Typology and Design of Parametric Cat-in-a-Box and Cat-in-a-Grid Triggers for Tropical Cyclone Risk Transfer

Guillermo Franco 1,* , Laura Lemke-Verderame 1, Roberto Guidotti 1, Ye Yuan 1, Gianbattista Bussi 2, Dag Lohmann 3 and Paolo Bazzurro 2,4

1 Guy Carpenter & Company LLC, 1166 Avenue of the Americas, New York, NY 10036, USA; laura.verderame@guycarp.com (L.L.-V.); roberto.guidotti@guycarp.com (R.G.); ye.yuan@guycarp.com (Y.Y.)
2 RED Risk Engineering + Development, Via Giuseppe Frank 38, 27100 Pavia, Italy; gianbattista.bussi@redrisk.com (G.B.); paolo.bazzurro@redrisk.com (P.B.)
3 KatRisk LLC, 2397 Shattuck Avenue, Suite 212, Berkeley, CA 94704, USA; dag.lohmann@katrisk.com
4 Istituto Universitario di Studi Superiori (IUSS) Pavia, Palazzo del Broletto, Piazza della Vittoria 15, 27100 Pavia, Italy
* Correspondence: guillermo.e.franco@guycarp.com

Abstract: The insurance industry has used parametric solutions to transfer catastrophe risks since the 1990s. Instead of relying on a lengthy process to assess a claim, these products pay the insured a pre-agreed amount if the physical characteristics of the event fulfill pre-defined conditions. Cat-in-a-box or cat-in-a-circle triggers, commonly used tools for tropical cyclone risk transfer, provide a payout to the insured if the track of a hurricane crosses the perimeter of a geographic area defined by a polygon or a circle with a certain intensity. Cat-in-a-grid solutions are novel and more sophisticated. They rely on a set of multiple cat-in-a-box triggers arranged on an orthogonal grid. The consideration of multiple geographic domains instead of a single box or circle is helpful to reduce basis risk, i.e., the difference between the parametric loss estimate and the target loss. In the case study for Miami presented here, for instance, a cat-in-a-grid solution showed 18.5% less basis risk than a typical cat-in-a-box alternative. To organize the different types of triggers within a common framework, we classify the existing alternatives based on whether they use a single geographic domain (like a box or a circle) or multiple domains (like a grid). We discuss their advantages and disadvantages and describe the process required to calibrate any one solution with the help of a catastrophe-risk model. We focus, in particular, on the analysis and construction of cat-in-a-grid triggers, the alternative that we believe offers the greatest potential for global standardization and adoption.

Keywords: parametric; insurance; hurricane; cyclone

MSC: 91G60; 91G70; 91G80

1. Introduction

Parametric insurance, or more generally, parametric risk transfer, arose in the late 1990s, in the form we know it today, as a mechanism for investors to deploy their capital in the earthquake insurance market [1]. This novel type of insurance tied the payouts of a risk transfer instrument to a set of parameters, determined with precision and speed, and contractually agreed upon as valid proxies for the damage sustained by the insured. The parameters chosen were often those published by scientific or governmental entities to describe the event shortly after its occurrence. Transacting risks in this manner unencumbered investors from the lengthy and costly process of the adjustment of claims, while also minimizing risk of litigation and the entrapment of their collateral during extended loss development periods. The fast and transparent response of these parametric solutions is also attractive for the insured.
Although parametric solutions have only been used sporadically since they first appeared in the 1990s, they have experienced a global renaissance in the last decade due to a combination of factors: (1) Earth observation technologies have made it easier to obtain more numerous and accurate measurements of physical events; (2) advances in electronic finance have made it possible to deploy parametric solutions more seamlessly through the internet; (3) the impact of climate change on the frequency and severity of events has spurred demand for more coverage of catastrophe losses not addressed by traditional insurance products; and (4) increases in premiums and reductions in coverage have inspired the creation of novel, alternative insurance solutions. In the complex landscape that has ensued, and notwithstanding their limitations, parametric solutions are being hailed as a helpful and sometime necessary complement to traditional products, filling gaps in coverage and attracting more capital to extend the reach of insurance [2,3].

Cat-in-a-box triggers were the first instances of parametric insurance to appear in the 1990s and they continue to be the most common implementation among the many alternatives in existence today. Specifically for hurricane risk transfer, these solutions assume that payouts for the insured are a function of the storm’s intensity and on whether the hurricane’s track (i.e. the geographic position of the event as it progresses) intersects certain geographic areas. These geographic areas often assume the shape of rectangular boxes or circles, inspiring the shorthand of “cat-in-a-box” or “cat-in-a-circle” solutions with the “cat” portion in the name referring to “catastrophe”. The intensity of the tropical cyclone is usually measured by its maximum wind speed or, sometimes, by its minimum central pressure, metrics that are correlated with the energy and damaging potential of the event [4–7]. When circumstances demand further simplification, payouts can be defined based on the Saffir–Simpson “category” of a hurricane, a discrete classification of the storm’s maximum wind speed [8,9].

Triggers that rely on broad event-level parameters, such as the ones described above, are often classified as “first-generation” solutions as opposed to “second-generation” mechanisms that rely on site-specific intensity metrics, e.g., the maximum wind speed measured at a particular geographic location. The ability to measure with a good level of accuracy event-level parameters, such as the storm’s track and its category throughout its evolution, was achieved before recent advances in technology allowed us, in some circumstances, to measure or estimate local intensities. This gave rise to the chronological reference to “first” and “second” generation solutions, which analogously applies to parametric triggers for earthquake risk transfer [10,11].

The quality of a parametric solution is a composite of its speed to pay, its transparency, its viability, the market support it enjoys, and, of course, its accuracy. The relative importance of these traits is perceived differently by the insured depending on the circumstances. For example, a client interested in the coverage of business interruption losses such as a hotel owner whose operations are hampered by the occurrence of a hurricane, may prioritize the speed of the recovery over the accuracy of the payout. Estimating the loss of income that a business may have suffered due to the incidence of a nearby disaster is still a rather arcane process that may take years. It is usually much more critical for the hotel owner to recover some money quickly to allow the business to continue operating, rather than waiting to estimate a possibly more accurate payout that may arrive too late. In contrast, an insured whose priority is to cover the physical damage of a particular building may not be satisfied with a quick, inaccurate recovery, and may have more tolerance for a slower-to-respond or more sophisticated parametric solution that produces a more appropriate payout.

The accuracy with which a parametric insurance solution approximates the damages suffered by the insured is referred to as “basis risk”, defined as the difference between the parametric payout received and the expected amount that should have been paid based on the actual losses incurred by the insured. The quantification of basis risk is a key component in the design and selection of an optimal parametric solution [12].

This paper focuses on the analysis and the minimization of basis risk of first-generation triggers. Even if it is generally recognized that second-generation parametric solutions
have the potential to be more accurate at representing physical damage at a site-specific level [13], first-generation triggers are simpler, faster to settle, more transparent, and more easily transacted globally. Comparisons between first- and second-generation solutions are scarce in the literature, but the analyses that exist for earthquake risks suggest relatively modest differences in performance, depending on the particular application [14–16].

In this paper, we present in Section 2 a general summary of the typologies of first-generation triggers used in the industry, which we split into single-domain and multi-domain solutions, depending on their geographic definition. We then describe in Section 3 a process to calibrate single-domain triggers based on a catastrophe-risk model, which we extend to a particular type of multi-domain solution that we refer to as “cat-in-a-grid”. In Section 4 we illustrate the construction of these solutions with a variety of case studies, including examples for both localized exposures and larger regions.

Throughout the text we use the terms tropical cyclone, hurricane, cyclone, or storm interchangeably, to refer to the same physical phenomenon: a tropical cyclone. Similarly, we use the term risk transfer or insurance indifferently, but note that risk transfer encompasses a broader class of instruments that includes insurance as well as derivatives, cat bonds, Insurance-Linked Securities (ILS), reinsurance, etc.

2. Typology of First-Generation Parametric Triggers Used in the Industry

Early parametric transactions were simple but, as more experience has been accrued, their design has evolved with the aim of improving their accuracy. The industry recognizes, however, that very complex solutions, regardless of their potential accuracy, tend to confuse insurers and insureds and have limited marketability. This results in a constant but healthy tension between the technical desire to design progressively more accurate and sophisticated transactions and the market’s demand to limit complexity.

There is currently no standard or formal classification of first-generation parametric solutions. Their nomenclature has followed a rather organic evolution guided by experimentation in practice. We propose here to split them into two sets: (a) single-domain triggers, which rely on only one geographic domain to determine a payout, and (b) multi-domain triggers, which rely on a number of interacting regions. This section describes both sets of triggers with their derived sub-classes, as shown in Figure 1. Their common trait is the definition of payouts as a function of the hurricane intensity along the track as it intersects certain geographic areas. In this section, we propose a mathematical formulation to represent them.

2.1. Single-Domain Triggers

Single-domain triggers are simple and easily understandable by all stakeholders involved in the transaction. This is the reason why they have been widely used in the industry to this day. They are, in general, more suited to transfer the risk of highly concentrated exposures like a particular specific property or exposures aggregated in small areas such as those related to a small town or county. Over the years, they have received different names depending on the shape of the geographic definition used (Figure 2):
SDT1. Cat-in-a-Circle: A circle, usually centered at the point of concentration of exposures and with a radius that varies typically between 25 km and 75 km.

SDT2. Cat-in-a-Box: A generic polygon that is designed to intersect hurricanes with tracks that can affect the set of exposures.

SDT3. Gate: A one-dimensional curve, usually a linear segment arranged along the coast, that defines a "landfall" zone for hurricanes that pose a threat to assets located onshore behind the gate.

Trigger types SDT1, SDT2, and SDT3 can be expressed mathematically as instances of a trigger function, \( \Lambda(x, S) \), that yields a payout for a tropical storm based on its physical characteristics collected in vector \( x \) and the definition of the geographic domain \( S \). A very basic cat-in-a-circle policy of type SDT1 can be expressed, for instance, as follows:

\[
\Lambda(x, S_C) = \text{USD 1000} \times \begin{cases} 
1; & \text{if track intersects circle } S_C \\
0; & \text{otherwise}
\end{cases}, \tag{1}
\]

where circle \( S_C \) is centered at coordinates \((x_{SC}, y_{SC})\) with radius \( r_{SC} \), and vector \( x \) contains the geometric definition of the track in the form of a sequence of \( N \) points, or vertices:

\[
x = [(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)], \tag{2}
\]

between which we assume that the track consists of linear segments. This trigger pays the insured USD 1000 if the storm track intersects circle \( S_C \) or 0 otherwise. Since function \( \Lambda \) only makes use of the storm’s track geometry but not of any intensity metric, the trigger might pay for any storms intersecting the circle’s boundary even though their damage potential may be negligible. This is usually unacceptable for an insurer as the correlation between payouts and actual damages is poor. We would say that this solution has large “positive” basis risk, paying more frequently than it should. A more refined version of this trigger that recognizes the variation of damage potential of the storm based on its intensity might look like this:

\[
\Lambda(x, S_C) = \text{USD 1000} \times \begin{cases} 
1; & w_{SC} \geq 137 \text{ kt} \\
\frac{w_{SC} - 64 \text{ kt}}{73 \text{ kt}}; & 64 \text{ kt} \leq w_{SC} < 137 \text{ kt} \\
0; & \text{otherwise}
\end{cases}, \tag{3}
\]

where \( w_{SC} \) is the storm’s maximum wind speed reported or interpolated at any point of the track within circle \( S_C \) and measured in knots. Vector \( x \) contains the geometry of the track, as before, but also the maximum wind speeds \( w_i \) reported at all track points \( i \) of the storm:

\[
x = [(x_1, y_1, w_1), (x_2, y_2, w_2), \ldots, (x_N, y_N, w_N)], \tag{4}
\]

with maximum wind speed, \( w \), assumed to vary linearly between track vertices. The payout of this new trigger is still zero if the storm track does not intersect circle \( S_C \), but in this case the payouts for tracks that intersect the circle grow linearly as a function of the maximum wind speed of the storm reported or interpolated within the circle. Even if the tropical cyclone crosses the circle, the payout of this trigger remains zero if the storm’s maximum wind speed reported or interpolated within the circle is lower than 64 kt (Category 1 hurricane). Once the maximum wind speed attains a value of 64 kt, the trigger starts paying a fraction of the USD 1000 limit. If the maximum wind speed reaches or surpasses 137 kt (Category 5 hurricane), the policy pays the full limit of USD 1000 and does not increase any further for higher maximum wind speeds. Instead of growing linearly, payout functions can be designed to grow in proportion to expected damages from hurricanes crossing the circle at different wind speeds. We discuss the calibration of these payout functions in Section 3.1.
Figure 2. Usage examples of single- and multi-domain triggers for an insured asset designated by the “star”: A typical cat-in-a-circle solution (SDT1) characterized by the circular domain ($S_C$) centered over the insured location. A typical cat-in-a-box solution (SDT2) characterized by the rectangular domain ($S$). A gate-based solution (SDT3) characterized by the one-dimensional domain ($S$) following the nearest coastline. A multiple circles solution (MDT1) characterized by two circular domains ($S_{C1}$ and $S_{C2}$) centered over two insured asset locations. A concentric circles solution (MDT2) characterized by three circular domains ($S_{C1}$, $S_{C2}$, and $S_{C3}$) of different radii centered over the insured location. A multiple-boxes solution (MDT3) characterized by three polygonal domains ($S_1$, $S_2$, and $S_3$). A cat-in-a-grid solution (MDT4) characterized by a regular grid of sixteen domains ($S_1$, $S_2$, ..., $S_{16}$). A multiple-gates solution (MDT5) characterized by three one-dimensional domains or gates ($S_1$, $S_2$, and $S_3$). A multiple-gates with thickness solution (MDT6), which expands the gates shown in MDT5 seaward by a constant distance. The definition of the geographic area is crucial. For example, the incoming hurricane from the south intersects domain $S_C$ in SDT1 and domain $S$ in SDT2, but it does not intersect domain $S$ in SDT3.
2.2. Multi-Domain Triggers

Parametric solutions that use single-domain triggers to estimate payouts consider any storm track that crosses the zone’s perimeter to belong to the same class, without differentiating hurricanes that travel through a large portion of the domain and those that just briefly intersect the perimeter. The limited ability of single-domain triggers to represent the varied effects of a hurricane as it approaches the exposures raises the need for more complex geometries that rely on multiple areas and that we define here as multi-domain triggers (Figure 2). Most recent transactions, especially those constructed for large risk transfer projects, rely on multi-domain approaches that follow one or a combination of these typologies:

MDT1. Multiple Circles: A set of cat-in-a-circle solutions (SDT1) spread out over a geography, covering distributed exposures and whose interaction is often represented by the sum of payouts for all the cat-in-a-circle solutions intersected by an event.

MDT2. Concentric Circles: A set of cat-in-a-circle solutions (SDT1) of different radii whose center is common to all of them, and which are typically characterized by diminishing payouts for the same intensity level as the radius increases. The insured usually receives the maximum payout dictated by any of the different concentric cat-in-a-circles for a given storm.

MDT3. Multiple Boxes: Analogous to the multiple circles solution (MDT2), but using arbitrary polygons rather than circles. Usually, the boxes are arranged to provide different levels of coverage for different distributions of exposure (see, for example, Figure 3 with an actual implementation for Mexico).

MDT4. Cat-in-a-Grid: Similar to the multiple-boxes solution (MDT3) with the peculiarity of arranging rectangular polygons across a regular orthogonal grid that covers the entirety of the space holding the exposures without discontinuity.

MDT5. Multiple Gates: Similar to the gate-based single-domain solution (SDT3), this version considers a series of gates that pay the maximum of all payouts dictated by the individual-gate solutions. Usually, gates are arranged to differentiate events according to their landfall location and the corresponding exposures they may affect.

MDT6. Multiple Gates with Thickness: This is an attempt to address the possibility that simple gates (MDT5) may miss bypassing storms. Its development follows a hybrid philosophy between the multiple-gates (MDT5) and the multiple-boxes solutions (MDT3).

A multi-domain trigger, \( \Lambda(x, S_1, \ldots, S_k) \), requires the definition of an interaction function, \( \Phi \), that dictates how the different domains influence the payout of the trigger:

\[
\Lambda(x, S_1, \ldots, S_k) = \Phi(\Lambda_1(x, S_1), \ldots, \Lambda_K(x, S_K)).
\] (5)

The MultiCat Mexico Ltd. (Series 2012-1) parametric risk transaction [17] in Figure 3 is one of the flagship examples of a multi-domain, multiple-boxes trigger (MDT3). It consists of three single-domain triggers of type SDT2, defined alongside their respective polygonal geographic domains. The intensity metric chosen for this transaction is the minimum central pressure, which we assume to vary linearly between the values \( p_i \) reported at all track points \( i = 1, 2, \ldots, N \) of the storm and included in vector \( x \), as follows:

\[
x = [(x_1, y_1, p_1), (x_2, y_2, p_2), \ldots, (x_N, y_N, p_N)].
\] (6)

The hurricane trigger is split into two tranches named “B” and “C” with USD 75 m and USD 100 m of limit of liability, respectively. The USD 75 m tranche B payout can be triggered by either of two parametric mechanisms, \( \Lambda_{B_1} \) or \( \Lambda_{B_2} \), according to:

\[
\Lambda_B(x, S_{B_1}, S_{B_2}) = \max(\Lambda_{B_1}(x, S_{B_1}), \Lambda_{B_2}(x, S_{B_2})),
\] (7)

where
\[ \Lambda_{B_1}(x,S_{B_1}) = \text{USD } 75 \text{ m} \times \begin{cases} 1; & p_{S_{B_1}} \leq 920 \text{ mb} \\ 0; & \text{otherwise} \end{cases}, \] 

(8)

\[ \Lambda_{B_2}(x,S_{B_2}) = \text{USD } 75 \text{ m} \times \begin{cases} 1; & p_{S_{B_2}} \leq 920 \text{ mb} \\ 0; & \text{otherwise} \end{cases}, \] 

(9)

while the USD 100 m tranche C is paid according to a third trigger definition:

\[ \Lambda_C(x,S_C) = \text{USD } 100 \text{ m} \times \begin{cases} 1; & p_{S_C} \leq 920 \text{ mb} \\ 0.5; & 920 \text{ mb} < p_{S_C} \leq 932 \text{ mb} \\ 0; & \text{otherwise} \end{cases}. \] 

(10)

Figure 3. The 2012 MultiCat transaction [17] consisted of one tranche of USD 140 m of parametric earthquake coverage and two tranches of USD 75 m and USD 100 m of parametric tropical cyclone coverage for the Caribbean and Pacific coasts of Mexico, respectively. Hurricane Patricia in 2015 (pictured) intersected the Pacific coverage polygon with a minimum central pressure sufficient to produce a 50% limit payout of the Pacific tranche, corresponding to USD 50 m.

The split of limits for MultiCat 2012 suggests that the exposures at risk along the Pacific coast of Mexico (Box \( S_C \), with a limit of USD 100 m) were considered to potentially cause a loss greater than those exposures at risk in the Caribbean (zones \( S_{B_1} \) and \( S_{B_2} \) with a combined limit of USD 75 m). Note that triggers \( \Lambda_{B_1} \) and \( \Lambda_{B_2} \) share the USD 75 m limit and interact according to function \( \Phi = \max(\)\).

In general, the multi-domain triggers presented in this section have been mostly used by insureds with exposures spread across large areas, such as governments, insurance companies, or corporations. See for example, the catastrophe bonds sponsored by Mexico in 2020 [18] and Jamaica in 2021 [19] or weather derivatives marketed by the Chicago Mercantile Exchange Group and previously commercialized as the Carvill Hurricane...
Some multi-domain triggers, however, have also been marketed to home and small business owners. For example, in 2018, StormPeace, a parametric insurance provider in the US, began offering policies that paid according to the shortest distance of the insured property to the hurricane’s center and the intensity of the hurricane at its closest point [21]. This type of policy relies on a system of concentric circles (solution of type MDT2) around the policy location, which are discrete in number if distances are classified according to pre-defined ranges, or continuous if the true distance from policy location to storm track is calculated.

2.3. Limitations of Cat-in-a-Box Triggers

Multi-domain triggers aim to reduce the basis risk that characterizes single-domain triggers, especially for sprawling portfolios of exposures. Nevertheless, both single- and multi-domain triggers can be susceptible to two important limitations that the designer or “structurer” needs to be aware of:

A. Missing Bypassing Storms: A storm capable of causing large damages due to wind or storm surge may run parallel to gates or skirt by circles and boxes causing large damages to the exposures onshore without actually crossing the domains’ perimeter (see Figure 4, left panel). This excludes the storm from triggering any monetary recovery in most of the mechanisms discussed and is a likely source of frustration for insureds, who are potentially exposed to all-or-nothing parametric responses hinging on the geographic definition of the track and the trigger domains.

B. Insufficient Resolution to Capture Evolving Impacts: It is feasible for two storms with very different evolution patterns and impacts to trigger the same parametric recovery. Picture, for instance, a gates-based solution (i.e. of type SDT3 or MDT5), in which two storms make landfall at the same gate and with identical intensity parameters, but one veers parallel to the coastline causing continued damages, while the second penetrates inland and dissipates quickly (see Figure 4, right panel).

![Figure 4](image-url). Hurricane Matthew in 2016 (left) caused large losses in Florida due to a combination of wind, storm surge, and rainfall, particularly in the eastern coastal counties [22]. As of March 2017, reported insurance claims in the state were USD 1.182b [23]. Matthew, however, runs parallel to the Florida coastline, potentially missing any gates or circles used to transfer risk within the state. Hurricane Bob in 1985 and the 1928 Okeechobee Hurricane (right), each made landfall as a Category 1 hurricane in South Carolina but following different trajectories over land and resulting in different impacts to the state’s coastal counties. Losses for the Okeechobee Hurricane were estimated at USD 25–38 m [24], while those from Hurricane Bob were approximately a fifth, USD 5 m, both in 1985 US dollars [25].
The limitations of first-generation triggers were brought to the forefront when Hurricanes Odile (2014) and Patricia (2015) made landfall on the Pacific Coast of Mexico. At the time of these events, MultiCat 2012 [17], depicted in Figure 3, was in place to provide parametric protection to Mexico, and initial data for both events indicated that Mexico might receive compensation [26]. However, in each case, when the official track data were released several months later, minimum central pressures near landfall were revised to be higher than initial data indicated, which ultimately implied an absence of payout for Odile and a reduced payout for Patricia, illustrating the sensitivity of these triggers to a few reported track points.

To address limitation A related to bypassing events, trigger domains need to extend throughout the entire geography that damaging hurricanes might traverse, ideally completely tessellating the region so as to not leave any gaps. To address limitation B related to evolving impacts, triggers need to be able to capture the influence of different track geometries throughout the life of the storm, and this can be implemented by using a large array of interacting single-domain triggers.

We favor a cat-in-a-grid typology to specifically address these two concerns. This type of solution consists of a large set of trigger domains arranged across a regular, orthogonal grid that completely tessellates the region of interest. This design, which provides widespread and continuous coverage, aims to reduce the chance of missing loss-causing events and is capable of tracking the evolution of the event within the region of interest, thus reducing basis risk and the sensitivity of the trigger response.

3. Design of Single-Domain and Cat-in-a-Grid Triggers

In this section, we first illustrate the calibration procedures for single-domain triggers followed by the formulation and construction of cat-in-a-grid solutions.

3.1. Calibration of Single-Domain Triggers of Types SDT1, SDT2, and SDT3

We have introduced in Section 2.1 the trigger function for a single geographic domain as \( \Lambda(x, S) \) and, in the examples provided, we have given this function a set of values depending on the characteristics of the storm collected in vector \( x \) and domain \( S \). It is customary to assign values to the trigger function depending on the estimated loss produced by the storm, \( L'(x, S) \). For example, a parametric policy can be designed to “mimic” a traditional indemnity policy if it is designed in this form:

\[
\Lambda(x, S) = \begin{cases} 
USD 100,000; & L'(x, S) \geq USD 150,000 \\
L'(x, S) - USD 50,000; & USD 50,000 < L'(x, S) < USD 150,000 \\
0; & L'(x, S) \leq USD 50,000
\end{cases},
\]

which pays a recovery equal to the loss estimate, \( L'(x, S) \), above a retention (or deductible) of USD 50,000 and with a limit of liability of USD 100,000. Structuring a parametric trigger in this manner makes it easier to align, within reason, a parametric recovery with the expected loss of the storm and, if required, with other indemnity policies in force.

The calibration of the trigger therefore relies on constructing the loss estimation function \( L'(x, S) \) that measures the impact of the hurricane on the exposures. It is possible to conceive a sophisticated form of this function that considers many parameters of the storm to approximate its loss with more fidelity. However, as mentioned previously, a higher complexity means more difficult marketability.

As we did for the cat-in-a-circle example of Equation (4), we will assume that the maximum wind speed of the tropical cyclone correlates well with loss. Existing research in structural vulnerability and fragility models suggests that damage (and loss) increases according to the cube of the wind speed above a certain threshold of wind speed [5,27]. Therefore, it is reasonable to define the loss estimation function as a cubic polynomial shifted along the horizontal axis as follows:
where, as before, vector \( \mathbf{w} \) includes the track geometry and intensity metrics at track vertices, \( \{S_1, \ldots, S_k\} \) represent the grid cell domains, and \( \{\alpha_1(x, S_1), \ldots, \alpha_k(x, S_k)\} \) are the coefficients used to weigh the corresponding trigger estimates, in turn defined as follows:

\[
\alpha_k(x, S_k) = \begin{cases} 
0; & \text{if track does not intersect } S_k \\
\hat{\alpha}_k; & \text{if track intersects } S_k
\end{cases}
\]  

This trigger formulation helps address the two concerns expressed in Section 2.3, namely: (A) The union of the \( K \) domains defines a coverage area such that it is extremely unlikely to miss a potentially damaging event and (B) individual triggers in each cell \( \Lambda_k(x, S_k) \) act as estimators of damage for the event when the track passes through their respective domains, which gives the ability to the cat-in-a-grid trigger to progressively correct over- or underestimations as the cyclone advances, reminiscent of a Bayesian approach.

We define the coefficient \( \hat{\alpha}_k \) as a measure of the quality of the corresponding single-domain trigger \( \Lambda_k(x, S_k) \) to estimate event losses. The cat-in-a-grid trigger function, \( \Lambda(x, S_1, \ldots, S_k) \), thus becomes, in essence, a weighted average of all the individual cat-in-a-box trigger functions for those domains that have been crossed by the storm, weighing more heavily those that have a better ability to estimate loss. This quality is expressed as the inverse of the Mean Square Error (MSE) relative to that of the null model (a reference trigger function identical to zero for all wind speeds), as shown in the following equation:

\[
\hat{\alpha}_k = \frac{N}{\sum_{i=1}^{N} (L(x) - L'(x, S))^2} - \frac{N}{\sum_{i=1}^{N} (L(x))^2}.
\]
4. Design Study Cases

In this section, we apply the formulation described in Section 3 to design a set of parametric triggers for insureds with localized exposures in Miami-Dade County and for insureds with widespread exposures over larger regions in Florida and Jamaica.

4.1. Study Case for Localized Exposures in Miami-Dade County, Florida

We use the Miami-Dade case study to explore and compare a single-domain cat-in-a-box trigger (type SDT2) vs. a cat-in-a-grid (type MDT4) solution.

4.1.1. Single-Domain Cat-in-a-Box Trigger

Let us start assuming that we aim to design a cat-in-a-box trigger to protect building assets in Miami-Dade County, collected in a database included in the KatRisk commercial catastrophe-risk model for U.S. tropical cyclone (see Appendix B for a description of this model). We define the domain of interest, \( S \), as a rectangular box with coordinates \( 80.5^\circ \text{W} \) to \( 80^\circ \text{W} \) and \( 25.5^\circ \text{N} \) to \( 26^\circ \text{N} \), as shown in Figure 5. Please note that the definition of the domain in single-domain triggers is rather critical due to the limitations described above, and should be carried out with care as we discuss in more detail in Appendix A.

Figure 5 includes a set of simulated tropical cyclone tracks. These synthetic events have an associated estimated loss for the exposures considered, which we obtain with the help of a model, as well as a maximum simulated wind speed in the domain. In order to calibrate the polynomial coefficients, we separate the data into \( Q \) groups according to wind speed, or wind bands, and perform a weighted regression exercise such that:

\[
\sum_{q=1}^{Q} W_q (\bar{L}^*(x_q, S) - \bar{L}(x_q))^2 \rightarrow 0, \tag{16}
\]

where the index \( q \) represents the wind bands \( q = 1, \ldots, Q \); \( \bar{L}(x_q) \) is the median loss of the simulated events within the wind band; and \( W_q \) is the weight proportional to the number of simulated events in the wind band. The purpose of the weights in the regression exercise is to avoid overfitting to a small number of high-loss modeled events at high wind speeds.

Figure 5. The map on the left shows the location of the domain \( S \) overlaid by a sample of the tropical cyclone tracks simulated with a catastrophe-risk model. Each of the cyclones shown has an associated data pair containing the loss produced by the storm and its maximum wind speed in domain \( S \). These data pairs are sorted into 5 kt wind bands and plotted as a box-plot on the right. For each band, the box depicts the 25% to 75% percentile modeled event losses, with the median shown as a horizontal line. Whiskers depict minimum and maximum modeled event losses. Black circle markers are used to show the cubic polynomial resulting from the weighted regression exercise and discretized according to a series of 5 knot wind bands. Note the group of points in the 0 kt wind speed band, denoting storms that cause a loss, \( L(x, S) \geq 0 \), to the exposures but which do not cross the domain \( S \) (contributing to basis risk), thus resulting in a zero maximum wind speed within the domain.
Figure 5 (right) shows the resulting loss estimation polynomial function, discretized into 5 kt wind bands. Observe that there is a group of data located directly in the 0 kt wind band. These correspond to events with a loss greater than zero that do not enter domain $S$. It is important to take note of these events as they contribute significantly to the mean square error (involved in Equation (15)) of the loss estimates. Based on the loss estimation function, $L'(x, S)$, the trigger can now be defined as suggested in Equation (11).

4.1.2. Multi-Domain Cat-in-a-Grid Trigger

Let us now apply the formulation described in Section 3.2 to the design of a cat-in-a-grid trigger for the same exposures, which allows us to compare the performance between the cat-in-a-box and the cat-in-a-grid approaches. We define the cat-in-a-grid trigger with nine domains of 0.5° by 0.5° resolution that cover an area ranging from 81° W to 79.5° W in longitude and from 25° N to 26.5° N in latitude, as depicted in Figure 6. This area extends the original cat-in-a-box domain introduced in Section 4.1.1 by 0.5° in each cardinal direction with the goal of capturing loss-causing events potentially missed by the single-domain trigger.

For each cell domain in the cat-in-a-grid solution, we build the loss estimation function following the same calibration procedure outlined for the single-domain trigger. We then evaluate the quality of each loss estimation function using Equation (15). Figure 6 presents the cat-in-a-grid trigger domain alongside the nine loss estimation functions, each calibrated to the corresponding domain. Note that the geographic definition of domain $S_5$ in the figure is identical to that of domain $S$ in the cat-in-a-box trigger (Figure 5), and therefore their loss estimation functions are the same. Table 1 lists, for each cell domain, the MSE and quality of the respective loss estimation functions (normalized to a maximum value of 100), which are the interaction function weights defined in Equation (15).

Figure 6. Individual cell domains considered for the cat-in-a-grid trigger for Miami-Dade County alongside their respective loss estimation functions, which depict the derived relationship between maximum wind speed within each domain and expected loss to the exposures. All loss estimation functions are discretized according to 5 kt wind bands. Note that the geographic definition of domain $S_5$ is identical to that of domain $S$ for the cat-in-a-box trigger. Therefore, the loss estimation function for domain $S_5$ is identical to that shown in Figure 5.
Table 1. MSE and loss estimation function quality for each cat-in-a-grid cell domain (normalized to a maximum of 100), which constitute the cell domain weights in the cat-in-a-grid interaction function.

<table>
<thead>
<tr>
<th>Domain</th>
<th>MSE ($10^{18}$)</th>
<th>Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>16.384</td>
<td>22.57</td>
</tr>
<tr>
<td>$S_2$</td>
<td>12.890</td>
<td>55.17</td>
</tr>
<tr>
<td>$S_3$</td>
<td>17.867</td>
<td>12.58</td>
</tr>
<tr>
<td>$S_4$</td>
<td>14.296</td>
<td>40.14</td>
</tr>
<tr>
<td>$S_5$</td>
<td>9.968</td>
<td>100.00</td>
</tr>
<tr>
<td>$S_6$</td>
<td>17.488</td>
<td>14.98</td>
</tr>
<tr>
<td>$S_7$</td>
<td>15.631</td>
<td>28.36</td>
</tr>
<tr>
<td>$S_8$</td>
<td>13.929</td>
<td>43.77</td>
</tr>
<tr>
<td>$S_9$</td>
<td>18.898</td>
<td>6.56</td>
</tr>
</tbody>
</table>

As an example, let us illustrate, step by step, how the cat-in-a-grid design would respond to Hurricane Katrina (2005): (1) We first obtain the track information from a reporting agency, such as the International Best Track Archive for Climate Stewardship (IBTrACS) database [28,29], which is listed in Table 2 and depicted in Figure 7. Track information includes the position of the center or eye of the tropical cyclone and its maximum sustained wind speed at 6-hour time intervals. (2) The track is discretized to determine the maximum wind speed attained in each cell domain, $S_1, \ldots, S_9$. (3) These wind speeds are used to estimate losses for each domain intersected by the track, according to the trigger functions defined in the respective domains (Figure 6). (4) The total loss estimate is calculated in accordance with the interaction function defined by Equation (13). This estimate may be monitored in real-time and updated each time an additional track point is reported. In Figure 7 (bottom), we observe that the loss estimate is zero until the event enters the covered area at Track Point 9. As more track points are reported and the event enters additional domains, the loss estimate is updated. The estimate stabilizes at a final loss result when the event leaves the covered area at Track Point 12.

Figure 7. Example of response of the cat-in-a-grid trigger for Hurricane Katrina (2005). (Top): Track progression in the vicinity of Florida. The first 19 reported track points are depicted before Katrina re-intensified and impacted Louisiana. The region of coverage of the cat-in-a-grid trigger is shown as a dotted rectangle, while the four individual domains that are intersected by the track are highlighted in solid black; (Center): Maximum wind speed along track with intersected domains shown in vertical bars shaded in gray tones proportional to the quality of the respective domain loss estimation function; (Bottom): Loss estimate computed at each reported track point according to the cat-in-a-grid trigger definition (Equation (13)).
Table 2. Progression of the cat-in-a-grid parametric response for the first nineteen track points reported during Hurricane Katrina (2005).

<table>
<thead>
<tr>
<th>Track Point</th>
<th>Lat</th>
<th>Lon</th>
<th>WS (kt)</th>
<th>Loss (USD m)</th>
<th>Track Point</th>
<th>Lat</th>
<th>Lon</th>
<th>WS (kt)</th>
<th>Loss (USD m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.1°</td>
<td>−75.1°</td>
<td>30</td>
<td>0</td>
<td>11</td>
<td>25.9°</td>
<td>−80.3°</td>
<td>70</td>
<td>483</td>
</tr>
<tr>
<td>2</td>
<td>23.4°</td>
<td>−75.7°</td>
<td>30</td>
<td>0</td>
<td>12</td>
<td>25.4°</td>
<td>−81.3°</td>
<td>65</td>
<td>378</td>
</tr>
<tr>
<td>3</td>
<td>23.8°</td>
<td>−76.2°</td>
<td>30</td>
<td>0</td>
<td>13</td>
<td>25.1°</td>
<td>−82.0°</td>
<td>75</td>
<td>378</td>
</tr>
<tr>
<td>4</td>
<td>24.5°</td>
<td>−76.5°</td>
<td>35</td>
<td>0</td>
<td>14</td>
<td>24.9°</td>
<td>−82.6°</td>
<td>85</td>
<td>378</td>
</tr>
<tr>
<td>5</td>
<td>25.4°</td>
<td>−76.9°</td>
<td>40</td>
<td>0</td>
<td>15</td>
<td>24.6°</td>
<td>−83.3°</td>
<td>90</td>
<td>378</td>
</tr>
<tr>
<td>6</td>
<td>26.0°</td>
<td>−77.7°</td>
<td>45</td>
<td>0</td>
<td>16</td>
<td>24.4°</td>
<td>−84.0°</td>
<td>95</td>
<td>378</td>
</tr>
<tr>
<td>7</td>
<td>26.1°</td>
<td>−78.4°</td>
<td>50</td>
<td>0</td>
<td>17</td>
<td>24.4°</td>
<td>−84.7°</td>
<td>100</td>
<td>378</td>
</tr>
<tr>
<td>8</td>
<td>26.2°</td>
<td>−79.0°</td>
<td>55</td>
<td>0</td>
<td>18</td>
<td>24.5°</td>
<td>−85.3°</td>
<td>100</td>
<td>378</td>
</tr>
<tr>
<td>9</td>
<td>26.2°</td>
<td>−79.6°</td>
<td>60</td>
<td>5</td>
<td>19</td>
<td>24.8°</td>
<td>−85.9°</td>
<td>100</td>
<td>378</td>
</tr>
<tr>
<td>10</td>
<td>26.0°</td>
<td>−80.1°</td>
<td>70</td>
<td>483</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.1.3. Single-Domain Cat-in-a-Box vs. Multi-Domain Cat-in-a-Grid Trigger

We compare the cat-in-a-box and the cat-in-a-grid solutions for Miami-Dade County according to how they address the limitations of first-generation triggers introduced in Section 2.3.

Limitation A focuses on the ability of the trigger to capture potentially damaging events. Figure 8 presents the number of events in the model catalog that intersect the domain of the cat-in-a-grid trigger but miss the single-domain trigger. It is evident that the cat-in-a-grid solution, which uses a larger domain than the cat-in-a-box solution, misses fewer potentially damaging events. The events are classified by the return period of the loss they produce. At very high loss levels, both triggers manage to capture all extreme events with losses corresponding to a 6000-year return period or higher. However, for lower return periods of 50 to 200 years, typical levels of loss for which an insurer or a corporation would seek protection, the cat-in-a-grid solution shows a better performance, capturing important events that the cat-in-a-box solution would miss. The largest modeled loss event missed by the single-domain trigger but captured by the cat-in-a-grid solution has a return period of 5952 years. The largest loss event that is missed by the cat-in-a-grid trigger has a significantly lower return period of 943 years.

![Figure 8](image-url)

**Figure 8.** Events that intersect the cat-in-a-grid domain but do not intersect the cat-in-a-box domain. Events are grouped by the return period of the corresponding event modeled loss. Both trigger domains capture all events with loss return periods of 6000 years and higher. The single-domain trigger misses more events with respect to the cat-in-a-grid domain as the loss return period decreases.

Limitation B is related to basis risk or the ability of the trigger to nuance the parametric response depending on the evolution of the event. We observe that the cat-in-a-grid design reproduces modeled losses with more fidelity and less volatility than the single-domain solution based on a variety of metrics:
i. The cat-in-a-grid trigger produces estimates of loss that agree better with modeled losses (MSE of $8.120 \times 10^{18}$) than those of the single-domain trigger (MSE of $9.968 \times 10^{18}$). Considering all modeled events, the basis risk of the cat-in-a-grid solution is therefore 18.5% lower than that of the cat-in-a-box.

ii. The single-domain trigger produces only a limited number of loss outcomes, as shown in Figure 9. This is in contrast to the cat-in-a-grid solution, which is able to produce a larger number of estimates of loss by considering multiple subdomains and ultimately generating a smoother parametric response. Figure 9 (top) illustrates the response of both triggers to Category 5 hurricanes that directly hit Miami-Dade County and intersect both the cat-in-a-grid and the single cat-in-a-box domains. The cat-in-a-grid trigger allows for a more nuanced response, in line with the modeled loss for these events (shown by the dashes). Figure 9 (bottom) shows the distribution of trigger response for events that intersect the cat-in-a-grid domain but which do not necessarily intersect the cat-in-a-box domain. Naturally, for those events that do not intersect the single-domain trigger, the estimated response of the cat-in-a-box is zero (giving rise to the large number of missed events to the left of the plot). The cat-in-a-grid, once again, manages to produce a nuanced response more in line with the modeled losses. Note also that the response of the single-domain trigger is limited, producing only a fraction of those outcomes possible with the cat-in-a-grid approach. While many of these events tend to produce lower losses than those directly hitting Miami-Dade, they are important to consider for the risk transfer of relatively more moderate losses between USD 10–60b.

iii. The comparison of return periods is often more useful than that of loss amounts. Return periods are routinely used to define the attachment of the parametric policy, i.e., the loss level at which the insured starts to receive a payout, as well as its exhaustion or loss level at which the payout reaches its maximum limit. Comparing return periods is equivalent to comparing the ranking of the events’ modeled and parametric losses. The Spearman’s Rank Correlation [30] between modeled and parametric losses for the single-domain trigger is $r_s = 0.324$, while the cat-in-a-grid achieves a significantly higher value of $r_s = 0.553$. Therefore, this metric suggests that the performance of the cat-in-a-grid at aligning the return periods of the parametric loss estimates and the modeled losses is higher than that of the cat-in-a-box.

iv. The single-domain trigger produces parametric loss estimates for modeled events that contribute 50.7% of the Average Annual Loss (AAL) (see Table 3). In contrast, the cat-in-a-grid trigger produces loss estimates for events contributing to 91.3% of the AAL. Assuming solutions that pay only above a certain probability level (higher probability means higher cost), the cat-in-a-grid solution captures a similar or greater portion of the AAL than the single-domain trigger at each tested attachment return period. The differences decrease as the attachment level increases, which means that both parametric solutions might be satisfactory if our purpose is to only obtain a payout for very large, rare events. However, if there is interest in obtaining a more nuanced response for moderate, more frequent events, a cat-in-a-grid solution performs better.

Table 3. Percent of Average Annual Loss (AAL) captured by each trigger assuming different return periods for the attachment of the parametric structure.

<table>
<thead>
<tr>
<th>Trigger</th>
<th>0-yr</th>
<th>15-yr</th>
<th>24-yr</th>
<th>33-yr</th>
<th>51-yr</th>
<th>74-yr</th>
<th>117-yr</th>
<th>203-yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Domain</td>
<td>50.7%</td>
<td>50.7%</td>
<td>48.8%</td>
<td>46.8%</td>
<td>42.8%</td>
<td>37.1%</td>
<td>29.3%</td>
<td>21.5%</td>
</tr>
<tr>
<td>Cat-in-a-Grid</td>
<td>91.3%</td>
<td>79.7%</td>
<td>69.4%</td>
<td>60.1%</td>
<td>48.7%</td>
<td>38.7%</td>
<td>29.6%</td>
<td>21.3%</td>
</tr>
</tbody>
</table>
Figure 9. Modeled losses and parametric loss estimates produced by the single-domain and cat-in-a-grid triggers for Category 5 modeled hurricanes. At the top, we consider all modeled events that attain Category 5 strength within the boundary of the single-domain trigger. At the bottom, we expand the selection to include all modeled events that attain Category 5 strength within the boundary of the cat-in-a-grid trigger. Note the large number of events missed by the cat-in-a-box in the bottom plot. In both plots, the cat-in-a-grid trigger shows a more nuanced response more in line with the modeled loss distribution than that of the cat-in-a-box.

4.2. Cat-in-a-Grid Design Study Cases for Larger Regions

Cat-in-a-grid structures seem particularly compelling to transfer risk for scattered portfolios across vast regions such as groups of exposures distributed among island nations or throughout large peninsulas that can be subjected to large variability in impacts from tropical storms depending on whether the storm travels parallel to the coast or makes a single or repeated landfalls. It would be hard or nearly impossible to track these different circumstances with a single-domain trigger. Likewise, it would be cumbersome and inefficient to use a collection of cat-in-a-boxes that do not completely tessellate the space (type MDT1) to capture the complex loss interaction between assets at different locations.

In this section, we present two examples for Florida and Jamaica, respectively, to showcase larger applications of the cat-in-a-grid trigger formulation presented in Section 3.2. The calibrations are performed based on modeling results kindly provided by KatRisk (www.kattrisk.com) and Risk Engineering + Development (www.redrisk.com), respectively. Appendix B provides a description of these models. Different models are used in order to illustrate the disassociation of the cat-in-a-grid calibration techniques from the model of choice to represent the risk. The construction process is mechanical and does not rely on subjective judgment to locate a series of boxes, circles, or gates. Therefore, it is highly automatable and reproducible.

4.2.1. A Cat-in-a-Grid Trigger for Florida

In this section, we present a prototype of a cat-in-a-grid trigger for the state of Florida. We consider a collection of assets representing the distribution of buildings across the state and included in KatRisk’s exposure database developed for their U.S. tropical cyclone model. The geographic area is defined from 88° W to 78° W and 23.5° N to 31.5° N and is tessellated by 320 domains of 0.5° by 0.5° resolution. This large area is designed to capture any tropical cyclone causing significant loss to Florida, whether storms make landfall or bypass the exposures along the coast and in Florida’s interior. According to
the modeling results, all events causing at least a 6-year return period loss to Florida pass through this area.

Figure 10 depicts the functions for each individual domain, calibrated following the procedure illustrated in Section 3, inset within the corresponding geographic area. The values shown within each domain correspond to the individual cell loss function’s quality normalized to a value of 100. We observe the highest loss functions and highest estimation qualities (with lower MSE and normalized values closer to 100) within domains near Miami, an area with a high concentration of exposure. Other domains estimating high losses and with a high loss function quality are observed along the western coast of the Florida peninsula, extending south from Tampa and St. Petersburg, and similarly correlated with large aggregations of population and exposures.

Figure 10. Map of domains considered for multi-domain cat-in-a-grid trigger designed for Florida. Domains are labeled with cell quality normalized to maximum value of 100. Loss functions depicting the derived relationship between the maximum wind speed in each cell and event model loss are shown within each domain. To depict these functions, each domain’s lower horizontal boundary represents the x-axis or wind speed, and the vertical left-most boundary represents the y-axis or event loss estimate.

4.2.2. A Cat-in-a-Grid Trigger for Jamaica

Similar to the exercise carried out for Florida, we replicate the process for Jamaica using a different model, Risk Engineering + Development’s hurricane model for the Caribbean, with its associated exposure database. The geographic area is defined from 80° W to 74° W and 16° N to 20° N and is tessellated by 96 domains of 0.5° by 0.5° resolution. According to the modeling results, all events causing at least a 20-year return period loss to Jamaica
pass through this area and, therefore, we assume that hurricane tracks that do not enter this rectangular area will not produce any significant loss to the country.

Generally, we observe cell domains with higher loss estimations and better quality near larger centers of population in Jamaica (Figure 11). Moderate cell model qualities of 10 to 15 are observed to the southeast and northwest of the island, indicating that many loss-producing tropical cyclones pass through these domains. In this region, it is common for tropical cyclones to pass through Jamaica with a SE–NW heading, and this is reflected in the cell domain calibrations. In fact, this circumstance may sometimes lead to model overfitting, which the designer of the parametric solution needs to carefully monitor.

![Figure 11. Map of cell domains considered for Jamaica. Domains are labeled with cell quality normalized to a value of 100. Loss estimation functions depicting the derived relationship between the maximum wind speed in each cell and event model loss are shown within the domains similarly to Figure 10.](image)

### 5. Conclusions

Parametric solutions for risk transfer are becoming more common in the insurance market. We expect this trend to continue, driven by demand for better and more responsive insurance coverage. Identifying what particular solutions work best for insureds in different circumstances is paramount.

We have proposed a novel classification of parametric first-generation solutions based on whether they use single or multiple domains. Within these broad classes, we have collected the most common typologies observed in market transactions to date. We have also presented a mathematical formulation and a methodology to calibrate any type of solution using a catastrophe-risk model.

Among the solutions collected, we have expressed our preference for cat-in-a-grid formulations as the most promising typology due to their regularity, potential for automation, objective calibration, and ability to capture a variety of impacts from bypassing or landfalling events to concentrated or scattered exposures. For a study case in Miami, the basis risk of a cat-in-a-grid solution was 18.5% lower than that of a cat-in-a-box trigger.
The efficiency of risk transfer (Average Annual Loss of events covered at a given return period) was systematically higher for the cat-in-a-grid approach, which suggests a higher utility for the insured (transferring more risk for the same cost). We have shown that a cat-in-a-box solution may be satisfactory in circumstances when (1) coverage is desired only for very large losses, (2) geographic exposures are highly concentrated, and (3) simplicity is required. Even in those cases, we recommend conducting a formal model-based calibration of the trigger (as shown in Section 3.1) accompanied by a geographic optimization exercise, such as the one illustrated in Appendix A.

There are some aspects related to this work that deserve further study, namely the process to formally determine the extension of the cat-in-a-grid mesh (which we have assumed here to be imposed by subjective design), the quantification of sensitivity to geographic resolution, pricing implications, or the usage of more advanced interaction functions for multi-domain triggers.


**Funding:** This research received no external funding.

**Data Availability Statement:** The data used for the case studies are not public and not readily available. The data were obtained from proprietary catastrophe-risk models owned by Risk Engineering + Development (www.redrisk.com, accessed on 11 March 2021) and KatRisk (www.katrisk.com, accessed on 13 June 2023).

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**Conflicts of Interest:** Authors Guillermo Franco, Laura Lemke-Verderame, Roberto Guidotti and Ye Yuan were employed by the Guy Carpenter & Company, LLC. Authors Gianbattista Bussi and Paolo Bazzurro were employed by the company RED Risk Engineering + Development. Author Dag Lohmann was employed by the KatRisk LLC. The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest. The Guy Carpenter & Company, LLC, RED Risk Engineering + Development and KatRisk LLC had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

**Abbreviations**

The following abbreviations are used in this manuscript:

- **AAL** Average Annual Loss
- **IBTrACS** International Best Track Archive for Climate Stewardship
- **ILS** Insurance Linked Securities
- **MSE** Mean Square Error
- **USD** United States Dollar

**Appendix A. Optimal Design of Cat-in-a-Circle Triggers**

Cat-in-a-circle triggers are among the most common types of parametric structures for tropical cyclone transactions (and often for earthquake transactions as well). Brokers, insurers, and reinsurers design and price them with ease for corporates and communities that desire to transfer risks associated with concentrated assets at a specific location. The usual process consists simply of placing a circle of a given radius with its center coincident with the location of the asset. This is intuitive but also potentially inefficient because hurricanes produce an asymmetric field of wind speeds and, consequently, asymmetric patterns of damage with respect to their center. For instance, a hurricane in the northern hemisphere,
typically advancing in a NW or NNW direction, tends to produce more damage on the northern side of the track because the translational speed of the hurricane is additive to the rotational speed on that side, whereas it is subtractive on the opposite side. As an example, asymmetrical patterns of wind damage were documented during Hurricane Andrew, which made landfall in southeastern Florida in 1992 [31,32]. In addition, both Hurricane Irma (2017) and Hurricane Ian (2022) saw asymmetrical patterns in storm surge flooding along the Florida coast due to differences in wind direction and speed [33,34]. For these reasons, a structure that assumes circular symmetry on the damaging effect of the tropical cyclone will incur an efficiency penalty, which we aim to quantify.

Modern catastrophe models like those used in the case studies in this paper capture this effect. Therefore, a trigger optimized to one such catastrophe model should reflect this asymmetry. The systematic process that we have presented in Section 3.2 to design cat-in-a-grid structures sheds some light on this effect. In solutions like those presented in Section 4.2, we have observed that the highest values of quality estimation are often found in cells that are slightly offset from the centers of aggregation of the largest exposures.

![Schematic: 6 of the 750 Tested Circles](image1.png)

**Figure A1.** (Left) Schematic of experiment that tested a series of 750 cat-in-a-circle solutions of radius 0.5°, centered on a dense grid with spacing of 0.1° (shown as crosses). Each solution was designed to capture events causing loss to a single asset in Miami Beach (shown as a star). Six of these cat-in-a-circle solutions, centered at points arranged on a dense grid in the region (Figure A1, left), are depicted. (Right) Contours derived from the MSE (10^9) of the 750 cat-in-a-circle solutions. The center of the contour map, corresponding to the center of the solution with the minimum MSE, is depicted. Distance between the two circles’ centers is calculated as 30 km.

To measure the effect of the asymmetries discussed, consider an asset at risk in Miami Beach, Florida, and a collection of different cat-in-a-circle solutions of radius 0.5° centered at points arranged on a dense grid in the region (Figure A1, left). We optimize the cat-in-a-circle solutions through the regression exercise described in Section 3.1 and derive a contour plot from the error of the optimized models (Figure A1, right). We observe that the minimum error is found at a distance of 30 km from the cat-in-a-circle nearest to the asset in Miami Beach, indicating that the optimal cat-in-a-circle solution should not be placed at the location of the asset but rather slightly offset, at the location pictured. The difference in error between the nearest cat-in-a-circle location and the optimal location is 3.2%. This is a relatively small error penalty, but it can be larger depending on the geographic position of the exposures of interest and on the wind regime of hurricanes passing nearby.
This lack of efficiency is compounded when multiple assets are considered in a portfolio transaction. If circumstances allow, insureds should consider avoiding cat-in-a-circle solutions (as well as any other single-domain solutions) and revert instead to cat-in-a-grid mechanisms, which increase the efficiency of the transaction and help mitigate all other concerns described above.

Appendix B. Model Descriptions

Appendix B.1. Florida Catastrophe Model

To model the impacts of storm surge, inland flooding, and wind, KatRisk has developed a tropical cyclone model for the Atlantic Basin. The following sections describe the generation of the track set and subsequent storm surge and wind modeling.

i. Track Development: Development of the tropical cyclone track model consisted of seven basic components: genesis (formation), propagation (tracks), termination (lysis), maximum sustained wind speed (Vmax), central pressure (CP), radius to maximum winds (Rmax), and landfall calibration. All the above components, apart from Rmax, are presented with varying degrees of detail in several publications. Genesis, propagation, and termination are discussed in Hall and Jewson (2007) [35], Hall and Yonekura (2011; 2014) [36,37], and Hall and Yonekura (2013) [38]. Vmax is discussed in Hall and Yonekura (2013) [38], and CP in Orton et al. (2016) [39]. Landfall calibration is discussed briefly in Hall and Tippett (2016) [40]. Two applications of the model are found in Hall and Sobel (2013) [41] and Hall and Hureid (2015) [42], and a comparison of the track-model landfall to local landfall estimation is found in Hall and Jewson (2008) [35].

The final track set consists of 50,000 years of tropical cyclone tracks conditioned on sea surface temperature (SST) and associated indices such as El Niño-Southern Oscillation (ENSO). Each of the 50,000 modeled years is associated with a climate state, making it possible to subset years and events and resulting model losses corresponding to a given climate state or range of climate states.

ii. Wind Field Modeling: For each track, a wind footprint was created utilizing the wind field parameterization described in Willoughby et al. (2006) [43]. For this model, overland roughness is modeled based on land use land cover data, and 3 s peak gust wind speeds are output on a 1 km grid.

iii. Storm Surge Modeling: The North American 50,000-year tropical cyclone track event set was then run with the NOAA SLOSH model, including tides, for operational basins along the gulf and east coasts, comprising over 800,000 track/basin simulations. Output from adjacent basins was then merged to create one continuous footprint for each storm. Water heights above datum from the SLOSH output have been adjusted using a parametric wave factor, as SLOSH does not include waves. The 10 m USGS National Elevation Dataset elevations were subtracted from the wave-adjusted SLOSH water heights to calculate surge heights above ground level for each event on a 10 m grid.

iv. Flood Vulnerability: KatRisk utilized the US Army Corps of Engineers (USACE-Galveston) and the Federal Insurance Administration (FIA) vulnerability curves as the basis for developing the vulnerability curves used for flood. Vulnerability curves for storm surge are the same as inland flood, with the exception that wave factors are applied close to the coast to account for momentum and water borne debris. The wave vulnerability factors are dependent on distance to coast, magnitude of the surge, and construction class.

v. Modeling Losses and Loss Uncertainty: Losses are modeled based on either KatRisk location level exposure or chosen by the user of SpatialKat, KatRisk’s loss modeling platform. Losses from wind, storm surge, or inland flood are computed at the same time and are combined based on user choice and insurance contracts. Uncertainty in modeled losses results from both unknown exposure characteristics that impact how exposures respond in an event and natural variability in the hazard. KatRisk accounts for uncertainty within its loss modeling framework by imposing a four-parameter beta distribution based on the mean damage ratio for every computed loss. This
Appendix B.2. Jamaica Catastrophe Model

The purpose of the probabilistic tropical cyclone risk model used in this study is to compute losses caused by tropical cyclone-induced wind and storm surge over the island of Jamaica. The model is composed of five modules: (i) the hazard module characterized by the stochastic catalog of tropical cyclones; (ii) the wind and storm surge physical modules; (iii) the vulnerability module; (iv) the exposure module; and (v) the loss computation module.

i. The Stochastic Catalog of Tropical Cyclones: We employed RED’s (Risk, Engineering + Development) commercial tropical cyclone catalog for the North Atlantic. This catalog contains an equivalent sample of 10,000 years and includes thousands of synthetic tropical cyclone tracks, each with a value of maximum wind speed, minimum sea level pressure, and radius of maximum wind associated to every 6-hour time step. The catalog was developed using an ensemble of literature methodology to reproduce the statistical properties of historical events recorded from 1931 to 2019. Key components included the following: the points of genesis, determined from a density map of historical points; movement in the x and y directions, modeled following James and Mason (2005) [45]; variation in along-track minimum sea level pressure, determined following Bloemendaal et al. (2020) [46]; variation in along-track maximum wind speed, linked to minimum central pressure through polynomial functions; variation in along-track radius of maximum wind, estimated by Vickery and Wadhera (2008) [47]; and inland decay, based on Kaplan and DeMaria (1995) [48].

ii. The Wind and Storm Surge Modules: The wind speed model of Holland et al. (2010) [49] estimates wind speeds on a regular grid of 360 × 360 arc-seconds over Jamaica, using the tropical cyclone properties at every time step of the track. The maximum wind speed map is obtained for each event as the maximum envelope of the time step-specific wind speed maps. The map is downscaled to a 1 km by 1 km grid and modified according to wind speed multipliers to account for roughness and topographic effects [50]. The storm surge model used is a modified version of GeoClaw [51,52], with the General Bathymetric Charts of the Oceans [53] as boundary conditions. A storm surge height is computed for every cell of the exposure grid as the difference between the sea level estimated by the model and the terrain elevation of the asset estimated from the Shuttle Radar Topography Mission digital elevation model with 1 arc-seconds resolution.

iii. The Exposure Module: The exposure database is a gridded representation of the assets at risk and was developed using input information including national building survey, land use maps, night-time lights imagery, population maps, and other satellite products. The database includes information on asset location, line of business, replacement cost, and physical characteristics. The database has a resolution of 100 m close to the coastline, where local effects are more likely, and 1 km inland.

iv. The Vulnerability Module: Wind vulnerability functions were derived for all the classes of assets from literature studies [54–60] and then adapted to the case study of Jamaica by adjusting them to obtain the best possible reproduction of the reported damage inflicted by selected historical tropical cyclones. Assets were divided into 12 occupancy categories, and for each occupancy class different typologies of assets were identified, producing a total of 150 wind vulnerability functions. Storm surge vulnerability functions for buildings were derived through the component-based methodology INSYDE [61], while literature functions were employed for infrastructure [62].

v. The Loss Computation Module: RED’s commercial loss computation engine, REDLoss, was used to compute losses for the whole exposure database based on maximum wind speed and storm surge height. Given that two concurrent perils can cause damage, a combination methodology was implemented to ensure that the total damage never exceeds the actual exposed value [63].
References


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