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Robust Adaptive Fault-Tolerant Control of Quadrotor Unmanned Aerial Vehicles

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Abstract: The paper introduces a robust adaptive fault-tolerant control system for the six-degree-of-freedom (six-DOF) dynamics of quadrotor unmanned aerial vehicles (UAVs), incorporating disturbances and abrupt actuator faults to represent real-world conditions. The proposed control scheme employs robust control terms to manage unknown disturbances. However, robust control performance may degrade due to sudden fault impacts. To handle this issue, we introduce adaptive laws to ensure continuous adaptation. The control architecture ensures the tracking system's stability by combining robust control using sliding-mode control (SMC) with adaptive control developed using the certainty equivalence principle. The sliding-surface error limits the adaptive laws, in which the convergence of estimated parameters to the actual unknown variables is not required as they fully rely on the convergence of the tracking error. We provide rigorous mathematics to validate the proposed control design. Furthermore, we conduct numerical simulations for a quadrotor UAV to showcase the effectiveness of the proposed scheme. The results demonstrate the efficacy of the proposed design in handling external disturbances and abrupt actuator faults.

Keywords: quadrotor UAV; robust adaptive control; sliding-mode control; fault-tolerant control; uncertain disturbances; abrupt actuator faults

MSC: 93-10; 93D21

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1. Introduction

The increasing presence of autonomous and semi-autonomous systems in many settings has attracted significant attention in recent years. Quadrotor unmanned aerial vehicles (UAVs) have become essential tools, especially for missions that involve dangerous or repetitive work like monitoring volcanoes, exploring geographical areas, mapping radiation, and conducting agricultural operations. Control engineers are becoming more interested in the possibility of using autonomous UAVs, either on their own or in interconnected networks, providing a promising area for research. The formulation of control challenges for UAVs, as demonstrated in studies such as [1–3], takes a cyber-physical system approach. It analyzes the obstacles and various control methods for both single and multiple UAV setups to enhance performance in challenging situations.

Quadrotors are notable among UAVs due to their adaptability and suitability for a wide range of activities. Nevertheless, their dynamic model presents distinct difficulties characterized by under-actuation, where there are fewer control inputs than output states. This intrinsic characteristic makes it difficult to create controllers that can guide the UAV within its operating area, which includes nonlinear dynamic behaviors. Given exact parameter knowledge, creating control strategies for UAVs using feedback linearization approaches would be trivial in an ideal scenario. However, practical implementations frequently encounter uncertainties caused by sensor noise and unobservable UAV states. This requires the investigation of alternate tactics, such as output feedback approaches, as discussed in [4]. Moreover, because of the inherent lack of control in UAVs, it is crucial to develop accurate control methods for following the desired path of movement and rotation. These control methods must consider the nonlinear nature of the dynamics involved. Unmanned aircraft systems (UASs) are extensively utilized in operations such as volcanic surveillance, geospatial exploration, and agricultural tasks to gather data for conventional uses, including mapping, or to aid in emergency response actions. The versatility and cost-effectiveness of UASs improve the capacity to monitor and analyze long-term changes in landscapes and the environment. This facilitates the integration of UAS data with in situ and satellite data sets [5,6].

Addressing the nonlinear dynamics presents key technical challenges in UAV controller design. This problem becomes more complicated as a UAV operates as an under-actuated system. Backstepping is one of the most prevalent robust research lines to handle uncertainties. This approach was studied for tracking control of an under-actuated UAV using Lyapunov stability theory in [7]. The technical challenges in implementing feedback control design become more complicated due to the presence of uncertain disturbances. To handle it, significant advancements in control strategies for UAVs are required to accommodate uncertainties. The concept behind this method is to design a controller by managing and mitigating uncertainties within a specified range to guarantee the stability of a closed-loop system. An intelligent fault tolerant control was introduced in [8] utilizing Nussbaum-type function to tackle unknown actuator faults. An optimal fuzzy FT controller was applied to stabilize a UAV based on bio-inspired optimization [9]. Robust backstepping control was designed in [10] for UAVs influenced by disturbances. To design a robust scheme, the absolute value of the maximum disturbance was considered in designing the controller. The backstepping method was improved in [11] to handle uncertain disturbances using the Lyapunov stability theory. Diverse robust control methodologies were employed in singular UAV scenarios [12–14], as well as in collaborative contexts [15,16].

SMC stands out as one of the foremost methodologies for robust control research. This technique has been implemented in diverse UAV applications across various settings [17,18]. Chattering poses a significant challenge in SMC methodologies. The extended SMC approach was studied to mitigate the chattering phenomenon [19]. Even though chattering can be attenuated in the above studies, these methods may lose asymptotic stability. This loss of stability is caused by the residual error in approximating the non-smooth function within the control structure. An alternative approach aimed at designing SMC for nonlinear UAVs is exploring a "finite-time" approach utilizing fractional-order controllers for enhanced performance and stability. For instance, in [20], a finite-time adaptive super-twisting SMC was investigated to achieve rapid convergence in both altitude and attitude control for a quadrotor. Furthermore, ref. [21] introduced a fractional-order integrator with a feedback derivative strategy designed to regulate all states of a UAV. These studies highlighted the potential of fractional-order control methodologies in enhancing the performance and agility of UAVs in various operational scenarios. A continuous SMC method and a hierarchical strategy were proposed to effectively control the path of a quadrotor, even with uncertainties in aerodynamics and external disturbances [22]. However, in cases where it is not possible to obtain accurate knowledge of the upper limit of uncertainty, the robust control strategy takes a cautious and risk-averse stance.

Adaptive control is a prevalent method to confront system dynamics characterized by parametric uncertainties. The certainty equivalence principle is the concept often utilized in developing adaptive methods. Herein, nonlinear phenomena incorporating parametric uncertainties can be handled through the estimation of their uncertain components. The derivation of the adaptive law, typically established from a Lyapunov-like function, facilitates the generation of estimated parameters, thus enabling the adaptation process to maintain uncertainties [23]. Model reference adaptive control (MRAC) is a famously adopted approach for estimating uncertain parameters in nonlinear dynamics [24]. This approach utilizes a reference model functioning as a state predictor, which is continually compared with measured states to deduce the mismatch between actual unknown parameters and estimated parameters. Several results utilizing adaptive control methodologies are documented in [25–27] within single-agent scenarios, and in [28–31] for collaborative environments within connected-network frameworks. These studies collectively contribute to the understanding and application of adaptive control strategies in diverse operational contexts.

Another common adaptive approach for addressing parametric uncertainties is through intelligent computation methods. For instance, a genetic algorithm was applied to optimize parameters for a robot manipulator in [32]. Neural networks were employed in [13] to manage unknown parameters within a single-agent scenario and in [33,34] for cooperative control. Intelligent computation techniques for UAVs were developed by incorporating reinforcement learning methods and algorithms [35]. However, these methodologies encounter two primary limitations, especially when deployed for onboard and real-time estimation purposes. Firstly, while they effectively manage unknown parameters, they may not achieve asymptotic tracking control, leading to residual errors due to the disparity between the approximated value and the actual value of the nonlinear term with parametric uncertainty. Secondly, intelligent computation approaches often contain algorithmic complexity. As a result, the required computational power often exceeds the capabilities of embedded UAV flight control systems.

A recent study developed an adaptive tracking controller and real-time nonlinear parameter estimation in [23] to address the challenges associated with implementing intelligent adaptive techniques on computing-constrained UAV devices. Even though these algorithms demonstrated the ability to maintain unknown parameters in real time with minimal control effort and low computational complexity, they operated under the assumption of a fault-free control input structure. The study referenced in [36] describes the development of a fault diagnosis and fault-tolerant control framework based on SMC developed for UAVs. They designed the system to tackle the problem of path planning. The issue addresses external disturbances as well as two distinct actuator malfunction cases: either multiple drops in rotor efficacy or a full failure of the rotating part. However, in practical scenarios, actuator faults or fault factors may arise within the control input structures, potentially degrading the performance of the design. Furthermore, the previous studies only considered the unknown magnitudes of external disturbances. Feedback linearization was studied to handle a rotor failure without external disturbances in [37]. This study aims to build upon these prior works by introducing abrupt actuator faults in translational and attitude dynamics. As these actuator faults manifest as separate nonlinear terms within the control input structure, the system is categorized as an unknown control gain. Moreover, the complexity of the control problem is heightened in the current study by the presence of external disturbances, adding another layer of complexity to the analysis and necessitating robust control strategies.

This paper proposes a robust adaptive control methodology specifically designed for a dynamic model of a quadrotor UAV with six DOFs. The proposed controller is crafted to effectively manage unknown time-varying disturbances while simultaneously addressing abrupt actuator faults impacting both translational and attitude dynamics. This design integrates robust control using SMC to handle uncertain disturbances, alongside adaptive control techniques to mitigate abrupt fault factors. The primary contribution of this work lies in the development of two adaptive, robust, fault-tolerant control approaches designed for both the translational and attitude dynamics of a nonlinear under-actuated quadrotor UAV, in the presence of abrupt fault factors and unknown disturbances. A comprehensive stability analysis of both the outer and inner loops is presented to formulate the proposed control protocol.

For better convenience in presentation, the contributions in the paper are listed as follows:

- An adaptive robust fault-tolerant control approach is developed for each translational and attitude dynamics of a quadrotor UAV.
- A robust control scheme is proposed to handle external disturbances, but its robustness may degrade in the presence of abrupt fault factors. An adaptive controller is designed to handle this control problem. The combination of adaptive and robust schemes is proposed to maintain tracking control stability in the presence of disturbances and abrupt fault factors.
- A comprehensive stability analysis and rigorous mathematical proof are provided in the proposed control protocol.
- Numerical simulations are presented to verify the effectiveness of the proposed design in tracking desired trajectories amid disturbances and fault factors. The results show the proposed design's ability for fault-tolerant control of quadrotor UAVs.

The forthcoming sections of the paper are structured as follows. Section 2 presents the dynamical model of the UAVs. Subsequently, Section 3 provides an overview of the tracking control design and stability analysis for system dynamics of the UAV. The effectiveness of the proposed design is demonstrated through simulation results in Section 4. Finally, Section 5 summarizes the key findings of this paper and suggests potential directions for future work.

2. System Dynamics of UAVs

Consider the dynamic model of a quadrotor UAV with 6 DOFs, consisting of translational dynamics (1) and attitude dynamics (2) [18].

$$\ddot{\eta}_t = -(g - \delta_z)z_e + \mathbf{R}z_e\frac{u_t}{m}, \tag{1}$$

$$\ddot{\eta}_r = wf(\eta_r) + \delta_r + I_m^{-1}\tau, \tag{2}$$

where

$$\begin{aligned}
w &= \text{diag}\begin{bmatrix} \frac{I_y - I_z}{I_x} & \frac{I_z - I_x}{I_y} & \frac{I_x - I_y}{I_z} \end{bmatrix} \\
f(\eta_r) &= \begin{bmatrix} \dot{\theta}\dot{\psi} & \dot{\phi}\dot{\psi} & \dot{\phi}\dot{\theta} \end{bmatrix}^T \\
I_m &= \text{diag}\begin{bmatrix} I_x & I_y & I_z \end{bmatrix} \\
z_e &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \\
\delta_r &= \begin{bmatrix} \delta_{r_1} & \delta_{r_2} & \delta_{r_3} \end{bmatrix}^T, \ \tau = \begin{bmatrix} \tau_1 & \tau_2 & \tau_3 \end{bmatrix}^T.
\end{aligned}$$

The vector $\eta_t = \begin{bmatrix} x & y & z \end{bmatrix}^\top$ represents the position in the earth/inertial frame, consisting of forward (x), lateral (y), and vertical (z) states. Vector $\eta_r = \begin{bmatrix} \phi & \theta & \psi \end{bmatrix}^\top$ denotes the orientation vector in the body frame, including roll (ϕ), pitch (θ), and yaw (ψ) states. The inertia parameters along the x, y, and z axes are denoted as I_x, I_y, and I_z, respectively. The gravitational acceleration is denoted by g and external disturbances influencing the dynamical model are denoted by δ_z and δ_r. Note that the UAV exhibits symmetry about the x and y axes, implying that the center of gravity aligns with the center of the UAV. Figure 1 depicts the coordinate frames of η_t and η_r.

The dynamic model of UAVs can be decomposed into two subsystems to streamline the control problem. The first subsystem encompasses translational dynamics, which include state spaces $\ddot{x}$, $\ddot{y}$, and $\ddot{z}$, governed by a single control input. The second subsystem is attitude or rotational dynamics, involving state spaces $\ddot{\phi}$, $\ddot{\theta}$, and $\ddot{\psi}$, controlled by three inputs. With fewer control inputs than the states, UAVs are classified as under-actuated systems. Disturbances in the x and y dynamics are implicitly addressed due to the quadrotor UAV's under-actuated nature. While control inputs cannot directly counteract disturbances along these axes, their influence on rotational dynamics is significant. This inherent coupling between translational and rotational states can lead to the accumulation of disturbances over time. Therefore, while not directly controlled, disturbances in x and y dynamics indirectly affect the rotational dynamics of the UAV.

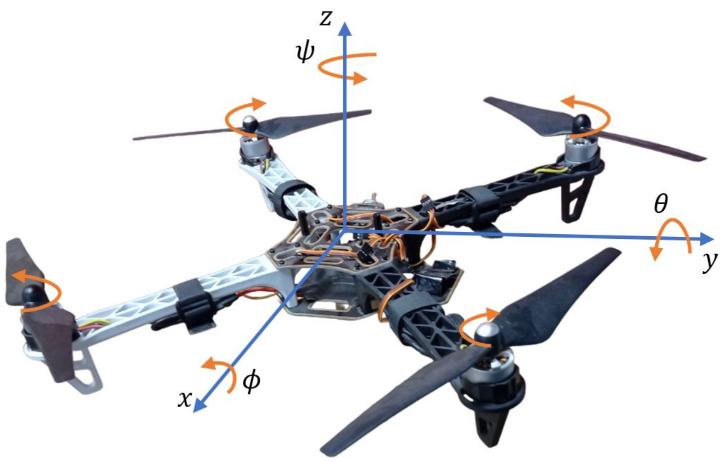

Figure 1. Illustration of a UAV with its earth and body-fixed reference.

Matrix I_m represents a diagonal matrix comprising inertia parameters along the diagonal, and $\mathbf{R}$ denotes a transformation matrix defined as

$$\mathbf{R} = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix}. \quad (3)$$

The values of roll and pitch angles range from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, ensuring non-zero values for $\cos\phi$ and $\cos\theta$. Consequently, $\mathbf{R}^{-1} = \mathbf{R}^\top$. The external disturbances added to both translational and attitude dynamics are represented by $\delta_z(t)$ and $\delta_r(t)$, respectively.

Due to the presence of fault factors in the control input, the dynamics of UAVs can be rewritten as

$$\ddot{\eta}_t = -(g - \delta_z)z_e + \mathbf{R}z_e\frac{\sigma_t u_t}{m}, \quad (4)$$

$$\ddot{\eta}_r = wf(\eta_r) + \delta_r + \sigma_r I_m^{-1}\tau, \quad (5)$$

where $\sigma_t(t)$ is multiple abrupt fault factors in the translational control input structure, and $\sigma_r(t) = \text{diag}\begin{bmatrix}\sigma_{r_1} & \sigma_{r_2} & \sigma_{r_3}\end{bmatrix}$ is a diagonal matrix representing multiple abrupt fault factors in the attitude dynamics. The fault factors in (4) and (5) are presented in a multiplicative form to represent more general cases, as also proposed by [38] in a different setting. Note that the values of fault factors satisfy the following assumption.

Assumption 1. *Both $\sigma_t(t)$ and $\sigma_r(t)$ are some positive piecewise-constant functions.*

External disturbances $\delta_z(t)$ and $\delta_r(t)$ influence the body of the UAV, conforming to Assumption 2.

Assumption 2. *The system dynamics of UAVs are subject to external disturbances with defined boundaries as given by*

$$|\delta_z(t)| \leq d_z \quad (6)$$

$$|\delta_r(t)| \leq d_r, \quad (7)$$

where d_z is a constant, and d_r is a constant vector.

If $\delta_z(t)$ and $\delta_r(t)$ are accessible to the controller, it enables the formulation of a direct, full, feedback linearization scheme, facilitating the stabilization of the closed-loop systems.

Real-world scenarios often involve unknown, time-varying disturbances, rendering both $\delta_z(t)$ and $\delta_r(t)$ inaccessible for feedback control design. As a result, an effective nonlinear control methodology is required to address the complexities of tracking control tasks. In this paper, only the worst cases of disturbances (d_z and d_r) are available for the feedback controllers, indicating that disturbances are constrained within a specific range. This situation closely resembles the practical environment of UAVs.

3. Proposed Control Scheme

This section outlines the proposed control scheme for UAVs, considering the impact of disturbances and abrupt fault factors. For clarity, it is divided into two subsections. The first subsection details the control design for translational dynamics, aiming to maintain position states along the x, y, and z axes. In the subsequent subsection, the proposed design is presented for the rotational dynamics of UAVs.

3.1. Position Control Design

The definition of the tracking error within the translational dynamics is given by

$$e_t = \eta_t - \eta_{t_d}, \tag{8}$$

where e_t and η_{t_d} represent the error vector position and the desired vector position, respectively. The position closed-loop control under ideal conditions is constructed as follows:

$$\ddot{e}_t = -K_D\dot{e}_t - K_P e_t, \tag{9}$$

where K_P and K_D are selected as positive definite matrices. Therefore, the system dynamics (9) satisfy the Routh–Hurwitz stability criterion, ensuring $\lim_{t\to\infty} e_t(t) = 0$. The dynamics (9) can be rewritten as

$$\ddot{\eta}_t = \ddot{\eta}_{t_d} - K_D\dot{e}_t - K_P e_t. \tag{10}$$

We define a virtual control input $U = \ddot{\eta}_t = \begin{bmatrix} U_1 & U_2 & U_3 \end{bmatrix}^T$. By employing a similar method as in [23,39], we can compute

$$\begin{aligned} \phi_d &= \arcsin\left(\frac{U_1\sin\psi_d - U_2\cos\psi_d}{\sqrt{U_1^2 + U_2^2 + (U_3+g)^2}}\right) \\ \theta_d &= \arctan\left(\frac{U_1\cos\psi_d + U_2\sin\psi_d}{U_3+g}\right), \end{aligned} \tag{11}$$

where ϕ_d and θ_d represent the desired ϕ and θ, respectively.

The altitude dynamics can be generated from (1) as expressed by

$$\ddot{z} = -g + \delta_z + \frac{\sigma_t u_t \cos\phi\cos\theta}{m}. \tag{12}$$

Let us define the sliding surface for dynamics (12) to be

$$s_z = \lambda e_z + \dot{e}_z, \tag{13}$$

where λ is a positive constant, $e_z = z - z_d$ is the tracking error along the z axis, and z_d is the desired altitude. Therefore, the sliding-surface dynamics (13) can be written as

$$\dot{s}_z = \lambda\dot{e}_z + \delta_z - g + \frac{\sigma_t u_t \cos\phi\cos\theta}{m} - \ddot{z}_d. \tag{14}$$

Successful tracking control of altitude dynamics is expressed by

$$\lim_{t\to\infty} s_z(t) = 0. \tag{15}$$

This implies that $\dot{e}_z = -\lambda e_z$.

A robust adaptive control scheme is formulated to attain the objective (15) as outlined in Theorem 1.

Theorem 1. *Consider the altitude dynamics (12) under Assumptions 1 and 2. Objective (15) can be achieved by selecting the control input*

$$u_t = -\frac{m\hat{\sigma}_t}{\cos\phi\cos\theta}\Big(k_1 s_z + k_2 sgn(s_z) - g - \ddot{z}_d + \lambda\dot{e}_z\Big), \tag{16}$$

where $\hat{\sigma}_t$ is computed as

$$\dot{\hat{\sigma}}_t = \gamma s_z(k_1 s_z + k_2 sgn(s_z) - g - \ddot{z}_d + \lambda\dot{e}_z) \tag{17}$$

with γ and k_1 being positive constants, and $k_2 \geq d_z$.

Proof. The dynamics of sliding surface (14) under control input (16) can be written as

$$\dot{s}_z = \lambda\dot{e}_z + \delta_z - g - \ddot{z}_d - \sigma_t\hat{\sigma}_t\Big(k_1 s_z + k_2\text{sgn}(s_z) - g - \ddot{z}_d + \lambda\dot{e}_z\Big) \tag{18}$$

Let the adaptive error be defined by $\tilde{\sigma}_t = \hat{\sigma}_t - \sigma_t^{-1}$. Hence, dynamics (18) can be written as

$$\begin{aligned}\dot{s}_z &= \lambda\dot{e}_z + \delta_z - g - \ddot{z}_d - \sigma_t(\tilde{\sigma}_t + \sigma_t^{-1})\Big(k_1 s_z + k_2\text{sgn}(s_z) - g - \ddot{z}_d + \lambda\dot{e}_z\Big)\\ &= \delta_z - k_1 s_z - k_2\text{sgn}(s_z) - \sigma_t\tilde{\sigma}_t\Big(k_1 s_z + k_2\text{sgn}(s_z) - g - \ddot{z}_d + \lambda\dot{e}_z\Big)\end{aligned} \tag{19}$$

The Lyapunov function for (19) is selected as

$$V_{s_z,\tilde{\sigma}_t} = \frac{1}{2}s_z^2 + \frac{\sigma_t}{2\gamma}\tilde{\sigma}_t^2. \tag{20}$$

The derivative of the Lyapunov function in (20) with respect to time can be determined as

$$\begin{aligned}\dot{V}_{s_z,\tilde{\sigma}_t} &= s_z\dot{s}_z + \frac{\sigma_t\tilde{\sigma}_t\dot{\hat{\sigma}}_t}{\gamma}\\ &= s_z\bigg(\delta_z - k_1 s_z - k_2\text{sgn}(s_z) - \sigma_t\tilde{\sigma}_t\Big(k_1 s_z + k_2\text{sgn}(s_z) - g - \ddot{z}_d + \lambda\dot{e}_z\Big)\bigg)\\ &\quad + \frac{\sigma_t\tilde{\sigma}_t\dot{\hat{\sigma}}_t}{\gamma}\\ &= s_z\Big(\delta_z - k_1 s_z - k_2\text{sgn}(s_z)\Big)\\ &\leq -k_1 s_z^2.\end{aligned} \tag{21}$$

Both s_z and $\tilde{\sigma}_t$ are bounded, as can be seen in (17) and (19). Therefore, the second time-derivative of (20) can be calculated to show the convergence of sliding surface s_z to zero, as expressed by

$$\ddot{V}_{s_z,\tilde{\sigma}_t} = -2k_1 s_z\dot{s}_z. \tag{22}$$

Hence, it can be concluded from (19) that s_z is uniformly bounded. Therefore, $\ddot{V}_{s_z,\tilde{\sigma}_t}$ is bounded. This situation demonstrates that $\dot{V}_{s_z,\tilde{\sigma}_t}$ is uniformly continuous. By applying Barbalat's Lemma, the tracking objective given in (15) is achieved. It means that $\lim_{t\to\infty} z(t) - z_d(t) = 0$. Thus, the proof is completed. □

The above formulation indicates that the convergence of estimation errors $\tilde{\sigma}_t(t)$ is fully dependent on $s_z(t)$. However, the values of $\tilde{\sigma}_t(t)\dot{\hat{\sigma}}_t(t)$ do not consistently remain negative, leading to continuous updates by the adaptation law (17) even upon reaching the actual value of σ. Conversely, updating ceases if $s_z(t)$ converges to zero. This situation demonstrates the attainment of asymptotic tracking control, even in cases where estimated parameters do not converge to their actual values. To handle the external disturbance, controller (16) incorporates a robust term $k_2\text{sgn}(s_z)$ to dominate $\delta_z(t)$.

Remark 1. *In Theorems 1, the unknown fault factor $\sigma_t(t)$ is characterized as a piecewise-constant function. However, treating it as a constant fault parameter does not compromise the validity of the proof. Specifically, during switching times, the value of the fault factor can be conceptualized as a sign function with a finite amplitude. This representation is akin to multiple bounded abrupt faults and aligns with the practical behavior of propeller motors. Propeller motors typically operate as low-pass filters, meaning they smooth out rapid changes and prevent sudden fluctuations in the system response. This characteristic is crucial because it implies that the system can inherently handle abrupt changes without significant instability. In practical scenarios, motor torques are naturally bounded, which means they cannot change instantaneously but rather within certain physical limits. These abrupt changes, when they occur, can be effectively modeled as bounded faults introduced into the weight parameter σ_t. To address these bounded faults, the adaptive control system incorporates a parameter γ within the adaptive law (17). By carefully adjusting γ, the control system can compensate for the effects of these faults. The adaptive law dynamically adjusts the control inputs to mitigate the impact of the faults, ensuring that the stability of the closed-loop system is maintained. This adaptive mechanism is particularly effective across switching intervals, where the system might otherwise be vulnerable to destabilizing influences due to abrupt parameter changes.*

3.2. Attitude Control Design

This section outlines the control design for attitude dynamics. The presence of fault factors and external disturbances within the attitude dynamics causes challenges for the tracking control of UAVs. To tackle this control challenge, a robust adaptive tracking control strategy was formulated to handle uncertainties.

Before presenting the primary outcomes, let us establish the desired trajectory as $\eta_{r_d} = \begin{bmatrix}\phi_d & \theta_d & \psi_d\end{bmatrix}^T$. The trajectory error denoted by e_r is then calculated as $e_r = \eta_r - \eta_{r_d}$. The sliding surface in tracking attitude dynamics is expressed by

$$s_r = \Lambda e_r + \dot{e}_r, \tag{23}$$

where Λ is a positive, constant, diagonal matrix. Similar to sliding surface (13), by driving s_r to zero as $t \to \infty$, $\dot{e}_r = -\Lambda e_r$. It is clear to see that the dynamics of e_r is stable for any $\Lambda > 0$. Therefore,

$$\dot{s}_r = \Lambda \dot{e}_r + wf(\eta_r) + \delta_r + \sigma_r I_m^{-1}\tau - \ddot{\eta}_{r_d}. \tag{24}$$

The primary outcome of the control design for attitude dynamics is summarized in the following theorem.

Theorem 2. *Consider the attitude dynamics Equation (5) under Assumptions 1 and 2. The sliding surface s_r converges to zero as $t \to \infty$ by computing τ as*

$$\tau = -\hat{\sigma}_r I_m \Big(K_1 s_r + K_2 sgn(s_r) + wf(\eta_r) - \ddot{\eta}_{r_d} + \Lambda \dot{e}_r\Big), \tag{25}$$

where $\hat{\sigma}_r$ is generated by the following adaptive law

$$\dot{\hat{\sigma}}_r = \Gamma F G \tag{26}$$

with

$$F = diag(s_r)$$

$$G = diag(K_1 s_r + K_2 sgn(s_r) - \ddot{\eta}_r + \Lambda \dot{e}_r).$$

The gains K_1 and Γ are some diagonal positive definite matrices, and $K_2 \geq diag(d_r)$.

Proof. The sliding-surface dynamics (24) under control input (25) can be written as

$$\dot{s}_r = \Lambda \dot{e}_r + wf(\eta_r) + \delta_r - \ddot{\eta}_{r_d} - \sigma_r \hat{\sigma}_r \big(K_1 s_r + K_2 \text{sgn}(s_r) + wf(\eta_r) - \ddot{\eta}_{r_d} + \Lambda \dot{e}_r\big). \tag{27}$$

Let $\tilde{\sigma}_r = \hat{\sigma}_r - \sigma_r^{-1}$, then (27) can be written as

$$\begin{aligned}\dot{s}_r &= \Lambda \dot{e}_r + wf(\eta_r) + \delta_r - \ddot{\eta}_{r_d} - \sigma_r(\tilde{\sigma}_r + \sigma_r^{-1})\big(K_1 s_r + K_2 \text{sgn}(s_r) + wf(\eta_r) \\ &\quad - \ddot{\eta}_{r_d} + \Lambda \dot{e}_r\big) \\ &= \delta_r - K_1 s_r - K_2 \text{sgn}(s_r) - \sigma_r \tilde{\sigma}_r \big(K_1 s_r + K_2 \text{sgn}(s_r) + wf(\eta_r) - \ddot{\eta}_{r_d} + \Lambda \dot{e}_r\big).\end{aligned} \tag{28}$$

The Lyapunov function for (28) is selected as

$$V_{s_r,\tilde{\sigma}_r} = \frac{1}{2} s_r^T s_r + \frac{1}{2} \text{tr}\Big(\Gamma^{-1} \sigma_r \tilde{\sigma}_r^2\Big), \tag{29}$$

where tr(.) is the trace of a square matrix. The derivative of Lyapunov function (29) with respect to time can be determined as

$$\begin{aligned}\dot{V}_{s_r,\tilde{\sigma}_r} &= s_r^T \dot{s}_r + \text{tr}\Big(\Gamma^{-1} \sigma_r \tilde{\sigma}_r \dot{\hat{\sigma}}_r\Big) \\ &= s_r^T \Big(\delta_r - K_1 s_r - K_2 \text{sgn}(s_r) - \sigma_r \tilde{\sigma}_r \big(K_1 s_r + K_2 \text{sgn}(s_r) + wf(\eta_r) \\ &\quad - \ddot{\eta}_{r_d} + \Lambda \dot{e}_r\big)\Big) + \text{tr}\Big(\Gamma^{-1} \sigma_r \tilde{\sigma}_r \dot{\hat{\sigma}}_r\Big) \\ &= s_r^T (\delta_r - K_1 s_r - K_2 \text{sgn}(s_r)) - s_r^T \sigma_r \tilde{\sigma}_r \big(K_1 s_r + K_2 \text{sgn}(s_r) + wf(\eta_r) \\ &\quad - \ddot{\eta}_{r_d} + \Lambda \dot{e}_r\big) + \text{tr}\Big(\Gamma^{-1} \sigma_r \tilde{\sigma}_r \dot{\hat{\sigma}}_r\Big) \\ &= s_r^T (\delta_r - K_1 s_r - K_2 \text{sgn}(s_r)) + \text{tr}\Big(\Gamma^{-1} \sigma_r \tilde{\sigma}_r \dot{\hat{\sigma}}_r - \sigma_r \tilde{\sigma}_r FG)\Big) \\ &\leq -s_r^T K_1 s_r.\end{aligned} \tag{30}$$

Both sliding surface s_r and adaptive error $\tilde{\sigma}_r$ are bounded. It can be seen from (28) and (26). It shows that s_r and $\tilde{\sigma}_r$ are bounded. To show the convergence of sliding surface s_z, the second time-derivative of Lyapunov function (29) can be computed as

$$\ddot{V}_{s_r,\tilde{\sigma}_r} = -s_r^T K_1 \dot{s}_r. \tag{31}$$

This situation shows that s_z is uniformly bounded. As a result, $\ddot{V}_{s_r,\tilde{\sigma}_r}$ is bounded and $\dot{V}_{s_r,\tilde{\sigma}_r}$ is uniformly continuous. By Barbalat's Lemma, s_z converges to zero as t goes to ∞. This implies that $\lim_{t \to \infty} \eta_r(t) = \eta_{r_d}(t)$. Thus, the proof is complete. □

Similarly to the design of translational control, it is evident that the convergence of estimation errors $\tilde{\sigma}_r(t)$ relies entirely on $s_r(t)$ as shown from the control formulation. However, the values of $\text{tr}(\tilde{\sigma}_r(t)\dot{\hat{\sigma}}_t(t))$ do not consistently remain negative. As a result, the adaptation law (26) persistently updates itself even upon reaching the actual value of σ_r. Conversely, it stops updating if $s_r(t)$ converges to zero, indicating the asymptotic stability of the tracking control even in the absence of convergence of estimated parameters to the actual values. The controller (25) has a robust term $K_2\text{sgn}(s_r)$ to handle external disturbances $\delta_r(t)$.

Remark 2. *In Theorem 2, the unknown fault factor $\sigma_r(t)$ is classified as a piecewise-constant function. This means that while $\sigma_r(t)$ may change values at certain points in time, it remains constant between these points. Similar to $\sigma_t(t)$, treating it as constant fault parameters within the diagonal matrix $\sigma_r(t)$ simplifies the analysis and does not affect the validity of the proof. Specifically, during the intervals when switching occurs, the fault factor's value can be modeled as a sign function with a finite amplitude. This representation mimics the behavior of multiple bounded abrupt faults, where sudden changes in the system can occur but within a bounded range. This approach is particularly relevant due to the inherent characteristics of propeller motors. These motors act as low-pass filters, meaning they do not allow rapid changes in the system's response, smoothing out abrupt shifts. Furthermore, in practical applications, motor torques are inherently bounded. This constraint ensures that even when abrupt changes occur, they are within a predictable and manageable range. These abrupt changes are bounded faults affecting the weight σ_t. By tuning the Γ gain in the adaptive law (26), we can mitigate the impact of these abrupt faults. Proper tuning ensures that the adaptive control system can maintain stability and achieve the desired tracking performance even during the switching intervals. The tuning of Γ is crucial because it determines the responsiveness and robustness of the adaptive control system. A well-tuned Γ gain can adjust for the disturbances introduced by the faults, maintaining the closed-loop system's stability. This ensures that the system continues to perform reliably, even in the presence of multiple and abrupt faults, by effectively managing the influence of these faults on the system dynamics. Thus, the overall control strategy remains robust and effective, providing stability and performance guarantees during all operational phases, including fault-induced disturbances.*

For enhanced clarity, Figure 2 visually outlines the control design schematic. Here, the translational and attitude controllers maintain the outer and inner loops, respectively. A specific adaptive control mechanism is introduced for each control input to ensure fault tolerance. This approach ensures system adaptability in anticipation of potential faults to enhance control performance.

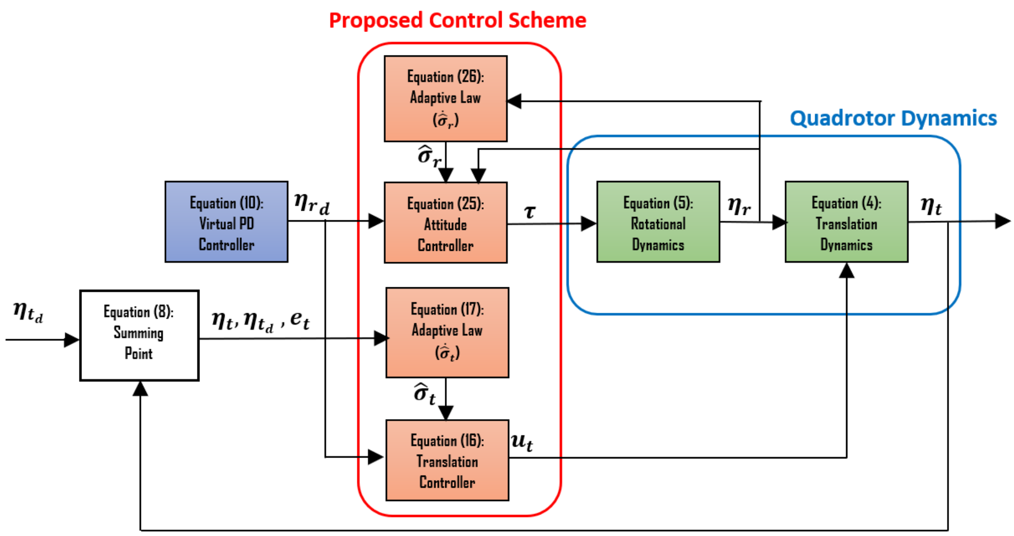

Figure 2. Robust adaptive fault-tolerant scheme for a 6-DOF UAV.

4. Simulation Results

In this section, we performed simulations utilizing Simulink/MATLAB (latest v. 2024a) to evaluate the performance of a UAV, utilizing the parameters specified in Table 1.

Table 1. The parameters of a quadcopter UAV [40].

Parameter Name	Notation	Value
Mass	m	1 kg
Gravity acceleration	g	9.8 m/s^2
Inertia of x-axis	I_x	0.1402 kg·m^2
Inertia of y-axis	I_y	0.1402 kg·m^2
Inertia of z-axis	I_z	0.3267 kg·m^2

The unknown external disturbances that influenced system dynamics were defined by

$$\begin{aligned} \delta_z &= 0.1\sin(t) \\ \delta_r &= \begin{bmatrix} 0.1\sin(t) & 0.12\cos(t) & 0.06\cos(t) \end{bmatrix}^T. \end{aligned} \tag{32}$$

Several abrupt faults added to the closed-loop system are illustrated in Figure 3.

The gains of the virtual PD controller were chosen as $K_P = 1$ and $K_D = 10I_3$, where $I_3 \in \mathbb{R}^{3\times3}$ is an identity matrix. The gains of the robust adaptive schemes were selected according to Theorems 1 and 2 as listed in Table 2.

Table 2. The gains of the robust adaptive scheme.

Translational Controller		Attitude Controller	
Gain	**Value**	**Gain**	**Value**
k_1	1000	K_1	$10^{-9}\, I_3$
k_2	0.12	K_2	$0.14\, I_3$
γ	1	Γ	$10^{19}\, I_3$
λ	500	Λ	$10^{-44}\,\mathrm{diag}(\begin{bmatrix}1 & 2 & 2\end{bmatrix})$

The simulation results presented in Figures 4–8 provide a comprehensive depiction of the performance of the proposed control scheme in following a desired trajectory. The effectiveness of the design in maintaining tracking control amidst external disturbances and various abrupt fault parameters is affirmed by Theorems 1 and 2. Specifically, Figure 4 depicts detailed insight into the tracking control performance for roll, pitch, and yaw, demonstrating the robust adaptive design in managing control without inducing excessive oscillations. This robustness extends to handling uncertain disturbances impacting UAV movement, as evidenced by the successful application of the proposed robust adaptive scheme.

The proposed adaptive control approach (17) adeptly addresses the presence of uncertain fault factors, as depicted in Figure 8. The profile of the adaptive law presented provides a visual representation of its dynamic adjustments to accommodate varying fault conditions. As stated in Section 3, the state of the adaptive law is not necessary to converge to the actual value of the fault factor, as the adaptive law fully relies on the sliding surface.

Figure 5 provides further clarity on the position of the UAV for the x, y, and z axes. It can be seen from the figure the successful execution of the desired tracking control mission. Similar to the translational controller, the control input (25) and adaptive law (26) effectively maintain tracking control without extreme oscillations. For enhanced visualization, the three-dimensional (3D) depiction of the UAV's performance is presented in Figure 7, offering a comprehensive understanding of its spatial dynamics.

Figure 6 illustrates the total thrust and torque of the UAV to track the desired path. The robust term in (25) handles the unknown disturbances in the attitude dynamics. Additionally, Figure 8 shows the profile of the adaptive laws employed to accommodate multiple abrupt factors. As mentioned in Section 3, the state of adaptive law (26) is not required to converge to the precise value of the unidentified fault to ensure the tracking control stability as it fully relies on the value of the designed sliding surface (23) along Lyapunov function (29).

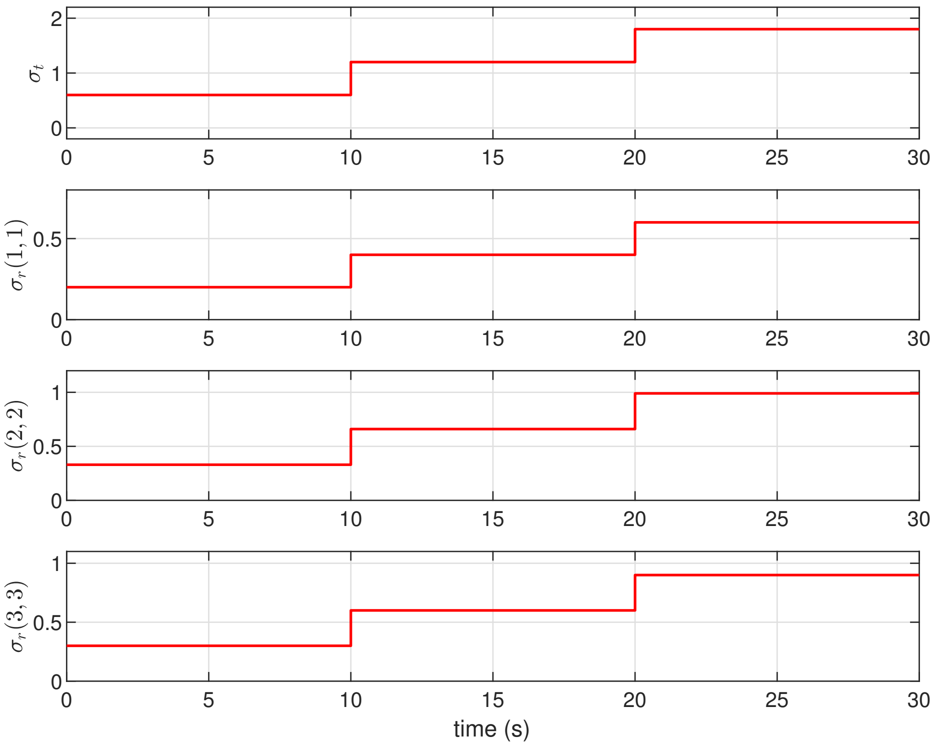

Figure 3. Profiles of multiple abrupt faults.

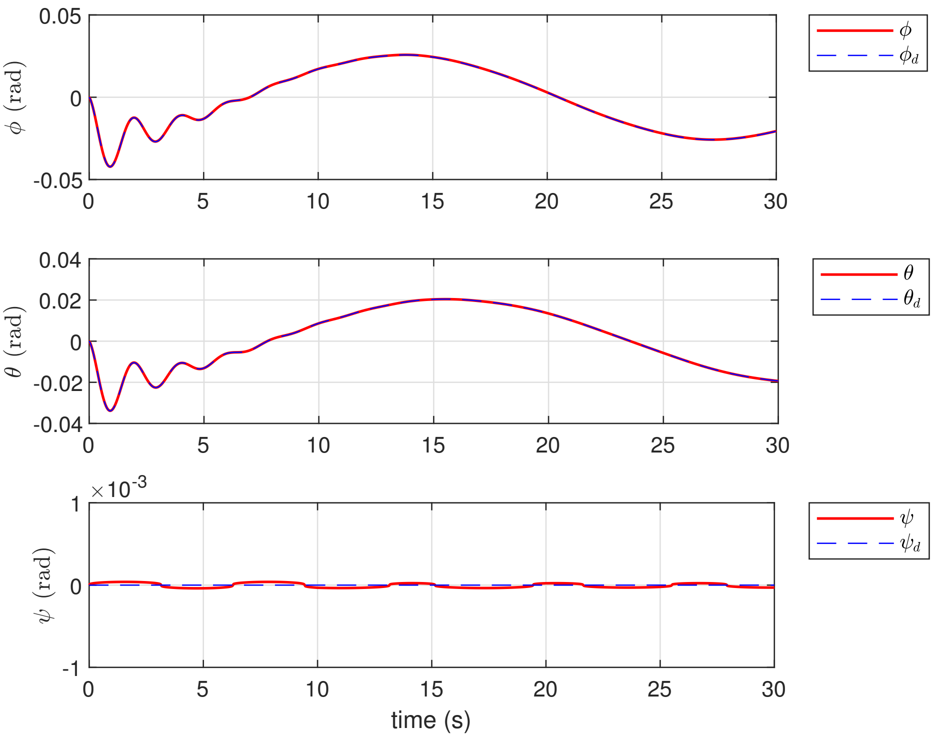

Figure 4. Profiles of ϕ, θ, and ψ.

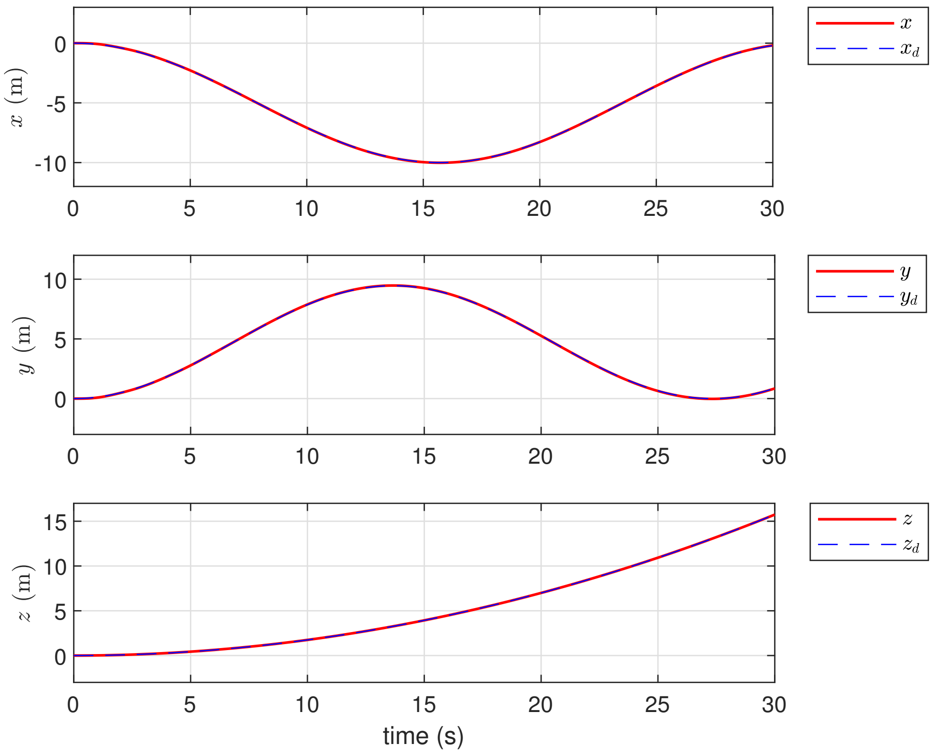

Figure 5. Profiles of x, y, and z.

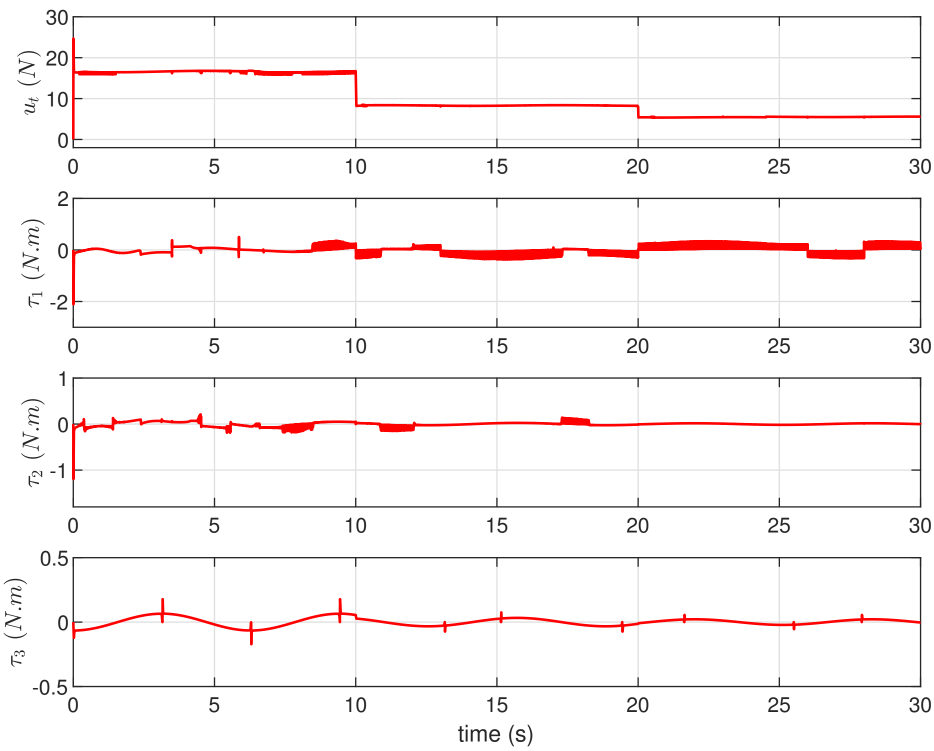

Figure 6. Profiles of u_t and τ.

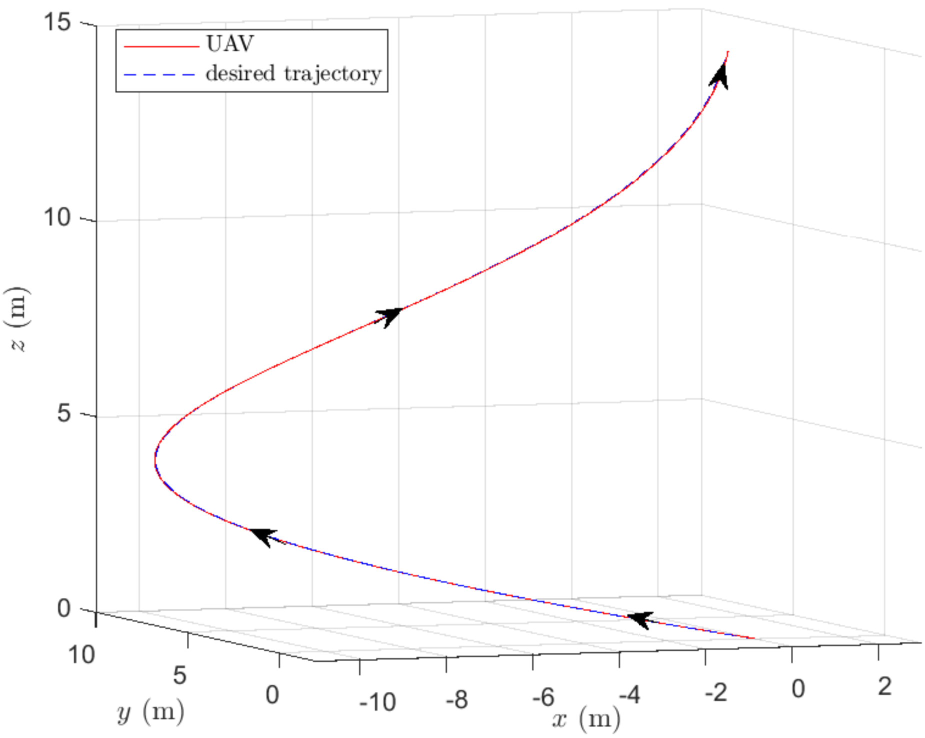

Figure 7. Profiles of x, y, and z in 3D.

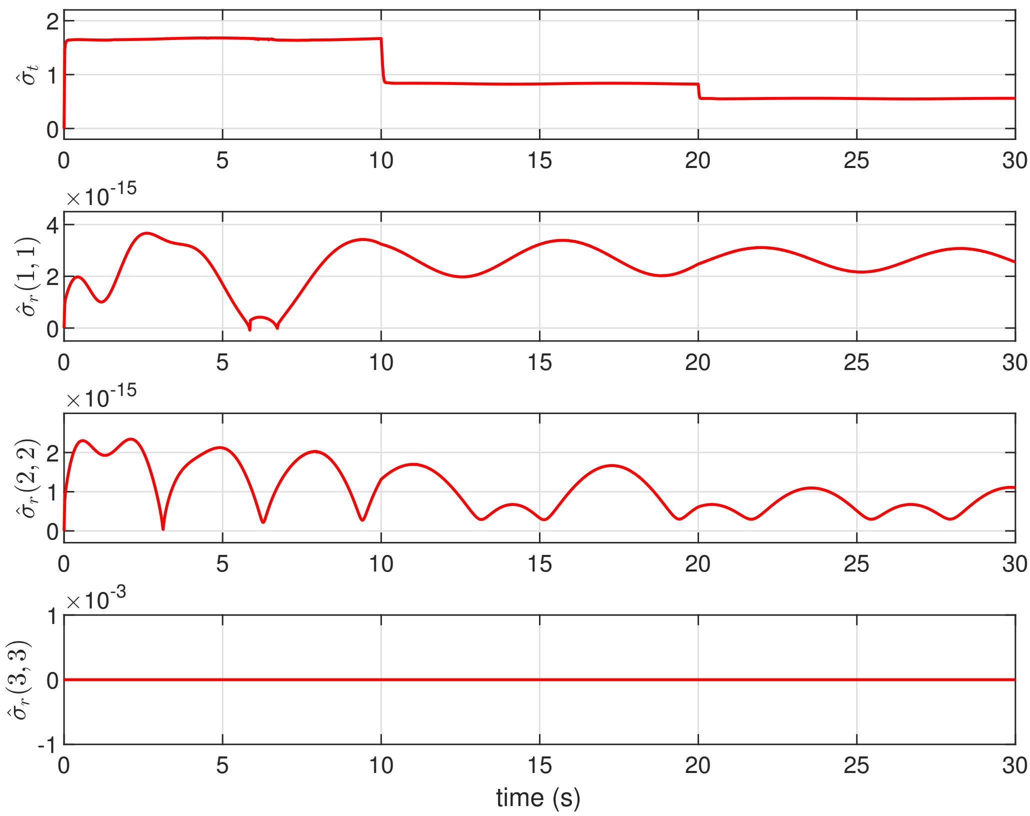

Figure 8. Profiles of the adaptive laws.

5. Conclusions and Future Research Directions

The paper presented a robust adaptive tracking control system designed for the six-DOF dynamics of UAVs. Disturbances were intentionally injected into both the translational and rotational dynamics of the UAVs to represent real-world applications. Additionally, abrupt fault factors were introduced into the control input structure, potentially impacting the performance of the control protocol. To address these challenges, robust control terms were integrated into both the outer and inner loops of the system, aiming to dominate the unknown disturbances effectively. However, the robust term alone may not sufficiently counteract the impact of abrupt fault factors. To tackle this issue, adaptive laws were introduced to address and adapt to these fault factors. By proposing robust control utilizing SMC and adaptive control designed based on certainty equivalence principles, the system guaranteed tracking control stability. In this approach, the sliding-surface error constrained the adaptive laws, ensuring that the adaptation process remained bounded. Consequently, the adaptive law operated without the necessity to estimate the precise values of the unknown abrupt factors. The proposed control scheme has a simple structure, making it more applicable in real-world scenarios. Numerous simulations involving a quadrotor UAV were conducted to validate the efficacy of our approach. Extending this scheme for a UAV under environmental constraints such as UAV singularities, actuator saturation, and environmental obstacles, and implementing it in real-world systems present compelling avenues for future exploration.

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