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Manufacturer vs. Retailer: A Comparative Analysis of Different Government Subsidy Strategies in a Dual-Channel Supply Chain Considering Green Quality and Channel Preferences

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Abstract: As escalating environmental pollution results from economic development, the green supply chain is vital in enhancing ecological quality. In light of the rapid growth of online shopping, the dual-channel supply chain has gained significant popularity. Governments have implemented policies to achieve carbon peaks and neutrality worldwide. Considering green quality and channel preferences, this study constructs a Stackelberg model led by the manufacturer within a dual-channel supply chain to compare the effects under two government subsidy strategies. A Stackelberg game-theoretic approach is applied to analyze the model. The findings indicate the following: (1) Increases in the subsidy coefficient can increase product greenness and the overall profit in both subsidy strategies, which makes sense; (2) When direct sales channels become dominant, subsidizing the manufacturer proves superior to subsidizing the retailer in terms of promoting green quality and overall profit, which extends existing studies of government subsidy decisions. Furthermore, subsidizing the retailer may negatively affect the total supply chain profit; (3) Consistent with previous literature, intense competition between manufacturers and retailers is expected to enhance the overall profit of the supply chain; (4) Interestingly, customer preference for online channels does not influence product green quality under manufacturer subsidization strategies but exhibits a negative impact under retailer subsidization strategies. Finally, this research provides managerial implications for decision-makers and potential issues for future research.

Keywords: government subsidy; dual-channel green supply chain; green quality; customer channel preference; comparative analysis

MSC: 90B06

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1. Introduction

In recent decades, the green supply chain (GSC) has become an increasingly important issue worldwide, and it has been integrated into firms' innovative strategies to achieve a competitive advantage [1]. The emergence of green supply chain management (GSCM) took place about a couple of decades ago. The definition and scope of GSCM in the literature has ranged from green purchasing to integrated green supply chains flowing from supplier to manufacturer to customer, and even to reverse logistics [1]. As this field is expanding rapidly, green and sustainable supply chain management practices have been developed, which are categorized into 16 aspects and include 58 practices such as carbon management, green manufacturing, and customer environmental collaboration.

[2]. Nowadays, the environmental pollution problem is being paid attention to by countries all over the world [3–5]. The International Energy Agency (IEA) stated that global carbon dioxide emissions reached a new record in 2023 and is continuing to rise [6]. The phenomenon above not only exacerbates the process of global warming but also imposes severe ecological damage [7,8]. The United Nations has officially endorsed the commitment to achieve net-zero emissions, urging all members (including companies, cities, regions, and institutions) to promptly and transparently implement measures to reduce global emissions by 45% by 2050. This endeavor aligns with the objectives of the Paris Agreement in fostering a more equitable world free from carbon pollution [9]. Consequently, to improve the environment and achieve the ambitious CO_2 emission reduction targets, an increasing number of governments have implemented policies regarding carbon peaking and carbon neutrality to incentivize enterprises toward adopting green production and promoting sustainable products [10–16].

Among the various measures being undertaken globally to reduce carbon emissions, government subsidies serve as a positive incentive for facilitating carbon emission reduction [17–19]. For instance, in 2012, a total of EUR 41.8 million was allocated by the European Commission toward promoting electric vehicles through a green campaign and supporting research and development in energy technology [20]. The German government has incentivized semiconductor chip manufacturers to produce environmentally sustainable semiconductor products by allocating EUR 14 billion in subsidies. Similarly, the Chinese government offers subsidies to major e-commerce platforms such as T-mall and JD.com based on the energy efficiency of their offerings, aiming to encourage retailers' active engagement in energy conservation and emission reduction [21]. From the above examples, governments have played a pivotal role in enhancing economic and environmental benefits by implementing different subsidy strategies. These strategies aim to mitigate the detrimental impacts of global warming and environmental pollution on sustainable development by providing subsidies to various stakeholders within the supply chain, including manufacturers, retailers, and so on [22–24].

Furthermore, with the rapid growth of internet technology and in the number of smartphone users, shopping online has become a common phenomenon among customers. Direct distribution via e-commerce platforms is convenient for customers, which prompts manufacturers to restructure their traditional channels by establishing online direct channels to meet the needs of various customer segments that are not accessible through conventional retail channels. This results in expanded market coverage, regulated sales prices, and increased profitability [25]. According to [26], retail e-commerce sales worldwide are expected to grow by 50%, reaching USD 7.4 trillion in 2025. Hence, numerous manufacturers like IBM, Nike, Interface, GREE, and others have adopted a dual-channel supply chain strategy by selling their products through both traditional retail channels (offline) and direct selling channels (online) [27–30]. Consequently, when faced with a dual-channel green supply chain, different subsidy strategies offered by the government will impact the enterprise's production and pricing decisions [11,18,31,32]. In addition, game theory is a widely used method in relevant research [11,12,14–16,18] to model the decisions among supply chain members. For example, Tian et al. [33] applied evolutionary game theory and found that government subsidies to manufacturers can promote the dissemination of green supply chain management more than subsidies to customers. Therefore, examining the effects of different government subsidies in a dual-channel green supply chain is essential to enhancing product sustainability and maximizing overall supply chain profits. In addition, according to [34], the emergence of customer channel preferences has significantly influenced the effects of different subsidy strategies. Hence, it is worthy of further study.

Previous studies using comparative analysis under different government subsidies have focused mainly on single channels, and the subsidy strategies primarily addressed either manufacturers or customers [35]. Recently, Meng and Li [11] investigated the pricing policies of standard and green products with or without government subsidies in a dual-channel green supply chain. Therefore, this study aims to contribute to the literature by filling the gap through an examination of how the government subsidy coefficient level and customer channel preferences affect the green quality of the product and the profits of the supply chain under strategies that subsidize manufacturers or retailers. Also, this research compares the effects of the supply chain under three strategies of no government subsidy, government subsidy to manufacturers, and subsidy to retailers, so that a decision can be made about whether to subsidize and how to subsidize.

To deal with the above issues, this study establishes a Stackelberg game model to study a dual-channel green supply chain, including a leading manufacturer and a following retailer, where the manufacturer has both a traditional retailer channel and a direct sale channel. Demand for each channel is a linear function of the online and offline selling prices of green products, green quality level, and customer channel preference. The manufacturer decides the product's online selling price and green quality level, while the retailer determines the offline selling price. Three analytical decision models are studied: (i) no government subsidy; (ii) a subsidy strategy to manufacturers; and (iii) a subsidy strategy to retailers. A comparative analysis and the equilibrium results are derived for the stated decision models. To our knowledge, among the different subsidy policies for the government, which one can provide the most benefits for the dual-channel green supply chain is still a problem that has not been previously examined.

The findings of this study are novel and intriguing. Firstly, regarding the existing research, government subsidies can increase product greenness and overall profit by subsidizing manufacturers, which makes sense. However, it is not always true that government subsidies positively affect the total supply chain profit in subsidizing retailers. Secondly, although much literature has focused on the influence of government subsidies in dual-channel supply chain decisions, most concentrate on pricing policies and coordinations for subsidizing manufacturers. There is still limited research that considers the effects of subsidizing retailers, and these rarely examine the effects of subsidizing manufacturers and retailers in a dual-channel supply chain. In this research, it is concluded that the strategy of subsidizing the manufacturer is better than that of subsidizing the retailer with regard to the green quality and the overall profit in most instances, which can guide the government in making decisions on subsidy strategies. Furthermore, consistent with previous literature, when faced with intense competition between manufacturers and retailers, regardless of whether to subsidize manufacturers or retailers, the supply chain's overall profit is expected to improve under identical circumstances, extending the related conclusion in the dual-channel green supply chain. In addition, different from the previous research showing that customers' preference for green online channels improves the product's greenness, it was surprising to observe that the customer preference for online channels does not influence the green quality of products under a manufacturer subsidization strategy. In contrast, it negatively impacts the retailer's subsidization strategy under certain conditions. Last but not least, the Stackelberg model effectively compares subsidy policies, which can solve the conflict of interests among enterprises, governments, and customers in green supply chain management and coordinate and balance economic and environmental benefits through the optimal behavioral decision-making of all parties.

The following sections are included in this work. Section 2 provides a summary of the existing literature. Section 3 presents the problem description, notations, and assumptions. Model formulations and analyses are presented in Section 4, followed by numerical analyses in Section 5. Section 6 provides concluding remarks and discusses future considerations. Furthermore, Appendix A at the end of this study includes proofs for all the technical results.

2. Literature Review

The literature concerning this topic can be divided into two categories. The first focuses on primary streams of green supply chains, while the other deals with various research studies on government subsidy strategies in the green supply chain.

2.1. Green Supply Chain

The Manufacturing Research Association of Michigan State University first proposed a green supply chain in 1996 in a study of "environmentally responsible manufacturing", which was a modern supply chain management model that considered both resource efficiency and the environmental impact. Scholars have conducted several literature reviews on green supply chain management [36,37]. Green supply chain management (GSCM) has become essential in academia and industry [38]. Implementing a green supply chain has yielded enhanced economic benefits and ecological advantages [21,39]. For example, Chen investigated the impact of green research and development (R&D) on manufacturer activities and proposed strategies to enhance enterprises' capabilities in green R&D that align with increasingly stringent environmental protection regulations. Zhang [40] demonstrated that manufacturers can boost their profits by producing green products driven by a growing awareness of green consumption. Wen [41] established three models to compare subsidy effects by considering customers' green preferences under different subsidy policies. Nielsen [42] formulated and analyzed twelve analytical models by considering the impact of single- and two-period procurement decisions on the sustainability goals of supply chain members, revealing that the optimal preference is highly contingent upon game structure and procurement decisions.

However, the rapid development of e-commerce has led to a significant surge in the scale of online sales in recent years. Many firms have expanded from single-channel to dual-channel supply chains to augment their market share [43]. Consequently, the dual-channel supply chain model, characterized by the coexistence of online direct sales channels and traditional retail channels, has emerged as the predominant distribution strategy for manufacturers.

Extensive research has been conducted on the dual-channel supply chain, primarily focusing on pricing strategy and channel coordination [44,45]. Barman et al. [18] developed a dual-channel supply chain model comprising a manufacturer and a retailer. They explored optimal pricing decisions and profit maximization by selling products online and in traditional retail settings. Zhou et al. [46] also examined pricing decisions using the Stackelberg game model. Additionally, there is growing attention to environmental concerns in dual-channel supply chains. Zhang et al. [47] investigated dynamic pricing strategies and green initiatives in a two-stage dual-channel supply chain, demonstrating that periodic price reviews can enhance profits and market demand for the entire supply chain. Li et al. [48] similarly analyzed pricing decisions within a dual-channel green supply chain.

In addition, a growing focus is on operational management within green supply chain management. According to [40], how firms improve the environmental quality of their products and how the government sets environmental policies are two main aspects. In terms of the ecological quality of the product, the standard methods to improve product environmental quality include increasing the technology investment (e.g., at the clean-up level or emission level), improving social responsibility, introducing eco-labeling, and so on [49–51]. In terms of government environmental policies, common measures (e.g., environmental standards, subsidies, and tax policies) are widely used across the world [52–54]. Regarding research methods, life cycle assessment (LCA) and the analytic hierarchy process (AHP) are qualitative research methods used with high frequency, in addition to fuzzy theory and game theory, which are also more commonly used methods. Among the above methods, game theory is widely used in the study of carbon emission reduction as an effective approach [11,42,52–55].

The above research about the green supply chain mainly focuses on elements of a traditional supply chain, such as pricing strategies and channel coordination. The existing literature rarely considers comparing economic and ecological benefits under a green dual-channel supply chain, especially under different government subsidies. Considering the product's green quality, this study compares and analyzes the impact of different government subsidies on optimal decision-making in the dual-channel green supply chain.

2.2. Government Subsidy Strategy in the Green Supply Chain

Another closely related literature stream is government subsidies. A subsidy policy regulates emissions by motivating firms to decrease the emissions discharged to the environment. Research on environmental subsidies to regulate emissions can be dated back to Lerner. Mitra [56] examined the effects of government subsidies as a means to promote remanufacturing activity, which involved government subsidies in the field of supply chains. Hojnik [57] believed that the government's economic incentives (e.g., preferential subsidies) could promote green innovation. In recent years, numerous scholars have extensively examined the impact of government subsidies on green supply chains. Based on the diverse recipients of these subsidies, manufacturers, retailers, and customers are identified as the most common beneficiaries. Several studies in the literature have conducted comparative analyses by employing game models.

Some scholars have paid attention to the impact of government subsidies on manufacturers in the green supply chain. For example, Zhu et al. [58] developed a game model that considers the environmental impact of products and government subsidies, which investigated the optimal subsidy coefficient per unit product provided by the government and the optimal level of greenness for manufacturers and product prices when two competing producers receive government subsidies. Shi and Min [59] examined the manufacturer's strategy of subsidizing unit products and found that subsidizing per unit product leads to higher efficiency in terms of subsidy allocation. Zhao, Zhu, and Cui [60] discovered that sharing government subsidies with customers can increase demand and generate more profits while constructing a subsidy strategy model for manufacturers. Han, Liu, and Jin [61] studied how retailers' fair concerns influence optimal decision-making within a dual-channel green supply chain under different scenarios.

In terms of government subsidies for retailers or customers, the influence of green subsidies has been discussed by scholars. For instance, Ma, Zhao, and Ke [62] examined customer subsidy strategies in the dual-channel supply chain and found that such subsidies mutually benefit manufacturers and retailers. Cohen, Lobel, and Perakis [24] investigated the effects of customer subsidy strategies under conditions of demand uncertainty. Their findings revealed that subsidy strategies can effectively coordinate product production quantities and sales prices. Sun and Miao [63] explored the differences between customer and manufacturer subsidies, demonstrating that it is more conducive to enhancing social welfare and expanding the market share when the government subsidizes customers. Bian, Zhang, and Zhou [35] investigated the influence of customer subsidies and manufacturer subsidies, and it was observed that customer subsidies lead to relatively lower emission reductions but higher levels of consumption. Furthermore, with the development of technology, subsidy recipients are increasingly diversified. Zhong, Lai, Guo, and Tang [23] studied the government subsidy strategy under the e-commerce of agricultural products by analyzing three different subsidy subjects: agricultural cooperatives, online shopping platforms, and customers. In the study by Mu, Li, Dai, Li, Zhang, Zhang, and Li [21], a platform supply chain composed of manufacturers and third-party platforms was constructed.

In general, although the previous literature has made a significant contribution to the knowledge of government subsidies in green supply chains, there is hardly any research that explores the effect of different subsidies between manufacturers and retailers in the dual-channel green supply chain (e.g., a horizon comparison between government subsidies for manufacturers and those for retailers). In reality, it is crucial and urgent for the government to make decisions for the following problems: Whether to subsidize? Which member to subsidize? How to subsidize? This study contributes to the existing research by comparing the product's green quality and the supply chain's overall profit, which considers the green quality and channel preferences. It also addresses the literature gaps on government decisions to improve the green quality and overall profits under strategies for subsidizing manufacturers or retailers in green supply chains. Table 1 summarizes the difference between our work and previous studies.

Table 1. Key related studies in the literature.

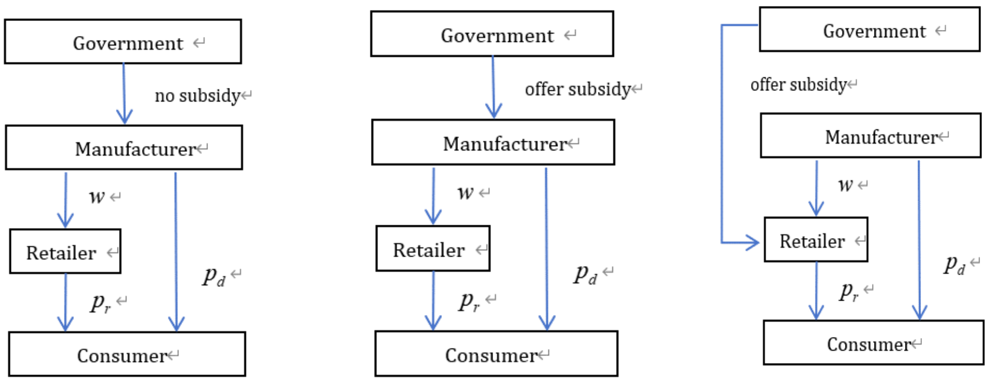
Key Related Literature	Subsidy Strategy		Government Subsidy Coefficient	Customers' Channel Preference	Dual-Channel
	Manufacturer	Retailer			
Sun, Wan [64]	√		√		
Wang, Huo [65]	√	√			
Chai, Sun [66]	√	√			
Hu, Wang [31]	√			√	√
Liu, Chang [34]			√	√	√
Zhang, Xue [67]	√	√	√		
Meng, Li [11]	√		√	√	√
Wang [68]	√		√	√	
Li, Guo [69]	√	√	√		
This study	√	√	√	√	√

3. Problem Description and Assumptions

3.1. Problem Description

In this study, we focus on three subsidy scenarios under two different subsidy strategies: no government subsidy in the supply chain (referred to as Mode N), government subsidization of the manufacturer (referred to as Mode M), and government subsidization of the retailer (referred to as Mode R), as illustrated in Figure 1. We limit our analysis to these three specific scenarios to determine which policy would enable the government to achieve higher levels of product greenness and overall profit.

Therefore, we construct a dual-channel green supply chain composed of one manufacturer and one retailer. Within the context of sustainable development, both the manufacturer and the retailer will intensify their efforts toward green production and sales promotion. To effectively stimulate the manufacturer and the retailer to implement green efforts, the government will provide subsidies based on the unit product's level of green quality [62]; the higher the product's green quality, the greener the subsidies that will be provided. There are two common government subsidy strategies: one is to subsidize the manufacturer, and the other is to subsidize the retailer.



(a) Model N (b) Model M (c) Model R

Figure 1. The structure of the dual-channel green supply chain.

This study applies the Stackelberg game model to investigate the influence of different government subsidies on pricing decisions in a dual-channel supply chain. In this model, it is assumed that the manufacturer only produces one type of green product. Simultaneously, the manufacturer wholesales products to the retailer through an offline retail channel and sells the green product directly to the market via an online direct channel. Referring to the existing literature, we designate the manufacturer as the leader who determines the wholesale price, the green quality level of the product, and the online sales price. The retailer acts as a follower who determines their own sales price.

3.2. Notation

The variables and their notations are shown in Table 2.

Before the formulation of the model, some notations are defined in Table 2. Parameters are dimensionless, and all the coefficients are no less than 0 except for the customer channel preference, alpha, and the customers' sensitivity coefficient to the green quality, which ranges between 0 and 1 [12].

Table 2. Related notation.

Parameters	
w	Unit wholesale price of the product
c	Unit production cost of the product
α	Customer preference to purchase through the online channel
$1-\alpha$	Customer preference to purchase through the offline channel
β	Own-price elasticity of demand
θ	The cross-price elasticity of demand
Q	Primary potential market demand for the product
k	Cost coefficient of green quality per unit
s	The government's subsidy coefficient
γ	Sensitivity coefficient of the customer to the green quality
D_r^i	Demand function of the offline channel ($i = N, M, R$)
D_d^i	Demand function of the online channel ($i = N, M, R$)
π_m^i	Profit function of the manufacturer ($i = N, M, R$)
π_r^i	Profit function of the retailer ($i = N, M, R$)
π_{sc}^i	Profit function of the supply chain ($i = N, M, R$)
Decision variables	
g^i	Green quality of the product
p_r	Offline sales price of the product
p_d	Online sales price of the product

In this section, the subscript represents the chain member or the entire supply chain, while the superscript identifies the different subsidy models. N represents no-subsidy strategies, M represents subsidies to manufacturers, and R represents subsidies to retailers.

3.3. Assumptions

Assumption 1. *Assume that the manufacturer is the leader, and the retailer is the follower; both are risk-neutral and seek to maximize their profits.*

Assumption 2. *The manufacturer only produces and sells one type of green product online, while the retailer wholesales and sells the green product to customers through offline channels. To avoid the trivial case, based on [70], it is assumed that* $p_r > w > c$ *and* $p_d > w > c$.

Assumption 3. *To simplify the calculation, refer to the previous research [11,70]; demand in each channel is expressed as a linear function affected by the price and the product's green quality. To simplify calculations, the retailer's order quantity is assumed to be the same as the demand function.*

Assumption 4. *Using the function form used by Ghosh and Shah [71], the cost function of the manufacturer to improve the green quality of the product is* $c(g) = \frac{1}{2}kg^2(k > 0)$*. It represents the cost that the manufacturer pays to improve the product's green quality while indicating the manufacturer's cost coefficient for this improvement.*

4. Model Formulation and Analysis

Equilibrium solutions are derived in three scenarios: no government subsidization strategy, the manufacturer subsidy strategy, or the retailer subsidy strategy. The impacts of the government's subsidy coefficient and the customer channel preference for improving green quality and profits are also analyzed. Finally, a comparison is made between the manufacturer and the retailer subsidy strategies to investigate which strategy is more effective in achieving a higher green quality and supply chain profit.

4.1. Demand and Profit Function

The study includes a manufacturer(M) and a retailer (R). Assuming that the product's green quality is denoted as g, the customer green preference coefficient as $\gamma(0 < \gamma < 1)$, and the customer preference to purchase through the offline channel as $\alpha(0 < \gamma < 1)$, then the customer preference to purchase through the offline channel is($1 - \alpha$). Based on [70] and Assumption 3, the market demand functions for both the offline channel D_r^i and the online channel D_d^i are formulated as follows:

$$D_r^i = (1 - \alpha)Q - \beta p_r + \theta p_d + \gamma g \quad (1)$$

$$D_d^i = \alpha Q - \beta p_d + \theta p_r + \gamma g \quad (2)$$

The market demand functions of the offline channel D_r^i and the online channel D_d^i are linear and are influenced by various factors, including the basic potential market demand, customers' inclination to purchase through offline and online channels, offline and online sales prices, the own-price elasticity of demand, the cross-price elasticity of demand, the green quality of the product, and the customer sensitivity coefficient toward the green quality.

Based on Assumptions 1, 2, and 4, the profit functions of the manufacturer and the retailer are established.

$$\pi_m = (w - c)D_r + (p_d - c)D_d - kg^2/2 \quad (3)$$

$$\pi_r = (p_r - w)D_r \quad (4)$$

Thus, the total profit of the supply chain is written as

$$\pi_c = \pi_m + \pi_r$$

The study is analyzed to derive the optimal results under three decision models: (i) models involving no government subsidization; (ii) models that involve subsidizing the manufacturer; and (iii) models that involve subsidizing the retailer.

4.2. Models involving No Government Subsidization

In this model, the supply chain is considered vertically integrated and centrally commanded by a single decision-maker who ensures the best value for all the decision variables (including the green quality of the product and the online and offline sales prices) by maximizing the total profit of the supply chain. Consequently, the supply chain profit function can be obtained as follows:

$$\pi_c^N = \pi_m^N + \pi_r^N = (p_r - c)D_r + (p_d - c)D_d - kg^2/2 \quad (5)$$

Lemma 1. *When the manufacturer and the retailer make centralized decisions, and when* $k > \frac{\gamma^2}{2\beta}$*, the equilibrium solutions* $(g^{N^*}, p_d^{N^*}, p_r^{N^*})$ *are below.*

$$g^{N^*} = -\frac{\gamma(2c(\theta-\beta)+Q)}{2(\gamma^2+k(\theta-\beta))} \quad (6)$$

$$p_d^{N^*} = \frac{1}{4}\left(\frac{2c\gamma^2-kQ}{\gamma^2+k(\theta-\beta)}+2c+\frac{(2\alpha-1)Q}{\beta+\theta}\right) \quad (7)$$

$$p_r^{N^*} = \frac{1}{4}\left(\frac{2c\gamma^2-kQ}{\gamma^2+k(\theta-\beta)}+2c+\frac{(1-2\alpha)Q}{\beta+\theta}\right) \quad (8)$$

By substituting the equilibrium solutions into (5), the optimal supply chain profit is

$$\pi_c^{N^*} = -\frac{2k(2c^2(\beta-\theta)^2(\beta+\theta)+2cQ(\theta^2-\beta^2)+Q^2(2(\alpha-1)\alpha\beta-2(\alpha-1)\alpha\theta+\beta))-(1-2\alpha)^2\gamma^2Q^2}{8(\beta+\theta)(\gamma^2+k(\theta-\beta))} \quad (9)$$

Lemma 1 implies that the supply chain can find equilibrium solutions when the related parameters reach a threshold without government subsidization.

Next, we analyze the effects of customers' channel preference on the green quality of products, the online and offline sales prices, and the optimal profits in Proposition 1.

Proposition 1. *In the model without government subsidies, the influence of customer preference of channel on the green quality of product, online and offline sales prices, and optimal profits is as follows:*

When $0 < c < \frac{Q}{2\beta-2\theta}$ *and* $k > \frac{\gamma^2}{\beta-\theta}$*, we have* (1) $\frac{\partial g^N}{\partial \alpha} = 0$, $\frac{\partial p_d^N}{\partial \alpha} > 0$, $\frac{\partial p_r^N}{\partial \alpha} > 0$*, and* $\frac{\partial \pi_c^N}{\partial \alpha} > 0$; (2) $0 < \alpha < \frac{1}{2}, p_d^N < p_r^N; \frac{1}{2} < \alpha < 1, p_d^N > p_r^N$.

Proposition 1 indicates that without government subsidies, the customer channel preference does not affect the green quality of products. However, given a certain level of production cost, both online and offline sales prices and overall profits are all positively related to the customer channel preference. Moreover, it implies that customer channel preferences can impact the magnitude of online and offline prices. The greater the customer channel preference, the higher the pricing associated with that channel will be.

4.3. Models involving Government Subsidization

There are two main strategies for the government subsidization of green products in the supply chain: (1) the strategy of subsidizing the manufacturer and (2) the strategy of subsidizing the retailer. Concerning the different government subsidy strategies, we first derived equilibrium solutions of the manufacturer and retailer subsidy strategies. Then, the impacts of the government's subsidy coefficient and the customer channel preference for improving green quality and profits are analyzed. Finally, a comparison is made between the manufacturer and the retailer subsidy strategies to investigate which approach is more effective in achieving a higher green quality and supply chain profit.

4.3.1. The Government's Strategy of Subsidizing the Manufacturer

Assuming that the unit price of government subsidies provided to the manufacturer is sg, the profit functions of supply chain members can be expressed as follows:

$$\pi_m^M = (w-c+sg)D_r + (p_d-c+sg)D_d - kg^2/2 \quad (10)$$

$$\pi_r^M = (p_d-w)D_r \quad (11)$$

$$\pi_c^M = \pi_m^M + \pi_r^M = (p_d-c+sg)D_r + (p_d-c+sg)D_d - kg^2/2 \quad (12)$$

Lemma 2. *When the manufacturer and the retailer make centralized decisions, and* $k > \frac{(\gamma+s(\beta-\theta))^2}{\beta-\theta}$*, then the equilibrium solutions* $(g^{M^*}, p_d^{M^*}, p_r^{M^*})$ *are as follows:*

$$g^{M^*} = \frac{[2c(\theta-\beta)+Q][\gamma+s(\beta-\theta)]}{2k(\beta-\theta)-2[\gamma+s(\beta-\theta)]^2} \tag{13}$$

$$p_d^{M^*} = \frac{2k(c\theta^2-\beta(\beta c+\alpha Q)+(\alpha-1)\theta Q)+(\gamma+s(\beta-\theta))(4c\gamma(\beta+\theta)))+(2\alpha-1)\gamma Q+Qs(2\alpha\beta+(3-2\alpha)\theta+\beta))}{4(\beta+\theta)(k(\theta-\beta)+(\gamma+s(\beta-\theta))^2)} \tag{14}$$

$$p_r^{M^*} = \frac{(\gamma+s(\beta-\theta))\left(4c\gamma(\beta+\theta)+Q(-2\alpha\gamma+\gamma+s(-2\alpha\beta+2\alpha\theta+3\beta+\theta))\right)-2k(c(\beta-\theta)(\beta+\theta)+Q(-\alpha\beta+\alpha\theta+\beta))}{4(\beta+\theta)(k(\theta-\beta)+(\gamma+s(\beta-\theta))^2)} \tag{15}$$

By substituting the optimal solutions into (12), the optimal supply chain profit is

$$\pi_c^{M^*} = \frac{(1-2\alpha)^2Q^2(\gamma+s(\beta-\theta))^2-2k(2c^2(\beta-\theta)^2(\beta+\theta)+2cQ(\theta^2-\beta^2)+Q^2(2(\alpha-1)\alpha\beta-2(\alpha-1)\alpha\theta+\beta))}{8(\beta+\theta)(k(\theta-\beta)+(\gamma+s(\beta-\theta))^2)} \tag{16}$$

Lemma 2 indicates that the supply chain can find the equilibrium solutions when the related parameters reach a threshold under the government subsidizing the manufacturer.

Proposition 2. *Under the model of government subsidies to the manufacturer, the influence of the subsidy coefficient on the green quality of the product, online and offline sales prices, and optimal profits is as follows:*

When $0 < c < \frac{Q}{2(\beta-\theta)}$ and $k > \frac{[\gamma+s(\beta-\theta)]^2}{\beta-\theta}$, *then we have* (1) $\frac{\partial g^M}{\partial s} > 0$; (2) $0 < s < \frac{\gamma}{\beta-\theta}$, $\frac{\partial p_r^M}{\partial s} < 0$ *and* $\frac{\partial p_d^M}{\partial s} < 0$; (3) $\frac{\partial \pi_c^M}{\partial s} > 0$.

Proposition 2 states that when the subsidy coefficient satisfies $0 < s < \frac{\gamma}{\beta-\theta}$, an increase in government subsidies leads to a concurrent enhancement in the green quality of the product, while online and offline prices demonstrate a negative relationship. This ultimately results in overall profit growth. This implies that the government strategy of subsidizing manufacturers is beneficial for improving both the green quality of products and overall supply chain profits.

Proposition 3. *Under the model of government subsidies to the manufacturer, the influence of customers' preference of channel on the green quality of product, online and offline sales prices, and optimal profits is as follows:*

(1) $\frac{\partial g^M}{\partial \alpha} = 0$, $\frac{\partial p_d^M}{\partial \alpha} > 0$ *and* $\frac{\partial p_r^M}{\partial \alpha} < 0$; (2) $0 < \alpha < \frac{1}{2}$, $\frac{\partial \pi_c^M}{\partial \alpha} < 0$; $\frac{1}{2} < \alpha < 1$, $\frac{\partial \pi_c^M}{\partial \alpha} > 0$.

Proposition 3 implies that under the models of subsidizing manufacturers, customers' channel preference does not affect the green quality of the product. Furthermore, it suggests that online sales prices are positively associated with customers' online channel preference, while offline sales prices exhibit a negative relationship. This reveals that customers' channel preferences will impact the pricing of different channels. Additionally, it indicates that as the customers' online channel preference increases, the overall supply chain's profit transitions from negative to positive. It can be inferred that when $\alpha = \frac{1}{2}$, indicating customers' equal preference for online and offline channels, the overall profit of the supply chain reaches the lowest point.

Proposition 4. *Under the model of government subsidies to the manufacturer, the impact of the customer sensitivity coefficient toward green quality and the cost elasticity coefficient of green quality, online and offline sales prices, as well as optimal profits, can be described as follows:*

When $0 < c < \frac{Q}{2(\beta-\theta)}$ *and* $k > \frac{[\gamma+s(\beta-\theta)]^2}{\beta-\theta}$, *then we have* (1) $\frac{\partial g^M}{\partial \gamma} > 0$, $\frac{\partial p_d^M}{\partial \gamma} > 0$, $\frac{\partial p_r^M}{\partial \gamma} > 0$ *and* $\frac{\partial \pi_c^M}{\partial \gamma} > 0$; (2) $\frac{\partial g^M}{\partial k} < 0$, $\frac{\partial p_d^M}{\partial k} < 0$, $\frac{\partial p_r^M}{\partial k} < 0$ *and* $\frac{\partial \pi_c^M}{\partial k} < 0$.

Proposition 4 implies that under the model of government subsidies to the manufacturer, the customer's sensitivity coefficient to green quality positively influences the product's green quality, online and offline prices, and overall profit. Conversely, the cost elasticity coefficient of green quality per unit negatively affects these factors. Therefore, improving the customer green preference coefficient and reducing the product green cost coefficient will contribute to the enhancement of sustainability in the supply chain.

4.3.2. The Government's Strategy of Subsidizing the Retailer

Assuming that the unit price of government subsidies provided to the retailer is sg, the profit functions of the members in the supply chain are as follows:

$$\pi_m^R = (w-c)D_r + (p_d-c)D_d - kg^2/2 \tag{17}$$

$$\pi_r^R = (p_d - w + sg)D_r \tag{18}$$

$$\pi_c^R = \pi_m^R + \pi_r^R = (p_d - c + sg)D_r + (p_d - c)D_d - kg^2/2 \tag{19}$$

Lemma 3. *When the manufacturer and the retailer make centralized decisions, and* $k > \frac{\gamma^2}{\beta-\theta} + \frac{\beta s^2}{2} + \gamma s$, *then the equilibrium solutions* $(g^{R^*},\ p_d^{R^*},\ p_r^{R^*})$ *are as follows:*

$$g^{R^*} = \frac{c(\beta-\theta)(2\gamma+s(\beta-\theta)) + Q((\alpha-1)s(\beta-\theta)-\gamma)}{2\gamma^2+2k(\theta-\beta)+\beta s^2(\beta-\theta)+2\gamma s(\beta-\theta)} \tag{20}$$

$$p_d^{R^*} = \frac{2k(c\theta^2-\beta(\beta c+\alpha Q)+(\alpha-1)\theta Q)+c(\beta+\theta)(4\gamma^2+\beta s^2(\beta-\theta)+3\gamma s(\beta-\theta))+Q(\gamma+\beta s)((2\alpha-1)\gamma+s(\alpha\beta-\alpha\theta+\theta))}{2(\beta+\theta)(2\gamma^2+2k(\theta-\beta)+\beta s^2(\beta-\theta)+2\gamma s(\beta-\theta))} \tag{21}$$

$$p_r^{R^*} = \frac{-2k(c(\beta-\theta)(\beta+\theta)+Q(-\alpha\beta+\alpha\theta+\beta))+c(\beta+\theta)(4\gamma^2+\theta s^2(\beta-\theta)}{2(\beta+\theta)\left(2\gamma^2+2k(\theta-\beta)+\beta s^2(\beta-\theta)+2\gamma s(\beta-\theta)\right)} + \frac{\gamma s(\beta-\theta))+Q((1-2\alpha)\gamma^2+s^2(-2(\alpha-1)\beta^2+\alpha\beta\theta+(\alpha-1)\theta^2)+\gamma s(-\alpha\beta+3\alpha\theta+2\beta))}{2(\beta+\theta)(2\gamma^2+2k(\theta-\beta)+\beta s^2(\beta-\theta)+2\gamma s(\beta-\theta))} \tag{22}$$

By substituting the optimal solutions into (19), the optimal profits are

$$\pi_c^{R^*} = \frac{(cs(\theta^2-\beta^2)+(2\alpha-1)\gamma Q+Qs(\alpha(\beta-\theta)+\theta))^2-2k(2c^2(\beta-\theta)^2(\beta+\theta)+2cQ(\theta^2-\beta^2)+Q^2(2(\alpha-1)\alpha\beta-2(\alpha-1)\alpha\theta+\beta))}{4(\beta+\theta)(2\gamma^2+2k(\theta-\beta)+\beta s^2(\beta-\theta)+2\gamma s(\beta-\theta))} \tag{23}$$

Lemma 3 indicates that the supply chain can find equilibrium solutions when the related parameters reach a threshold when the government subsidizes the retailer.

Proposition 5. *Under the model of government subsidies to the retailer, the impact of the subsidy coefficient on the green quality of product, online and offline sales prices, and optimal profits is as follows:*

When $0 < c < c_1, k > \frac{\gamma^2}{\beta-\theta} + \frac{\beta s^2}{2} + \gamma s$ *and* $\frac{\beta}{2+\sqrt{5}} < \theta < \frac{3\beta}{2+\sqrt{13}}$, *then we have* (1) $\frac{\partial g^R}{\partial s} > 0$ *and* $\frac{\partial p_d^R}{\partial s} > 0$; (2) $0 < s < \frac{\gamma}{\beta-\theta}$ *and* $0 < \alpha < \frac{1}{2}$, $\frac{\partial p_r^R}{\partial s} < 0(0 < c < c_2)$, $\frac{\partial p_r^R}{\partial s} > 0(c_2 < c < c_1)$; $0 < s < \frac{\gamma}{\beta-\theta}$ *and* $\frac{1}{2} < \alpha < 1$, $\frac{\partial p_r^R}{\partial s} < 0(0 < c < c_1)$; (3) $0 < c < c_3$, $\frac{\partial \pi_c^R}{\partial s} > 0$,

where

$$c_1 = \frac{Q(2\alpha\gamma^2 - 2(\alpha-1)k(\beta-\theta) - ((\alpha-1)\beta s^2(\beta-\theta)) + 2\beta\gamma s)}{(\beta-\theta)(2\gamma^2 + 2k(\beta-\theta) + \beta s^2(\beta-\theta) + 4\beta\gamma s)} \tag{24}$$

$$c_2 = \frac{Q(\gamma(-2(\alpha+1)\gamma^2 + s^2(\beta-\theta)(3\alpha\beta - 2\alpha\theta - 2\beta + 2\theta) + 2\gamma s(2\alpha\beta - 2\alpha\theta - 3\beta + 2\theta)) - 2k(\beta-\theta)(2(\alpha-1)s(\beta-\theta) - \alpha\gamma))}{(\beta-\theta)(2k(\beta-\theta)(\gamma + 2s(\beta-\theta)) - \gamma(6\gamma^2 + s^2(\beta-2\theta)(\beta-\theta) + s(8\beta\gamma - 4\gamma\theta)))} \tag{25}$$

$$c_3 = \frac{Q(\gamma((2\alpha-1)\gamma + s(\alpha\beta - \alpha\theta + \theta)) - 2(\alpha-1)k(\beta-\theta))}{(\beta-\theta)(2k(\beta-\theta) + \gamma s(\beta+\theta))} \tag{26}$$

Proposition 5 states that under the model of government subsidies to the retailer, an increase in government subsidies leads to a simultaneous enhancement of the green quality of the product, online prices, and overall profit. However, the impact on offline prices varies depending on different production costs. Different from the manufacturer's subsidy strategy, when online channels are small, as production costs rise, the retailer's subsidy strategy leads to a shift from a negative correlation to a positive correlation between the subsidy coefficient and offline prices, as the retailer's higher prices compensate for the rising production cost.

Proposition 6. *Under the model of government subsidies to the retailer, the impact of customers' preference of channel on the green quality of product, online and offline sales prices, and optimal profits is as follows:*

When $k > \frac{\gamma^2}{\beta-\theta} + \frac{\beta s^2}{2} + \gamma s$ *and* $\frac{\beta}{2+\sqrt{5}} < \theta < \frac{3\beta}{2+\sqrt{13}}$, *then we have* (1) $\frac{\partial g^R}{\partial \alpha} < 0$, $\frac{\partial p_d^R}{\partial \alpha} < 0$; (2) $0 < s < \frac{\gamma}{\beta-\theta}$, $\frac{\partial p_r^R}{\partial \alpha} < 0$; (3) $0 < c < c_4$, $\frac{\partial \pi_C^R}{\partial \alpha} < 0$; $c > c_4$, $\frac{\partial \pi_C^R}{\partial \alpha} > 0$,

where

$$c_4 = \frac{Q((2\gamma + s(\beta-\theta))((2\alpha-1)\gamma + s(\alpha\beta - \alpha\theta + \theta)) - 2(2\alpha-1)k(\beta-\theta))}{s(\beta^2-\theta^2)(2\gamma + s(\beta-\theta))} \tag{27}$$

Proposition 6 implies that a product's green quality as well as its online and offline sales prices exhibit a negative influence on the customer channel preference under the retailer's subsidy strategy, which is different from the manufacturer's subsidy strategy. What may explain this result is that the increase in customers' preference for online channels primarily benefits manufacturers. Additionally, when the product cost meets certain conditions, the impact of customer preference on the channel's overall profits transitions from negative to positive.

Proposition 7. *Under the model of government subsidies to the retailer, the impact of the customer sensitivity coefficient toward green quality and the cost elasticity coefficient per unit of green quality on the product's green quality, online and offline sales prices, as well as optimal profits can be described as follows:*

When $0 < c < \frac{Q}{2(\beta-\theta)}$, $k > \frac{\gamma^2}{\beta-\theta} + \frac{\beta s^2}{2} + \gamma s$, $0 < \alpha \le \frac{1}{2}$, $0 < s < \frac{\gamma}{\beta-\theta}$ *and* $\frac{\beta}{2+\sqrt{5}} < \theta < \frac{3\beta}{2+\sqrt{13}}$, *then we have* (1) $\frac{\partial g^R}{\partial \gamma} > 0$, $\frac{\partial p_d^R}{\partial \gamma} > 0$, $\frac{\partial p_r^R}{\partial \gamma} > 0$ *and* $\frac{\partial \pi_C^R}{\partial \gamma} > 0$; (2) $\frac{\partial g^R}{\partial k} < 0$, $\frac{\partial p_d^R}{\partial k} < 0$, $\frac{\partial p_r^R}{\partial k} < 0$ *and* $\frac{\partial \pi_C^R}{\partial k} < 0$.

Proposition 7 implies that the green quality of the product, the online and offline prices, and the overall profit are all positively related to the sensitivity coefficient of the customer to green quality. In contrast, they are all negatively associated with the cost elasticity coefficient of green quality per unit. Proposition 7 reveals that the influence of both the customer green sensitivity coefficient and the green product cost elasticity coefficient on the optimal decision remains consistent for the two different subsidy strategies. Irrespective of the subsidy strategy adopted, enhancing the customer's green sensitivity coefficient and reducing the green product cost elasticity coefficient will foster sustainable development within the supply chain.

4.3.3. Comparative Analysis of the Two Decision Models

This subsection mainly compares and analyzes the effects of the two strategies of subsidizing manufacturers or retailers, investigating whether to subsidize and which strategies can improve the green quality of the product and supply chain profits.

Proposition 8. The comparison and analysis of the optimal solution for green quality products in two models of subsidizing manufacturers or retailers are as follows:

When $0 < c < \frac{Q}{2(\beta-\theta)}$, $k > \frac{[\gamma+s(\beta-\theta)]^2}{\beta-\theta}$, $\beta > \frac{4}{3}\theta$, $0 < s < \frac{\gamma}{\beta-\theta}$, (1) $0 < \alpha < \frac{1}{2}$ and $0 < c < c_5$, $g^M > g^R$; (2) $0 < \alpha < \frac{1}{2}$ and $c_5 < c < \frac{Q}{2(\beta-\theta)}$, $g^M < g^R$; (3) $\frac{1}{2} < \alpha < 1$ and $0 < c < \frac{Q}{2(\beta-\theta)}$, $g^M > g^R$,

where

$$c_5 = \frac{Q(2\alpha k(\theta-\beta) + (\gamma + s(\beta-\theta))(2(\alpha-1)\gamma + (2\alpha-1)\beta s - 2(\alpha-1)\theta s))}{2(\beta-\theta)(k(\theta-\beta) + (\theta s - \gamma)(\gamma + s(\beta-\theta)))} \quad (28)$$

According to Proposition 8, under the fulfillment of the above conditions, when both customers' online channel preference and production cost are low, the green quality of products is higher under a government subsidy strategy toward manufacturers compared with one toward retailers. However, as production costs increase, subsidizing the retailer can lead to a higher level of green quality. Nevertheless, the proportion of customers' online channel preferences has surpassed 50%, and subsidizing the manufacturer can yield better results in terms of green quality products. These findings suggest that channel preference influences government subsidy strategies: when there is a high preference for online channels, it is more beneficial to subsidize manufacturers. In contrast, when there is a limited inclination toward online channels, the choice of subsidy strategy depends on production costs.

Proposition 9. *The comparison and analysis of the optimal solution for the overall profits in the two models of subsidizing manufacturers or retailers are as follows:*

When $k > \frac{[\gamma+s(\beta-\theta)]^2}{\beta-\theta}$, $\beta > \frac{4}{3}\theta$, $0 < s < \frac{\gamma}{\beta-\theta}$, *(1)* $0 < \alpha < \frac{1}{2}$ *and* $0 < c < c_6$, $\pi_c^M > \pi_c^R$; *(2)* $0 < \alpha < \frac{1}{2}$ *and* $c_6 < c < \frac{Q}{2(\beta-\theta)}$, $\pi_c^M < \pi_c^R$; *(3)* $\frac{1}{2} < \alpha < 1$ *and* $0 < c < c_1$, $\pi_c^M > \pi_c^R$.

$$c_6 = \frac{Q((\gamma+s(\beta-\theta))^2((2\alpha-1)\gamma + s(\alpha\beta - \alpha\theta + \theta)) - k(\beta-\theta)(2\alpha\gamma + \gamma + (\alpha+1)s(\beta-\theta)))}{(\beta-\theta)(s(\beta+\theta)(\gamma+s(\beta-\theta))^2 - k(\beta-\theta)(4\gamma + 3s(\beta-\theta)))} - \frac{\sqrt{\frac{(1-2\alpha)^2 Q^2(\gamma+s(\beta-\theta))^2(k(\theta-\beta) + (\gamma+s(\beta-\theta))^2)(2\gamma^2 + 2k(\theta-\beta) + \beta s^2(\beta-\theta) + 2\gamma s(\beta-\theta))}{(\beta-\theta)^2(k(\beta-\theta)(4\gamma+3s(\beta-\theta)) - s(\beta+\theta)(\gamma+s(\beta-\theta))^2)^2}}}{\sqrt{2}} \quad (29)$$

According to Proposition 9, when the above conditions are satisfied, firstly, both customers' online channel preference and production costs are low; under the subsidy strategy for manufacturers, the overall profit is higher compared with subsidizing retailers. Secondly, as the production cost increases, the results will change, and subsidizing the retailer can lead to higher levels. Last but not least, subsidizing manufacturers can yield higher supply chain profits if the preference for online channels among customers is dominant. These findings indicate that channel preference influences government subsidy strategies: when there is a high preference for online channels, subsidizing manufacturers is more favorable, whereas when there is a low preference for online channels, the choice of subsidy strategy depends on production costs.

In conclusion, when the preference for online channels among customers is dominant, subsidizing manufacturers is generally considered a more favorable strategy in both government subsidy modes, as it can result in higher product green quality and overall profit within the dual-channel green supply chain. However, when the customers' preference for online channels is limited, the selection of a subsidy strategy depends on the cost of production. Furthermore, if there is no difference in customer online and offline channel preferences, the overall profit of the supply chain would reach the lowest point under the strategy of subsidizing manufacturers. Therefore, the government should encourage manufacturers and retailers to engage in healthy competition within the market while also devising appropriate subsidy strategies based on customer channel preferences and fluctuations in product production costs to promote the improvement of product green quality and supply chain profits.

4.4. Results and Discussion

The theoretical results can be derived based on Propositions 1–9, and some of them are obtained through the comparative analysis of the nine propositions, as follows.

4.4.1. Impacts of Government Subsidy on the Supply Chain Decision Variables and Profits

According to Propositions 2 and 5, the green product quality exhibits improvement while online sales prices decline as the subsidy coefficient increases. However, the impact of the two strategies on retail price and overall profit varies.

Under the manufacturer subsidization strategy, Proposition 2 shows that an increase in the subsidy coefficient negatively affects retail price while positively affecting total profit, aligning with previous research findings.

Conversely, under the retailer subsidization strategy, Proposition 5 indicates that the relationship between retail price and total profit concerning the subsidy coefficient is inconsistent, as their relationship is also influenced by customer channel preference and production cost. When the customers' online channel preference is dominant, all the decision variables and the overall profit also show the same relationship as the strategy of subsidizing the manufacturer.

In summary, the raising of the government's subsidy stimulates manufacturers to improve the green quality of products, thereby generating greater benefits for manufacturers, retailers, and the entire supply chain. Consequently, it becomes crucial to reinforce government subsidies, especially when both subsidy levels and production costs are low, in order to improve the green quality of the product and the overall profit.

4.4.2. Impacts of Channel Preference on the Supply Chain Decision Variables and Profits

Propositions 1, 3, 6, 8, and 9 demonstrate that customers' increasing online channel preferences under different subsidy strategies influence the decision variables and overall profit. When both the online channel preference and the production cost are low, subsidizing the manufacturer proves superior to the retailer across three distinct modes. Therefore, governments must consider customers' channel preferences when formulating policies.

Propositions 1, 3, and 6 suggest that customers' increasing preference for online channels under different subsidy strategies influences the decision variables and overall profit. This aspect will be further discussed below.

Propositions 1 and 3 state that online channel preference does not affect the green quality without a government subsidy or manufacturer subsidization strategy. However, according to Proposition 6, when government subsidies are given to retailers, customers' online channel preference negatively influences the green quality. The primary reason for this phenomenon is the provision of government subsidies to retailers, which diminishes manufacturers' motivation for enhancing the green quality of products.

Secondly, regarding online and offline prices, the impact of online channel preference on them varies among three modes. Without government subsidies, online and offline prices positively correlate with the online channel preference, whereas under the manufacturer subsidization strategy, all decision variables demonstrate a negative association with the online channel preference. Nevertheless, when the retailer receives government subsidies, online channel preference is positively correlated with the online sales price while being negatively correlated with the offline sales price. This is because channel preference directly impacts channel pricing, and an increase in channel preference leads to a corresponding increase in the product price of that channel.

Last but not least, as the preference for online channels continues to expand, the impact of this preference on overall profits under the two government subsidy strategies transitions from negative to positive.

Propositions 8 and 9 also propose a comparative analysis of the green quality and overall profit. When the preference for online channels is low, subsidizing the manufacturer leads to a higher green quality and overall profit compared with subsidizing the retailer. However, as production costs increase, subsidizing the retailer yields a higher green quality and overall profit than subsidizing the manufacturer. Furthermore, when the online channel preference becomes dominant, subsidizing the manufacturer proves more effective in enhancing both the green quality and the overall profit.

Therefore, when the preference for online channels is dominant, subsidizing manufacturers is more effective in achieving better green quality and overall profit. However, when the preference for online channels is limited, the selection of subsidy strategies mainly depends on the cost of the product. Based on the findings above, in the initial stages of green product production or promotion, or when online channel preferences prevail, the government could subsidize the manufacturer to improve the product's green quality and the supply chain profit.

4.4.3. Impacts of the Sensitivity Coefficient of the Customer and the Cost Coefficient of Green Products on the Supply Chain Decision Variables and Profits

The implications of Propositions 4 and 7 suggest a positive relationship between the customer's sensitivity coefficient and both the decision variables and profits while indicating a negative association with the cost coefficient of green products. An increase in customers' awareness of green products or a decrease in the costs associated with green products would enhance both the green quality and profit within the supply chain. Therefore, it becomes imperative to elevate customers' perception of green products in order to foster sustainable development within the supply chain.

5. Numerical Analysis

This section aims to illustrate the behavior of the proposed models using numerical examples and to analyze the effects of key parameters.

5.1. Comparisons of Green Quality and Overall Profits under the Two Subsidies

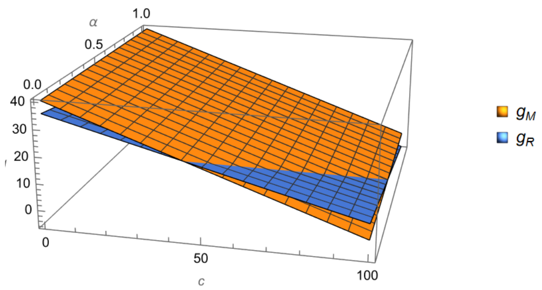
The following numerical examples were conducted to gain further insights from the models. With reference to the setting of related parameters in the literature [11,41,70] and the parameters satisfying the above conditions, each parameter and its assignment are presented in Table 3.

Table 3. Parameter assignment.

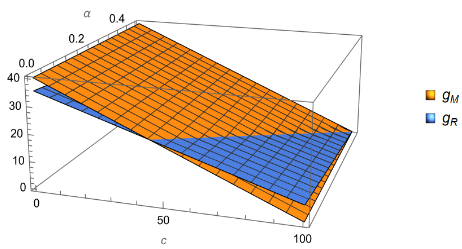
Parameter	Value	Parameter	Value
Q	100	s	0.5
β	1	θ	0.5
γ	0.5	k	3

5.1.1. Comparisons of the Green Quality under the Different Subsidies

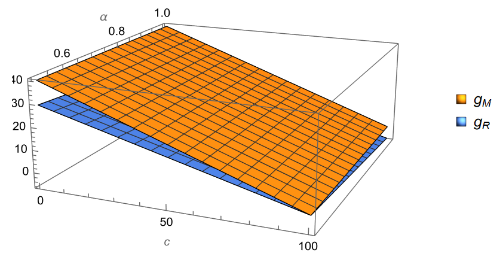
Figure 2 demonstrates the changes in customers' online channel preference under the manufacturer's or retailer's subsidization. Figure 2a shows that when government subsidy and production costs reach a certain level, there is an increasing trend toward green product quality as the customer online channel preference increases. In Figure 2b, when the customer's online channel preference is limited (less than 50%) and the production cost is relatively low, subsidizing the manufacturer leads to better green product quality. However, when production cost exceeds a certain threshold ($c_5 = \frac{Q(2\alpha k(\theta-\beta)+(\gamma+s(\beta-\theta))(2(\alpha-1)\gamma+(2\alpha-1)\beta s-2(\alpha-1)\theta s))}{2(\beta-\theta)(k(\theta-\beta)+(\theta s-\gamma)(\gamma+s(\beta-\theta)))}$), subsidizing the retailer instead can achieve higher green product quality. Furthermore, Figure 2c shows that regardless of which subsidy strategy is employed, when the customer online channel preference is dominant (more than 50%), better green product quality is obtained by subsidizing the manufacturer. Therefore, in situations where both the customer online channel preference and production cost are low or the customer online channel preference has become dominant, the government should subsidize manufacturers to achieve superior green product quality.



(**a**) Comparison of the green quality under the different subsidies $(\alpha \in (0,1))$.



(**b**) Comparison of the green quality under the different subsidies $(\alpha \in (0, 0.5])$.



(**c**) Comparison of the green quality under the different subsidies $(\alpha \in (0.5,1))$.

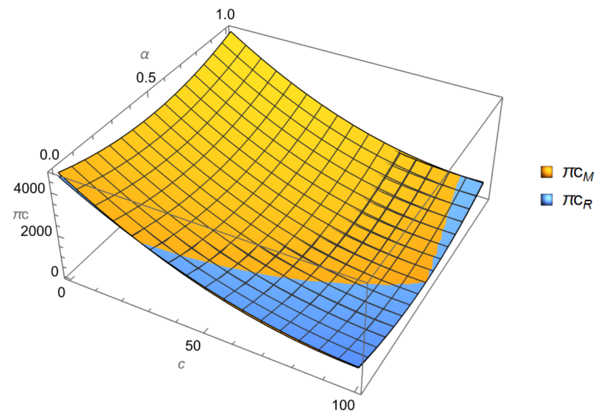
Figure 2. Comparisons of the green quality under the manufacturer's subsidy and the retailer's subsidy.

5.1.2. Comparison of the Overall Profit under the Different Subsidies

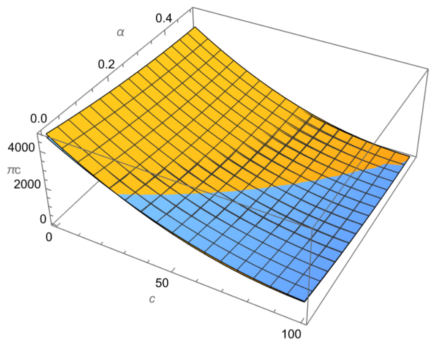
In terms of centralized decision-making, the overall profits of the supply chain are depicted in Figure 3 under the two government subsidy strategies: subsidies for manufacturers and subsidies for retailers.

Similarly, as shown in Figure 3a, the impact of the customer online channel preference on overall profits is examined under two different subsidy strategies. Figure 3b shows that when the customers' online channel preference is limited (less than 50%) and production costs are relatively low, subsidizing the manufacturer leads to higher supply chain profits. However, subsidizing the retailer results in a higher overall profit when production costs exceed a certain threshold. Furthermore, Figure 3c shows that when the customers' online channel preference is dominant (more than 50%), regardless of the subsidy strategy employed, subsidizing the manufacturer yields a better supply chain profit. Therefore, government subsidies to manufacturers can enhance the overall supply chain profit when customers' online channel preference and production costs are low or when the customer's online channel preference has become dominant.

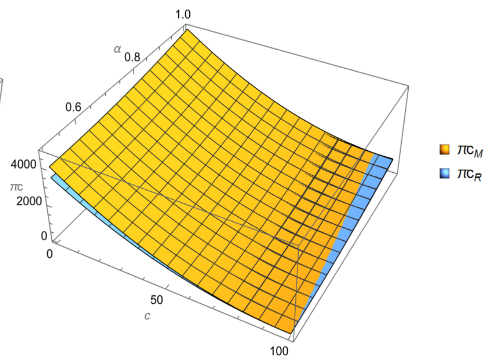
In conclusion, different government subsidy strategies impact the product's green quality and overall profit. During the initial stage of subsidy implementation, when both production costs and the customer preference for online channels are low, or when most customers prefer to purchase online, it is advisable to adopt a strategy of subsidizing manufacturers. However, when the customers' online channel preference is limited, the government should consider the product cost implications to determine an appropriate subsidy strategy.



(**a**) Comparison of the overall profit of the product under the different subsidies $(\alpha \in (0,1))$.



(**b**) Comparison of the overall profit under the different subsidies $(\alpha \in (0, 0.5])$.



(**c**) Comparison of the overall profit under the different subsidies $(\alpha \in (0.5,1))$.

Figure 3. Comparisons of the overall profit under the manufacturer's subsidy and the retailer's subsidy.

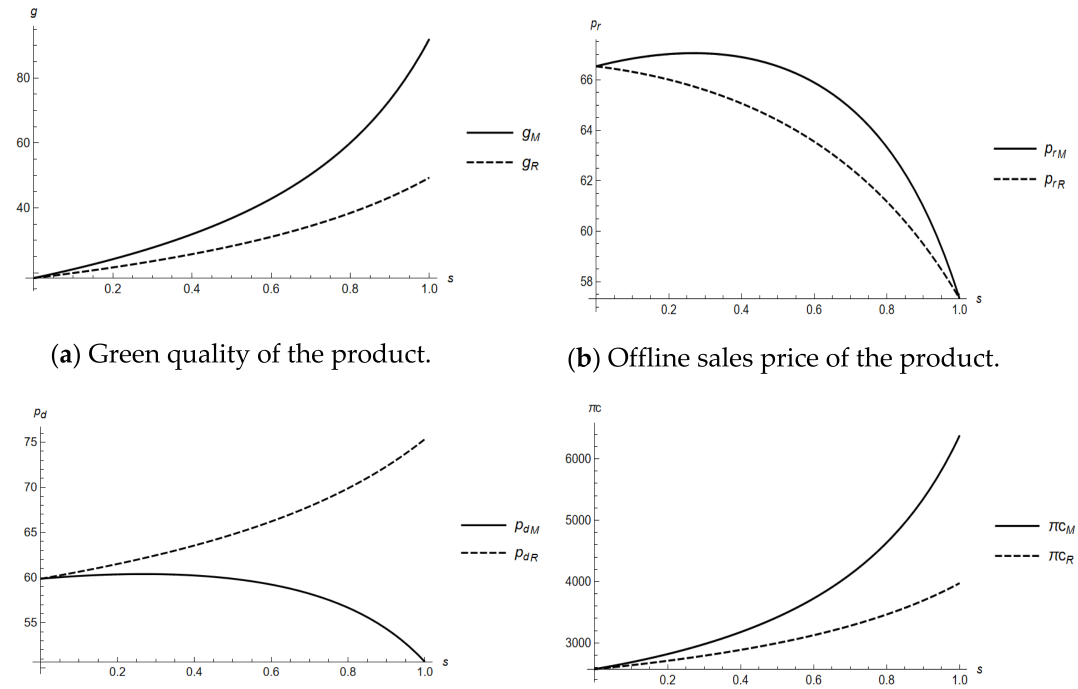
5.2. Sensitivity Analysis

A sensitivity analysis was conducted to investigate the impact of the subsidy coefficient on the green quality level (s) and customers' channel reference (α). Both s and α

are the most critical parameters affecting the optimal results of the supply chain. The numerical values of the critical parameters are considered as $Q = 100, \alpha = 0.4, \beta = 1, \theta = 0.5, \gamma = 0.5,$ and $k = 3$ [11,41,70]. To present the sensitivity analysis, solid and dashed lines represent the results corresponding to the subsidy for manufacturers and retailers, respectively.

5.2.1. Effects of the Government's Subsidy Coefficient (s)

To study the effects of the government's subsidy coefficient(s), the value of s is set from 0 to 1. For the given values of parameters, Figure 4 demonstrates the effects of the subsidy coefficient for all the decision variables and the overall profit.



(**a**) Green quality of the product. (**b**) Offline sales price of the product.

(**c**) Online sales price of the product. (**d**) Profit of the total supply chain.

Figure 4. The effects of the government's subsidy coefficient on the optimal results.

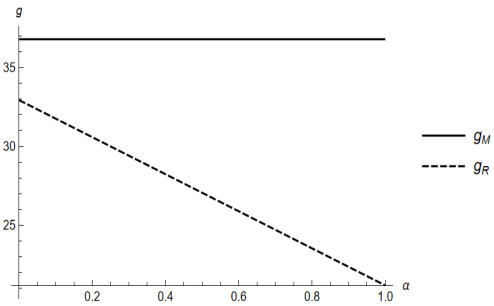
Figure 4 demonstrates the variations in the optimal results with s. From Figures 4a and 4d, it can be observed that the green quality and the overall profit both increase as the government's subsidy coefficient increases under the two subsidy strategies. Therefore, the government's subsidy strategy will improve the product's green quality and the supply chain's overall profit. Also, if the subsidy coefficient increases, the retailer will reduce the retail price for customers (Figure 4b). According to Figure 4c, as the subsidy coefficient rises, the manufacturer will offer a lower price to customers when governments subsidize them; however, the manufacturer will charge customers a higher price when governments subsidize retailers.

5.2.2. Effects of the Customers' Channel Reference (α)

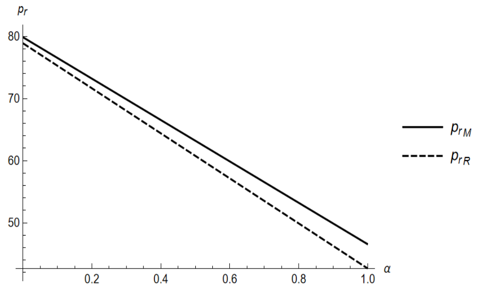
The customers' channel reference (α) is also an important parameter in this study, affecting optimal decisions. For this purpose, the value of the government's subsidy coefficient is fixed at 0.5. The effects of k on the decision variables and profits are shown in Figure 5.

Figure 5a indicates that it is not affected by the online channel preference under the manufacturer subsidization strategy. However, when government subsidies are given to retailers, customers' online channel preference negatively influences the green quality.

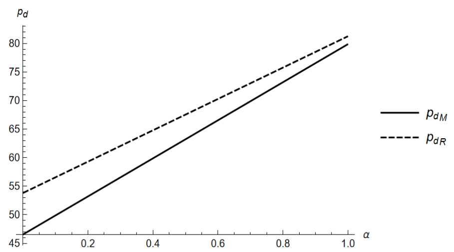
The primary reason for this phenomenon is the provision of government subsidies to retailers, which diminishes manufacturers' motivation to enhance the green quality of products. Moreover, as the customers' online channel preferences continue to rise, online prices are expected to increase (see Figure 5c), while offline prices are anticipated to decrease (see Figure 5b). In addition, according to Figure 5d, in the two models, it can be observed that the supply chain profit first decreases and then increases with the increase in customers' channel preference for purchasing through the online channel.



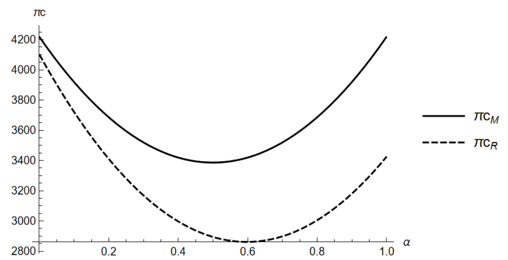
(**a**) Green quality of the product.



(**b**) Offline sales price of the product.



(**c**) Online sales price of the product.



(**d**) Profit of the total supply chain.

Figure 5. The effects of the customers' channel reference on the optimal results.

6. Conclusions and Future Research

6.1. Conclusions

This study constructs a game model led by the manufacturer to investigate the optimal decisions of the dual-channel green supply chain under three different subsidy strategies: no government subsidy, manufacturer subsidization, and retailer subsidization. Based on the model, we introduce the government subsidy coefficient and the customer's online channel preference to analyze their impact on decision variables and overall profits within the supply chain. The objective is to guide governments in formulating appropriate subsidy strategies that enhance product sustainability while maximizing overall profit. The key findings can be summarized as follows:

(1) A generally accepted conclusion in the existing green supply chain literature is that considering government subsidies can advance green product sales and promote green development, which can bring benefits to environmentally sustainable development [16]. This paper also obtains a similar conclusion under the strategy of subsidizing manufacturers. However, if the impacts of different government subsidy strategies are considered, then we find that the influences of the subsidy coefficient on the offline price and overall profit are different under the strategy of subsidizing retailers. That is, with the increase in customer online channel preference and production cost under certain conditions, subsidizing retailers may result in reduced offline prices, which could also lead to a decrease in overall profits. The potential explanation for this phenomenon lies in the fact that government subsidies to retailers can effectively reduce retail prices in order to attract customers, particularly when product costs are high and the customer's online channel

preferences are over 50%. In summary, government subsidies can enhance the environmental sustainability of products and the overall profit in most cases. However, careful consideration must be given to potential adverse effects resulting from alterations in the subsidy coefficient and production cost under the retailer subsidization strategy.

(2) Comparing the green quality under the two subsidy strategies, the government should consider the subsidy coefficient and the customer online channel preference within a dual-channel supply chain. When the customer preference for online channels is small (less than 50%) with a low production cost (from 44 to 100 in the numerical examples in Figure 3b), or when there is a larger market share for online channels (over than 50% in the numerical examples in Figure 3c), strategies for subsidizing the manufacturer are preferred. However, when production costs are high, strategies for subsidizing the retailer become more effective in achieving a higher green quality of the product. When customer preference for online channels becomes dominant (over 50%), subsidizing manufacturers has more advantages in improving the green quality. Conversely, when the preference for online channels is limited (less than 50%), the choice of subsidy strategy is influenced by production costs. The findings underscore the importance of governments developing effective subsidy strategies that consider customers' channel preferences and production costs to promote sustainable development within the supply chain.

(3) To maximize overall profits, the government intends to find the appropriate subsidy strategies to achieve this goal. Similarly, the selection of government subsidies depends not only on the subsidy coefficient but also on the customer preference for online channels. When direct sales channels make up a relatively small portion (less than 50%) and the production costs are low (from 28 to 100 in the numerical examples in Figure 4b), or when direct sales channels account for a significant proportion (more than 50% in the numerical examples in Figure 4c), the government may subsidize the manufacturer to achieve a higher total profit in the supply chain. Conversely, when production costs grow, the government may subsidize the retailer to attain greater total supply chain profits. In short, when the customer preference for online channels becomes predominant (more than 50%), subsidizing manufacturers would be a better option for enhancing the overall profit. Nevertheless, when the preference for online channels is low (less than 50%), the selection of subsidy strategy is influenced by the production costs. The managerial implications of this finding suggest to the government that subsidizing manufacturers would be a preferable strategy for enhancing both the product's greenness and overall profit when online channels become dominant among customers.

(4) Under two different subsidy strategies, an increase in customers' green preference or a decrease in the cost elasticity coefficient of green quality will increase the green quality of the product, the online and offline selling prices, the market demand, and the overall profit of the supply chain. It reminds the government and customers that enhancing customers' environmental awareness will contribute to the sustainability and development of the supply chain.

(5) Under the two different subsidy strategies, an increase in the preference for online channels leads to a corresponding decrease in offline selling prices, and the overall profit shows a U-shape trend, which decreases first and increases later. However, the preference for online channels has no influence on the green quality under strategies involving subsidizing the manufacturer, while it has a negative influence under strategies involving subsidizing the retailer. Therefore, we should notice the negative impact on the green quality of subsidizing the retailer; on the other hand, irrespective of customer channel preference, the cooperation of online and offline channels can lead to enhanced overall profit. Consequently, as for the enterprises, there is no necessity for adversarial competition between online and offline channels; instead, collaboration can be fostered to drive overall profit growth. Governments should foster a healthy market environment by guiding market expectations and stimulating market vitality, which can mitigate cut-throat competition among enterprises operating through different channels so as to contribute to the profit of the supply chain.

6.2. Future Research

Although this study contributes to the dual-channel green supply chain under different subsidies, it has some limitations, and further research can be performed. Firstly, the demand function is linear and affected by many factors. This paper only considers the green quality and the online and offline selling prices in centralized decision-making. Further research can involve more factors influencing decentralized decision-making and nonlinear demand functions. Secondly, the supply chain usually has more than two echelon members in practical applications. This may include suppliers, manufacturers, retailers, platforms, etc. Multi-echelon members will be involved in more complex decisions. We can study more complex supply chains in the following study to find more insights. Finally, this article explored incentive measures implemented by the government using different subsidy strategies. More effective measures, such as taxes, are also worth studying further.

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Appendix A

Proof of Lemma 1. Firstly, the first-order partial derivatives of π_c^N over g_c^N, ${p_d}_c^N$, and ${p_r}_c^N$ in Equation (5) are as follows:

$$\frac{\partial \pi_c^N}{\partial g_c^N} = \frac{-\gamma(c(3\beta+\theta)+(\alpha-1)Q)+2\gamma(\beta+\theta)p_d+g(\gamma^2-2\beta k)}{2\beta}$$

$$\frac{\partial \pi_c^N}{\partial {p_r}_c^N} = \beta c - c\theta + 2\theta p_d + \gamma g - 2\beta p_r - \alpha Q + Q$$

$$\frac{\partial \pi_c^N}{\partial {p_d}_c^N} = \frac{2k\big(c(\beta-\theta)(\beta+\theta)+Q(\alpha\beta-\alpha\theta+\theta)\big)}{2\beta k-\gamma^2} + \frac{\gamma^2(-4c(\beta+\theta)-2\alpha Q+Q)+4(\beta+\theta)p_d\big(\gamma^2+k(\theta-\beta)\big)}{2\beta k-\gamma^2}$$

Secondly, the Hessian matrix is $H = \begin{bmatrix} \frac{\partial^2\pi_c}{\partial g^2} & \frac{\partial^2\pi_c}{\partial g\partial p_r} & \frac{\partial^2\pi_c}{\partial g\partial p_d} \\ \frac{\partial^2\pi_c}{\partial p_r\partial g} & \frac{\partial^2\pi_c}{\partial {p_r}^2} & \frac{\partial^2\pi_c}{\partial p_r\partial p_d} \\ \frac{\partial^2\pi_c}{\partial p_d\partial g} & \frac{\partial^2\pi_c}{\partial p_d\partial p_r} & \frac{\partial^2\pi_c}{\partial {p_d}^2} \end{bmatrix} = \begin{bmatrix} \frac{\gamma^2}{2\beta}-k & 0 & \frac{\gamma(\beta+\theta)}{\beta} \\ \gamma & -2\beta & 2\theta \\ 0 & 0 & \frac{4(\beta+\theta)(\gamma^2+k(\theta-\beta))}{2\beta k-\gamma^2} \end{bmatrix}$ and its first-order, second-order and third-order master sub-determinants are $|H_1| = \frac{\gamma^2}{2\beta}-k < 0$, $\lceil H_2 \rceil = (-2\beta)\left(\frac{\gamma^2}{2\beta}-k\right) > 0$ and $\lceil H_3 \rceil =$

$(-2\beta)\left(\frac{\gamma^2}{2\beta}-k\right)\left[\frac{4(\beta+\theta)(\gamma^2+k(\theta-\beta))}{2\beta k-\gamma^2}\right]<0$, respectively. Then the Hessian matrix is negative definite when $k>\frac{\gamma^2}{\beta-\theta}$ holds. It is easy to show that the second-order partial derivatives of π_c^N over g_c^N, ${p_d}_c^N$, and ${p_r}_c^N$ in Equation (5) are as follows:

$$\frac{\partial^2\pi_c^N}{\partial g^2}=\frac{\gamma^2}{2\beta}-k<0$$

$$\frac{\partial^2\pi_c^N}{\partial {p_r}^2}=-2\beta<0$$

$$\frac{\partial^2\pi_c^N}{\partial {p_d}^2}=\frac{4(\beta+\theta)(\gamma^2+k(\theta-\beta))}{2\beta k-\gamma^2}<0$$

Hence, there is a unique optimal solution (g,p_r,p_d), making the π_c global maximal. Thirdly, let the first derivative of π_c with respect to g,p_r,p_d be equal to 0. That is,

$$\frac{\partial\pi_c^N}{\partial g_c^N}=\frac{-\gamma(c(3\beta+\theta)+(\alpha-1)Q)+2\gamma(\beta+\theta)p_d+g(\gamma^2-2\beta k)}{2\beta}=0$$

$$\frac{\partial\pi_c^N}{\partial {p_r}_c^N}=\beta c-c\theta+2\theta p_d+\gamma g-2\beta p_r-\alpha Q+Q=0$$

$$\frac{\partial\pi_c^N}{\partial {p_d}_c^N}=\frac{2k(c(\beta-\theta)(\beta+\theta)+Q(\alpha\beta-\alpha\theta+\theta))}{2\beta k-\gamma^2}+$$

$$\frac{\gamma^2(-4c(\beta+\theta)-2\alpha Q+Q)+4(\beta+\theta)p_d(\gamma^2+k(\theta-\beta))}{2\beta k-\gamma^2}=0$$

Hence, we have $g^{N^*}=-\frac{\gamma(2c(\theta-\beta)+Q)}{2(\gamma^2+k(\theta-\beta))}$

$$p_d^{N^*}=\frac{1}{4}\left(\frac{2c\gamma^2-kQ}{\gamma^2+k(\theta-\beta)}+2c+\frac{(2\alpha-1)Q}{\beta+\theta}\right)$$

$$p_r^{N^*}=\frac{1}{4}\left(\frac{2c\gamma^2-kQ}{\gamma^2+k(\theta-\beta)}+2c+\frac{(1-2\alpha)Q}{\beta+\theta}\right)$$

Substituting g^{N^*}, $p_d^{N^*}$, and $p_r^{N^*}$ into Equation (5), the total profit of the entire system is expressed as

$$\pi_C^{N^*}=-\frac{2k\left(2c^2(\beta-\theta)^2(\beta+\theta)+2cQ(\theta^2-\beta^2)+Q^2(2(\alpha-1)\alpha\beta-2(\alpha-1)\alpha\theta+\beta)\right)}{8(\beta+\theta)\left(\gamma^2+k(\theta-\beta)\right)}-\frac{(1-2\alpha)^2\gamma^2Q^2}{8(\beta+\theta)\left(\gamma^2+k(\theta-\beta)\right)}$$

□

Proof of Proposition 1. Based on Lemma 1, when it holds for $\beta>\theta>0, Q>0$, the first derivative of g_c,p_d,p_r, and π_c with respect to α are as follows:

$$\frac{\partial g_c}{\partial\alpha}=0,\ \frac{\partial p_d}{\partial\alpha}=\frac{Q}{2(\beta+\theta)}>0,\ \frac{\partial p_r}{\partial\alpha}=-\frac{Q}{2(\beta+\theta)}<0,$$

$$\frac{\partial\pi_r}{\partial\alpha}=\frac{(2\alpha-1)Q^2}{2(\beta+\theta)}$$

Furthermore, because $0<\alpha<1$, when $0<\alpha\le\frac{1}{2}$, $\frac{\partial\pi_r}{\partial\alpha}=\frac{(2\alpha-1)Q^2}{2(\beta+\theta)}\le 0$;

$$p_d - p_r = \frac{1}{4}\left(\frac{2c\gamma^2 - kQ}{\gamma^2 + k(\theta - \beta)} + 2c + \frac{(2\alpha - 1)Q}{\beta + \theta}\right) - \frac{1}{4}\left(\frac{2c\gamma^2 - kQ}{\gamma^2 + k(\theta - \beta)} + 2c + \frac{Q - 2\alpha Q}{\beta + \theta}\right) = \frac{(2\alpha - 1)Q}{2(\beta + \theta)} > 0$$

When $\frac{1}{2} < \alpha < 1$, $\frac{\partial \pi_r}{\partial \alpha} = \frac{(2\alpha - 1)Q^2}{2(\beta + \theta)} > 0$,

$$p_d - p_r = \frac{1}{4}\left(\frac{2c\gamma^2 - kQ}{\gamma^2 + k(\theta - \beta)} + 2c + \frac{(2\alpha - 1)Q}{\beta + \theta}\right) - \frac{1}{4}\left(\frac{2c\gamma^2 - kQ}{\gamma^2 + k(\theta - \beta)} + 2c + \frac{Q - 2\alpha Q}{\beta + \theta}\right) = \frac{(2\alpha - 1)Q}{2(\beta + \theta)} < 0$$

That is, $0 < \alpha \le \frac{1}{2}$; $p_d > p_r$; $\frac{1}{2} < \alpha < 1$; and $p_d < p_r$. □

Proof of Lemma 2. Similar to the proof for Lemma 1, the first derivatives of π_c^M with respect to ${g_c}^M$, ${p_d}^M$, and ${p_r}^M$ can be found as follows:

$$\frac{\partial \pi_c^M}{\partial {g_c}^M} = \frac{-c[\gamma(3\beta + \theta) + s(\beta - \theta)^2] + 2p_d(\beta + \theta)[\gamma + s(\theta - \beta)]}{2\beta} + \frac{g[\gamma^2 - 2\beta k + s^2(\beta - \theta)^2 + 2\gamma s(3\beta + \theta)] + Q[\gamma - \alpha\gamma + s(\alpha\beta - \alpha\theta + \beta + \theta)]}{2\beta}$$

$$\frac{\partial \pi_c^M}{\partial {p_d}^M} = -\frac{2k\big(c(\beta - \theta)(\beta + \theta) + Q(\alpha\beta - \alpha\theta + \theta)\big)}{\gamma^2 - 2\beta k + s^2(\beta - \theta)^2 + 2\gamma s(3\beta + \theta)} - \frac{\Big(\big(\gamma + s(\beta - \theta)\big)\big(4c\gamma(\beta + \theta) + (2\alpha - 1)\gamma Q + Qs(2\alpha\beta + (3 - 2\alpha)\theta + \beta)\big)\Big)}{\gamma^2 - 2\beta k + s^2(\beta - \theta)^2 + 2\gamma s(3\beta + \theta)} + \frac{+4(\beta + \theta)p_d(k(\theta - \beta) + (\gamma + \beta s - \theta s)^2)}{\gamma^2 - 2\beta k + s^2(\beta - \theta)^2 + 2\gamma s(3\beta + \theta)}$$

$$\frac{\partial \pi_c^M}{\partial {p_r}^M} = c(\beta - \theta) + 2\theta p_d + g(\gamma + s(\theta - \beta)) - 2\beta p_r - \alpha Q + Q$$

Then, we obtain the Hessian matrix,

$$\text{Hessian}(\pi_c^M) = \begin{bmatrix} \frac{\gamma^2 - 2\beta k + s^2(\beta - \theta)^2 + 2\gamma s(3\beta + \theta)}{2\beta} & 0 & \frac{(\beta + \theta)(\gamma + s(\theta - \beta))}{\beta} \\ \gamma + s(\theta - \beta) & -2\beta & 2\theta \\ 0 & 0 & -\frac{4(\beta + \theta)(k(\theta - \beta) + (\gamma + s(\beta - \theta))^2)}{\gamma^2 - 2\beta k + s^2(\beta - \theta)^2 + 2\gamma s(3\beta + \theta)} \end{bmatrix}$$

and its first-order, second-order, and third-order master sub-determinants are $|H_1| = \frac{\gamma^2 - 2\beta k + s^2(\beta - \theta)^2 + 2\gamma s(3\beta + \theta)}{2\beta} < 0$, $|H_2| = \left[\frac{\gamma^2 - 2\beta k + s^2(\beta - \theta)^2 + 2\gamma s(3\beta + \theta)}{2\beta}\right](-2\beta) > 0$, $|H_3| = \left[\frac{\gamma^2 - 2\beta k + s^2(\beta - \theta)^2 + 2\gamma s(3\beta + \theta)}{2\beta}\right](-2\beta)\left[-\frac{4(\beta + \theta)(k(\theta - \beta) + (\gamma + s(\beta - \theta))^2)}{\gamma^2 - 2\beta k + s^2(\beta - \theta)^2 + 2\gamma s(3\beta + \theta)}\right] < 0$, respectively. The Hessian matrix is negative definite when $k > \frac{(\gamma + s(\beta - \theta))^2}{\beta - \theta}$ holds. It is easy to show that the second-order partial derivatives of π_c^M with respect to ${g_c}^M$, ${p_d}^M$, and ${p_r}^M$ are as follows:

$$\frac{\partial^2 \pi_c^M}{\partial g^2} = \frac{\gamma^2 - 2\beta k + s^2(\beta - \theta)^2 + 2\gamma s(3\beta + \theta)}{2\beta} < 0$$

$$\frac{\partial^2 \pi_c^M}{\partial {p_r}^2} = -2\beta < 0$$

$$\frac{\partial^2 \pi_c^M}{\partial {p_d}^2} = -\frac{4(\beta + \theta)(k(\theta - \beta) + (\gamma + s(\beta - \theta))^2)}{\gamma^2 - 2\beta k + s^2(\beta - \theta)^2 + 2\gamma s(3\beta + \theta)} < 0$$

Hence, there is a unique optimal solution (g, p_r, p_d) making the π_c global maximum.

Letting $\frac{\partial \pi_c^M}{\partial {g_c}^M} = 0$, $\frac{\partial \pi_c^M}{\partial {p_d}^M} = 0$, $\frac{\partial \pi_c^M}{\partial {p_r}^M} = 0$, we have

$$g^{M*} = \frac{(2c(\theta - \beta) + Q)(\gamma + s(\beta - \theta))}{2k(\beta - \theta) - 2(\gamma + s(\beta - \theta))^2}$$

$$p_d^{M^*} = \frac{2k(c\theta^2 - \beta(\beta c + \alpha Q) + (\alpha - 1)\theta Q)}{4(\beta + \theta)\left(k(\theta - \beta) + \left(\gamma + s(\beta - \theta)\right)^2\right)}$$
$$+ \frac{\left(\gamma + s(\beta - \theta)\right)\left(4c\gamma(\beta + \theta) + (2\alpha - 1)\gamma Q + Qs(2\alpha\beta + (3 - 2\alpha)\theta + \beta)\right)}{4(\beta + \theta)\left(k(\theta - \beta) + \left(\gamma + s(\beta - \theta)\right)^2\right)}$$

$$p_r^{M^*} = \frac{\left(\gamma + s(\beta - \theta)\right)\left(4c\gamma(\beta + \theta) + Q\left(-2\alpha\gamma + \gamma + s(-2\alpha\beta + 2\alpha\theta + 3\beta + \theta)\right)\right)}{4(\beta + \theta)\left(k(\theta - \beta) + \left(\gamma + s(\beta - \theta)\right)^2\right)}$$
$$- \frac{2k\left(c(\beta - \theta)(\beta + \theta) + Q(-\alpha\beta + \alpha\theta + \beta)\right)}{4(\beta + \theta)\left(k(\theta - \beta) + \left(\gamma + s(\beta - \theta)\right)^2\right)}$$

Then, the overall profit is

$$\pi_c^{M^*} = \frac{(1 - 2\alpha)^2 Q^2 (\gamma + s(\beta - \theta))^2 - 2k(2c^2(\beta - \theta)^2(\beta + \theta) + 2cQ(\theta^2 - \beta^2)}{8(\beta + \theta)(k(\theta - \beta) + (\gamma + s(\beta - \theta))^2)}$$
$$+ \frac{Q^2(2(\alpha - 1)\alpha\beta - 2(\alpha - 1)\alpha\theta + \beta))}{8(\beta + \theta)(k(\theta - \beta) + (\gamma + s(\beta - \theta))^2)}$$

□

Proof of Proposition 2. Based on Lemma 2, when it holds for $0 < \alpha < 1, \beta > \theta > 0, Q > 0, 0 < \gamma < 1, s > 0, 0 < c < \frac{Q}{2(\beta - \theta)}$, and $k > \frac{[\gamma + s(\beta - \theta)]^2}{\beta - \theta}$, the first derivative of $g^M, {p_d}^M, {p_r}^M$, and ${\pi_c}^M$ with respect to s are as follows:

$$\frac{\partial g^M}{\partial s} = \frac{(\beta - \theta)(2c(\theta - \beta) + Q)\left(k(\beta - \theta) + \left(\gamma + s(\beta - \theta)\right)^2\right)}{2\left(k(\theta - \beta) + \left(\gamma + s(\beta - \theta)\right)^2\right)^2} > 0$$

$$\frac{\partial {\pi_c}^M}{\partial s} = \frac{k(\beta - \theta)(2c(\theta - \beta) + Q)^2(\gamma + s(\beta - \theta))}{4(k(\theta - \beta) + (\gamma + s(\beta - \theta))^2)^2} > 0$$

$$\frac{\partial {p_r}^M}{\partial s} = -\frac{(2c(\theta - \beta) + Q)(ks(\beta - \theta)^2 - \gamma(\gamma + s(\beta - \theta))^2)}{2(k(\theta - \beta) + (\gamma + s(\beta - \theta))^2)^2}$$

$$\frac{\partial {p_d}^M}{\partial s} = -\frac{(2c(\theta - \beta) + Q)(ks(\beta - \theta)^2 - \gamma(\gamma + s(\beta - \theta))^2)}{2(k(\theta - \beta) + (\gamma + s(\beta - \theta))^2)^2}$$

where $0 < s < \frac{\gamma}{\beta - \theta}, \frac{\left(\gamma + s(\beta - \theta)\right)^2}{\beta - \theta} < k < \frac{\gamma\left(\gamma + s(\beta - \theta)\right)^2}{s(\beta - \theta)^2}, \frac{\partial {p_r}^M}{\partial s} > 0$, and $\frac{\partial {p_d}^M}{\partial s} > 0$. □

Proof of Proposition 3. Similar to the proof for Proposition 1, based on Lemma 2, when it holds for $\beta > \theta > 0, Q > 0$, the first derivatives of $g_c, p_d, p_r,$ and π_c with respect to α are as follows:

$$\frac{\partial g^M}{\partial \alpha} = 0, \frac{\partial {p_r}^M}{\partial \alpha} = -\frac{Q}{2(\beta + \theta)} < 0, \frac{\partial {p_d}^M}{\partial \alpha} = \frac{Q}{2(\beta + \theta)} > 0,$$

$$\frac{\partial {\pi_c}^M}{\partial \alpha} = \frac{(2\alpha - 1)Q^2}{2(\beta + \theta)}$$

Furthermore, because $0 < \alpha < 1$, when $0 < \alpha \le \frac{1}{2}$, $\frac{\partial {\pi_c}^M}{\partial \alpha} = \frac{(2\alpha - 1)Q^2}{2(\beta + \theta)} \le 0$; when $\frac{1}{2} < \alpha < 1$, $\frac{\partial {\pi_c}^M}{\partial \alpha} = \frac{(2\alpha - 1)Q^2}{2(\beta + \theta)} > 0$.

That is, $0 < \alpha \le \frac{1}{2}$, $\frac{\partial {\pi_c}^M}{\partial \alpha} \le 0$; $\frac{1}{2} < \alpha < 1$, $\frac{\partial {\pi_c}^M}{\partial \alpha} > 0$. □

Proof of Proposition 4. Based on Lemma 2, when it holds for $\beta > \theta > 0, Q > 0, 0 < c < \frac{Q}{2(\beta-\theta)}$, and $k > \frac{[\gamma+s(\beta-\theta)]^2}{\beta-\theta}$, the first derivatives of $g_c, p_d, p_r,$ and π_c with respect to γ and k are as follows:

$$\frac{\partial g^M}{\partial \gamma} = \frac{(2c(\theta-\beta)+Q)(k(\beta-\theta)+(\gamma+s(\beta-\theta))^2)}{2(k(\theta-\beta)+(\gamma+s(\beta-\theta))^2)^2} > 0$$

$$\frac{\partial g^M}{\partial k} = -\frac{(\beta-\theta)(2c(\theta-\beta)+Q)(\gamma+s(\beta-\theta))}{2(k(\theta-\beta)+(\gamma+s(\beta-\theta))^2)^2} < 0$$

$$\frac{\partial {\pi_c}^M}{\partial \gamma} = \frac{k(2c(\theta-\beta)+Q)^2\big(\gamma+s(\beta-\theta)\big)}{4\Big(k(\theta-\beta)+\big(\gamma+s(\beta-\theta)\big)^2\Big)^2} > 0$$

$$\frac{\partial {\pi_c}^M}{\partial k} = -\frac{(2c(\theta-\beta)+Q)^2(\gamma+s(\beta-\theta))^2}{8(k(\theta-\beta)+(\gamma+s(\beta-\theta))^2)^2} < 0$$

When $0 < s < \frac{\gamma}{\beta-\theta}$, $\frac{\partial {p_d}^M}{\partial \gamma} = -\frac{(2c(\theta-\beta)+Q)\left(s\left(\gamma+s(\beta-\theta)\right)^2-\gamma k\right)}{2\left(k(\theta-\beta)+\left(\gamma+s(\beta-\theta)\right)^2\right)^2} > 0$

$$\frac{\partial {p_d}^M}{\partial k} = \frac{(2c(\beta-\theta)-Q)(\gamma+s(\beta-\theta))(\gamma+s(\theta-\beta))}{4(k(\theta-\beta)+(\gamma+s(\beta-\theta))^2)^2} < 0$$

$$\frac{\partial {p_r}^M}{\partial \gamma} = -\frac{(2c(\theta-\beta)+Q)(s(\gamma+s(\beta-\theta))^2-\gamma k)}{2(k(\theta-\beta)+(\gamma+s(\beta-\theta))^2)^2} > 0$$

$$\frac{\partial {p_r}^M}{\partial k} = \frac{(2c(\theta-\beta)+Q)(s^2(\beta-\theta)^2-\gamma^2)}{4(k(\theta-\beta)+(\gamma+s(\beta-\theta))^2)^2} < 0$$

Proof of Lemma 3. The proof is similar to that for Lemma 2. □

Proof of Proposition 5. The proof is similar to that for Proposition 2. □

Proof of Proposition 6. The proof is similar to that for Proposition 3. □

Proof of Proposition 7. The proof is similar to that for Proposition 4. □

Proof of Proposition 8. Based on Lemmas 2 and 3, we obtain

$$g_C^{M^*} = \frac{[2c(\theta-\beta)+Q][\gamma+s(\beta-\theta)]}{2k(\beta-\theta)-2[\gamma+s(\beta-\theta)]^2}$$

$$g^{R^*} = \frac{c(\beta-\theta)\big(2\gamma+s(\beta-\theta)\big)+Q\big((\alpha-1)s(\beta-\theta)-\gamma\big)}{2\gamma^2+2k(\theta-\beta)+\beta s^2(\beta-\theta)+2\gamma s(\beta-\theta)}$$

When it holds for $0 < c < \frac{Q}{2(\beta-\theta)}, k > \frac{[\gamma+s(\beta-\theta)]^2}{\beta-\theta}, \beta > \frac{4}{3}\theta, 0 < s < \frac{\gamma}{\beta-\theta}$,

$$g_c^{M^*} - g_c^{R^*} = \frac{Q\big(\gamma-(\alpha-1)s(\beta-\theta)\big)-c(\beta-\theta)\big(2\gamma+s(\beta-\theta)\big)}{2\gamma^2+2k(\theta-\beta)+\beta s^2(\beta-\theta)+2\gamma s(\beta-\theta)} + \frac{(2c(\theta-\beta)+Q)\big(\gamma+s(\beta-\theta)\big)}{2k(\beta-\theta)-2\big(\gamma+s(\beta-\theta)\big)^2}$$

When $0 < \alpha < \frac{1}{2}$, if $0 < c < c_5, g^M - g^R > 0$; $c_5 < c < \frac{Q}{2(\beta-\theta)}, g^M - g^R < 0$; when $\frac{1}{2} < \alpha < 1$, if $0 < c < \frac{Q}{2(\beta-\theta)}, g^M - g^R > 0$.

That is, when $0 < \alpha < \frac{1}{2}$, if $0 < c < c_5, g^M > g^R$, $c_5 < c < \frac{Q}{2(\beta-\theta)}, g^M < g^R$; when $\frac{1}{2} < \alpha < 1$, if $0 < c < \frac{Q}{2(\beta-\theta)}, g^M > g^R$. □

Proof of Proposition 9. The proof is similar to that for Proposition 8. □

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