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Abstract: New energy vehicles (NEVs) are the main direction for the development of the global automobile industry. Evaluating and analyzing the competitiveness of new energy vehicle enterprises (NEVEs) is of great significance for promoting their development. In order to explore the current situation of NEVEs in Henan Province, this paper firstly constructs a competitiveness evaluation index system for NEVEs, comprising both quantitative and qualitative indexes. Then, a new definition of consistency, the consistency measure level, and corresponding improvement methods for interval multiplicative preference relations (IMPRs) are proposed. On this basis, fuzzy group decision-making models with IMPRs are constructed to deal with the ambiguity and uncertainty of the decision information, where consistency and consensus are both considered. In our case study, decision results are derived using Lingo 11.0 software. The results of this paper show that the degree of specialization has the greatest impact on the competitiveness of NEVEs, and some NEVEs are deficient in this regard. Related suggestions based on expert evaluation results are also provided. In addition, a comparison with other consistency improvement methods of IMPRs reveals that the methods proposed utilize the original information provided to decision-makers to the utmost degree.

Keywords: group decision-making; new energy vehicle; competitiveness evaluation; interval multiplicative preference relation; consistency

MSC: 03E72; 90-10

1. Introduction

The development of NEVs is the main direction of progress in the global automobile industry, as well as China’s initiative to cope with climate change and promote green development. Improving the competitiveness of NEVs is of far-reaching significance for ensuring the sustainable and healthy development of the industry, developing the low-carbon economy, and perfecting the future energy structure. In recent years, the world’s major traditional automobile manufacturers have taken NEVs as an important development direction and now they are striving to seize the international market share of NEVs. Compared with other countries, China took up the development of the NEV industry as a key national strategy much earlier. China’s progress in establishing a NEV industry started at the beginning of the 21st century. With the worsening of the energy crisis and the proposal of the “dual-carbon” goal, China’s traditional car enterprises have begun the strategic transformation to NEVEs. Since 2009, supportive policies for the development of NEVs have been introduced intensively. China has applied government subsidies, tax incentives, and other powerful means to optimize the business environment and strongly support the development of NEVs. Nowadays, the competition in the NEV market is increasingly fierce. In face of this severe business environment, NEVEs’ strengths and weaknesses must be explored, and we should compile sustainable development plans to enhance the competitiveness of NEV enterprises.
Henan Province is a Chinese province with a large population and economic scale, but it has no advantage over some of the other developed provinces in terms of output, industrial matching, talent cultivation, and so on. In order to enhance the competitiveness of NEVEs in Henan Province and promote the sustainable and healthy development of NEVs, this paper establishes an index system for evaluating the competitiveness of NEVEs and, specifically, constructs group decision-making models to evaluate the competitiveness of four typical NEVEs in Henan Province.

NEVs constitute an emerging industry, and the existing studies on the competitiveness of NEVEs are not enough. This section of the article mainly analyzes the relevant literature on enterprise competitiveness. In the 1980s, Michael Porter [1] derived five forces that determine the intensity of competition and market attractiveness of an industry, that is, cost, product differentiation, management, human resources, and innovation. Voudouris et al. [2] classified the sources of enterprises’ competitiveness into factors such as high specialization, service quality, and continuous innovation. Sukumar et al. [3] examined the potential relationship between innovation and competitiveness. Matinaro et al. [4] found that sustainable business performance is the key for enterprises to gain competitive advantages. Uvarova et al. [5] revealed the importance of operations management for enterprise competitiveness. Concalves et al. [6] emphasized the important role of innovation and human resources in enhancing the competitiveness of enterprises. Salun and Palanyachka [7] studied the competitiveness of industrial firms in terms of market level, economic level, and product level. Ciniciogluet et al. [8] found that the development of specialized technology in the automobile industry can significantly improve the competitiveness of enterprises. As can be seen from the above literature, many scholars have pointed out that innovation capability, operations management, and business performance can influence enterprises’ competitiveness. In addition, scholars believe that specialization technology, human resources, and product differentiation also have an impact on competitiveness.

In terms of evaluation methods, Guo and Lu [9] used data-driven principal component analysis to reduce the dimensionality of indicators and measure enterprises’ competitiveness. Wu et al. [10] used structural equation modeling to investigate factors that enhance the dynamic capability of enterprises. Widyawati et al. [11] used descriptive analysis and factor analysis to analyze the impact of social capital and organizational health on the competitive advantages of enterprises. Uwizeyemungu et al. [12] analyzed the competitive performances of enterprises through cluster analysis. Gouveia et al. [13] used data envelopment analysis to assess the competitiveness of enterprises in different beneficiary regions of the European Union. These contributions to the literature mainly used quantitative analysis to measure enterprises’ competitiveness. As we know, indicators for competitiveness evaluation are not always quantitative and may be difficult to quantify precisely. These methods may have certain difficulty in dealing with qualitative data and cannot make full use of experts’ personal experience and judgment, although the experience and intuition of the expert are sometimes more reliable.

On the contrary, fuzzy sets, such as interval fuzzy numbers, have the natural advantage in dealing with the uncertainty and fuzziness in decision-making. Moreover, in practical decision-making problems, the assessment results derived from pairwise comparisons are supposedly more reliable [14]. Accordingly, studies on preference relations based on interval values, i.e., interval multiplicative preference relations (IMPRs), have been widely carried out. For example, Xia and Xu [15] built a goal programming model to derive the weights of interval preference relations. Chandran et al. [16] proposed a method for weight estimation of generated IMPRs within the framework of analytic hierarchy process. For models with preference relations, consistency ensures decision makers’ judgments are logical [17]. High inconsistency may lead to illogical or even erroneous conclusions [18]. In order to solve the problem of inconsistency, many scholars have proposed different definitions of consistency for IMPRs. Li et al. [19] proposed a new concept of acceptable IMPRs based on the geometric weighted average method. Liu [20] proposed the concept of acceptable consistency and divided IMPRs into two clear preference relations based on
upper and lower bounds. Then, Xu [21] amended the consistency model of Liu [20] to use a more reasonable equivalent constraint. Meng and Tan [22] introduced the concept of quasi-consistency of IMPRs. In order to measure the degree of inconsistency of IMPRs, Saaty [23] proposed a consistency index and consistency ratio. Aguaron et al. [24] defined the geometric consistency index and gave the threshold value. Entani and Tanaka [25] introduced interval probability to measure the degree of uncertainty. In addition, when compared with other evaluation methods, it has been proposed that the preference relation-based method is more suitable for dealing with group decision-making problems. Group consensus has received a lot of attention in the research on group decision-making based on IMPRs. Xu and Liu [26] established a group consensus model of IMPRs. Meng et al. [27] defined a group consensus index to measure the consensus level and introduced methods to improve the consensus level. Pérez et al. [28] proposed a new consensus model for heterogeneous group decision-making guided by heterogeneity criteria. Wan et al. [29] studied group decision-making for IMPRs based on geometric consistency. Among the group decision-making applications of IMPRs, Xia and Chen [30] studied the group decision problem of IMPRs and applied it to the problem of choosing partners for virtual enterprises. Xu and Cai [31], meanwhile, explored group decision problems involving complete and incomplete IMPRs and applied them to the selection of Internet service providers.

As discussed above, many achievements have been made in regard to competitiveness evaluation of NEVEs, but there are still some gaps in the related research that need to be filled, as follows.

1. When analyzing the literature discussed above, it can be seen that many scholars have constructed evaluation indexes of enterprise competitiveness from the perspectives of innovation ability, operation management, and business performance. Meanwhile, the literature on competitiveness evaluation systems of NEVEs is still relatively scarce. Using existing literature achievements as references, this paper constructs a competitiveness evaluation index system for NEVEs, in which quantitative and qualitative indexes are both involved.

2. Although many scholars have provided insights into the definition of IMPRs from different perspectives [19–22], no one definition has yet been universally recognized. In addition, existing group decision-making models usually consider the group consensus and ignore individual consistency [26–28]. Therefore, this paper naturally extends the consistency definition of Saaty [12], propose a new consistency definition for IMPRs, and establish group decision-making models that simultaneously control individual consistency and the group consensus when solving fuzzy group decision-making problems.

3. In the literature [9–13], the evaluation of enterprise competitiveness mainly uses principal component analysis, factor analysis, and cluster analysis. However, indicators for evaluating competitiveness are not always quantitative and may be difficult to quantify precisely. These methods that can be applied may face some difficulties in dealing with qualitative data and do not make full use of the personal experience and judgment of experts. In contrast, interval values and IMPRs can do well at characterizing the ambiguity of decision-making information. In addition, due to the increasing complexity of decision problems, the evaluation results obtained by a single individual are sometimes one-sided. To derive reliable evaluation results for NEVEs, the wisdom of the group should be utilized. Therefore, in order to further enhance the competitiveness of NEVEs in Henan Province, this paper proposes fuzzy group decision-making models based on IMPRs to hone the evaluation process.

4. There are two main research questions in this paper. One is, how can the competitiveness of NEVEs be effectively evaluated? This firstly requires a suitable treatment of the qualitative indicators in the evaluation. Secondly, the weights of the indicators must be determined scientifically. By doing so, the one-sidedness of individual decision-making in the evaluation process is circumvented. The other question is, to what extent does the proposed fuzzy group decision-making model based on IMPRs
contribute to enhancing the clarity and precision of this evaluation? Since IMPRs express the preferences of decision makers in the form of interval numbers, they can do well at describing the vagueness and uncertainty in qualitative indicators. In addition, based on group decision-making and sound decision-making procedures, the impact of individual mistakes or errors can be reduced, which makes the weight allocation and evaluation results more reliable.

The paper is structured as follows: The research methodology is outlined in Section 2. The results and implications are revealed in Section 3. Lastly, Section 4 summarizes the findings, limitations, and future research directions.

2. Methods

2.1. The Construction of an Evaluation Index System

Using existing literature achievements on enterprise competitiveness evaluation as references, in this study, we constructed the following two-level evaluation index system for NEVEs, in which quantitative and qualitative indexes are both involved. The specifics are shown in Table 1.

Table 1. Evaluation index system.

<table>
<thead>
<tr>
<th>Primary Indicator</th>
<th>Secondary Indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree of specialization</td>
<td>Specialization of the technology (A11: 0.2729)</td>
</tr>
<tr>
<td>(A1: 0.3600)</td>
<td>Specialization of the product or service (A12: 0.3612)</td>
</tr>
<tr>
<td></td>
<td>Specialization of the staff team (A13: 0.3659)</td>
</tr>
<tr>
<td>Business performance</td>
<td>Main business income (A21: 0.5556)</td>
</tr>
<tr>
<td>(A2: 0.2800)</td>
<td>Asset–liability ratio (A22: 0.2192)</td>
</tr>
<tr>
<td></td>
<td>Labor productivity (A23: 0.2252)</td>
</tr>
<tr>
<td>Innovation ability</td>
<td>Intensity of research and development (A31: 0.1111)</td>
</tr>
<tr>
<td>(A3: 0.1630)</td>
<td>Construction of innovation team (A32: 0.3333)</td>
</tr>
<tr>
<td></td>
<td>Number of patents (A33: 0.5556)</td>
</tr>
<tr>
<td>Operation management</td>
<td>Digital level (A41: 0.3333)</td>
</tr>
<tr>
<td>(A4: 0.0400)</td>
<td>Quality management (A42: 0.5556)</td>
</tr>
<tr>
<td></td>
<td>Supply chain relationship (A43: 0.1111)</td>
</tr>
<tr>
<td>Product features</td>
<td>Market position (A51: 0.1111)</td>
</tr>
<tr>
<td>(A5: 0.1570)</td>
<td>Substitutability (A52: 0.4444)</td>
</tr>
<tr>
<td></td>
<td>Brand reputation (A53: 0.4444)</td>
</tr>
</tbody>
</table>

Five primary indicators are included in the evaluation index system. The degree of specialization mainly reflects the level of specialization of technology, which is subdivided into three secondary indicators: specialization of technology, specialization of products or services, and specialization of the staff teams. Only with the aid of specialized technology can NEVEs obtain an obvious advantage in a specific field and maintain a leading position in this field. Specialization of products or services can better meet the customers’ specific demands and gradually establish a differentiated image from the competitors. Specialized staff teams are the foundation of specialization and conducive to the long-term development of NEVEs.

Business performance reflects the state of operation within a certain period of time, which is subdivided into three secondary indicators: main business income, asset–liability ratio, and labor productivity. The main business income reflects the focus level on the main business. The scale of the main business income also represents the market share and position occupied by the enterprise in the field. Asset–liability ratio reflects the financial stability and solvency of NEVEs. Too high an asset–liability ratio will increase the financial risk and limit the development space of enterprises. Efficient labor productivity can save time costs and bring greater profits.
Innovation ability is always the important driving factor for business growth of NEVEs, which is subdivided into three secondary indicators: intensity of research and development, construction of innovation team, and number of patents. The intensity of research and development denotes the ratio of research and development investment to revenue. Innovation ability is affected by the compiling of an innovation team. When members of the team have different backgrounds, perspectives, and ways of thinking, this is more likely to produce innovative results. The number of patents is the important direct indicator of the innovation ability.

Operation management reflects the overall ability to arrange various resources, which is subdivided into three secondary indicators: digital level, quality management, and supply chain relationships. Digitization is an effective measure for improvements to NEVEs’ management level, with digital transformation urgently needed for NEVEs in the information age. Effective quality management is indispensable to improve the quality of products and gain customers’ confidence. Supply chain relationships reflect the competitive–cooperative relationships with other supply chain members. Strong supply chain relationships are very important in ensuring win–win cooperation.

Product features are important factors for NEVEs to outperform competitors and win orders; these are subdivided into three secondary indicators: market position, substitutability, and brand reputation. Market positioning reflects how when developing NEVEs, limited resources are focused on specific market segments. Substitutability of the product means the product has incomparable advantages over competitors in certain product features. Brand reputation represents customer trust and loyalty. Customers are often more willing to choose the products with good reputations and popularity in the market.

2.2. Fuzzy Group Decision Method Based on IMPR

When performing a competitiveness evaluation for NEVEs, in order to make full use of collective wisdom and manage the uncertainty of decision information, a fuzzy group decision method based on IMPRs was constructed.

For ease of reading, a description of the symbols commonly used in this article is listed in Table 2.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>$R$</td>
<td>Multiplicative preference relations</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>Interval multiplicative preference relations</td>
</tr>
<tr>
<td>$\bar{R}^*$</td>
<td>Improved interval multiplicative preference relations</td>
</tr>
<tr>
<td>$\bar{R}_p$</td>
<td>Individual interval multiplicative preference relations</td>
</tr>
<tr>
<td>$\bar{R}_g$</td>
<td>Group interval multiplicative preference relations</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>Weight vector</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>Standardized weight vector</td>
</tr>
<tr>
<td>$\bar{W}$</td>
<td>Interval multiplicative preference relations constructed using weight vectors</td>
</tr>
<tr>
<td>$\bar{P}$</td>
<td>Probability degree matrix</td>
</tr>
</tbody>
</table>

2.2.1. Basic Concept

Let $N = \{1, 2, \ldots, n\}$ and $X = \{x_i | i \in N\}$ be a finite set of alternatives.

**Definition 1** ([14]). A multiplicative preference relation (MPR) $R$ on a set of alternatives $X$ is denoted by a reciprocal matrix $R = (r_{ij})_{n \times n}$ with

$$r_{ij} > 0, \ r_{ij} \cdot r_{ji} = 1, \ r_{ii} = 1, \ \forall i, j \in N \quad (1)$$

In Definition 1, $r_{ij}$ represents the ratio of preference intensity between alternatives $x_i$ and $x_j$, and $0 < r_{ij} < 1$ indicates that alternative $x_j$ is better than $x_i$. $r_{ij} = 1$ indicates that
there is no difference between \( x_i \) and \( x_j \). The larger the value of \( r_{ij} \), the stronger the degree of superiority of \( x_i \) to \( x_j \).

Saaty [14] also proposed the consistent definition of MPR.

**Definition 2 ([14])**. An MPR \( R = (r_{ij})_{n \times n} \) is consistent if

\[
r_{ik} = r_{ij} \cdot r_{jk}, \forall i, j, k \in N
\]

Furthermore, \( R \) is consistent if there is a weight vector \( w = (w_1, w_2, \ldots, w_n) \) with \( w_i > 0 \) such that

\[
r_{ij} = w_i / w_j, \ i, j \in N
\]

Due to the complexity of the decision-making environment and the limited ability of individuals, it is difficult for decision makers to express their preferences in terms of crisp values; instead, they prefer to make judgements in the form of interval values. Therefore, there is the following definition of IMPRs.

**Definition 3 ([23])**. An IMPR \( \tilde{R} \) on a set of alternatives \( X \) is denoted by an interval reciprocal matrix \( \tilde{R} = (\tilde{r}_{ij})_{n \times n} \) with

\[
\tilde{r}_{ij} = [r_{ij}^-, r_{ij}^+], \ r_{ij}^- \leq r_{ij}^+, r_{ij}^- r_{ji}^+ = 1, r_{ii}^- = r_{ii}^+ = 1, \forall i, j \in N
\]

In Definition 3, \( \tilde{r}_{ij} \) is the preference degree of \( x_i \) and \( x_j \). \( \tilde{r}_{ij} = [1, 1] \) indicates that there is no difference between \( x_i \) and \( x_j \).

If the IMPR \( \tilde{R} \) is consistent, there is a weight vector satisfying the following definition.

**Definition 4 ([32])**. An IMPR \( \tilde{R} \) is consistent if there is a weight vector \( \tilde{w} = [w_i^-, w_i^+] \) with \( 0 < w_i^- < w_i^+ \), such that

\[
r_{ij}^- = w_i^- / w_j^+, r_{ij}^+ = w_i^+ / w_j^-
\]

In order to provide comparability of weight vectors, Li et al. [19] proposed the definition of normalized weight vectors shown in Definition 5.

**Definition 5 ([19])**. Let \( \tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n) \) be an interval weight vector with \( \tilde{w} = [w_i^-, w_i^+] \) and \( 0 < w_i^- < w_i^+, i = 1, \ldots, n \). \( \tilde{w} \) is called a normalized weight vector if

\[
w_i^- \cdot \prod_{j \neq i} w_j^- \leq 1, w_i^- \cdot \prod_{j \neq i} w_j^+ \geq 1, \ i, j = 1, 2, \ldots, n
\]

The following possibility degree formula can be constructed from the weight vector \( \tilde{w}_i \).

**Definition 6 ([19])**. Let \( \tilde{w}_i = [w_i^-, w_i^+] \) and \( \tilde{w}_j = [w_j^-, w_j^+] \) be any two interval weights; then, the possibility degree of \( \tilde{w}_i \geq \tilde{w}_j \) is defined as

\[
P(\tilde{w}_i \geq \tilde{w}_j) = \max\left\{ 0, \ln w_i^+ - \ln w_j^- \right\} - \max\left\{ 0, \ln w_i^- - \ln w_j^+ \right\}, \tilde{w}_i \geq 0, \tilde{w}_j \geq 0
\]

The possibility matrix \( P = (p_{ij})_{n \times n} \) is constructed using Equation (7) to obtain the final ranking results and obtain the optimal decision.
We can construct a new IMPR \( \tilde{W} = \begin{bmatrix} w_{ij}^+, w_{ij}^- \end{bmatrix}_{n \times n} = \begin{bmatrix} w_i^- / w_i^+, w_i^+ / w_i^- \end{bmatrix}_{n \times n} \) based on the weight vector \( \tilde{w} \) by using Definition 4. To measure the distance between the original IMPR \( R \) and \( \tilde{W} \), Cheng et al. [32] propose the following definition.

**Definition 7** ([32]). Let \( R \) be an IMPR and \( \tilde{W} \) be the IMPR obtained by using the weight vector; the distance between \( R \) and \( \tilde{W} \) is

\[
D(\tilde{W}, R) = \sum_{i=1}^{n} \sum_{i<j} \left( \ln w_{ij}^- - \ln r_{ij}^- + \ln w_{ij}^+ - \ln r_{ij}^+ \right)
\]

(8)

Definition 7 can be used to measure the degree of change between two IMPRs.

2.2.2. A New Consistency Definition for IMPRs

In the decision-making process, the consistency of IMPRs is a key factor to determine the accuracy and reliability of decision results. However, since experts have different preferences for different alternatives, the given preference relations may not satisfy the consistency condition. To solve the related problems, this section naturally extends the consistency definition of MPRs and proposes the consistency definition of IMPRs, and constructs a model to improve consistency.

Let \( R \) be a consistent MPR according to Definition 2, based on which we construct an IMPR \( \tilde{R} \), \( r_{ij}^- = r_{ij} / D_{ij}^- , r_{ij}^+ = r_{ij} \cdot D_{ij}^+ , D_{ij}^- , D_{ij}^+ \geq 1 \). \( D_{ij}^- \) and \( D_{ij}^+ \) denote the upward and downward fluctuation extents of \( r_{ij} \), respectively. If \( D_{ij}^- = D_{ij}^+ = D, \forall i \neq j \), IMPR \( \tilde{R} \) is consistent. Based on this idea, this paper presents a new consistency definition for IMPRs.

**Definition 8.** An IMPR \( \tilde{R} \) is consistent if there is a positive real number \( D \) with \( D > 1 \) such that

\[
\begin{cases}
    r_{ij}^+ \cdot D = r_{ik}^+ \cdot r_{kj}^- & \forall i \neq j \neq k, i, j, k \in N \ \text{and} \ D > 1 \\
    r_{ij}^- / r_{ij}^+ = D^2 & \forall i \neq j \neq k, i, j, k \in N \ \text{and} \ D > 1
\end{cases}
\]

(9)

Definition 8 is defined from the perspective of the upper limit of the interval. A similar definition can be given from the lower limit of the interval.

**Definition 9.** An IMPR \( \tilde{R} \) is consistent if there is a positive real number \( D \) with \( D > 1 \) such that

\[
\begin{cases}
    r_{ij}^- / D = r_{ik}^- \cdot r_{kj}^+ & \forall i \neq j \neq k, i, j, k \in N \ \text{and} \ D > 1 \\
    r_{ij}^+ / r_{ij}^- = D^2 & \forall i \neq j \neq k, i, j, k \in N \ \text{and} \ D > 1
\end{cases}
\]

(10)

**Theorem 1.** Definition 9 is equivalent to Definition 8.

**Proof of Theorem 1.** According to Definition 8, we have \( r_{ij}^+ = r_{ij}^- \cdot D^2 \). Thus, \( r_{ij}^+ \cdot D = r_{ik}^- \cdot r_{kj}^+ \) can be converted to \( r_{ij}^- \cdot D^2 \cdot D = r_{ik}^- \cdot D^2 \cdot r_{kj}^+ \cdot D^2 \). Then, \( r_{ij}^- / D = r_{ik}^- \cdot r_{kj}^+ \) \( \Box \)

Based on Definition 8, we built a model that improves any inconsistent IMPR into a consistent IMPR. Let \( \tilde{R} \) and \( \tilde{R}^* \) be the original and improved IMPR, respectively.

**Model**

\[
\begin{cases}
    \text{Min} \frac{2}{w_i^- w_i^+} \sum_{i<j} \left( \ln r_{ij}^- - \ln r_{ij}^- + \ln r_{ij}^+ - \ln r_{ij}^+ \right) \\
    \ln r_{ij}^- + \ln D = \ln r_{ik}^- + \ln r_{kj}^+ \\
    \ln r_{ij}^+ = \ln r_{ij}^- + 2 \ln D \\
    r_{ij}^- \leq r_{ij}^+
\end{cases}
\]
In Model 1, the objective function is to minimize the amount of change between the improved IMPR $\tilde{R}$ and the original IMPR $R$. The first two constraints ensure that $\tilde{R}$ meets the consistency requirements in Definition 8. The last constraint indicates that the lower limit of the interval is no greater than the upper limit of the interval.

By solving this model, we can obtain a consistent IMPR $\tilde{R}$ with minimal adjustments.

2.2.3. Consistency Measurement Method

In practical decision problems, due to the limitations of subjective and objective conditions, it is often difficult for decision makers to give a completely consistent IMPR. If the IMPR given by the decision maker lacks consistency, it can lead to illogical or even incorrect results. Therefore, the initial IMPR given by decision makers often needs to be tested and improved for consistency.

For an IMPR $R$, the geometric mean of all $D_{ij}^-$ and $D_{ij}^+$ can be expressed as

$$\left(\prod_{i<j} (D_{ij}^- \cdot D_{ij}^+)\right)^{\frac{1}{n(n-1)}} = \left(\prod_{i<j} \left(\frac{r_{ij}^+}{r_{ij}^-}\right)\right)^{\frac{1}{n(n-1)}}. $$

If $R$ is consistent according to Definition 8, we have $r_{ij}^+ \cdot D = r_{ij}^- + r_{ij}^+$ and $r_{ij}^+ / r_{ij}^- = D^2$. Thus, we derive

$$\left(\frac{r_{ik}^+ \cdot r_{kj}^-}{r_{ij}^-}\right) / r_{ij}^+ = \left(\frac{r_{ij}^-}{r_{ij}^-}\right)^{\frac{1}{2}} \quad (11)$$

$$\left(\frac{r_{ij}^+}{r_{ij}^-}\right)^{\frac{1}{2}} = \left(\prod_{i<j} \left(\frac{r_{ij}^-}{r_{ij}^-}\right)\right)^{\frac{1}{n(n-1)}} \quad (12)$$

Equations (11) and (12) can be equivalently rewritten as:

$$\left|\ln r_{jk}^+ + \ln r_{kj}^- - \ln r_{ij}^- - \frac{1}{2} (\ln r_{ij}^- - \ln r_{ij}^-)\right| = 0 \quad (13)$$

$$\left|\frac{1}{2} (\ln r_{ij}^- - \ln r_{ij}^-) - \frac{1}{n(n-1)} \sum_{i<j} (\ln r_{ij}^- - \ln r_{ij}^-)\right| = 0 \quad (14)$$

In real life, it is difficult for decision makers to provide decision information that fully satisfies consistency. Therefore, disturbance terms $s_{ij}$ and $t_{ij}$ need to be added. Considering all locations in the IMPR, the overall disturbance terms can be expressed as

$$\sum_{i<j} \left|\frac{1}{2} (\ln r_{ij}^+ - \ln r_{ij}^-) - \frac{1}{n(n-1)} \sum_{i<j} (\ln r_{ij}^+ - \ln r_{ij}^-)\right| = \sum_{i<j} t_{ij} \quad (15)$$

$$\sum_{i<k,k<j} \left|\ln r_{ik}^+ + \ln r_{kj}^- - \ln r_{ij}^- - \frac{1}{2} (\ln r_{ij}^- - \ln r_{ij}^-)\right| = \sum_{i<j} s_{ij} \quad (16)$$

Then, the average of disturbance terms can be represented as

$$S(\tilde{R}) = \frac{6}{n(n-1)(n-2)} \sum_{i<j} s_{ij} \quad (17)$$

$$T(\tilde{R}) = \frac{2}{n(n-1)} \sum_{i<j} t_{ij} \quad (18)$$

The smaller the values of $S(\tilde{R})$ and $T(\tilde{R})$, the more consistent is IMPR $\tilde{R}$. 
**Definition 10.** For an IMPR $\bar{R}$, its consistency level is defined as

$$ CL = S(\bar{R}) + T(\bar{R}) $$

(19)

Given a threshold $CL$, when $CL = 0$, $\bar{R}$ is consistent. When $0 < CL(\bar{R}) < CL$, $\bar{R}$ is acceptably consistent. Otherwise, $\bar{R}$ is unacceptably consistent and needs to be improved.

In this section, Monte Carlo simulation is applied to determine the threshold $CL$, where 100,000 IMPRs are randomly generated and their average consistency level is calculated and regarded as a threshold. The specific results are shown in Table 3.

<table>
<thead>
<tr>
<th>Table 3. Consistency levels of IMPRs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
</tr>
<tr>
<td>$CL$</td>
</tr>
</tbody>
</table>

Then, a model that adjusts an IMPR $\bar{R}$ to $\bar{R}^*$ with acceptable consistency is constructed as follows.

**Model 2**

$$ Min \frac{2}{n(n-1)} \sum_{i<j} \left( |\ln r_{ij}^* - \ln r_{ij}^-| + |\ln r_{ij}^+ - \ln r_{ij}^+| \right) $$

$$ \left\{ \begin{array}{l}
\sum_{i<k,k<j} \left( \sum_{i<k} \ln r_{ik}^* + \ln r_{kj}^* - \ln r_{ij}^+ - \ln r_{ij}^- - \frac{1}{2} \left( \ln r_{ij}^* - \ln r_{ij}^- \right) \right) = \sum_{i<j} s_{ij} \\
\sum_{i<j} \frac{1}{2} \left( \ln r_{ij}^+ - \ln r_{ij}^- \right) - \frac{1}{n(n-1)} \sum_{i<j} \left( \ln r_{ij}^+ - \ln r_{ij}^- \right) = \sum_{i<j} t_{ij} \\
CL = \frac{6}{n(n-1)(n-2)} \sum_{i<j} s_{ij} + \frac{2}{n(n-1)} \sum_{i<j} t_{ij}
\end{array} \right. $$

$$ CL \leq CL, \quad r_{ij}^- \leq r_{ij}^* $$

In Model 2, the objective function is to minimize the amount of change between the improved IMPR $\bar{R}^*$ and the original IMPR $\bar{R}$. The first to third constraints are used to calculate the consistency level of $\bar{R}^*$. The fourth constraint ensures that $\bar{R}^*$ is acceptably consistent. The fifth constraint ensures that the lower limit of the improved interval is no greater than the upper limit of the interval.

After obtaining an acceptably consistent IMPR $\bar{R}^*$, Model 3 is constructed for deriving the weight vector using Definitions 4, 5, and 7.

**Model 3**

$$ Min \frac{2}{n(n-1)} \sum_{i<j} \left[ |\ln w_{ij}^* - \ln w_{ij}^-| + |\ln w_{ij}^+ - \ln w_{ij}^+| \right] $$

$$ \left\{ \begin{array}{l}
\ln w_{ij}^- - \ln w_{ij}^+ = \ln w_{ij}^- \\
\ln w_{ij}^+ - \ln w_{ij}^- = \ln w_{ij}^+ \\
\ln w_{ij}^+ + \sum_{i \neq j} \ln w_{ij}^- \leq 0 \\
\ln w_{ij}^- + \sum_{i \neq j} \ln w_{ij}^+ \geq 0 \\
\ln w_{ij}^- < \ln w_{ij}^+
\end{array} \right. $$
In Model 3, the objective function is to minimize the amount of change between IMPR $\tilde{R}$ and $\tilde{W}$. The first two constraints represent the IMPR $\tilde{W}$ formed by the weight vector $\bar{w}$. The third to fourth constraints are the requirements for the normalization of the weight vector $\bar{w}$. The last constraint ensures that the lower limit of the interval is no greater than the upper limit.

Let $\tilde{R}$ be an IMPR. For an individual decision-making problem, the individual decision method is given as follows:

Step 1. Set an acceptable consistency threshold of an n-order IMPR using Table 3.
Step 2. Check whether $\tilde{R}$ is acceptably consistent using Equation (20).
Step 3. If $\tilde{R}$ satisfies acceptable consistency, then go to step 5; otherwise, continue with the next step.
Step 4. Input $\tilde{R}$ into Model 2 to obtain an IMPR $\tilde{R}^*$ that satisfies acceptable consistency.
Step 5. Obtain the normalized weight vector $\bar{w}$ according to Model 3, and then construct the possibility matrix using Equation (7) and output the final ranking result.

2.2.4. Group Decision Method

Due to the gradually increasing complexity of the decision-making environment, individual decision makers are often unable to consider problems in a comprehensive manner. Therefore, it is necessary to utilize group wisdom and apply a group decision-making method. In group decision problem-solving, after obtaining acceptably consistent individual IMPRs, we need to aggregate multiple individual IMPRs into a group IMPR by using an aggregation operator. Then, a model that considers both individual consistency and group consensus is constructed.

Let $e_R^p = [r_{ij,p}^- r_{ij,p}^+]_{n \times n}$ be an IMPR of decision maker $e_p$ ($p = 1, 2, \ldots, m$) with the weight $\lambda_p \geq 0$, $\sum_{p=1}^{m} \lambda_p = 1$. By means of a weighted geometric averaging operator [27], the corresponding group IMPR $\tilde{R}_g = [r_{ij,g}^- r_{ij,g}^+]_{n \times n}$ is derived, i.e., $r_{ij,g}^- = \prod_{p=1}^{m} (r_{ij,p}^-)^{\lambda_p}$, $r_{ij,g}^+ = \prod_{p=1}^{m} (r_{ij,p}^+)^{\lambda_p}$. The individual consensus level for $e_R^p$ is measured as the divergence between $\tilde{R}_p$ and $\tilde{R}_g$.

\[
ICL(\tilde{R}_p) = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{1}{2} \left( |r_{ij,p}^- - r_{ij,g}^-| + |r_{ij,p}^+ - r_{ij,g}^+| \right) (20)
\]

And the group consensus level, $GCL = GCL\{\tilde{R}_1, \ldots, \tilde{R}_m\}$, is defined as

\[
GCL = \max_{p=1}^{m} ICL(\tilde{R}_p) (21)
\]

Given a threshold $GCL$, the group consensus level is acceptable if $GCL \leq GCL$. In group decision-making, if each individual IMPR is of acceptable consistency, the aggregated group IMPR is generally supposed to be of acceptable consistency.

**Theorem 2.** Let $\tilde{R}_p = [r_{ij,p}^- r_{ij,p}^+]_{n \times n}$, $p = 1, 2, \ldots, m$ be the individual IMPR and $\tilde{R}_g = [r_{ij,g}^- r_{ij,g}^+]_{n \times n}$ be the group IMPR obtained by using a weighted geometric averaging operator. If $CL(\tilde{R}_p) \leq CL, p = 1, 2, \ldots, m$, then $CL(\tilde{R}_g) \leq CL$.

**Proof of Theorem 2.** According to (17) and (18), we have
\[ S(\bar{R}_g) = \frac{6}{n(n-1)(n-2)} \sum_{i < k < j} \left| \frac{1}{m} \sum_{p=1}^{m} (r_{ik,p}^+) - \frac{1}{m} \sum_{p=1}^{m} (r_{ij,p}^+) \right| \]

\[ = \frac{6}{n(n-1)(n-2)} \sum_{i < k < j} \sum_{p=1}^{m} \lambda_p \left( \ln r_{ik,p}^+ - \ln r_{ij,p}^+ - \frac{3}{2} \ln r_{ij,p}^+ + \frac{1}{2} \ln r_{ij,p}^- \right) \]

\[ \leq \max \frac{6}{n(n-1)(n-2)} \sum_{i < k < j} \sum_{p=1}^{m} \ln r_{ik,p}^+ + \ln r_{ij,p}^+ - \frac{3}{2} \ln r_{ij,p}^+ + \frac{1}{2} \ln r_{ij,p}^- = \max S(\bar{R}_p) \]

\[ T(\bar{R}_g) = \frac{2}{n(n-1)} \sum_{i < j} \left( \frac{1}{m} \left( \ln r_{ij,p}^+ - \ln r_{ij,p}^- \right) - \frac{1}{n(n-1)} \sum_{i < j} \left( \ln r_{ij,p}^+ - \ln r_{ij,p}^- \right) \right) \]

\[ = \frac{2}{n(n-1)} \sum_{i < j} \sum_{p=1}^{m} \lambda_p \left( \ln r_{ij,p}^+ - \ln r_{ij,p}^- \right) - \frac{1}{n(n-1)} \sum_{i < j} \sum_{p=1}^{m} \lambda_p \left( \ln r_{ij,p}^+ - \ln r_{ij,p}^- \right) \]

\[ \leq \max \frac{2}{n(n-1)} \sum_{i < j} \sum_{p=1}^{m} \left( \ln r_{ij,p}^+ - \ln r_{ij,p}^- \right) - \frac{1}{n(n-1)} \sum_{i < j} \sum_{p=1}^{m} \left( \ln r_{ij,p}^+ - \ln r_{ij,p}^- \right) = \max T(\bar{R}_p) \]

Then, we obtain \( CL(\bar{R}_g) = S(\bar{R}_g) + T(\bar{R}_g) \leq \max CL(\bar{R}_p) \). Thus, if \( CL(\bar{R}_p) \leq CL \), \( (p = 1, 2, \ldots, m) \), we have \( CL(\bar{R}_g) \leq CL \). □

Theorem 2 shows the group IMPR is acceptably consistent if all the individual IMPRs are acceptably consistent. Based on the above analysis, the following model is constructed to obtain the group IMPR with acceptable individual consistency and group consensus.

**Model 4**

\[ \text{Min} \frac{2}{n(n-1)} \sum_{i < j} \sum_{p=1}^{m} \left( |\ln r_{ij,p}^- - \ln r_{ij,p}^-| + |\ln r_{ij,p}^+ - \ln r_{ij,p}^+| \right) \]

\[ \text{subject to:} \]

\[ \sum_{i < j} \sum_{p=1}^{m} \ln r_{ik,p}^+ + \ln r_{kj,p}^+ - \ln r_{ij,p}^- - \frac{1}{2} \left( \ln r_{ij,p}^+ - \ln r_{ij,p}^- \right) = \sum_{i < j} s_{ij,p} \]

\[ \sum_{i < j} \sum_{p=1}^{m} \frac{1}{2} \left( \ln r_{ij,p}^+ - \ln r_{ij,p}^- \right) - \frac{1}{n(n-1)} \sum_{i < j} \sum_{p=1}^{m} \left( \ln r_{ij,p}^+ - \ln r_{ij,p}^- \right) = \sum_{i < j} t_{ij,p} \]

\[ \text{CL}(\bar{R}_p) = \frac{6}{n(n-1)(n-2)} \sum_{i < j} s_{ij,p} + \frac{2}{n(n-1)} \sum_{i < j} t_{ij,p} \]

\[ \text{CL}(\bar{R}_g) \leq CL \]

\[ r_{ij}^- = \prod_{p=1}^{m} \left( r_{ij,p}^- \right)^{\lambda_p} \]

\[ r_{ij}^+ = \prod_{p=1}^{m} \left( r_{ij,p}^+ \right)^{\lambda_p} \]

\[ \text{GCL}(\bar{R}_g) = \max_{p=1}^{m} \frac{2}{n(n-1)} \sum_{j=1}^{n} \sum_{i=1}^{n} \frac{1}{2} \left( |r_{ij,p}^- - r_{ij,p}^-| + |r_{ij,p}^+ - r_{ij,p}^+| \right) \]

\[ \text{GCL}(\bar{R}_g) \leq \text{GCL}(\bar{R}_g) \leq CL \]

\[ r_{ij}^+ \leq r_{ij,p}^+ \]

In Model 4, the objective function is to minimize the amount of change between the improved individual IMPRs \( \bar{R}_p^* \) and the original individual IMPRs \( \bar{R}_p \). The first three constraints are the calculation methods of individual consistency level. The fourth constraint ensures that the improved individual IMPRs meet acceptable consistency. The fifth and sixth constraints are the aggregation of multiple individual IMPRs into a group IMPR by using an aggregation operator. The seventh constraint is the calculation method of the
group consensus level. The eighth constraint ensures that the resulting group IMPR meets an acceptable consensus. The last constraint is the constraint on the interval.

Let $\tilde{R}_p(p = 1, 2, \ldots, m)$ be the multiple individual IMPRs. For a group decision problem, the group decision method is given as follows:

Step 1. Set $C_L$ of an n-order IMPR by using Table 3 and $C_CL = 0.1$.

Step 2. Check whether $\tilde{R}_p(p = 1, 2, \ldots, m)$ is acceptably consistent by using Equation (20).

Step 3. If all $\tilde{R}_p(p = 1, 2, \ldots, m)$ satisfy acceptable consistency, then go to step 5; otherwise, continue with the next step.

Step 4. Input $\tilde{R}_p(p = 1, 2, \ldots, m)$ into Model 2 to obtain IMPRs $\tilde{R}_p^+(p = 1, 2, \ldots, m)$ that satisfy acceptable consistency.

Step 5. Aggregate multiple individual IMPRs into group IMPR $\tilde{R}_g$ using the aggregation operator.

Step 6. Check whether $\tilde{R}_g$ is an acceptable consensus by using Equation (21).

Step 7. If $\tilde{R}_g$ satisfies an acceptable consensus, then proceed to Step 9; otherwise, continue with the next step.

Step 8. Input $\tilde{R}_g$ into Model 4 to obtain IMPR $\tilde{R}_g^+$ that satisfies both an acceptable consistency and acceptable group consensus.

Step 9. Obtain the normalized weight vector $\overline{\omega}$ by using Model 3, and then construct the possibility matrix by using Equation (7), and output the final ranking result.

The above group decision framework based on IMPRs is graphically depicted in Figure 1.

**Figure 1.** Decision-making flowchart.
2.2.5. Numerical Examples

Example 1 ([33]). Consider the following IMPR \( \tilde{R} \).

\[
\tilde{R} = \begin{bmatrix}
[1, 3] & [1/2, 1] & [2, 3] & [1, 1]
\end{bmatrix}
\]

Step 1. Set the consistency threshold \( CL = 1.6500 \) for the four-order IMPR according to Table 3.

Step 2. According to Definition 10, we can calculate that \( CL(\tilde{R}) = 1.6947 > CL \).

Step 3. \( \tilde{R} \) is inconsistent and needs to be improved by using Model 2. The adjusted IMPR \( \tilde{R}^* \) is derived as follows.

\[
\tilde{R}^* = \begin{bmatrix}
[1, 1] & [4, 6] & [0.5, 1] & [0.33, 1] \\
[0.17, 0.26] & [1, 1] & [0.33, 0.5] & [1, 1.88] \\
[1, 2] & [2, 3] & [1, 1] & [0.33, 0.5] \\
[1, 3] & [0.53, 1] & [2, 3] & [1, 1]
\end{bmatrix}
\]

Step 4. The weight vector \( \tilde{w} \) is solved according to Model 3.

\[
\tilde{w}_1 = [0.91, 1.43], \quad \tilde{w}_2 = [0.40, 0.50], \quad \tilde{w}_3 = [1.01, 1.21], \quad \tilde{w}_4 = [1.43, 2.73].
\]

Then, we can calculate the possibility matrix \( P \) by using Equation (7).

\[
P = \begin{bmatrix}
0.5 & 1 & 0.55 & 0 \\
0 & 0.5 & 0 & 0 \\
0.45 & 1 & 0.5 & 0 \\
1 & 1 & 1 & 0.5
\end{bmatrix}
\]

After adding each row of matrix \( P \), we obtain \( P_1 = 2.05, P_2 = 0.5, P_3 = 1.95, P_4 = 3.5 \). The ranking result of the four alternatives is \( x_4 \succ x_1 \succ x_3 \succ x_2 \).

The decision-making methods of Liu [20] and Meng et al. [33] are used to produce the ranking results of IMPR \( R \), and they are compared with the method in this paper, with the comparison results shown in Table 4.

<table>
<thead>
<tr>
<th>Method</th>
<th>Weight Vector</th>
<th>Sort Result</th>
<th>( D(\tilde{w}, \tilde{R}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liu [20]</td>
<td>([0.66, 6.10], [0.10, 0.19], [0.87, 1.74], [0.98, 8.89])</td>
<td>( x_4 \succ x_1 \succ x_3 \succ x_2 )</td>
<td>19.96</td>
</tr>
<tr>
<td>Meng et al. [33]</td>
<td>([0.19, 0.35], [0.07, 0.13], [0.19, 0.30], [0.30, 0.43])</td>
<td>( x_4 \succ x_1 \succ x_3 \succ x_2 )</td>
<td>7.41</td>
</tr>
<tr>
<td>This article</td>
<td>([0.91, 1.43], [0.40, 0.50], [1.01, 1.21], [1.43, 2.73])</td>
<td>( x_4 \succ x_3 \succ x_1 \succ x_2 )</td>
<td>6.37</td>
</tr>
</tbody>
</table>

As can be seen from Table 4, in the ranking results of the method in this paper, alternative \( x_3 \) is superior to alternative \( x_1 \), while on the contrary, in the methods of Liu [20] and Meng et al. [33], alternative \( x_1 \) is superior to alternative \( x_3 \). According to the initial IMPR \( \tilde{R} \), \( \tilde{r}_{13} = [1/2, 1] \) indicates that alternative \( x_3 \) is superior to alternative \( x_1 \). This is consistent with the result in this paper, which shows that the method in this paper better retains the initial preference information of the decision maker and better reflects the initial willingness of the decision maker than the methods of Liu [20] and Meng et al. [33].
Meanwhile, analyzing the value of $D(\tilde{W}, \tilde{R})$ shows that the $D(\tilde{W}, \tilde{R})$ in this paper is smaller than those of Liu [20] and Meng et al. [33]. This indicates that the distance between the IMPR $\tilde{W}$ obtained in this paper and the initial IMPR $\tilde{R}$ is smaller than those of Liu [20] and Meng et al. [33]. It also shows that the decision-making method in this paper retains as much of the decision maker’s original preference information as possible.

Furthermore, the consistency definition of IMPRs should be independent of the ordering of the compared objects. However, the consistency definition of Liu [20] relies on the ordering of the compared objects and is, therefore, not stable or robust.

To demonstrate the validity of the method proposed, another numerical example for comparison is performed.

**Example 2 ([29]).** Consider the following IMPR $\tilde{R}$.

$$
\tilde{R} = \begin{bmatrix}
[1,1] & [1,2] & [1,2] & [2,3] \\
[1/2,1] & [1,3] & [1,1] & [6,8] \\
\end{bmatrix}
$$

Step 1. Set the consistency threshold $CL = 1.6500$ for the four-order IMPR according to Table 3.

Step 2. According to Definition 10, we can calculate that $CL(\tilde{R}) = 1.5075 < CL$.

Step 3. $\tilde{R}$ satisfies acceptable consistency. The weight vector can be solved directly.

Step 4. The weight vector $\tilde{w}$ is solved according to Model 3.

$$
\tilde{w}_1 = [1.23, 1.56], \tilde{w}_2 = [1.81, 1.81], \tilde{w}_3 = [0.78, 1.23], \tilde{w}_4 = [0.36, 0.45]
$$

Then, we can calculate the possibility matrix $P$ by using Equation (7).

$$
P = \begin{bmatrix}
0.5 & 0 & 1 & 1 \\
1 & 0.5 & 1 & 1 \\
0 & 0 & 0.5 & 1 \\
0 & 0 & 0 & 0.5
\end{bmatrix}
$$

Add each row of matrix $P$ to obtain $P_1 = 2.5$, $P_2 = 3.5$, $P_3 = 1.5$, $P_4 = 0.5$. The ranking result of the four alternatives is $x_2 = x_1 > x_3 > x_4$.

The decision-making method of Wan et al. [29] is used to produce the ranking result of $\tilde{R}$, and that method is compared with the one in this paper, with the comparison result shown in Table 5.

**Table 5.** Comparison of results with another method.

<table>
<thead>
<tr>
<th>Method</th>
<th>Weight Vector</th>
<th>Sort Result</th>
<th>$D(\tilde{W}, \tilde{R})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wan et al. [29]</td>
<td>[1.16, 1.80], [1.57, 2.31], [0.87, 1.22], [0.28, 0.35]</td>
<td>$x_2 &gt; x_1 &gt; x_3 &gt; x_4$</td>
<td>6.18</td>
</tr>
<tr>
<td>This article</td>
<td>[1.23, 1.56], [1.81, 1.81], [0.78, 1.23], [0.36, 0.45]</td>
<td>$x_2 &gt; x_1 &gt; x_3 &gt; x_4$</td>
<td>5.49</td>
</tr>
</tbody>
</table>

In Table 5, it can be seen that the ranking result in this paper is the same as that of Wan et al. [29]. However, in the result of Wan et al. [29], the distance between the IMPR composed of weight vectors and the initial IMPR is larger. This is because Wan et al. [29] used a stricter definition of consistency, which led to the loss of more original information in the consistency improvement process. According to the decision-making method in this paper, the IMPR satisfies acceptable consistency. Therefore, we propose that the consistency
definition and improvement method in this paper retain as much original information of the decision maker as possible.

In addition, the consistency improvement method of Wan et al. [29] is realized by using an iterative algorithm with many more steps, while in this paper, the model is simpler and easier to solve.

3. Case Study
3.1. Weight Determination Based on IMPR Processing

Four NEVEs in Henan Province were selected. These four NEVEs are involved in charging and replacing equipment, power batteries, vehicle manufacturing, and the automotive aftermarket. Three experts with weight of \(\lambda = (1/3, 1/3, 1/3)\) were invited to provide preference information. The main process was as follows.

The first step in evaluating the competitiveness of NEVEs was to determine the weight of each indicator. The specific steps were as follows:

Step 1. We invited three experts to make pairwise comparisons of the importance of each primary indicator. The corresponding IMPRs \(\tilde{R}_p^1, \tilde{R}_p^2\), and \(\tilde{R}_p^3\) are shown below.

\[
\tilde{R}_p^1 = \begin{bmatrix}
0.2,0.25 & [1,1] & [3,4] & [4,5] \\
0.11,0.2 & [0.33,1] & [1,1] & [3,4] & [5,6] \\
0.25,0.33 & [0.25,0.33] & [0.25,0.33] & [1,1] & [0.17,0.2] \\
0.33,0.5 & [0.2,0.25] & [0.17,0.2] & [5,6] & [1,1]
\end{bmatrix}
\]

\[
\tilde{R}_p^2 = \begin{bmatrix}
0.13,0.17 & [1,1] & [4,7] & [1,2] & [7,9] \\
0.2,0.33 & [0.14,0.25] & [1,1] & [0.5,4] & [2,3] \\
0.17,0.2 & [0.5,1] & [0.25,0.2] & [1,1] & [0.11,0.2] \\
0.33,0.5 & [0.11,0.14] & [0.33,0.5] & [5,9] & [1,1]
\end{bmatrix}
\]

\[
\tilde{R}_p^3 = \begin{bmatrix}
0.25,0.33 & [0.14,0.5] & [1,1] & [3,1] & [1,2] \\
0.13,0.14 & [0.14,0.2] & [0.33,1] & [1,1] & [0.5,1] \\
0.2,0.25 & [0.2,0.33] & [0.5,1] & [1,2] & [1,1]
\end{bmatrix}
\]

Step 2. \(\overline{C_L} = 1.6675\) and \(G\overline{C_L} = 0.1\) were set according to Table 3 and the previous literature [34], respectively. The corresponding group IMPR \(\tilde{R}_g\), with acceptable consistency and consensus, was obtained through Model 4.

\[
\tilde{R}_g = \begin{bmatrix}
0.23,0.35 & [1,1] & [2.5,2.8] & [2.47,3.83] & [4.38,6.08] \\
0.18,0.28 & [0.19,0.5] & [1,1] & [1.15,3.63] & [2.15,3.09] \\
0.15,0.21 & [0.26,0.41] & [0.28,0.87] & [1,1] & [0.21,0.34] \\
0.28,0.40 & [0.16,0.23] & [0.32,0.46] & [2.92,4.76] & [1,1]
\end{bmatrix}
\]

Step 3. We solved the weight vector by using Model 3 to obtain the weight vector of each primary indicator.

\[
\bar{\omega}_1 = [2.44, 2.44], \quad \bar{\omega}_2 = [1.28, 1.38], \quad \bar{\omega}_3 = [0.59, 1.21], \quad \bar{\omega}_4 = [0.36, 0.52], \quad \bar{\omega}_5 = [0.69, 0.97]
\]

Step 4. We constructed the possibility degree matrix by using Equation (7).

\[
P = \begin{bmatrix}
0.5 & 1 & 1 & 1 & 1 \\
0 & 0.5 & 1 & 1 & 1 \\
0 & 0 & 0.5 & 1 & 0.54 \\
0 & 0 & 0 & 0.5 & 0 \\
0 & 0 & 0.46 & 1 & 0.5
\end{bmatrix}
\]

\(P_{1} = 4.5, \quad P_{2} = 3.5, \quad P_{3} = 2.04, \quad P_{4} = 0.5, \quad P_{5} = 1.96.\)
Step 5. We standardized the possibility degree of each primary indicator to derive the weight of each primary indicator.

\[
w_1 = \frac{p_1}{p_1 + p_2 + p_3 + p_4 + p_5} = 0.360, \quad w_2 = \frac{p_2}{p_1 + p_2 + p_3 + p_4 + p_5} = 0.280, \\
w_3 = \frac{p_3}{p_1 + p_2 + p_3 + p_4 + p_5} = 0.163, \quad w_4 = \frac{p_4}{p_1 + p_2 + p_3 + p_4 + p_5} = 0.040, \\
w_5 = \frac{p_5}{p_1 + p_2 + p_3 + p_4 + p_5} = 0.157.
\]

In this way, the weights of the primary indicators were determined to be 0.360, 0.280, 0.163, 0.040, and 0.157. In a similar manner to the processing method of primary indicators, the weights of each secondary indicator were derived separately, and the specific results are shown in Table 1.

3.2. Decision-Making Process

After obtaining results for the weights of the indicators, three experts were invited to evaluate the four NEVEs from the perspectives of different indicators, with the specific steps shown below.

Step 1. The three experts evaluated the competitiveness of the four NEVEs through the indicator $A_{11}$. The resulting IMPRs $e_{R_{11}}$, $e_{R_{21}}$, and $e_{R_{31}}$ are shown below.

\[
\begin{align*}
e_{R_{11}} &= \begin{bmatrix} [1, 1] & [0.3, 0.5] & [6, 8] & [3, 6] \\ [2.3, 3.3] & [1, 1] & [3, 5] & [2, 4] \\ [0.13, 0.17] & [0.2, 0.33] & [1, 1] & [4, 7] \\ [0.17, 0.33] & [0.25, 0.5] & [0.14, 0.25] & [1, 1] \end{bmatrix} \\
e_{R_{21}} &= \begin{bmatrix} [1, 1] & [0.25, 2] & [4, 6] & [1, 3] \\ [0.5, 4] & [1, 1] & [7, 9] & [5, 7] \\ [0.17, 0.25] & [0.11, 0.14] & [1, 1] & [0.14, 0.25] \\ [0.33, 1] & [0.14, 0.2] & [4, 7] & [1, 1] \end{bmatrix} \\
e_{R_{31}} &= \begin{bmatrix} [1, 1] & [1.2] & [5, 6] & [3, 5] \\ [0.5, 1] & [1, 1] & [7, 9] & [7, 8] \\ [0.17, 0.2] & [0.11, 0.14] & [1, 1] & [0.5, 1] \\ [0.2, 0.33] & [0.13, 0.14] & [1, 2] & [1, 1] \end{bmatrix}
\end{align*}
\]

Step 2. $\overline{CL} = 1.6500$ and $\overline{GCL} = 0.1$ were set according to Table 3 and the previous literature [34], respectively. Substituting IMPRs into Model 4 yielded group IMPR $\tilde{G}_{11}$ that satisfied acceptable consistency and consensus.

\[
\tilde{G}_{11} = \begin{bmatrix} [1, 1] & [0.42, 1.27] & [4.93, 6.60] & [2.08, 4.48] \\ [0.79, 2.37] & [1, 1] & [5.28, 7.40] & [4.12, 6.07] \\ [0.15, 0.20] & [0.14, 0.19] & [1, 1] & [0.66, 1.20] \\ [0.22, 0.48] & [0.17, 0.24] & [0.84, 1.52] & [1, 1] \end{bmatrix}
\]
Step 3. The three experts evaluated the competitiveness of the four NEVEs through the indicator A12. The resulting IMPRs $\hat{R}_{12}^1$, $\hat{R}_{12}^2$, and $\hat{R}_{12}^3$ are shown below.

$$
\hat{R}_{12}^1 = \begin{bmatrix}
[1,1] & [0.4,0.6] & [0.7,1.5] & [1,3] \\
[1.67,1.43] & [0.13,0.2] & [1,1] & [1,2] \\
[0.33,1] & [0.17,0.25] & [0.5,1] & [1,1]
\end{bmatrix}
$$

$$
\hat{R}_{12}^2 = \begin{bmatrix}
[1,1] & [0.33,0.5] & [0.5,2.5] & [2,5] \\
[0.4,2] & [0.2,0.33] & [1,1] & [0.33,1] \\
[0.2,0.5] & [0.25,0.5] & [1,3] & [1,1]
\end{bmatrix}
$$

$$
\hat{R}_{12}^3 = \begin{bmatrix}
[1,1] & [0.14,0.25] & [2,4] & [6,7] \\
[0.25,0.5] & [0.13,0.25] & [1,1] & [0.5,3] \\
[0.14,0.17] & [0.11,0.14] & [0.33,2] & [1,1]
\end{bmatrix}
$$

Step 4. The corresponding group IMPR $\tilde{G}_{12}$ satisfying acceptable consistency and consensus was obtained through Model 4.

$$
\tilde{G}_{12} = \begin{bmatrix}
[1,1] & [0.27,0.42] & [0.89,2.47] & [2.29,4.72] \\
[0.41,1.13] & [0.15,0.26] & [1,1] & [0.55,1.82] \\
[0.21,0.44] & [0.17,0.26] & [0.55,1.82] & [1,1]
\end{bmatrix}
$$

Step 5. The three experts evaluated the competitiveness of the four NEVEs through the indicator A13. The resulting IMPRs $\hat{R}_{13}^1$, $\hat{R}_{13}^2$, and $\hat{R}_{13}^3$ are shown below.

$$
\hat{R}_{13}^1 = \begin{bmatrix}
[1,1] & [0.4,0.6] & [7,9] & [1,3] \\
[0.11,0.14] & [0.13,0.2] & [1,1] & [6,7] \\
[0.33,1] & [0.17,0.25] & [0.14,0.17] & [1,1]
\end{bmatrix}
$$

$$
\hat{R}_{13}^2 = \begin{bmatrix}
[1,1] & [0.17,0.33] & [3,7] & [4,7] \\
[0.14,0.33] & [0.17,0.71] & [1,1] & [3,6] \\
[0.14,0.25] & [0.17,0.2] & [0.17,0.33] & [1,1]
\end{bmatrix}
$$

$$
\hat{R}_{13}^3 = \begin{bmatrix}
[1,1] & [0.17,0.2] & [4,7] & [8,9] \\
[0.14,0.25] & [0.14,0.2] & [1,1] & [0.17,1] \\
[0.11,0.13] & [0.11,0.14] & [1,6] & [1,1]
\end{bmatrix}
$$

Step 6. The corresponding group IMPR $\tilde{G}_{13}$ satisfying acceptable consistency and consensus was obtained through Model 4.

$$
\tilde{G}_{13} = \begin{bmatrix}
[1,1] & [0.22,0.34] & [4.38,7.30] & [3.18,5.74] \\
[0.14,0.23] & [0.14,0.33] & [1,1] & [1.44,3.48] \\
[0.17,0.32] & [0.15,0.19] & [0.29,0.69] & [1,1]
\end{bmatrix}
$$
Step 7. We aggregated $\tilde{G}_{11}$, $\tilde{G}_{12}$, and $\tilde{G}_{13}$ according to the weights of secondary indicators, and the result was

$$\tilde{G}_1 = \begin{bmatrix} [1, 1] & [0.28, 0.53] & [2.54, 4.80] & [2.51, 5.00] \\ [1.90, 3.53] & [1, 1] & [3.98, 7.03] & [4.37, 6.33] \\ [0.21, 0.39] & [0.14, 0.25] & [1, 1] & [0.82, 2.06] \\ [0.20, 0.40] & [0.16, 0.23] & [0.49, 1.22] & [1, 1] \end{bmatrix}$$

Step 8. The weight vector of $\tilde{G}_1$ was solved by using Model 3.

$$\mathbf{w}_1 = [1.40, 1.84], \mathbf{w}_2 = [2.70, 2.70], \mathbf{w}_3 = [0.38, 0.68], \mathbf{w}_4 = [0.37, 0.56]$$

Step 9. We constructed the possibility degree matrix by using Equation (7).

$$P = \begin{bmatrix} 0.5 & 0 & 1 & 1 \\ 1 & 0.5 & 1 & 1 \\ 0 & 0 & 0.5 & 0.6216 \\ 0 & 0 & 0.3784 & 0.5 \end{bmatrix}, \ p_1 = 2.5, \ p_2 = 3.5, \ p_3 = 1.1216, \ p_4 = 0.8784$$

Given $p_2 > p_1 > p_3 > p_4$, the sorting result was $x_2 \succ x_1 \succ x_3 \succ x_4$. In terms of the degree of specialization, $x_2$ performed the best, while $x_4$ performed poorly.

Similarly, following the above steps, the optimal results under the evaluation of various indicators could be obtained. For business performance, $x_1$ and $x_4$ performed better and $x_3$ performed worse; for innovation ability, the performance of the four enterprises was comparable; for operation management, $x_2$ and $x_4$ performed better and $x_1$ performed the worst; for product features, $x_3$ performed best and $x_1$ performed the worst.

### 3.3. Results and Findings

Through the evaluation results of various evaluation indicators, it can be seen that the degree of specialization has a greater impact on the competitiveness of NEVEs, while the impact of operation management is smaller.

In addition, according to the evaluation results of the four NEVEs, enterprise $x_1$ lacks effective operation management and unique product features. In addition, it is recommended that enterprise $x_1$ integrates its own style and philosophy into the product or service to form a unique product image. Compared with other NEVEs, $x_2$ performs well in terms of the degree of specialization, business performance, innovation ability, operation management, and product features. For enterprise $x_3$, the business performance is not good. $x_3$ should pay more attention to the innovation of its marketing strategy, optimize the product structure, and reduce unnecessary resource consumption. For enterprise $x_4$, the degree of specialization is relatively weak. $x_4$ should further focus resources on the most familiar business areas and create specialized products. Moreover, $x_4$ also need to enhance the degree of specialization of the staff team and strengthen the training of employees.

### 3.4. Discussion and Implications

With the increasingly serious pollution of the environment caused by tailpipe emissions from traditional fuel vehicles and the gradual shortage of oil reserves, NEVs, which use clean energy as a source of power for automobiles, have gradually become the mainstream of the development of the world automobile industry.

This paper has evaluated the competitiveness of NEVEs, which not only enriches the research on enterprise competitiveness but also, in general, supports the long-term development of NEVEs. From the social point of view, this study can clarify the key factors that promote the improvement of enterprises’ competitiveness, so that enterprises can objectively understand their own status quo and clearly recognize their advantages and disadvantages in regard to the market competition. In terms of business implications, for the case study firms, these enterprises’ strategy makers can now focus their attention on...
the indicators with poor results according to the evaluation results, which is conducive to
guiding the reallocation of resources and improving areas of weakness. In addition, the
research in this paper provides a certain reference for other NEVEs, which should support
them to improve their levels of competitiveness.

4. Conclusions

4.1. Research Contribution

In order to better understand the development status of NEVEs in Henan Province and
promote the competitiveness of NEVEs, in this study, we have constructed an evaluation
index system and utilized a group decision-making method based on IMPRs to evaluate
the competitiveness of NEVEs. The main factors affecting the competitiveness of different
NEVEs have been identified, and corresponding suggestions have been put forward. The
research in this paper offers up the following key results:

(1) Based on the development characteristics of NEVEs, in this study, we attempted
to construct a competitiveness evaluation index based on five aspects: degree of
specialization, business performance, innovation ability, operation management, and
product features. Since the qualitative indicators contain fuzzy information, it is
difficult to quantify them accurately. Therefore, we mainly evaluated competitiveness
based on the group decision-making method of IMPRs, which allows experts to give
evaluation information in the form of interval values according to the actual situation
and preference.

(2) A new consistency definition and consistency level measurement method based
on IMPRs have been proposed. A consistency improvement model, weight vector
solution model, and group decision-making model were also constructed. Numerical
examples have shown that the method in this paper can retain as much of the original
information of decision makers as possible. In addition, the group decision-making
model in this paper uses the concept of acceptability and considers both individual
consistency and group consensus, which means the evaluation results have some
scientific basis.

(3) From our determination of the weights of the indicators, it is concluded that the degree
of specialization has the greatest influence on the competitiveness of enterprises.
Meanwhile, four NEVEs in Henan Province were evaluated. Combined with the
enterprises’ own characteristics, this paper puts forward suggestions for other relevant
factors to consider: the degree of specialization, business performance, innovation
ability, operation management, and product characteristics. The results provide a
reference for the development of other NEVEs.

4.2. Research Limitations and Future Studies

Here, we have studied the competitiveness of NEVEs based on IMPRs, which provides
a useful reference for the development of NEVEs, but our work has some shortcomings, as
based on those, we highlight areas that should be further studied and improved.

(1) The selection of indicators remains to be improved. Research on the competitiveness
of NEVEs involves a very wide range of fields, and the upstream and downstream
industrial chains are also very complicated. Therefore, there is still a lot of work worth
pursuing in the future. For example, researchers may analyze the impact of factors
such as the government or related support industries on NEVEs, or they may explore
how corporate social responsibility will affect the competitiveness of enterprises.

(2) The evaluation method needs to be improved. The group decision-making method
based on IMPRs mainly relies on expert evaluation, which has certain subjectivity.
Furthermore, there are limitations in that sometimes the indicators cannot be evaluated
using the group decision-making model of IMPRs. In order to avoid subjectivity, in
the future, we will reduce the influence of individual experts’ opinions by increasing
the number of people involved in group decision-making.
(3) With the gradually increasing complexity of the decision-making environment, the size of the group involved in decision-making is getting larger and larger. Accordingly, how IMPRs can be utilized to solve complex large-scale group decision-making problems is an important direction for our future research.

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References
7. Salun, M.; Palyanychka, Y. Features and principles of monitoring of industrial enterprise competitiveness. Econ. Dev. 2018, 17, 74–82. [CrossRef]
11. Widyawati, F.; Soemaryani, I.; Muizzi, W.O.Z. The effect of social capital and organizational health on competitive advantages of culinary and craft SMEs in Samarinda city. Sustainability 2023, 15, 7945. [CrossRef]


30. Xia, M.M.; Chen, J. Studies on interval multiplicative preference relations and their application to group decision making. *Group DecisNegot.* 2015, 24, 115–144. [CrossRef]


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