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An Improved Inverse DEA for Assessing Economic Growth and Environmental Sustainability in OPEC Member Nations

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Abstract: Economic growth is essential for nations endowed with natural resources as it reflects how well those resources are utilized in an efficient and sustainable way. For instance, OPEC member nations, which hold a large proportion of the world's oil and gas reserves, may require a frequent evaluation of economic growth patterns to ensure that the natural resources are best used. For this purpose, this study proposes an inverse data envelopment analysis model for assessing the optimal increase in input resources required for economic growth among OPEC member nations. In this context, economic growth is reflected in the GDP per capita, taking into account possible environmental degradation. Such a model is applied to the selected OPEC member nations, which suggests that in terms of increasing the GDP per capita, only one member was able to achieve the best efficiency (i.e., reaching the efficiency frontier), resulting in a hierarchy or dominance within the sample countries. The analysis results further identify the economic growth potential for each member country. For the case of Indonesia, the analysis suggests that further economic growth may be achieved for Indonesia without additional input resources. This calls for diversification of the nation's economy or investment in other input resources. In addition, the overall results indicated that each member nation could increase its GDP per capita while experiencing minimal environmental degradation. Our analysis not only benchmarks the growth efficiency of countries, but also identifies opportunities for more efficient and sustainable growth.

Keywords: inverse DEA; economic growth; GDP per capita; growth potential; environmental degradation

MSC: 90-10

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1. Introduction

Economic growth is a measure of the increase in the production of goods and services in an economy over time. One of the most used indicators of economic growth is the gross domestic product (GDP), which is the total value of all final goods and services produced within a country in a given year. Another indicator is the GDP per capita, which is the GDP divided by the population of the country and reflects the average income and living standards of the people.

As regards OPEC member nations, Orisaremi et al. [1] provided a clear explanation of the correlation between the GDP of each member nation and its current account balance (i.e., an input resource that represents liquidity). It is presumed that a member nation with a surplus current account balance is a net exporter, while a member nation with a deficit account balance is a net importer [1]. In particular, a surplus current account balance correlates with a higher GDP, whereas a deficit current account balance correlates with a lower GDP [1]. Additionally, Ostic et al. [2] conducted a multiple regression analysis to investigate the relationship between the economic growth (i.e., GDP growth rate) of OPEC member nations and their carbon emission levels. In their study, economic growth and carbon emissions for OPEC members were found to be positively correlated and statistically significant.

According to the World Bank, the GDP of OPEC member countries increased from USD 2.8 trillion in 2000 to USD 6.5 trillion in 2019, while the GDP per capita increased from USD 8700 to USD 15,600 in the same period. However, these aggregate figures mask significant variations among different OPEC members, as some countries experienced faster growth than others and some faced economic contraction due to political instability, conflicts, sanctions, or low oil prices. For example, according to a study by Pekarčíková et al. [3], GDP growth is positively impacted by oil production, exports, and prices, but negatively impacted by unemployment and exchange rates. Moreover, the GDP per capita levels of OPEC members also varied widely, from USD 1030 in Congo to USD 69,420 in Qatar in 2019. The augmentation of economies within OPEC nations can be evaluated via an examination of their Gross Domestic Product (GDP) and per capita GDP. Such an examination has often been carried out using some methodologies from the field of both applied mathematics and microeconomics. For example, an investigation by Abdulqadir [4] scrutinized the correlation between economic growth and environmental degradation among OPEC member nations via the pool mean group (PMG) technique. Analytical instruments such as panel co-integration assessments, dynamic OLS (DOLS), and the fully modified OLS (FMOLS) method offer valuable insights into fiscal expansion. These methodologies facilitate an exploration of the enduring influence of the petroleum output on GDP expansion and provide estimates for short-term coefficients and causality relationships. Nonetheless, these tools are not without their constraints. They necessitate extensive data sets and may not accurately encapsulate the intricacies of fiscal expansion within OPEC nations.

1.1. A Brief Overview of Economic Hardship in Some OPEC Member Nations

It is evident that economic hardship is a significant issue in some OPEC member nations, including Nigeria, Venezuela, Libya, and Iraq. A number of economic challenges have faced these countries, including currency devaluations, high poverty levels, energy poverty, and a lack of basic amenities. Due to an abrupt increase in fuel prices, the cost-of-living crisis in Nigeria is worsening daily [5,6]. The hike in fuel prices has resulted in a shortage of passengers for transporters, indicating the impact on the day-to-day lives of the citizens [7]. There were 35 million more Nigerians living in poverty in 2022 as compared to 2021, and some analysts have even labelled the nation as the "poverty capital of the world" [8].

Venezuela is experiencing a severe humanitarian crisis, with millions unable to access basic healthcare, energy supply, and adequate nutrition [9–11]. During the last six years, the economy of the country has stagnated, growing by less than 1% cumulatively, far below the population growth rate of 2.6%. In addition, about 40% of the population of about 200 million lives in poverty. In Libya, the economy is gradually recovering from the twin shocks of the pandemic and the collapse of oil prices in 2020. There is, however, an increase in food insecurity due to the rise in global wheat prices [12]. According to the African Economic Outlook, Libya's population is estimated to contain 2% of multidimensionally poor individuals and is 11.4% at risk of being multidimensionally poor. Iraq's economy is gradually recovering from both the pandemic and the decline in oil prices in 2020. Nevertheless, climate change challenges and the spread of new COVID-19 variants are significant headwinds. It is clear that the fragility of the country has far-reaching economic and social consequences.

Economic hardships in these countries are complex and multifaceted, often accompanied by political instability and other socio-economic factors. Economic experts in some institutions in these nations, such as the Central Bank of Nigeria (CBN), have commonly agreed that addressing some of these challenges can be accomplished via better utilization of current resources or by investing in additional resources (i.e., the diversification of the economy). These challenges must be addressed via comprehensive and inclusive economic policies. Therefore, the purpose of this study is to develop an economic framework for assessing the potential for economic growth in these nations from the standpoint of the efficient utilization of input resources.

1.2. Factors Hindering Economic Growth in OPEC Member Nations

One of the factors that hinder economic growth in OPEC member countries is their high dependence on oil revenues, which makes them vulnerable to external shocks and price volatility. According to a study by Mahmood and Saqib [13], rising oil rents have positive effects on CO_2 emissions in eight OPEC members (Saudi Arabia, Angola, Congo, Equatorial Guinea, Iran, Iraq, Kuwait, and Libya), and negative impacts on three OPEC members (Algeria, Nigeria, and the UAE). This implies that oil rents can have both beneficial and detrimental effects on economic growth and environmental quality depending on how they are managed and invested. Moreover, oil rents can also create rent-seeking behavior, corruption, political instability, and conflict among different groups competing for resource wealth. Another factor that hinders economic growth in OPEC member countries is their low level of human capital development and diversification. Human capital refers to the skills, knowledge, health, and creativity of the labor force that contribute to productivity and innovation. Diversification refers to the expansion of economic activities into new sectors and markets that reduce dependence on a single source of income and increase resilience to shocks. Abid and Alotaibi [14] have also reiterated that human capital development and diversification are positively associated with economic growth in OPEC member countries. However, these countries face many challenges in improving their human capital and diversification indicators due to inadequate investment in education, health care, infrastructure, research and development (R&D), technology transfer (TT), entrepreneurship (ENT), trade openness (TO), institutional quality (IQ), etc.

Other impediments to economic growth within OPEC nations encompass complications in petroleum output and disparities in oil export capacities owing to production costs, reserves, geological characteristics, and political situations. Contemporary global challenges such as the pandemic outbreak, heightened inflation, unprecedented debt levels, and escalating income inequality also present significant risks. Incorporating a variety of inputs and outputs is crucial when utilizing mathematical techniques to evaluate economic growth within OPEC nations. Several inputs involved in production procedures capital, labor, and natural resources over an extended period. Outputs could encompass aspects like GDP growth rates, values of goods and services' exports, values of petroleum exports, and values of goods and services', among others. By including multiple inputs and outputs in a mathematical technique, one can capture the complexity and diversity of economic growth in OPEC member countries and identify the strengths and weaknesses of each country in terms of their sources and uses of economic resources.

1.3. Mathematical Tools for Economic Analysis

To assess economic growth in OPEC member countries, various mathematical tools can be used to analyze the data and identify the factors that influence growth. One such tool is regression analysis, which is a statistical method that estimates the relationship between a dependent variable (such as GDP growth) and one or more independent variables (such as oil rents, foreign direct investment, human capital, etc.). Regression analysis can help to test hypotheses, measure causal effects, control confounding factors, and make predictions based on historical data. However, regression analysis also has some limitations, such as potential problems of multicollinearity, endogeneity, heteroscedasticity, autocorrelation, omitted variables, measurement errors, or model misspecification.

Another mathematical tool that can be used to assess economic growth in OPEC member countries is input-output analysis, which is an economic model that describes the interdependent relationships between industrial sectors within an economy. It shows how the outputs of one sector flow into another sector as inputs and how changes in final demand affect the production and consumption of goods and services across sectors. Inputoutput analysis can help to measure the direct and indirect impacts of economic shocks (such as changes in oil prices or production) on output, income, employment, and emissions in different sectors and regions. However, input-output analysis also has some limitations, such as requiring large amounts of data and assumptions, ignoring price effects and market adjustments, assuming fixed coefficients and linear relationships, and neglecting dynamic and behavioral aspects. In light of these limitations, there is a need to seek other alternatives or mathematical techniques that can accurately reflect the latent potential for economic growth in OPEC member nations.

Consequently, it is imperative to emphasize that, in this context, data envelopment analysis (DEA), as well as its recent developments in the form of multistage/network DEA, non-homogenous DEA, and inverse DEA, have shown beyond a reasonable doubt that they are quite robust in handling multiple case studies involving multiple inputs and outputs. Due to their concept of relative efficiency, DEA models are capable of addressing the limitations of traditional input–output models and other economic models. In comparison with other mathematical techniques, Data Envelopment Analysis (DEA) offers a number of advantages:

- *No Functional Form Required*: DEA does not require any specific assumptions or functional forms regarding the production process. This makes it a versatile tool for determining the relative efficiency of decision-making units (DMUs) in a variety of fields.
- *Handling Multiple Variables*: DEA can handle multiple inputs and outputs, making it applicable to a wide range of industries, including banking, healthcare, education, manufacturing, and public administration.
- *Analysis of Comparability*: DEA allows for comparative analysis by comparing the efficiency of units with those that perform best. Furthermore, it facilitates the comparison of multiple DMUs by assessing their ability to convert multiple inputs into multiple outputs.
- *Adaptability*: DEA's flexibility in not requiring a predetermined functional relationship between inputs and outputs makes it suitable for analyzing complex systems and assessing performance.
- *Strategic Planning*: DEA can be used by organizations for strategic planning, goal setting, and identifying areas of underperformance to enhance the output or outcome of performance measurement.
- *Scale Efficiency*: DEA incorporates the concept of returns to scale in efficiency calculations, allowing for the consideration of increasing or decreasing efficiency based on the size and output levels.
- *Inefficiency Analysis*: DEA allows for the analysis and quantification of the sources of inefficiency for each evaluated unit.

These features make DEA a robust tool for efficiency analysis across a broad range of applications. In particular, the inverse DEA is even more robust than the conventional DEA for addressing inverse problems across a variety of production and service work systems.

In the context of the inverse DEA methodology, an inverse problem often entails determining the optimal changes to be made to the inputs and/or outputs of a decision-making unit (DMU) such that the predetermined efficiency score of the unit remains unchanged at all times [15–17]. To put it another way, a conventional DEA model is first employed to determine the efficiency of a unit, and then the inverse DEA is used to improve its relative performance by either increasing or decreasing some or all of its inputs/outputs. This is a relatively complex problem that can be solved via reverse or backward optimization (i.e., inverse optimization). In this study, we also address an inverse problem associated with the petroleum industry in terms of computing the optimal changes required in input resources to boost economic growth in terms of GDP per capita. Thus, this study extends the gas flaring trilogy [1,11,16], as well as the novel study by Wegner and Amin [17]. Therefore, it is necessary to highlight some recent applications of both DEA and inverse DEA in order to justify the choice of our methodology.

Recent Advancements and Applications of DEA

Data envelopment analysis (DEA) has been applied to a wide variety of industrial work systems as a reliable tool for performance evaluation. It is noteworthy, however, that there have been pioneering advancements in the application of DEA to solve optimization problems in the industry. Multi-stage/network DEA and inverse DEA are notable advancements that address much more complex and challenging real-world problems. Accordingly, in this section, we will briefly discuss a number of recent and pertinent applications of DEA, including some of its recent advancements.

Inverse DEA has been described comprehensively by Emrouznejad and Amin [18], outlining its significance, current state of knowledge, and future research needs. Moreover, Emrouznejad et al. [19] provided an in-depth review covering the latest and practical advancements in inverse DEA, as well as highlighting promising future research directions. Hybrid approaches, including optimization techniques and inverse DEA, are suggested to address challenges like infeasibility and nonlinearity. Orisaremi et al. [11] have applied the directional distance DEA to the design of an optimal energy mix for Venezuela in the presence of both positive and negative data. Oukil [20] presented a novel application of inverse DEA for the investigation of merger gains in the agricultural sector. Moghaddas et al. [21] have developed a network inverse DEA model to evaluate the performance of sustainable supply chains (SSCs) under varying network configurations. Zhang et al. [22] employed DEA and machine learning in order to predict CO_2 emissions in China with a high degree of accuracy. DEA was employed by Lantara [23] to analyze Indonesia's Islamic banking performance before and after the COVID-19 pandemic. Ghiyasi et al. [24] have developed an inverse DEA for assessing the performance of 130 public hospitals in Iran. The DEA and Malmquist productivity techniques were combined by Firsova et al. [25] to assess the efficiency of the knowledge-intensive services sector. Ashuri et al. [26] have developed a new DEA model for benchmarking building energy use. A summary of some significant milestones in the advancement of conventional DEA models is presented in Cooper et al. [27] and Charnes et al. [28].

1.4. Problem Statement

In light of the impediments hindering economic growth in OPEC member nations and the limitations associated with the use of various mathematical tools for assessing economic growth, it is imperative that a more robust technique be developed in order to capture the potential for economic growth in OPEC nations. Considering that the GDP and GDP per capita are both positively related to economic growth, it is essential that such a robust technique incorporates the crucial inputs and outputs for the calculation of the GDP growth rate. This study aims to fill this substantial gap in the literature via the development of a novel inverse DEA.

While Orisaremi et al. [1] developed an inverse DEA model to analyze the gas-to-wire (GTW) process in OPEC member nations using the GDP per capita and flared gas as outputs, they did not assess the potential for GDP growth (i.e., the increase in the GDP per capita). Their study is limited to evaluating reductions in gas flaring at a constant GDP per capita without an increase in production. It is important to note, however, that in order to measure GDP growth, production must be increased. Unfortunately, an increase in production often leads to an increase in environmental degradation via the increase in flared gases [16]. Thus, we intend to address this problem by minimizing the environmental degradation associated with gas flaring in order to increase the GDP per capita on a targeted basis. This study aims to answer the following research questions in the form of inverse problems:

- *Considering a set of OPEC member nations, what would be the minimum increase in the flare gas levels associated with a desired increase in the GDP per capita, so that the predetermined efficiency score of each nation remains unchanged?*

- *To achieve a desirable rise in the GDP per capita, what is the minimum increase in the flare gas levels for an OPEC member nation?*
- *Given the same level of production efficiency, what is the optimal increase in input resources to achieve a specified increase in the GDP per capita for an OPEC member nation?*

It should be noted that the three inverse problems discussed above are somewhat similar to the one expressed in Wegner and Amin [17], but their study only concentrated on this inverse problem for a set or group of producers. In addition, their study did not examine the influence of negative data on their novel model. To address both limitations, we postulate a second and third inverse problem for a single producer and also consider all three problems in the context of negative data, as demonstrated by Orisaremi et al. [1].

This paper contributes to the literature in the following ways:

1. This paper proposes an inverse DEA to examine the economic growth potential of OPEC member nations.
2. In order to achieve economic growth, the proposed model is capable of calculating the optimal increase in variegated input resources (i.e., positive and negative values).
3. Moreover, the model serves as a template for assessing the resource utilization levels, which might lead to investment in other sectors or diversification.
4. Through mathematical transitivity, the study establishes a correlation between economic growth and environmental degradation in OPEC member nations.

The remainder of this study is summarized in this section. The second section reviews the DEA model's preliminary background knowledge. Section 3 introduces the research methodology. A discussion of the results is presented in Section 4. In Section 5, the study's economic implications are discussed, and in Section 6, the conclusions are summarized.

2. Preliminaries

Imagine a production system with n decision-making units that utilize m inputs to generate two different sets of outputs, namely good outputs r and bad outputs q. In order to increase the quantity of good outputs and decrease the quantity of bad outputs, Chung et al. [29] proposed the following DEA model:

$$M1: \quad \theta_k^* = \max\theta$$

$$s.t.$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik} \qquad i = 1, \ldots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{g} \geq (1+\theta) y_{rk}^{g} \qquad r = 1, \ldots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{pj}^{b} = (1-\theta) y_{pk}^{b} \qquad p = 1, \ldots, q$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \qquad j = 1, \ldots, n$$

In the special case where every input i is positive for some units and negative for others, Orisaremi et al. [1] modified $M1$ to give model $M2$ in terms of how the LINGO source code is modified to handle negative inputs (i.e., multiplied by -1) as follows:

$$M2: \quad \theta_k^* = \max\theta$$

$$s.t.$$

$$\sum_{j=1} \lambda_j x_{ij}^+ \leq x_{ik}^+ \qquad i = 1, \ldots, m$$

$$\sum_{j=1} \lambda_j x_{ij}^- \leq x_{ik}^- \qquad i = 1, \ldots, m \quad (multiply\ by - 1)$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^g \geq (1+\theta) y_{rk}^g \qquad r = 1, \ldots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{pj}^b = (1-\theta) y_{pk}^b \qquad p = 1, \ldots, q$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \qquad j = 1, \ldots, n$$

Furthermore, in another special case of one input having a negative value for at least one unit, Orisaremi et al. [11] partitioned the input set using the SORM technique introduced by Emrouznejad et al. [30] to give the following model:

$$M3: \quad \theta_k^* = \max\theta$$

$$s.t.$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik} \qquad i = 1, \ldots, m \qquad i \,\epsilon\, I_1$$

$$\sum_{j=1} \lambda_j x_{ij}^+ \leq x_{ik}^+ \qquad i = 1, \ldots, m \qquad i \,\epsilon\, I_2$$

$$\sum_{j=1} \lambda_j x_{ij}^- \leq x_{ik}^- \qquad i = 1, \ldots, m \qquad i \,\epsilon\, I_2 \ (multiply\ by - 1)$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^g \geq (1+\theta) y_{rk}^g \qquad r = 1, \ldots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{pj}^b = (1-\theta) y_{pk}^b \qquad p = 1, \ldots, q$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \qquad j = 1, \ldots, n$$

Note that I_1 is the set of inputs that take only positive values for all $DMUs$. I_2 is the set of inputs that take negative values for at least one DMU Orisaremi et al. [11]. As a result, $M3$ is the SORM version of $M2$ based on the data type. In the absence of the line command applicable to the generated source code for solving model $M3$ (i.e., multiplying the set of negative input constraints by -1), we can transform $M3$ to $M4$ as follows:

$$M4: \quad \theta_k^* = \max\theta$$

$$s.t.$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik} \qquad i = 1, \ldots, m \qquad i \,\epsilon\, I_1$$

$$\sum_{j=1} \lambda_j x_{ij}^{+} \leq x_{ik}^{+} \qquad i = 1, \ldots, m \qquad i \,\epsilon\, I_2$$

$$\sum_{j=1} \lambda_j x_{ij}^{-} \geq x_{ik}^{-} \qquad i = 1, \ldots, m \qquad i \,\epsilon\, I_2$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{g} \geq (1+\theta) y_{rk}^{g} \qquad r = 1, \ldots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{pj}^{b} = (1-\theta) y_{pk}^{b} \qquad p = 1, \ldots, q$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \qquad j = 1, \ldots, n$$

It is important to reiterate here that both models $M3$ and $M4$ are the same or equivalent (i.e., same optimal solution in the presence of negative inputs). $M3$ was only structured in its current form for easy application of the generated code. On the other hand, $M4$ is the better representation of negative inputs as evident in current studies.

3. Materials and Methods

Since this study extends Orisaremi et al. [1] and Orisaremi et al. [11], it is imperative that we employ the same input and output data that both studies gathered from the Annual Statistical Bulletin (ASB) of OPEC. The entire data were also selected and classified in consultation with experts in the petroleum industry. The data consists of five inputs (i.e., *the current account balance*, *wells completed*, *producing wells*, *active rigs*, and *refining capacity*) and two outputs (i.e., *GDP per capita*, to reflect economic growth, and *flared gas*, to reflect environmental degradation and economic loss). Both studies incorporate the current account balance of each member nation as a positive or negative input. A positive input implies surplus balance, and a negative input implies deficit balance. Detailed explanations of all inputs and outputs are provided by Orisaremi et al. [1].

For a better understanding of our research methodology, we need to recall the three key research questions for this study stated in Section 1.4. As previously mentioned, these research questions are the inverse problems of the study:

Inverse problem one (P1):

Considering a set of OPEC member nations, what would be the minimum increase in the flare gas levels associated with a desired increase in the GDP per capita, so that the predetermined efficiency score of each nation remains unchanged?

Inverse problem two (P2):

To achieve a desirable rise in the GDP per capita, what is the minimum increase in the flare gas levels for an OPEC member nation?

Inverse problem three (P3):

Given the same level of production efficiency, what is the optimal increase in input resources to achieve a specified increase in the GDP per capita for an OPEC member nation?

In order to solve problem P1, an inverse DEA model needs to be developed to set growth targets in the form of an increased GDP per capita while minimizing the increase in environmental waste (i.e., carbon emissions). This proposal has great significance since economic growth in OPEC member nations is positively correlated with carbon emissions from gas flaring Ostic et al. [2]. Furthermore, such an inverse DEA must be able to accommodate both positive and negative data, as the input data in this study can be both positive and negative. As a result, the assumption that all DEA models should accommodate only positive data has been relaxed.

To formulate an inverse DEA for problem P1, first we need to briefly introduce the inverse DEA developed by Wegner and Amin [17] for minimizing the increase in GHG emissions as follows:

$$M5: \quad \mathrm{Min}\gamma = (\gamma_{11}, \ldots, \gamma_{q1}, \ldots, \gamma_{1t}, \ldots, \gamma_{qt})$$

$$s.t.$$

$$\sum_{j \epsilon F} \lambda_j^k x_{ij} + \sum_{l \epsilon G} \hat{\lambda}_l^k (\alpha_{il} + x_{il}) - (\alpha_{ik} + x_{ik}) \leq 0 \quad \forall k \in S, \quad i = 1, \ldots, m$$

$$\sum_{j \epsilon F} \lambda_j^k y_{rj}^g + \sum_{l \epsilon G} \hat{\lambda}_l^k \left(\beta_{rl} + y_{rl}^g\right) - (1 + \hat{\theta}_k) \times \left(\beta_{rk} + y_{rk}^g\right) \geq 0 \quad \forall k \in S, \quad r = 1, \ldots, s$$

$$\sum_{j \epsilon F} \lambda_j^k y_{pj}^b + \sum_{l \epsilon G} \hat{\lambda}_l^k \left(\gamma_{pl} + y_{pl}^b\right) - (1 - \hat{\theta}_k) \times \left(\gamma_{pk} + y_{pk}^b\right) = 0 \quad \forall k \in S, \ p = 1, \ldots, q$$

$$\sum_{j \epsilon F} \lambda_j^k + \sum_{l \epsilon G} \hat{\lambda}_l^k = 1 \quad \forall k \in S$$

$$\sum_{k \epsilon S} \beta_{rk} = \hat{y}_r^g \quad r = 1, \ldots, s$$

$$\alpha_{ik} \geq 0, \beta_{rk} \geq 0, \gamma_{pk} \geq 0 \ \ \forall k \in S, \ i = 1, \ldots, m \ \ r = 1, \ldots, s \ \ p = 1, \ldots, q$$

$$\lambda_j^k \geq 0, \hat{\lambda}_l^k \geq 0 \ \ \forall k, l \in G, \quad \forall j \in F$$

To preserve the efficiency score of DMU_k, one must set $\hat{\theta}_k \leq \theta_k^*$, where θ_k^* is the optimal solution of model $M1$. Thus, $M5$ was developed using $M1$ as a base model in the presence of only positive data for a group/set of oil firms. Due to the occurrence of negative real-world data, and the need for computing reductions in gas flaring for a single oil firm/producer, by Orisaremi et al. [1] developed a novel inverse DEA based on model $M2$ that accomplishes both goals. However, their related model is limited in the sense that the gas flaring reductions were computed at the current level of the good output (i.e., a constant GDP per capita). Their model lacked the capability to estimate growth in the GDP per capita. Hence, the novelty in this study lies in addressing this limitation by developing a new inverse DEA for analyzing potential growth in the GDP per capita for two different cases—for a group of oil producers and for a single producer. The first step towards the development of our inverse DEA would be to incorporate the negative input data into model $M5$ using the approach demonstrated by Orisaremi et al. [1] to give a new model $M6$ as follows:

$$M6: \quad \mathrm{Min}\gamma = (\gamma_1 + \gamma_2 +, \ldots + \gamma_t)$$

$$s.t.$$

$$\sum_{j \epsilon F} \lambda_j^k x_{ij}^+ - (\alpha_{ik}^+ + x_{ik}^+) \leq 0 \quad \forall k \in S, \quad i = 1, \ldots, m$$

$$\sum_{j\epsilon F}\lambda_j^k x_{ij}^- - \left(\alpha_{ik}^- + x_{ik}^-\right) \geq 0 \qquad \forall k \in S, \qquad i = 1,\ldots,m$$

$$\sum_{j\epsilon F}\lambda_j^k y_{rj}^g - \left(1+\hat{\theta}_k\right) \times \left(\beta_{rk} + y_{rk}^g\right) \geq 0 \qquad \forall k \in S, \qquad r = 1,\ldots,s$$

$$\sum_{j\epsilon F}\lambda_j^k y_{pj}^b - \left(1-\hat{\theta}_k\right) \times \left(\gamma_{pk} + y_{pk}^b\right) = 0 \qquad \forall k \in S,\ p = 1,\ldots,q$$

$$\sum_{j\epsilon F}\lambda_j^k = 1 \qquad \forall k \in S$$

$$\sum_{k\epsilon S}\beta_{rk} = \hat{y}_r^g \qquad r = 1,\ldots,s$$

$$\alpha_{ik} \geq 0, \beta_{rk} \geq 0, \gamma_{pk} \geq 0\ \ \forall k \in S,\ i = 1,\ldots,m \quad r = 1,\ldots,s \quad p = 1,\ldots,q$$

$$\lambda_j^k \geq 0, \hat{\lambda}_l^k \geq 0 \quad \forall k,l \in G,\ \ \forall j \in F$$

Model $M6$ has the same structure as $M2$ and this is makes model $M2$ its base model. Now, according to by Orisaremi et al. [1], both models are based on the assumption that every input i takes both positive and negative values for each decision-making unit. If this is not the case, then the application of both models will require partitioning the input set via the SORM technique. At this point, let us consider the final SORM model $M4$ as the base optimization platform on which we can develop a new inverse DEA for addressing our research questions. In this connection, we follow by Orisaremi et al. [11] by partitioning the input set of model $M6$, to give a new model $M7$ as follows:

$$M7: \quad \text{Min}\gamma = (\gamma_1 + \gamma_2 +, \ldots + \gamma_t)$$

$$s.t.$$

$$\sum_{j\epsilon F}\lambda_j^k x_{ij} - \left(\alpha_{ik} + x_{ik}\right) \leq 0 \qquad \forall k \in S, \qquad i = 1,\ldots,m \quad i\,\epsilon\, I_1$$

$$\sum_{j\epsilon F}\lambda_j^k x_{ij}^+ - \left(\alpha_{ik}^+ + x_{ik}^+\right) \leq 0 \qquad \forall k \in S, \qquad i = 1,\ldots,m \quad i\,\epsilon\, I_2$$

$$\sum_{j\epsilon F}\lambda_j^k x_{ij}^- - \left(\alpha_{ik}^- + x_{ik}^-\right) \geq 0 \qquad \forall k \in S, \qquad i = 1,\ldots,m \quad i\,\epsilon\, I_2$$

$$\sum_{j\epsilon F}\lambda_j^k y_{rj}^g - \left(1+\hat{\theta}_k\right) \times \left(\beta_{rk} + y_{rk}^g\right) \geq 0 \qquad \forall k \in S, \qquad r = 1,\ldots,s$$

$$\sum_{j\epsilon F}\lambda_j^k y_{pj}^b - \left(1-\hat{\theta}_k\right) \times \left(\gamma_{pk} + y_{pk}^b\right) = 0 \qquad \forall k \in S, \qquad p = 1,\ldots,q$$

$$\sum_{j\epsilon F}\lambda_j^k = 1 \qquad \forall k \in S$$

$$\sum_{k\epsilon S}\beta_{rk} = \hat{y}_r^g \qquad r = 1,\ldots,s$$

$$\alpha_{ik} \geq 0, \beta_{rk} \geq 0, \gamma_{pk} \geq 0\ \ \forall k \in S,\ i = 1,\ldots,m \quad r = 1,\ldots,s \quad p = 1,\ldots,q$$

$$\lambda_j^k \geq 0, \hat{\lambda}_l^k \geq 0 \quad \forall k,l \in G,\ \ \forall j \in F$$

In its current form, model $M7$ is fully capable of minimizing the increase in the bad output (i.e., gas flaring) while computing the optimal increase in inputs required to achieve a specified target/increase in the good output (i.e., GDP per capita). Thus, $M7$ fully addresses the limitation of the study conducted by Orisaremi et al. [1]. Furthermore, $M7$ applies to a group of firms/producers unlike the inverse DEA developed by Orisaremi et al. [1] that applies to a single firm/producer. Most importantly, unlike the original model $M5$ that can only accommodate positive data, $M7$ incorporates negative input data. Note that $M4$ is the proper base model of $M7$. As model $M7$ refers to a group of inefficient producers predetermined via base model $M4$, the application of $M7$ answers the first research question (i.e., the inverse problem P1). Thus, in the subsequent sections, we will apply both $M4$ and $M7$ to provide the needed solutions to problem P1.

By reducing the cardinality of the set of inefficient producers in model $M7$ to one, we can easily solve the second inverse problem P2. Thus, for problem P2, we aim to find the minimum amount of environmental waste while setting growth targets for one producer or member nation at a time. Mathematically, it requires replacing the objective function of $M7$ with $min\gamma = \gamma_k$ for each $DMU_k \in S$, where S is the set of the inefficient $DMUs$ or producers determined via application of the base model $M4$, where there are t $DMUs$ in S (refer to Wegner and Amin [17]).

The solution to the third inverse problem P3 can be derived from the solution to the second inverse problem P2. This involves obtaining from the solution report of P2 the increase in all the inputs (i.e., α_{ik}, $\alpha^{+}_{ik}, \alpha^{-}_{ik}$) for each DMU or producer. As part of this study, we will utilize LINGO 20 software to solve our proposed models $M4$ and $M7$.

Summary of Research Methodology

This section summarizes the entire methodology of this study using some important definitions and an algorithm with detailed steps as follows:

Definition 1. *An inefficient decision-making unit (DMU) in this study refers to an OPEC member nation in need of economic growth.*

Definition 2. *Economic growth is defined in this study as an increase in the GDP per capita of the member nation.*

Definition 3. *In the OPEC annual statistical bulletin (ASB), GDP is defined as the GDP of each member nation at current market prices (i.e., nominal GDP). The GDP per capita is calculated by dividing GDP at current oil prices by the population of each member nation Orisaremi et al. [1]*

Definition 4. *By applying our proposed base model M4, an inefficient DMU_k or member nation is one with $\hat{\theta}_k > 0$. Therefore, an efficient member nation is one with $\hat{\theta}_k = 0$, indicating a relatively strong economy (i.e., an ideal benchmark for inefficient member nations).*

Definition 5. *With the application of our inverse DEA, M7, we propose that environmental sustainability in a best-case scenario is achieved when at least one member nation in a specified group can increase its GDP per capita with no increase in environmental waste across all the other group member(s) (i.e., a scenario in which $\gamma_1 = \gamma_2 = \cdots = \gamma_t = 0$).*

Definition 6. *For a member nation, diversification or investment in other input resources is recommended if no additional inputs are required to achieve a specified rise in the GDP per capita. Mathematically speaking, if by applying model M7 to a single member nation, it is determined that $\alpha_{ik} = \alpha^{+}_{ik} = \alpha^{-}_{ik} = 0, \ \forall k \in S, \ i = 1, \ldots, m \quad i \in I_1, I_2$, it follows that all input resources have been properly utilized.*

Based on the proposed Algorithm 1, all three inverse problems in this study are answered. Initially, the universal set (U) of inefficient or underperforming units in need of economic growth is determined using model $M4$. At random, a subset S is formulated with at least two units or member nations from the universal set. The next step involves the application of model $M7$ in two different ways—initially to the original subset S, and then to only one member of subset S (i.e., reducing the cardinality of the subset S to 1). As a result of these steps, solutions to problems P1 and P2 are provided. For problem P3, the solution lies in obtaining the increments in input resources from the solution to problem P2. Finally, economic diversification is recommended for any unit or member nation without an increase in input resources.

Algorithm 1: Brief summary of methodology

Step 1: Through $M4$ evaluate θ_k^* for each DMU$_j$ $\forall j = 1, 2, \ldots, n$.
Step 2: Create a universal set U for all decision-making units (DMUs) with $\theta_k^* > 0$.
Step 3: To solve the inverse problem P1, apply $M7$ to any subset $S \in U$, where $|S| > 1$.
Step 4: Let $|S| = 1$ and reapply $M7$ to solve the inverse problem P2, $\forall$ $DMU \in S$.
Step 5: Repeat step 4 $\forall$ $DMU_k \in U$, provided $DMU_k \notin S$, to obtain solution to problem P3.
Step 6: Through Definition 6, determine the set $T \in U$ $\forall$ DMU_k with α_{ik}= $\alpha_{ik}^+ = \alpha_{ik}^- = 0$

4. Application of Proposed Models

This section discusses the application of our proposed models $M4$ and $M7$ in accordance with Algorithm 1 stated in the preceding section. This section will be divided into three Sections 4.1–4.3 in order to simplify the overall analysis. In Section 4.1, we apply model $M4$ to the production data provided by Orisaremi et al. [1]. The data set consists of five inputs and two outputs, as described earlier in Section 3. Following that, we apply our inverse DEA model, $M7$, in Sections 4.2 and 4.3 to provide solutions to the three inverse problems, P1, P2, and P3.

4.1. Discussion of Preliminary Results

The aim here is to obtain the universal set of inefficient DMUs or member nations in need of economic growth. Inefficient DMUs have inefficiency scores greater than zero, whereas efficient ones have zero inefficiency scores. As we are using the same data as contained in Orisaremi et al. [1], the application of model $M4$ will yield same preliminary results. Hence, we extract the tabulated results of the inefficiency scores from Orisaremi et al. [1]. These results are presented in Table 1. Based on the tabulated results, U is the universal set of inefficient DMUs, expressed as $U = \{DMU_1, DMU_4 DMU_5, DMU_8, DMU_{10}$ $DMU_{11}, \}$.

Thus, every $DMU \in U$ has more room for minimizing environmental waste (i.e., gas flaring) compared to the efficient $DMUs$ with zero inefficiency scores. In this connection, a decision maker can select any number of inefficient $DMUs$ from the universal set U to form a smaller subset S with a cardinality of t (i.e., $t = |S|$). We will present two different scenarios in Sections 4.2 and 4.3 for solving the inverse problems, P1, P2, and P3. We propose a first scenario in which we focus only on a subset S of inefficient $DMUs$ in Section 4.2 to solve inverse problem P1. In Section 4.3, we propose a second scenario in which we analyze one DMU at a time, thereby providing solutions to problems P2 and P3.

Table 1. Production data and inefficiency scores for 11 OPEC members.

DMU	Current Account Balance (m USD)	Wells Completed	Producing Wells	Active Rigs	Refining Capacity (1000b/cd)	GDP per Capita (USD/Person)	Routinely Flared Gas (M cu m)	Ineff. (θ)
1-Algeria	17,770	249	2010	33	592	5453.5	3604	0.83
2-Angola	13,085	112	1476	22	65	4666.95	7183	0
3-Ecuador	−402	207	3079	39	188.4	5193.04	539	0
4-Indonesia	1685	838	10423	80	1125	3121	2452	0.78
5-Iraq	26,365	76	1695	59	810	5571.55	9612	0.91
6-Kuwait	65,743	523	1798	32	936	41672	217	0
7-Libya	3173	76	609	55	380	5858	1302	0
8-Nigeria	10,757	124	2116	38	445	2451.75	14,270	0.91
9-Qatar	51,906	29	517	6	283	97,983.27	558	0
10-UAE	50,948	266	1592	19	675	40,819.31	982	0.55
11-Venezuela	16,342	1050	14915	116	1872	10,283.2	9284	0.94

4.2. Discussion of Main Results (Scenario 1)

Following Wegner and Amin [17], we consider the smallest subset S with two random units (i.e., $|S| > 1$ as stated in the algorithm we have proposed) and gradually increase $|S|$ to the total number of inefficient units in the universal set, U. As an example, let $S = \{DMU_8, DMU_{11}\}$. When real-life scenarios are considered, an increase in the GDP per capita of an oil-producing nation is largely dependent upon an increase in production. The reason for this is the fact that the revenues generated from the exportation of crude oil and refined petroleum products take up a substantial portion of their GDP [1]. There is, however, a tendency for such an increase in production to be accompanied by an increase in environmental waste (i.e., carbon emissions via gas flaring) as well [16]. Therefore, this subsection will aim to determine, based on the application of the proposed inverse DEA, $M7$, an optimal increase in the GDP per capita for the group of producers in S that will minimize an increase in waste or gas flaring.

Table 2 presents three different scenarios for potential growth in the GDP per capita for the selected producers. To avoid overestimation of inherent potentials in terms of GDP per capita, it is pertinent to mention here that the target growth rates in Table 2 were determined by consulting with industry experts as well as considering the fact that more than 70% of Nigerians live below the poverty line of USD 730 per year Oyedepo [31]. Furthermore, the GDP per capita in this study is calculated on an annual basis based on the data obtained from OPEC. From Table 2, it is evident that both producers can achieve a combined GDP per capita growth rate of USD 100/person, USD 200/person, and USD 300/person, without increasing their flare gas volumes (i.e., $\gamma_{1,8} = \gamma_{1,11} = 0$). Accordingly, this represents an optimal solution to the first inverse problem P1 (i.e., a best-case scenario) without an additional increase in environmental waste as evidenced in the zero values of γ for the producers.

Table 2. Optimal volumes of flare gas for two producers.

Increase in GDP per Capita	$\beta_{1,8}$	$\beta_{1,11}$	$\gamma_{1,8}$	$\gamma_{1,11}$
$\hat{y}_1^g = 100$	100	0.0000	0.0000	0.0000
$\hat{y}_1^g = 200$	200	0.0000	0.0000	0.0000
$\hat{y}_1^g = 300$	300	0.0000	0.0000	0.0000

Among the three scenarios in Table 2, only DMU_8 (Nigeria) achieved growth in the GDP per capita (i.e., $\beta_{1,8} = \hat{y}_1^g$). However, we see that DMU_{11} (Venezuela) has no potential for an increase in the GDP per capita (i.e., $\beta_{1,11} = 0$) under the same scenarios. Considering the results of Table 2, Nigeria currently dominates Venezuela when both producers are tasked with meeting a combined growth in the GDP per capita. Therefore, we interpret these results as zero values for γ as representing no additional increase in waste associated with an increase in the GDP per capita. By contrast, a zero value for β indicates that the producer in the subset S cannot increase their output as reflected in the GDP per capita due to the dominance of another producer. Factors such as varying production capacities and financial capabilities of producers can easily explain this phenomenon.

Consequently, it is imperative to investigate further by extending the cardinality of the subset S from two to six $DMUs$. With the addition of DMU_1(Algeria) to the set S, there was no increase in their current levels of flare gas as shown in Table 3 (i.e., $\gamma_{1,1} = \gamma_{1,8} = \gamma_{1,11} = 0$). However, like Table 2, only DMU_1 could achieve an increase in the GDP per capita in each scenario. Accordingly, the other two producers (i.e., DMU_8 and DMU_{11}) cannot boost the GDP per capita without a corresponding increase in flare gas. It can be argued that DMU_1 has the greatest potential for GDP growth while maintaining its current flare gas level. The sample size of S was increased to four, five, and six to determine if this dominance of DMU_1 remains true for other combinations with the same increases in the GDP per capita. These other three samples had the same results, with DMU_1 being the only producer to increase the GDP per capita, although no increase in the flare gas levels was observed in any of the scenarios. Therefore, it is imperative to also investigate this dominance in the reverse direction by testing sample sizes of S, beginning with $t,\ \ t-1,\ \ t-2,\ldots,1$. The findings revealed a consistent pattern of dominance where the inefficient DMU with the lowest value of k is assigned to dominate the other DMUs in S. For instance, at $t = 6$, just like the original set, G, DMU_1 (i.e., having the smallest k value of 1) was the only producer that achieved growth in the GDP per capita. When DMU_1 is removed from set S, the smallest k value in the set becomes 4. As a result, DMU_4 becomes the dominant unit. This resulted in DMU_{11} being placed at the bottom of the dominance hierarchy because of this pattern. Therefore, the dominance hierarchy created via the proposed model, $M7$, can be summarized as follows:

$$DMU_1 > DMU_4 > DMU_5 > DMU_8 > DMU_{10} > DMU_{11}$$

Table 3. Optimal volumes of flare gas for three producers.

Increase in GDP per Capita	$\beta_{1,1}$	$\beta_{1,8}$	$\beta_{1,11}$	$\gamma_{1,1}$	$\gamma_{1,8}$	$\gamma_{1,11}$
$\hat{y}_1^g = 100$	100	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{y}_1^g = 200$	200	0.0000	0.0000	0.0000	0.0000	0.0000
$\hat{y}_1^g = 300$	300	0.0000	0.0000	0.0000	0.0000	0.0000

Generally, there are many other combinations of $DMUs$ that can form the subset S, but we can see from the examples presented here that only one DMU per combination dominates or has room for growth in the GDP per capita. This further explains why, in the first example, only DMU_8 with a lower k value increased the GDP per capita in comparison to DMU_{11} with a higher k value. To put it another way, a lower k value indicates a dominant DMU. This can be expressed mathematically as follows:

Definition 7. *Let DMU_p and DMU_q be two inefficient DMUs in S, then via the application of model M7, DMU_p is dominant over DMU_q, if and only if $p < q$. The converse also holds for both DMUs.*

Based on the above definition, the second scenario of this study assesses only one DMU separately to examine its potential for an increase in the GDP per capita, flare gas minimization abilities, and input increase requirements.

4.3. Discussion of Main Results (Scenario 2)

Here, a separate analysis is conducted for each $DMU \in S$ or a case where $|S| = 1$. In this second case, every DMU is evaluated for its potential to increase the GDP per capita by USD 100. The proposed model $M7$ also calculates the change in inputs (i.e., α_{ik}, α_{ik}^{+}, α_{ik}^{-}) required by each DMU to increase its GDP per capita. Tables 4–9 present the results obtained. Clearly, the results within Tables 4–9 indicate that each DMU or producer can increase its GDP per capita by USD 100/person, without causing an increase in environmental waste (flare gas). Interestingly, Table 5 revealed that Indonesia could achieve this growth in the GDP per capita even without increasing inputs. For Indonesia, this means that all inputs are fully utilized, and to increase its GDP per capita, it would be necessary to improve production efficiency or reduce production losses. An example of a production loss can be attributed to oil spills that occur during the oil extraction process. In turn, this results in a loss of oil revenue, which contributes substantially to the GDP of the nation.

Table 4. Optimal changes in inputs and flare gas volume for Algeria.

Increase in GDP per Capita	$\alpha_{1,1}$	$\alpha_{2,1}$	$\alpha_{3,1}$	$\alpha_{4,1}$	$\alpha_{5,1}$	$\gamma_{1,1}$
$\beta_{1,1} = 100$	25,370.47	112.6204	0.0000	7.3036	143.2683	0.0000

Table 5. Optimal changes in inputs and flare gas volume for Indonesia.

Increase in GDP per Capita	$\alpha_{1,4}$	$\alpha_{2,4}$	$\alpha_{3,4}$	$\alpha_{4,4}$	$\alpha_{5,4}$	$\gamma_{1,4}$
$\beta_{1,4} = 100$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 6. Optimal changes in inputs and flare gas volume for Iraq.

Increase in GDP per Capita	$\alpha_{1,5}$	$\alpha_{2,5}$	$\alpha_{3,5}$	$\alpha_{4,5}$	$\alpha_{5,5}$	$\gamma_{1,5}$
$\beta_{1,5} = 100$	0.0000	175.8459	1107.407	0.0000	0.0000	0.0000

Table 7. Optimal changes in inputs and flare gas volume for Nigeria.

Increase in GDP per Capita	$\alpha_{1,8}$	$\alpha_{2,8}$	$\alpha_{3,8}$	$\alpha_{4,8}$	$\alpha_{5,8}$	$\gamma_{1,8}$
$\beta_{1,8} = 100$	0.0000	72.83294	791.4443	0.0000	0.0000	0.0000

Table 8. Optimal changes in inputs and flare gas volume for UAE.

Increase in GDP per Capita	$\alpha_{1,10}$	$\alpha_{2,10}$	$\alpha_{3,10}$	$\alpha_{4,10}$	$\alpha_{5,10}$	$\gamma_{1,10}$
$\beta_{1,10} = 100$	16,329.65	134.6359	0.0000	7.130176	136.1426	0.0000

Table 9. Optimal changes in inputs and flare gas volume for Venezuela.

Increase in GDP per Capita	$\alpha_{1,11}$	$\alpha_{2,11}$	$\alpha_{3,11}$	$\alpha_{4,11}$	$\alpha_{5,11}$	$\gamma_{1,11}$
$\beta_{1,11} = 100$	10,913.23	0.0000	0.0000	0.0000	0.0000	0.0000

The obtained results suggests that, since the proposed inverse DEA, $M7$, was successful in preventing an increase in flare gas while increasing the GDP per capita for each producer, then via mathematical transitivity, it is practically possible to reduce flare gas at the current GDP per capita. This was demonstrated in the study conducted by Orisaremi et al. [1]. Thus, this study serves as a prequel to that performed by Orisaremi et al. [1]. It provides a rationale for the logic adopted and implemented in their research. As outlined above, both studies propose two robust options for OPEC member nations: one for economic growth with minimal environmental degradation, and another for cleaner production to address global warming.

5. Economic Implications

Despite the much-anticipated transition to 100 percent renewable energy in the not-too-distant future, OPEC continues to play a pivotal role in global energy exports. As such, it would appear haphazard for OPEC member nations not to assess their potential for economic growth in relation to their quality of life during and after this transition period. Economic growth is often assessed in terms of the GDP per capita by analysts. There is, however, a tendency to overlook the important input resources that are required to stimulate economic growth.

This study examines economic growth in OPEC member nations from the perspective of the efficient use of input resources. The importance of this is highlighted by the fact that most of these members are developing countries with a large number of resources (both natural and human) that are currently underutilized. Additionally, some member nations continue to face a variety of economic challenges, including currency devaluations, political instability, low direct foreign investment, energy poverty, low purchasing power, and large populations living in poverty. According to experts, a more efficient use of resources, and, if possible, economic diversification, would be a possible way of alleviating some of these challenges. In spite of this, there is a lack of a clear-cut strategy or economic tool to address these concerns. For this reason, and within the scope of this research, we developed an inverse DEA capable of determining the optimal increase in variegated input resources (i.e., with positive and negative values) necessary to achieve economic growth within OPEC member nations. The application of our methodology, however, has some implications for managers and economists.

It is important to acknowledge that economic growth in OPEC member nations is not without challenges when considering some internal and external factors involved in oil production. Previous research findings have indicated that fiscal expansion within these nations, as reflected in an increase in the GDP per capita, is highly dependent on increased oil production. There is, however, a classic domino effect involved here because increased production often results in an increase in production waste, such as gas flaring. Thus, it is necessary for OPEC member nations to utilize more input resources efficiently while minimizing their combined environmental wastes. Towards this goal, targets were established for a subset of member nations/producers with the objective of estimating optimal input increments with minimal amounts of associated waste.

Furthermore, we find that estimating potential growth for a given subset of producers leads only to one type of scenario. This scenario describes the case where only one producer can meet the specified growth targets with optimal input increments. This clearly implies that our developed inverse DEA is best suited for analyzing the growth potential of a single producer at a time. Hence, the separate analysis for each producer will certainly provide decision makers and managers with a more thorough and detailed understanding of input resources that are properly utilized. With this knowledge, management will be able to plan both short- and long-term for more efficient resource utilization and transformation, as well as, when necessary, invest in other input resources for diversification.

6. Conclusions

In this study, an inverse DEA model was developed to estimate the potential for economic growth among OPEC member nations. Our objective was to determine the optimal level of input resources that would enable GDP per capita growth to be achieved. There is, however, a positive correlation between economic growth and environmental pollution related to carbon emissions in OPEC member countries, as reported in previous studies. Thus, there is a need to strike a balance between economic growth and environmental sustainability in these countries.

To achieve that much-needed balance, we applied our proposed model to a random set of member nations predetermined to be inefficient. It was found that only one member nation within this set achieved the stated goal of increasing the GDP per capita with minimal environmental waste. As a result, there was some sort of hierarchy within the set driven by the varying production capacities and financial capabilities of each nation. One of the most intriguing findings of this study was that minimal environmental degradation in each nation was calculated at every point in time to be zero increases in environmental waste, which would suggest a best-case scenario.

For further validation, we considered an alternative scenario by decomposing the set of member nations and performing separate analyses for each nation. The goal was to determine the optimal increase in input resources that each member nation needed in order to reach the targeted level of GDP per capita. The findings from this scenario were the most promising for Indonesia due to the fact that no increase in input resources would be required to boost economic growth. Therefore, it is evident that the inputs are well utilized. Thus, diversification, or rather, substantial investments in other sectors, would be beneficial to the nation's economic growth. Our research findings suggest that our proposed model is a reliable tool for evaluating how efficiently input resources are being utilized in an environment where production waste is a significant contributor.

The recent advances and applications of data envelopment analysis are currently expanding and gaining the attention of researchers across many fields. As a reliable and powerful mathematical tool, it continues to pave the way for further advancements and/or modifications that address emerging and novel problems across a wide range of industries. In light of this, it may be worthwhile to consider novel hybrid approaches consisting of DEA and other mathematical tools in order to tackle complex problems. Therefore, the approach we propose in this study should be extended and/or combined with other novel techniques. In this way, we can address more pressing global issues beyond climate change and environmental degradation.

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Nomenclature

General Parameters :

n:	number of decision-making units (DMUs)
t:	number of inefficient decision-making units (DMUs)
m:	number of inputs of each DMU
s:	number of good outputs of each DMU
q:	number of bad outputs of each DMU

Data Parameters :	
x_{ij}:	ith input of DMU_j $(j = 1, \ldots, n)$
y^g_{rj}:	rth good output of $\text{DMU}_j(\ j = 1, \ldots, n\ )$
y^b_{pj}:	pth bad output of $\text{DMU}_j(\ j = 1, \ldots, n\ )$
$\hat{y}^g_r$:	desired production quantity of rth good output by inefficient DMUs
Decision Variables :	
θ_k:	inefficiency score of $\text{DMU}_k(\ k = 1, \ldots, n\ )$
λ_j:	weight assigned to DMU_j $(j = 1, \ldots, n)$
α_{ik}:	change in ith input of $\text{DMU}_k(k = 1, \ldots, t)$
β_{rk}:	change in rth good output of $\text{DMU}_k(k = 1, \ldots, t)$
γ_{pk}:	change in pth bad output of $\text{DMU}_k(\ k = 1, \ldots, t)$

References

1. Orisaremi, K.K.; Chan, F.T.; Chung, N.S. Potential reductions in global gas flaring for determining the optimal sizing of gas-to-wire (GTW) process: An inverse DEA approach. *J. Nat. Gas Sci. Eng.* **2021**, *93*, 103995. [CrossRef]
2. Ostic, D.; Twum, A.K.; Agyemang, A.O.; Boahen, H.A. Assessing the impact of oil and gas trading, foreign direct investment inflows, and economic growth on carbon emission for OPEC member countries. *Environ. Sci. Pollut. Res.* **2022**, *29*, 43089–43101. [CrossRef]
3. Pekarčíková, K.; Vaněk, M.; Sousedíková, R. Determinants of economic growth: Panel data analysis of OPEC. *Resour. Policy* **2022**, *79*, 103129. [CrossRef]
4. Abdulqadir, I.A. CO_2 emissions policy thresholds for renewable energy consumption on economic growth in OPEC member countries. *Int. J. Energy Sect. Manag.* **2022**, *17*, 1074–1091. [CrossRef]
5. Agbonifo, J. Fuel subsidy protests in Nigeria: The promise and mirage of empowerment. *Extr. Ind. Soc.* **2023**, *16*, 101333. [CrossRef]
6. Evans, O.; Nwaogwugwu, I.; Vincent, O.; Wale-Awe, O.; Mesagan, E.; Ojapinwa, T. *The Socio-Economics of the 2023 Fuel Subsidy Removal in Nigeria*; Munich Personal RePEc Archive: Munich, Germany, 2023.
7. Obulezi, O.J.; Chinedu, E.Q.; Oramulu, D.O.; Etaga, H.O.; Onyeizu, N.M.; Chukwuogo, O.E. Machine Learning Models for Predicting Transportation Costs Inflated by Fuel Subsidy Removal Policy in Nigeria. *Int. Res. J. Mod. Eng. Technol. Sci.* **2023**, *5*, 1053–1070.
8. Abubakar, I.R. Multidimensional poverty among Nigerian households: Sustainable development implications. *Soc. Indic. Res.* **2022**, *164*, 993–1014. [CrossRef]
9. González-Rivas, J.P.; Infante-García, M.M.; Nieto-Martinez, R.; Mechanick, J.I.; Danaei, G. Feasibility and Effectiveness of a Preventive Care Program during the Compound Humanitarian Crisis and COVID-19 Pandemic in Venezuela. *Nutrients* **2022**, *14*, 939. [CrossRef]
10. Lozada, M. Political Transition and Social Reparation in Venezuela: Challenges of Democratic Reconstruction. In *Human Rights Violations in Latin America: Reparation and Rehabilitation*; Springer: Berlin/Heidelberg, Germany, 2022; pp. 239–252.
11. Orisaremi, K.K.; Chan, F.T.; Fu, X.; Chung, N.S. Maximizing flare gas power generation for the design of an optimal energy mix. *J. Clean. Prod.* **2023**, *391*, 136164. [CrossRef]
12. Bertassello, L.; Winters, P.; Müller, M.F. Access to global wheat reserves determines country-level vulnerability to conflict-induced Ukrainian wheat supply disruption. *Nat. Food* **2023**, *4*, 673–676. [CrossRef]
13. Mahmood, H.; Saqib, N. Oil rents, economic growth, and CO_2 emissions in 13 OPEC member economies: Asymmetry analyses. *Front. Environ. Sci.* **2022**, *10*, 1025756. [CrossRef]
14. Abid, M.; Alotaibi, M.N. Crude oil price and private sector of Saudi Arabia: Do globalization and financial development matter? *New Evid. Comb. Cointegration Test. Resour. Policy* **2020**, *69*, 101774.
15. Ghiyasi, M. Industrial sector environmental planning and energy efficiency of Iranian provinces. *J. Clean. Prod.* **2017**, *142*, 2328–2339. [CrossRef]
16. Orisaremi, K.K.; Chan, F.T.; Chung, S.; Fu, X. A sustainable lean production framework based on inverse DEA for mitigating gas flaring. *Expert Syst. Appl.* **2022**, *206*, 117856. [CrossRef]
17. Wegener, M.; Amin, G.R. Minimizing greenhouse gas emissions using inverse DEA with an application in oil and gas. *Expert Syst. Appl.* **2019**, *122*, 369–375. [CrossRef]
18. Emrouznejad, A.; Amin, G.R. *Advances in Inverse Data Envelopment Analysis: Empowering Performance Assessment*; Oxford University Press: Oxford, UK, 2023; Volume 34, pp. 415–419.
19. Emrouznejad, A.; Amin, G.R.; Ghiyasi, M.; Michali, M. A review of inverse data envelopment analysis: Origins, development, and future directions. *IMA J. Manag. Math.* **2023**, *34*, 421–440. [CrossRef]
20. Oukil, A. Investigating prospective gains from mergers in the agricultural sector through Inverse DEA. *IMA J. Manag. Math.* **2023**, *34*, 465–490. [CrossRef]
21. Moghaddas, Z.; Tosarkani, B.M.; Yousefi, S. Resource reallocation for improving sustainable supply chain performance: An inverse data envelopment analysis. *Int. J. Prod. Econ.* **2022**, *252*, 108560. [CrossRef]

22. Zhang, Z.; Xiao, Y.; Niu, H. DEA and Machine Learning for Performance Prediction. *Mathematics* **2022**, *10*, 1776. [CrossRef]
23. Lantara, D.; Junaidi, J.; Rauf, N.; Pawennari, A.; Achmad, R.N. Indonesian Islamic banks: A review of the financial state before and after the COVID-19 pandemic. *Banks Bank Syst.* **2022**, *17*, 12–24. [CrossRef]
24. Ghiyasi, M.; Soltanifar, M.; Sharafi, H. A novel inverse DEA-R model with application in hospital efficiency. *Socio-Economic Plan. Sci.* **2022**, *84*, 101427. [CrossRef]
25. Firsova, A.; Chernyshova, G.; Tugusheva, R. Approaches to efficiency assessing of regional knowledge-intensive services sector development using data envelopment analysis. *Mathematics* **2022**, *10*, 173. [CrossRef]
26. Ashuri, B.; Wang, J.; Shahandashti, M.; Baek, M. A data envelopment analysis (DEA) model for building energy benchmarking. *J. Eng. Des. Technol.* **2019**, *17*, 747–768. [CrossRef]
27. Cooper, W.W.; Seiford, L.M.; Zhu, J. *Handbook on Data Envelopment Analysis*; Springer: Berlin/Heidelberg, Germany, 2011.
28. Charnes, A.; Cooper, W.; Lewin, A.Y.; Seiford, L.M. Data envelopment analysis theory, methodology and applications. *J. Oper. Res. Soc.* **1997**, *48*, 332–333. [CrossRef]
29. Chung, Y.H.; Färe, R.; Grosskopf, S. Productivity and undesirable outputs: A directional distance function approach. *J. Environ. Manag.* **1997**, *51*, 229–240. [CrossRef]
30. Emrouznejad, A.; Anouze, A.L.; Thanassoulis, E. A semi-oriented radial measure for measuring the efficiency of decision making units with negative data, using DEA. *Eur. J. Oper. Res.* **2010**, *200*, 297–304. [CrossRef]
31. Oyedepo, S.O. On energy for sustainable development in Nigeria. *Renew. Sustain. Energy Rev.* **2012**, *16*, 2583–2598. [CrossRef]