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Complex t-Intuitionistic Fuzzy Graph with Applications of Rubber Industrial Water Wastes

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Abstract: The main concept involved in this study explains the theme of complex t-intuitionistic fuzzy graphs (CTIFGs), which act as a powerful tool in analyzing and displaying the relationships among various applications that are difficult to recognize. The manuscript also demonstrates the capability of CTIFGs to create complex associations with multiple domains when considering a physical situation. Following this, the basic set of operations for CTIFGs is projected. The ideas on isomorphism and homomorphism of the CTIFGs are also presented. Moreover, the manuscript describes the importance of the above-mentioned technique in an effective way, giving a solution to the practical application associated with rubber processing industrial wastewater. The contributing factors and corresponding interdependencies are considered when calibrating the complex nature of industrial wastewater associated with the CTIFGs. The results highlight the adaptability and possible efficiencies of CTIFGs, which act as a decision-making tool and also indicate their importance for policy planners in important societal issues.

Keywords: graph theory; t-intuitionistic graph; complex t-intuitionistic fuzzy graph; decision making and sustainability

MSC: 05C72, 03B52, 68R10, 05C60, 05C90.

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1. Introduction

1.1. Fuzzy Set and Graphs

Real-world problems can be meticulously addressed with the support of mathematical concepts using the applications associated with fuzzy graph theory. In [1], Zadeh depicts the importance of assigning membership values running from zero to one, which are used for understanding the set theory, where judgment, fuzziness, and human opinion are enforced for predictions. The fuzzy-based mathematical concept emerged in the 1960s and 1970s, and various research in the field of applications-focused theory in the area of tube trains, video cameras, and washing machines was conducted. Simultaneously, formal theories are used to obtain the qualitative outcome. The review paper [2] highlights the significant factors associated with fuzzy set theory, where natural and formal models are used to predict uncertainties with the support of modeling. The fuzzy set in this work is denoted in the form of

$$A = \{\alpha, \mu_A(\alpha) | \alpha \epsilon A\}$$

The pairs indicate the fuzzy set, $\mu_A(\alpha)$ epresents membership value, and it is denoted as $0 \leq \mu_A(\alpha) \leq 1$.

The work in [3] described the role of variables linked with a potential prospective distribution and a probability distribution. The work of Zadeh and their teammates has demonstrated an idea in decision-making using the probability and possibility concept. They have established unique concepts in the name of the Zadeh fuzzy set, n-dimensional fuzzy set, and finite value fuzzy sets [4]. For an assigned methodology, introducing the algorithms supported by fuzzy graph theory gives the output on evaluating the multiple species-based fishery dynamics and will also provide a systematic introductory interpretation in the form of patterns. Fuzzy means it is represented as not having a clear vision in normal conditions. Most of the daily life problems can be modeled using fuzzy graphs [5].

The linear Diophantine fuzzy graphs play a significant role in categorizing the properties of various applications. Some of the most predominant outcomes are in the field of bridges, trees, forests, cut vertices, and cycles [6]. An algorithm is commonly involved in evaluating the number of components associated with rooted trees that are covered using fuzzy graphs and in addition, it also determines the complexity analysis [7]. There might be alternative solutions for the algorithms that are favored. The application of optimization with the support of algorithms will be an alternative and more advantageous than the application of neural network-based fuzzy models [8]. The application of a fuzzy model can even predict the functioning of the human heart and can be easily plotted using graphs for easy understanding the graphs [9]. Fuzzy methods play a significant role in compiling graphical content, which includes edge weight for balance, edge connectedness for continuity, edge existence for surveying, and vertex existence and are plotted in order to give concluding remarks in fuzziness [10,11]. These data are normally used in maintaining the ranking by fuzzy numbers. The introduction of a fuzzy hyper-based wiener index helps in calibrating various graphical similarities [12]. The role of the fuzzy cognitive map and its associated structure of output concepts minimizes all concepts of complicated connections in the graph [13].

1.2. Intuitionistic Fuzzy Set and Graphs

The advanced stage of conventional fuzzy set theory and the corresponding graphs are representations through intuitionistic fuzzy sets (IFSs) and (IFGs) intuitionistic fuzzy Graphs. Given the flexibility in making models, the Krassimir Atanassov pattern was introduced in 1980 when it addressed the inclusion and removal of a set of values. In cases where decisions are made in critical situations, the IFS models are considered as suitable solutions for correcting alternatives through graphical representation [14–16]. In decision-making, initially, the level of resistance is measured, and then the membership value perspectives and non-membership value viewpoints are considered when making flexible decisions with more information through the graphs [17,18].

In the case of technical content, the mathematical concept associated with IFSs and IFGs plays a significant role in developing algorithms and aggregation strategies. The major components involved in accurate modeling are networking, utilizing image recognition, clustering of data, and control systems, which provide significant changes to the decision-making process [19]. Framing a hypothesis for the IFG graph deliberately signifies the self-complimentary displays [20]. The importance of IFG components is examined and explained with the support of various typical examples [21]. In IFGs, the applicability of the vertex set is assessed through the upgraded algorithm [22]. Product operations are defined in the IFG with the support of graphs [23]. With the energy of a fuzzy graph, it is further transformed into the IFG concept for effective outcomes [24]. The combined effect of fuzzy and IFG clustering roles significantly considers the clustering of vertices [25]. Connectivity plays a predominant role in the study of IFGs, which has been discussed briefly [26]. Connectivity indexing supports IFG research by giving better outputs [27].

Multi-person thinking, multi-criteria-based decision-making support, and reliability scores are the outcomes of the intuitionistic fuzzy model [28].

1.3. Complex Fuzzy Sets (CFS) and Complex Fuzzy Graphs (CIFG)

In complex fuzzy theory, $\mu(x)$ has been highlighted as the membership function that covers the unit circle of the complex plane that has range values higher than [0, 1] with r as an identifier. Meanwhile $\mu(x)$ indicates the complex-valued function, which is represented as $v(x)e^{iax}, i = \sqrt{-1}$ as a membership grade. In due course, amplitude term $v(x)$, has been disclosed with the unit interval [0, 1], and similarly, the phase term represented as a periodic term has (x) with the interval [0, 2π], as both the components united to form a membership in the CFS. With uniqueness in the phase term terminology, the CFS model is elevated. Membership degree has been utilized as the complex plane where the unit circle consists of additional components diverging for the fuzzy characterization in the CFS module. The importance of intersection in the compound, discussions in the complement, and support of unions comes from the complex parameters associated with the CFS examples for simplification, as discussed by Ramot et al. [29]. The reputation of CFS and their systematic analysis are detailed in a clear sketch by Yazdanbakhsh and Dick [30]. The non-membership degree developed by Alkouri and Salleh [31] connecting the CIFS and CFS database is given as $v(x) = se^{i\beta x}$ and is further transformed to restriction $r + s \leq 1$. In this case, the role of managing periodicity and considering uncertainty are significant factors that boost the knowledge concurrently.

Hesitation in concluding the decision may be reduced with the support of complex-valued truth factors and considering falsity membership degrees for effective discussion. Meanwhile, the application of physical phenomena such as wave functions, problem-solving situations, and resistance in the electrical sector significantly express uncertainty. Even though CIFS and CFS work together concept-wise in the field of cylindrical extensions, followed by projections and distant measurements, the individuality differs in the part of introducing a group of phase words, as CFS will always have one more additional term. The concept of IFG and its extensive application and evidence have been cited [32–35]. The recent research work by Asima Razzaque [36] in the area of TIFGs reveals solutions to complicated problems with the support of multiple factors considering the physical situations. Under a fundamental set of working conditions, the ideas of homomorphism and isomorphism were experimented with. Specifically, the work proposed a systematic view of the elimination of poverty in an identified community. In their new definition of SA, Shao et al. [37] discussed how applications were used in water supply systems and the degree of connectedness in IFGs. The first of its kind, the new video processing algorithm by Chen, Zhihua et al. [38] uses temporal intuitionistic fuzzy sets to enhance films via suitable examples in interval-valued IFGs and introduce some of the specific concepts in Xiaoli Qiang et al. [39], such as covering, matching, and paired domination via strong arc [40,41]. Interval-valued (S, T)-fuzzy graphs that are regular and fully regular were first conceptualized by Rashmanlou and Borzooei. Talebi, Kosari, and Shi et al. model challenges associated with this network and provide solutions to some of them, including the neutrality state [42], thus extending the energy idea on the fuzzy graph picture. Guan et al. [43] examine domination in intuitionistic fuzzy directed graphs (IFDGs) to discover dominant nodes with implications for social networks and network security. Imran et al. [44] create novel operations on intuitionistic fuzzy graphs that use the Sombor index to improve internet routing efficiency and reliability.

Sakander Hayat et al. [45] present two novel temperature-based topological indices for predicting the physicochemical features of polycyclic aromatic hydrocarbons, which apply to silicon carbide nanotubes. Sakander Hayat et al. use eigenvalue-based graphical indices to forecast the thermodynamic parameters of polycyclic aromatic hydrocarbons, with an emphasis on polyacenes [46]. Sakander Hayat et al. compare temperature-based graphical indicators to determine total π-electron energy in benzenoid hydrocarbons [47]. Sakander Hayat et al. [48] investigate structure-property modeling using temperature-based topological indices to predict the thermodynamic characteristics of benzenoid hydrocarbons. Sakander Hayat [49] uses distance-based graphical indices to forecast the thermodynamic parameters of benzene hydrocarbons, displaying a variety of applications. Sakander Hayat et al. investigate the statistical importance of valency-based topological descriptors in predicting the thermodynamic parameters of benzenoid hydrocarbons [50].

In this study, the CTIFG outperforms IFSs in handling ambiguity and uncertainty. This tactic is beneficial since it provides an adaptable approach to managing the ambiguity and uncertainty that accompany decision-making. Due to their efforts to bridge the gap between classical computational models used in technology and the sciences and symbolic models used in expert systems, complicated ITF models are becoming more and more valuable.

The CTIFG theory is a useful instrument for defining and elucidating difficult and ambiguous issues that emerge in real-world situations. This phenomenon can be explained by its capacity to convey the innate qualities of complexity, imprecision, ambiguity, and predictableness that are associated with the items that fall under these categories. However, in order to address the actual issues pertaining to membership and non-membership functions, these approaches must be rewritten using particular numerical values. We introduced the idea of a CTIFG, which makes use of the linear t-norm and t-conform operators, to get around this restriction. The implementation of the CTIFG was prompted by the need for an organized and flexible technique to manage ambiguity and facilitate decision-making under the direction of established parameters.

Here, the use of the parameter "t" makes the process easier to understand by defining specific standards for determining the level of membership or non-membership. It becomes necessary to base decisions on varying degrees of confidence in several real-life scenarios. The goal of adding the parameter "t" to the CTIFG is to get around the IFG's limitations. This parameter provides distinct thresholds for decision-making, improves customization, decreases uncertainty, and gives exact control over stringency. Because of the aforementioned advantages, the CTIFG is an extremely useful method for illustrating uncertainty and assisting in informed decision-making in situations where a customized and controlled approach to uncertainty management is required. When classical IFG is insufficient, the CTIFG makes complicated decision environments easier to grasp and manipulate.

In the context of difficult decision-making, the impact of complex, ambiguous relationships cannot be overstated. These graphs provide decision-makers with strong tools for evaluating and analyzing various options by providing a detailed description of the intricate interaction between input and output factors. Decision-makers can determine choices fully and systematically by taking into account several criteria and their interdependencies, thanks to complex fuzzy connections. This makes it easier to handle complex decision-making challenges holistically. Decision-making has advanced significantly with the help of this sophisticated technique, especially when complex membership, non-membership, and parameter t are present. It opens the door for improved decision-making precision and denotes a departure from the constraints of binary logic.

1.4. Motivation for the Research

- The main reason for employing CTIFGs is their ability to manage complex and uncertain situations with hesitant and variable elemental interactions.
- These graphs, which incorporate the "t" parameter, provide a framework for evaluating and simulating various degrees of connection confidence and uncertainty.
- A technique for managing the conjunction and disjunction of uncertain information is provided by including t-norms and t-conforms. This approach is specifically intended for decision-making scenarios involving a variety of inputs along with results in real-world scenarios.

- This methodology is utilized in various domains, such as risk assessment, decision analysis, and systems optimization, where the aim is to attain a trade-off between pragmatic utility and unresolved relationships.

1.5. Novelties of the Work

- The "t" parameter is a threshold of reluctance that allows for the formation of a new, structured representation of ambiguous connections.
- Adding the "t" option can improve the representation of interactions in which choosing nodes and their edges is contingent upon adhering to a protective confidence level.
- This approach would enable a more methodical treatment of ambiguity by providing a more exact difference between effective and sensitive relationships.
- Multi-layered analysis, in which distinct graph layers are linked to different parameter values "t", is possible with a CTIFG. Using this method would allow for a complete analysis of the graph's relationships while accounting for varying levels of assurance. It makes the fundamental framework easier to comprehend.

1.6. Primary Goals for This Article Are to Make the Following Contributions

- Propose the idea of the CTIFG. This phenomenon is advantageous in that it offers a flexible paradigm for describing the uncertainty and ambiguity inherent in decision-making. Moreover, it plays a significant role in various disciplines such as computer science, economics, chemistry, medicine, and engineering.
- Examine and demonstrate several important characteristics of the recently defined CTIFG set theory procedures. These functions make it possible to integrate data, investigate relationships, and support well-informed decision-making in a variety of application domains.
- Explain what homomorphism and isomorphism of CTIFG mean, and give examples of some recently defined important characteristics. This idea is utilized to make conducting comparative analysis and data transmission more comfortable in situations where there are uncertain and hesitant graph topologies.
- Introduce the concept of the complement of a CTIFG and demonstrate several essential aspects of this approach. The concept of uncertainty highlights inverse linkages that the initial graph might not have made clear. Applications for this technology include analyzing decisions, network verification, and error detection.
- Utilizing the recently defined technique, determine the essential elements for mitigating poverty within a certain community. By strengthening representation, identifying vulnerable populations, assigning resources, monitoring and assessing results, and developing thoughtful policies, this strategy will aid in the reduction of rubber processing industrial wastewater.
- Examines the complexities and uncertainties surrounding industrial wastewater, culminating in an evaluation of its roots, evolution, and effects.

1.7. Strengths and Weaknesses

Finally, while complex t-intuitionistic fuzzy networks provide an effective framework for representing ambiguity and uncertainty in graph theory, they also present computational difficulties and necessitate meticulous parameter adjustment for best interpretation and performance. Their strengths are their strong mathematical foundation and their capacity to deal with a variety of uncertainty types; their shortcomings are their increased computing complexity and interpretability problems.

1.8. Structure of the Paper

A quick overview of the CTIFG (see Table 1) is followed by the remainder of the article, which is organized as follows: The "Preliminaries of CTIFGs" section offers some basic definitions to aid the reader in understanding the uniqueness of the work that is described in this article. Numerous set-theoretical operations of CTIFGs are examined and illustrated graphically in the section "Operations on CTIFGs". The definitions of homomorphisms and isomorphisms of CTIFGs are given in the section "Isomorphism of CTIFGs". The freshly defined approach is used to create a mechanism for lowering poverty in a particular society in the "Application of CTIFGs" section. The final portions of the paper are titled "Comparative analysis" and "Conclusion", respectively, and they include some comparative analysis and specific findings.

Table 1. The list of abbreviations used in this article is shown in the table below.

IFS	Intuitionistic Fuzzy Set
TIFS	t-Intuitionistic Fuzzy Set
CIFS	Complex Intuitionistic Fuzzy Set
IFG	Intuitionistic Fuzzy Graph
CTISs	Complex t-Intuitionistic Subset
CTIFGs	Complex t-Intuitionistic Fuzzy Graphs

2. Preliminaries CTIFGs

Definition 1. *Given a universal set U, let G be its intuitionistic fuzzy set (IFS) and t∈ [0, 1]. Known as a t-intuitionistic fuzzy set (TIFS), the* $IFS_{\breve{G}_t}$ *of U is defined as* $\mu_{\breve{G}_t}(\upsilon_1) = \wedge\{\mu_{\breve{G}_t}(\upsilon_1), t\}$, *and* $\rho_{\breve{G}_t}(\upsilon_1) = \vee\{\rho_{\breve{G}_t}(\upsilon_1), 1-t\}, \forall\, \upsilon_1 \in U$. *The form of TIFS is* $\breve{G}_t = \{\upsilon_1, \mu_{\breve{G}_t}(\upsilon_1), \rho_{\breve{G}_t}(\upsilon_1) : \upsilon_1 \in U\}$ *where* $\mu_{\breve{G}_t}$ *and* $\rho_{\breve{G}_t}$ *are functions that assign a degree of truth membership and falsity membership, respectively. Moreover, the functions* $\mu_{\breve{G}_t}$ *and* $\rho_{\breve{G}_t}$ *satisfy the condition* $0 \le \mu_{\breve{G}_t}(\upsilon_1) + \rho_{\breve{G}_t}(\upsilon_1) \le 1$.

Definition 2. *A complex intuitionistic fuzzy set (CIFS) A, defined on a universe of discourse X is an objective of the form* $A = \left\{\upsilon_1, \mu_{\breve{G}_{At}}(\upsilon_1)e^{i\mu\varpi_{\breve{G}_{At}}(\upsilon_1)}, \rho_{\breve{G}_{At}}(\upsilon_1)e^{i\rho\varsigma_{\breve{G}_t}(\upsilon_1)}\right\}$, *here* $i = \sqrt{-1}$, $(\mu_{\breve{G}_{At}}(\upsilon_1), \rho_{\breve{G}_{At}}(\upsilon_1)) \in [0,1], 0 \le \mu\varpi_{\breve{G}_{At}}(\upsilon_1), \rho\varsigma_{\breve{G}_{At}}(\upsilon_1) \le 2\pi$.

Definition 3. *For a given simple graph* $\mathcal{G}$ *= (V,E), let* $\breve{G}_t = (A_t, B_t)$ *be a t-intuitionistic fuzzy graph (TIFG). The notation* $\breve{G}_t = (A_t, B_t)$ *denotes a CTIFG, where* $A_t = \left\{(\upsilon_i, \mu_{\breve{G}_{At}}(\upsilon_i)e^{i\mu\varpi_{\breve{G}_{At}}(\upsilon_i)}, \rho_{\breve{G}_{At}}(\upsilon_i)e^{i\rho\varsigma_{\breve{G}_{At}}(\upsilon_i)}) : \upsilon_i \in V\right\}$ *is a CTIFS on V and* $B_t = \left\{\left((\upsilon_i, \upsilon_j), \mu_{\breve{G}_{Bt}}(\upsilon_i, \upsilon_j)e^{i\mu\varpi_{\breve{G}_{Bt}}(\upsilon_i, \upsilon_j)}, \rho_{\breve{G}_{Bt}}(\upsilon_i, \upsilon_j)e^{i\rho\varsigma_{\breve{G}_{Bt}}(\upsilon_i, \upsilon_j)}\right) : (\upsilon_i, \upsilon_j) \in E\right\}$ *is a CTIFS on* $E \subseteq V \times V$, *such that* $\forall(\upsilon_i, \upsilon_j) \in E$.

$$\mu_{\breve{G}_{At}}(\upsilon_i, \upsilon_j)e^{i\mu\varpi_{\breve{G}_{At}}(\upsilon_i, \upsilon_j)} \le \wedge\{\mu_{\breve{G}_{At}}(\upsilon_i), \mu_{\breve{G}_{At}}(\upsilon_j)\}e^{i\wedge\{\mu\varpi_{\breve{G}_{At}}(\upsilon_i), \mu\varpi_{\breve{G}_{At}}(\upsilon_j)\}}$$

$$\rho_{\breve{G}_{Bt}}(\upsilon_i, \upsilon_j)e^{i\rho\varsigma_{\breve{G}_{Bt}}(\upsilon_i, \upsilon_j)} \le \vee\{\rho_{\breve{G}_{Bt}}(\upsilon_i), \rho_{\breve{G}_{Bt}}(\upsilon_j)\}e^{i\vee\{\rho\varsigma_{\breve{G}_{Bt}}(\upsilon_i), \rho\varsigma_{\breve{G}_{Bt}}(\upsilon_j)\}}$$

for all $\upsilon_i, \upsilon_j \in V$.

Example 1. *Examine the* $G' = (V, E)$ *in which* $V = \{a, b, c, d\}$ *and* $E = \{ab, bc, cd, da\}$. *Let A be a complex t-intuitionistic subset (CTIS) of V and B be a CTIS of* $E \subseteq V \times V$, *as given at* $t = 0.80$ *in Figure 1.*

$$A_{0.80} = \left\{\begin{matrix} (a, 0.6\, e^{i\,0.5\pi}, 0.4e^{i\,0.5\pi}) \\ (b, 0.7e^{i0.4\pi}, 0.3e^{i0.7\pi}), \\ (c, 0.8e^{i0.4\pi}, 0.2e^{i0.3\pi}) \\ (d, 0.6e^{i0.3\pi}, 0.4e^{i0.4\pi}) \end{matrix}\right\}$$

and

$$Ɓ_{0.8} = \begin{Bmatrix} (ab, 0.6e^{i0.4\pi}, 0.4e^{i0.7\pi}) \\ (bc, 0.7e^{i0.4\pi}, 0.3e^{i0.7\pi}) \\ (cd, 0.6e^{i0.3\pi}, 0.4e^{i0.4\pi}) \\ (da, 0.5e^{i0.2\pi}, 0.3e^{i0.5\pi}) \end{Bmatrix}$$

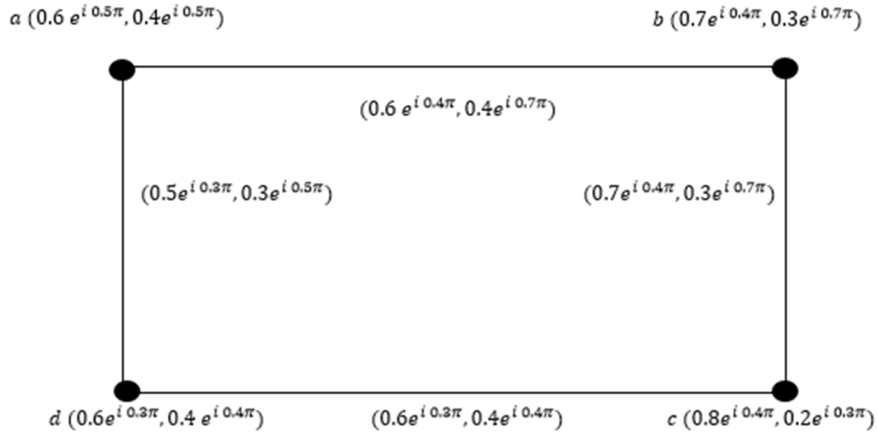

Figure 1. CTIFG*,* where $\mathcal{G}_{0.8}$.

Definition 4. *Let* $Ğ_t = (Ą_t, Ɓ_t)$ *be a CTIFG. Then* $\mathcal{H}_t = (Ą'_t, Ɓ'_t)$ *is considered a complex intuitionistic subgraph (CTISG) if* $Ą'_t \subseteq Ą_t$ *and* $Ɓ'_t \subseteq Ɓ_t$.

Definition 5. *A CTIFG* $Ğ_t = (Ą_t, Ɓ_t)$ *is termed a complete CTIFG if it satisfies the following conditions:*

$$\mu_{Ğ_{Ɓt}}(\upsilon_1, \upsilon_2)e^{i\mu\varpi_{Ğ_{Ɓt}}(\upsilon_1,\upsilon_2)} = \wedge\left\{\mu_{Ğ_{Ąt}}(\upsilon_1), \mu_{Ğ_{Ąt}}(\upsilon_2)\right\}e^{i\wedge\left\{\mu\varpi_{Ğ_{Ąt}}(\upsilon_1),\mu\varpi_{Ğ_{Ąt}}(\upsilon_2)\right\}}$$

$$\rho_{Ğ_{Ɓt}}(\upsilon_1, \upsilon_2)e^{i\rho\varsigma_{Ğ_{Ɓt}}(\upsilon_1,\upsilon_2)} = \vee\left\{\rho_{Ğ_{Ąt}}(\upsilon_1), \rho_{Ğ_{Ąt}}(\upsilon_2)\right\}\ e^{i\vee\left\{\rho\varsigma_{Ğ_{Ąt}}(\upsilon_1),\rho\varsigma_{Ğ_{Ąt}}(\upsilon_2)\right\}}, \forall\ (\upsilon_1, \upsilon_2) \in E.$$

Example 2. *Figure 2 illustrates the entire -CTIFG* $Ğ_{0.7}$.

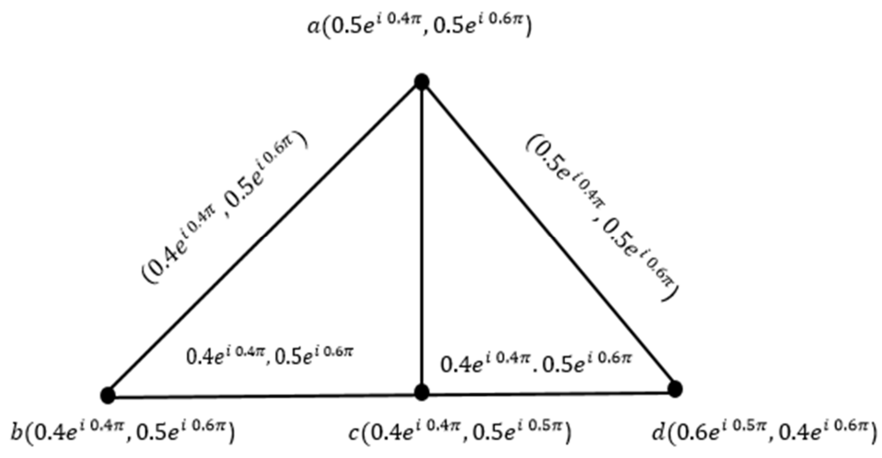

Figure 2. CTIFG $Ğ_{0.7}$.

Definition 6. *In CTIFGs, the order is defined as follows*

$$O(\breve{G}_t) = \begin{pmatrix} \sum_{\upsilon_1 \in V} \mu_{\breve{G}_{At}}(\upsilon_1)\, e^{i \sum_{\upsilon_1 \in V} \mu\varpi_{\breve{G}_{At}}(\upsilon_1)} \\ , \\ \sum_{\upsilon_1 \in V} \rho_{\breve{G}_{At}}(\upsilon_1)\, e^{i \sum_{\upsilon_1 \in V} \rho\varsigma_{\breve{G}_{At}}(\upsilon_1)} \end{pmatrix}$$

Example 3. *The order of the CTIFG* $\breve{G}_t$ *is* $(2.7e^{i1.6\pi}, 1.3e^{i1.9\pi})$ *from Example 1.*

Definition 7. *The CTIFG has a size defined by*

$$S(\breve{G}_t) = \begin{pmatrix} \sum_{(\upsilon_1,\upsilon_2) \in E} \mu_{\breve{G}_{At}}(\upsilon_1, \upsilon_2) e^{i\, \mu\varpi_{\breve{G}_{At}}(\upsilon_1,\upsilon_2)} \\ , \\ \sum_{(\upsilon_1,\upsilon_2) \in E} \rho_{\breve{G}_{At}}(\upsilon_1, \upsilon_2) e^{i\, \rho\varsigma_{\breve{G}_{At}}(\upsilon_1,\upsilon_2)} \end{pmatrix}$$

Definition 8. *In CTIFGs, the degree of vertex* υ_1 *in* $\breve{G}_t$ *is defined as follows,*

1. $deg_{\breve{G}_t}(\upsilon_1) = \left(deg_{\mu_{\breve{G}_{Bt}}}(\upsilon_1), deg_{\rho_{\breve{G}_{Bt}}}(\upsilon_1) \right)$

$$deg_{\breve{G}_t}(\upsilon_1) = \begin{pmatrix} \sum_{(\upsilon_1,\upsilon_2) \in E} \mu_{\breve{G}_{Bt}}(\upsilon_1, \upsilon_2) e^{i\, \mu\varpi_{\breve{G}_{Bt}}(\upsilon_1,\upsilon_2)} \\ , \\ \sum_{(\upsilon_1,\upsilon_2) \in E} \rho_{\breve{G}_{Bt}}(\upsilon_1, \upsilon_2)\, e^{i\, \rho\varsigma_{\breve{G}_{Bt}}(\upsilon_1,\upsilon_2)} \end{pmatrix}$$

2. *The minimum degree* $\delta(\breve{G}_t)$ *of CTIFG is given by*

$$\delta(\breve{G}_t) = \left(\delta_{\mu_{\breve{G}_{Bt}}}(\breve{G}_t) e^{i(\delta_{\mu\varpi_{\breve{G}_{Bt}}}(\breve{G}_t)}, \delta_{\rho_{\breve{G}_{Bt}}}(\breve{G}_t) e^{i(\delta_{\rho\varsigma_{\breve{G}_{Bt}}}(\breve{G}_t))} \right)$$

$$\delta(\breve{G}_t) = \begin{pmatrix} \wedge \left\{ deg_{\ \mu_{\breve{G}_{Bt}}}(\upsilon_1) \right\} e^{i \wedge \{deg_{\ \mu\varpi_{\breve{G}_{Bt}}}(\upsilon_1)\}} \\ , \\ \wedge \left\{ deg_{\ \rho_{\breve{G}_{Bt}}}(\upsilon_1) \right\} e^{i \wedge \left\{deg_{\ \rho\varsigma_{\breve{G}_{Bt}}}(\upsilon_1)\right\}} \end{pmatrix} \upsilon_1 \in V$$

3. *The maximum degree* $\Delta(\breve{G}_t)$ *of CTIFG is given by*

$$\Delta(\breve{G}_t) = \left(\Delta_{\mu_{\breve{G}_{Bt}}}(\breve{G}_t) \mathrm{e}^{\mathrm{i}(\Delta_{\mu\varpi_{\breve{G}_{Bt}}}(\breve{G}_t)}, \Delta_{\rho_{\breve{G}_{Bt}}}(\breve{G}_t) \mathrm{e}^{\mathrm{i}(\Delta_{\mu\varpi_{\breve{G}_{Bt}}}(\breve{G}_t))} \right)$$

$$\Delta(\breve{G}_t) = \begin{pmatrix} \vee \left\{ \mathrm{deg}_{\mu_{\breve{G}_{Bt}}}(\upsilon_1) \right\} \mathrm{e}^{\mathrm{i} \vee \{\mathrm{deg}_{\ \mu\varpi_{\breve{G}_{Bt}}}(\upsilon_1)\}}, \\ , \\ \vee \left\{ \mathrm{deg}_{\ \rho_{\breve{G}_{Bt}}}(\upsilon_1) \right\} \mathrm{e}^{\mathrm{i} \vee \left\{\mathrm{deg}_{\ \rho_{\breve{G}_{Bt}}}(\upsilon_1)\right\}} \end{pmatrix} \upsilon_1 \in V$$

Example 4. *From Example 1, The degree of a vertex in* $\breve{G}_t$ *is*

$$deg_{\breve{G}_t}(a) = (1.1e^{i0.7\pi}, 0.7e^{i1.2\pi});\ deg_{\breve{G}_t}(b) = (1.3e^{i1.8\pi}, 0.7e^{i1.4\pi});$$

$$deg_{\breve{G}_t}(c) = (1.3e^{i0.7\pi}, 0.7e^{i1.1\pi});\ deg_{\breve{G}_t}(d) = (1.1e^{i0.6\pi}, 0.7e^{i0.9\pi}).$$

3. Operations on CTIFG

3.1. Cartesian Product of CTIFG

Definition 9. *Let* $\breve{G}_t = (A_t, B_t)$ *and* $\breve{G}_t' = (A_t', B_t')$ *be any two CTIFGs of G = (V, E) and G′= (V′, E′), respectively. The Cartesian product* $\breve{G}_t \times \breve{G}_t'$ *of two CTIFG,* $\breve{G}_t$ *and* $\breve{G}_t'$ *is defined by* $(A_t \times A_t', B_t \times B_t')$, *where* $A_t \times A_t'$ *and* $B_t \times B_t'$ *are CTISs on* $V \times V' = \{(\upsilon_1, \omega_1), (\upsilon_2, \omega_2) : \upsilon_1, \upsilon_2 \in V; \omega_1, \omega_2 \in V'\}$ *and* $E \times E' = \{(\upsilon_1, \omega_1), (\upsilon_2, \omega_2) : \upsilon_1 = \upsilon_2, \upsilon_1, \upsilon_2 \in V, (\omega_1, \omega_2) \in E'\}\ U\ \{(\upsilon_1, \omega_1), (\upsilon_2, \omega_2) : \omega_1 = \omega_2, \omega_1, \omega_2 \in V', (\upsilon_1, \upsilon_2) \in E\}\ U\ \{(\upsilon_1, \omega_1), (\upsilon_2, \omega_2) : \omega_1 \neq \omega_2, \upsilon_1 \neq \upsilon_2, (\omega_1, \omega_2) \in E', (\upsilon_1, \upsilon_2) \in E\}$, *respectively, which satisfy the following conditions.*

1. $\forall\ (\upsilon_1, \omega_1) \in V \times V'$

   (a) $$\mu_{\breve{G}_{A_t \times A'_t}}(\upsilon_1, \omega_1)\, e^{i\mu\varpi_{\breve{G}_{A_t \times A'_t}}(\upsilon_1, \omega_1)} = \wedge \left\{\mu_{\breve{G}_{A_t}}(\upsilon_1), \mu_{\breve{G}_{A'_t}}(\omega_1)\right\} e^{i \wedge \left\{\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1),\, \mu\varpi_{\breve{G}_{A'_t}}(\omega_1)\right\}}$$

   (b) $$\rho_{\breve{G}_{A_t \times A'_t}}(\upsilon_1, \omega_1) e^{i\rho\varsigma_{\breve{G}_{A_t \times A'_t}}(\upsilon_1, \omega_1)} = \vee \left\{\rho_{\breve{G}_{A_t}}(\upsilon_1),\, \rho\vartheta_{\breve{G}_{A'_t}}(\omega_1)\right\} e^{i \vee \left\{\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1),\, \rho\varsigma_{\breve{G}_{A'_t}}(\omega_1)\right\}}$$

2. *If* $\upsilon_1 = \upsilon_2$ *and* $\forall\ (\omega_1, \omega_2) \in E'$

   (a) $$\mu_{\breve{G}_{B_t \times B'_t}}\big((\upsilon_1, \omega_1), (\upsilon_2, \omega_2)\big) e^{i\mu\varpi_{\breve{G}_{B_t \times B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))}$$
   $$= \wedge \left\{\mu_{\breve{G}_{A_t}}(\upsilon_1), \mu_{\breve{G}_{B'_t}}(\omega_1, \omega_2)\right\} e^{i \wedge \left\{\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1), \mu\varpi_{\breve{G}_{B'_t}}(\omega_1, \omega_2)\right\}}$$

   (b) $$\rho_{\breve{G}_{B_t \times B'_t}}\big((\upsilon_1, \omega_1), (\upsilon_2, \omega_2)\big) e^{i\rho\varsigma_{\breve{G}_{B_t \times B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))}$$
   $$= \vee \left\{\rho_{\breve{G}_{A_t}}(\upsilon_1), \rho_{\breve{G}_{B'_t}}(\omega_1, \omega_2)\right\} e^{i \vee \left\{\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1), \rho\varsigma_{\breve{G}_{B'_t}}(\omega_1, \omega_2)\right\}}$$

3. *If* $\omega_1 = \omega_2$ *and* $\forall\ (\upsilon_1, \upsilon_2) \in E$

   (a) $$\mu_{\breve{G}_{B_t \times B'_t}}\big((\upsilon_1, \omega_1), (\upsilon_2, \omega_2)\big) e^{i\mu\varpi_{\breve{G}_{B_t \times B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))}$$
   $$= \wedge \left\{\mu_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \mu_{\breve{G}_{A'_t}}(\omega_1)\right\} e^{i \wedge \left\{\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \mu\varpi_{\breve{G}_{A'_t}}(\omega_1)\right\}}$$

   (b) $$\rho_{\breve{G}_{B_t \times B'_t}}\big((\upsilon_1, \omega_1), (\upsilon_2, \omega_2)\big) e^{i\rho\varsigma_{\breve{G}_{B_t \times B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))}$$
   $$= \vee \left\{\rho_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \rho_{\breve{G}_{A'_t}}(\omega_1)\right\} e^{i \vee \left\{\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \rho\varsigma_{\breve{G}_{A'_t}}(\omega_1)\right\}}$$

Example 5. *Figures 3 and 4 illustrate two CTIFGs,* $\breve{G}_t$ *and* $\breve{G}_t'$, *which are the elements of consideration. The Cartesian product* $\breve{G}_{0.6} \times \breve{G}_{0.6}'$, *which corresponds to them, is seen in Figure 5.*

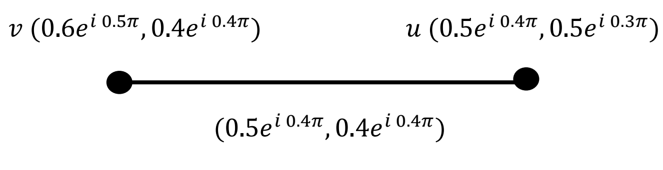

Figure 3. CTIF $\breve{G}_{0.6}$.

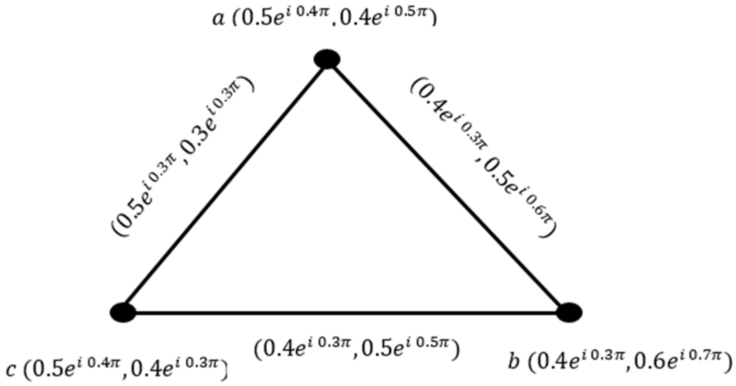

Figure 4. CTIF $\breve{G}'_{0.6}$.

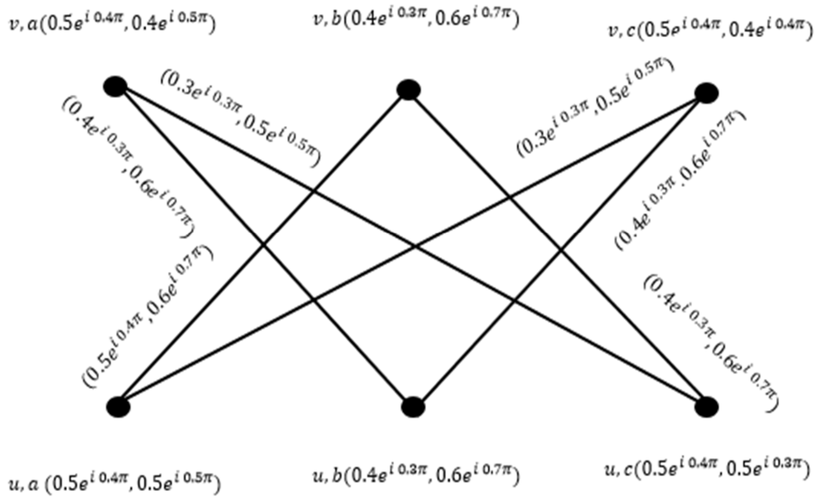

Figure 5. Shows their corresponding Cartesian product $\breve{G}_{0.6}\times\breve{G}_{0.6}{}'$.

Definition 10. *The degree of a vertex in* $\breve{G}_t \times \breve{G}_t{}'$ *is defined as follows: for any* $(\upsilon_1, \omega_1) \in V \times V'$,

$$deg_{\mathcal{G}_t\times\mathcal{G}_t'}(\upsilon_1,\omega_1) = \begin{pmatrix} deg\left\{ɱ_{\breve{G}_{B_t\times B'_t}}\big((\upsilon_1,\omega_1),(\upsilon_2,\omega_2)\big)\right\} e^{iɱ\varpi_{\breve{G}_{B_t\times B'_t}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))}, \\ , \\ deg\left\{ϱ_{\breve{G}_{B_t\times B'_t}}\big((\upsilon_1,\omega_1),(\upsilon_2,\omega_2)\big)\right\} e^{iϱς_{\breve{G}_{B_t\times B'_t}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))} \end{pmatrix}$$

where

$$deg\left\{ɱ_{\breve{G}_{B_t\times B'_t}}\big((\upsilon_1,\omega_1),(\upsilon_2,\omega_2)\big)\right\} e^{iɱ\varpi_{\breve{G}_{B_t\times B'_t}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))}$$

$$= \sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} \wedge\left\{ɱ_{\breve{G}_{A_t}}(\upsilon_1), ɱ_{\breve{G}_{B'_t}}(\omega_1,\omega_2)\right\} e^{i\sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} \wedge\left\{ɱ\varpi_{\breve{G}_{A_t}}(\upsilon_1), ɱ\varpi_{\breve{G}_{B'_t}}(\omega_1,\omega_2)\right\}}$$

$$+ \sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} \wedge\left\{ɱ_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2), ɱ_{\breve{G}_{A_t{}'_t}}(\omega_1)\right\} e^{i\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} \wedge\left\{ɱ\varpi_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2), ɱ\varpi_{\breve{G}_{A_t{}'_t}}(\omega_1)\right\}}$$

and

$$deg\left\{\rho_{\breve{G}_{B_t\times B'_t}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\right\}e^{i\,\rho\varsigma_{\breve{G}_{B_t\times B'_t}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))}$$

$$=\sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'}\vee\left\{\rho_{\breve{G}_{A_t}}(\upsilon_1),\rho_{\breve{G}_{B'_t}}(\omega_1,\omega_2)\right\}e^{i\sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'}\vee\left\{\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1),\rho\varsigma_{\breve{G}_{B'_t}}(\omega_1,\omega_2)\right\}}$$

$$+\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E}\vee\left\{\rho_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2),\rho_{\breve{G}_{A_t{}'_t}}(\omega_1)\right\}e^{i\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E}\vee\left\{\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2),\rho\varsigma_{\breve{G}_{A_t{}'_t}}(\omega_1)\right\}}$$

Theorem 1. *Two CTIFGs are Cartesian products, and the result is another CTIFG.*

Proof. For $A_t\times A'_t$ the condition is obvious. Assuming that $\upsilon_1\in V$ and $(\omega_1,\omega_2)\in E'$ then,

$$\mu_{\breve{G}_{B_t\times B'_t}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_2))e^{i\,\mu\varpi_{\breve{G}_{B_t\times B'_t}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_2))}$$

$$=\wedge\left\{\mu_{\breve{G}_{A_t}}(\upsilon_1),\mu_{\breve{G}_{B'_t}}(\omega_1,\omega_2)\right\}e^{i\wedge\left\{\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1),\mu\varpi_{\breve{G}_{B'_t}}(\omega_1,\omega_2)\right\}}$$

$$\le\wedge\left\{\mu_{\breve{G}_{A_t}}(u_1),\wedge\{\mu_{\breve{G}_{A_t}{}'}(\omega_1),\mu_{\breve{G}_{A_t}{}'}(\omega_2)\}\right\}e^{i\wedge\left\{\mu\varpi_{\breve{G}_{A_t}}(u_1),\wedge\{\mu\varpi_{\breve{G}_{A_t}{}'}(\omega_1),\mu\varpi_{\breve{G}_{A_t}{}'}(\omega_2)\}\right\}}$$

$$\le\wedge\left\{\wedge\left\{\mu_{\breve{G}_{A_t}}(\upsilon_1),\mu_{\breve{G}_{A_t}{}'}(\omega_1)\right\},\wedge\left\{\mu_{\breve{G}_{A_t}}(\upsilon_1),\mu_{\breve{G}_{A_t}{}'}(\omega_2)\right\}\right\}$$

$$e^{i\wedge\left\{\wedge\left\{\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1),\mu\varpi_{\breve{G}_{A_t}{}'}(\omega_1)\right\},\wedge\{\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1),\mu\varpi_{\breve{G}_{A_t}{}'}(\omega_2)\right\}}$$

$$=\wedge\left\{\mu_{\breve{G}_{A_t\times A'_t}}(\upsilon_1,\omega_1),\mu_{\breve{G}_{A_t\times A'_t}}(\omega_1,\omega_2)\right\}e^{i\wedge\left\{\mu\varpi_{\breve{G}_{A_t\times A'_t}}(\upsilon_1,\omega_1),\mu\varpi_{\breve{G}_{A_t\times A'_t}}(\omega_1,\omega_2)\right\}}$$

Consequently,

$$\mu_{\breve{G}_{B_t\times B'_t}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_2))e^{i\,\mu\varpi_{\breve{G}_{B_t\times B'_t}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_2))}$$

$$\le\wedge\left\{\mu_{\breve{G}_{A_t\times A'_t}}(\upsilon_1,\omega_1),\mu_{\breve{G}_{A_t\times A'_t}}(\omega_1,\omega_2)\right\}e^{i\wedge\left\{\mu\varpi_{\breve{G}_{A_t\times A'_t}}(\upsilon_1,\omega_1),\mu\varpi_{\breve{G}_{A_t\times A'_t}}(\omega_1,\omega_2)\right\}}$$

if $\upsilon_1\in V$, $(\omega_1,\omega_2)\in E'$. Similarly for,

$$\rho_{\breve{G}_{B_t\times B'_t}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_2))e^{i\,\rho\varsigma_{\breve{G}_{B_t\times B'_t}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_2))}$$

$$\le\vee\left\{\rho_{\breve{G}_{A_t\times A'_t}}(\upsilon_1,\omega_1),\rho_{\breve{G}_{A_t\times A'_t}}(\omega_1,\omega_2)\right\}e^{i\vee\left\{\rho\varsigma_{\breve{G}_{A_t\times A'_t}}(\upsilon_1,\omega_1),\rho\varsigma_{\breve{G}_{A_t\times A'_t}}(\omega_1,\omega_2)\right\}}$$

if $\upsilon_1\in V$, $(\omega_1,\omega_2)\in E'$. Likewise, we can demonstrate it for $w_1\in V'$,$(\upsilon_1,\upsilon_2)\in E$. □

3.2. Composistion of CTIFG

Definition 11. *The composition* $\breve{G}_t\circ\breve{G}_t{}'$ *of two CTIFGs,* $\breve{G}_t$ *and* $\breve{G}_t{}'$*, is a CTIFG and defined as a pair* $(A_t\circ A'_t,\ B_t\circ B'_t)$ *. Where* $(A_t\circ A'_t)$ *and* $(B_t\circ B'_t)$ *are CTISs on* $V\times V'=\{(\upsilon_1,\omega_1),(\upsilon_2,\omega_2):\upsilon_1,\upsilon_2\in V;\omega_1,\omega_2\in V'\}$ *and* $E\times E'=\{(\upsilon_1,\omega_1),(\upsilon_2,\omega_2):\upsilon_1=\upsilon_2,\upsilon_1,\upsilon_2\in V,(\omega_1,\omega_2)\in E'\}\ U\ \{(\upsilon_1,\omega_1),(\upsilon_2,\omega_2):\omega_1=\omega_2,\omega_1,\omega_2\in V',(\upsilon_1,\upsilon_2)\in E\}\ U\ \{(\upsilon_1,\omega_1),(\upsilon_2,\omega_2):\omega_1\ne\omega_2,\upsilon_1\ne\upsilon_2,(\omega_1,\omega_2)\in E',(\upsilon_1,\upsilon_2)\in E\}$*, respectively, which satisfies the following condition*

1. $\forall((\upsilon_1,\omega_1)\in V\circ V'$

(a) $\mu_{\breve{G}_{A_t \circ A'_t}}(\upsilon_1, \omega_1) e^{i\, \mu\varpi_{\breve{G}_{A_t \circ A'_t}}(\upsilon_1, \omega_1)} = \wedge\left\{\mu_{\breve{G}_{A_t}}(\upsilon_1), \mu_{\breve{G}_{A'_t}}(\omega_1)\right\} e^{i \wedge\left\{\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1), \mu\varpi_{\breve{G}_{A'_t}}(\omega_1)\right\}}$

(b) $\varrho_{\breve{G}_{A_t \circ A'_t}}(\upsilon_1, \omega_1) e^{i\, \varrho\varsigma_{\breve{G}_{A_t \circ A'_t}}(\upsilon_1, \omega_1)} = \vee\left\{\varrho_{\breve{G}_{A_t}}(\upsilon_1), \varrho_{\breve{G}_{A'_t}}(\omega_1)\right\} e^{i \vee\left\{\varrho\varsigma_{\breve{G}_{A_t}}(\upsilon_1), \varrho\varsigma_{\breve{G}_{A'_t}}(\omega_1)\right\}}$

2. *If* $\upsilon_1 = \upsilon_2$ *and* $\forall\ (\omega_1, \omega_2) \in E'$,

(i) $\mu_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2)) e^{i\, \mu\varpi_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))}$

$= \wedge\left\{\mu_{\breve{G}_{A_t}}(\upsilon_1), \mu_{\breve{G}_{B'_t}}(\omega_1, \omega_2)\right\} e^{i \wedge\left\{\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1), \mu\varpi_{\breve{G}_{B'_t}}(\omega_1, \omega_2)\right\}}$

(ii) $\varrho_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2)) e^{i\, \varrho\varsigma_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))}$

$= \vee\left\{\varrho_{\breve{G}_{A_t}}(\upsilon_1), \varrho_{\breve{G}_{B'_t}}(\omega_1, \omega_2)\right\} e^{i \vee\left\{\varrho\varsigma_{\breve{G}_{A_t}}(\upsilon_1), \varrho\varsigma_{\breve{G}_{B'_t}}(\omega_1, \omega_2)\right\}}$

3. *If* $\omega_1 = \omega_2$ *and* $\forall\ (\upsilon_1, \upsilon_2) \in E$,

(a) $\mu_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2)) e^{i \mu\varpi_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))}$

$= \wedge\left\{\mu_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \mu_{\breve{G}_{A'_t}}(\omega_1)\right\} e^{i \wedge\left\{\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \mu\varpi_{\breve{G}_{A'_t}}(\omega_1)\right\}}$

(b) $\varrho_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2)) e^{i \varrho\varsigma_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))}$

$= \vee\left\{\varrho_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \varrho_{\breve{G}_{A'_t}}(\omega_1)\right\} e^{i \vee\left\{\varrho\varsigma_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \varrho\varsigma_{\breve{G}_{A'_t}}(\omega_1)\right\}}$

4. *If* $\omega_1 \neq \omega_2$ *and* $\forall\ (\upsilon_1, \upsilon_2) \in E$,

(a) $\mu_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2)) e^{i\, \mu\varpi_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))}$

$= \wedge\left\{\mu_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \mu_{\breve{G}_{A'_t}}(\omega_1), \mu_{\breve{G}_{A'_t}}(\omega_2)\right\} e^{\wedge i\left\{\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \mu\varpi_{\breve{G}_{A'_t}}(\omega_1), \mu\varpi_{\breve{G}_{A'_t}}(\omega_2)\right\}}$

(b) $\varrho_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2)) e^{i\, \varrho\varsigma_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))}$

$= \vee\left\{\varrho_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \varrho_{\breve{G}_{A'_t}}(\omega_1), \varrho_{\breve{G}_{A'_t}}(\omega_2)\right\} e^{i \vee\left\{\varrho\varsigma_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \varrho\varsigma_{\breve{G}_{A'_t}}(\omega_1), \varrho\varsigma_{\breve{G}_{A'_t}}(\omega_2)\right\}}$

Example 6. *Consider the two CTIFG* $\breve{G}_t$ *and* $\breve{G}_t{}'$ *illustrated in Figure 6. Then, their corresponding composition* $\breve{G}_t{}' \circ \breve{G}_t{}'$ *is in Figure 7.*

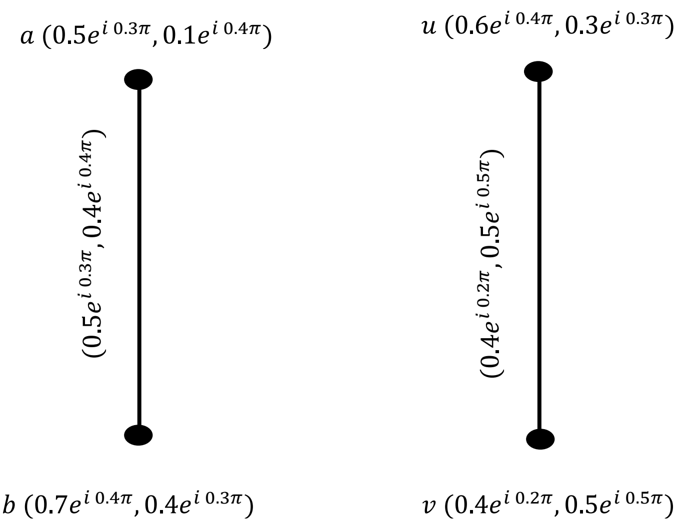

Figure 6. $\breve{G}_{0.7}$ *and* $\breve{G}'_{0.7}$.

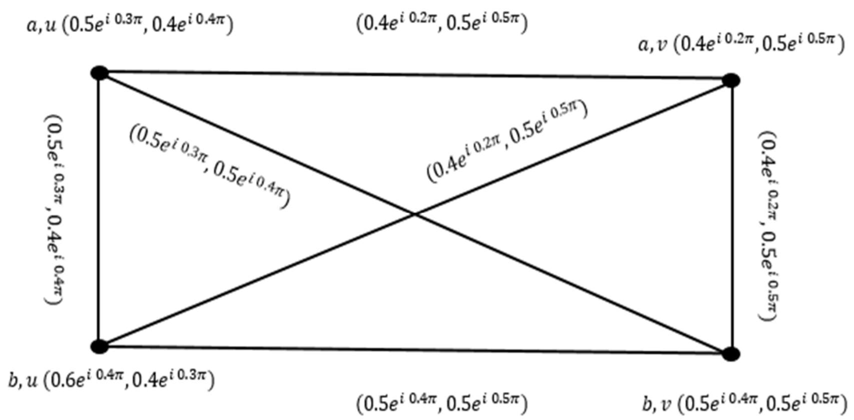

Figure 7. $\breve{G}_{0.7} \circ \breve{G}'_{0.7}$.

Definition 12. *The following defines the degree of a vertex in* $\breve{G}_t \circ \breve{G}_t{}'$ *for any*

$$(\upsilon_1, \omega_1) \in V \times V';\ deg_{\breve{G}_t \circ \breve{G}_t'}(\upsilon_1, \omega_1) = \begin{pmatrix} deg\left\{\mu_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))\right\} e^{i\left\{\mu\varpi_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\right\}}, \\ , \\ deg\left\{\rho_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))\right\} e^{i\,\rho\varsigma_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))}. \end{pmatrix}$$

where

$$deg\left\{\mu_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))\right\} e^{i\left\{\mu\varpi_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\right\}}$$

$$= \sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} \wedge\left\{\mu_{\breve{G}_{A_t}}(\upsilon_1), \mu_{\breve{G}_{B'_t}}(\omega_1, \omega_2)\right\} e^{i\sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} \wedge\left\{\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1),\, \mu\varpi_{\breve{G}_{B'_t}}(\omega_1,\omega_2)\right\}} +$$

$$\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} \wedge\left\{\mu_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \mu_{\breve{G}_{A'_t}}(\omega_1)\right\} e^{i\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} \wedge\left\{\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2),\, \mu\varpi_{\breve{G}_{A'_t}}(\omega_1)\right\}} +$$

$$\sum_{\omega_1\neq\omega_2,(\upsilon_1,\upsilon_2)\in E} \wedge\left\{\mu_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \mu_{\breve{G}_{A'_t}}(\omega_1), \mu_{\breve{G}_{A'_t}}(\omega_2)\right\} e^{i\sum_{\omega_1\neq\omega_2,(\upsilon_1,\upsilon_2)\in E} \wedge\left\{\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2),\, \mu\varpi_{\breve{G}_{A'_t}}(\omega_1),\, \mu\varpi_{\breve{G}_{A'_t}}(\omega_2)\right\}}$$

and

$$deg\left\{\rho_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1, \omega_1), (\upsilon_2, \omega_2))\right\} e^{i\left\{\rho\varsigma_{\breve{G}_{B_t \circ B'_t}}((\upsilon_1,\omega_1),(\upsilon_2,\omega_2))\right\}}$$

$$= \sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} \vee\left\{\rho_{\breve{G}_{A_t}}(\upsilon_1), \rho_{\breve{G}_{B'_t}}(\omega_1, \omega_2)\right\} e^{i\sum_{\upsilon_1=\upsilon_2,(\omega_1,\omega_2)\in E'} \vee\left\{\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1),\, \rho\varsigma_{\breve{G}_{B'_t}}(\omega_1,\omega_2)\right\}}$$

$$+ \sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} \vee\left\{\rho_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \rho_{\breve{G}_{A'_t}}(\omega_1)\right\} e^{i\sum_{\omega_1=\omega_2,(\upsilon_1,\upsilon_2)\in E} \vee\left\{\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2),\rho\varsigma_{\breve{G}_{A'_t}}(\omega_1)\right\}}$$

$$+ \sum_{\omega_1\neq\omega_2,(\upsilon_1,\upsilon_2)\in E} \vee\left\{\rho_{\breve{G}_{B_t}}(\upsilon_1, \upsilon_2), \rho_{\breve{G}_{A'_t}}(\omega_1), \rho_{\breve{G}_{A'_t}}(\omega_2)\right\} e^{i\sum_{\omega_1\neq\omega_2,(\upsilon_1,\upsilon_2)\in E} \vee\left\{\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2),\rho\varsigma_{\breve{G}_{A'_t}}(\omega_1),\rho\varsigma_{\breve{G}_{A'_t}}(\omega_2)\right\}}$$

3.3. Union of CTIFG

Definition 13. *Let G= (V, E) and G′= (V′, E′) be any two CTIFGs, such that* $\breve{G}_t = (A_t, B_t)$ *and* $\breve{G}_t{}' = (A'_t, B'_t)$. *The union* $\breve{G}_t \cup \breve{G}_t{}'$ *of these two CTIFGs is defined, under certain assumptions, as* $(A_t \cup A'_t, B_t \cup B'_t)$, *where* $A_t \cup A'_t$ *and* $B_t \cup B'_t$, *respectively, represent CTIS on* $V \cup V'$ *and* $E \cup E'$, *which satisfies the following condition,*

(1) *If* $\upsilon_1 \in V$ *and* $\upsilon_1 \notin V'$

(a) $\mu_{\breve{G}_{A_t \cup A'_t}}(\upsilon_1) e^{i\,\mu\varpi_{\breve{G}_{A_t \cup A'_t}}(\upsilon_1)} = \mu_{\breve{G}_{A_t}}(\upsilon_1) e^{i\,\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1)}$

(b) $\rho_{\breve{G}_{A_t \cup A'_t}}(\upsilon_1) e^{i\,\rho\varsigma_{\breve{G}_{A_t \cup A'_t}}(\upsilon_1)} = \rho_{\breve{G}_{A_t}}(\upsilon_1) e^{i\,\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1)}$

(2) *If* $\upsilon_1 \notin V$ *and* $\upsilon_1 \in V'$

(a) $\mu_{\breve{G}_{A_t \cup A'_t}}(\upsilon_1) e^{i\,\mu\varpi_{\breve{G}_{A'_t}}(\upsilon_1)} = \mu_{\breve{G}_{A'_t}}(\upsilon_1) e^{i\,\mu\varpi_{\breve{G}_{A'_t}}(\upsilon_1)}$

(b) $\rho_{\breve{G}_{A_t \cup A'_t}}(\upsilon_1) e^{i\,\rho\varsigma_{\breve{G}_{A_t \cup A'_t}}(\upsilon_1)} = \rho_{\breve{G}_{A'_t}}(\upsilon_1) e^{i\,\rho\varsigma_{\breve{G}_{A'_t}}(\upsilon_1)}$

(3) *If* $\upsilon_1 \in V \cap V'$

(a) $\mu_{\breve{G}_{A_t \cup A'_t}}(\upsilon_1) e^{i\,\mu\varpi_{\breve{G}_{A_t \cup A'_t}}(\upsilon_1)} = \vee\{\mu_{\breve{G}_{A_t}}(\upsilon_1), \mu_{\breve{G}_{A'_t}}(\upsilon_1)\} e^{i\,\vee\{\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1), \mu\varpi_{\breve{G}_{A'_t}}(\upsilon_1)\}}$

(b) $\rho_{\breve{G}_{A_t \cup A'_t}}(\upsilon_1)\, e^{i\,\rho\varsigma_{\breve{G}_{A_t \cup A'_t}}(\upsilon_1)} = \wedge\{\rho_{\breve{G}_{A_t}}(\upsilon_1), \rho_{\breve{G}_{A'_t}}(\upsilon_1)\} e^{i\,\wedge\{\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1), \rho\varsigma_{\breve{G}_{A'_t}}(\upsilon_1)\}}$

(4) If $(\upsilon_1, \omega_1) \in E$ and $(\upsilon_1, \omega_1) \notin E'$

(a) $\mu_{\breve{G}_{B_t \cup B'_t}}(\upsilon_1, \omega_1) e^{i\,\mu\varpi_{\breve{G}_{B_t \cup B'_t}}(\upsilon_1,\omega_1)} = \mu_{\breve{G}_{B_t}}(\upsilon_1, \omega_1) e^{i\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)}$

(b) $\rho_{\breve{G}_{B_t \cup B'_t}}(\upsilon_1, \omega_1) e^{i\,\rho\varsigma_{\breve{G}_{B_t \cup B'_t}}(\upsilon_1,\omega_1)} = \rho_{\breve{G}_{B_t}}(\upsilon_1, \omega_1) e^{i\,\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)}$

(5) *If* $(\upsilon_1, \omega_1) \notin E$ *and* $(\upsilon_1, \omega_1) \in E'$

(a) $\mu_{\breve{G}_{B_t \cup B'_t}}(\upsilon_1, \omega_1) e^{i\mu\varpi_{\breve{G}_{B_t \cup B'_t}}(\upsilon_1,\omega_1)} = \mu_{\breve{G}_{B'_t}}(\upsilon_1, \omega_1) e^{i\mu\varpi_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)}$

(b) $\rho_{\breve{G}_{B_t \cup B'_t}}(\upsilon_1, \omega_1) e^{i\rho\varsigma_{\breve{G}_{B_t \cup B'_t}}(\upsilon_1,\omega_1)} = \rho_{\breve{G}_{B'_t}}(\upsilon_1, \omega_1) e^{i\rho\varsigma_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)}$

(6) *If* $(\upsilon_1, \omega_1) \in E \cap E'$

(a) $\mu_{\breve{G}_{B_t \cup B'_t}}(\upsilon_1, \omega_1) e^{i\mu\varpi_{\breve{G}_{B_t \cup B'_t}}(\upsilon_1,\omega_1)}$

$= \vee\{\mu_{\breve{G}_{B_t}}(\upsilon_1, \omega_1), \mu_{\breve{G}_{B'_t}}(\upsilon_1, \omega_1)\} e^{\vee i\{\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1,\omega_1),\, \mu\varpi_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)\}}$

(b) $\rho_{\breve{G}_{B_t \cup B'_t}}(\upsilon_1, \omega_1) e^{i\rho\varsigma_{\breve{G}_{B_t \cup B'_t}}(\upsilon_1,\omega_1)}$

$= \wedge\{\rho_{\breve{G}_{B_t}}(\upsilon_1, \omega_1), \rho_{\breve{G}_{B'_t}}(\upsilon_1, \omega_1)\} e^{\wedge i\{\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\omega_1), \rho\varsigma_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)\}}$

Example 7. *Consider the two* $0.7e^{i0.6\pi}$*-CTIFG* $\breve{G}_t$ *and* $\breve{G}_t{}'$ *shown in Figures 8 and 9. Figure 10 depicts the graphical representation of the union.* $\breve{G}_{0.7} \cup \breve{G}'_{0.7}$ *of two 0.7-CTIFG* $\breve{G}_{0.7}$ *and* $\breve{G}'_{0.7}$.

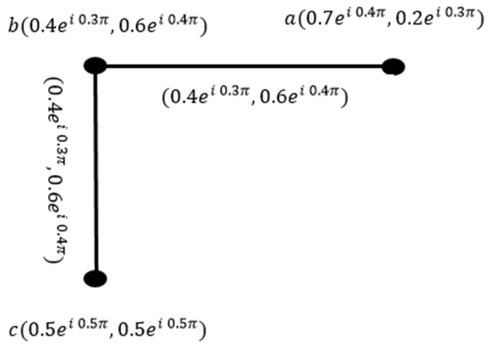

Figure 8. ICTFG $\breve{G}_{0.7}$.

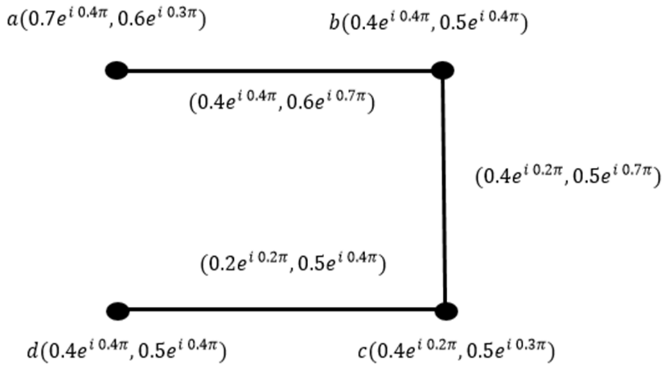

Figure 9. ICTFG $\breve{G}'_{0.7}$.

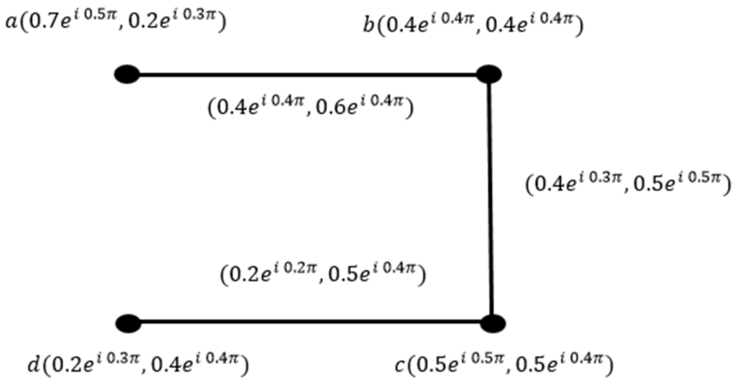

Figure 10. $\breve{G}_{0.7} \cup \breve{G}'_{0.7}$.

Definition 14. *The degree of vertex* (υ_1, ω_1) *at a CTIFG for any* $(\upsilon_1, \omega_1) \in V \times V'$

$$deg_{\breve{G}_{ŧ} \cup \breve{G}_{ŧ}'}(\upsilon_1, \omega_1) = \begin{pmatrix} deg\left\{\mu_{\breve{G}_{B_ŧ \cup B'_ŧ}}(\upsilon_1, \omega_1)\right\} e^{i\left\{\mu\varpi_{\breve{G}_{B_ŧ \cup B'_ŧ}}(\upsilon_1, \omega_1)\right\}}, \\ deg\left\{\rho_{\breve{G}_{B_ŧ \cup B'_ŧ}}(\upsilon_1, \omega_1)\right\} e^{i\left\{\rho\varsigma_{\breve{G}_{B_ŧ \cup B'_ŧ}}(\upsilon_1, \omega_1)\right\}} \end{pmatrix}$$

where

$$deg\left\{\mathrm{M}_{\breve{G}_{B_t\cup B'_t}}(\upsilon_1,\omega_1)e^{i\,\mathrm{M}\varpi_{\breve{G}_{B_t\cup B'_t}}(\upsilon_1,\omega_1)}\right\}$$

$$=\sum\nolimits_{(\upsilon_1,\,\omega_1)\in E\,,(\upsilon_1,\,\omega_1)\notin E'}\mathrm{M}_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)\,e^{i\sum_{(\upsilon_1,\omega_1)\in E\,,(\upsilon_1,\omega_1)\notin E'}\mathrm{M}\varpi_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)}$$

$$+\sum\nolimits_{(\upsilon_1,\,\omega_1)\notin E,(\upsilon_1,\omega_1)\in E'}\mathrm{M}_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)e^{i\sum_{(\upsilon_1,\omega_1)\notin E,(\upsilon_1,\omega_1)\in E'}\mathrm{M}\varpi_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)}$$

$$+\sum\nolimits_{(\upsilon_1,\,\omega_1)\in E\cap E'}\vee\left\{\mathrm{M}_{\breve{G}_{B_t}}(\upsilon_1,\omega_1),\mathrm{M}_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)\right\}e^{i\sum_{(\upsilon_1,\omega_1)\in E\cap E'}\vee\{\mathrm{M}\varpi_{\breve{G}_{B_t}}(\upsilon_1,\omega_1),\,\mathrm{M}\varpi_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)\}},$$

and

$$deg\left\{\rho_{\breve{G}_{B_t\cup B'_t}}(\upsilon_1,\omega_1)e^{i\,\rho\varsigma_{\breve{G}_{B_t\cup B'_t}}(\upsilon_1,\omega_1)}\right\}$$

$$=\sum\nolimits_{(\upsilon_1,\omega_1)\in E\,,(\upsilon_1,\omega_1)\notin E'}\rho_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)\,e^{i\sum_{(\upsilon_1,\omega_1)\in E\,,(\upsilon_1,\omega_1)\notin E'}\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\,\omega_1)}$$

$$+\sum\nolimits_{(\upsilon_1,\omega_1)\notin E,(\upsilon_1,\omega_1)\in E'}\rho_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)e^{i\sum_{(\upsilon_1,\omega_1)\notin E,(\upsilon_1,\omega_1)\in E'}\rho\varsigma_{\breve{G}_{B'_t}}(\upsilon_1,\,\omega_1)}$$

$$+\sum\nolimits_{(\upsilon_1,\omega_1)\in E\cap E'}\wedge\left\{\rho_{\breve{G}_{B_t}}(\upsilon_1,\omega_1),\rho_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)\right\}e^{i\sum_{(\upsilon_1,\omega_1)\in E\cap E'}\wedge\{\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\omega_1),\,\rho\varsigma_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)\}}$$

3.4. Join of CTIFG

Definition 15. *Consider two CTIFGs* $\breve{G}_t=(A_t,B_t)$ *and* $\breve{G}_t{}'=(A'_t,B'_t)$. *The CTIFGs' join operation* $\breve{G}_t+\breve{G}_t{}'$ *is described as* $(A_t+A'_t,B_t+B'_t)$, *where* $A_t+A'_t$ *produces a CTIFG on* $V\cup V'$ *and* $B_t+B'_t$ *forms a CTIFG on* $E\cup E'\cup E''$, *subject to certain requirements.*

(1) *If* $\upsilon_1\in V$ *and* $\upsilon_1\notin V'$

(a) $\mathrm{M}_{\breve{G}_{A_t+A'_t}}(\upsilon_1)e^{i\mathrm{M}\varpi_{\breve{G}_{A_t+A'_t}}(\upsilon_1)}=\mathrm{M}_{\breve{G}_{A_t}}(\upsilon_1)\,e^{i\mathrm{M}\varpi_{\breve{G}_{A_t}}(\upsilon_1)}$

(b) $\rho_{\breve{G}_{A_t+A'_t}}(\upsilon_1)e^{i\rho\varsigma_{\breve{G}_{A_t+A'_t}}(\upsilon_1)}=\rho_{\breve{G}_{A_t}}(\upsilon_1)e^{i\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1)}$

(2) *If* $\upsilon_1\notin V$ *and* $\upsilon_1\in V'$

(a) $\mathrm{M}_{\breve{G}_{A_t+A'_t}}(\upsilon_1)e^{i\mathrm{M}\varpi_{\breve{G}_{A_t+A'_t}}(\upsilon_1)}=\mathrm{M}\vartheta_{A'_t}(\upsilon_1)e^{i\mathrm{M}\varpi_{\breve{G}_{A'_t}}(\upsilon_1)}$

(b) $\rho_{\breve{G}_{A_t+A'_t}}(\upsilon_1)e^{i\rho\varsigma_{\breve{G}_{A_t+A'_t}}(\upsilon_1)}=\rho\vartheta_{A'_t}(\upsilon_1)e^{i\rho\varsigma_{\breve{G}_{A'_t}}(\upsilon_1)}$

(3) *If* $\upsilon_1\in V\cap V'$

(a) $\mathrm{M}_{\breve{G}_{A_t+A'_t}}(\upsilon_1)\,e^{i\mathrm{M}\varpi_{\breve{G}_{A_t+A'_t}}(\upsilon_1)}=\vee\left\{\mathrm{M}_{\breve{G}_{A_t}}(\upsilon_1),\mathrm{M}_{\breve{G}_{A'_t}}(\upsilon_1)\right\}e^{i\vee\{\mathrm{M}\varpi_{\breve{G}_{A_t}}(\upsilon_1),\,\mathrm{M}\varpi_{\breve{G}_{A'_t}}(\upsilon_1)\}}$

(b) $\rho_{\breve{G}_{A_t+A'_t}}(\upsilon_1)e^{i\rho\varsigma_{\breve{G}_{A_t+A'_t}}(\upsilon_1)}=\wedge\left\{\rho_{\breve{G}_{A_t}}(\upsilon_1),\rho_{\breve{G}_{A'_t}}(\upsilon_1)\right\}e^{i\wedge\{\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1),\rho\varsigma_{\breve{G}_{A'_t}}(\upsilon_1)\}}$

(4) *If* $(\upsilon_1,\omega_1)\in E$ *and* $(\upsilon_1,\omega_1)\notin E'$

(a) $\mathrm{M}_{\breve{G}_{B_t+B'_t}}(\upsilon_1,\omega_1)e^{i\,\mathrm{M}\varpi_{\breve{G}_{B_t+B'_t}}(\upsilon_1,\omega_1)}=\mathrm{M}_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)e^{i\,\mathrm{M}\varpi_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)}$

(b) $\rho_{\breve{G}_{B_t+B'_t}}(\upsilon_1,\omega_1)e^{i\,\rho\varsigma_{\breve{G}_{B_t+B'_t}}(\upsilon_1,\omega_1)}=\rho_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)e^{i\,\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)}$

(5) *If* $(\upsilon_1,\omega_1)\notin E$ *and* $(\upsilon_1,\omega_1)\in E'$

(a) $\mathrm{M}_{\breve{G}_{B_t+B'_t}}(\upsilon_1,\omega_1)e^{i\,\mathrm{M}\varpi_{\breve{G}_{B_t+B'_t}}(\upsilon_1,\omega_1)}=\mathrm{M}_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)e^{i\,\mathrm{M}\varpi_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)}$

(b) $\rho_{\breve{G}_{B_t+B'_t}}(\upsilon_1,\omega_1)e^{i\,\rho\varsigma_{\breve{G}_{B_t+B'_t}}(\upsilon_1,\omega_1)}=\rho\vartheta_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)e^{i\,\rho\varsigma_{\breve{G}_{B'_t}}(\upsilon_1,\omega_1)}$

(6) *If* $(\upsilon_1) \in E \cap E'$

(a) $\text{ɱ}_{\breve{G}_{\mathcal{B}_t+\mathcal{B}'_t}}(\upsilon_1,\omega_1)e^{i\,\text{ɱ}\varpi_{\breve{G}_{\mathcal{B}_t+\mathcal{B}'_t}}(\upsilon_1,\omega_1)} = \vee \{\text{ɱ}_{\breve{G}_{\mathcal{B}_t}}(\upsilon_1,\omega_1), \text{ɱ}_{\breve{G}_{\mathcal{B}'_t}}(\upsilon_1,\omega_1)\}e^{i\vee\{\text{ɱ}\varpi_{\breve{G}_{\mathcal{B}_t}}(\upsilon_1,\omega_1),\text{ɱ}\varpi_{\breve{G}_{\mathcal{B}'_t}}(\upsilon_1,\omega_1)\}}$

(b) $\varrho_{\breve{G}_{\mathcal{B}_t+\mathcal{B}'_t}}(\upsilon_1,\omega_1)e^{i\,\varrho\varsigma_{\breve{G}_{\mathcal{B}_t+\mathcal{B}'_t}}(\upsilon_1,\omega_1)} = \wedge \{\varrho_{\breve{G}_{\mathcal{B}_t}}(\upsilon_1,\omega_1), \varrho_{\breve{G}_{\mathcal{B}'_t}}(\upsilon_1,\omega_1)\}e^{i\wedge\{\varrho\varsigma_{\breve{G}_{\mathcal{B}_t}}(\upsilon_1,\omega_1),\,\varrho\varsigma_{\breve{G}_{\mathcal{B}'_t}}(\upsilon_1,\omega_1)\}}$

(7) *If* $(\upsilon_1,\omega_1) \in E''$

(a) $\text{ɱ}_{\breve{G}_{\mathcal{B}_t+\mathcal{B}'_t}}(\upsilon_1,\omega_1)\; e^{i\text{ɱ}\varpi_{\breve{G}_{\mathcal{B}_t+\mathcal{B}'_t}}(\upsilon_1,\omega_1)} = \vee\, \{\text{ɱ}_{\breve{G}_{A_t}}(\upsilon_1), \text{ɱ}_{\breve{G}_{A'_t}}(\omega_1)\}e^{i\,\vee\{\text{ɱ}\varpi_{\breve{G}_{A_t}}(\upsilon_1),\text{ɱ}\varpi_{\text{ɱ}_{\breve{G}_{A'_t}}}(\omega_1)\}}$

(b) $\varrho_{\breve{G}_{\mathcal{B}_t+\mathcal{B}'_t}}(\upsilon_1,\omega_1)e^{i\varrho\varsigma_{\breve{G}_{\mathcal{B}_t+\mathcal{B}'_t}}(\upsilon_1,\omega_1)} = \wedge\, \{\varrho_{\breve{G}_{A_t}}(\upsilon_1), \varrho_{\text{ɱ}_{\breve{G}_{A'_t}}}(\omega_1)\}e^{i\,\wedge\{\varrho\varsigma_{\breve{G}_{A_t}}(\upsilon_1),\,\varrho\varsigma_{\text{ɱ}_{\breve{G}_{A'_t}}}(\omega_1)\}}$

Example 8. *The graphical depiction of CTIFG* $\breve{G}_t + \breve{G}_t'$ *in Figure 11 is from Example 7.*

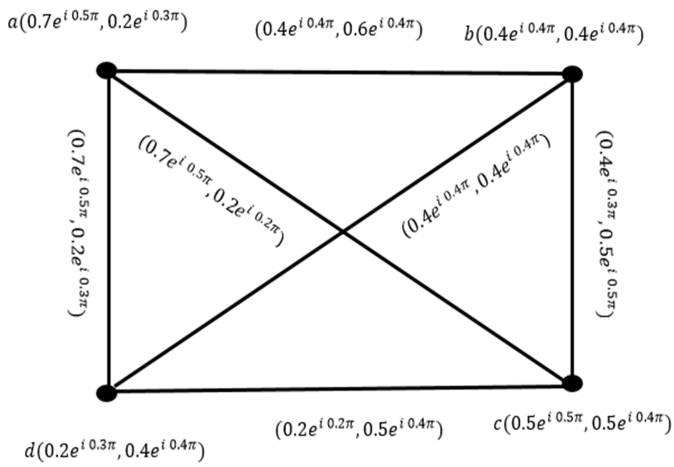

Figure 11. $\breve{G}_{0.7} + \breve{G}_{0.7}'$.

Definition 16. *Consider the following two CTIFGs* $\breve{G}_t$ *and* $\breve{G}_t'$. *The vertex degree in the CTIFG* $\breve{G}_t + \breve{G}_t'$ *is described below. For any* $(\upsilon_1,\omega_1) \in V \times V'$.

$$deg_{\breve{G}_t+\breve{G}_{t'}}(\upsilon_1,\omega_1) = \begin{pmatrix} deg\left\{\text{ɱ}_{\breve{G}_{\mathcal{B}_t+\mathcal{B}'_t}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_1))\right\}e^{i\left\{\text{ɱ}\varpi_{\breve{G}_{\mathcal{B}_t+\mathcal{B}'_t}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_1))\right\}} \\ deg\left\{\varrho_{\breve{G}_{\mathcal{B}_t+\mathcal{B}'_t}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_1))\right\}e^{i\left\{\varrho\varsigma_{\breve{G}_{\mathcal{B}_t+\mathcal{B}'_t}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_1))\right\}} \end{pmatrix}$$

where

$$deg\left\{ \mu_{\breve{G}_{B_t+B'_t}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_1))\right\} e^{i\left\{\mu\varpi_{\breve{G}_{B_t+B'_t}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_1))\right\}}$$

$$= \left(\sum_{\upsilon_1\in V\times V'} \mu_{\breve{G}_{A_t\cup A'_t}}(\upsilon_1)\, e^{i\sum_{\upsilon_1\in V\times V'}\mu\varpi_{\breve{G}_{A_t\cup A'_t}}(\upsilon_1)} + \sum_{\upsilon_1,\omega_1\in E\cap E'} \mu_{\breve{G}_{B_t\cup B'_t}}(\upsilon_1,\omega_1)\, e^{i\sum_{\upsilon_1,\omega_1\in E\cap E'}\mu\varpi_{\breve{G}_{B_t\cup B'_t}}(\upsilon_1,\omega_1)} + \sum_{\upsilon_1,\omega_1\in E''} \vee\left\{\mu_{\breve{G}_{A_t}}(\upsilon_1),\mu_{\breve{G}_{A'_t}}(\omega_1)\right\} e^{i\sum_{(\upsilon_1,\omega_1)\in E''}\vee\{\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1),\mu\varpi_{\breve{G}_{A'_t}}(\omega_1)\}} \right)$$

and

$$deg\left\{ \rho_{\breve{G}_{B_t+B'_t}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_1))\right\} e^{i\left\{\rho\varsigma_{\breve{G}_{B_t+B'_t}}((\upsilon_1,\omega_1),(\upsilon_1,\omega_1))\right\}}$$

$$= \left(\sum_{\upsilon_1\in V\times V'} \rho_{\breve{G}_{A_t\cup A'_t}}(\upsilon_1)\, e^{i\sum_{\upsilon_1\in V\times V'}\rho\varsigma_{\breve{G}_{A_t\cup A'_t}}(\upsilon_1)} + \sum_{\upsilon_1,\omega_1\in E\cap E'} \rho_{\breve{G}_{B_t\cup B'_t}}(\upsilon_1,\omega_1)\, e^{i\sum_{\upsilon_1,\omega_1\in E\cap E'}\rho\varsigma_{\breve{G}_{B_t\cup B'_t}}(\upsilon_1,\omega_1)} + \sum_{\upsilon_1,\omega_1\in E''} \vee\left\{\rho_{\breve{G}_{A_t}}(\upsilon_1),\rho_{\breve{G}_{A'_t}}(\omega_1)\right\} e^{i\sum_{(\upsilon_1,\omega_1)\in E''}\vee\{\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1),\rho\varsigma_{\breve{G}_{A'_t}}(\omega_1)\}} \right)$$

Theorem 2. *For any two CTIFGs* $\breve{G}_t = (A_t, B_t)$ *and* $\breve{G}'_t = (A'_t, B'_t)$ *of G = (V,E) and G′ = (V′,E′), respectively, where* $V \cap V' \neq \emptyset$*, their union* $\breve{G}_t \cup \breve{G}'_t = (A_t \cup A'_t, B_t \cup B'_t)$ *is a CTIFG of G = G∪G′ iff* $\mathcal{G}_t$ *and* $\mathcal{G}'_t$ *are CTIFG of G and G′, respectively.*

Proof. Let us assume a CTIFG, $\breve{G}_t \cup \breve{G}'_t$. Let $(\upsilon_1,\omega_1) \in E, (\upsilon_1,\omega_1) \notin E'$, and $(\upsilon_1,\omega_1) \in V - V'$.

Consider

$$\mu_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)e^{i\,\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)} = \mu_{\breve{G}_{B_t\cap B'_t}}(\upsilon_1,\omega_1)\; e^{i\,\mu\varpi_{\breve{G}_{B_t\cap B'_t}}(\upsilon_1,\omega_1)}$$

$$\leq \wedge\left\{\mu_{\breve{G}_{A_t\cap A'_t}}(\upsilon_1), \mu_{\breve{G}_{A_t\cap A'_t}}(\omega_1)\right\} e^{i\wedge\{\mu\varpi_{\breve{G}_{A_t\cap A'_t}}(\upsilon_1),\mu\varpi_{\breve{G}_{A_t\cap A'_t}}(\omega_1)\}}$$

$$= \wedge\left\{\mu_{\breve{G}_{A_t}}(\upsilon_1), \mu_{\breve{G}_{A_t}}(\omega_1)\right\} e^{i\wedge\{\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1),\mu\varpi_{\breve{G}_{A_t}}(\omega_1)\}}$$

Consequently,

$$\rho_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)e^{i\,\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)} = \rho_{\breve{G}_{B_t\cap B'_t}}(\upsilon_1,\omega_1)\; e^{i\,\rho\varsigma_{\breve{G}_{B_t\cap B'_t}}(\upsilon_1,\omega_1)}$$

$$\leq \vee\left\{\rho_{\breve{G}_{A_t\cap A'_t}}(\upsilon_1), \rho_{\breve{G}_{A_t\cap A'_t}}(\omega_1)\right\} e^{i\vee\{\rho\varsigma_{\breve{G}_{A_t\cap A'_t}}(\upsilon_1),\rho\varsigma_{\breve{G}_{A_t\cap A'_t}}(\omega_1)\}}$$

$$= \vee\left\{\rho_{\breve{G}_{A_t}}(\upsilon_1), \rho_{\breve{G}_{A_t}}(\omega_1)\right\} e^{i\vee\{\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1),\rho\varsigma_{\breve{G}_{A_t}}(\omega_1)\}}$$

This establishes $\breve{G}_t = (A_t, B_t)$ as a CTIFG. Similarly, we conclude that $\breve{G}'_t = (A'_t, B'_t)$ is a CTIFG of G''. Assuming $\breve{G}_t$ and $\breve{G}'_t$ and understanding that the merging of two CTIFGs generates a CTIFG, it follows that $\breve{G}_t \cup \breve{G}'_t$. □

4. Isomorphism of CTIFGs

This section introduces the concepts of homomorphism and isomorphism of CTIFG and explores the essential properties of these ideas.

Definition 17. *Let* $\breve{G}_t = (A_t, B_t)$ *and* $\breve{G}'_t = (A'_t, B'_t)$ *be any two CTIFG of G = (V, E) and G′ = (V′, E′) respectively. A homomorphism* θ *from CTIFG* $\breve{G}'_t$ *and* $\breve{G}'_t$ *is a mapping* $\theta: V \to V'$ *satisfying the following conditions:*

1. $\mu_{\breve{G}_{A_t}}(\upsilon_1)\, e^{i\,\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1)} \leq \mu_{\breve{G}_{A'_t}}(\theta(\upsilon_1))e^{i\,\mu\varpi_{\breve{G}_{A'_t}}(\theta(\upsilon_1))}$
   $\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1)\, e^{i\,\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1)} \leq \rho_{\breve{G}_{A'_t}}(\theta(\upsilon_1))e^{i\,\rho\varsigma_{\breve{G}_{A'_t}}(\theta(\upsilon_1))}$; $\forall \upsilon_1 \in V$.
2. $\mu_{\breve{G}_{B_t}}(\upsilon_1, \omega_1)e^{i\,\mu\varpi_{\rho_{\breve{G}_{B_t}}}(\upsilon_1,\omega_1)} \leq \mu_{\breve{G}_{B'_t}}(\theta(\upsilon_1), \theta(\omega_1))\, e^{i\,\mu\varpi_{\breve{G}_{B'_t}}(\theta(\upsilon_1),\theta(\omega_1))}$,
   $\rho_{\breve{G}_{B_t}}(\upsilon_1, \omega_1)e^{i\,\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)} \leq \rho_{\breve{G}_{B'_t}}(\theta(\upsilon_1), \theta(\omega_1))e^{i\,\rho\varsigma_{\breve{G}_{B'_t}}(\theta(\upsilon_1),\theta(\omega_1))}$; $\forall(\upsilon_1, \omega_1) \in E$.

Definition 18. *A weak isomorphism* $\theta: V \to V'$, *from CTIFG* $\breve{G}_t$ *to* $\breve{G}'_t$ *must meet the following conditions the following condition:*

$$\mu_{\breve{G}_{A_t}}(\upsilon_1)e^{i\,\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1)} \leq \mu_{\breve{G}_{A'_t}}(\theta(\upsilon_1))e^{i\,\mu\varpi_{\breve{G}_{A'_t}}(\theta(\upsilon_1))},$$

$$\rho_{\breve{G}_{A_t}}(\upsilon_1)e^{i\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1)} \leq \rho_{\breve{G}_{A'_t}}(\theta(\upsilon_1))e^{i\,\rho\varsigma_{\breve{G}_{A'_t}}(\theta(\upsilon_1))};\ \forall \upsilon_1 \in V.$$

Definition 19. *A strong co-isomorphism is defined as a bijective mapping* $\theta: V \to V'$ *between any two CTIFGs,* $\breve{G}_t = (A_t, B_t)$ *and* $\breve{G}'_t = (A'_t, B'_t)$ *of G = (V,E) and G′ = (V′,E′), respectively, that meets the following conditions:*

1. $\mu_{\breve{G}_{A_t}}(\upsilon_1)e^{i\,\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1)} \leq \mu_{\breve{G}_{A'_t}}(\theta(\upsilon_1))e^{\mu\varpi_{\breve{G}_{A'_t}}(\theta(\upsilon_1))}$,
   $\rho_{\breve{G}_{A_t}}(\upsilon_1)e^{i\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1)} \leq \rho_{\breve{G}_{A'_t}}(\theta(\upsilon_1))e^{i\,\rho\varsigma_{\breve{G}_{A'_t}}(\theta(\upsilon_1))}$; $\forall \upsilon_1 \in V$.
2. $\mu_{\breve{G}_{B_t}}(\upsilon_1, \omega_1)e^{i\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)} \leq \mu_{\breve{G}_{B'_t}}(\theta(\upsilon_1), \theta(\omega_1))\, e^{i\mu\varpi_{\breve{G}_{B'_t}}(\theta(\upsilon_1),\theta(\omega_1))}$,
   $\rho_{\breve{G}_{B_t}}(\upsilon_1, \omega_1)e^{i\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)} \leq \rho_{\breve{G}_{B'_t}}(\theta(\upsilon_1), \theta(\omega_1))e^{i\rho\varsigma_{\breve{G}_{B'_t}}(\theta(\upsilon_1),\theta(\omega_1))}$, $\forall(\upsilon_1, \omega_1) \in E$.
3. $\mu_{\breve{G}_{B_t}}(\upsilon_1, \omega_1)e^{i\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)} = \mu_{\breve{G}_{B'_t}}(\theta(\upsilon_1), \theta(\omega_1))e^{i\mu\varpi_{\breve{G}_{B'_t}}(\theta(\upsilon_1),\theta(\omega_1))}$,
   $\rho_{\breve{G}_{B_t}}(\upsilon_1, \omega_1)e^{i\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)} = \rho_{\breve{G}_{B'_t}}(\theta(\upsilon_1), \theta(\omega_1))e^{i\rho\varsigma_{\breve{G}_{B'_t}}(\theta(\upsilon_1),\theta(\omega_1))}$, $\forall(\upsilon_1, \omega_1) \in E$.

Definition 20. *An isomorphism between CTIFGs* $\breve{G}_t = (A_t, B_t)$ *and* $\breve{G}'_t = (A'_t, B'_t)$ *is a bijective homomorphism mapping* $\theta: V \to V'$ *(written as* $\breve{G}_t \approx \breve{G}'_t$*) that satisfies the following condition:*

1. $\mu_{\breve{G}_{A_t}}(\upsilon_1)e^{i\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1)} \leq \mu_{\breve{G}_{A'_t}}(\theta(\upsilon_1))e^{\mu\varpi_{\breve{G}_{A'_t}}(\theta(\upsilon_1))}$,

$$\rho_{\breve{G}_{A_t}}(\upsilon_1)e^{i\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1)} \leq \rho_{\breve{G}_{A_t'}}(\theta(\upsilon_1)),\ \forall\upsilon_1 \in V.$$

2. $$\mu_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)e^{i\,\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)} = \mu_{\breve{G}_{B'_t}}(\theta(\upsilon_1),\theta(\omega_1))e^{i\mu\varpi_{\breve{G}_{B'_t}}(\theta(\upsilon_1),\theta(\omega_1))},$$
$$\rho_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)e^{i\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\omega_1)} = \rho_{\breve{G}_{B'_t}}(\theta(\upsilon_1),\theta(\omega_1)), \forall(\upsilon_1,\omega_1) \in E.$$

Example 9. *Take into consideration* $\breve{G}_t$ *and* $\breve{G}'_t$ *as displayed in Figures 12 and 13 based on the following figures.*

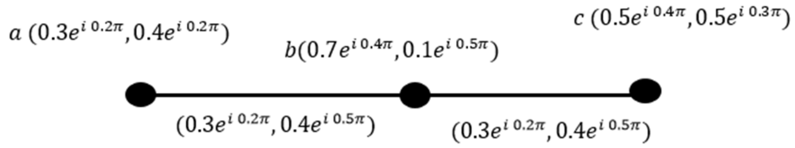

Figure 12. CTIFG $\breve{G}'_{0.6}$.

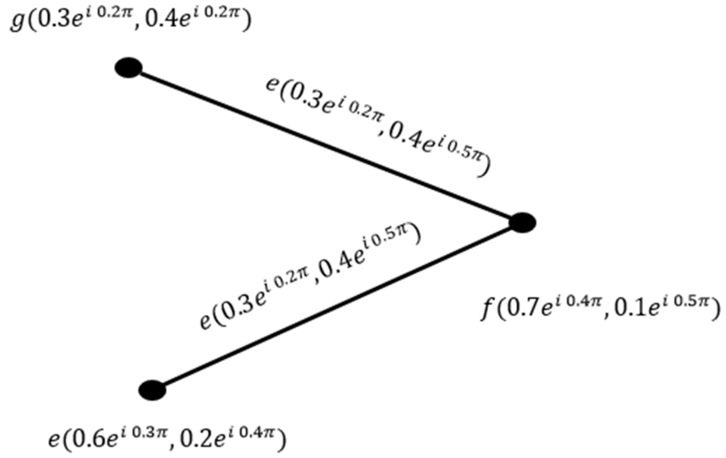

Figure 13. CTIF $\breve{G}'_{0.6}$.

According to definition (20), the mapping ζ(a) = g, ζ(b) = f, and ζ(c) = e gives us $\breve{G}_{0.8} \approx \breve{G}'_{0.8}$.

Theorem 3. *The characteristics of an equivalence relation are satisfied by the connection of isomorphism between CTIFGs.*

Proof. Both symmetry and reflexivity are clear. The isomorphism of $\breve{G}_t$ onto $\breve{G}'_t$ and $\breve{G}'_t$ onto $\breve{G}''_t$, respectively, are denoted by the notations $\varphi: V \to V'$ and $\theta: V' \to V''$. Accordingly, $\theta \circ \varphi: V \to V''$ is a bijective map from V' to V'', and it is defined as follows:

$$(\theta \circ \varphi)(\upsilon_1) = \theta(\varphi(\upsilon_1)), \forall\upsilon_1 \in V.$$

For a map $\varphi: V \to V'$ defined by $\varphi(\upsilon_1) = \omega_1, \forall\upsilon_1 \in V$, it is an isomorphism.

Considering definition (20), we have

$$\mu_{\breve{G}_{A_t}}(\upsilon_1)e^{i\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1)} = \mu_{\breve{G}_{A_t'}}(\varphi(\upsilon_1))e^{i\mu\varpi_{\breve{G}_{A_t'}}(\varphi(\upsilon_1))} = \mu_{\breve{G}_{A_t'}}(\omega_1)e^{i\mu\varpi_{\breve{G}_{A_t'}}(\varphi(\upsilon_1))}, \forall\upsilon_1 \in V \quad (1)$$

$$\rho_{\breve{G}_{A_t}}(\upsilon_1)e^{i\rho\varsigma_{A_t}(\upsilon_1)} = \rho_{\breve{G}_{A_t'}}\big(\varphi(\upsilon_1)\big)e^{i\rho\varsigma_{\breve{G}_{A_t'}}(\varphi(\upsilon_1))} = \rho_{\breve{G}_{A_t'}}(\omega_1)e^{i\rho\varsigma_{\breve{G}_{A_t'}}(\omega_1)}, \forall\upsilon_1 \in V \quad (2)$$

and

$$\begin{aligned}\mu_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)e^{i\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)} &= \mu_{\breve{G}_{B'_t}}\big(\varphi(\upsilon_1),\varphi(\upsilon_2)\big)e^{i\mu\varpi_{\breve{G}_{B'_t}}(\varphi(\upsilon_1),\varphi(\upsilon_2))}\\ &= \mu_{\breve{G}_{B'_t}}(\omega_1,\omega_2)e^{i\mu\varpi_{\breve{G}_{B'_t}}(\omega_1,\omega_2)}, \forall(\upsilon_1,\upsilon_2) \in E\end{aligned} \quad (3)$$

$$\begin{aligned}\rho_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)e^{i\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)} &= \rho_{\breve{G}_{B'_t}}\big(\varphi(\upsilon_1),\varphi(\upsilon_2)\big)e^{i\rho\varsigma_{\breve{G}_{B'_t}}(\varphi(\upsilon_1),\varphi(\upsilon_2))}\\ &= \rho_{\breve{G}_{B'_t}}(\omega_1,\omega_2)e^{i\rho\varsigma_{\breve{G}_{B'_t}}(\omega_1,\omega_2)} \forall(\upsilon_1,\upsilon_2) \in E\end{aligned} \quad (4)$$

In the same way, we obtained that

$$\mu_{\breve{G}_{A_t'}}(\omega_1)e^{i\mu\varpi_{\breve{G}_{A_t'}}(\omega_1)} = \mu_{\breve{G}_{A_t''}}(v_1)e^{i\mu\varpi_{\breve{G}_{A_t''}}(v_1)}, \forall\ \omega_1 \in V' \quad (5)$$

$$\rho_{\breve{G}_{A_t'}}(\omega_1)e^{i\rho\varsigma_{\breve{G}_{A_t'}}(\omega_1)} = \rho_{\breve{G}_{A_t''}}(v_1)e^{i\rho\varsigma_{\breve{G}_{A_t''}}(v_1)}, \forall\ \omega_1 \in V' \quad (6)$$

and

$$\mu_{\breve{G}_{B'_t}}(\omega_1,\omega_2)e^{i\mu\varpi_{\breve{G}_{B'_t}}(\omega_1,\omega_2)} = \mu_{\breve{G}_{B''_t}}(v_1,v_2)e^{i\mu\varpi_{\breve{G}_{B''_t}}(v_1,v_2)}, \forall(\omega_1,\omega_2) \in E' \quad (7)$$

$$\rho_{\breve{G}_{B'_t}}(\omega_1,\omega_2)e^{i\rho\varsigma_{\breve{G}_{B'_t}}(\omega_1,\omega_2)} = \rho_{\breve{G}_{B''_t}}(v_1,v_2)e^{i\rho\varsigma_{\breve{G}_{B''_t}}(v_1,v_2)}, \forall(\omega_1,\omega_2) \in E' \quad (8)$$

By using the relations (1), (3), and $\varphi(\upsilon_1) = \omega_1, \forall\ \upsilon_1 \in V$, we have

$$\begin{aligned}\mu_{\breve{G}_{A_t}}(\upsilon_1)e^{\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1)} &= \mu_{\breve{G}_{A_t'}}\big(\varphi(\upsilon_1)\big)e^{i\mu\varpi_{\breve{G}_{A_t'}}(\varphi(\upsilon_1))}\\ &= \mu_{\breve{G}_{A_t'}}(\omega_1)e^{i\mu\varpi_{\breve{G}_{A_t'}}(\omega_1)}\\ &= \mu_{\breve{G}_{A_t''}}\big(\theta(\omega_1)\big)e^{i\mu\varpi_{\breve{G}_{A_t''}}(\theta(\omega_1))}\\ &= \mu_{\breve{G}_{A_t''}}\Big(\theta\big(\varphi(\upsilon_1)\big)\Big)e^{i\,\mu\varpi_{\breve{G}_{A_t''}}(\theta(\varphi(\upsilon_1)))}\end{aligned}$$

By using the relations (3), (7), and $\varphi(\upsilon_1) = \omega_1, \forall\ \upsilon_1 \in V$, we have

$$\begin{aligned}\rho_{\breve{G}_{A_t}}(\upsilon_1)e^{\rho\varsigma_{\breve{G}_{A_t}}(\upsilon_1)} &= \rho_{\breve{G}_{A_t'}}\big(\varphi(\upsilon_1)\big)e^{i\rho\varsigma_{\breve{G}_{A_t'}}(\varphi(\upsilon_1))}\\ &= \rho_{\breve{G}_{A_t'}}(\omega_1)e^{i\rho\varsigma_{\breve{G}_{A_t'}}(\omega_1)}\\ &= \rho_{\breve{G}_{A_t''}}\big(\theta(\omega_1)\big)e^{i\rho\varsigma_{\breve{G}_{A_t''}}(\theta(\omega_1))}\\ &= \rho_{\breve{G}_{A_t''}}\Big(\theta\big(\varphi(\upsilon_1)\big)\Big)e^{i\,\rho\varsigma_{\breve{G}_{A_t''}}(\theta(\varphi(\upsilon_1)))}\end{aligned}$$

When using the relations (4) and (10), the outcome is

$$\mu_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)e^{i\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)} = \mu_{\breve{G}_{B'_t}}(\omega_1,\omega_2)e^{i\mu\varpi_{\breve{G}_{B'_t}}(\omega_1,\omega_2)}$$

$$= \mu_{\breve{G}_{B''_t}}(\theta(\omega_1),\theta(\omega_2))e^{\mu\varpi_{\breve{G}_{B''_t}}(\theta(\omega_1),\theta(\omega_2))}$$

$$= \mu_{\breve{G}_{B''_t}}\left(\theta(\varphi(\upsilon_1)),\theta(\varphi(\upsilon_2))\right)e^{i\mu\varpi_{\breve{G}_{B''_t}}(\theta(\varphi(\upsilon_1)),\theta(\varphi(\upsilon_2)))}$$

When using the relations (6) and (12), the out come is

$$\rho_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)e^{i\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)} = \rho_{\breve{G}_{B'_t}}(\omega_1,\omega_2)e^{i\rho\varsigma_{\breve{G}_{B'_t}}(\omega_1,\omega_2)}$$

$$= \rho_{\breve{G}_{B''_t}}(\theta(\omega_1),\theta(\omega_2))e^{\rho\varsigma_{\breve{G}_{B''_t}}(\theta(\omega_1),\theta(\omega_2))}$$

$$= \rho_{\breve{G}_{B''_t}}\left(\theta(\varphi(\upsilon_1)),\theta(\varphi(\upsilon_2))\right)e^{i\rho\varsigma_{\breve{G}_{B''_t}}(\theta(\varphi(\upsilon_1)),\theta(\varphi(\upsilon_2)))}$$

Hence, $\breve{G}_t$ and $\breve{G}''_t$ are isomorphic to each other via $\theta \circ \varphi$. As a result, the proof is finished.

□

Theorem 4. *A partial ordering relation between CTIFGs is a weak isomorphism.*

Proof. The reflexivity and transitivity are obvious. To prove anti-symmetry, let $\varphi: V \to V'$ be a strong isomorphism of $\breve{G}_t$ onto $\breve{G}'_t$. Then, φ is a bijective map defined by $\varphi(\upsilon_1) = \upsilon_2$, for all $\upsilon_1 \in V$ satisfying

$$\mu_{\breve{G}_{A_t}}(\upsilon_1)e^{i\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1)} = \mu_{\breve{G}_{A'_t}}(\varphi(\upsilon_1))e^{i\mu\varpi_{\breve{G}_{A'_t}}(\varphi(\upsilon_1))}, \forall \upsilon_1 \in V$$

$$\rho_{\breve{G}_{A_t}}(\upsilon_1)e^{i\rho\varsigma_{A_t}(\upsilon_1)} = \rho_{\breve{G}_{A'_t}}(\varphi(\upsilon_1))e^{i\rho\varsigma_{\breve{G}_{A'_t}}(\varphi(\upsilon_1))}, \forall \upsilon_1 \in V$$

and

$$\mu_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)e^{i\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)} = \mu_{\breve{G}_{B'_t}}(\varphi(\upsilon_1),\varphi(\upsilon_2))e^{i\mu\varpi_{\breve{G}_{B'_t}}(\varphi(\upsilon_1),\varphi(\upsilon_2))}, \forall(\upsilon_1,\upsilon_2) \in E \quad (9)$$

$$\rho_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)e^{i\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)} = \rho_{\breve{G}_{B'_t}}(\varphi(\upsilon_1),\varphi(\upsilon_2))e^{i\rho\varsigma_{\breve{G}_{B'_t}}(\varphi(\upsilon_1),\varphi(\upsilon_2))} \forall(\upsilon_1,\upsilon_2) \in E \quad (10)$$

Let $\theta: V' \to V$ be a strong isomorphism of $\breve{G}_t$ and $\breve{G}'_t$. Then, θ is a bijective map defined by $\theta(\upsilon_2) = \upsilon_1$, for all $\upsilon_2 \in V'$ satisfying

$$\mu_{\breve{G}_{A_t}}(\upsilon_1)e^{i\mu\varpi_{\breve{G}_{A_t}}(\upsilon_1)} = \mu_{\breve{G}_{A'_t}}(\theta(\upsilon_1))e^{i\mu\varpi_{\breve{G}_{A'_t}}(\theta(\upsilon_1))}, \forall \upsilon_2 \in V'$$

$$\rho_{\breve{G}_{A_t}}(\upsilon_1)e^{i\rho\varsigma_{A_t}(\upsilon_1)} = \rho_{\breve{G}_{A'_t}}(\theta(\upsilon_1))e^{i\rho\varsigma_{\breve{G}_{A'_t}}(\theta(\upsilon_1))}, \forall \upsilon_2 \in V'$$

and

$$\mu_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)e^{i\mu\varpi_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)} = \mu_{\breve{G}_{B'_t}}(\theta(\upsilon_1),\theta(\upsilon_2))e^{i\mu\varpi_{\breve{G}_{B'_t}}(\theta(\upsilon_1),\theta(\upsilon_2))}, \forall(\upsilon_1,\upsilon_2) \in E' \quad (11)$$

$$\rho_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)e^{i\rho\varsigma_{\breve{G}_{B_t}}(\upsilon_1,\upsilon_2)} = \rho_{\breve{G}_{B'_t}}(\theta(\upsilon_1),\theta(\upsilon_2))e^{i\rho\varsigma_{\breve{G}_{B'_t}}(\theta(\upsilon_1),\theta(\upsilon_2))} \forall(\upsilon_1,\upsilon_2) \in E' \quad (12)$$

Only in cases when $\breve{G}_t$ and $\breve{G}'_t$ have an equal number of edges and corresponding edges with the same weight do the inequalities (9), (10), (11), and (12) hold on the finite sets V and V'. $\mathcal{G}_t$ and $\mathcal{G}'_t$ are hence the same. As a result, $\breve{G}_t$ and $\breve{G}''_t$ are strongly isomorphized by $\varphi \circ \theta$. The proof is now complete. □

5. Real World Applications of Rubber Industrial Waste Water

This paper focuses on the application of industrial wastewater that has been treated at different parameters using CTIFGs. The major physicochemical parameters that are considered for stabilizing the rubber industrial waste water are turbidity, BOD, and COD, where they play an active role in identifying the pollutant level. Using an easy approach with the support of an intuitionistic fuzzy graph, the performance of the treatment is assessed.

5.1. Experiment Description

In this experiment, the CTIFGs play a significant role in investigating the rubber industrial wastewater parameters. The CTIFGs are incorporated with decision-making support, which enables the evaluation of complicated data sets and typically estimates the importance of effluent parameters such as Biological Oxygen Demand (BOD), Chemical Oxygen Demand (COD), pH, alkalinity, nitrogen, phosphorous, and turbidity.

The CTIFGs in this stage outlined the truth-membership and the falsity-membership roles for the characterization of parameters connected with the degree of alignment or the factors considering divergence.

Findings

1. CTIFGs can initiate the identification of the critical factors that directly contribute to the treatment of rubber industry effluent for fixing the decision-making components in association with complex parameters related to the effluents.
2. CTIFGs can flexibly used to examine the comparison among effluent parameters and give prior importance to interventions that allow the decision-making module to effectively identify more competitive decisions.
3. The parameter that highlights 't' in CTIFGs naturally causes the decision-makers to plot a graph associated with location-specific regional knowledge on effluent and to identify the typical problem associated with the domain, which naturally leads to identifying the optional targeted interventions and simultaneously giving preference for effluent treatment.
4. The visualization of CTIFGs naturally provides a view of the complex relationships interlinked between the various effluent characterizations that are directly contributing to the formation of an effective pollution-free treated effluent that favors a toxic free zone environment.

5.2. Application of CTIFGs in Evaluating the Rubber Industrial Effluents

The pollutants in the synthesis of rubber industry naturally emit a larger volume of wastewater that is naturally treated before being subjected to final disposal in water bodies. The need for rubber and allied products is increasing day by day due to its vast applications. Addressing the mixed composition of waste materials in the rubber-synthesized effluent needs a meticulous treatment strategy that naturally controls the pollutants. Using the CTIFG, the role of decision-making parameters associated with effluent gives a clear picture of how to eliminate the pollutant. In addition to this, the importance of individual physicochemical parameters linked with the effluent is to be evaluated with the support of decision-making strategies, and necessary treatment strategies are also suggested. The important key factor associated with rubber effluent is BOD.

Regarding BOD (P_1), COD (P_2), pH(P_3), alkalinity (P_4), nitrogen (P_5), phosphorous (P_6), and turbidity (P_7), let $\mathcal{M} = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$ be the physicochemical parameters that strongly contribute to the elimination of pollutants in the effluent. The values in each corner stone represents the characteristics of effluent and their linkage are connected with t-intuitionistic values. CTIFGs play an active role in covering the truth-membership factor and falsity-membership functions, which highlight the relationship among positive node and negative node of a single parameter. The parameter 't' permits the decision-making process to give valuable suggestions through the CTIFGs, which will be a focused result for further processing. In addition, it will give an indication of risk factors and the possibility of uncertaininity. This process will give a clear picture through visual representation for an effective pollutant removal strategy. The important factors associated with the work are directly interlinked with truth-membership and falsity-membership, which determine certain features that are connected with the range of confidence level and concern confidence level. The importance of parameter 't' in this stage normally covers the entire circumference and sensitivity parameters. The change in 't' values at consecutive zones in the intuitionistic graphs determines the factor of acceptance of values at one end and simultaneously accepting the rejection of values at the other end. In addition to this, they also cover the factors of risk and uncertainity. During customization with CTIFGs, the role of 't' plays a significant role as it may focus on numerous factors associated with the parameter in a concluding favorable path and, at the same time, consider the opposing point of view. These concepts may be crucial when facing unacceptable effluent discharge. Finalizing the threshold value with the CTIFGs is comfortable in identifying the sensitivity levels of the effluents. Table 2 displays the set of CIF and 0.8-CTIFs connected to the vertices and graphical representation given in Figure 14.

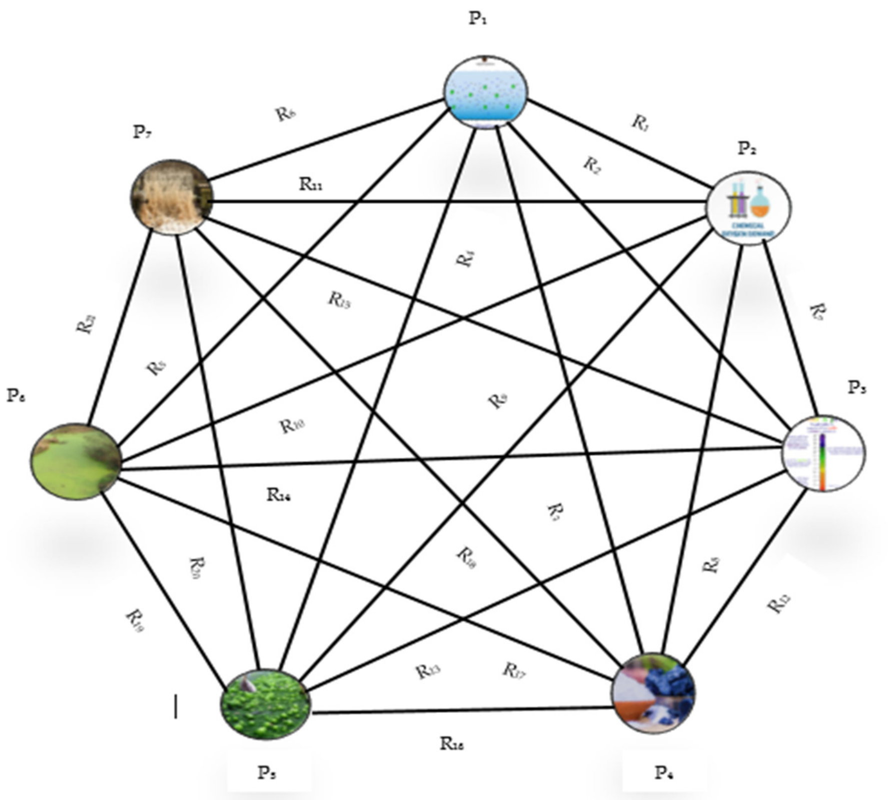

Figure 14. Graphical representation of CTIFGs.

Algorithm to Investigate Rubber Industrial Wastewater Parameters Using CTIFGs

Step 1. Define parameters $\mathcal{M} = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$ (P_1 is BOD, P_2 is COD, P_3 is pH, P_4 is alkalinity, P_5 is nitrogen, P_6 is phosphorus, and P_7 is turbidity). Initialize the CTIFGs framework.
Step 2. Obtain effluent data from rubber industry samples. Preprocess data (manage missing values and outliers). Normalize the data for consistency.
Step 3. Establish truth-membership (t), indeterminacy (i), and falsity-membership (f) for each parameter.
Step 4. Use CTIFGs to determine parameter relevance. Calculate the alignment and divergence for each parameter.
Step 5. Use CTIFGs to compare parameters, prioritize actions, and emphasize the relevance of parameter t when visualizing effluent concerns.
Step 6. Use CTIFGs to visualize parameter correlations, make graphs and charts, detect issues, and recommend targeted remedies.
Step 7. Implement recommended treatment procedures, regularly check efficacy, and adapt tactics in response to data and feedback.

Table 2. Edges of CTIFS.

Edges	Complex 0.8-IFS
R_1	$(0.6e^{i0.4\pi}, 0.3e^{i\,0.3\pi})$
R_2	$(0.8e^{i0.4\pi}, 0.2e^{i\,0.4\pi})$
R_3	$(0.6e^{i0.5\pi}, 0.4e^{i\,0.4\pi})$
R_4	$(0.7e^{i0.4\pi}, 0.3e^{i\,0.4\pi})$
R_5	$(0.8e^{i0.5\pi}, 0.2e^{i\,0.5\pi})$
R_6	$(0.6e^{i0.4\pi}, 0.4e^{i\,0.5\pi})$
R_7	$(0.7e^{i0.4\pi}, 0.3e^{i\,0.4\pi})$
R_8	$(0.6e^{i0.4\pi}, 0.4e^{i\,0.4\pi})$
R_9	$(0.5e^{i0.4\pi}, 0.5e^{i\,0.4\pi})$
R_{10}	$(0.6e^{i0.4\pi}, 0.3e^{i\,0.5\pi})$
R_{11}	$(0.5e^{i0.4\pi}, 0.4e^{i\,0.5\pi})$
R_{12}	$(0.6e^{i0.5\pi}, 0.4e^{i\,0.4\pi})$
R_{13}	$(0.7e^{i0.5\pi}, 0.3e^{i\,0.4\pi})$
R_{14}	$(0.8e^{i0.5\pi}, 0.4e^{i\,0.5\pi})$
R_{15}	$(0.6e^{i0.5\pi}, 0.5e^{i\,0.5\pi})$
R_{16}	$(0.6e^{i0.5\pi}, 0.4e^{i\,0.4\pi})$
R_{17}	$(0.6e^{i0.5\pi}, 0.4e^{i\,0.5\pi})$
R_{18}	$(0.6e^{i0.4\pi}, 0.4e^{i\,0.5\pi})$
R_{19}	$(0.7e^{i0.5\pi}, 0.3e^{i\,0.5\pi})$
R_{20}	$(0.6e^{i0.4\pi}, 0.4e^{i\,0.5\pi})$
R_{21}	$(0.6e^{i0.5\pi}, 0.4e^{i\,0.5\pi})$

Table 2 displays the values identified in the edge R_{13}, which normally connects the physicochemical parameters that determine the removal of pollutants in rubber industrial wastewater. In the assigned framework of the module, the edge $R_{13} = (0.7e^{i0.5\pi}, 0.3e^{i\,0.4\pi})$ and its truth membership value is $0.7e^{i0.5\pi}$ and the corresponding falsity membership is $0.3e^{i\,0.4\pi}$ these values indicate the correlation efforts. The outcome of this module connected with 't' highlights an 80% possible reduction in pollutant removal.

Table 3 focuses on the application part (1) that correlated with the definition (8), and the respective outcomes are described.

Table 3. Table of truth and falsity membership degree of each factor.

Factor	Degree of Each Factor
P_1	$\deg(P_1) = (4.1e^{i\,3.4\pi}, 2.3e^{i\,3.0\pi})$
P_2	$\deg(P_2) = (3.5e^{i\,2.4\pi}, 2.5e^{i\,2.5\pi})$
P_3	$\deg(P_3) = (4.2e^{i\,2.8\pi}, 2.5e^{i\,3.0\pi})$
P_4	$\deg(P_4) = (3.6e^{i\,2.8\pi}, 2.5e^{i\,3.0\pi})$
P_5	$\deg(P_5) = (3.5e^{i\,2.6\pi}, 2.5e^{i\,3.0\pi})$
P_6	$\deg(P_6) = (3.8e^{i\,3.2\pi}, 2.2e^{i\,2.6\pi})$
P_7	$\deg(P_7) = (4.1e^{i\,2.8\pi}, 3.0e^{i\,3.0\pi})$

The score value of each edge has been defined based on the below-mentioned equation.

$$SV = \sqrt{\mu^2 + (1-\rho)^2 + \frac{1}{2\pi}(\varpi^2 + (1-\varsigma)^2)},\ 1 \le j \le 7.$$

With respect to Table 3 and using the scoring function formula, the results are obtained and have been highlighted in Table 4. The values in Table 4 are further considered for the visual depiction that has been represented in Figure 15, which is supported through the score function. Turbidity value $P_7 = 5.1046$ is obtained as the greatest value that naturally highlights the most influential factor for the removal of pollutants in the rubber industrial wastewater that is represented in terms of (parameter 't'). Table 4 clearly illustrates that the respective values associated with the module have a huge impact on treating industrial wastewater.

Table 4. Score value of CTIFGs.

Factor	Score Value of CTIFG
BOD (P_1)	4.3025
COD(P_2)	4.2031
pH(P_3)	4.7600
Alkalinity(P_4)	4.5230
Nitrogen(P_5)	4.4244
Phosphorous(P_6)	4.4799
Turbidity(P_7)	5.1046

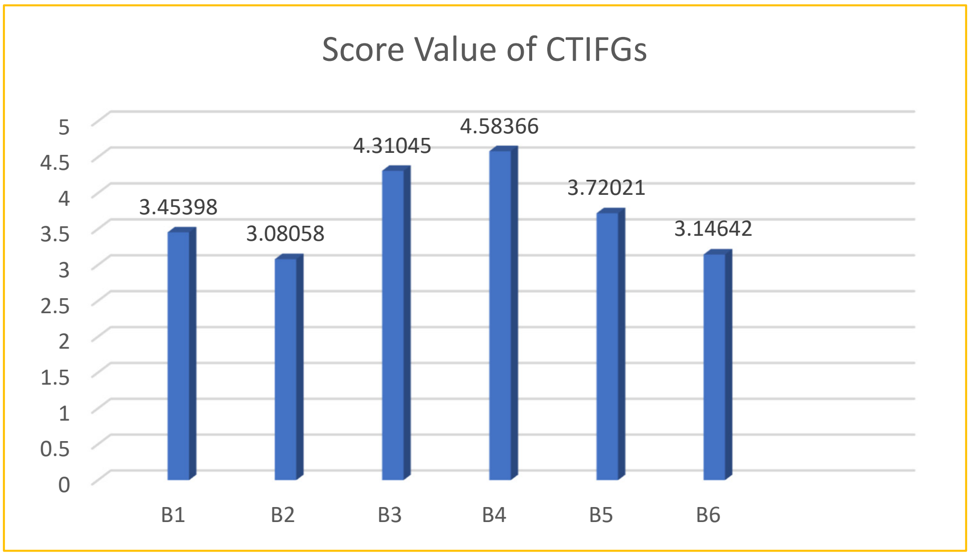

Figure 15. Graphical representation of score values of CTIFGs.

5.3. Performance Comparative Analysis

The CTIFGs play a significant role in fine-tuning the uncertainity module corresponding to individual parameters. A range of parameters, when subjected to diverse representation, gives a systematic and optimistic problem-solving source. The CTIFGs role is crucial in observing the change in degrees of vagueness and unwillingness. The graphical representation clearly highlights the uniformity in all elements that are considered equal links in the study, which correlates with the independence of truth, followed by falsity membership levels. In connection with this, the truth membership has been assigned a 0.6 value, which suggests a strong link; meanwhile, the falsity membership is assigned a 0.4 value, which implies a weak link among elements. The determination of uncertainity among the parameters in the linking factor shows consistency across and in all dimensions, highlighting greater accuracy in defining solutions for the removal of industrial pollution. The score values corresponding to the parameters associated with CTIFGs are highlighted in Table 4 and are graphically illustrated in Figure 15.

5.4. Sensitivity Analysis

The sensitivity analysis of the study is focused on considering the parameter ‘t’ in the CTIFGs that normally highlight the importance of the removal of pollutants from industrial wastewater. The major source incorporated in this analysis normally covers ‘t’, which facilitates the decision-makers to observe a balanced view on each elemental acceptance and rejection, which systematically observes the role of different levels of uncertainity and risk. The importance of decision-makers in the experimentation on varying situations is commonly associated with the value of ‘t’, which highlights the important key factors in suggesting the favorite confidence levels. The flexibility in the experimentation promotes effective decision-making support in unpredicted contexts and varying situations, which normally plays a significant path in handling the difficulties aligned with the pollutant removal from the rubber industrial wastewater. Further, the ability to adapt to ‘t’ helps the treatment of industrial effluent from a different perspective connected to risk and uncertainity; in this case, the decision-maker’s role plays a major part in prioritizing the operations.

6. Conclusions

The concept of CTIFGs has been presented in this study, and the fundamentals related to the rubber industrial wastewater physicochemical parameters have been clearly investigated. The CTIFGs comprise various studies and demonstrations that are incorporated in the graphical format, which is a representation of various sets of theoretical operations such as the Cartesian product, compositions, unions, and joints. In addition to that, a graphical representation of the above-mentioned operations is also depicted. The homomorphism and isomorphism concepts in the CTIFGs have been neatly presented. With the support of these advanced techniques, the application part has been studied to evaluate the pollutant removal from industrial rubber wastewater.

Current pollution prevention strategies have given a wide-ranging statement on the standard for effluent disposal and the norms have been followed in this observational study; when the value of parameter ‘t’ near 0 is used, distinct specificity is lacking. At the same time, a most relevant and superficial link has been generated for effective pollutant removal when the parameter ‘t’ value is associated closer to 1. It is clear that the parameter ‘t’ value in the CTIFGs indicates the level of degree of confidence or considers doubt on the rate of success relevant to the removal of pollutants from the industrial wastewater. These two factors play a significant role in classifying the metrics that determine the strongest association or considering the negligible impact pertaining to the desired outcome. Meanwhile, by engaging with this calibrated parameter, the decision-maker’s role may be accurately observed with regard to how the values on uncertainty are clearly portrayed, and at the same time, it also predicts the impacts on the analytical outcomes. Theses factors naturally result in obtaining a more complex and, at the same time, an adaptable strategical framework, which has been predicted for clearly addressing the fact on complex challenges that is associated with the removal of pollutant from industrial wastewater.

In future, the topological indicies of CTIFGs will be studied with suitable real time applications in decision-making problems like neural networks, Machine learning, and Artificial Intelligence.

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