Review

Application of Mass Service Theory to Economic Systems Optimization Problems—A Review

Farida F. Galimulina and Naira V. Barsegyan *

Logistics and Management Department, Kazan National Research Technological University, 420015 Kazan, Russia; galimulinaaff@corp.knrtu.ru
* Correspondence: n.v.barsegyan@yandex.ru or barsegyannv@corp.knrtu.ru

Abstract: An interdisciplinary approach to management allows for the integration of knowledge and tools of different fields of science into a unified methodology in order to improve the efficiency of resource management of different kinds of systems. In the conditions of global transformations, it is economic systems that have been significantly affected by external destabilizing factors. This determines the focus of attention on the need to develop tools for the modeling and optimization of economic systems, both in terms of organizational structure and in the context of resource management. The purpose of this review study is to identify the current gaps (shortcomings) in the scientific literature devoted to the issues of the modeling and optimization of economic systems using the tools of mass service theory. This article presents a critical analysis of approaches for the formulation of provisions on mass service systems in the context of resource management. On the one hand, modern works are characterized by the inclusion of an extensive number of random factors that determine the performance and efficiency of economic systems: the probability of delays and interruptions in mobile networks; the integration of order, inventory, and production management processes; the cost estimation of multi-server system operation; and randomness factors, customer activity, and resource constraints, among others. On the other hand, controversial points are identified. The analytical study carried out allows us to state that the prevailing majority of mass service models applied in relation to economic systems and resource supply optimization are devoted to Markov chain modeling. In terms of the chronology of the problems studied, there is a marked transition from modeling simple systems to complex mass service networks. In addition, we conclude that the complex architecture of modern economic systems opens up a wide research field for finding a methodology for assessing the dependence of the enterprise performance on the effect of optimization provided by using the provisions of mass service theory. This statement can be the basis for future research.

Keywords: modeling; productivity and efficiency of economic systems; mass service theory; resource supply; complex networks; optimization; organizational structure

MSC: 37N40; 60K20; 90B22

1. Introduction

Modeling is generally recognized as an effective tool for making managerial decisions and realizing the hidden potential of the management object. According to the theory of decision making, the basis for the justified choice of the best alternative for solving a problem situation is the use of mathematical tools; the situation itself is considered as the object of research. Managerial decisions are performed on the scale of economic, organizational, technical, and other types of systems. An interdisciplinary approach to management allows for the integration of the knowledge and tools of different fields of science into a unified methodology in order to improve the efficiency of resource management of different types of systems. In the context of global transformations, the focus is economic systems that have been significantly affected by external destabilizing factors. The importance of
optimizing flows in global supply chains has increased significantly. The Russian economy is at the stage of the radical revision of import routes, which is accompanied by changes in the load on border crossing points. As a consequence, queues are inevitable due to the large traffic flow. Similar problems are characteristic not only for macroeconomic systems, but also for meso- (region, industrial complex) and micro-level systems (business entity).

The abovementioned factors determine the focus of attention on the need to develop tools for the modeling and optimization of economic systems, both in terms of organizational structure and in the context of resource management.

Modern economic systems are a complex structure, a set of interrelated elements united by flows of resources and information, processes, and operations. The key problem of the functioning of economic systems is limited resources, which determines the emergence of «bottlenecks» and queues. As a consequence, it affects the efficiency of the whole system and its competitiveness.

There is a wide variety of mathematical methods used in economics to forecast processes, operations, and indicators, such as statistical models and dynamic models (linear and nonlinear) [1]. The methods that provide the search for optimal solutions include the following: linear or nonlinear programming problems, combinatorial problems, production functions, mass service systems, and others. However, there are disadvantages that are compensated by the theory of mass service. The main difference of the latter is the ability to effectively manage the flow (of information, customers, orders, etc.), while other methods focus on optimizing the functioning of individual links. An alternative modeling method can be the problem of optimizing the distribution of requests for servicing the flow of container cargoes by time slots (a time slot is a time window for receiving and servicing requests). However, this method ignores the definition of probabilities (denial of service, system downtime, queue formation), which allows us to judge the leveling by the theory of mass service on the drawbacks of other methods of optimization.

Summarizing the above, we emphasize that the study of the provisions of the mass service theory and the study of practicing its application in modeling economic systems of different levels (macro-, meso- and microsystems) is a topical issue. However, under the conditions of the complexity increasing in economic systems structures, the intensive development of technologies, as well as the transition to Industry 5.0, the interest in flexible complex automated solutions is growing. Such solutions can be built on the basis of combined (covering several constraints, including resource constraints) and network (combined systems of several processes) mass service systems. It is necessary to study scientific approaches of the application of provisions adapted for mass service theory in the management of complex organized systems and their optimization. Therefore, the purpose of this review study is to identify the current gaps (shortcomings) in the scientific literature devoted to the issues of the modeling and optimization of economic systems using the tools of mass service theory.

2. Materials and Methods

The main method of this research was the content analysis of scientific papers devoted to the problem under study. The international bibliographic platforms Scopus, Web of Science, Science Direct, Springer journals, as well as the official website of MDPI publishing house served as the knowledge base.

The research algorithm covers three key steps:

1. Searching for and studying scientific papers;
2. The systematization of approaches to the application of mass service theory in the context of the optimization of economic systems through the prism:
   - Resourcing of processes and operations;
   - Of the organizational structure of management;
3. The formulation of identified gaps, whose filling is planned to be realized by the authors in future research.
A more detailed outline of the research work is reflected in the Figure 1.

3. An Analytical Review of the Current State of Scientific Research

Mass service systems are present in various fields of activity and can project production systems, computing systems, order and sales management systems, purchasing, warehouses, transport, organizational structures, border points, etc. The key categories of mass service theory are requests for service and service channels, while the parameters of evaluation are the intensity of the flow of lost or served requests, the probability of one or another system state (denial of service, queuing, system idle time), and the duration of service.
The fundamentals of mass service theory were outlined by Erlang in his work «Sandsynlighedsregning og Telefonsamtaler», and the probability of hitting \( x \) events in a time interval \( a \) (probability \( S_x \)) is usually defined as follows [2]:

\[
S_x = \frac{(na)^x}{x!} e^{-na}
\]

(1)

where \( n \) is the flow intensity (average number of calls during the device operation time); \( a \) is the specified time interval; and \( x \) is the number of events.

Subsequent adaptation of the theory of mass service to various spheres of activity represents modifications aimed at optimizing the distribution of resources (personnel, stocks, production capacity, transport, etc.), etc.

3.1. Resource Management Based on a Mass Maintenance System

The functioning of the economic system relies on the use of a wide range of resources, the combination of which is determined by the level of management and industry specifics. In our study, we focus on types of resources such as:

- Personnel;
- Material;
- Technical (devices, servers, appliances);
- Capital (buildings, structures, machinery, equipment).

The mass service system is considered at the microeconomic level and refers to manufacturing, warehousing, transport, service, or other types of economic systems. In each of these areas, queues occur. It is necessary to distinguish three types of models: (1) the maintenance of excess capacities provides immediate customer service (no queue is formed, supply exceeds demand); (2) the capacities (resources) for customer service are loaded almost completely and used more efficiently, but a queue is formed due to available prices (demand is higher than supply); (3) the capacities (resources) are idle, and a queue is formed (high fluctuations of demand during the day, week, seasons). On this basis, the mass service system is applicable to the models of the second and third types.

From the point of view of the management object, all known approaches are proposed to be classified into four categories: management of technical resources, resource pool, capital, and material resources. A further review of the scientific literature within this section is differentiated by the four blocks.

3.1.1. Optimization of Technical Resources

An extensive body of scientific research focuses directly on technical devices, devices, servers, and information-computing systems. These are the key resources in mass service models describing the behavior of the economic system in the conditions of improving business processes and queuing. Mobile networks, order management systems (in warehousing, production, catering), the resource capacity of nodes in service channels, banking systems, the operation of post offices, and other aspects are the subject area of the research. The listed systems are mostly discrete in nature, i.e., the system parameters change only in case of customer arrival (incoming request). It is reasonable to classify all resource capacity management models into two types: single-server and multi-server.

Single-server mass service models (model notation—M/M/1) are simple with a single service unit, a Poisson flow of incoming requests, and an exponential distribution of service times [3–6]. The advantage of building the simplest models is the ability to conduct preliminary studies before proceeding to modeling complex multi-server systems.

However, modern economic systems focused on high productivity, high level of service quality, and profit maximization are not limited to a single service channel; on the contrary, they are complex multi-channel systems. In this regard, multi-server service models are more popular. Next, let us focus on the coverage of this type of system in the literature.
The theory of mass service finds application in the field of mobile communication system optimization, which is caused by mass flows of requests for information transfer requiring organization. In this case, the input flow is described by a Markov process. Modeling the user service of mobile networks has been the subject of research [7–9]. Thus, the system model of Makeeva et al. is characterized by taking into account the probability of delays and interruptions in the operation of the 5G network as well as the retransmission of data. The system of repeated calls in the categories of mass service theory, according to the authors’ idea, is described by a Markov chain with a continuous process [7]:

\[
X(t) = (N_m(t), N_u(t), Q_1(t), Q_2(t)), t \geq 0, \chi = \{(n_m, n_u, q_1, q_2) \mid n_m \geq 0, n_u \geq 0, 0 \leq q_1 \leq C_1, 0 \leq q_2 \leq C_2, b n_m + n_u \leq C, n_m + q_2 \leq N\}, 
\]

where \( \chi \) is the state space; \( N_m(t) \) is the number of active extended mobile broadband sessions; \( N_u(t) \) is the number of active URLLC (ultra-reliable low-latency communication) sessions; \( Q_1(t) \) is the number of pending extended mobile broadband sessions; \( Q_2(t) \) is the number of interrupted sessions; \( C_i \) is the capacity of buffer 1 (for storing pending sessions) and buffer 2 (for storing interrupted sessions); and \( N \) is the number of resource blocks.

Multi-server models find applications in order and production inventory management. In the work of Shajin et al., the optimization of the number of required servers is based on the total cost minimization function [10]. The advantage of the author’s solution, in our opinion, is the consideration of the causal relationship between orders, inventories, and the production process. Researchers have determined that when the probability of fulfilling a customer order is 50%, the optimal number of servers stabilizes, and the cost of the order management system decreases. In this case, the system is described by a continuous Markov chain [10]:

\[
\Omega = \{(N(t), I(t), C(t), J(t), t \geq 0)\}, 
\]

where \( N(t) \) is the number of customers in the system; \( I(t) \) is the number of goods in stock; \( J(t) \) is the production phase (production on/off); and \( C(t) \) is a component taking the value «0» when production is «off» and the value «1» when production is «on».

A number of multi-server customer service systems are based on setting aside time for maintenance and upgrades to ensure uninterrupted operation of the system («vacation»—a period of time during which the server is unavailable for maintenance) [11–14]. Jeganathan et al. consider a mass service system with server holidays with respect to warehouse order management [13]. According to the study, the server independently determines the necessity of going on holiday (when insufficient number of customers is detected) independently of other servers. The practical value of the authors’ research results is due to the cost estimation of the functioning of a multi-server system with an adjustable number of servers. Also, the team of scientists led by Jeganathan developed a model of mass service based on a two-level service system: junior and senior servers. Such a model takes place when the junior servers do not have the data to solve the problem and they turn to the senior server [15].

The problems of optimal server resource allocation are widespread. In the work of R. Yang et al., the authors study three cases with a characteristic optimal level of resources: in the first case, the studied service centers are independent of each other; in the second case, there is a distribution of resources between service centers (when launching a new task); and in the third case, resources are redistributed during the execution of operations. Scientists have revealed the following regularity: the optimal number of server resources has the highest value for the first case and the lowest for the third model [16]. In the example of heterogeneous resource-based mass service systems, Pankratova et al. offer a method for determining the optimal level of resource provision for the k-th channel of the system [17]:

\[
V_{opt}^k = a_k + r \sqrt{K_{sk}}, 
\]

where \( a \) is the mathematical expectation; \( r \) is the radius of the hyperellipsoid due to the probability of losing customers; and \( K_k \) is the state of the Markov chain.
At the same time, the authors propose a solution for mass service systems with an unlimited number of servers. However, the question remains open as to how cybersecure and capital- and energy-intensive such a solution is for large industrial enterprises. After all, an industrial system is a set of interconnected elements and processes united by commodities, energy, information, and financial and service flows that contribute to the production of industrial products.

Modern catering businesses are also mass service systems that integrate combinations of online and offline ordering. Zhan et al. determined the optimal throughput of an industry enterprise for different service channels [18].

Methods of mass service theory are widely used in the banking sector, as the bank system is built on the management of incoming requests, i.e., customer service through N channels (bank specialists). With the application of mass service models, the problems of managing the requests of multiple classes of customers [19], stability of the mass service network [20], and reengineering of business processes in order to eliminate «bottlenecks» [21] are solved. In the latter case, Hao and Yifei, relying on the classical provisions of the theory of mass service, offer a model for optimizing the number of servers based on simulation. The key criteria of optimization are the growth of the customer satisfaction level and reduction of waiting time. It is noteworthy that the authors based their study on the assumption that it is economically inexpedient to open a large number of servers.

Another area of service subject to the provisions of mass service theory is the operation of post offices. The processes in such systems are described by their stochastic nature, infinite queues, and the need to optimize the number of service counters. An example of optimizing the operation of post offices is the simulation modeling of a mass service system [22]. The ambiguity of the author’s approach is that the result is based on finding the optimal number of service counters «manually». The imperfection of this method lies in the risk of missing the true optimal value in a set of search iterations.

The principles of mass service theory are also realized in the sphere of medical services. The research and modeling of this category of systems are covered in the works of [23,24].

Summarizing the highlighted points, we summarize the numerical superiority of manuscripts devoted to multi-server mass service models. The complex architecture of such systems opens a wide research field but at the same time requires taking into account many different factors that determine a high level of entropy in service systems.

3.1.2. The Object of the Mass Service Model Is a Pool of Resources

This group of scientific works includes studies of universal character, which do not focus on a specific type of resource, but are not limited to the sphere of application of mass service models. A special place in the literature devoted to the practical application of mass service theory is occupied by the works of Naumov and Samouylov. In the work of [25], the scientists, in the context of resource management, discuss the occurrence of negative resources; these occur in the case of excess demand over supply, but the total amount of resources should not be negative and should correspond to the established threshold level. In another study [26], the authors propose a method for analyzing mass service systems that provides accurate convolution power calculations and is based on the detection of the pattern of approximation error from the level of resource load. As a consequence, the authors conclude that the accuracy of calculations increases with increasing resource load. Summarizing the ideas of Naumov and Samouylov, it should be emphasized that the accuracy of calculations increases with increasing resource load. For Samouylov, it should be emphasized that the factors of randomness, client activity, and resource constraints are taken into account.

Kim and Yeun, in terms of a sharing economy, justified the mass service model of the G/M/1 type on the example of a technological (online) platform. The key task of this model is to find a balance of interests of the key participants of the model—resource owners and consumers. The authors take into account the term of the contract and distinguish two types of contracts: individual (one-time) and permanent. The peculiarity of the mass service
system examined by the authors is as follows: the first subsystem is aimed at checking suppliers and estimating the time between their arrival in a single-channel queue; the second subsystem—customer (consumer) service—is built on Markovian service time [27]. The advantage of the model proposed by the authors is its universal character and its applicability to the management of any type of resource.

3.1.3. Focus of Mass Maintenance Models on Capital Resources

The optimization of capital resource management is less represented in the global scientific literature than the modeling of server resources. However, the published works are undoubtedly practically significant and cover an extensive sphere of economic systems. Such works include the ANFIS model for the optimization of a queuing system in warehouses [28], a model of railway stations [29,30], an analytical study of the capacity of Cairo International Airport [31], etc.

A separate niche in the section of research devoted to the study of fixed assets and their management is occupied by works focused on modeling logistics systems. A logistics system is a set of harmoniously interacting subsystems oriented towards the management of material and service flows with an optimal level of costs and maximum customer satisfaction. As a consequence, a logistics system is a set of mass service systems, the efficiency of which is conditioned by rational management of queues of transport units, passenger flow, and material flow.

Stojčić et al. propose a model for managing the time component in the context of logistics system optimization: the arrival time of vehicles and the service time of logistics operations (loading and unloading) [28]. The authors conclude that the time spent in the system determines the total vehicle dwell time in the warehouse, total logistics costs, and efficiency of the microeconomic system as a whole. Servers are associated with loading zones where queuing is possible, and the modeled warehouse system includes two such servers. The peculiarity of the solution proposed by the authors is the combination of the principles of mass service theory and artificial neural networks (adaptive neuro-fuzzy inference system, ANFIS). The input variables are: (1) time between vehicle arrivals; (2) total dwell time; and (3) service time. The output variable is the time spent in the system. The advantages of the model include: the ability to take into account the uncertainty factor in the management of logistics processes; and adaptability, which is achieved in the process of training neural networks.

In the example of railway stations, the theory of mass service is disclosed in the papers by Bychkov et al. [29] and Kazakov et al. [30]. The station operation system covers four subsystems (arrival, accumulation, loading, departure), each of which represents a separate mass service system [29]. A special merit of the authors lies in the development of a model of mass service networks serving as a basis for routing within the logistics system. The solution makes it possible to optimize the service channels (number of service crews, locomotives) and contribute to the reduction of service time. The mathematical apparatus is based on the BMAP (batch Markovian arrival process) model and allows for the control of the intensity of request processing, which is described as follows [29]:

\[
(D_0)_{v,v'} = -\lambda_v, \quad v = \overline{0,W}, \quad (D_0)_{v,v'} = \lambda_v p_0(v,v'), \quad v, v' = \overline{0,W}, \quad (D_k)_{v,v'} = \lambda_v p_k(v,v'), \quad v, v' = \overline{0,W}, \quad k \geq 1 (5)
\]

where \(v_t\) is a Markov chain with continuous time and state \(\overline{0,1,\ldots,W}\); \(\lambda_0\) is the query intensity; and \(p_k(v, v')\) is the probability that the chain transitions to state \(v'\).

The problem of modeling the airport service system has been reflected in a few scientific studies [31–36]. The classical mass service system for the example of the infrastructure of Cairo International Airport was studied by Abdulaziz Alnowibet et al. [31]. In the work of the scientists, it is emphasized that check-in counters are an extremely limited resource, the irrational management of which leads to an increase in the time of passenger flow service. The authors described the notations (Kendall) of mass service systems, differentiated for different links of passenger flow passage:
• “Check-in” subsystem—model M/M/s/GD/N/N/N;
• “Security Check” subsystem—M/G/s/FCFS/∞∞;
• “Immigration” subsystem—M/G/s/GD/N/∞;
• “Boarding” subsystem—M/G/s/GD/N/N/N.

This approach can provide the flexibility of an automated system for managing scarce airport resources but ignores the baggage service channels that are an integral part of the flight service process.

A related area of economic systems research is the modeling of taxi operation. The service device in such a mass service system is a taxi itself, and a taxi call is a request. The taxi service is a model with an unlimited queue, an example of a Markov process. In this case, the system is characterized by non-uniformity and randomness of flow (unlike an airport, where the schedule of aircraft departures and arrivals is known in advance). Accordingly, the main purpose of solving taxi service optimization problems is the modeling of dynamic queues [37,38]. Yang et al. see the solution of this problem as considering random factors such as the coincidence of passenger flow and free taxis, the pattern of passenger arrival (taxis on the airport territory), etc. [39].

In addition to logistic systems, the block of capital resource research includes production processes. A technological system as a set of machines and production equipment with the flow of products and the probability of equipment failure (in case of failure) can serve as a mass service system. Scientists have developed a number of solutions to improve the efficiency of the production system: the method of identifying «bottlenecks» in the technological process in the conditions of Industry 4.0 [40], a model for the accurate forecasting of queue length [41], a model of a repairable mass service system [42], etc.

Martyn et al. rightly point out that the productivity of a technological process is determined by the productivity of its weakest link, and they also determine the optima intensity of material feeding based on the following methodology [40]:

\[
p = \frac{\lambda}{\mu},
\]

\[
\psi(n) = \frac{p}{n},
\]

where \( p \)—loading intensity of the production line (channel); \( \lambda \)—average processing time; \( \mu \)—intensity of material processing in the channel; and \( \psi(n) \)—intensity of material supply for \( n \) channels.

Complex production systems are characterized by the type of production (continuous, direct flow) and intensity of material flow. From the point of view of mass service theory, material flow is a queue that is characterized by unstable size (length). Scientists see the solution to the problem of queue management as optimizing its length. So, May et al., in the example of semiconductor production using machine learning tools, offer the following heuristic approach [41]:

\[
\text{Queue}(j,t_1) = \text{Queue}(j,t_0) - \text{LeavingJobs} + \text{ArrivingJobs},
\]

where \( t_0 \) is the time before the material arrives in the system (on the machine); \( t_1 \) is the time after machining is completed on this machine; and \( \text{LeavingJobs}, \text{ArrivingJobs} \) are outgoing and incoming machining jobs.

However, the statements formulated by May et al. are only based on machine learning and require comparative analysis with alternative methods of mass service system prediction.

In general, approaches to capital resource management require consideration of a greater number of random factors, which is firstly due to the dynamism of the development of new technologies and secondly due to the high degree of uncertainty of external factors.


The theory of mass service considers the material flow as a flow of requirements (orders, stocks, material resources, finished products, etc.), and the efficiency of the management of queues of requirements determines the level (quality) of service of the system.
The stock level optimization problem is a classical problem in logistics systems management. From the perspective of a mass service system, the problem is to determine the inventory level (and the number of customers) such that the server will self-regulate going on and off holiday. This approach ensures that it is possible to pre-process information about the product that is booked during the holiday [5].

Various studies of the inventory management system are presented in the works of Melikov. These works cover the modeling of the system with the formation of stock shortages due to production delays [10]; the problems of stock shortage and the search for the optimal threshold level of stocks [43]; a warehouse system with catastrophes (in the form of two-dimensional Markov chains) and the optimization of the reorder point [44]; and systems with flows of primary customers, repeat sales customers, repeat service customers, and destructive customers (customers who do not purchase an item but destroy stock, after which stock is reduced by one) [45], among others. The models proposed by the author represent a Markov chain based on random factors and are aimed at minimizing expected total costs and improving the efficiency of the inventory management system. The random variables are order fulfilment time, inventory volume, and customer flow.

The time for processing orders, stocks, and delivery of the order to the distributor is also considered as a random variable in the works of Jeganathan et al. (using the example of asynchronous holidays in a multi-server system) [13], Baek and Moon (the independence of the level of production stocks and queue length is proven) [46], Atnowibet et al. (mass service system with impatient customers leaving the queue for various reasons) [47], Dissa and Ushakumari (in modeling the flows of perishable products with random lifetime) [48], and other works. Random demands on the stock of manufactured products are considered in the model of Chang and Lu; the researchers optimize the base stock level in a hybrid production system for standard and ad hoc needs [49].

Limitations—Earlier, we noted that a modern economic system is a rather complex structure that combines many subsystems and requires simultaneous consideration of a wide range of random factors. Scientists have presented serious attempts to develop hybrid models that integrate mass service models with machine learning or neuro-fuzzy models, which are able to jointly solve control and optimization problems. At the same time, digitalization of business processes requires capacious capacities for collecting, storing, and processing big data. And the economic system is a set of connected mass service models that regulate the functioning of the individual links of the system. To service such a volume of information flows requires significant energy resources and capacity, which entails significant costs. A number of authors address the issues of the total costs of the functioning of the subsystems under study [10,13,16,22,43]. However, the numerical measurement of the dependence of enterprise performance (including profitability) on the optimization effect (achieved through simulation) is poorly represented in the literature and represents a promising area of research.

3.2. Optimization of Organizational Structure on the Basis of Mass Service Systems

The practices in recent decades have shown that the Russian industry is facing the acute task of increasing competitiveness and efficiency. It is possible to achieve these tasks only on the basis of innovative transformations of both the production system itself and its structural elements.

The organizational structure of management is one of the tools for improving the management system, the units of which are subject to constant changes and adjustments—creation, reduction, division, and unification of links—as enterprises develop and grow. Consequently, there is a need for an evaluation to identify a more effective and optimal organizational structure, which requires the development of appropriate techniques or a set of them.

The design methods, their essence, and the approaches that are used in each method are summarized and shown in Table 1.
Table 1. Methods of designing organizational management structures (summarized by the authors).

<table>
<thead>
<tr>
<th>Methods of Designing Organizational Management Structures</th>
<th>Approaches/Models/Stages</th>
<th>Characterization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analogies Method [50–55]</td>
<td>Building an organizational management structure based on best practices</td>
<td>Use of best practices for enterprises with similar parameters; recommendations to enterprises based on typical organizational structures</td>
</tr>
<tr>
<td></td>
<td>Development of model organizational management structures</td>
<td></td>
</tr>
<tr>
<td>Expert-analytical method [50–55]</td>
<td>Quantitative and qualitative evaluation methods, questionnaires</td>
<td>Identification by experts and managers of the peculiarities and failures in the functioning of organizational structures and development of measures to improve the methods of their management</td>
</tr>
<tr>
<td>Structurization purposes method [54–58]</td>
<td>Developing a goal tree</td>
<td>Analyzing management organizational structures in terms of whether the functional mix is fit for the purpose</td>
</tr>
<tr>
<td></td>
<td>Expert analysis of structured objectives</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Development of key performance indicators to achieve objectives</td>
<td></td>
</tr>
<tr>
<td>Organizational modeling method [57–61]</td>
<td>Mathematical and cybernetic models</td>
<td>Designing organizational management structures based on the use of mathematical, graphical, and machine models for the purpose of the optimal distribution of functional powers</td>
</tr>
<tr>
<td></td>
<td>Graphoanalytical models</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Natural models</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mathematical and statistical models</td>
<td></td>
</tr>
</tbody>
</table>

The application of a set of these methods makes it possible to evaluate management organizational structures in order to determine their efficiency and identify weak links in the structure that need to be improved and changed. However, it is necessary to adapt these methods for the management organizational structures of petrochemical enterprises focused on saving resources.

An analysis of their advantages and disadvantages should be carried out to determine the methodology that will enable the design of the optimal organizational management structure (Figure 2).

The analysis allows us to conclude that for the design of flexible organizational management structures, it is not enough to use one method. It is necessary to use a set of methods, focusing on the positive characteristics of each of them. To verify the correctness of the selected methods of the design and development of organizational structures, it is also necessary to determine the methods of evaluation, conducting their comparative analyses (Figure 3).

One of the important indicators for assessing the effectiveness of the management hierarchy in the chain «supervisor—subordinates» is the norm of controllability.

In the theory of personnel management, the norms of manageability for management levels have been derived. Numerous empirical studies and the law of rational range of management derived by the American scientist Graicunas, presented in the research Calvo and Wellisz [62], were the basis for the development of management norms. This law states...
that the number of unit, direct, group, and cross relations in an organization is in geometric progression and obeys Formula (8):

\[ N = n \left[ 2^{n-1} + (n - 1) \right], \quad (8) \]

where \( N \) is the number of possible interrelationships of the unit’s employees, units; and \( n \) is the number of subordinates of the manager, people.

---

**Figure 2.** Advantages and disadvantages of methods of designing organizational management structures (summarized by the authors).

<table>
<thead>
<tr>
<th>advantages</th>
<th>disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>THE METHOD OF ANALOGIES:</td>
<td>THE METHOD OF ANALOGIES:</td>
</tr>
<tr>
<td>- unification and standardization of organizational solutions;</td>
<td>- unreliability of the description of management mechanisms and organizational structure of management</td>
</tr>
<tr>
<td>- multivariance</td>
<td></td>
</tr>
<tr>
<td>EXPERT-ANALYTICAL METHOD:</td>
<td>EXPERT-ANALYTICAL METHOD:</td>
</tr>
<tr>
<td>- use of various tools;</td>
<td>- subjectivity of expert assessments;</td>
</tr>
<tr>
<td>- flexibility</td>
<td>- additional expenses in connection with the involvement of consultants</td>
</tr>
<tr>
<td>THE METHOD OF STRUCTURING GOALS:</td>
<td>THE METHOD OF STRUCTURING GOALS:</td>
</tr>
<tr>
<td>- analysis taking into account the factors of the external and internal environment</td>
<td>- high risks due to changes in environmental factors</td>
</tr>
<tr>
<td>THE METHOD OF ORGANIZATIONAL MODELING</td>
<td>THE METHOD OF ORGANIZATIONAL MODELING</td>
</tr>
<tr>
<td>- use of various tools;</td>
<td>- the need to use the model of this method in conjunction with other methods for an objective assessment</td>
</tr>
<tr>
<td>- design of linear and horizontal connections</td>
<td></td>
</tr>
</tbody>
</table>

---

**Figure 3.** Outlying parameters and performance indicators of methods for assessing organizational management structures (summarized by the authors).

It is very difficult to represent this dependence graphically, but we provide the estimated number of possible relationships according to the Graicunas formula for 10 subordinate employees (the index at the bottom is the number of subordinates).
\[ N_1 = 1; N_2 = 6; N_3 = 18; N_4 = 44; N_5 = 100; N_6 = 182; N_7 = 504; N_8 = 1080; N_9 = 2306; N_{10} = 5210. \]

The calculation shows that with 5 subordinates, the number of links is 100, and with 10 subordinates, the number of links is 5210, i.e., when the number of subordinates is doubled, the number of links increases more than 50 times.

The design of optimal organizational structures characterizes the efficiency of management and the speed of processes within each enterprise. In this regard, it is very important to determine the speed of the processes running within each unit and evaluate their efficiency.

One method of analyzing organizational structure, as already presented in Figure 3, is to represent structures in the form of mass service networks. The use of a method based on the mass service theory makes it possible to perform an evaluation using indicators such as the average number of operations performed by an employee, the average time to perform an operation, the average number of operations waiting to be performed, and the average waiting time to perform an operation, which makes it possible to find problem areas in the management structure and in individual departments.

Often, the assessment is carried out by integrated indicators; for example, the ratio of management costs per one manager and the share of the number of management employees in the stock output and stock capacity is calculated. However, such calculations do not allow for characterization of the organizational structure itself and the processes that take place in it.

The use of a method based on the theory of mass service provides an opportunity to estimate the speed of information passage in the control system of petrochemical enterprises.

The representation of the organizational structure in the form of a mass service system model is justified by the fact that org structures are affected by the multitude of tasks that the system must solve given the requirements placed on the system (Figure 4).

![Figure 4. Organizational structure model in the form of a mass service system [50].](image)

In Figure 4, \( \lambda \) characterizes the average speed of arrival of tasks to be performed, adapting it to the management org structure of a petrochemical enterprise; it is an indicator that characterizes the requirements of petrochemical consumers. \( \mu \) is the average speed of operations per unit of time, i.e., the satisfaction of needs. Individual structures of the control system can be represented as a mass service network.

In the case of a lean strategy of petrochemical enterprises development and a development strategy aimed at the development of new petrochemical products, divisional and project types of organizational management structures are recommended. In this regard, let us present these structures in the form of mass service networks (Figure 5a,b).

In the presented schemes, the general director of the enterprise, line and functional managers, and the project manager are single-channel mass service systems, and the other units of the organizational structure are multi-channel systems. Each employee is a part of a multi-channel system. And it should be noted that the organizational structure of management is a mass service network without queue limitation, which means that the incoming request to the enterprise will be fulfilled sooner or later.
Figure 5. Modeling of the organizational structure of management in the form of a mass service network: (a) mass service network of the divisional organizational structure of management of a petrochemical enterprise; (b) mass service network of the project organizational structure of management of a petrochemical enterprise (proposed by the authors).

The main characteristics of a classical mass service system, like any mass service system, are the following numerical characteristics of it:

- Probability of waiting;
- Average number of claims under service (related to the service unit utilization rate);
- Average queue length;
- Average number of requirements in the system;
- Average length of the real queue.

For single-channel mass service systems, the calculation of the main indicators can be presented in the form of the following formulas, which are specified in the work of Prof. Kirpichnikov [63].

Waiting probability, i.e., the probability that an incoming demand will be in the queue (find the serving device busy):

\[ p_{\text{exp}} = \sum_{k=1}^{\infty} p_k = \sum_{k=0}^{\infty} p_k - p_0 = 1 - (1 - \rho) = \rho \]  \ 

(9)

where \( p_{\text{exp}} \)—the probability of waiting for the application to be completed; \( p_0 \)—the maximum probability that the system is free; and \( p_k \)—the maximum probability that the system is busy servicing existing applications.

Respectively, the probability of immediate servicing of the request received in the system:

\[ p_{\text{ser}} = 1 - p_{\text{exp}} = 1 - \rho = p_0 \]  \ 

(10)

where \( p_{\text{ser}} \)—the probability of maintenance.
The average number of demands simultaneously under service (in this case, i.e., for the single-channel model, which coincides with the load factor) is obvious:

$$l.f. = \bar{m} = 0 \cdot p_0 + 1 \cdot p_1 + 1 \cdot p_2 + \ldots = 1 - p_0 = \rho$$ (11)

where $l.f.$—load factor.

The variance of this value is:

$$\sigma_m^2 = 0^2 p_0 + 1^2 (p_1 + p_2 + \ldots) - \bar{m}^2 = 1 - p_0 - \rho^2 = \rho - \rho^2$$ (12)

Further, if the service flow is a Poisson flow, then the probability $B_k(t)$ that $k$ requests are served by the system at time $t$ is defined by Formula (1).

The average number of claims in the queue (actual average queue length) is:

$$T = 0 \cdot p_0 + 0 \cdot p_1 + 1 \cdot p_2 + 2 \cdot p_3 + 3 \cdot p_4 + \ldots = \sum_{k=1}^{\infty} (k-1)p_k$$ (13)

The average number of demands in the mass service system as a whole (both in queue and under service) is:

$$\bar{k} = \sum_{k=0}^{\infty} kp_k = \frac{\rho}{1 - \rho}$$ (14)

The waiting probability in a multi-channel mass service system, i.e., the probability that an incoming demand finds all channels occupied, is obviously given by the formula:

$$p_{\text{exp}} = \sum_{k=m}^{\infty} p_k = \frac{p_0}{m!} \sum_{k=m}^{\infty} \frac{\rho^k}{m^{k-m}} = \frac{\rho^m p_0}{m!} \left(1 + \frac{\rho}{m} + \frac{\rho^2}{m^2} + \ldots\right) = \frac{\rho^m p_0}{(m-1)! (m - \rho)}$$ (15)

This formula in the USA is called the Erlang C formula and is denoted as $C(m, \rho)$, while in Europe it has a different designation—$E_2, m(\rho)$, where it is called Erlang’s second formula.

To carry out the assessment of organizational management structures of industrial enterprises, which are complex systems, it is necessary to apply these methods comprehensively, taking into account different criteria and indicators. And it should be noted that the organizational structure of management is a mass service network without queue limitation, which means that the incoming request to the enterprise will be fulfilled sooner or later.

4. Discussion

The conducted analytical study allows us to state that the prevailing majority of mass service models applied in relation to economic systems and resource supply optimization are devoted to the modeling of Markov chains. In terms of the chronology of the problems studied, there is a noticeable transition from modeling simple systems to the creation of complex mass service networks.

At the same time, certain peculiarities of approaches to the formulation of provisions on mass service systems in the context of resource management have been found. On the one hand, modern works are characterized by the inclusion of a large number of random factors that determine the performance and efficiency of economic systems: probabilities of delays and interruptions of mobile networks [7]; the integration of order, inventory and production management processes [10]; cost estimation of multi-server system performance [13]; and randomness, customer activity, and resource constraints [25,26], among others. On the other hand, controversial points have been identified. For example, there is a contradiction in the approaches of some authors: E. Pankratova et al. propose a mass service model with an unlimited number of servers [17], and Hao and Yifei are based on the idea that it is economically inexpedient to open a large number of servers [21]; in optimization issues, scientists rely on the development of alternative hypotheses and models [22], which is not always able to reflect the single best solution; and in the context
of forecasting, a number of alternative prediction methods should be applied rather than limiting to comparing only two different models, as was conducted in May et al. [41].

As a consequence, the literature review revealed the need for a systematic consideration of a number of optimal criteria: profit, costs (energy, material, and labor costs), production capacity, and others. Only such a multi-criteria approach can overcome the difficulties of the effective application of the provisions of the theory of mass service in the optimization of economic systems. The solution of this problem can help to increase the efficiency of business process automation and form a highly efficient digital corporate platform.

In modern conditions of the transformation of the production structure of economic management, deployment of the Fourth Industrial Revolution, technological modernization of the industrial complex, and new approaches to the organization of resource-saving production systems are required. One of the tools for solving such an important problem is the design of organizational structures for managing resource-saving production systems in the conditions of digitalization of industry, as well as the development of approaches assessing their effectiveness. An effective method of designing optimal organizational structures, which characterizes the efficiency of management and the speed of processes within each enterprise, has been proposed to represent management structures in the form of mass service networks. This method makes it possible to perform evaluations using indicators such as the average number of operations performed by an employee, the average time to perform an operation, the average number of operations waiting to be performed, and the average waiting time to perform an operation, which makes it possible to find problem areas in the management structure and in individual departments.

Implementation of the policy of resource-saving production technologies finds practical application in all sectors of industry, which predetermines the importance and relevance of their use to achieve sustainable development and improve the competitiveness of industrial enterprises. At present, it is impossible to ignore the fact of the complex nature of this issue, which involves competitiveness management, the development of product and process innovations, networking models, and ensuring the sustainable development of industry. It is not possible to achieve a solution to these problems without a proper assessment of existing organizational structures and their optimization, if necessary.

5. Conclusions

Thus, the following gaps in the research of mass service systems have been revealed: the principles of system analysis (unity of elements, connectivity, hierarchy, etc.) are often ignored when modeling functional subsystems interconnected within the contour of an economic system (for example, an industrial enterprise); calculations of the optimal level of resource consumption should be based on the analysis of the maximum number of solutions (search iterations) within the modeled mass service system and should not be limited to alternative ones; causal relationships between the eco-economic effect of the practical implementation of the provisions of the theory of mass service (in the context of processes) and the level of competitiveness of organizations are insufficiently presented and substantiated; and studies devoted to the construction of an effective organizational structure on the basis of the mass service system are poorly presented.

Regarding digital platforms, which are flexible and scalable, it becomes possible to plan the production of new goods. The organizational structure of management is one of the tools to improve the management system, the units of which are subject to constant changes and adjustments—the creation, reduction, division, and unification of links—in the process of the development and growth of enterprises. Each type of organizational structure has its advantages and disadvantages and can be formed in accordance with a particular strategy of enterprise development. The identification of a more effective and optimal organizational structure requires the development of appropriate methods or a set of them. Each employee is a part of a multi-caliber system. In this regard, the systematized toolkit of industrial production efficiency management based on the methodology of mass service
system, the technology of designing the organizational structure of management, and the formed indicators for assessing its effectiveness allows us to improve the performance of the existing organizational structure of the enterprise as a whole. The organizational structure of management is a network of mass service without queue limitation, which means that the incoming request to the enterprise will be fulfilled sooner or later. The main characteristics of a classical mass service system are its numerical characteristics: the waiting probability; average number of requirements under service (related to the load of the servicing unit); average queue length; average number of requirements in the system; and average length of the real queue, which requires further testing as data are accumulated. For correct management decision making and evaluation of the efficiency of the management system, we propose to take into account, for example, the following criteria: the coefficient of manageability; the share of personnel involved in innovative management projects; the coefficient of the labor efficiency of the staff; the coefficient of the economic efficiency of managerial activity; the output of the target product; and the cost of production, among others.

In addition, we conclude that the complex architecture of modern economic systems opens a wide research field for finding a methodology for assessing the dependence of enterprise performance on the effect of optimization provided by using the provisions of the theory of mass service. The application of this theory to resource and organizational-structural components seems to be a promising direction of research. This statement can be used as the basis for future research aimed at formalizing the mathematical dependence of the effective indicators of the organization’s functioning (profit, profitability, etc.) on the optimization effect (reducing customer service costs, order management, process and operation management, organizational structure rationalization, etc.).

Author Contributions: Conceptualization, N.V. B.; methodology, F.F. G.; formal analysis, F.F. G. and N.V. B.; investigation, F.F. G. and N.V. B.; writing—original draft preparation, F.F. G. and N.V. B.; writing—review and editing, F.F. G. and N.V. B. All authors have read and agreed to the published version of the manuscript.

Funding: This research was carried out within the framework of the grant of the President of the Russian Federation for state support of leading scientific schools of the Russian Federation, project number NSh-1886.2022.2, agreement 075-15-2022-836.

Data Availability Statement: No new data were created or analyzed in this study. Data sharing is not applicable to this article.

Conflicts of Interest: The authors declare no conflicts of interest.

References
5. Krishnamoorthy, A.; Joshua, A.N.; Kozyrev, D. Analysis of a Batch Arrival, Batch Service Queueing-Inventory System with Processing of Inventory While on Vacation. Mathematics 2021, 9, 419. [CrossRef]


23. Zychlinski, N. Applications of fluid models in service operations management. *Queueing Syst. 2023*, 103, 161–185. [CrossRef]


36. Zhao, X.; Wang, Y.; Li, L.; Delahaye, D. A Queuing Network Model of a Multi-Airport System Based on Point-Wise Stationary Approximation. *Aerospace 2022*, 9, 390. [CrossRef]


45. Melikov, A.; Aliyeva, S.; Nair, S.S.; Kumar, B.K. Retrial Queuing-Inventory Systems with Delayed Feedback and Instantaneous Damaging of Items. *Axioms* 2022, 11, 241. [CrossRef]
48. Dissa, S.; Ushakumari, P.V. Two commodity Queuing inventory system with random common lifetime, two demand classes and pool of customers. *Heliyon* 2023, 9, e21478. [CrossRef] [PubMed]
58. Gritans, Y.M. Organizational design and restructuring (reengineering) of enterprises and holdings. In *Economic, Managerial and Legal Aspects*; Wolters Kluwer Russia: Moscow, Russia, 2006; Volume 205. (In Russian)

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.