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Stability Analysis of the Credit Market in Supply Chain Finance Based on Stochastic Evolutionary Game Theory

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Abstract: The rapid development of supply chain finance (SCF) has significantly alleviated the financing difficulties of small and medium-sized enterprises (SMEs). However, it is important to recognize that within the accounts receivable financing segment of the SCF credit market, the credit risk associated with SMEs poses a serious challenge and potential threat to the stability, health, and sustainable development of the SCF system. This paper pays special attention to the stability of the two-party evolutionary game between SMEs and financial institutions (FIs) within the context of the Chinese SCF credit market. To identify a pathway to reduce credit risks for SMEs while simultaneously enhancing system stability, this paper adopts the stochastic evolutionary game (SEG) model and combines the fixed-point method to determine the conditions that satisfy the stability of the system's index p mean square of the system. This study has made attempts in various aspects, such as the innovative construction and investigation of a nonlinear SEG model, the endeavor to study the stability of SEG systems using fixed-point methods, and the innovative construction of a more realistic two-player SEG system. The data and simulation results generated from hypothetical scenarios show that the conclusions of the article are credible and feasible. Through the study, we conclude that the higher credit ratio from FI and the higher penalty intensity from core enterprises (CEs) will accelerate the stability of the system. Based on solid data and modeling analysis, insights into the regulation of FI are provided.

Keywords: stochastic evolutionary game; credit market of supply chain finance; p-exponential stability in the mean square; fixed-point method

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1. Introduction

During economic downturns, SMEs that are heavily reliant on bank loans face tremendous liquidity pressure [1–3]. SCF has emerged as a financing solution, capable of alleviating financial strains by reducing interest rates, extending payment terms, and providing more loans, thereby benefiting supply chain partners (see [4–6]). The central aim revolves around fostering a decrease in financial risk within the supply chain context, achieved through enhancements in the collaborative cash-to-cash cycle and optimization of working capital [7].

SCF has received extensive attention from the business community, academia, and government since its introduction [8]. Currently, studies related to SCF are mainly categorized into two major genres: financial supply chain management and trade credit (see [7,9]).

The financial theory of agency, pioneered by Ross [10], elucidates the dynamics between shareholders and corporate managers. This theory was subsequently refined by Jensen and Meckling [11], who characterized the agency relationship as a contractual construct aimed at analyzing the interplay of conflicting interests and cooperative endeavors between agents and principals. Within the realm of the SCF credit market, a plethora of participants, including FIs, CEs, and SMEs, engender a complex web of agency relationships. Concurrently, scholars have focused on the equilibrium dynamics among the primary stakeholders in the SCF credit market. Employing game-theoretic methodologies, they endeavor to devise strategies to manage systemic risks, thereby fostering enduring and resilient development within these agency relationships.

The realm of SCF exhibits a conspicuously game-like essence. Within this domain, scholars have extensively delved into game-theoretic analyses, exploring various facets of SCF dynamics [12]. In the realm of "SCF credit market" research, apart from a small number of studies [13] employing alternative game methods, such as differential games, the majority utilize evolutionary game methods. Li X. [14] constructed an evolutionary game model of SMEs' credit market under both the SCF and traditional financing models to explore the impact of SCF on the SME credit market; Tang D. [15] developed an evolutionary game model that included family farms, e-commerce platforms, and rural credit unions, summarizing the importance of several core factors such as the production volume of e-orders for agricultural products, the loan pledge rate, the availability of autonomous repayment mechanisms, the cost of e-commerce platforms, and the loan interest rate in shaping the dynamics of the financing game in the context of e-commerce-based SCF; Geng Z. [16] investigated agricultural supply chain receivable financing on commodity trading platforms through the construction of a tripartite game model involving banks, agricultural product buyers, and sellers; Xu L. [17] also focused on agricultural supply chain receivable financing in commodity trading platforms, and constructed a similar tripartite game model involving banks, agricultural product buyers, and sellers; Xu Y. et al. [18] explored the game relationship between regulators and FIs in the SCF credit market; Wang X. [19] developed a three-party game model involving FIs, CEs, and SMEs in the SCF credit market.

The SEG theory, a branch of game theory, explores the evolution of strategies and behaviors of game participants in uncertain environments influenced by random events. Initially, scholars recognized the significant presence of uncertainty in the objective world and incorporated it into game theory studies. Foster and Young P [20] were the first to introduce the concept of SEG and highlighted the significance of stochastic stabilization strategies. The SEG aims to simulate the strategic interactions among these stakeholders under uncertain and evolving conditions. By constructing the model, it is possible to explore the dynamic evolution of stakeholder strategies, the emergence of cooperation or competition, and the formation of market structures. Additionally, the model can facilitate the analysis of how uncertainties and randomness impact the stability and equilibrium of the market. Thus, numerous experts have leveraged SEG to investigate decision-making challenges across the social, economic, and financial spheres. Examples include examining collaborative governance strategies amid urban public crises [21], devising optimal data-sharing approaches within digital government frameworks [22], analyzing the dynamics of competition and cooperation in online supply-chain finance [23], exploring endogenous asset payoffs [24], and addressing risks within financial networks [25] and so on.

SEG theory, a branch of game theory, explores the evolution of strategies and behaviors among game participants in uncertain environments, influenced by random events. In this model, participants' behaviors are shaped not only by the strategies of others but also by stochastic factors, such as environmental changes or uncertainties in information.

In the realm of SCF research, SEG is considered an innovative and effective method. However, due to its complex mathematical reasoning, most studies have focused on deterministic evolutionary game models (e.g., [26–29]), and there are few results on the application of SEG in this field (see [12,23,30–32]). However, after a comprehensive review of databases such as Google Scholar and the China National Knowledge Infrastructure, we found that fewer studies have used the SEG approach to study the financing of the SCF credit market (only the source [12]).

Many studies on stochastic dynamical systems focus on the stability of these systems. This is because stability is crucial for understanding the long-term behavior of systems and for dealing with external disturbances ([33]). SCF is a complex financial ecosystem involving multiple participants such as suppliers, manufacturers, distributors, and FIs, and the stability of this system plays a crucial role in the healthy development of the entire supply chain ecosystem. Therefore, most research studies on the SCF credit market focus on exploring the conditions for maintaining the stability of this system. When studying the stability of SEG models, almost all scholars have adopted the Lyapunov direct method (see [12,21–25,30–32]). However, this method often encounters certain difficulties, such as the requirement of a bounded time lag [34]. Recently, the fixed point method has gained attention for analyzing the stability of various dynamical systems. The findings indicate that the fixed-point method can overcome the limitations associated with stability analysis (see [34–41]). Considering the advantages of the fixed-point method, we will adopt the Banach fixed-point method to investigate the p-exponential stability in the mean square condition of system (7) in this paper.

Since Luo ([34]) first applied the fixed-point method to stochastic dynamical systems, this approach has been widely utilized in theoretical investigations across various stochastic dynamical systems. While SEG has found extensive application, the use of the fixed-point method in studying the stability of the SEG model remains unexplored. In order to obtain the conditions satisfied by the stability of the SCF credit market, this paper will establish a two-party SEG system involving "FI-SME" within the SCF credit market. We will then consider the p-exponential stability in the mean square of this system using the fixed-point method. Kabak. et al. [42] argue for the potential and role of simulation modeling in operations management theory building from both positivist and post-positivist perspectives. Based on this, we will utilize MATLAB to validate the model's conclusions with real-world cases and perform sensitivity analysis on various parameters of the system in the next step. Although it is not possible to fully compare the advantages and disadvantages of the fixed-point method and the Lyapunov direct method [34], the research in this paper provides a new method and idea for the stability of the credit market in SCF.

The subsequent sections of the paper are organized as follows: Section 2 details the construction of the SCF credit market evolutionary game model and its analysis; Section 3 further develops the SEG model from Section 2 and discusses its stability; and Section 4 offers numerical simulation examples. Finally, in Section 5, we present the conclusions drawn from this study.

2. Construction and Analysis of Evolutionary Game Models

2.1. Description of the Problem

SCF deals with the management of financial flows along the supply chain [6]. The main objective of SCF is to support and enhance cash flows, as well as to reduce financial risks in the supply chain by enhancing the collaborative cash-to-cash cycle and optimizing working capital [7]. In order to maximize their own interests and those of the entire supply chain, CEs often use their own credit, such as through financing guarantees in supply chain finance, to assist SMEs in the supply chain in obtaining loans from FIs.

In the accounts receivable financing model of SCF, SMEs act as upstream suppliers and typically provide products or services to CEs on credit. Once a transaction is completed, the CE issues accounts receivable vouchers to the SME, which then transfers these vouchers to a financial institution (FI) to obtain financing support. At this point, the responsibility for collecting the receivables shifts from the SME to the FI, and the CE is required to repay the accounts receivable before the due date.

Based on publicly available data from China, FIs have strict qualification criteria for credit institutions in the SCF process. According to the findings of Gao G. et al. [43], the existing literature characterizes CE decisions as "guaranteed" and "unguaranteed". But according to the regulations of the management measures for the accounts receivable financing business under the CE project of a certain state-owned commercial bank, SMEs applying for accounts receivable loans from banks are required to obtain credit from CEs. Therefore, in the SCF credit market, the only viable decision for CEs can be "guaranteed".

In the recourse accounts receivable credit market, credit risk primarily arises from SMEs and CEs. The credit risk associated with SMEs is largely derived from operational risks. In the implementation of accounts receivable financing, CEs use their creditworthiness to extend credit to SMEs. However, the primary responsibility for repaying the loan remains with the SME, despite the involvement of the CE. The operational health and debt-servicing capability of SMEs impact the potential for financial institutions to recover the loan amount. Additionally, the credit risk for CEs encompasses risk-sharing between principal and agent as well as operational risks. On the one hand, during the credit approval process, CEs and SMEs may exploit information asymmetry to engage in deliberate collusion, credit fraud, and falsification of information to secure loans from FIs, which may fail to accurately assess the business risk. On the other hand, FIs extend loans based on the solid reputation of the CEs. If a CE fails to deposit the principal and interest into the designated supervisory account within the agreed-upon timeframe, the FI can only pursue recovery of these amounts from the SME and impose fines. Given the CE's strong repayment capability, the risk of CE default mainly stems from potential joint fraudulent behavior with the SME.

This article focuses on the accounts receivable financing model in SCF, and utilizes the SEG framework to construct a two-party evolution model based on "FI-SME". This work aims to explore strategies to achieve the stable stochastic evolution of the system and provide relevant recommendations.

2.2. Model Assumptions

Based on the conditions of the evolutionary game research problem, the following assumptions common to the construction of SEG systems are proposed.

Hypothesis 1 (H1): *This study uses the FI and SME in the supply chain to serve as two participants in a game, with the credit market formed by these two participants as the subject of the research. It is assumed that both stakeholders have bounded rationality.*

Hypothesis 2 (H2): *Due to the differences in analytical and judgment abilities between the two participants in the game, there exists information asymmetry during the game process, leading to stochastic outcomes and behavioral interactions.*

Hypothesis 3 (H3): *The two participants in the game have different choices. FI can choose to not provide loans (suppose the probability is $x \in [0,1]$) or provide loans; SME can choose default (suppose the probability is $y \in [0,1]$) or non-default. When an SME chooses "Default", it is faced with the choice of whether or not to make a joint fraudulent loan with CE.*

2.3. Analysis of the Game Model

Suppose the total accounts receivable amount is M. When FI approves the business application, it is required to pay the SME an amount of λM ($\lambda \in [0.5, 0.8]$). Then the SME transfers the accounts receivable documents to FI, resulting in a transfer of debt ownership. If FI chooses not to accept the business application after conducting a credit check on the SME, the allocated funds can be redirected for other purposes, such as short-term loans. Suppose the interest rate on loans is r_1 and the interest rate on deposits is r_2. The process of FI lending necessitates a meticulous credit assessment of both the CE and SME. This includes scrutinizing their financial records, examining their credit history, and evaluating their ability to repay the loan. It is important to note that conducting such a comprehensive check incurs a certain cost known as the credit check cost (assumed to be C_F). Assume that the production cost of the SME is C, the default cost (or punitive intensity) is C_S, and the rate of return on normal production is r_3. Assume that when CE and SME jointly cheat on a loan, the SME's allocation is β. The reasons why a CE may not make a payment to the designated supervisory account as agreed upon typically fall into two categories. On the one hand, if the products supplied by an SME do not meet the CE's requirements, the CE will refuse to pay for the goods. On the other hand, if the SME and CE collude in loan fraud, the CE will also withhold payment. Assume that the probability of the CE refusing to pay due to issues with the SME's products is κ, while the probability of joint loan fraud is $1-\kappa$. According to Sun R. et al. [44], collusive behavior between firms is the most significant source of CE credit risk, accounting for more than 90% of the total. Therefore, in general, $\kappa < 0.1$.

2.4. Payment Matrix Construction

The symbols of the relevant parameters are explained in Table 1.

Table 1. Accounts receivable factoring business model parameter settings.

Parameters	Clarification
x	Probability of not lending by financial institutions
y	Probability of default for SMEs
M	Level of accounts receivable, currency unit
λ	Accounts receivable factoring credit rate (lending ratio)
κ	The probability that SMEs choose not to engage in joint fraudulent loans with CEs
r_1	Interest rates on loans from financial institutions
r_2	Deposit rates for financial institutions
r_3	Rate of return on SME financing used for normal production
C	Production costs for SMEs, currency unit
C_S	Punitive intensity (penalty from CEs), currency unit
C_F	Cost of credit to financial institutions, currency unit
β	Distribution ratio of benefits to SMEs in case of joint fraudulent loans

From the previous assumptions, it is clear that FI's decision consists of "provide loan" and "do not provide loan", while SME's decision consists of "Default" and "Non-default". However, when an SME chooses "Default", it is faced with the choice of whether or not to make a joint fraudulent loan with CE. In addition, the latest Civil Code of China stipulates that in credit markets with recourse rights, FIs have the right to sequentially trace claims to both CEs and SMEs in the event of a default. In such cases, FIs can legally pursue debts from both the borrowing entity and the SME to safeguard their legitimate interests. Such regulations promote the stable development of the credit market and enhance the overall efficiency of the financial system. Therefore, Table 2 illustrates the distribution of benefits between FIs and SMEs in the SCF for the recourse accounts receivable credit market.

Table 2. Matrix of benefits for both financial institutions and SMEs.

Strategic Combination	FI	SME
(Provide loans, not joint)	$(1-\lambda)M - C_F - \lambda M r_2$	$\lambda M(1+r_3) - C - C_S$
(Provide loans, Joint)	$-\lambda M(1+r_2) - C_F$	$\lambda\beta M(1+r_3)$
(Provide loans, Not default)	$(1-\lambda)M - C_F - \lambda M r_2$	$\lambda M(1+r_3) - C$
(Not provide loans, not joint)	$\lambda M(r_1 - r_2) - C_F$	$M - C - C_S$
(Not provide loans, joint)	$\lambda M(r_1 - r_2) - C_F$	0
(Not provide loans, not default)	$\lambda M(r_1 - r_2) - C_F$	$M - C$

Note: 'Not joint' means that the SME chooses to default but does not make a joint fraudulent loan with CE. Correspondingly, "Joint" means that the SME chooses to default and defraud the loan jointly with the CE.

Given that the anticipated yield on loans extended by the FI is denoted as E_{11}, the expected return on loans not extended is represented by E_{12}, with the average expected return being E_1, with the following:

$$\begin{aligned} E_{11} &= (\kappa y - \lambda - \lambda r_2 - y + 1)M - C_F. \\ E_{12} &= \lambda M(r_1 - r_2) - C_F. \\ E_1 &= (1-x)E_{11} + xE_{12} \end{aligned} \tag{1}$$

Let $a(t) = (\kappa y + \lambda r_1 - 2\lambda r_2 - \lambda - y + 1)M$. Obviously, x is a function of t. So, the equation describing the replication dynamics of the FI is derived as follows:

$$dx(t) = x(t)(1 - x(t))(E_{11} - E_{12})dt = x(t)(1 - x(t))a(t)dt. \tag{2}$$

Given that the expected return on SME default is denoted as E_{21} and the expected return on non-default is denoted as E_{22}, the average expected return is E_2, with the following:

$$\begin{aligned} E_{21} &= (\kappa\lambda r_3 + \kappa\lambda + \lambda\beta r_3 + \lambda\beta - \lambda\beta\kappa r_3 - \lambda\beta\kappa - \kappa)(1-x)M + \kappa(M - C - C_S). \\ E_{22} &= (-\lambda r_3 x + \lambda r_3 - \lambda x + \lambda + x)M - C. \\ E_2 &= (1-y)E_{21} + yE_{22} \end{aligned} \tag{3}$$

Let $b(t) = (\kappa\lambda r_3 + \kappa\lambda + \lambda\beta r_3 + \lambda\beta - \lambda\beta\kappa r_3 - \lambda\beta\kappa - \kappa - \lambda r_3 - \lambda + 1)(1-x)M - (1-\kappa)M + (1-\kappa)C - \kappa C_S$. Obviously, y is a function of t. So, the equation describing the replication dynamics of the SME is derived as follows:

$$dy(t) = y(t)(1 - y(t))(E_{21} - E_{22})dt = y(t)(1 - y(t))b(t)dt. \tag{4}$$

In order to better reflect the impact of various uncertainties from the external environment on the decision-making of FIs and SMEs, this paper introduces random factors into models (2) and (4). As a result of these modifications, we obtain an SEG dynamics system for the participating entities.

3. Construction of SEG Models and Analysis of Stabilization Strategies

3.1. Construction of SEG Systems

Based on the above deterministic evolutionary game dynamics system, (2) and (4), this paper introduces the Wiener process into the dynamic evolutionary game model of the SCF credit market; this reflects the impact of various types of external stochastic uncertainties on the decision-making processes of FIs and SMEs. Therefore, the SEG model of FI is as follows.

$$\begin{cases} dx(t) = x(t)(1 - x(t))a(t)dt + \beta(x(t))d\omega(t), & t \geq 0, \\ x(t) = 0, & t = 0. \end{cases} \tag{5}$$

The stochastic dynamic evolutionary game model of SME is as follows:

$$\begin{cases} dy(t) = y(t)(1 - y(t))b(t)dt + \beta(y(t))d\omega(t), & t \geq 0, \\ y(t) = 0, & t = 0. \end{cases} \tag{6}$$

Drawing on the methodology by Chen et al. [45], we set $\beta(x(t)) = \delta x(t)(1 - x(t))$ and $\beta(y(t)) = \delta y(t)(1 - y(t))$, where δ denotes the intensity of the random disturbance and is positive. Clearly, when $x(t)$ or $y(t)$ equals 0.5, random disturbance reaches its maximum. Consequently, each factor affecting system stability will not have a decisive impact. Systems (5) and (6) which involve multiple participants such as FIs, CEs, and SMEs, are critical to the healthy development of the entire supply chain ecosystem. Next, we will employ fixed-point methods to examine the stability of these systems.

3.2. Stability Theory of Stochastic Dynamical Systems

3.2.1. Overview of the Lemma

Let $R = (-\infty, +\infty), R^+ = (0, +\infty)$, $C(A, \Omega)$ is a continuous function from A to Ω. $\{\Omega, F, P\}$ is a complete probability space with the flow F_t satisfying the usual conditions. $\omega(t)$ is the standard Wiener process defined on this space, $d\omega(t) = \omega(t)\sqrt{dt}$, with functions $a(t), b(t) \in C(R^+, R)$. Consider a class of nonlinear stochastic dynamical systems, as follows:

$$\begin{cases} d\psi(t) = f(\psi(t))dt + g(\psi(t))d\omega(t), t \geq 0; \\ \psi(t) = \phi(0), \qquad\qquad t = 0. \end{cases} \tag{7}$$

where $\psi(t) \in R$. $f(x), g(x)$ are functions that satisfy Lipschitz coefficients of F, G.

Definition 1 (Xiazhou [46])**.** *The zero solution of system (7) exhibits p-exponential stability in the mean square if, for any initial value $\phi(0)$ and constant $p \geq 2$, there exist positive numbers δ and M, such that, when $t \geq 0$, there is $E|x(t)|^p \leq M|\phi(0)|^p e^{-\delta t}$.*

3.2.2. Stability Theorem and Proof for Stochastic Dynamical Systems

Theorem 1. *Let there exist constants α, ρ and a continuous function $h(t) : [0, \infty) \to R^+$ that satisfies $h(t) \in [\alpha/p, +\infty)$. When $t \geq 0$, we have the following:*

$$\int_0^t e^{-\int_s^t h(\mu)d\mu}|h(s) + F|ds + \left(\int_0^t e^{-2\int_s^t h(\mu)d\mu} G^2 ds\right)^{1/2} \leq \rho < 1.$$

Then, the zero solution of system (7) exhibits p-exponential stability in the mean square. So, there exists a constant $M > 0$ satisfying $E|x(t)|^p \leq M|\phi(0)|^p e^{-\alpha t}$.

Proof. S denotes the Banach space consisting of F-adaptation processes $\psi(t, \omega) : [0, \infty) \times \Omega \to R$. For a fixed $\omega \in \Omega$, $\psi(t, \omega)$ is required to be continuous for almost all t. Additionally, suppose that when $\psi(0, \omega) = \phi(0)$ and as $t \to \infty$, there exists a positive constant α, such that $e^{\alpha t} E \sup_{s \in [0,t)} |\psi(s, \omega)|^p \to 0$.

The operator $\Psi : S \to S$ is defined as follows:

(I) When $t = 0$, $(\Psi\varphi)(0) = \phi(0)$;
(II) When $t \geq 0$,

$$\begin{aligned} (\Psi\varphi)(t) = \phi(0)e^{-\int_0^t h(\mu)d\mu} + \int_0^t e^{-\int_s^t h(\mu)d\mu}[h(s)\varphi(s) + f(\varphi(s))]ds \\ + \int_0^t e^{-\int_s^t h(\mu)d\mu} g(\varphi(s))d\omega(s). \end{aligned} \tag{8}$$

Let $(\Psi\varphi)(t) = \sum_{j=1}^3 I_j$. It is clear that Ψ is mean-square continuous on $[0, +\infty)$. Next, it will be shown that $\Psi(S) \subset S$. Since

$$e^{\alpha t} E|(\Psi\varphi)(t)|^p \leq 3^{p-1} \sum_{j=1}^3 e^{\alpha t} E|I_j(t)|^p.$$

So, by $h(t) \in (\alpha/p, +\infty)$ (as $t \to \infty$) and the conditions of Theorem 1, we have the following:

$$e^{\alpha t} E|I_1(t)|^p = e^{\alpha t} E\left|\phi(0)e^{-\int_0^t h(\mu)d\mu}\right|^p \leq e^{\alpha t - p\int_0^t h(\mu)d\mu} E|\phi(0)|^p \to 0,$$

$$e^{\alpha t} E|I_2(t)|^p = e^{\alpha t} E\left|\int_0^t e^{-\int_s^t h(\mu)d\mu}[h(s)\varphi(s) + f(\varphi(s))]ds\right|^p \leq \beta e^{\alpha t} E(\sup_{s \in [0,t)} |\varphi(s)|^p)ds \to 0.$$

This follows from Hölder's inequality:

$$e^{\alpha t}E|I_3(t)|^p = e^{\alpha t}E\left|\int_0^t e^{-\int_s^t \alpha(\mu)d\mu}G(\varphi(s))d\omega(s)\right|^p \leq e^{\alpha t}E\left(\int_0^t e^{-2\int_s^t h(\mu)d\mu}G^2|\varphi(s)|^2 ds\right)^{p/2}$$
$$\leq e^{\alpha t}E(\sup_{s\in[0,t]}|\varphi(s)|^p\Big)\left|\int_0^t e^{-2\int_s^t \alpha(\mu)d\mu}G^2 ds\right|^{p/2} \leq \beta e^{\alpha t}E(\sup_{s\in[0,t)}|\varphi(s)|^p\Big) \to 0.$$

So, $\Psi(S) \subset S$.

Finally, by the conditions of Theorem 1, there exists a positive constant M satisfying the following:

$$(1+\frac{1}{M})\left[\int_0^t e^{-\int_s^t h(\mu)d\mu}|h(s)+F|ds\right]^2 + (1+M)\int_0^t e^{-2\int_s^t h(\mu)d\mu}G^2 ds \leq \beta^2 < 1. \quad (9)$$

By (9), Ψ is a compression mapping. By the compressive mapping theorem, Ψ has a unique immovable point $x(t)$ in the space (S) that satisfies $x(0) = \phi(0)$ and there exists a positive constant α, such that when $t \to \infty$, $e^{\alpha t}E|x(t)|^p \to 0$. So, the zero solution of system (7) is p-exponential in the mean square. Therefore, this completes the proof. □

4. Numerical Simulation Analysis and Parameter Sensitivity Analysis

In order to verify the feasibility of the article's findings, the relevant parameters of model (7) are assumed as follows.

Assume that the accounts receivable amounting to M is 1 million, and FI's credit ratio is λ. From the previous analysis, $k \leq 0.1$. Here, we take $\kappa = 0.1$. Based on the latest data, the annual interest rate for accounts receivable financing with a maturity of up to (and including) one year is 5.22%, which is assumed to be $r_1 = 0.05$ here. According to the latest data, the interest rate on one-year deposits is 1.48% per annum. Adding the portion for demand deposits, the interest rate can be assumed to be $r_2 = 0.01$. The return on normal SME production is assumed to be $r_3 = 0.2$. Based on a profit margin of 35%, C is approximately 0.7 million. Assume that when the SME defaults, the CE penalty is C_S million. Assume that the cost of credit to FI is $C_F = 0.01$ million. In the case of a joint fraudulent loan, the SME will generally be allocated a lower percentage than the CE, so it can be assumed that $\beta = 0.3$. Then, $a(t) = -0.9y(t) - 1.07\lambda + 1$, and $b(t) = -0.756\lambda(1-x(t)) + 0.9(1-x(t)) - 0.1C_S - 0.27$.

4.1. Credit Ratio Factor

With other parameters being held constant, the effect of 'Credit Ratio λ' on the evolutionary path of FI and SME strategies is investigated. Let $C_S = 0.5, \delta = 0.5$.

The random dynamic evolutionary game system between FI and SME can be described as follows based on the analysis above:

$$\begin{cases} dx(t) = x(t)(1-x(t))[(-0.9y(t) - 1.07\lambda + 1)dt + 0.5d\omega(t)], & t \geq 0, \\ x(t) = 0, & t = 0. \end{cases} \quad (10)$$

$$\begin{cases} dy(t) = y(t)(1-y(t))[(-0.756\lambda(1-x(t)) + 0.9(1-x(t)) - 0.32)dt + 0.5d\omega(t)], & t \geq 0, \\ y(t) = 0, & t = 0. \end{cases} \quad (11)$$

For systems (10) and (11), it is evident that there exists a trivial solution, i.e., the zero solution. In fact, at the initial moment of the game, the probabilities of all parties participating in the alliance are zero. Therefore, systems (10) and (11) have a zero solution, and in the absence of external disturbances, this zero solution remains stable along its trajectory, thus serving as the equilibrium solution of the system.

$x(1-x)$ is a function that satisfies the Lipschitz coefficient of 1. Therefore, we can obtain the Theorems 2 and 3 from Theorem 1.

Theorem 2. *Let there exist constants α, ρ and a continuous function $h(t) : [0, \infty) \to R^+$ satisfying $h(t) \in [\alpha/p, +\infty)$. If $t \geq 0$, we have the following:*

(**A.1**)

$$\int_0^t e^{-\int_s^t h(\mu)d\mu}|h(s) - 0.9y(s) - 1.07\lambda + 1|ds + \left(\int_0^t e^{-2\int_s^t h(\mu)d\mu} 0.25ds\right)^{1/2} \leq \rho < 1,$$

(**A.2**)

$$\int_0^t e^{-\int_s^t h(\mu)d\mu}|h(s) + 0.9(1 - x(s)) - 0.756\lambda(1 - x(s)) - 0.32|ds + \left(\int_0^t e^{-2\int_s^t h(\mu)d\mu} 0.25ds\right)^{1/2} \leq \rho < 1.$$

If condition $A1$ holds, then the zero solution of system (10) is p-exponential stable in the mean square. Similarly, if condition $A2$ holds, the zero solution of system (11) is also p-exponential stable in the mean square.

Remark 1. *In fact, assuming $y(t) = 0.5$ (which is a realistic assumption), setting $h(t) = 1.07, \lambda - 0.55$, the condition for p-exponential stability in the mean square of system (10) is $\frac{0.5}{\sqrt{2.14\lambda - 1.1}} < 1$. Of course, the range of λ would be more precise if the appropriate $h(s)$ was chosen. Similarly, assuming $x(t) = 0.5$ (which is a realistic assumption), and setting $h(t) = 0.378\lambda - 0.13$, the condition for p-exponential stability in the mean square of system (11) is $\frac{0.5}{\sqrt{0.756\lambda - 0.26}} < 1$. Under normal circumstances, the lending ratio λ of FI tends to be no less than 0.7. Hence, Figures 1 and 2 illustrate the impact of variations in the lending ratio λ, which is not less than 0.7, on the decision-making evolution paths of both FIs and SMEs. The numerical simulation graph is shown in Figure 2. The simulation results in Figures 1 and 2 demonstrate the validity of the conclusions in this paper.*

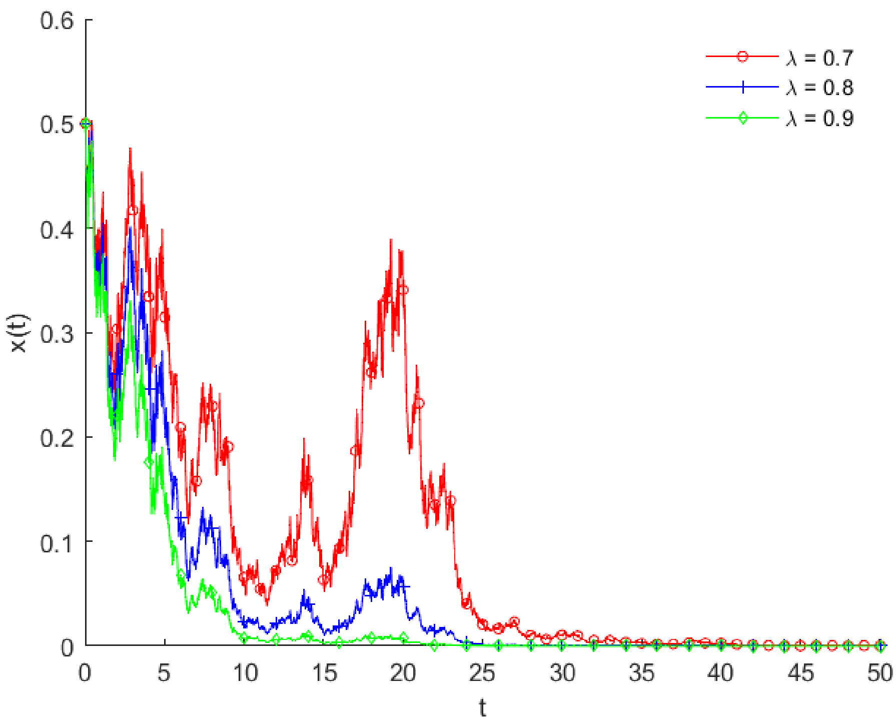

Figure 1. The graph of function $x(t)$ of system (10).

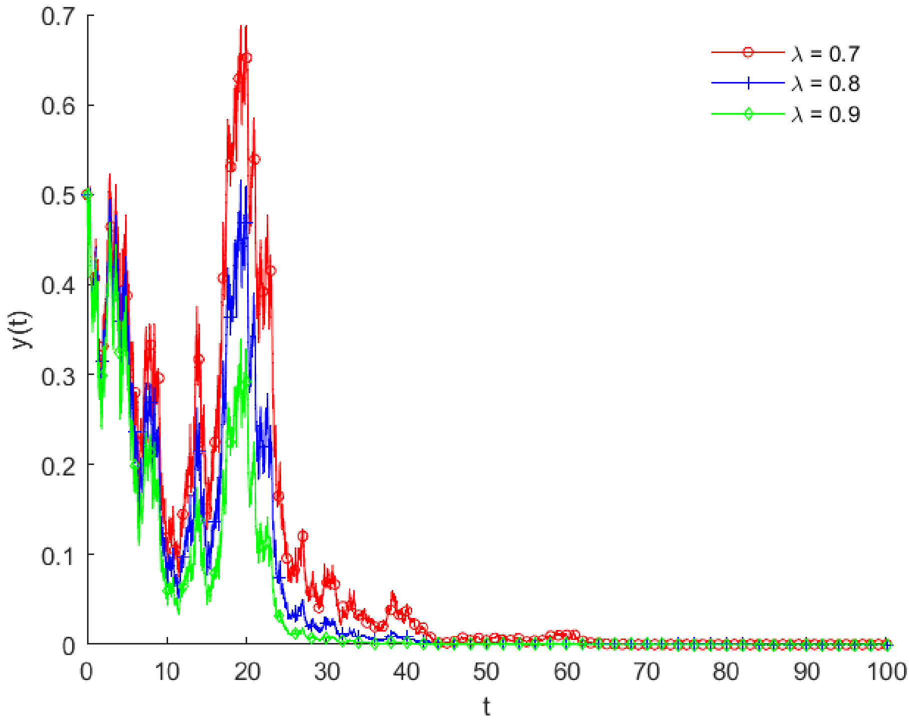

Figure 2. The graph of function $y(t)$ of system (11).

Remark 2. *According to the results from Figures 1 and 2, as the lending ratio increases, both FIs and SMEs exhibit significantly accelerated convergence speeds, demonstrating a clear consistency. This could be because the lending ratio of FIs is correlated with the credit ratings of SMEs—the higher the credit rating of SMEs, the higher the lending ratio of FIs. Therefore, a higher lending ratio implies a decrease in the probability of default for all parties involved. Additionally, it is noteworthy that the convergence time of FIs is notably earlier than that for SMEs. This implies that within this system, FIs bear greater risks. Consequently, the lower lending ratio of FIs for SMEs with lower credit ratings can be explained.*

4.2. Penalty Intensity Factor

From the previous analysis, we set $\lambda = 0.8$, $\delta = 0.5$. The SEG system between FI and SME can be described as follows based on the analysis above:

$$\begin{cases} dy(t) = y(t)(1-y(t))[(0.2952x(t) - 0.1C_S - 0.27)dt + 0.5d\omega(t)], & t \geq 0, \\ y(t) = 0, & t = 0. \end{cases} \tag{12}$$

Theorem 3. *Let there exist constants α, ρ and a continuous function $h(t) : [0, \infty) \to R^+$ satisfying $h(t) \in [\alpha/p, +\infty)$. If $t \geq 0$, we have the following:*

$$\int_0^t e^{-\int_s^t h(\mu)d\mu} |h(s) + 0.2952x(s) - 0.1C_S - 0.27| ds + \left(\int_0^t e^{-2\int_s^t h(\mu)d\mu} 0.25 ds \right)^{1/2} \leq \rho < 1.$$

Then, the zero solution of system (12) is also p-exponential stable in the mean square.

Remark 3. *As can be seen from the results of the simulation in Figure 3, the greater the punitive intensity from the CE, the faster the SME converges to zero. Specifically, when the punitive intensity reaches 0.9 (calculated by CS/M), the convergence speed of the SME is remarkably rapid. Conversely, when the punitive intensity is 0.1, noticeable fluctuations appear in the evolutionary game trajectory of SME decisions. Therefore, to ensure the long-term healthy development of the FI-SME credit market game system, adopting high-level punitive measures is imperative.*

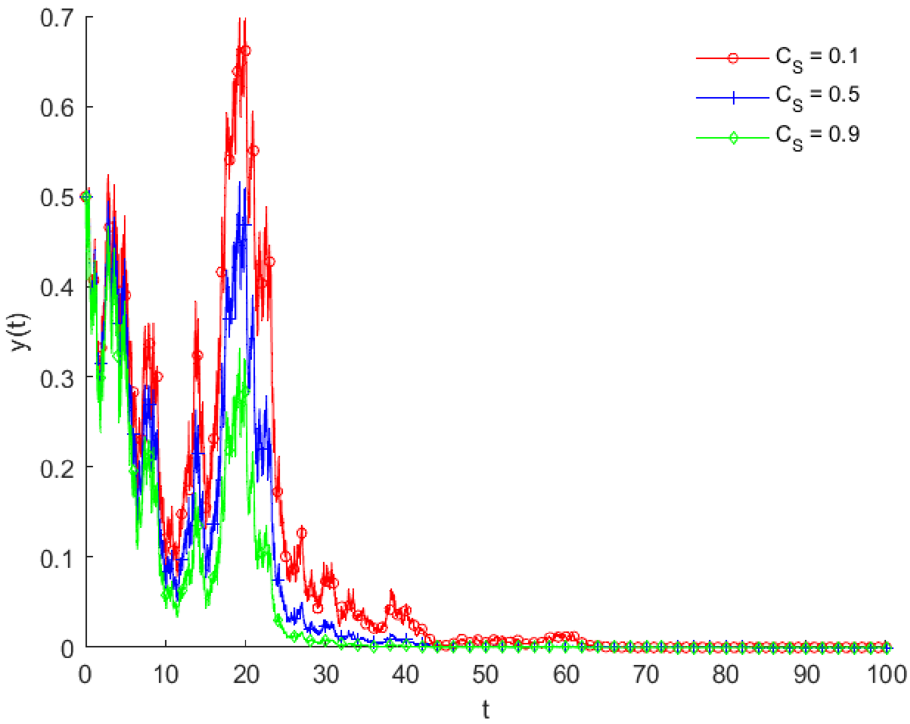

Figure 3. The graph of function $y(t)$ of system (12).

4.3. Intensity of the Random Disturbance Factor

Set $\lambda = 0.8, C_S = 0.5$. The SEG system between FI and SME can be described as follows based on the analysis above:

$$\begin{cases} dx(t) = x(t)(1-x(t))[(-0.9y(t)+0.144)dt + \delta d\omega(t)], & t \geq 0, \\ x(t) = 0, & t = 0. \end{cases} \tag{13}$$

$$\begin{cases} dy(t) = y(t)(1-y(t))[(0.2952x(t)-0.32)dt + \delta d\omega(t)], & t \geq 0, \\ y(t) = 0, & t = 0. \end{cases} \tag{14}$$

Theorem 4. *Let there exist constants α, ρ and a continuous function $h(t) : [0, \infty) \to R^+$ satisfying $h(t) \in [\alpha/p, +\infty)$. If $t \geq 0$, we have the following:*

(**B.1**)

$$\int_0^t e^{-\int_s^t h(\mu)d\mu} |h(s) - 0.9y(s) + 0.144| ds + \left(\int_0^t e^{-2\int_s^t h(\mu)d\mu} \delta^2 ds \right)^{1/2} \leq \rho < 1,$$

(**B.2**)

$$\int_0^t e^{-\int_s^t h(\mu)d\mu} |h(s) + 0.2952x(s) - 0.32| ds + \left(\int_0^t e^{-2\int_s^t h(\mu)d\mu} \delta^2 ds \right)^{1/2} \leq \rho < 1.$$

If condition B1 holds, the zero solution of system (13) is p-exponential stable in the mean square. Similarly, if condition B2 holds, the zero solution of system (14) is also p-exponential stable in the mean square.

Remark 4. *As can be seen from Figures 4 and 5, the greater the intensity of the random disturbance δ, the more variable the systems appear, i.e., the more pronounced effect of the random disturbance term on system stability. In Figures 4 and 5, when the intensity of random disturbance is 0.9, the systems show obvious sharp fluctuations. This fully illustrates the impact of the random disturbance* *factor on the stability of the SEG system. Therefore, when making decisions, both FIs and SMEs need to fully consider such random uncontrollable factors in order to minimize losses as much as possible.*

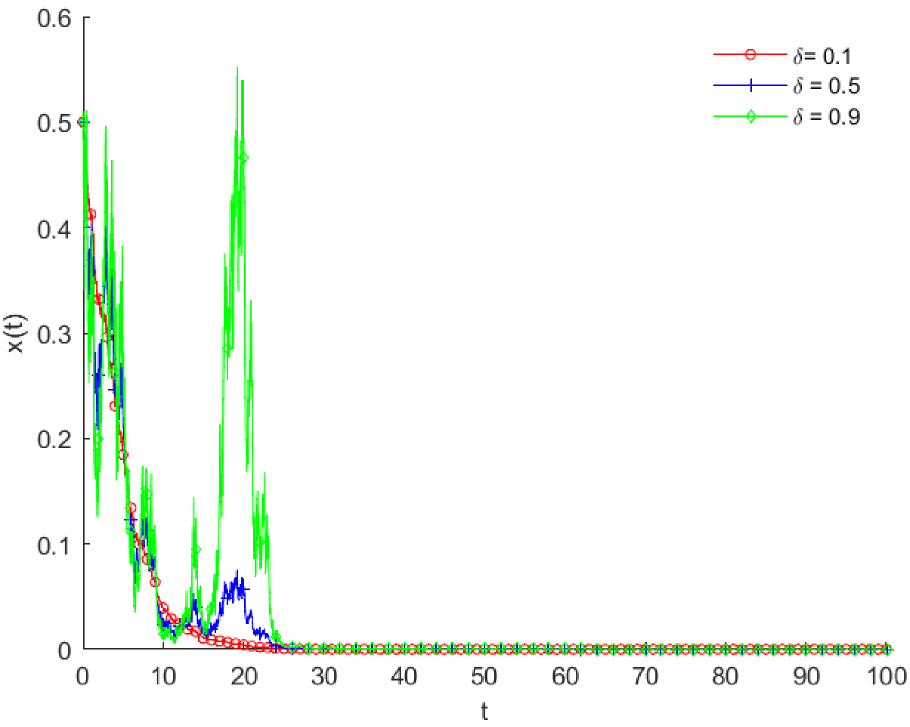

Figure 4. The graph of function $x(t)$ of system (13).

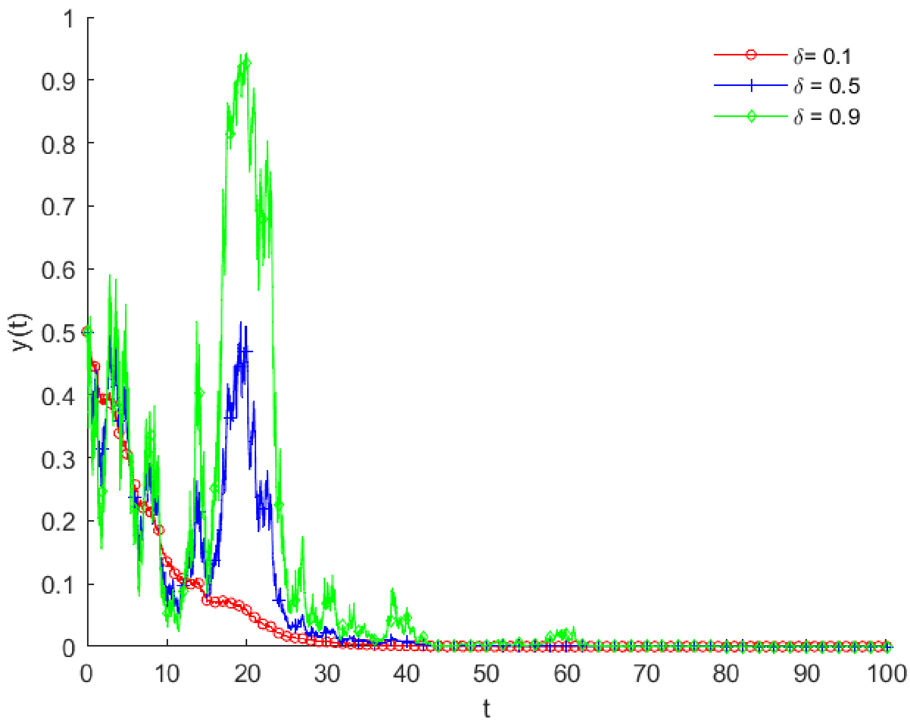

Figure 5. The graph of function $y(t)$ of system (14).

5. Conclusions and Discussions

5.1. Conclusions

This paper investigates the stability of the two-party evolutionary game "FI-SME" in the SCF credit market using SEG models and derives the conditions for the system to exhibit stable mean square index p using the fixed-point method. Subsequently, based on relevant data, MATLAB is employed to simulate the conclusions drawn. Building on previous research findings, this paper introduces partial innovations in the research model and methodology, focusing on the following aspects:

1. Currently, the methods for studying game theory issues in the SCF credit market are mostly deterministic evolutionary games (see [13–19]), with few considering more realistic SEG methods. The conclusions of this paper partially extend the application of the SEG method in the two-party evolutionary game "FI-SME" in the SCF credit market.

2. Most literature studies employing the SEG model to study decision-making problems use Lyapunov's direct method to analyze system stability (see [12,21–25,30–32]). However, when using the Lyapunov direct method to study the stability of time-lagged dynamical systems, it often encounters difficulties such as the requirement for bounded time lags and coefficient functions that exhibit a fixed sign, but the use of the fixed-point method can overcome the above difficulties to a certain extent (see [34,36,37,39] in the example analysis section). In view of the superiority of the fixed-point method in studying the stability of stochastic dynamical systems, this paper introduces the fixed-point method to the stability study of SEG models for the first time. This study fills the gap in the use of the fixed-point method in the stability analysis of SEG models. In addition, since the choice of $h(s)$ is more flexible when the immobile point method is used to study the stability, this makes the stability study more simple and easy to implement (See [34,36,37]).

3. Nearly all the literature studies have omitted the $(1 - x)$ term in the constructed evolutionary models (see [13–25,30–32]), thereby simplifying the nonlinear differential equations into relatively simpler linear ones. However, this approach is questionable, and original references supporting it are scarce. In contrast to other authors' methods, this paper considers the original nonlinear SEG system before any transformations. Then, with the help of the properties of the Lipschitz function, the nonlinear SEG model system is transformed into a linear stochastic system. Mathematically, this treatment is more scientific and has wider applicability (See [40,41]).

4. When investigating the stability of the SCF credit market using the Lyapunov direct method, numerous requirements imposed by the Lyapunov function often render its selection arduous [34]. Moreover, studies of the SCF credit market frequently encounter issues pertaining to time delays, wherein the actions of one party are influenced by the preceding actions of another. Under such circumstances, the fixed-point method demonstrates its superiority (see [34,36,37,39]). Although the Lyapunov direct method has its advantages, the research in this paper provides a new method and idea for the study of the stability of SEG models in SCF credit markets.

5. The findings of this paper show that higher credit ratios by FIs to SMEs and increased penalties by CEs for SME defaults can significantly accelerate the stabilization of the system.

5.2. Suggestions

The decision-making behaviors of FIs and SMEs are crucial for the long-term healthy development of the credit market. Firstly, FIs should conduct meticulous assessments of the credit levels of SMEs and CEs, and determine lending ratios based on the assessment results. Secondly, high-level default penalties are essential safeguards for SME compliance, and FIs should establish penalty standards according to the credit levels of SMEs and CEs. Lastly, unpredictable stochastic disturbances are factors that cannot be overlooked. FIs and SMEs need to fully consider the impact of these disturbances on their decision-making processes.

5.3. Limitations

Finally, there are still many issues that can be further explored in the research presented in this paper, which will be the future research topics for the project team.

1. The decision-making of FI agents is influenced by the previous behaviors of SMEs in the initial stages, and conversely, SME behavior is also affected by previous decisions made by FIs. Therefore, in subsequent research, the authors will attempt to introduce an SEG system with time delays to explore the stability of decision-making among various entities in the SCF credit market.

2. Additionally, the government is increasingly recognizing the importance of the SCF credit market. Therefore, the role of the government cannot be overlooked. In future studies, the authors plan to incorporate the role of the government into the existing "FI-SME" game model to explore the impact of government behavior on system stability.

3. Furthermore, blockchain technology is gradually reshaping the traditional landscape of SCF credit markets. With the continuous maturation and application of blockchain technology, profound changes are expected in the strategic interactions among stakeholders in the SCF domain. Considering the factors of blockchain when studying the evolutionary game behaviors of various entities in the SCF credit market will be a promising research topic.

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