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Starlike Functions of the Miller–Ross-Type Poisson Distribution in the Janowski Domain

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Abstract: In this paper, considering the various important applications of Miller–Ross functions in the fields of applied sciences, we introduced a new class of analytic functions f, utilizing the concept of Miller–Ross functions in the region of the Janowski domain. Furthermore, we obtained initial coefficients of Taylor series expansion of f, coefficient inequalities for f^{-1} and the Fekete–Szegö problem. We also covered some key geometric properties for functions f in this newly formed class, such as the necessary and sufficient condition, convex combination, sequential subordination and partial sum findings.

Keywords: analytic function; starlike function; subordination; Fekete–Szegö inequality; Miller–Ross distribution series

MSC: 30C80; 30C45

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1. Introduction

Let $\mathcal{A}_0$ represent the collections of analytic functions f inside open unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ with normalization $f(0) = 0 = f'(0) - 1$ and be of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \ z \in \mathbb{D}. \tag{1}$$

Furthermore, all functions that are univalent in $\mathbb{D}$ comprise the class denoted by $\mathcal{S}$, a subclass of $\mathcal{A}_0$. If for every point in set $B \subset \mathbb{C}$ the line segment joining origin to that point lies inside B, then that set is said to be starlike with respect to origin. Starlike functions are defined as functions $f \in \mathcal{A}_0$ that map $\mathbb{D}$ to a starlike domain, and $\mathcal{S}^*$ designates this class of functions. Analytically, a function $f \in \mathcal{A}_0$ is called a starlike function if

$$\mathrm{Re}\left(\frac{zf'(z)}{f(z)}\right) > 0, \ (z \in \mathbb{D}).$$

If the line segment connecting any two points in set B, z_1 and z_2, lies inside B, then the set $B \subset \mathbb{C}$ is convex. A set $B \subset \mathbb{C}$ is convex if the line segment joining any two points in set $z_1, z_2 \in B$, falls inside convex domain B. Analytically, a function $f \in \mathcal{A}_0$ is called a convex function if

$$\mathrm{Re}\left(\frac{(zf'(z))'}{f'(z)}\right) > 0, \ (z \in \mathbb{D}).$$

The coefficients of functions f in a particular subclass of $\mathcal{A}_0$ have been the subject of attention for numerous mathematicians since the early 1900s. De Branges solved the most important and inspirational problem, the Bieberbach hypothesis, seventy years after it was

first put forth in 1984. Over time, many interesting subclasses have emerged with respect to these coefficients. The Fekete–Szegö inequalities introduced in 1933 (see [1]) and functional $\left|a_3 - a_2^2\right|$ are also among the important findings for the coefficients of the functions f. The Fekete–Szegö problem is to maximize $\left|a_3 - \mu a_2^2\right|$ for a real as well as complex μ. Fekete and Szegö gave sharp estimates of $\left|a_3 - \mu a_2^2\right|$ for a real μ and $f \in \mathcal{S}$, the class of univalent functions [2,3].

Miller and Ross [4] proposed the special function as the basis of the solution of fractional order initial value problem, which is called the Miller–Ross function, defined as

$$\mathbf{E}_{\nu,\mu}(z) = z^{\nu} e^{\mu z} \mathrm{Y}^*(\nu, \mu z), \ \nu, \mu, z \in \mathbb{C}, \text{ with } \operatorname{Re}\nu > 0, \ \operatorname{Re}\mu > 0,$$

where Y^* is the incomplete gamma function (p. 314, [4]). Using the properties of the incomplete gamma functions, the Miller–Ross function can easily be written as

$$\mathbf{E}_{\nu,\mu}(z) := z^{\nu} \sum_{n=0}^{\infty} \frac{(\mu z)^n}{\Gamma(n+\nu+1)}, \ \nu, \mu, z \in \mathbb{C}, \text{ with } \operatorname{Re}\nu > 0, \ \operatorname{Re}\mu > 0, \tag{2}$$

which can be stated as

$$\mathbf{E}_{\nu,\mu}(z) \equiv z^{\nu} \mathbf{E}_{1,1+\nu}(\mu z)$$

where in the right hand member, $\mathbf{E}_{1,1+\nu}(\mu z)$ is the Mittag–Leffler function of two parameters [5]. Some of special values of the Miller–Ross functions can be given as follows:

$$\begin{aligned} \mathbf{E}_{\nu,\mu}(0) &= 0, \quad \operatorname{Re}(\nu) > 0, \\ \mathbf{E}_{0,\mu}(z) &= e^{\mu z}, \\ \mathbf{E}_{0,1}(z) &= e^{z}. \end{aligned}$$

Recently, Eker and Ece [6] showed that for $\mu > 0$ and if $\nu > 2\mu - 1$, the normalized Miller–Ross function $\mathbf{E}_{\nu,\mu}$ is univalent and starlike in $\mathbb{D}_{\frac{1}{2}} = \{z \in \mathbb{C} : |z| < \frac{1}{2}\}$. They also proved that if $\nu > (2+\sqrt{2})\mu - 1$, then the normalized Miller–Ross function $\mathbf{E}_{\nu,\mu}$ is univalent and convex in $\mathbb{D}_{\frac{1}{2}}$. For more details, refer to Miller and Ross [4].

Simple distributions, including the Pascal, Poisson, logarithmic, binomial and beta-negative binomial, have been substantially examined from a theoretical perspective in geometric function theory; for detailed study, refer to [7–12]. The probability mass function of the Miller–Ross-type Poisson distribution is given by

$$\mathcal{P}_{\nu,\mu}(m,k) := \frac{(m\mu)^k \, m^{\nu}}{\mathbf{E}_{\nu,\mu}(m)\Gamma(\nu+k+1)}, \quad k = 0, 1, 2, \ldots, \tag{3}$$

where $\nu > -1, \mu > 0$ and $\mathbf{E}_{\nu,\mu}$ is the Miller–Ross function given in (2). The normalized form of the Miller–Ross-type Poisson distribution is given by

$$\mathcal{M}_{\nu,\mu}^{m}(z) = z + \sum_{n=2}^{\infty} \frac{(m\mu)^{n-1} \, m^{\nu}}{\mathbf{E}_{\nu,\mu}(m)\Gamma(\nu+n)} z^n. \tag{4}$$

In the study of geometric function theory, operators are a crucial subject. The convolution of specific analytic functions can be used to express a wide variety of differential and integral operators. It has been noted that this formalism facilitates more mathematical investigation and aids in a better understanding of the symmetric and geometric characteristics of such operators. The work of [13,14] makes it easy to see the significance of convolution in the theory of operators. We consider the linear operator

$$\mathcal{Q}_{\nu,\mu}^{m} : \mathcal{A} \to \mathcal{A}$$

as below:

$$\begin{aligned} \mathcal{Q}^{m}_{\nu,\mu} f(z) &= f(z) * \mathcal{M}^{m}_{\nu,\mu}(z) \\ &= z + \sum_{n=2}^{\infty} \Upsilon_n a_n z^n, \end{aligned} \tag{5}$$

where

$$\Upsilon_n = \frac{(m\mu)^{n-1}\, m^{\nu}}{\mathbf{E}_{\nu,\mu}(m)\Gamma(\nu+n)} \tag{6}$$

and the symbol $*$ specifies the Hadamard product (convolution) of two series.

Moreover, for two analytic functions f and g in $\mathbb{D}$, we say that the function f is subordinate to the function g and write it as

$$f \prec g \quad \text{or} \quad f(z) \prec g(z),$$

if there exists a Schwarz function w which is analytic in $\mathbb{D}$ with

$$w(0) = 0 \qquad \text{and} \qquad |w(z)| < 1,$$

such that

$$f(z) = g(w(z)).$$

Furthermore, if the function g is univalent in $\mathbb{D}$, then it follows that:

$$f(z) \prec g(z) \quad (z \in \mathbb{D}) \Rightarrow f(0) = g(0) \quad \text{and} \quad f(\mathbb{D}) \subset g(\mathbb{D}).$$

Janowski [15] introduced the subclass of starlike functions as follows:

$$\mathcal{S}^*(A,B) = \left\{ f(z) \in \mathcal{A} : \frac{zf'(z)}{f(z)} \prec \frac{1+Az}{1+Bz} \ (-1 \le B < A \le 1; z \in \mathbb{D}) \right\}. \tag{7}$$

Note that $\dfrac{1+Az}{1+Nz}$ maps $\mathbb{D}$ conformally onto a symmetrical disc with respect to the real axis, which is centered at $\frac{1-AB}{1-B^2}$ $(B \neq \pm 1)$, and with a radius $\frac{A-B}{1-B^2}$ $(B \neq \pm 1)$.

Inspired by all of the aforementioned discussions and recent work by Khan et al. [16], wherein they presented a class of analytic functions with Mittag–Leffler-type Poisson distribution in the Janowski domain; analytic functions with Mittag–Leffler-type Borel distribution [17]; and the work presented in the articles [18–20], we now present a new class of analytic functions using operator (5) as follows:

$$\mathcal{Q}^{m}_{\nu,\mu}(A,B) = \left\{ f \in \mathcal{A} : \frac{z(\mathcal{Q}^{m}_{\nu,\mu} f(z))'}{\mathcal{Q}^{m}_{\nu,\mu} f(z)} \prec \frac{1+Az}{1+Bz} \right\} \quad (z \in \mathbb{D}), \tag{8}$$

where $-1 \le B < A \le 1$. By fixing the values of A and B, one can state new classes $\mathcal{Q}^{m}_{\nu,\mu}(1-2\alpha,-1) = \mathcal{Q}^{m}_{\nu,\mu}(\alpha)$, analogues to the classes studied in [21] and $\mathcal{Q}^{m}_{\nu,\mu}(1,-1) = \mathcal{Q}^{m}_{\nu,\mu}(\varphi)$ where $\varphi(z) = \frac{1+z}{1-z}$.

In the following sections, for this newly defined function class, we determine the well-known results, like the Fekete–Szegö inequalities, necessary and sufficient conditions, growth and distortion bounds, convex combination, sequential subordination and partial-sum-type results.

2. Initial Coefficient Bounds and Fekete–Szegö Problem for $f \in \mathcal{Q}^{m}_{\nu,\wp}(A,B)$

To find initial estimates and the Fekete–Szegö problem, the following lemma is required.

Lemma 1 ([22,23]). *Let*

$$h(z) = 1 + p_1 z + p_2 z^2 + \dots$$

be in the class $\mathcal{P}$ of functions of the positive real part in $\mathbb{D}$, then

$$|p_n| \leq 2, \ n \geq 1, \tag{9}$$

and for any complex number v

$$\left| p_2 - v p_1^2 \right| \leq 2 \max\{1, |1 - 2v|\}. \tag{10}$$

In particular, if v is a real parameter, then

$$|p_2 - v p_1^2| \leq \begin{cases} -4v + 2, & \text{if} \quad v \leq 0, \\ 2, & \text{if} \quad 0 \leq v \leq 1, \\ 4v - 2, & \text{if} \quad v \geq 1. \end{cases} \tag{11}$$

When $v < 0$ or $v > 1$, the equality holds true in (1) if and only if

$$h(z) = \frac{1+z}{1-z}$$

or one of its rotations. If $0 < v < 1$, then the equality holds true in (1) if and only if

$$h(z) = \frac{1+z^2}{1-z^2}$$

or one of its rotations. If $v = 0$, the equality holds true in (1) if and only if

$$h(z) = \left(\frac{1+\rho}{2}\right)\frac{1+z}{1-z} + \left(\frac{1-\rho}{2}\right)\frac{1-z}{1+z} \qquad (0 \leq \rho \leq 1)$$

or one of its rotations. If $v = 1$, then the equality in (1) holds true if $h(z)$ is a reciprocal of one of the functions such that the equality holds true in the case when $v = 0$.

Theorem 1. *Let $f \in \mathcal{A}_0$ be assigned to the class $\mathcal{Q}^m_{\nu,\mu}(A, B)$. Then,*

$$|a_2| \leq \frac{A - B}{\mathrm{Y}_2}, \tag{12}$$

$$|a_3| \leq \frac{A - B}{2\mathrm{Y}_3} \max\{1, |A - 2B|\}. \tag{13}$$

Furthermore, for a complex number $\aleph$,

$$\left| a_3 - \aleph a_2^2 \right| \leq \frac{A - B}{2\mathrm{Y}_3} \max\{1, |\Theta(\aleph, A, B) - 1|\}, \tag{14}$$

where

$$\Theta(\aleph, A, B) = (1 + 2B - A) + \frac{2\aleph(A - B)\mathrm{Y}_3}{\mathrm{Y}_2^2}.$$

$$\mathrm{Y}_2 = \frac{(m\mu)\ m^\nu}{\mathbf{E}_{\nu,\mu}(m)\Gamma(\nu + 2)} \quad \text{and} \quad \mathrm{Y}_2 = \frac{(m\mu)^2\ m^\nu}{\mathbf{E}_{\nu,\mu}(m)\Gamma(\nu + 3)}. \tag{15}$$

Proof. We begin by showing that the inequalities (12)–(14) hold true for $f \in \mathcal{Q}^m_{\nu,\mu}(A, B)$. Since $f \in \mathcal{Q}^m_{\nu,\mu}(A, B)$, we have the following subordination:

$$\frac{z(\mathcal{Q}^m_{\nu,\mu} f(z))'}{\mathcal{Q}^m_{\nu,\mu} f(z)} \prec \frac{1 + Az}{1 + Bz}. \tag{16}$$

The above subordination can also be written as:

$$\frac{z(\mathcal{Q}^m_{\nu,\mu}f(z))'}{\mathcal{Q}^m_{\nu,\mu}f(z)} = \frac{1+Aw(z)}{1+Bw(z)} = G(w(z)), \quad (-1 \le B < A \le 1).$$

Now, $w(z)$ can be written as follows:

$$w(z) = \frac{1-h(z)}{1+h(z)} = \frac{p_1 z + p_2 z^2 + p_3 z^3 + \cdots}{2 + p_1 z + p_2 z^2 + p_3 z^3 + \cdots}.$$

Now,

$$G(w(z)) = 1 + \frac{1}{2}(A-B)p_1 z + \frac{1}{4}\Big(2(A-B)p_2 - (A-B)(1+B)p_1^2\Big)z^2 + \cdots. \tag{17}$$

And

$$\frac{z(\mathcal{Q}^m_{\nu,\mu}f(z))'}{\mathcal{Q}^m_{\nu,\mu}f(z)} = 1 + \Upsilon_2 a_2 z + \Big(2\Upsilon_3 a_3 - \Upsilon_2^2 a_2^2\Big)z^2 + \cdots. \tag{18}$$

After comparing (17) and (18), we obtain

$$a_2 = \frac{A-B}{2\Upsilon_2}p_1, \tag{19}$$

$$a_3 = \frac{A-B}{4\Upsilon_3}\left(p_2 - \frac{p_1^2}{2}(1+2B-A)\right). \tag{20}$$

Applying (9) to (19) and (10) to (20), we obtain

$$|a_2| \le \frac{A-B}{\Upsilon_2}, \tag{21}$$

$$|a_3| \le \frac{A-B}{2\Upsilon_3}\max\{1, |A-2B|\}. \tag{22}$$

Furthermore, from (19) and (20), we obtain

$$\left|a_3 - \aleph a_2^2\right| = \frac{A-B}{4\Upsilon_3}\left|p_2 - \frac{p_1^2}{2}\Theta(\aleph, A, B)\right|, \tag{23}$$

where

$$\Theta(\aleph, A, B) = (1+2B-A) + \frac{2\aleph(A-B)\Upsilon_3}{\Upsilon_2^2}. \tag{24}$$

Applying (10) to the above (23), we obtain the required results. □

Furthermore, for the real $\aleph$, when applying (11) to the above (23), we obtain the following results.

Theorem 2. *Let $f \in \mathcal{A}_0$ be assigned to the class $\mathcal{Q}^m_{\nu,\mu}(A,B)$. Then, for a real parameter $\aleph$,*

$$\left|a_3 - \aleph a_2^2\right| \le \begin{cases} 2\left[\frac{(A-2B)\Upsilon_2^2 - 2\aleph(A-B)\Upsilon_3}{\Upsilon_2^2}\right] & \left(\aleph < -\frac{(1+2B-A)\Upsilon_2^2}{2(A-B)\Upsilon_3}\right), \\ 2 & \left(-\frac{(1+2B-A)\Upsilon_2^2}{2(A-B)\Upsilon_3} \le \aleph \le \frac{(1-2B+A)\Upsilon_2^2}{2(A-B)\Upsilon_3}\right), \\ 2\left[\frac{(2B-A)\Upsilon_2^2 + 2\aleph(A-B)\Upsilon_3}{\Upsilon_2^2}\right] & \left(\aleph > \frac{(1-2B+A)\Upsilon_2^2}{2(A-B)\Upsilon_3}\right) \end{cases} \tag{25}$$

where Υ_2 and Υ_3 are as in (15).

3. Coefficient Inequalities for $f^{-1} \in \mathcal{Q}^m_{\nu,\mu}(A,B)$

The Koebe one quarter theorem [24] ensures that the image of $\mathbb{D}$ for every univalent function $f \in \mathcal{A}_0$ contains a disk of radius $\frac{1}{4}$. Thus, every univalent function f has an inverse f^{-1} satisfying

$$f^{-1}(f(z)) = z, \ (z \in \mathbb{D}) \text{ and } \ f(f^{-1}(w)) = w, (|w| < r_0(f), \ r_0(f) \geq \frac{1}{4}).$$

A function $f \in \mathcal{A}_0$ is said to be bi-univalent in $\mathbb{D}$ if both f and f^{-1} are univalent in $\mathbb{D}$. We notice that the class of bi-univalent functions defined in the unit disk $\mathbb{D}$ is not empty. For example, the functions z, $\frac{z}{1-z}$, $-\log(1-z)$ and $\frac{1}{2}\log\frac{1+z}{1-z}$ are members of the bi-univalent function class; however, the Koebe function is not a member.

Theorem 3. *If $f \in \mathcal{Q}^m_{\nu,\mu}(A,B)$ and $f^{-1}(w) = w + \sum_{n=2}^{\infty} d_n w^n$ is the inverse function of f with $\left(|w| < r_0(f); r_0(f) \geq \frac{1}{4}\right)$ the Koebe domain of the class $f \in \mathcal{Q}^m_{\nu,\mu}(A,B)$, then*

$$|d_2| \leq \frac{A-B}{\mathrm{Y}_2}, \tag{26}$$

$$|d_3| \leq \frac{A-B}{2\mathrm{Y}_3}\max\{1, |\Theta(2,A,B)-1|\}, \tag{27}$$

and for any complex number $\hbar$, we have

$$|\, d_3 - \hbar d_2^2 \,| \leq \frac{A-B}{2\mathrm{Y}_3}\max\{1, |\Theta(2,A,B) + \hbar\frac{2(A-B)\mathrm{Y}_3}{\mathrm{Y}_2^2} - 1|\}, \tag{28}$$

where $\Theta(2,A,B) = (1+2B-A) + \frac{4(A-B)\mathrm{Y}_3}{\mathrm{Y}_2^2}$; Y_2 and Y_3 are as in (15).

Proof. As

$$f^{-1}(w) = w + \sum_{n=2}^{\infty} d_n w^n \tag{29}$$

is the inverse function of f, it can be seen that

$$f^{-1}(f(z)) = f\{f^{-1}(z)\} = z. \tag{30}$$

From (1) and (30), we obtain

$$f^{-1}(z + \sum_{n=2}^{\infty} a_n z^n) = z. \tag{31}$$

From Equations (30) and (31), one can obtain

$$z + (a_2 + d_2)z^2 + (a_3 + 2a_2 d_2 + d_3)z^3 + \cdots = z. \tag{32}$$

By equating the corresponding coefficients of (32), we have

$$d_2 = -a_2, \tag{33}$$

$$d_3 = 2a_2^2 - a_3. \tag{34}$$

From relations (19) and (33)

$$d_2 = -\frac{A-B}{2\mathrm{Y}_2}p_1; \tag{35}$$

$$|d_2| \leq \frac{A-B}{\mathrm{Y}_2}.$$

To find $|d_3|$, from (34), we set $\aleph = 2$ in (23). From (23) we have

$$\left|a_3 - \aleph a_2^2\right| = \frac{A-B}{4\Upsilon_3}\left|p_2 - \frac{p_1^2}{2}\Theta(\aleph, A, B)\right|,$$

thus

$$\begin{aligned} |d_3| = \left|a_3 - 2a_2^2\right| &= \frac{A-B}{4\Upsilon_3}\left|p_2 - \frac{p_1^2}{2}\Theta(2, A, B)\right|; \\ &\leq \frac{A-B}{2\Upsilon_3}\max\{1, |\Theta(2, A, B) - 1|\} \end{aligned} \tag{36}$$

where

$$\Theta(2, A, B) = (1 + 2B - A) + \frac{4(A-B)\Upsilon_3}{\Upsilon_2^2}. \tag{37}$$

For any complex number $\hbar$, consider

$$\begin{aligned} d_3 - \hbar d_2^2 &= \frac{A-B}{4\Upsilon_3}\left(p_2 - \frac{p_1^2}{2}\Theta(2, A, B)\right) - \hbar\frac{(A-B)^2}{4\Upsilon_2^2}p_1^2; \\ &= \frac{A-B}{4\Upsilon_3}\left(p_2 - \frac{p_1^2}{2}\left[\Theta(2, A, B) + \hbar\frac{2(A-B)\Upsilon_3}{\Upsilon_2^2}\right]\right). \end{aligned} \tag{38}$$

Taking the modulus on both sides and by applying Lemma 1 and (9), on the right hand side of (38), one can obtain the result

$$|d_3 - \hbar d_2^2| \leq \frac{A-B}{2\Upsilon_3}\max\{1, |\Theta(2, A, B) + \hbar\frac{2(A-B)\Upsilon_3}{\Upsilon_2^2} - 1|\} \tag{39}$$

as in (28). Hence, this completes the proof. □

4. Initial Logarithmic Coefficient Bounds and Fekete–Szegö Problem for $f \in \mathcal{Q}_{\nu,\wp}^m(A, B)$

Inspired by recent works like [25,26], in this section, we determine the coefficient bounds and Fekete–Szegö problem associated with the logarithmic function.

If the function f is analytic in $\mathbb{D}$, such that $\frac{f(z)}{z} \neq 0$ for all $z \in \mathbb{D}$, then the well-known logarithmic coefficients $d_n := d_n(f)$, $n \in \mathbb{N}$, of f are given by

$$\log\frac{f(z)}{z} = 2\sum_{n=1}^{\infty} d_n z^n, \; z \in \mathbb{D}. \tag{40}$$

For a function $f \in \mathcal{Q}_{\nu,\wp}^m(A, B)$, the left hand side of the subordination of the function defined in (8) should be an analytic function in $\mathbb{D}$; hence, $\frac{f(z)}{z} \neq 0$ for all $z \in \mathbb{D}$. Therefore, for all functions $f \in \mathcal{Q}_{\nu,\wp}^{m,n}(A, B)$, the relation (40) is well defined.

Theorem 4. *Let $f \in \mathcal{Q}_{\nu,\wp}^m(A, B)$ with the logarithmic coefficients given by (40). Then,*

$$\begin{aligned} |d_1| &\leq \frac{A-B}{2\Upsilon_2}, \\ |d_2| &\leq \frac{A-B}{4\Upsilon_3}\max\left\{1; \left|2B - A + \frac{(A-B)\Upsilon_3}{\Upsilon_2^2}\right|\right\} \end{aligned}$$

and for $\mu \in \mathbb{C}$ *we have*

$$|d_2 - \mu d_1^2| \leq \frac{A-B}{4\mathrm{Y}_3} \max\left\{1; \left|2B - A + \frac{(1+\mu)(A-B)\mathrm{Y}_3}{\mathrm{Y}_2^2}\right|\right\}$$

where Y_2 *and* Y_3 *are as in* (15).

Proof. If $f \in \mathcal{Q}_{\nu,\wp}^{m,n}(A,B)$ has the form (8), equating the first two coefficients of the relation (40), we obtain

$$d_1 = \frac{a_2}{2},$$
$$d_2 = \frac{1}{2}\left(a_3 - \frac{a_2^2}{2}\right).$$

Replacing a_2 and a_3 in the above equalities with those of (19) and (20), we obtain

$$\begin{aligned} d_1 =& \frac{A-B}{4\mathrm{Y}_2} p_1, \\ d_2 =& \frac{1}{2}\left(a_3 - \frac{a_2^2}{2}\right) \\ =& \frac{A-B}{8\mathrm{Y}_3}\left(p_2 - \frac{p_1^2}{2}[1+2B-A]\right) - \frac{(A-B)^2}{16\mathrm{Y}_2^2} p_1^2 \\ =& \frac{A-B}{8\mathrm{Y}_3}\left(p_2 - \frac{p_1^2}{2}\left[1+2B-A+\frac{(A-B)\mathrm{Y}_3}{\mathrm{Y}_2^2}\right]\right). \end{aligned}$$

Using (9), it follows that

$$|d_1| \leq \frac{A-B}{2\mathrm{Y}_2},$$

and using (10), the last equality leads to

$$|d_2| \leq \frac{A-B}{4\mathrm{Y}_3} \max\left\{1; \left|2B - A + \frac{(A-B)\mathrm{Y}_3}{\mathrm{Y}_2^2}\right|\right\}.$$

Furthermore, we have

$$\begin{aligned} d_2 - \mu d_1^2 &= \frac{A-B}{8\mathrm{Y}_3}\left(p_2 - \frac{p_1^2}{2}\left[1+2B-A+\frac{(A-B)\mathrm{Y}_3}{\mathrm{Y}_2^2}\right]\right) - \mu\frac{(A-B)^2}{16\mathrm{Y}_2^2} p_1^2 \\ &= \frac{A-B}{8\mathrm{Y}_3}\left(p_2 - \frac{p_1^2}{2}\left[1+2B-A+\frac{(1+\mu)(A-B)\mathrm{Y}_3}{\mathrm{Y}_2^2}\right]\right) \end{aligned}$$

and in view of (10), we obtain desired result. □

5. Characterization Properties

In this section, we obtain the necessary and sufficient conditions, growth and distortion bounds and convex combination for the newly defined class.

Theorem 5. *Let* $f \in \mathcal{A}_0$ *be assigned to the class* $\mathcal{Q}_{\nu,\mu}^{m}(A,B)$ *if it fulfills the inequality*

$$\sum_{n=2}^{\infty} (|A - Bn| + (n-1))\mathrm{Y}_n |a_n| \leq A - B, \tag{41}$$

equivalently, we may write

$$\sum_{n=2}^{\infty} \Xi_n |a_n| \leq A - B,$$

where

$$\Xi_n = [|A - Bn| + (n-1)]\Upsilon_n \tag{42}$$

and Υ_n *as given in* (6). *Inequality* (41) *is sharp.*

Proof. Assume that inequality (41) holds, and $|z| = 1$. Then, one can put (8) in the form of a Schwarz function $w(z)$ as

$$\frac{z(\mathcal{Q}^m_{\nu,\mu} f(z))'}{\mathcal{Q}^m_{\nu,\mu} f(z)} = \frac{1 + Aw(z)}{1 + Bw(z)} \ (z \in \mathbb{D}). \tag{43}$$

It must be shown that the values satisfy the condition

$$\left| \frac{z(\mathcal{Q}^m_{\nu,\mu} f(z))' - \mathcal{Q}^m_{\nu,\mu} f(z)}{A\mathcal{Q}^m_{\nu,\mu} f(z) - Bz(\mathcal{Q}^m_{\nu,\mu} f(z))'} \right| < 1.$$

Then, we have

$$\begin{aligned} \left| \frac{z(\mathcal{Q}^m_{\nu,\mu} f(z))' - \mathcal{Q}^m_{\nu,\mu} f(z)}{A\mathcal{Q}^m_{\nu,\mu} f(z) - Bz(\mathcal{Q}^m_{\nu,\mu} f(z))'} \right| &= \left| \frac{\sum_{n=2}^{\infty} (n-1)\Upsilon_n a_n z^n}{(A-B)z + \sum_{n=2}^{\infty} (A - Bn)\Upsilon_n a_n z^n} \right| \\ &\leq \frac{\sum_{n=2}^{\infty} (n-1)\Upsilon_n |a_n|}{A - B - \sum_{n=2}^{\infty} |A - Bn|\Upsilon_n |a_n|} \\ &< 1. \end{aligned}$$

This shows that the values of $\frac{z(\mathcal{Q}^m_{\nu,\mu} f(z))'}{\mathcal{Q}^m_{\nu,\mu} f(z)}$ lie in a circle centered at $w = 1$ whose radius is 1. Hence, $f(z) \in \mathcal{Q}^m_{\nu,\mu}(A, B)$.

Now, for the function

$$f(z) = z + \sum_{n=2}^{\infty} \frac{A - B}{[|A - Bn| + (n-1)]\Upsilon_n} k_n z^n, \ z \in \mathbb{D},$$

such that $\sum_{n=2}^{\infty} k_n = 1$, we have

$$\begin{aligned} &\sum_{n=2}^{\infty} [|A - Bn| + (n-1)]\Upsilon_n |a_n| \\ = &\sum_{n=2}^{\infty} [|A - Bn| + (n-1)]\Upsilon_n \left(\frac{A - B}{[|A - Bn| + (n-1)]\Upsilon_n} k_n \right) \\ = &\ (A - B) \sum_{n=2}^{\infty} k_n = A - B. \end{aligned}$$

Thus, $f(z) \in \mathcal{Q}^m_{\nu,\mu}(A, B)$, and the approximation (41) is sharp. □

A function $f(z)$ defined by (1) and belonging to the class $\mathcal{A}_0$ is said to be in the class $\overline{\mathcal{Q}}^m_{\nu,\mu}(A, B)$ if it is also satisfies the coefficient inequality (41).

Using the technique of proof of distortion theorems given by Silverman [27], we state the following results without proof.

Theorem 6. *If a function* $f \in \overline{\mathcal{Q}}^m_{\nu,\mu}(A, B)$, *then*

$$|z| - \frac{(A - B)}{[|A - 2B| + 1]\Upsilon_2} |z|^2 \leq |f(z)| \leq |z| + \frac{(A - B)}{[|A - 2B| + 1]\Upsilon_2} |z|^2. \tag{44}$$

The approximation is sharp for the function defined as:

$$f(z) = z + \frac{(A-B)}{[|A-2B|+1]\Upsilon_2} z^2. \tag{45}$$

Theorem 7. *If a function* $f \in \overline{\mathcal{Q}}^m_{\nu,\mu}(A,B)$, *then*

$$1 - \frac{2(A-B)}{[|A-2B|+1]\Upsilon_2}|z| \leq \left|f'(z)\right| \leq 1 + \frac{2(A-B)}{[|A-2B|+1]\Upsilon_2}|z|. \tag{46}$$

The result is sharp for the extreme function defined in (45).

Theorem 8. *Let* $f_i \in \overline{\mathcal{Q}}^m_{\nu,\mu}(A,B)$ *and have the form*

$$f_i(z) = z + \sum_{n=2}^{\infty} a_{i,n} z^n, \text{ for } i = 1,2,3,...,k. \tag{47}$$

Then, $H \in \overline{\mathcal{Q}}^m_{\nu,\mu}(A,B)$, *where*

$$H(z) = \sum_{i=1}^{k} c_i f_i(z) \text{ with } \sum_{i=1}^{k} c_i = 1. \tag{48}$$

Proof. Consider $f_i \in \overline{\mathcal{Q}}^m_{\nu,\mu}(A,B)$. Then, we can write

$$\sum_{n=2}^{\infty} [|A-Bn| + (n-1)]\Upsilon_n |a_n| \leq A - B.$$

Furthermore,

$$\begin{aligned} H(z) &= \sum_{i=1}^{k} c_i \left(z + \sum_{n=2}^{\infty} a_{i,n} z^n \right) \\ &= z + \sum_{n=2}^{\infty} \left(\sum_{i=1}^{k} c_i a_{i,n} \right) z^n, \end{aligned}$$

therefore

$$\begin{aligned} & \sum_{n=2}^{\infty} [|A-Bn| + (n-1)]\Upsilon_n \left| \sum_{i=1}^{k} c_i a_{i,n} \right| \\ = & \sum_{i=1}^{k} \left[\sum_{n=2}^{\infty} [|A-Bn| + (n-1)]\Upsilon_n |a_{i,n}| \right] c_i \\ \leq & \sum_{i=1}^{k} (A-B) c_i = (A-B) \sum_{i=1}^{k} c_i = A - B, \end{aligned}$$

thus $H(z) \in \overline{\mathcal{Q}}^m_{\nu,\mu}(A,B)$. □

Theorem 9. *Let* $f_i \in \overline{\mathcal{Q}}^m_{\nu,\mu}(A,B)$, *for* $i = 1,2,...,j$. *Then, the arithmetic mean* $\mathcal{G}$ *of* f_i *is given by*

$$\mathcal{G}(z) = \frac{1}{j} \sum_{n=1}^{j} f_i(z), \tag{49}$$

and also belongs to class $\overline{\mathcal{Q}}^m_{\nu,\mu}(A,B)$.

Proof. From (49), we can write

$$\begin{aligned}\mathcal{G}(z) &= \frac{1}{j}\sum_{n=1}^{j} f_i(z)\\ &= \frac{1}{j}\sum_{n=1}^{j}\left(z+\sum_{n=2}^{\infty} a_{j,n}z^n\right)\\ &= z+\sum_{n=2}^{\infty}\left(\frac{1}{j}\sum_{n=1}^{j} a_{j,n}\right)z^n.\end{aligned}$$

To show $\mathcal{G}(z)$ belongs to $\overline{\mathcal{Q}}^{m}_{\nu,\mu}(A,B)$, it is sufficient to show that

$$\sum_{n=2}^{\infty}[|A-Bn|+(n-1)]\Upsilon_n\left(\frac{1}{j}\sum_{n=1}^{j}|a_{j,n}|\right)\leq A-B.$$

Consider

$$\begin{aligned}&\sum_{n=2}^{\infty}[|A-Bn|+(n-1)]\Upsilon_n\left(\frac{1}{j}\sum_{n=1}^{j}|a_{j,n}|\right)\\ =\;& \frac{1}{j}\sum_{n=1}^{j}\left(\sum_{n=2}^{\infty}[|A-Bn|+(n-1)]\Upsilon_n|a_{j,n}|\right)\\ \leq\;& \frac{1}{j}\sum_{n=1}^{j}(A-B)=A-B,\end{aligned}$$

which yields $\mathcal{G}(z)\in\overline{\mathcal{Q}}^{m}_{\nu,\mu}(A,B)$. □

6. Subordination Results

Now, we recall the following results of Wilf [28], which are very much needed for our study.

Definition 1 (subordinating factor sequence). *A sequence $\{b_n\}_{n=1}^{\infty}$ of complex numbers is said to be a subordinating sequence if, whenever $f(z)=\sum_{n=1}^{\infty}a_nz^n$, $a_1=1$ is regular, univalent and convex in $\mathcal{U}$, we have*

$$\sum_{n=1}^{\infty}b_na_nz^n\prec f(z),\quad z\in\mathbb{D}. \tag{50}$$

Lemma 2. *The sequence $\{b_n\}_{n=1}^{\infty}$ is a subordinating factor sequence if and only if*

$$Re\left\{1+2\sum_{n=1}^{\infty}b_nz^n\right\}>0,\quad z\in\mathbb{D}. \tag{51}$$

Theorem 10. *Let $f\in\overline{\mathcal{Q}}^{m}_{\nu,\mu}(A,B)$ and $g(z)$ be any function in the usual class of convex functions $\mathcal{C}$, then*

$$\frac{\Xi_2}{2[A-B+\Xi_2]}(f*g)(z)\prec g(z) \tag{52}$$

and

$$Re\,\{f(z)\}>-\frac{[A-B+\Xi_2]}{\Xi_2},\quad z\in\mathbb{D}. \tag{53}$$

where

$$\Xi_2 = (|A-2B|+1)\Upsilon_2 = (|A-2B|+1)\frac{(m\mu)\,m^{\nu}}{\mathbf{E}_{\nu,\mu}(m)\Gamma(\nu+2)} \tag{54}$$

The constant factor $\frac{\Xi_2}{2[A-B+\Xi_2]}$ *in* (52) *cannot be replaced by a larger number.*

Proof. Let $f \in \overline{\mathcal{Q}}^{m}_{\nu,\mu}(A,B)$ and suppose that $g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in C$. Then,

$$\frac{\Xi_2}{2[A-B+\Xi_2]}(f*g)(z) = \frac{\Xi_2}{2[A-B+\Xi_2]}\left(z + \sum_{n=2}^{\infty} b_n a_n z^n\right). \tag{55}$$

Thus, by Definition 1, the subordination result holds true if

$$\left\{\frac{\Xi_2}{2[A-B+\Xi_2]}\right\}_{n=1}^{\infty}$$

is a subordinating factor sequence, with $a_1 = 1$. In view of Lemma 2, this is equivalent to the following inequality

$$\mathrm{Re}\left\{1 + \sum_{n=1}^{\infty} \frac{\Xi_2}{[A-B+\Xi_2]} a_n z^n\right\} > 0, \quad z \in \mathbb{D}. \tag{56}$$

By noting the fact that $\frac{\Xi_2}{[A-B+\Xi_2]}$ is an increasing function for $n \geq 2$, and in particular

$$\frac{\Xi_2}{A-B} \leq \frac{\Xi_n}{A-B}, \quad n \geq 2,$$

then for $|z| = r < 1$, we have

$$\begin{aligned}
&\mathrm{Re}\left\{1 + \frac{\Xi_2}{[A-B+\Xi_2]} \sum_{n=1}^{\infty} a_n z^n\right\} \\
&= \mathrm{Re}\left\{1 + \frac{\Xi_2}{[A-B+\Xi_2]} z + \frac{\sum_{n=2}^{\infty} \Xi_2 a_n z^n}{[A-B+\Xi_2]}\right\} \\
&\geq 1 - \frac{\Xi_2}{[A-B+\Xi_2]} r - \frac{\sum_{n=2}^{\infty} |\Xi_n a_n| r^n}{[A-B+\Xi_2]} \\
&\geq 1 - \frac{\Xi_2}{[A-B+\Xi_2]} r - \frac{A-B}{[A-B+\Xi_2]} r \\
&> 0, \quad |z| = r < 1,
\end{aligned}$$

where we have also made use of assertion (41) of Theorem 5. This evidently proves inequality (56) and hence also the subordination result (52) asserted by (41).

Inequality (53) follows from (52) by taking

$$g(z) = \frac{z}{1-z} = z + \sum_{n=2}^{\infty} z^n \in C.$$

Next, we consider the function

$$F(z) := z - \frac{A-B}{\Xi_2} z^2$$

where Ξ_2 is given by (54). Clearly $F \in \overline{\mathcal{Q}}^m_{\nu,\mu}(A,B)$. For this function, (52) becomes

$$\frac{\Xi_2}{2[A-B+\Xi_2]} F(z) \prec \frac{z}{1-z}.$$

It is easily verified that

$$\min\left\{ \mathrm{Re}\left(\frac{\Xi_2}{2[A-B+\Xi_2]} F(z) \right) \right\} = -\frac{1}{2}, \quad z \in \mathbb{D}.$$

This shows that the constant $\frac{\Xi_2}{2[A-B+\Xi_2]}$ cannot be replaced by any larger one. □

7. Partial Sums

In 1997, Silverman [29] examined partial sums results for the class of starlike and convex functions f given by (1) and established through

$$\begin{aligned} f_1(z) &= z, \\ f_j(z) &= z + \sum_{n=2}^{\infty} a_n z^n, \ j = 2,3,4,\cdots. \end{aligned}$$

Many authors have investigated partial sums for different subclasses; for some recent investigations, we refer to [16,30] and references cited therein.

Theorem 11. *If f of the form (1) satisfies condition (41), then*

$$\Re\left(\frac{f(z)}{f_j(z)} \right) \geq 1 - \frac{1}{\Lambda_{j+1}} \quad (\forall z \in \mathbb{D}) \tag{57}$$

and

$$\Re\left(\frac{f_j(z)}{f(z)} \right) \geq \frac{\Lambda_{j+1}}{1+\Lambda_{j+1}} \quad (\forall z \in \mathbb{D}), \tag{58}$$

where

$$\Lambda_j = \frac{[|A-Bn| + (n-1)]}{(A-B)} \Upsilon_n. \tag{59}$$

Proof. To prove the approximation (57), we put:

$$\begin{aligned} \Lambda_{j+1}\left[\frac{f(z)}{f_j(z)} - \left(1 - \frac{1}{\Lambda_{j+1}} \right) \right] &= \frac{1 + \sum\limits_{n=2}^{j} a_n z^{n-1} + \Lambda_{j+1} \sum\limits_{n=j+1}^{\infty} a_n z^{n-1}}{1 + \sum\limits_{n=2}^{j} a_n z^{n-1}} \\ &= \frac{1+\psi_1(z)}{1+\psi_2(z)}. \end{aligned}$$

We now set:

$$\frac{1+\psi_1(z)}{1+\psi_2(z)} = \frac{1+w(z)}{1-w(z)}.$$

Then, we find after some worthwhile simplification:

$$w(z) = \frac{\psi_1(z) - \psi_2(z)}{2 + \psi_1(z) + \psi_2(z)}.$$

Thus, clearly, we find that:

$$w(z) = \frac{\Lambda_{j+1} \sum_{n=j+1}^{\infty} a_n z^{n-1}}{2 + 2 \sum_{n=2}^{j} a_n z^{n-1} + \Lambda_{j+1} \sum_{n=j+1}^{\infty} a_n z^{n-1}}$$

By implementation of the triangle inequalities with $|z| < 1$, we arrive at the following inequality:

$$|w(z)| \leq \frac{\Lambda_{j+1} \sum_{n=j+1}^{\infty} |a_n|}{2 - 2 \sum_{n=2}^{j} |a_n| - \Lambda_{j+1} \sum_{n=j+1}^{\infty} |a_n|}.$$

We can now see that:

$$|w(z)| \leq 1$$

if and only if

$$2\Lambda_{j+1} \sum_{n=j+1}^{\infty} |a_n| \leq 2 - 2 \sum_{n=2}^{j} |a_n|,$$

which hints that:

$$\sum_{n=2}^{j} |a_n| + \Lambda_{j+1} \sum_{n=j+1}^{\infty} |a_n| \leq 1. \tag{60}$$

Finally, to prove the inequality in (57), it suffices to show that the left hand side of (60) is bounded above by the following sum:

$$\sum_{n=2}^{\infty} \Lambda_n |a_n|,$$

which is equivalent to

$$\sum_{n=2}^{j} (\Lambda_n - 1)|a_n| + \sum_{n=j+1}^{\infty} (\Lambda_n - \Lambda_{j+1})|a_n| \geq 0. \tag{61}$$

In light of (61), this is evidence that the proof of the inequality in (57) is now completed.

Next, in order to prove inequality (58), we set:

$$(1 + \Lambda_{j+1}) \left(\frac{f_j(z)}{f(z)} - \frac{\Lambda_{j+1}}{1 + \Lambda_{j+1}} \right) = \frac{1 + \sum_{n=2}^{j} a_n z^{n-1} - \Lambda_{j+1} \sum_{n=j+1}^{\infty} a_n z^{n-1}}{1 + \sum_{n=2}^{\infty} a_n z^{n-1}}$$

$$= \frac{1 + w(z)}{1 - w(z)},$$

where

$$|w(z)| \leq \frac{\left(1+\Lambda_{j+1}\right)\sum\limits_{n=j+1}^{\infty}|a_n|}{2-2\sum\limits_{n=2}^{j}|a_n|-\left(\Lambda_{j+1}-1\right)\sum\limits_{n=j+1}^{\infty}|a_n|} \leq 1. \tag{62}$$

This last inequality in (62) is equivalent to

$$\sum_{n=2}^{j}|a_n|+\Lambda_{j+1}\sum_{n=j+1}^{\infty}|a_n| \leq 1. \tag{63}$$

Finally, we can see that the left hand side of the inequality in (63) is bounded above by the following sum:

$$\sum_{n=2}^{\infty}\Lambda_n|a_n|,$$

so we have completed the proof of assertion (58), which completes the proof of Theorem 11. □

We next turn to ratios involving derivatives.

Theorem 12. *If f of the form* (1) *satisfies condition* (41)*, then*

$$\Re\left(\frac{f'(z)}{f_j'(z)}\right) \geq 1-\frac{j+1}{\Lambda_{j+1}} \quad (\forall z \in \mathbb{D}) \tag{64}$$

and

$$\Re\left(\frac{f_j'(z)}{f'(z)}\right) \geq \frac{\Lambda_{j+1}}{\Lambda_{j+1}+j+1} \quad (\forall z \in \mathbb{D}), \tag{65}$$

where Λ_j is given by (59)*.*

Proof. The proof of Theorem 12 is similar to that of Theorem 11; we here choose to omit the analogous details. □

8. Conclusions

In this paper, for this newly defined functions class, we have examined several well-known results, including the Fekete–Szegö inequalities, necessary and sufficient conditions, growth and distortion bounds, convex combination, sequential subordination and partial-sum-type results. Furthermore, we believe that this study will motivate a number of researchers to extend this idea for meromorphic functions and harmonic functions. One may also apply this idea to the shell-like and petal-shaped domains instead of the Janowski domain. Several analytic function classes involving Miller–Ross functions have been developed using the concept of subordination based on the geometrical interpretation of their image domains, including the right half plane, circular disc, oval- and petal-type domains, conic domain, leaf-like domain and generalized conic domain, which have all been defined and studied (see [31–37] for details).

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References

1. Fekete, M.; Szegő, G. Eine Bemerkung über ungerade schlichte Funktionen. *J. Lond. Math. Soc.* **1933**, *8*, 85–89. [CrossRef]
2. Dziok, J. A general solution of the Fekete-Szegő problem. *Bound. Value Probl.* **2013**, *98*, 13. [CrossRef]
3. Kanas, S. An unified approach to the Fekete-Szegő problem. *Appl. Math. Comput.* **2012**, *218*, 8453–8461. [CrossRef]
4. Miller, K.S.; Ross, B. *An Introduction to the Fractional Calculus and Fractional Differentials*; John Wiley and Sons, New York Press: New York, NY, USA, 1993.
5. Wiman, A. Über die Nullstellen der Funktionen Ea(x). *Acta Math.* **1905**, *29*, 217–234. [CrossRef]
6. Sümer Eker, S.; Ece, S. Geometric Properties of the Miller-Ross Functions. *Iran. J. Sci. Technol. Trans. Sci.* **2022**, *46*, 631–636. [CrossRef]
7. Altınkaya, Ş.; Yalçın, S. Poisson distribution series for certain subclasses of starlike functions with negative coefficients. *Annal. Oradea Univ. Math. Fascicola* **2017**, *24*, 5–8.
8. El-Deeb, S.M.; Bulboaca, T.; Dziok, J. Pascal distribution series connected with certain subclasses of univalent functions. *Kyungpook Math. J.* **2019**, *59*, 301–314.
9. Nazeer, W.; Mehmood, Q.; Kang, S.M.; Haq, A.U. An application of Bionomial distribution series on certain analytic functions. *J. Comput. Anal. Appl.* **2019**, *26*, 11–17.
10. Porwal, S.; Kumar, M. A unified study on starlike and convex functions associated with Poisson distribution series. *Afr. Mat.* **2016**, *27*, 10–21. [CrossRef]
11. Wanas, A.K.; Al-Ziadi, N.A. Applications of beta negative binomial distribution series on holomorphic funxtions. *Earthline J. Math. Sci.* **2021**, 6, 271–292. [CrossRef]
12. Wanas, A.K.; Khuttar, J.A. Applications of Borel distribution series on analytic functions. *Earthline J. Math. Sci.* **2020**, *4*, 71–82. [CrossRef]
13. Ahmad, B.; Khan, M.G.; Aouf, M.K.; Mashwani, W.K.; Salleh, Z.; Tang, H. Applications of a new q-difference operator in Janowski-type meromorphic convex functions. *J. Funct.* **2021**, *2021*, 5534357. [CrossRef]
14. Srivastava, H.M.; Murugusundaramoorthy, G.; Sivasubramanian, S. Hypergeometric functions in the parabolic starlike and uniformly convex domains. *Integr. Transf. Spec. Func.* **2007**, *18*, 511–520. [CrossRef]
15. Janowski, W. Some extremal problems for certain families of analytic functions. *Bull. Acad. Plolon. Sci. Ser. Sci. Math. Astronomy* **1973**, *21*, 17–25. [CrossRef]
16. Khan, M.G.; Ahmad, B.; Khan, N.; Mashwani, W.K.; Arjika, S.; Khan, B.; Chinram, R. Applications of Mittag–Leffler type poisson distribution to a subclass of analytic functions involving conic-type regions. *J. Funct. Spaces* **2021**, *2021*, 4343163.
17. Srivastava, H.M.; El-Deeb, S.M. Fuzzy differential subordinations based upon the Mittag–Leffler type Borel distribution. *Symmetry* **2021**, *13*, 1023. [CrossRef]
18. El-Deeb, S.M.; Murugusundaramoorty, G.; Alburaikan, A. A bi-Bazilevič functions based on the Mittag–Leffler-Type Borel distribution associated with Legendre polynomials. *J. Math. Comput. Sci.* **2021**, *24*, 235–245. [CrossRef]
19. Srivastava, H.M.; Murugusundaramoorty, G.; El-Deeb, S.M. Faber polynomial coefficient estimates of bi-close-to-convex functions connected with Borel distribution of the Mittag–Leffler-type. *J. Nonlinear Var. Anal.* **2021**, *5*, 103–118.
20. Sümer Eker, S.; Murugusundaramoorthy, G.; Şeker, B.; Çekiç, B. Spiral-like functions associated with Miller–Ross-type Poisson distribution series. *Boletin Soc. Matematica Mex.* **2023**, *29*, 16. [CrossRef]
21. Robertson, M.S. Certain classes of starlike functions. *Mich. Math. J.* **1985**, *32*, 135–140. [CrossRef]
22. Keogh, F.R.; Merkes, E.P. A coefficient inequality for certain classes of analytic functions. *Proc. Am. Math. Soc.* **1969**, *20*, 8–12. [CrossRef]
23. Ma, W.; Minda, D. A unified treatment of some special classes of univalent functions. In Proceedings of the Conference on Complex Analysis, Tianjin, China, 19–23 June 1992; Li, Z., Ren, F., Yang, L., Zhang, S., Eds.; International Press: Cambridge, MA, USA, 1994; pp. 157–169.
24. Duren, P.L. Univalent Functions. In *Grundlehren der Mathematischen Wissenschaften*; Band 259, Springer: New York, NY, USA; Berlin/Heidelberg, Germany; Tokyo, Japan, 1983.
25. Alimohammadi, D.; Adegani, E.A.; Bulboaca, T.; Cho, N.E. Logarithmic coefficients for classes related to convex functions. *Bull. Malays. Math. Sci. Soc.* **2021**, *44*, 2659–2673. [CrossRef]
26. Alimohammadi, D.; Adegani, E.A.; Bulboaca, T.; Cho, N.E. Logarithmic coefficients bounds and coefficient conjectures for classes associated with convex functions. *J. Funct. Spaces* **2021**, *2021*, 6690027. [CrossRef]
27. Silverman, H. Univalent functions with negative coefficients. *Proc. Am. Math. Soc.* **1975**, *51*, 109–116. [CrossRef]
28. Wilf, H.S. Subordinating factor sequence for convex maps of the unit circle. *Proc. Am. Math. Soc.* **1961**, *12*, 689–693. [CrossRef]
29. Silverman, H. Partial sums of starlike and convex functions. *J. Math. Anal. Appl.* **1997**, *209*, 221–227. [CrossRef]
30. Rosy, T.; Subramanian, K.G.; Murugusundaramoorthy, G. Neighbourhoods and Partial sums of Starlike Functions Based on Ruscheweyh Derivatives. *J. Ineq. Pure Appl. Math.* **2003**, *64*, 4.
31. Cho, N.E.; Kumar, S.; Kumar, V.; Ravichandran, V. Radius problemsfor starlike functions associated with the sine function. *Bull. Iran. Math. Soc.* **2019**, *45*, 213–232. [CrossRef]
32. Mendiratta, R.; Nagpal, S.; Ravichandran, V. On a subclass ofstrongly starlike functions associated with exponential functions. *Bull. Malays. Math. Sci. Soc.* **2015**, *38*, 365–386. [CrossRef]
33. Sharma, K.; Jain, N.K.; Ravichandran, V. Starlike functionsassociated with cardioid. *Afrika Math.* **2016**, *27*, 923–939. [CrossRef]

34. Wani, L.A.; Swaminathan, A. Starlike and convex functionsassociated with a Nephroid domain. *Bull. Malays. Math. Sci. Soc.* **2021**, *44*, 79–104. [CrossRef]
35. Sokól,J. Radius problem in the class $\mathcal{SL}^*$. *Appl. Math. Comput.* **2009**, *214*, 569–573.[CrossRef]
36. Raina, R.K.; Sokól, J. On Coefficient estimates for acertain class of starlike functions. *Hacettepe. J. Math. Statist.* **2015**, *44*, 1427–1433. [CrossRef]
37. Gandhi, S. Radius estimates for three leaf functions and convexcombination of starlike functions. In *Mathematical Analysis 1: Approximation Theory*; Deo N., Gupta V., Acu A., Agrawal P., Eds.; Springer: Singapore, 2018; Volume 306.

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