Review

Exploring the Contributions to Mathematical Economics: A Bibliometric Analysis Using Bibliometrix and VOSviewer

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Abstract: From Cournot, Walras, and Pareto’s research to what followed in the form of marginalist economics, chaos theory, agent-based modeling, game theory, and econophysics, the interpretation and analysis of economic systems have been carried out using a broad range of higher mathematics methods. The evolution of mathematical economics is associated with the most productive and influential authors, sources, and countries, as well as the identification of interactions between the authors and research topics. Bibliometric analysis provides journal-, author-, document-, and country-level metrics. In the present study, a bibliometric overview of mathematical economics came from a screening performed in September 2023, covering the timespan 1898–2023. About 6477 documents on mathematical economics were retrieved and extracted from the Scopus academic database for analysis. The Bibliometrix package in the statistical programming language R was employed to perform a bibliometric analysis of scientific literature and citation data indexed in the Scopus database. VOSviewer (version 1.6.19) was used for the visualization of similarities using several bibliometric techniques, including bibliographic coupling, co-citation, and co-occurrence of keywords. The analysis traced the most influential papers, keywords, countries, and journals among high-quality studies in mathematical economics.

Keywords: mathematical economics; Bibliometrix; VOSviewer; bibliometric analysis; science mapping

MSC: 00A64; 00A06; 91-00; 91B02

1. Introduction

Chiang [1] defines mathematical economics as “Mathematical Economics refers to the application of mathematics to the purely theoretical aspects of economic analysis, with little or no concern about such statistical problems as the errors of measurement of the variables under study”. Aligned with this view of the role of mathematical economics, this paper screens the beginning, evolution, and recent trends of the relevant scientific literature.

Cournot, Walras Jevons, Pigou, Fisher, and Edgeworth are considered the precursors to modern mathematical economics [2,3]. The origin of mathematical economics dates back to the late 19th century and is attributed to pioneers like Jevons, Walras, and Fisher.

Modern quantitative economic analyses focus more on technical analysis and less on the social problems posed by classical political economy [4]. The modern role of mathematical model-building is to primarily serve the neoclassical economic school of thought and marginalism, which consists largely of higher mathematics. In recent years, the complexities that arise when interpreting structural changes in economic systems have been tackled by chaos theory, agent-based modeling, game theory, and econophysics.

Some concerns about the appropriateness of mathematical reasoning in economics and the limitations of the associated assumptions have been raised over time (some reported in [2,4]). The connection of mathematical economics with real-world economics has been doubted even from the early stages of its development (see more details in [3]).
Highly distinguished and widely respected researchers in economics, like Cournot [5,6], Walras [7], Pareto, Marshall, Samuelson, Johnson [8], Edgeworth [9,10], Wald [3], Debreu [11–15], and recent studies by Robertson [16], Nobel laureate Hurwitz [17], Theocharis [18,19], Quddus and Rashid [2], Katzner [20], Tarasov [21,22], and Mokhov et al. [23] have addressed the criticism about the unsuitability of mathematics for the representation of economic processes. Dow [24] discussed the pro-math/anti-math debate separating pure theory from applied economics, taking a stand on the proper use of mathematics in economics. Additionally, the great number of publications in respected scientific economic journals from 1886 to present proves the usefulness of mathematical model building in economics.

Jeschke et al. [25] proved empirically that knowledge of mathematics facilitates an understanding of economics. In [25], the authors analyzed the subject-specific professional knowledge of (prospective) upper-secondary teachers trained in mathematics and economics. They estimated content knowledge (CK) in mathematics and economics for a sample of pre- and in-service teachers and found a significant moderate relationship between Mathematics-CK and Economics-CK. Espinosa et al. [26] investigated scholars’ career prospects in relation to the training and use of mathematics. They found that knowledge in mathematics has a positive correlation with the probability of winning a Nobel Prize in certain cases.

The development of mathematical economics was already under investigation back in 1990, when Nobel laureate Prof. Gerard Debreu presented before the American Economic Association the progress of the mathematization of economic theory [15]. Prof. Debreu, in his presidential address entitled “The Mathematization of Economic Theory” [15], estimated the development of mathematical economics by calculating the total number of pages of the leading periodicals Econometrica, the Review of Economic Studies, International Economic Review, Journal of Economic Theory, and the Journal of Mathematical Economics. Another index used by Prof. Debreu to estimate the progress of the mathematization of economics was the percentage of refereed pages of each volume in the American Economic Review from 1940 to 1990 that ventured to include mathematical expressions.

Nobel laureate George Stigler et al. [27] examined the use of mathematical techniques in four leading journals of economics. As reported in [28], George Stigler et al. found a dramatic decrease in articles that use neither geometric representations nor mathematical notation, from 95% in 1892 to 5.3% in 1990.

Espinosa et al. [26] compiled and reviewed all the articles available on JSTOR. JSTOR was considered the most convenient database for their article as it contains articles written before 1900, journals dedicated to other sciences and areas of knowledge outside of economics, and journals that have been discontinued or completely absorbed by other publications. Their scientometrics approach was based on counting the number of equations per article, the number of equations per footnote, the number of econometric results, and the number of mathematical appendices per paper.

While these former evaluations were made with techniques and sources of the time, in this paper, we highlight the leading trends in mathematical economics occurring from its genesis to the present, using bibliometric analysis via modern technologies that encourage communication, sharing, and other interactions with research output. Bibliometrics is a research field of library and information science [29,30] that studies bibliographic material with quantitative methods. Academic databases, along with bibliometric software, categorize and analyze all the bibliographic information and citation data of high-quality studies. Nowadays, modern databases provide (1) multidisciplinary content coverage, (2) a detailed classification of academic research, and (3) categorized metrics for clearer understanding and analysis. In the second step, bibliometric analysis and visualization software take the Scopus (or Web of Science) output files as the input to provide journal-, author-, article-, and country-level metrics to create bibliometric networks (often referred to as “science mapping”).

This work uses the Scopus database, which combines an academic’s proximity to the social sciences with high-quality bibliographic material. The Scopus database also provides
Boolean operators for queries that can return the appropriate search results. To assess the so-called “mathematization of economics”, we involve open-access bibliometric software that evaluates the scientometrics of a large number of bibliographic records. Thus, we upgrade the past approaches of [15,26,27] by effectively reviewing a great number of documents and considering established bibliometric data analysis. The 6477 documents that were reviewed come from 424 sources, authored by 6138 authors, and have received 126,832 references. All retrieved documents are targeted at mathematical economics, covering a time span of about 125 years. In contrast to the work of [22,23] and to other review articles on mathematical economics, the present approach is data-driven, fully computational, more quantitative, and less narrative. Its computational character makes the study objective and transparent.

The major contributions of this work include:

i. an overview of the contributions to mathematical economics over time and the estimation of the related scientometrics;

ii. Creation of global structural networks and their application to classifying mathematical economics research topics, authors, and documents that have influenced the research community;

iii. Investigation of the global structure of mathematical economics to determine co-authorship relations among authors, co-citation, collaboration among countries, and bibliographic coupling of sources.

The rest of the paper is organized as follows: The next section briefly reviews the data extraction methods, the methodological approach, and the software used in the paper. Section 3 presents the results obtained by bibliometric analysis. The first part categorizes the metrics for a clearer understanding and analysis of the bibliographic material, while the second part develops structure maps of the source-coupling networks of journals, scholars, keywords, or countries using citations, bibliographic coupling, co-citations, or co-authorship relations. Section 4 summarizes the main findings and conclusions of this paper. The last section points out the limitations of this study.

2. Materials and Methods

2.1. Academic Publishing in the Field of Mathematical Economics

The academic journals in the field of mathematical economics at the date of the present publication include Journal of Mathematical Economics published by Elsevier (Online ISSN: 1873-1538 Print ISSN: 0304-4068), Mathematics and Financial Economics published by Springer (Electronic ISSN 1862-9660 Print ISSN 1862-9679), Insurance: Mathematics and Economics published by Elsevier (Print ISSN: 0167-6687, Online ISSN: 1873-5959), Mathematical Economic Letters (not listed in the Scopus database (published from 29 October 2013 to 30 November 2014)) published by De Gruyter (Online ISSN: 2195-4623), and the Journal of Mathematical Economics and Finance (not listed in the Scopus database) published by Asers publishing (Online ISSN: 2458-0813).

The Journal of Mathematical Economics, as stated on the journal’s website, publishes “work in economic theory which expresses economic ideas using formal mathematical reasoning”. The journal covers a wide range of topics in mathematical economics, including but not limited to general equilibrium theory, game theory, mathematical optimization, dynamic economic modeling, decision theory, and econometrics.

Mathematics and Financial Economics is an academic journal that focuses on the intersection of mathematics, economics, and finance. The journal is dedicated to publishing research articles that apply mathematical and quantitative techniques to address issues in financial economics and related fields. According to its website, the primary objective of the journal “is to provide a forum for work in finance which expresses economic ideas using formal mathematical reasoning”.

Insurance: Mathematics and Economics focuses on the intersection of mathematics, statistics, and economics as applied to the field of insurance and risk management. The journal is dedicated to publishing research articles that contribute to the understanding and analysis of various aspects of insurance and risk. According to its editors [31], the subject matter of Insurance: Mathematics and Economics includes theory models and methods of actuarial
Science and insurance applications of results from related fields, such as probability and statistics, computer science and numerical analysis, quantitative economics, mathematical finance, operations research and management science, and, in particular, quantitative risk management.

2.2. Data Extraction and Analysis

In this paper, searches were performed in the English language and conducted in the Scopus academic database. Scopus was considered the most convenient database for the purpose of this article as, according to its developers, it “indexes over 14,000 STM and social science titles from 4000 publishers, stating that it is the largest single abstract and indexing database ever built” [32]. In addition to American journals, it includes European and Asia Pacific literature in both English and non-English, covering publications from 1788 to 2019, making it among the largest curated bibliographic abstract and citation databases in the world [32,33]. Scopus covers a vast range of scientific disciplines and considers a wide range of bibliometric indicators, citation information, abstracts, and keywords, funding details, and references per paper.

The bibliometric analysis was applied to the Scopus bibliometric data produced by a search string consisting of four search fields that were connected by the Boolean operator “OR”. The search fields used included (1) the print ISSN of the Journal of Mathematical Economics, (2) the print ISSN of Mathematics and Financial Economics, (3) the print ISSN of Insurance: Mathematics and Economics, and (4) the string “mathematical economics” as a keyword in the “title, abstract, or keywords” of the searchable document. We did not restrict the material type or publication years.

Through this Boolean search, a total of 6477 documents were deemed valid for this analysis. A total of 6436 documents from 424 sources of relevant scientific literature (journals, books, etc.) were traced after cleaning data from duplicates. A total of 677 documents did not come from journals dedicated to mathematical economics (as defined in Section 2.1). Since chaos theory, econophysics, game theory, and stochastic modeling provide tools suitable for describing economic phenomena, it was highly expected to trace documents on mathematical economics in a great variety of economic- and non-economic-oriented sources.


A bibliometric analysis was performed according to the methodology proposed in [34] via the Bibliometrix package in R [35]. Bibliometrix is an R package used for bibliometric analysis, which is the quantitative study of publications, particularly in the field of science and research. This package provides tools and functions to analyze and visualize bibliographic data, such as publication records, citations, and co-authorship networks.

Duplicated records in the bibliographic data frame were removed by the R command duplicatedMatching of the Bibliometrix package. Erroneous entries concerning the indication of authors’ first names (using initials) were corrected to standardize the authors’ names.

VOSviewer (version 1.6.19) [36] was employed to construct and visualize bibliometric networks. VOSviewer is a free and open-source software tool used for bibliometric and scientometric analysis, as well as for visualizing and mapping research networks. VOSviewer provides several bibliometric techniques concerning co-authorship, co-occurrence of keywords, citation among documents, sources, authors, organizations, countries, bibliographic...
coupling of documents, sources, authors, organizations, countries, co-citation of cited references, cited sources, and cited authors [37]. The topic of network and density visualization is explained thoroughly in [38].

By combining Bibliometrix and VOSviewer, we streamlined the process of conducting bibliometric analyses and created visually informative representations of the collected data.

3. Results

The techniques for bibliometric analysis manifest across the following two categories: (1) performance analysis and (2) science mapping.

3.1. Performance Analysis

The bibliometric analysis in the present section includes (1) some descriptive statistics about the dataset, such as publication counts over time; (2) citation analysis (citation patterns, including the most cited publications and authors); (3) authorship analysis (author productivity report and authorship indicators); (4) countries’ performance analysis (the productivity of countries in terms of publications and identification of core countries in documents and citations); and (5) keyword analysis (trace the most frequently used keywords and analyze trends in keyword usage).

3.1.1. Descriptive Statistics of General Bibliometric Analysis

The Scopus database started reporting documents in mathematical economics in 1898. Since then, the field has been growing and establishing itself as a popular thematic area in economics. Table 1 summarizes the statistics for the bibliometric dataset.

Table 1. Main information about the data.

<table>
<thead>
<tr>
<th>Description</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Timespan</td>
<td>1898–September 2023</td>
</tr>
<tr>
<td>Sources (Journals, Books, etc.)</td>
<td>424</td>
</tr>
<tr>
<td>Documents</td>
<td>6436</td>
</tr>
<tr>
<td>References</td>
<td>126,832</td>
</tr>
<tr>
<td>Authors</td>
<td>6138</td>
</tr>
<tr>
<td>Authors of single-authored documents</td>
<td>1586</td>
</tr>
<tr>
<td>Single-authored documents</td>
<td>2339</td>
</tr>
<tr>
<td>Documents per Author</td>
<td>1.05</td>
</tr>
<tr>
<td>Co-Authors per Doc</td>
<td>1.97</td>
</tr>
<tr>
<td>International co-authorships %</td>
<td>27.28</td>
</tr>
</tbody>
</table>

Gerard Debreu noticed in his presidential speech in 1990, “During the period 1944–1977, the index more than doubled every nine years. By that measure, 1944 was a sharp turning point in the history of mathematical economics” [15].

Analyzing our Scopus-extracted results, the topic has increased in prominence in later years. The publication evolution through time is depicted in Figure 1, where the annual number of papers published in the fields of mathematical economics since 1898 is depicted. From 1999 and onwards, three-digit numbers count the number of published documents. The years 2005, 2010, 2018, and 2021 were turning points for sudden increases (see Figure 1). The annual percentage growth rate of productivity was calculated to be 4.03.

3.1.2. Citation Analysis

This section focuses on the total number of citations, citation patterns, and the most cited publications and authors. Figure 2 presents the annual citation structure, showing the average number of article citations (Figure 2a) and the average number of total citations received (Figure 2b). The average number of citations per document was calculated to be 16.39. The average number of citations per year per document was calculated to be 1.087.
The annual number of papers published in the fields of mathematical economics since 1898 is depicted. From 1999 and onwards, three-digit numbers count the number of published documents. The years 2005, 2010, 2018, and 2021 were turning points for sudden increases (see Figure 1). The annual percentage growth rate was calculated to be 4.03.

Figure 1. Annual scientific production based on 6436 Scopus search results between 1898 and 2023. The annual percentage growth rate is 4.03.

The top 15 cited documents in mathematical economics are displayed in Table 2.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Title/Source</th>
<th>Author/s</th>
<th>Year</th>
<th>TC</th>
<th>TC/Y</th>
<th>NTC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Maxmin expected utility with non-unique prior, Journal of Mathematical Economics</td>
<td>Gilboa, I.; Schmeidler, D.</td>
<td>1989</td>
<td>2355</td>
<td>67.29</td>
<td>43.41</td>
</tr>
<tr>
<td>4</td>
<td>Goodness-of-fit tests for copulas: A review and a power study, Insurance: Mathematics and Economics</td>
<td>Genest, C.; Rémillard, B,; Beaudoin, D.</td>
<td>2009</td>
<td>960</td>
<td>64.00</td>
<td>33.62</td>
</tr>
<tr>
<td>8</td>
<td>Expected utility with purely subjective non-additive probabilities, Journal of Mathematical Economics</td>
<td>Gilboa, I.</td>
<td>1987</td>
<td>499</td>
<td>13.49</td>
<td>17.10</td>
</tr>
</tbody>
</table>

Figure 2. (a) Average article citations per year; (b) average total citations per year.
Table 2. Top 15 most global cited documents.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Title/Source</th>
<th>Author/s</th>
<th>Year</th>
<th>TC</th>
<th>TC/Y</th>
<th>NTC</th>
</tr>
</thead>
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<tr>
<td>1</td>
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<td>1987</td>
<td>499</td>
<td>13.49</td>
<td>17.10</td>
</tr>
</tbody>
</table>

1 Normalized total citation: the article’s TCs/mean of all articles’ TCs.

The obvious observation from Table 2 is the fact that the *Journal of Mathematical Economics* is the dominant source for referencing (it published 8 out of the 15 top-cited documents). Articles in the journal *Insurance: Mathematics and Economics* are highly cited in the relevant literature (10 entries in the top 25 list), although this could be an expected and natural result since 2886 out of 6436 (44.8%) of the published documents are in this journal.

Next, let us look into the most frequently cited papers of all time. The aim is to identify those studies that have more influence and impact on the scientific community. Table 3 presents a list of the top 15.

It is worth noting that in second place among the most frequently cited documents is Debreu’s “Theory of Value”, which has been criticized for being a theoretical mathematical model of no practical use. However, it is well-cited in the academic community. The third place in the list in Table 3 was “General Competitive Analysis”, co-authored by Kenneth J. Arrow and Frank H. Hahn, which is an approach that is criticized for not being realistic concerning the conclusions of stability and the uniqueness of the equilibrium state. Moreover, documents with the main theme of the states of economic equilibria, namely the book *Core and Equilibria of a Large Economy* by Werner Hildenbrand and the paper

Table 3. The most frequently cited manuscripts in our dataset.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Reference</th>
<th>Total Citations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hildenbrand W., Core and Equilibria of a Large Economy (1974)</td>
<td>169</td>
</tr>
<tr>
<td>2</td>
<td>Debreu G., Theory of Value (1959)</td>
<td>134</td>
</tr>
<tr>
<td>3</td>
<td>Arrow K.J., Hahn F.H., General Competitive Analysis (1971)</td>
<td>98</td>
</tr>
<tr>
<td>4</td>
<td>Rockafellar R.T., Convex Analysis (1970)</td>
<td>98</td>
</tr>
<tr>
<td>5</td>
<td>Joe H., Multivariate Models and Dependence Concepts (1997)</td>
<td>87</td>
</tr>
<tr>
<td>7</td>
<td>Grandell J., Aspects of Risk Theory (1991)</td>
<td>75</td>
</tr>
<tr>
<td>8</td>
<td>Buhlmann H., Mathematical Methods in Risk Theory (1970)</td>
<td>72</td>
</tr>
<tr>
<td>9</td>
<td>Rockafellar R.T., Convex Analysis (1970)</td>
<td>98</td>
</tr>
<tr>
<td>10</td>
<td>Arrow K. J. and Debreu G., Economies with a Finite Set of Equilibria,</td>
<td>67</td>
</tr>
<tr>
<td>12</td>
<td>Joe H., Multivariate Models and Dependence Concepts (1997)</td>
<td>87</td>
</tr>
<tr>
<td>13</td>
<td>Buhlmann H., Mathematical Methods in Risk Theory (1970)</td>
<td>72</td>
</tr>
<tr>
<td>14</td>
<td>Rockafellar R.T., Convex Analysis (1970)</td>
<td>98</td>
</tr>
<tr>
<td>15</td>
<td>Arrow K. J. and Debreu G., Economies with a Finite Set of Equilibria,</td>
<td>67</td>
</tr>
</tbody>
</table>

In recent years (1999–2007), the list in Table 3 includes titles from the field of stochastic modeling and probabilities (documents ranked 6th, 12th, 13th, and 15th), thus reflecting an intense focus on this field.

Another interesting issue is analyzing the authors of the most cited papers. To do so, Bibliometrix counts the citations of the papers and assigns them to the first author of the document. Table 4 presents a list of the fifteen most frequently cited first authors in mathematical economics between 1898 and September 2023.

Table 4. The most frequently cited first authors.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Author</th>
<th>Total Citations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Goovaerts M. J.</td>
<td>1601</td>
</tr>
<tr>
<td>2</td>
<td>Gerber H. U.</td>
<td>1361</td>
</tr>
<tr>
<td>3</td>
<td>Denuit M.</td>
<td>1325</td>
</tr>
<tr>
<td>4</td>
<td>Dhaene J.</td>
<td>1288</td>
</tr>
<tr>
<td>5</td>
<td>Kaas R.</td>
<td>1017</td>
</tr>
<tr>
<td>6</td>
<td>Debreu G.</td>
<td>960</td>
</tr>
<tr>
<td>7</td>
<td>Blake D.</td>
<td>834</td>
</tr>
<tr>
<td>8</td>
<td>Haberman S.</td>
<td>782</td>
</tr>
<tr>
<td>9</td>
<td>Arrow K. J.</td>
<td>758</td>
</tr>
<tr>
<td>10</td>
<td>Embrechts P.</td>
<td>749</td>
</tr>
<tr>
<td>11</td>
<td>Willmot G. E.</td>
<td>706</td>
</tr>
<tr>
<td>12</td>
<td>Dowd K.</td>
<td>663</td>
</tr>
<tr>
<td>13</td>
<td>Young V R.</td>
<td>646</td>
</tr>
<tr>
<td>14</td>
<td>Delbaen F.</td>
<td>611</td>
</tr>
<tr>
<td>15</td>
<td>Mas Colell A.</td>
<td>582</td>
</tr>
</tbody>
</table>

Arrow and Debreu, being both committed promoters of mathematical formalism, are included in the list of the top 10 most frequently cited authors. Arrow and Debreu, along with Hicks, Samuelson, and Hahn, are also the main representatives of the general equilibrium theory (with Debreu being the dean of the modern general equilibrium theory), which is not widely accepted as realistic and applicable to economic phenomena. Their inclusion in the list of most frequently cited first authors proves that authors of mathemati-
cal economics papers do not share the doubts following the mathematical formalism and general equilibrium approach.

3.1.3. Authorship Analysis

Many authors have made significant contributions to mathematical economics. In order to identify the most productive authors, Table 5 presents a list of the fifteen authors with the highest number of papers published in relevant journals. To provide a complete overview of the most productive authors, fractional counting is also applied to the number of articles per author. The idea of fractional counting of articles, which is widely used in bibliometric analyses, is to reduce the influence of documents with many authors. When fractional counting is used, the strength of a co-authorship link between two authors is determined not only by the number of documents co-authored by the authors but also by the total number of authors of each of the co-authored documents [37].

Table 5. Top 15 most productive authors in mathematical economics.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Author Name</th>
<th>Articles</th>
<th>Author Name</th>
<th>Articles</th>
<th>Fractionalized</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Goovaerts, M.J.</td>
<td>80</td>
<td>Goovaerts, M.J.</td>
<td>32.0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Haberman, S.</td>
<td>59</td>
<td>Haberman, S.</td>
<td>29.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Denuit, M.</td>
<td>47</td>
<td>Willmot, G.E.</td>
<td>23.5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Young, V.R.</td>
<td>42</td>
<td>Gerber, H.U.</td>
<td>22.9</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Willmot, G.E.</td>
<td>41</td>
<td>Young, V.R.</td>
<td>21.9</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Dhaene, J.</td>
<td>40</td>
<td>Denuit, M.</td>
<td>20.6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Gerber, H.U.</td>
<td>38</td>
<td>De Vylder, F.E.</td>
<td>18.1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Kaas, R.</td>
<td>38</td>
<td>Hürlimann, W.</td>
<td>17.0</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Liang, Z.</td>
<td>34</td>
<td>Sundt, B.</td>
<td>16.5</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>De Vylder, F.E.</td>
<td>33</td>
<td>Liang, Z.</td>
<td>15.4</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Yang, H.</td>
<td>33</td>
<td>Kaas, R.</td>
<td>15.1</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Li, J.</td>
<td>28</td>
<td>Cheung, K.C.</td>
<td>15.0</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Cheung, K.C.</td>
<td>26</td>
<td>Cornel, B.</td>
<td>14.8</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Landriault, D.</td>
<td>26</td>
<td>Li, J.</td>
<td>14.5</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Li, S.</td>
<td>25</td>
<td>Yang, H.</td>
<td>13.5</td>
<td></td>
</tr>
</tbody>
</table>

1 The fractionalized frequency distribution of articles. In the case of fractional counting, when an author has co-authored a document with n other authors, this yields a strength of 1/n for each of the n co-authorship links [38].

The top prolific author is Goovaerts, M.J., Katholieke Universiteit Leuven, Belgium (80 documents). In this list in Table 5, there are three authors from Belgium, two authors from each of the Netherlands, Canada, China, and Hong Kong, and one author from each of the UK, USA, Australia, and Switzerland.

Table 6 shows other indicators such as the number of single-authored, multi-authored, and first-authored papers, the dominance factor (DF). Note that in the case of Table 6, the ranking is according to the dominance factor.

Table 6. Top 15 authors’ dominance ranking as proposed by Kumar and Kumar [39].

<table>
<thead>
<tr>
<th>Author</th>
<th>Dominance Factor</th>
<th>Tot Articles</th>
<th>Single-Authored</th>
<th>Multi-Authored</th>
<th>First-Authored</th>
<th>Rank by Articles</th>
<th>Rank by DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>De Vylder F.E.</td>
<td>0.78</td>
<td>33</td>
<td>6</td>
<td>27</td>
<td>21</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Cheung K.C.</td>
<td>0.61</td>
<td>26</td>
<td>8</td>
<td>18</td>
<td>11</td>
<td>13</td>
<td>2</td>
</tr>
<tr>
<td>Denuit M.</td>
<td>0.57</td>
<td>47</td>
<td>3</td>
<td>44</td>
<td>25</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Landriault D.</td>
<td>0.52</td>
<td>26</td>
<td>1</td>
<td>25</td>
<td>13</td>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>Gerber H.U.</td>
<td>0.52</td>
<td>38</td>
<td>11</td>
<td>27</td>
<td>14</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Kaas R.</td>
<td>0.44</td>
<td>38</td>
<td>2</td>
<td>36</td>
<td>16</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Liang Z.</td>
<td>0.36</td>
<td>34</td>
<td>1</td>
<td>33</td>
<td>12</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>Dhaene J.</td>
<td>0.33</td>
<td>40</td>
<td>0</td>
<td>40</td>
<td>13</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Li S.</td>
<td>0.32</td>
<td>25</td>
<td>0</td>
<td>25</td>
<td>8</td>
<td>15</td>
<td>9</td>
</tr>
<tr>
<td>Li J.</td>
<td>0.26</td>
<td>28</td>
<td>5</td>
<td>23</td>
<td>6</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>Goovaerts M.J.</td>
<td>0.25</td>
<td>80</td>
<td>0</td>
<td>80</td>
<td>20</td>
<td>1</td>
<td>11</td>
</tr>
</tbody>
</table>
Table 6. Cont.

<table>
<thead>
<tr>
<th>Author</th>
<th>Dominance Factor</th>
<th>Tot Articles</th>
<th>Single-Authored</th>
<th>Multi-Authored</th>
<th>First-Authored</th>
<th>Rank by Articles</th>
<th>Rank by DF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Willmot G.E.</td>
<td>0.23</td>
<td>41</td>
<td>11</td>
<td>30</td>
<td>7</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>Haberman S.</td>
<td>0.21</td>
<td>59</td>
<td>7</td>
<td>52</td>
<td>11</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>Yang H.</td>
<td>0.13</td>
<td>33</td>
<td>1</td>
<td>32</td>
<td>4</td>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>Young V.R.</td>
<td>0.05</td>
<td>42</td>
<td>5</td>
<td>37</td>
<td>2</td>
<td>4</td>
<td>15</td>
</tr>
</tbody>
</table>

1 The dominance factor (DF) is a ratio indicating the fraction of multi-authored articles in which a scholar appears as the first author [40].

Authorship analysis continues with exploring authorship patterns and collaboration networks in Section 3.2.

In order to provide a complete picture of each of the leading authors in mathematical economics, considering both their impact and productivity rate over time, Figure 3 combines their total number of articles with the total citations over the time span 1898–2023.

Figure 3. How the 15 leading authors have evolved their productivity (in terms of the number of publications and total citations per year) over time.

3.1.4. Country Performance Analysis

Table 7 shows the fifteen most productive corresponding author’s countries between 1898 and 2023. Most of the publications on mathematical economics come from authors working at USA’s institutions.

The countries that stand at a high publication level per multiple country publications (MCP) ratio are Hong Kong, Switzerland, Belgium, Australia, and the Netherlands (see MPC ratio in Table 7 highlighted in bold type).

The results of Table 7 are illustrated in Figure 4.
**Table 7.** Top 15 most productive corresponding author’s countries on mathematical economics.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Country</th>
<th>Articles</th>
<th>Frequency</th>
<th>SCP 1</th>
<th>MCP 2</th>
<th>MCP_Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>USA</td>
<td>648</td>
<td>0.1532</td>
<td>435</td>
<td>213</td>
<td>0.329</td>
</tr>
<tr>
<td>2</td>
<td>CHINA</td>
<td>442</td>
<td>0.1045</td>
<td>302</td>
<td>140</td>
<td>0.317</td>
</tr>
<tr>
<td>3</td>
<td>CANADA</td>
<td>332</td>
<td>0.0785</td>
<td>212</td>
<td>120</td>
<td>0.361</td>
</tr>
<tr>
<td>4</td>
<td>FRANCE</td>
<td>331</td>
<td>0.0783</td>
<td>235</td>
<td>96</td>
<td>0.308</td>
</tr>
<tr>
<td>5</td>
<td>UNITED KINGDOM</td>
<td>285</td>
<td>0.0674</td>
<td>193</td>
<td>92</td>
<td>0.323</td>
</tr>
<tr>
<td>6</td>
<td>GERMANY</td>
<td>260</td>
<td>0.0615</td>
<td>180</td>
<td>80</td>
<td>0.308</td>
</tr>
<tr>
<td>7</td>
<td>ITALY</td>
<td>212</td>
<td>0.0501</td>
<td>142</td>
<td>70</td>
<td>0.361</td>
</tr>
<tr>
<td>8</td>
<td>NETHERLANDS</td>
<td>166</td>
<td>0.0392</td>
<td>104</td>
<td>62</td>
<td>0.373</td>
</tr>
<tr>
<td>9</td>
<td>AUSTRALIA</td>
<td>164</td>
<td>0.0388</td>
<td>87</td>
<td>77</td>
<td>0.470</td>
</tr>
<tr>
<td>10</td>
<td>SPAIN</td>
<td>164</td>
<td>0.0388</td>
<td>126</td>
<td>38</td>
<td>0.232</td>
</tr>
<tr>
<td>11</td>
<td>JAPAN</td>
<td>136</td>
<td>0.0322</td>
<td>111</td>
<td>25</td>
<td>0.184</td>
</tr>
<tr>
<td>12</td>
<td>SWITZERLAND</td>
<td>125</td>
<td>0.0296</td>
<td>60</td>
<td>65</td>
<td>0.520</td>
</tr>
<tr>
<td>13</td>
<td>BELGIUM</td>
<td>121</td>
<td>0.0286</td>
<td>61</td>
<td>60</td>
<td>0.496</td>
</tr>
<tr>
<td>14</td>
<td>HONG KONG</td>
<td>104</td>
<td>0.0246</td>
<td>47</td>
<td>57</td>
<td>0.548</td>
</tr>
<tr>
<td>15</td>
<td>ISRAEL</td>
<td>65</td>
<td>0.0154</td>
<td>46</td>
<td>19</td>
<td>0.292</td>
</tr>
</tbody>
</table>

1 SCP: single country publications. 2 MCP: multiple country publications.

**Figure 4.** Top 15 corresponding author’s countries. SCP stands for single country publications and MCP stands for multiple country publications. Counting was based on 6436 Scopus advanced search results.

A further interesting aspect is to identify which countries provide citations to mathematical economics. This analysis is referred to in Table 8. The USA is clearly the country providing the most citations, followed by China, Canada, the UK, Germany, and Italy. Ten of the top fifteen countries on the list in Table 8 are European countries. In the top fifteen most-cited countries, 29,424 citations from non-European countries were in the same order of magnitude as compared to 27,323 citations from European countries (see also Figure 5). The country ranking changes according to average article citations. The top five average article citations scores are highlighted in bold in Table 8.
Table 8. Top 15 most cited countries on mathematical economics.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Country</th>
<th>Total Citations</th>
<th>Average Article Citations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>USA</td>
<td>9170</td>
<td>14.151</td>
</tr>
<tr>
<td>2</td>
<td>CHINA</td>
<td>7977</td>
<td>18.048</td>
</tr>
<tr>
<td>3</td>
<td>CANADA</td>
<td>7771</td>
<td>23.407</td>
</tr>
<tr>
<td>4</td>
<td>UNITED KINGDOM</td>
<td>4983</td>
<td>17.484</td>
</tr>
<tr>
<td>5</td>
<td>GERMANY</td>
<td>3747</td>
<td>14.412</td>
</tr>
<tr>
<td>6</td>
<td>ITALY</td>
<td>3408</td>
<td>16.075</td>
</tr>
<tr>
<td>7</td>
<td>FRANCE</td>
<td>3186</td>
<td>9.625</td>
</tr>
<tr>
<td>8</td>
<td>BELGIUM</td>
<td>2560</td>
<td>21.157</td>
</tr>
<tr>
<td>9</td>
<td>AUSTRALIA</td>
<td>2368</td>
<td>14.439</td>
</tr>
<tr>
<td>10</td>
<td>NETHERLANDS</td>
<td>2306</td>
<td>13.892</td>
</tr>
<tr>
<td>11</td>
<td>SPAIN</td>
<td>2206</td>
<td>13.451</td>
</tr>
<tr>
<td>12</td>
<td>SWITZERLAND</td>
<td>2155</td>
<td>17.240</td>
</tr>
<tr>
<td>13</td>
<td>HONG KONG</td>
<td>2138</td>
<td>20.558</td>
</tr>
<tr>
<td>14</td>
<td>NORWAY</td>
<td>1698</td>
<td>56.600</td>
</tr>
<tr>
<td>15</td>
<td>DENMARK</td>
<td>1074</td>
<td>21.059</td>
</tr>
</tbody>
</table>

Figure 5. Total citations per country. Counting was based on metadata from 6436 Scopus advanced search results.

3.1.5. Keyword Analysis

In Table 9, we list the most frequently used keywords from the 6436 Scopus documents retrieved.

A point worth mentioning is that mathematical economics utilizes creative computing. Keywords such as “algorithms” and “computation theory” are ranked 8th and 14th in the top fifteen list of most frequently used terms, but “computational complexity”, “computer programming”, “dynamic programming”, and “linear programming” occur further in the list.
Table 9. Top 15 most relevant keywords in documents on mathematical economics.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Author Keywords ¹</th>
<th>Articles</th>
<th>Keywords-Plus ²</th>
<th>Articles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>RUIN PROBABILITY</td>
<td>137</td>
<td>MATHEMATICAL ECONOMICS</td>
<td>126</td>
</tr>
<tr>
<td>2</td>
<td>MATHEMATICAL ECONOMICS</td>
<td>104</td>
<td>ECONOMICS</td>
<td>64</td>
</tr>
<tr>
<td>3</td>
<td>INCOMPLETE MARKETS</td>
<td>94</td>
<td>OPTIMIZATION</td>
<td>31</td>
</tr>
<tr>
<td>4</td>
<td>GENERAL EQUILIBRIUM</td>
<td>87</td>
<td>GAME THEORY</td>
<td>27</td>
</tr>
<tr>
<td>5</td>
<td>COMONOTONICITY</td>
<td>68</td>
<td>MATHEMATICAL MODELS</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>LONGEVITY RISK</td>
<td>67</td>
<td>SET THEORY</td>
<td>18</td>
</tr>
<tr>
<td>7</td>
<td>REINSURANCE</td>
<td>67</td>
<td>PROBLEM SOLVING</td>
<td>16</td>
</tr>
<tr>
<td>8</td>
<td>EQUILIBRIUM</td>
<td>63</td>
<td>ALGORITHMS</td>
<td>15</td>
</tr>
<tr>
<td>9</td>
<td>STOCHASTIC CONTROL</td>
<td>59</td>
<td>COMMERCE</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>VALUE-AT-RISK</td>
<td>57</td>
<td>DECISION MAKING</td>
<td>14</td>
</tr>
<tr>
<td>11</td>
<td>DEPENDENCE</td>
<td>54</td>
<td>FUNCTIONS</td>
<td>14</td>
</tr>
<tr>
<td>12</td>
<td>LIFE INSURANCE</td>
<td>54</td>
<td>COSTS</td>
<td>12</td>
</tr>
<tr>
<td>13</td>
<td>RISK MEASURES</td>
<td>52</td>
<td>TOPOLOGY</td>
<td>12</td>
</tr>
<tr>
<td>14</td>
<td>COPULA</td>
<td>51</td>
<td>COMPUTATION THEORY</td>
<td>11</td>
</tr>
<tr>
<td>15</td>
<td>CORE</td>
<td>51</td>
<td>THEOREM PROVING</td>
<td>11</td>
</tr>
</tbody>
</table>

¹ The keywords on the title page. ² Index keywords, the most common keywords and those that appear more frequently in the same documents.

The major research areas in mathematical economics lie in pure mathematics, as “set theory”, “topology”, “optimization”, and “functions” appear in the top places in Table 9. Additionally, theoretical methodologies of “theorem proving” are frequently a key component of the documents.

3.2. Science Mapping

In this section, network visualizations are employed to represent relationships between authors, countries, journals, and keywords in our bibliographic dataset. Visualizing and exploring bibliometric networks, including co-authorship, co-citation, bibliographic coupling, and co-occurrence networks, can be useful for understanding research areas and trends.

For network visualization and clustering, VOSviewer [37] was used as a standalone software tool, but it was also used via the net2VOSviewer (https://search.r-project.org/CRAN/refmans/bibliometrix/html/net2VOSviewer.html) command of the Bibliometrix package to import processed data and indicators generated by Bibliometrix. This way, Bibliometrix and VOSviewer can be used together to perform comprehensive bibliometric analyses and gain deeper insights into the scientific literature and citation data.

Here, we focus on the following three types of scrutiny based on network visualizations:

− Collaboration network of the 150 most productive authors, depicted in Figure 6;
− Co-citation network of the 150 most productive authors, depicted in Figure 7;
− Network visualization of country collaborations (co-authorship), depicted in Figure 8;
− Co-occurrence network of the most common terms used together in publications, with the option index keywords, as depicted in Figure 9;
− Co-occurrence network of the most common terms used together in publications, with the option author keywords, as depicted in Figure 10;
− Network visualization of bibliographic coupling of sources of articles (journals) on mathematical economics, as depicted in Figure 11.

Co-authorship is a type of scientific collaboration between scholars in a specific field of research. The co-authorship network of authors in mathematical economics is displayed in Figure 6. The map includes the 150 most productive authors, as the global network of a total of 6138 active authors was impossible to map. VOSviewer categorized the 150 authors into 16 clusters, with 6 clusters containing only 2 authors. The first cluster (red one) includes 30 authors, the second cluster (green one) includes 19 authors, the third cluster (blue one) includes 18 authors, the fourth cluster (yellow one) includes 17 authors, the fifth cluster...
(purple one) includes 14 authors, and the sixth cluster (light blue) includes 9 authors. The strongest links were between Goovaerts M.J. and Kaas R. (54) and between Goovaerts M.J. and De Vylder F.E. (42) in cluster 8. The strongest link was recorded to the benefit of Goovaerts M.J. (80). This author has the largest number of links with other authors (14) and the most documents (80).

![Authors' collaboration network](image1)

**Figure 6.** Authors’ collaboration network of the top 150 most productive authors in mathematical economics. The size of the node indicates the documents of the author and the thickness of the link between any two authors is an indicator of the strength of the collaboration between the two scholars.

![Co-citation network](image2)

**Figure 7.** Co-citation of cited authors from 6436 Scopus search results for mathematical economics research. Fractional counting was selected, meaning that publications with a long reference list (e.g., review articles) play a less important role in the construction of a co-citation network.

The co-citation network of cited authors in mathematical economics is displayed in Figure 7. The minimum number of citations for an author is set at 20. Of the 53,444 authors, 2342 meet the threshold. VOSviewer selected 1000 authors with the greatest total link strength and categorized them into seven clusters. The first cluster (red one) includes 379 authors, the second cluster (green one) includes 254 authors, the third cluster (blue one) includes 162 authors, the fourth cluster (yellow one) includes 81 authors, the fifth cluster (purple one) includes 64 authors, the sixth cluster (in light blue) includes 58 authors, and the seventh cluster (in orange) includes 2 authors.
mathematical and computational methods employed to address economic issues and problems. The number of items (1706) met the set threshold. The index keyword co-occurrence network in Figure 9 reveals the occurrence analysis is useful to identify the most common terms used related to each other (i.e., within the same cluster).

Figure 8. Country collaboration network of the 81 productive countries on mathematical economics research from 6436 Scopus search results. The thickness of the link between any two countries is an indicator of the strength of the collaboration between the two countries. The size of the node is an indicator of the contribution of the country (i.e., the larger the node, the higher the contribution of the country in terms of co-authorship). The items with the same color indicate that these items are related to each other (i.e., within the same cluster).

Figure 9. Co-occurrence of the index keywords from the 6436 Scopus search results for mathematical economics research. Each node in the network represents a Keywords: The bigger the node, the more frequently the keyword appears in the bibliographic data frame. Occurrences were calculated using the full counting methodology. Lines represent the number of occurrences of two keywords together (how often they appear together). (a) VOSviewer identified six clusters of related keywords with different colors; (b) VOSviewer colored keywords according to their occurrences per year.
with the highest total link strength were the USA with 648 documents and a link strength (i.e., the number of times that the keywords co-occur or occur together). The link between the nodes represents networkPlot (i.e., the number of times that the keyword occurs); the link between the nodes represents the networks of 213, followed by China with 442 documents and a link strength of 140. Canada was in the field of mathematical economics; the thickness and length of the links between countries reflects the collaboration between countries in terms of documents as well as the degree of collaboration. The larger nodes represent the most productive countries in terms of documents on mathematical economics; the thickness and length of the links between nodes represent the number of co-authorship connections between countries. The countries with the highest total link strength were the USA with 648 documents and a link strength of 213, followed by China with 442 documents and a link strength of 140. Canada was in the third position with 332 documents and a link strength of 120 (see also Table 6).

Figure 8 provides a VOSviewer visualization of the 81 productive countries concerning mathematical economics research in 20 collaboration clusters indicated by different colors. The network was created with networkPlot in Bibliometrix and plotted in VOSviewer via the net2VOSviewer command. As shown in Figure 8, the co-authorship analysis of the countries reflects the collaboration between countries in terms of documents as well as the degree of collaboration. The larger nodes represent the most productive countries in terms of documents on mathematical economics; the thickness and length of the links between nodes represent the number of co-authorship connections between countries. The countries with the highest total link strength were the USA with 648 documents and a link strength of 213, followed by China with 442 documents and a link strength of 140. Canada was in the third position with 332 documents and a link strength of 120 (see also Table 6).

We calculated the main statistics of the network in the Bibliometrix package using the networkStat command to describe the structural properties of the country collaboration network. The results are as follows:

- Network density (i.e., the proportion of present edges among all possible edges in the network) equaled 0.126, indicating a sparse network. In practical terms, this result implies weak inter-country collaboration across the globe.
- Network transitivity (i.e., the ratio of triangles to connected triples) equals 0.507.
- The normalized degree of the overall network was 0.437. That is the normalized count of how many co-authorship connections (i.e., edges) a country in this collaboration network has.

Keyword co-occurrence analysis is useful to identify the most common terms used together in publications. The size of the node indicates the occurrence of the keyword (i.e., the number of times that the keyword occurs); the link between the nodes represents the co-occurrence between keywords (i.e., keywords that co-occur or occur together); and the thickness of the link signals the occurrence of co-occurrences between keywords (i.e., the number of times that the keywords co-occur or occur together).
Figure 11. Bibliographic coupling of sources of mathematical economics based on 6436 Scopus search results. The size of the node indicates the number of articles published by a source. The number of bibliographic coupling links between two documents equals the number of pairs of commonly cited references in the two sources. Bibliographic coupling links were calculated using the fractional counting methodology. (a) VOSviewer identified seven thematic clusters of similar sources. (b) VOSviewer-colored keywords based on their bibliographic coupling relations per year.

The keyword co-occurrence network reveals research themes and emerging topics. In the global and temporal analyses of Figure 9a,b, respectively, the minimum number of co-occurrences of an index keyword used was 5. Out of the 1706 index keywords, 53 met the set threshold. The index keyword co-occurrence network in Figure 9 reveals the mathematical and computational methods employed to address economic issues and problems. The number of items (1706) indicates the diversity of subjects. As mentioned in Table 9, the strongest link strength was recorded by the “mathematical economics” keyword (126). This was followed by the link strength of “economics” (64), “optimization” (31), “game theory” (27), “mathematical models” (21), “set theory” (18), “problem solving” (16), “algorithms” (15), “commerce” (15), and “decision making” (14). In recent periods (yellow color
nodes), the active topics include “decision making”, “sustainable development”, “economic analysis”, “nonlinear analysis”, “investments”, “stochastic systems”, and “budget control”.

The visualization map of the co-occurrence of the author keywords is depicted in Figure 10. For a keyword, the minimum number of occurrences was set to five occurrences. A total of 790 keywords out of 14,312 keywords met the set threshold. The size of the node in the visualization map is an indicator of the contribution of the keyword (the larger the size of the node, the greater the occurrence of the keyword). The thickness of the link signals the occurrence of co-occurrences between keywords (i.e., the number of times that the keywords co-occur or occur together). The results are consistent with Table 9. The term “ruin probability” was the keyword with the most occurrences (137). This was followed by “mathematical economics” (104), “incomplete markets” (94), “general equilibrium” (87), “comonotonicity” (68), “longevity risk” and “reinsurance” (67), “equilibrium” (63), “stochastic control” (59), “value-at-risk” (57), “dependence” and “life insurance” (54). In recent periods (yellow color nodes), the active topics include “value-at-risk”, “ambiguity”, “Nash equilibrium”, “strategy proofness”, “capital allocation”, “risk measures”, and “systemic risk” (see Figure 10b).

Another interesting issue is analyzing the bibliographic coupling of sources that publish research on mathematical economics. According to van Eck and Waltman [38], “bibliographic coupling is about the overlap in the reference lists of publications. The larger the number of references two publications have in common, the stronger the bibliographic coupling relationship between the publications”. Figure 11 presents the bibliographic coupling of sources, considering a minimum threshold of five documents and one hundred connections. Figure 11 presents the results that indicate the most productive sources with the volume of nodes and points, which tend to cite the same material with the thickness of the links. Figure 11a identifies seven clusters of similar items. Cluster 1 contains seven sources, namely the Journal of Mathematical Analysis and Applications; Journal of Mathematical Economics; Journal of Optimization Theory and Applications; Journal of Statistical Mechanics: Theory and Experiment; and Mathematical Methods of Operations Research, Mathematics, and Optimization. Cluster 2 contains six sources, namely European Journal of the History of Economic Thought, History of Political Economy, Journal of Economic Methodology, Cambridge Journal of Economics, Journal of the History of Economic Thought, and Journal of Post Keynesian Economics. Cluster 3 contains three sources: Journal of Mathematical Sciences, Lecture Notes in Computer Science (including the subseries Lecture Notes in Artificial Intelligence), and Mathematics of Operations Research. Cluster 4 contains two sources: the Bulletin of the American Mathematical Society and Mathematical Social Sciences. Cluster 5 contains two sources: Insurance: Mathematics and Economics and Mathematics and Financial Economics. Cluster 6 contains the Journal of Physics: Conference Series, and cluster 7 contains Positivity. Figure 11b reveals the older and newer sources. In yellowish colors and distinguishable sizes appear the journals Mathematics and Financial Economics, Journal of Physics: Conference Series, Mathematics, Mathematical Methods of Operations Research, History of Political Economy, and Journal of the History of Economic Thought.

4. Conclusions

This paper reviewed developments in mathematical economics between 1898 and 2023. The work is motivated by the Special Issue “Latest Advances in Mathematical Economics” in the Mathematics journal.

The study conducted bibliometric and scientometric analysis in the fields of mathematical economics, including the source of data acquisition, performance analysis, and science mapping, with the use of the most recommended software tools [40].

The analysis collected the bibliographic data from 6477 results of an advanced Scopus search between 1898 and September 2023. The aim was to report on the impact of the research, gain a deeper understanding of the structure and dynamics of mathematical economics, and identify key authors, publications, and research trends. The analysis
further defined the limits of mathematical economics, pointed out the issues of interest in the research field, and outlined the way researchers collaborate or influence each other.

The Scopus database has traced records since 1898. The research field has been growing rapidly during recent years, with the highest annual scientific production of 285 documents in 2021, compared to 113 documents in 2001 and 21 documents in 1981.

Of the 6436 documents left after data cleaning, 677 documents did not come from journals dedicated to mathematical economics. Keyword analysis revealed a wide range of subjects.

The results show a strong dominance by the USA, which currently publishes most of the papers on the field. Therefore, it has the most productive and influential authors and has published the majority of highly cited papers on mathematical economics. Authors from European institutions have also made some significant contributions to mathematical economics. The scholars affiliated with institutions in France, the United Kingdom, Germany, Italy, the Netherlands, Spain, Switzerland, and Belgium stand out in the global network of mathematical economics research papers.

Most of the analyzed research publications were collaborative.


“Optimization”, “game theory”, “mathematical models”, “set theory”, “problem solving”, “algorithms”, “commerce”, and “decision making” were identified as emerging knowledge domains in the field of mathematical economics.

Overall, we can say that the countries and research constituents in the field (e.g., authors, institutions, countries, and journals) having good metrics in bibliometric indices are able to get the academic credit in the field. This study could then provide guidance to researchers interested in mathematical economics, pinpoint scholars or chairs for potential collaboration, give directions for study resources, reveal countries and institutions for focused research.

The present methodology can be adapted to carry out further types of studies, e.g., focused studies on specific thematic fields of mathematical economics, such as game theory and graph theory, for problems that represent and explain economic behavior, income and wealth distribution, topological issues, input-output structures with matrix algebra formulations, and ICT tools for teaching mathematical economics courses. Apart from carrying out additional analysis with data from academic databases, the study results can be expanded to include publication years beyond 2023.

5. Limitations of the Study

Nevertheless, we note some limitations when developing a bibliometric analysis.

First, the paper used the Scopus abstract and citation database, while academic databases are not exclusively designed for bibliometric analysis. For example, various failures concerning the affiliation in the author field of entry and misstatements of the authors’ initials occurred.

Second, the results analyzed publications since 1898, with very little data available in Scopus during the period 1898–1973. Moreover, the search approach was based on the “mathematical economics” keyword and dedicated journals in the field, while economists who promoted mathematics in economic theory (like Vilfredo Pareto, Léon Walras, Alfred Marshall, and Paul Samuelson, as well as the living mathematical economists Robert Solow, Alpha Chiang, and Hal Varian) were not included in the analysis.

Moreover, the sources of the search were the dedicated academic journals in mathematical economics. However, two of them, namely Mathematics and Financial Economics published by Springer and Insurance: Mathematics and Economics published by Elsevier, do not specialize solely in mathematical economics. Thus, some documents that were analyzed in the present bibliometric analysis might not serve the specific field of mathematical economics.
Finally, we note that academic research includes a lot of exceptional papers by well-respected authors that do not come through strongly in bibliometric analysis results and may not even appear in the rankings [41].

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