

Article

Kelly Criterion Extension: Advanced Gambling Strategy

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Abstract: This article introduces an innovative extension of the Kelly criterion, which has traditionally been used in gambling, sports wagering, and investment contexts. The *Kelly criterion extension* (KCE) refines the traditional capital growth function to better suit dynamic market conditions. The KCE improves the traditional approach to accommodate the complexities of financial markets, particularly in stock and commodity trading. This innovative method focuses on crafting strategies based on market conditions and player actions rather than direct asset investments, which enhances its practical application by minimizing risks associated with volatile investments. This paper is structured to first outline the foundational concepts of the Kelly criterion, followed by a detailed presentation of the KCE and its advantages in practical scenarios, including a case study on its application to blackjack strategy optimization. The mathematical framework and real-world applicability of the KCE are thoroughly discussed, demonstrating its potential to bridge the gap between theoretical finance and actual trading outcomes.

Keywords: Kelly criterion; money management; multivariate portfolios; fractional Kelly strategies; gambling strategy; probability-based betting

MSC: 60G40; 60G42; 90B30; 90B50; 91B28; 93E20



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1. Introduction

The Kelly criterion (KC) is a mathematical equation that offers guidance on the ideal proportion of assets to dedicate to a particular investment or trade. It stands out as a financial management strategy that surpasses many others in numerous aspects. While the KC model is inspired by the real-world scenario of gambling, there is potential for it to be applicable to certain other economic circumstances [1,2]. The approach hinges on the concept of utilizing the likelihood of an event occurrence and the odds presented on a wager to identify the ideal degree of risk. The applications of the KC are typically found in the contexts of investment, gambling, and sports wagering. Employing the KC can aid investors to enhance their portfolio performance while diminishing the possibility of losses. Its equivalence with logarithmic utility has proven to be exceedingly practical, notably in maximizing the expected growth rate and the median of terminal wealth [3]. Given that the optimal fraction remains steady, it is independent of the number of trades, leading such utility functions to be labelled as myopic, as each trade is optimized as if it is the final one [4,5]. Separate research proposed a straightforward and extraordinarily rapid algorithm to maximize log investment return; however, it is inappropriate for fractional Kelly strategies [6]. Additional research assumes normal asset returns and scrutinizes Kelly strategies within the context of outperforming the return of a target benchmark portfolio [3,7].

Money management is a crucial aspect of financial trading [8]. The theoretical foundation of money management relies on the Kelly criterion (KC) for capital allocation and the optimization of position management [1,8,9]. According to the KC, gamblers should first identify betting opportunities that present a positive mathematical expectation, indicating a positive expected profit [10,11]. Regarding the application of the KC to American roulette, it

results in a negative expectation. This implies that, from a theoretical standpoint, gamblers are advised against participating in this form of gambling as they are not expected to emerge as winners in the long term when this strategy is employed [9]. A discrepancy between theory and practical outcomes is acknowledged [9,12]. Numerous studies on the KC have highlighted its limitations and drawbacks when applied to trading [13–15]. The primary challenge lies in the fact that the profit and loss distribution in financial trading deviates from the binary outcomes (one being profit and the other loss) typical of traditional gambling and is subject to change over time. Ralph Vince introduced the concept of the optimal fraction to address such issues. This approach involves comparing the differences between the traditional KC and the optimal f by back-testing actual financial transaction data [16]. Other researchers have explored the feasibility of directly applying the Kelly formula to the profit-and-loss distribution in actual trading strategies [9]. The optimal f is presented as a method for optimizing the betting ratio in relation to profit and loss [9]. A significant challenge in developing trading strategies is that using historical data for back-testing does not necessarily predict future outcomes. It is assumed that the expected profit and loss can be estimated from the sample variation of profit and loss over previous periods [17,18].

This article introduces an innovative extension of the Kelly criterion (KC) that includes a unique capital growth function sensitive to market conditions. The *Kelly criterion extension* (KCE) has successfully narrowed the divide between theoretical constructs and practical applications by offering a methodology for determining the optimal fraction. An advanced gambling strategy is an innovative framework for an investment strategy for a player, not an asset by itself. This strategic framework provides a strategy to reduce the risks of direct asset investments. This framework outlines an option of a buy-side trading structure specifically tailored to a simple index futures trading strategy. The KCE encompasses multiple winning bet categories that depend on prevailing market conditions and players. For instance, an investor should prioritize holding stocks over cash when stock prices are rising but should opt for holding cash when stock prices are declining. Unlike the original KC, the KCE does not invest in stocks directly; instead, it constructs appropriate winning strategies based on the actions of players who are buying or selling stocks. KCE offers a clear solution that specifies the appropriate expected capital growth. This extension is applicable to investments in dynamic market environments, such as the stock and commodities markets, including oil and gold.

The paper is organized as follows: Section 2 provides the preliminary background of the Kelly criterion and its expansion. This section also includes a real-world case study on the Kelly criterion and its optimization for the blackjack gambling strategy. The extension of the Kelly criterion (KCE) is introduced in Section 3. The mathematical explanation of an innovative gambling strategy for players is covered in this section. Finally, the conclusion is presented in Section 4.

2. Kelly Criterion Basics

The necessary conditions for the validity of the Kelly criterion include the capability to reinvest profits and the flexibility to control or adjust the invested amount across different categories. The channel of the theory might correspond to a real communication channel or simply to the totality of inside information available to the investor [1,19]. The quality function derived from the KC equates to exponential capital growth [1,2]. Let us denote the exponential rate of growth of capital G as follows [1]:

$$G = \lim_{n \rightarrow \infty} \frac{1}{n} \left(\log_2 \frac{V_n}{V_0} \right), \quad (1)$$

where V_n represents the capital of the gambler after n bets, while V_0 denotes their initial capital. The expected value of the gambler's capital can be defined as follows:

$$\mathbb{E}[V_n] = (2\sigma_1)^n V_0, \quad (2)$$

where σ_1 is the probability of winning the bet. Assuming the gambler bets a fraction f of his capital each time, we can derive the following from (2):

$$\mathbb{E}[V_n] = \{(1-f) + \alpha f\}^k \{(1-f) - \beta f\}^{n-k}, \quad (3)$$

where α is the total reward portion (i.e., $\alpha \geq 1$), and β is the total lose portion (i.e., $0 \leq \beta \leq 1$) after k winning bets out of n total bets. As has been mentioned, f is the fraction of the gambler's capital each time, and k is the number of wins within the total number of investments n from (3). Let us assign the following:

$$\Gamma(f) := V_n = w(f) \cdot l(f), \quad (4)$$

which represents the capital of the gambler from (3). Then we have the following:

$$w(f) = \{1 + w_0 f\}^{np}, l(f) = \{1 - l_0 f\}^{nq}, \quad (5)$$

where

$$w_0 = (\alpha - 1), l_0 = (1 - \beta), \quad (6)$$

$$p = \frac{k}{n}, q = \frac{n-k}{n}. \quad (7)$$

Let us maximize the Γ with respect to the fraction f . We could determine the maximum capital by taking the derivative of Γ as follows:

$$\left. \frac{d\Gamma(f)}{df} \right|_{f=f^*} = 0, \quad (8)$$

and the optimal value of f could be defined as follows:

$$f^* = \left\{ \exists f \left| \frac{d\Gamma(f)}{df} = 0, f \in [0, 1] \right. \right\}. \quad (9)$$

From (4) and (8), we obtain the following:

$$\begin{aligned} \frac{d\Gamma(f)}{df} &= \frac{d}{df} \{w(f) \cdot l(f)\} \\ &= n(1 - w_0 f)^{np-1} (1 - l_0 f)^{nq-1} \cdot \{pw_0(1 - l_0 f) - ql_0(1 - w_0 f)\}. \end{aligned}$$

Then, we obtain the following:

$$\left. \frac{d\Gamma(f)}{df} \right|_{f=f^*} = \{(pw_0 - ql_0) - w_0 l_0 f\} = 0, \quad (10)$$

and the optimal fraction f^* , which is the Kelly criterion, could be found as follows:

$$f^* = \frac{pw_0 - ql_0}{w_0 l_0} = \frac{p}{l_0} - \frac{q}{w_0}, f^* \in [0, 1]. \quad (11)$$

It is noted that the fraction of the capital should be more than zero (i.e., $f^* \geq 0$), and the optimization of the capital growth through KC f^* is illustrated in Figure 1. It indicates that the growth is optimized to 26% growth when the investment portion is 18.2% (i.e., $f^* = 0.182$) within the specified conditions. From (11), we obtain the following:

$$\frac{p}{q} \geq \frac{l_0}{w_0}, \quad (12)$$

which implies that the ratio of wins to losses should be greater than the ratio of penalties to rewards. Figure 1 depicts the payoff rate Γ as a function of the investment portion f using

the KC, with a revenue ratio of 1.26. The shape in the figure demonstrates the relationship between the investment portion and the payoff rate. Initially, as the investment portion increases, the payoff rate also rises, peaking at a specific point. The optimal fraction of the bankroll to invest to maximize growth occurs at $f^* = 0.182$. Beyond this optimal point, further increases in the investment portion lead to a sharp decline in the payoff rate, illustrating the diminishing returns and increased risk associated with larger bets. The graph emphasizes the importance of the Kelly criterion in determining the optimal bet size to achieve the highest growth rate while managing risk effectively.

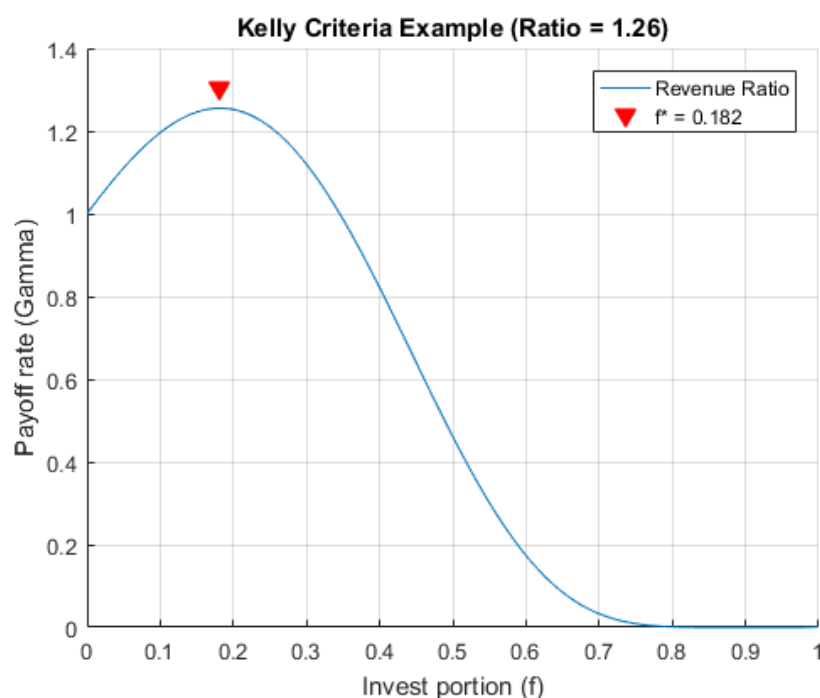


Figure 1. Optimization example for the Kelly criterion.

2.1. Optimizing Blackjack Betting

Blackjack, a card game enjoyed globally, brings an annual net profit of approximately USD 8 million to Nevada casinos. Given the typical price/earnings ratio of 15 for current common stocks, blackjack operations in Nevada could be likened to a corporation valued at USD 1.2 billion [20]. As a basic approximation, blackjack can be considered a coin toss, where the successful probability p is chosen independently from a known distribution F and announced prior to each trial. The game commences with a dealer randomly shuffling n decks of cards and players placing their bets. This betting usually imposes a maximum and minimum limit [21].

Flat betting is a simple blackjack strategy where you consistently bet the same amount in each round. This approach is easy to understand and implement, making it particularly suitable for beginners or intermediate players aiming to mitigate risk while enhancing their gameplay. By combining flat betting with a basic blackjack strategy, it is possible to keep the house edge below 0.5% (i.e., $\frac{p}{q} \geq \frac{1}{1.005}$). However, flat betting may not be suitable for all players, especially those employing advanced techniques like card counting. The expected benefit based on a binomial distribution after betting n times is as follows:

$$\mathbb{E}[V_n] = 2np \cdot v = 2p \cdot V_0, \quad (13)$$

where v is the value of flat betting and p is the winning ratio of blackjack. From (13), we can determine that the winning rate should be greater than 0.5. Otherwise, the gambler eventually loses his capital all the time (i.e., $V_n \leq V_0$). Despite the limitations of the flat betting, its simplicity makes it possible to be applied to various casino games including

baccarat and roulette. It can be effectively utilized with good live casino sites offering quality tables for games like blackjack and baccarat.

Alternatively, the KC could be employed for more effective investment. From (11), the fraction of the blackjack bet can be defined as follows:

$$f^* = p - q = 2p - 1, l_0 = w_0 = 1. \quad (14)$$

Typical blackjack betting rules stipulate that the player loses his entire bet (i.e., $l_0 = 1$) but receives double the bet when winning (i.e., $w_0 = 1$). Equation (14) can be derived from (11) by applying typical blackjack rules. From (3) and (14), the expected capital after n trials could be found as follows:

$$\mathbb{E}[V_n] = \{2p\}^{np} \{3 - 4p\}^{n(1-p)} V_0. \quad (15)$$

Figure 2 illustrates the expected investment ratio for blackjack gambling for 50 rounds, comparing flat betting and the Kelly criterion (KC) across varying winning probabilities p . Flat betting, shown as a straight line in Figure 2, maintains a linear relationship with the winning probability, indicating a steady increase in the expected investment ratio as the probability of winning rises. The KC, depicted by the curve, exhibits a non-linear growth. Initially, it closely follows the flat betting line but begins to rise sharply as the winning probability exceeds approximately 0.65. The optimal winning probability ($p^* = 0.685$) leads to a higher expected investment ratio. This figure highlights that while flat betting provides a consistent but modest return, the KC offers substantial gains, particularly when the probability of winning is high, demonstrating its efficiency in maximizing growth over repeated bets.

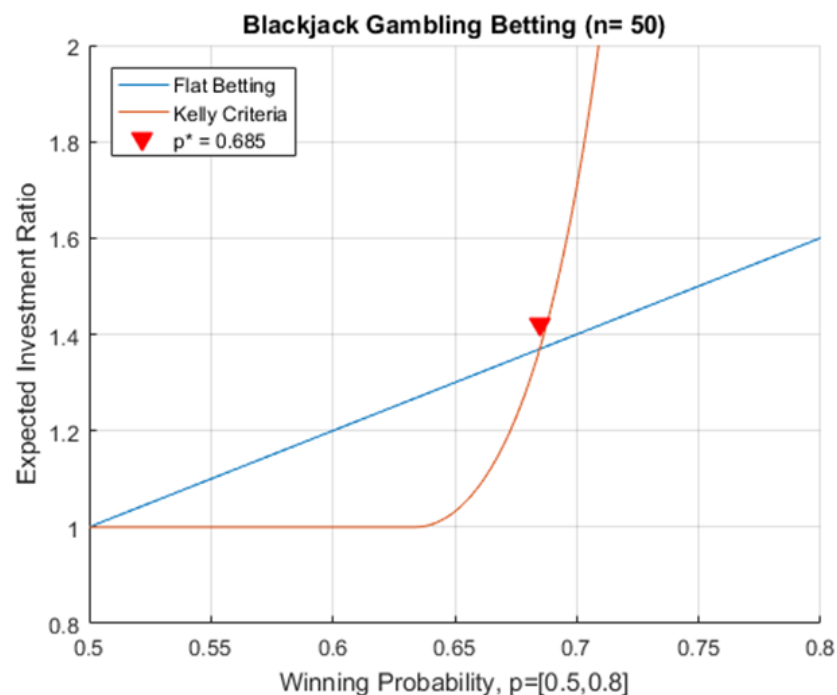


Figure 2. Betting comparison between flat betting and Kelly criterion betting.

As illustrated in the above figure, the proper fraction of flat betting is 37% of the last investment when the winning rate of the gambler is smaller than 0.685 for betting 50 times (i.e., $f^* = 0.685 - 0.315 = 0.37$). It is noted that flat betting in blackjack depends on your individual gaming goals and strategies but the strategic decision is determined by the winning rate p . Then, the optimal portion of betting f^* could be calculated from (14).

2.2. Optimizing Winning Reward Rate

It is noted that the gambler cannot make positive capital growth when the optimal fraction is less than zero (i.e., $f^* \leq 0$). In the practice of the KC, we need to determine the condition of the reward rate which can make the KC feasible to generate profit. Let us consider every bet is win and lose, with a player winning rate of p when betting n times, which follows the binomial distribution. This binomial distribution could be approximated as the the Gaussian distribution with a fairly large number of trials,

$$B(n, p) \sim N(np, npq), \quad (16)$$

where

$$\mu = np, \sigma^2 = npq, q = 1 - p. \quad (17)$$

From (16), the minimum number of winning bets could be found as follows:

$$k^* = \mu - 1.96\sigma = np - 1.96\sqrt{npq}, \quad (18)$$

where we consider 95% confidence with a 2σ level. Let us assume you are a blackjack gambler who has a 40% winning rate and you are a flat better and doing the flat betting 50 times. From (18), we obtain $k^* \simeq 13.21$. From (12) and (18), the optimal reward ratio ρ^* could be calculated as follows:

$$\rho^* := \left(\frac{w_0}{l_0} \right) \geq \frac{q}{p} = \frac{(50 - 13.21)}{(13.21)} = 2.79. \quad (19)$$

Given our example configuration, it is derived from gambling or investment scenarios. It represents the net expected return per unit of risk, calculated as the ratio, where w_0 is the gain factor when a bet is won and l_0 is the loss factor when a bet is lost. The expression. In the given numerical example, it is anticipated that a net benefit ρ^* will be realized when the reward rate is a minimum of 2.8 times greater than the loss rate.

3. Kelly Criterion Extension

The Kelly criterion extension follows the same assumptions which Kelly originally pointed out: reinvested winnings and a fairly large number of bets (>10). Kelly also mentioned that his strategy is equivalent to the maximization of logarithmic utility but he explicitly avoided any argumentation based on utility functions [3,22]. Additionally, the KCE has a different capital growth function, which reflects the market status. This extension could be applied for investments in flexible market environments such as the stock market, oil, and gold. These markets are good and bad situations, and each situation requires a different betting strategy. The KCE could be adapted for indirect investment in these flexible markets, such as a mutual fund and an ETF (exchange-traded fund). A mutual fund is an investment fund that pools money from many investors to purchase securities. Mutual funds are often classified by their principal investments. The primary structures of mutual funds are open-end funds, closed-end funds, and unit investment trusts [23]. Although the advantages of mutual funds include economies of scale, diversification, liquidity, and professional management, they still have the risk of losing funded money [24].

The core concept of KCE is investing in the (mutual) fund manager himself, who has a certain winning rate. Investors allocate their money to a fund manager (or a firm) in the form of mutual funds. The fund manager then invests these funds into various stocks as a portfolio. When the stocks appreciate, the funds should be channelled into those stocks. Conversely, when the stocks depreciate, the funds should be transferred back to the fund bank account. Possessing stocks is considered a win when the market is thriving (stocks are appreciating), while holding onto money is advantageous when the market is declining.

Gambling the Mutual Fund Investment

Let us consider you are an investing fund manager, putting in some funds to obtain profit. Unlike the previous gambling case, you are not directly involved in this game. You are betting on the player (i.e., a fund manager), not directly betting on the game (i.e., the stock market). The expected value of the capital from (2) is reconstructed as follows:

$$\mathbb{E}[V] = \left\{ (2\Delta_1)^m (2\Delta_2)^{N-m} \right\} V_0, \quad (20)$$

where Δ_1 is the probability of winning the bet when the market is good (i.e., stocks go up), and Δ_2 is the winning probability when the market is bad (i.e., stocks go down). When the market is good, let us assume that an investor bets a fraction of his capital f each time, then the expected benefit when the stock market goes up could be as follows:

$$V_m = \left\{ w_1^u \cdot l_1^{m-u} \right\} V_0, \quad (21)$$

where

$$w_1 = (1 - f) + (1 + w_0)f, l_1 = 1, \quad (22)$$

where m is the total number of the good market, u is the total number of winning bets within the good stock market status, and w_0 is the rate of increasing stocks within mutual funds. When stocks go up, there is no loss in capital although there is a betting loss. When the stock market goes bad, the expected benefit as follows:

$$V_n = \left\{ w_2^d \cdot l_2^{n-d} \right\} V_0, \quad (23)$$

where

$$w_2 = 1, l_2 = (1 - f) - l_0 f, \quad (24)$$

where n is the total number of good markets, d is the total number of winning bets within the bad stock market status, and l_0 is the rate of decreasing stocks within mutual funds. When stocks go down, the best betting does not lose the capital of the fund manager. From (21)–(24), the expected capital after N -th betting is as follows:

$$\mathbb{E}[V] = [\Psi(f)] V_0, \quad (25)$$

where

$$\Psi(f) = \{1 + w_0 f\}^u \{1 - (1 + l_0)f\}^{n-d}. \quad (26)$$

Similar to (8), let us maximize the Ψ with respect to fraction f . From (26) and (8), the maximum capital growth could be determined as follows:

$$f^* = \left\{ \exists f \mid \frac{d\Psi(f)}{df} = 0, f \in [0, 1] \right\}. \quad (27)$$

From (27), we need to determine the optimal fraction f^* :

$$\left. \frac{d\Psi(f)}{df} \right|_{f=f^*} = \frac{d}{df} \left(\{1 + w_0 f\}^u \{1 - (1 + l_0)f\}^{n-d} \right) = 0, \quad (28)$$

then,

$$f^* = \frac{u}{(1 + l_0)\{u + (n - d)\}} - \frac{(n - d)}{w_0\{u + (n - d)\}}. \quad (29)$$

Let us consider that Δ_1 and Δ_0 are the winning betting rates, which depend on the stock market status. From (29),

$$f^* = \frac{\Delta_1}{(1 + l_0) \cdot m} - \frac{1 - \Delta_0}{w_0 \cdot n}, \quad (30)$$

where $u = \Delta_1 \cdot m, d = \Delta_0 \cdot n$. Since, $f^* \geq 0$ from (11), we obtain the following:

$$\frac{\Delta_1 \cdot m}{(1 - \Delta_0)n} \geq \frac{1 + l_0}{w_0}, w_0, l_0 \in [0, 1]. \quad (31)$$

It is noted that Δ_1 indicates the winning betting rate when the stock goes up and Δ_0 is the winning betting rate when the stock goes down. Let us consider that the winning rate of the fund manager is statistically even whenever the stock market status is good or bad (i.e., $\Delta := \Delta_0 = \Delta_1$); then, the optimal fraction f^* could be reconstructed as follows:

$$f^* = \frac{\Delta \cdot m}{(1 + l_0)\{\Delta(m - n) + n\}} - \frac{n(1 - \Delta)}{w_0\{\Delta(m - n) + n\}}, \quad (32)$$

where $u = \Delta \cdot m, d = \Delta \cdot n$. From (32), we obtain the following:

$$\frac{\Delta \cdot m}{(1 - \Delta)n} \geq \frac{1 + l_0}{w_0}, \quad (33)$$

and

$$\Delta \geq \frac{(1 + l_0)n}{w_0m + (1 + l_0)n}, w_0, l_0 \in [0, 1]. \quad (34)$$

The winning probability of the player Δ from (34) is influenced by several factors, namely the reward rate denoted as w_0 , the loss rate symbolized as l_0 , the number of favorable markets represented by m , and the count of unfavorable markets indicated by n . The primary distinction between the KC and KCE lies in the latter's enhanced methodology, which is designed for effective winning strategies based on the behaviors of market participants who buy or sell stocks rather than invest directly in stocks. More importantly, the KCE does not directly invest in stocks, potentially lowering investment and gambling risks. This extension is suited to dynamic market environments, such as the stock and commodities markets, including oil and gold, where it aims to minimize risk for direct investments.

4. Conclusions

This research deals with the innovative extension of the Kelly criterion (KCE) that suggests an optimal response to market fluctuations. It offers an explicit solution for expected capital growth, making it a valuable tool for investors navigating volatile markets like stocks, oil, and gold. The KCE allows us to adjust to both favorable and unfavorable market conditions, each requiring distinct investment strategies. The core concept of the KCE revolves around investing in a fund manager with a proven winning rate. When the market thrives, funds should be invested in stocks, and when it falters, they should be kept in reserve. Therefore, using the KCE can significantly enhance the potential for profit, regardless of the market status, making it a highly promising strategy for future investment practice. It is noted that an investor whom you want to bet on does not need to be a human. The KCE can feasibly be incorporated into AI for stock market predictions. Consequently, the KCE provides a significant measure for evaluating a variety of machine learning-based prediction systems, including those for stock markets. Integrating the KCE with machine learning algorithms for time-series analysis of assets such as oil, stocks, and currency exchanges can enhance the forecasting framework. Adapting this innovative combined approach into real-world applications for optimizing real-time forecasting models in machine learning represents a promising direction for future research topics.

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