E \). If \((X_E, \tau, \mathcal{I})\) is soft ideal semi-IRS, then \((Y_E, \tau_{Y_E}, \mathcal{I}_{Y_E})\) is soft ideal semi-IRS whenever \( Y_E = U_E - A_E \), for every non-null soft open set \( U_E \) and \( A_E \in \mathcal{I} \).

**Proof.** Assume that \( D_E \) and \( G_E \) are soft ideal dense sets in \((Y_E, \tau_{Y_E}, \mathcal{I}_{Y_E})\). Then, by Theorem 11, \( D_E = (U_E - A_E) \cap D_E' \) and \( G_E = (U_E - A_E) \cap D_E'' \), where \( D_E' \) and \( D_E'' \) are soft ideal dense sets in \((X_E, \tau, \mathcal{I})\). Hence, \( D_E \cap G_E = (U_E - A_E) \cap D_E' \cap D_E'' \) and, since \( D_E' \cap D_E'' \) is a soft ideal dense set in \((X_E, \tau, \mathcal{I})\), once more by Theorem 11, \( D_E \cap G_E \) is soft ideal dense in \((Y_E, \tau_{Y_E}, \mathcal{I}_{Y_E})\). So, by Theorem 12, \((Y_E, \tau_{Y_E}, \mathcal{I}_{Y_E})\) is soft ideal semi-IRS. □

**Definition 10.** A SITS \((X_E, \tau, \mathcal{I})\) is said to be soft ideal semi-\(HC\) if each \( U_E - A_E \neq \phi_E \), where \( U_E \) is a soft open set and \( A_E \in \mathcal{I} \) is a soft ideal dense set.

**Theorem 14.** A SITS \((X_E, \tau, \mathcal{I})\) is soft ideal semi-\(HC\) iff it is soft ideal \(HC\) and \( \tau \cap \mathcal{I} = \phi_E \).

**Proof.** Let \((X_E, \tau, \mathcal{I})\) be soft ideal semi-\(HC\). Clearly, \((X_E, \tau, \mathcal{I})\) is soft ideal \(HC\). Let \( U_E \neq \phi_E \) be a non-null soft open set and a member of the soft ideal \( \mathcal{I} \). Then, \( U_E^{-1} = X_E \) since \((X_E, \tau, \mathcal{I})\) is soft ideal \(HC\). Conversely, yet, since \( U_E \in \mathcal{I} \), \( U_E^{-1} = \phi_E \), it is paradoxical. So \( \tau \cap \mathcal{I} = \phi_E \).

Conversely let \((X_E, \tau, \mathcal{I})\) be a soft ideal \(HC\) and \( \tau \cap \mathcal{I} = \phi_E \). Let \( U_E - A_E \) where \( U_E \) is a non-null soft open set and \( A_E \in \mathcal{I} \). Then \( U_E - A_E \neq \phi_E \) because \( \tau \cap \mathcal{I} = \phi_E \).

We show that \( U_E - A_E \) is soft ideal dense. Let \( x_E \in X_E \) and \( V_E \) be a soft open set containing \( x_E \). By Lemma 2, \((X_E, \tau)\) is soft \(HC\) and \( V_E \cap (U_E - A_E) \neq \phi_E \) because \( V_E \cap (U_E - A_E) = V_E \cap U_E - A_E \neq \phi_E \) and \( \tau \cap \mathcal{I} = \phi_E \). Thus, \((X_E, \tau, \mathcal{I})\) is soft ideal semi-\(HC\). □

**Example 3.** Let \((X_E, \tau, \mathcal{I})\) be a SITS, where \( X = \{h_1, h_2, h_3\} \), \( E = \{e\} \). Consider \( \tau = \{X_E, \phi_E, \{(e, \{h_1, h_2\})\} \) and \( \mathcal{I} = \{\phi_E, \{(e, \{h_2, h_3\})\}, \{(e, \{h_1, h_3\})\} \). Then

1. \( \tau \cap \mathcal{I} = \phi_E \).
2. Every non-null soft open set is soft ideal dense. So, \((X_E, \tau, \mathcal{I})\) is soft ideal \(HC\).

Hence, by Theorem 14, \((X_E, \tau, \mathcal{I})\) is soft ideal semi-IRS.

**Theorem 15.** If a SITS \((X_E, \tau, \mathcal{I})\) is soft ideal semi-\(HC\) and soft ideal IRS, then it is soft ideal semi-IRS.
We demonstrate that $D_E'$ and $D_E''$ are two soft ideal dense sets in $(X_E, \tau, \mathcal{I})$. We require to demonstrate that $D_E' \cap D_E''$ is soft ideal dense. By Theorem 4, it suffices to demonstrate that $(D_E' \cap D_E'') \cap (U_E - A_E) \neq \phi_E$ for all non-null soft open sets $U_E$ and $A_E \in \mathcal{I}$. So, since $(X_E, \tau, \mathcal{I})$ is soft ideal IRS, by Corollary 1, there exists a non-null soft open set $V_E$ and $B_E \in \mathcal{I}$ such that $\phi_E \neq V_E - B_E \subseteq D_E'$. Similarly, there exists a non-null soft open set $W_E$ and $C_E \in \mathcal{I}$ such that $\phi_E \neq W_E - C_E \subseteq D_E''$. Now, $(X_E, \tau)$ is soft HHC by Lemma 2 and Theorem 14; we have $V_E \cap W_E \neq \phi_E$. Since $\tau \cap \mathcal{I} = \phi_E, (V_E - B_E) \cap (W_E - C_E) = (V_E \cap W_E) - (B_E \cup C_E) \neq \phi_E$ and, hence, $(V_E \cap W_E) - (B_E \cup C_E) \subseteq D_E' \cap D_E''$. Therefore, by the soft ideal semi-HHC property of $(X_E, \tau, \mathcal{I})$, $(V_E \cap W_E) - (B_E \cup C_E)$ is soft ideal dense and, by Theorem 4, we have $\phi_E \neq (U_E - A_E) \cap [(V_E \cap W_E) - (B_E \cup C_E)]$ and, hence, $(U_E - A_E) \cap (D_E' \cap D_E'') \neq \phi_E$. Therefore, $D_E' \cap D_E''$ is soft ideal dense. So, by Theorem 12, $(X_E, \tau, \mathcal{I})$ is soft ideal semi-IRS.

**Remark 3.** For a SITS $(X_E, \tau, \mathcal{I})$, if $\tau \cap \mathcal{I} \neq \phi_E$. Then, no soft ideal dense set exists, because, if $\tau \cap \mathcal{I} \neq \phi_E$ and there exists $D_E$, any soft ideal dense, then $D_E^* = X_E$, so by Remark 1 we have $X_E^* = X_E$. Hence, by Theorem 1, $\tau \cap \mathcal{I} = \phi_E$, which is a contradiction. Therefore, if $\tau \cap \mathcal{I} \neq \phi_E$ then no soft ideal dense set exists.

**Question:** Is there any example of soft ideal topological space such that $\tau \cap \mathcal{I} \neq \phi_E$, and Theorems 10, 11, 12, 13, and 14 are true?

**7. Conclusions and Future Work**

As an extension of the classical (crisp) topology, the idea of a soft topology on a universal set was independently proven by Shabir and Naz [3], and Çağman et al. [30]. The study of this topological generalization has becoming more fascinating. Numerous techniques for building soft topologies have been documented in the literature. We have added to the body of knowledge in soft topology by delving into the ideas of soft hyperconnected modulo ideal, soft ideal resolvable, and soft ideal semi-irresolvable spaces. This research is based on the hyperconnectedness and resolvability of soft ideal spaces. We spoken about several fundamental operations on soft ideal spaces. A concept of a soft ideal semi-irresolvable space and an overview of its properties are provided. Furthermore, we have determined the basic characteristics of soft ideal resolvable spaces and connections between the other concepts. The findings presented in this work are preliminary and further research will examine additional aspects of the soft ideal resolvable space. By integrating these two approaches, our work creates opportunities for potential contributions to this trend using hyperconnectedness and resolvability structures with generalized rough approximation spaces, as well as the resolvability of primal soft topologies and the resolvability of fuzzy soft topologies in classical and soft settings.

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Abbreviations

The following abbreviations are utilized in this document:

SITS  soft ideal topological space

HC  hyperconnected

IRS  irresolvable

RS  resolvable

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