mathematics

Article

Command-Filtered Nussbaum Design for Nonlinear Systems with Unknown Control Direction and Input Constraints

Yuxuan Liu

School of Electronic and Electrical Engineering, University of Leeds, Leeds LS2 9IT, UK; el23yl@leeds.ac.uk

Abstract: This paper studies the problem of adaptive fuzzy control based on command filtering for a class of nonlinear systems characterized by an input dead zone, input saturation, and unknown control direction. First, this paper proposes a novel equivalent transformation technique that simplifies the design complexity of multiple input constraints by converting the input dead zone and saturation nonlinearities into a unified functional form. Subsequently, a fuzzy logic system is utilized to handle the unknown nonlinear functions, and the command-filtering method is employed to address the issue of complexity explosion, while the Nussbaum function is utilized to resolve the challenge of an unknown control direction. Based on Lyapunov stability, it is proven that the tracking error converges to a small neighborhood around the origin, and all closed-loop signals are bounded. Finally, a numerical simulation result and an actual simulation result of a pendulum are presented to verify the feasibility and effectiveness of the proposed control strategy.

Keywords: unknown control direction; adaptive fuzzy control; dead zone and saturation; command filter

MSC: 93-10; 93C10; 93C40; 93D21

check for updates

Citation: Liu, Y. Command-Filtered Nussbaum Design for Nonlinear Systems with Unknown Control Direction and Input Constraints. *Mathematics* **2024**, *12*, 2167. https://doi.org/10.3390/math12142167

Academic Editor: Quanxin Zhu

Received: 21 May 2024
Revised: 3 July 2024
Accepted: 9 July 2024
Published: 10 July 2024

1. Introduction

It is widely acknowledged that stability analysis and controller design for nonlinear systems have been subjects of ongoing research and interest for several decades [1–3]. This technology is often applied to robots [4], quadcopter UAVs [5], noise data classification [6], aerospace systems [7], etc. Recently, the combination of backstepping methodology with adaptive control techniques to address nonlinear systems has undergone significant development and practical application. In earlier studies, nonlinear terms of the system were often assumed to be known a priori or linearly parameterizable. However, for many practical systems, this assumption is considered unrealistic. To solve this problem, neural networks (NNs) and fuzzy logic systems (FLS) have been used to approximate unknown system dynamics [8,9]. For example, [10] introduced an adaptive fuzzy controller grounded in sliding mode control theory. In [11–13], the authors introduced several intelligent control methodologies for nonlinear systems featuring pure feedback structures by amalgamating neural networks or fuzzy logic systems with adaptive backstepping approaches.

However, the most common drawback of backstepping techniques is the complexity explosion caused by repeatedly differentiating the virtual controller. To address this issue, Ref. [14] proposed a dynamic surface control (DSC) scheme, which incorporated a first-order filter dynamic surface at each stage of the backstepping control design process, thereby obviating the need for calculating the derivatives of the virtual controller. Dynamic filtering technology was introduced by [15] to investigate event-triggered tracking control of a category of uncertain nonlinear systems. However, DSC technology failed to account for the error introduced by the filter, consequently diminishing the control performance of the system. Refs. [16–19] applied command-filtering technology to nonlinear systems under different constraints, which not only solved the problem of complexity explosion in the backstepping design process, but also established an error compensation system to make up for the shortcomings of DSC technology.

Despite the success of command-filter adaptive control in nonlinear systems, the presence of various constraints in practical engineering contexts, such as input dead zones, input saturation, unknown control direction, etc., can influence the system's stability. To solve the dead-zone problem, some related results are provided in [20–22]. Ref. [20] proposed an adaptive dead-zone inverse technology. Ref. [21] developed a corresponding disturbance observer for estimation based on the unknown approximation error and the impact of unknown dead zones and external disturbances. In [22], the system was converted into n-step predictors, and an adaptive compensation term was introduced to overcome the asymmetric dead zone existing in the system. Apart from the presence of dead zone input, the presence of input saturation can also cause performance degradation of nonlinear systems and signal delay or loss. The study of input saturated systems has also been an important topic in recent years [23,24].

On the other hand, when researching adaptive control of nonlinear systems, it is often necessary to know the control direction representing the direction of motion in advance [25,26]. However, the direction of controlling gain is mostly unknown in practical applications. The Nussbaum gain method is an effective tool for processing unknown signals. Characterized by its values and integral oscillating infinitely between positive and negative, the Nussbaum function allows the control system to adjust its strategy automatically, despite uncertainty about the sign of the control gain, ensuring that the system can stably achieve the desired state. Building upon this technology, numerous control strategies have been formulated [27–30]. Ref. [27] introduced the Nussbaum function to compensate for the impact of the unknown direction problem and designed an adaptive tracking controller based on a command filter. For systems featuring multiple unknown high-frequency gains, Ref. [28] introduced a novel command-filtered Nussbaum design. A novel Nussbaum function was devised by [29] to address the tracking problem encountered within a category ofstochastic strict feedback nonlinear systems. By using an improved Nussbaum function, [30] extended previous research results to cover a broader range of nonlinear systems, characterized by unknown variations in both the sign and magnitude of the control gain over time. However, to the best of our knowledge, there is a scarcity of papers that concurrently address nonlinear systems with input dead zones, input saturation, and uncertain control directions. This scarcity partly motivated the research presented in this paper.

Based on the previous discussion, the main contributions of this article, in contrast to existing research outcomes, can be encapsulated as follows:

1. Compared with the nonlinear systems studied in [25,26], where the control direction was known, this paper considers a broader situation in which the control direction is unknown, and it designs adaptive fuzzy control using the Nussbaum function.
2. This paper proposes a novel transformation method to eliminate the impact of the input dead zone and saturation on the system, and uses command-filtering technology to solve the problem of complexity explosion in traditional backstepping design.

2. Preliminary Knowledge and Problem Statement

2.1. System Model

Consider the following nonlinear system

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + \lambda_i g_i(\bar{x}_i) x_{i+1},\ i = 1, \dots, n-1, \\ \dot{x}_n = f_n(\bar{x}_n) + \lambda_n g_n(\bar{x}_n) u, \\ y = x_1, \end{cases} \tag{1}$$

where $x = [x_1, x_2, \dots x_n]^T \in R^n$ represents the state vector with $\bar{x}_i = [x_1, x_2, \dots x_i]^T \in R^i$, and $y \in R$ denotes the system output; $\lambda_i = 1$ (or -1) represents an unknown control direction; $g_i(\bar{x}_i)$ are bounded continuous functions with $0 < \hbar_i \leq g_i(\cdot) \leq \Theta_i$, $\hbar_i$ and Θ_i

represent two constants; and $f_i(\bar{x}_i)$ signifies the unknown smooth function. The control input $u \in R$ is specified as

$$u(t) = \begin{cases} u_h, & v(t) > u_h, \\ k_h(v(t) - m_h), & m_h \leq v(t) < u_h, \\ 0, & -m_l < v(t) \leq m_h, \\ k_l(v(t) + m_l), & -u_l < v(t) \leq -m_l, \\ -u_l, & v(t) < -u_l, \end{cases} \tag{2}$$

where $v(t)$ is the input of the dead zone; u_h and u_l are positive parameters and represent unknown saturation values; $k_h > 0$, $k_l > 0$, $m_h > 0$, and $m_l > 0$ are the unknown zone parameters; and the dead-zone slopes in positive and negative region are same, i.e., $k_h = k_l = k$.

Assumption 1 ([22]). *The dead-zone parameters of m_h, m_l, and k are bounded. This implies that there are known parameters $m_{h\max}$, $m_{h\min}$, $m_{l\max}$, $m_{l\min}$, $k_{\max}$, and $k_{\min}$, such that $m_h \in [m_{h\min}, m_{h\max}]$, $m_l \in [m_{l\min}, m_{l\max}]$, and $k \in [k_{\min}, k_{\max}]$.*

For the development of a robust control scheme, (2) is reformulated as follows:

$$u(t) = \pi\big(v(t)\big)v(t) + \vartheta\big(v(t)\big). \tag{3}$$

Based on Assumption 1, one can conclude that $\vartheta\big(v(t)\big)$ is bounded, while satisfies $\big|\vartheta\big(v(t)\big)\big| \leq L_p$, where L_p represents the upper bound.

Assumption 2 ([25]). *In this article, considering that the input signal v is limited in actual situations, $\pi\big(v(t)\big)$ satisfies the following inequality*

$$0 < \Im \leq \min\left\{\frac{u_h}{v(t)_{\max}}, k\right\} \leq \pi\big(v(t)\big) \leq \max\{1, k\}, \tag{4}$$

2.2. Fuzzy Logic Systems

FLS consists of four primary components: the knowledge base, fuzzifier, fuzzy inference engine, and defuzzifier. The knowledge base houses a comprehensive set of fuzzy if-then rules, which are defined as follows:

R^l: IF x_1 is P_1^j, and x_2 is P_2^j, and x_n is P_n^j, then y is Q^j,$j = 1, 2, \ldots, \wp$, where $x = [x_1, x_2, \ldots, x_n]^T$, and y are the FLSs input and system output, respectively; P_m^j, Q^j denote the fuzzy sets for x and y, respecitvley; an equivalent expression of FLS can be obtained as

$$y(x) = \frac{\sum_{j=1}^{\wp} \bar{y}_j \prod_{m=1}^{n} \mu_{F_m^j}(x_m)}{\sum_{j=1}^{\wp}\left[\prod_{m=1}^{n} \mu_{F_m^j}(x_m)\right]}, \tag{5}$$

with $\bar{y}_j = \max_{y\in R} \mu_{Q^j}(y)$, where $\mu_{P_m^j}(x_m)$ and $\mu_{Q^j}(y)$ are the membership functions. Denote $W = [\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_\wp]^T = [W_1, W_2, \ldots, W_\wp]^T$ and $\psi(x) = [\psi_1(x), \psi_2(x), \ldots, \psi_\wp(x)]^T$, the membership functions, which are defined as $\psi_j(x) = \frac{\prod_{m=1}^{n} \mu_{F_m^j}(x_m)}{\sum_{j=1}^{\wp}\left[\prod_{m=1}^{n} \mu_{F_m^j}(x_m)\right]}$. Consequently, FLS can be succinctly described as follows

$$y(x) = W^T \psi(x). \tag{6}$$

Lemma 1 ([27]). *The following inequality holds for any smooth function $f(x)$ defined on the compact set Ω if there is a sufficiently tiny positive scalar ε:*

$$\sup_{x\in\Omega}\left|f(x)-W^{\mathrm{T}}\psi(x)\right|\leq\varepsilon. \tag{7}$$

Definition 1 ([29]). *The Nussbaum function $N(\zeta):\mathbb{R}\rightarrow\mathbb{R}$ has the properties*

$$\begin{aligned}&\lim_{\ell\rightarrow\infty}\sup\frac{1}{\ell}\int_0^{\ell}N(\zeta)d\zeta=+\infty,\\&\lim_{\ell\rightarrow-\infty}\inf\frac{1}{\ell}\int_0^{\ell}N(\zeta)d\zeta=-\infty.\end{aligned} \tag{8}$$

Lemma 2 ([29]). *Consider $\zeta(t)$ and $V(t)\geq 0$ are smooth functions on $[0,t_f)$, and $N(\zeta(t))$ is an even smooth Nussbaum-type function. Suppose*

$$V(t)\leq e^{-\Upsilon_1 t}\int_0^t\big(w(\bar{x}(\tau))N(\zeta(\tau))+1\big)\dot{\zeta}(\tau)e^{\Upsilon_1\tau}d\tau+D, \tag{9}$$

in which $D>0$ and $\Upsilon_1>0$, and $V(t)$, $\zeta(t)$ and $\int_0^t w(\bar{x}(\tau))N(\zeta(\tau))\dot{\zeta}(\tau)d\tau$ remain bounded on $[0,t_f)$.

Lemma 3 ([25]). *The command filter is defined as*

$$\begin{cases}\dot{\omega}_i=\varpi\omega_{i,2},\\\dot{\omega}_{i,2}=-2\varphi\varpi\omega_{i,2}-\varpi(\omega_i-\alpha_{i-1}),\end{cases} \tag{10}$$

where α_{i-1} and ω_i represent the input and output of the command filter, respectively, $\omega_i(0)=\alpha_{i-1}$ and $\omega_{i,2}(0)=0$, $\varphi\in(0,1]$, and $\varpi>0$.

Remark 1. *The command-filtering approach is a control strategy that simplifies the design and implementation of complex control systems. By incorporating a filter between the controller and actuator, it smooths the control signals, preventing performance degradation due to overly complex control strategies. This method effectively reduces system complexity and avoids "complexity explosion" caused by high-frequency control updates and excessive regulation.*

Assumption 3 ([26]). *The reference signal x_d and its first-order derivative $\dot{x}_d$ are continuous and bounded.*

3. Controller Design and Stability Analysis

3.1. Controller Design

In this section, an adaptive command-filter controller is designed for the nonlinear system (1) by integrating the Nussbaum function with the back-stepping technique. Coordinate changes are introduced to facilitate controller design:

$$\begin{cases}e_1=x_1-y_d,\\e_i=x_i-\omega_i,\quad i=1,\ldots,n,\end{cases} \tag{11}$$

where e_i represents the tracking error and ω_i denotes the output of the filter.

Remark 2. *It is noteworthy that the error induced by the command filter exacerbates the system error. To address this drawback, a compensation signal, denoted as β, is introduced to mitigate the adverse effects of the filter error $\omega_i-\alpha_{i-1}$ on the system.*

Design the compensation signal β_i to eliminate the error caused by the command filter as follows

$$\begin{cases} \dot{\beta}_i = -k_i\beta_i + g_i\beta_{i+1} + g_i(\omega_{i+1} - \alpha_i), \\ \dot{\beta}_n = -k_n\beta_n, \end{cases} \tag{12}$$

where k_i is a given positive constant and $\beta_i(0) = 0$.

Subsequently, the compensated tracking error signals can be expressed as follows

$$\xi_i = e_i - \beta_i, \quad i = 1, 2, \ldots, n. \tag{13}$$

Step 1: Taking the derivative of ξ_i as

$$\dot{\xi}_1 = \dot{e}_1 - \dot{\beta}_1 = \dot{x}_1 - \dot{y}_d - \dot{\beta}_1 = f_1 + \lambda_1 g_1 x_2 - \dot{y}_d - \dot{\beta}_1. \tag{14}$$

The Lyapunov function is chosen as

$$V_1 = \frac{1}{2}\xi_1^2 + \frac{1}{2\Gamma_1}\tilde{\theta}_1^2, \tag{15}$$

where Γ_1 represents the positive parameter to be constructed, and in order to solve the parameter estimation problem, the parameter estimation error is $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$, and the constant is defined as $\theta_i = \|W_i\|^2$.

Based on (11), (13), (14) and (15) the time derivative of V_1 is shown as

$$\begin{aligned} \dot{V}_1 &= \xi_1\dot{\xi}_1 - \frac{1}{\Gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 \\ &= \xi_1\big(f_1 + \lambda_1 g_1(\omega_2 + \xi_2 + \beta_2) - \dot{y}_d - \dot{\beta}_1\big) - \frac{1}{\Gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 \\ &= \xi_1 f_1 + \lambda_1 g_1\xi_1\omega_2 + \lambda_1 g_1\xi_1\xi_2 + \lambda_1 g_1\xi_1\beta_2 - \xi_1\dot{y}_d - \xi_1\dot{\beta}_1 - \frac{1}{\Gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1. \end{aligned} \tag{16}$$

As function $f_1(x)$ is unknown, the direct design of the virtual control signal α_1 is not feasible. Thus, according to Lemma 1, for any given number $\varepsilon_1 > 0$, there are

$$f_1(B_1) = W_1^T\psi_1 + \delta_1(B_1), \|\delta_1(B_1)\| < \varepsilon_1,$$

in which $\|\delta_1(B_1)\|$ denotes the estimation error.

By applying Young's inequality, the following formula can be derived

$$\xi_1 f_1 \leq \frac{\xi_1^2\theta_1 S_1^T S_1}{2a_1^2} + \frac{1}{2}a_1^2 + \frac{1}{2}\xi_1^2 + \frac{1}{2}\varepsilon_1^2, \tag{17}$$

where a_1 is a given positive scalar.

Consider a compensating signal $\dot{\beta}_1$ as

$$\dot{\beta}_1 = -k_1\beta_1 + g_1\beta_2 + g_1(\omega_2 - \alpha_1). \tag{18}$$

Next, after combining formulas (16)–(18), it can be easily obtained that

$$\begin{aligned} \dot{V}_1 \leq& \frac{\xi_1^2\theta_1 S_1^T S_1}{2a_1^2} + \frac{1}{2}a_1^2 + \frac{1}{2}\xi_1^2 + \frac{1}{2}\varepsilon_1^2 + \lambda_1 g_1\xi_1\omega_2 + \lambda_1 g_1\xi_1\xi_2 + \lambda_1 g_1\xi_1\beta_2 \\ &+ k_1\xi_1\beta_1 - g_1\xi_1\beta_2 - g_1\xi_1\omega_2 + g_1\xi_1\alpha_1 - \frac{1}{\Gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 - \xi_1\dot{y}_d \\ \leq& \frac{\xi_1^2\theta_1 S_1^T S_1}{2a_1^2} + \frac{1}{2}a_1^2 + \frac{1}{2}\xi_1^2 + \frac{1}{2}\varepsilon_1^2 + \lambda_1 g_1\xi_1\xi_2 + k_1\xi_1\beta_1 + g_1\xi_1\alpha_1 - \frac{1}{\Gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 \\ &+ (\lambda_1 - 1)g_1\xi_1(\beta_2 + \omega_2) - \xi_1\dot{y}_d. \end{aligned} \tag{19}$$

In this article, $\lambda_1 = 1(or-1)$ represents the unknown control direction. Applying Young's inequality, one can obtain

$$
\begin{aligned}
(\lambda_1-1)g_1\xi_1(\beta_2+\omega_2) = (\lambda_1-1)g_1\xi_1(x_2-\xi_2) = 0 \le 2g_1^2\xi_1^2 + x_2^2 + \xi_2^2, \quad \lambda_1 = 1, \\
(\lambda_1-1)g_1\xi_1(\beta_2+\omega_2) = (\lambda_1-1)g_1\xi_1(x_2-\xi_2) \le 2g_1^2\xi_1^2 + x_2^2 + \xi_2^2, \quad \lambda_1 = -1, \\
\lambda_1 g_1\xi_1\xi_2 \le \frac{1}{2}g_1^2\xi_1^2 + \frac{1}{2}\xi_2^2, \quad \lambda_1 = 1(or-1).
\end{aligned} \tag{20}
$$

Substituting (20) into (19) produces

$$
\begin{aligned}
\dot{V}_1 \le & \frac{\xi_1^2\theta_1\psi_1^T\psi_1}{2a_1^2} + \frac{1}{2}a_1^2 + \frac{1}{2}\xi_1^2 + \frac{1}{2}\varepsilon_1^2 + \frac{5}{2}g_1^2\xi_1^2 + \frac{3}{2}\xi_2^2 \\
& + k_1\xi_1\beta_1 + g_1\xi_1\alpha_1 - \frac{1}{\Gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 + x_2^2 - \xi_1\dot{y}_d.
\end{aligned} \tag{21}
$$

Next, the virtual control signal α_1 and the Nussbaum-type gain ζ_1 are developed as follows

$$
\begin{cases}
\alpha_1 = N(\zeta_1)\left(\frac{\xi_1\hat{\theta}_1 S_1^T S_1}{2a_1^2} + k_1 e_1 + \frac{1}{2}\xi_1 + \frac{5}{2}g_1^2\xi_1 - \dot{y}_d\right), \\
\dot{\zeta}_1 = \xi_1\left(\frac{\xi_1\hat{\theta}_1 S_1^T S_1}{2a_1^2} + k_1 e_1 + \frac{1}{2}\xi_1 + \frac{5}{2}g_1^2\xi_1 - \dot{y}_d\right).
\end{cases} \tag{22}
$$

By amalgamating the aforementioned equation, (21) can be reformulated as

$$
\begin{aligned}
\dot{V}_1 \le & \frac{\xi_1^2\theta_1\psi_1^T\psi_1}{2a_1^2} + \frac{1}{2}a_1^2 + \frac{1}{2}\xi_1^2 + \frac{1}{2}\varepsilon_1^2 + \frac{5}{2}g_1^2\xi_1^2 + \frac{3}{2}\xi_2^2 + k_1\xi_1\beta_1 + g_1\xi_1\alpha_1 - \frac{1}{\Gamma_1}\tilde{\theta}_1\dot{\hat{\theta}}_1 + x_2^2 - \xi_1\dot{y}_d \\
\le & -k_1\xi_1^2 + g_1 N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 + \frac{\tilde{\theta}_1}{\Gamma_1}\left(\frac{\Gamma_1\xi_1^2\psi_1^T\psi_1}{2a_1^2} - \dot{\hat{\theta}}_1\right) + \frac{1}{2}a_1^2 + \frac{1}{2}\varepsilon_1^2 + \frac{3}{2}\xi_2^2 + x_2^2.
\end{aligned} \tag{23}
$$

Next, the adaptive law $\dot{\hat{\theta}}_1$ is designed as $\dot{\hat{\theta}}_1 = \frac{\Gamma_1\xi_1^2\psi_1^T\psi_1}{2a_1^2} - \sigma_1\hat{\theta}_1$, and with the help of Young's inequality $\frac{\sigma_1\tilde{\theta}_1\hat{\theta}_1}{\Gamma_1} \le \frac{\sigma_1\theta_1^2}{2\Gamma_1} - \frac{\sigma_1\tilde{\theta}_1^2}{2\Gamma_1}$, one obtain

$$
\dot{V}_1 \le -k_1\xi_1^2 + g_1 N(\zeta_1)\dot{\zeta}_1 + \dot{\zeta}_1 - \frac{\sigma_1\tilde{\theta}_1^2}{2\Gamma_1} + N_1, \tag{24}
$$

where $N_1 = \frac{1}{2}a_1^2 + \frac{1}{2}\varepsilon_1^2 + \frac{3}{2}\xi_2^2 + x_2^2 + \frac{\sigma_1\theta_1^2}{2\Gamma_1}$.

Step i: $(2 \le i \le n-1)$: According to the differential rules, the following expression is derived

$$
\dot{\xi}_i = \dot{e}_i - \dot{\beta}_i = \dot{x}_i - \dot{\omega}_i - \dot{\beta}_i = f_i + \lambda_i g_i x_{i+1} - \dot{\omega}_i - \dot{\beta}_i. \tag{25}
$$

Choose a Lyapunov function candidate function, as follows

$$
V_i = V_{i-1} + \frac{1}{2}\xi_i^2 + \frac{1}{2\Gamma_i}\tilde{\theta}_i^2. \tag{26}
$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$.

By differentiating V_i, the following formula holds

$$
\begin{aligned}
\dot{V}_i &= \dot{V}_{i-1} + \xi_i\dot{\xi}_i - \frac{1}{\Gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i \\
&= \dot{V}_{i-1} + \xi_i\big(f_i + \lambda_i g_i(\omega_{i+1} + \xi_{i+1} + \beta_{i+1}) - \dot{\omega}_i - \dot{\beta}_i\big) - \frac{1}{\Gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i \\
&= \dot{V}_{i-1} + \xi_i f_i + \lambda_i g_i\xi_i\omega_{i+1} + \lambda_i g_i\xi_i\xi_{i+1} + \lambda_i g_i\xi_i\beta_{i+1} - \xi_i\dot{\omega}_i - \xi_i\dot{\beta}_i - \frac{1}{\Gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i.
\end{aligned} \tag{27}
$$

According to Lemma 1 again, for any given number $\varepsilon_i > 0$, there are

$$f_i(B_i) = W_i^T \psi_i + \delta_i(B_i), \|\delta_i(B_i)\| < \varepsilon_i,$$

in which $\|\delta_i(B_i)\|$ denotes the estimation error.

By applying Young's inequality again, the following formula can be derived

$$\xi_i f_i \leq \frac{\xi_i^2 \theta_i \psi_i^T \psi_i}{2a_i^2} + \frac{1}{2}a_i^2 + \frac{1}{2}\xi_i^2 + \frac{1}{2}\varepsilon_i^2, \tag{28}$$

where a_i is a given positive scalar.

The compensation signal $\dot{\beta}_i$ is designed to be

$$\dot{\beta}_i = -k_i\beta_i + g_i\beta_{i+1} + g_i(\omega_{i+1} - \alpha_i). \tag{29}$$

Incorporating Equations (28) and (29) into (27), one can obtain

$$\begin{aligned} \dot{V}_i \leq& \dot{V}_{i-1} + \frac{\xi_i^2 \theta_i \psi_i^T \psi_i}{2a_i^2} + \frac{1}{2}a_i^2 + \frac{1}{2}\xi_i^2 + \frac{1}{2}\varepsilon_i^2 + \lambda_i g_i \xi_i \xi_{i+1} + k_i \xi_i \beta_i + g_i \xi_i \alpha_i - \frac{1}{\Gamma_i}\tilde{\theta}_i \dot{\hat{\theta}}_i \\ &+ \left[\lambda_i g_i \xi_i \omega_{i+1} + \lambda_i g_i \xi_i \beta_{i+1} - g_i \xi_i \beta_{i+1} - g_i \xi_i \omega_{i+1}\right] - \xi_i \dot{\omega}_i \\ \leq& \dot{V}_{i-1} + \frac{\xi_i^2 \theta_i \psi_i^T \psi_i}{2a_i^2} + \frac{1}{2}a_i^2 + \frac{1}{2}\xi_i^2 + \frac{1}{2}\varepsilon_i^2 + \lambda_i g_i \xi_i \xi_{i+1} + k_i \xi_i \beta_i + g_i \xi_i \alpha_i - \frac{1}{\Gamma_i}\tilde{\theta}_i \dot{\hat{\theta}}_i \\ &+ (\lambda_i - 1) g_i \xi_i \big(\beta_{i+1} + \omega_{i+1}\big) - \xi_i \dot{\omega}_i. \end{aligned} \tag{30}$$

Similar to (20), one can obtain

$$\begin{aligned} \lambda_i g_i \xi_i \xi_{i+1} &\leq \frac{1}{2} g_i^2 \xi_i^2 + \frac{1}{2}\xi_{i+1}^2, \\ (\lambda_i - 1) g_i \xi_i (\beta_{i+1} + \omega_{i+1}) &\leq 2 g_i^2 \xi_i^2 + x_{i+1}^2 + \xi_{i+1}^2. \end{aligned} \tag{31}$$

Then, (30) is rewritten as

$$\begin{aligned} \dot{V}_1 \leq& \dot{V}_{i-1} + \frac{\xi_i^2 \theta_i \psi_i^T \psi_i}{2a_i^2} + \frac{1}{2}a_i^2 + \frac{1}{2}\xi_i^2 + \frac{1}{2}\varepsilon_i^2 + \frac{5}{2}g_i^2\xi_i^2 + \frac{3}{2}\xi_{i+1}^2 \\ &+ k_i \xi_i \beta_i + g_i \xi_i \alpha_i - \frac{1}{\Gamma_i}\tilde{\theta}_i \dot{\hat{\theta}}_i + x_{i+1}^2 - \xi_i \dot{\omega}_i. \end{aligned} \tag{32}$$

The virtual control signal α_1 and the Nussbaum-type gain ζ_1 are designed as

$$\begin{cases} \alpha_i = N(\zeta_i)\left(\frac{\xi_i \hat{\theta}_i \psi_i^T \psi_i}{2a_i^2} + k_i e_i + \frac{1}{2}\xi_i + \frac{5}{2}g_i^2 \xi_i - \dot{\omega}_i\right), \\ \dot{\zeta}_i = \xi_i\left(\frac{\xi_i \hat{\theta}_i \psi_i^T \psi_i}{2a_i^2} + k_i e_i + \frac{1}{2}\xi_i + \frac{5}{2}g_i^2 \xi_i - \dot{\omega}_i\right). \end{cases} \tag{33}$$

Combining the above equation, (32) can be rewritten as

$$\dot{V}_i \leq \dot{V}_{i-1} - k_i \xi_i^2 + g_i N(\zeta_i)\dot{\zeta}_i + \dot{\zeta}_i + \frac{\tilde{\theta}_i}{\Gamma_i}\left(\frac{\Gamma_i \xi_i^2 \psi_i^T \psi_i}{2a_i^2} - \dot{\hat{\theta}}_i\right) + \frac{1}{2}a_i^2 + \frac{1}{2}\varepsilon_i^2 + \frac{3}{2}\xi_{i+1}^2 + x_{i+1}^2. \tag{34}$$

Next, the adaptive law $\dot{\hat{\theta}}_i$ is designed as $\dot{\hat{\theta}}_i = \frac{\Gamma_i \xi_i^2 \psi_i^T \psi_i}{2a_i^2} - \sigma_i \hat{\theta}_i$, and with the help of Young's inequality $\frac{\sigma_i \tilde{\theta}_i \hat{\theta}_i}{\Gamma_i} \leq \frac{\sigma_i \theta_i^2}{2\Gamma_i} - \frac{\sigma_i \tilde{\theta}_i^2}{2\Gamma_i}$, one obtain

$$\begin{aligned}\dot{V}_i \leq& \dot{V}_{i-1} - k_i \xi_i^2 + g_i N(\zeta_i)\dot{\zeta}_i + \dot{\zeta}_i - \frac{\sigma_i \tilde{\theta}_i^2}{2\Gamma_i} + \frac{1}{2}a_i^2 + \frac{1}{2}\varepsilon_i^2 + \frac{3}{2}\xi_{i+1}^2 + x_{i+1}^2 + \frac{\sigma_i \theta_i^2}{2\Gamma_i} \\ \leq& -\sum_{j=1}^{i} k_j \xi_j^2 - \sum_{j=1}^{i} \frac{1}{2}\frac{\sigma_j \tilde{\theta}_j^2}{\Gamma_j} + \sum_{j=1}^{i} g_j N(\zeta_j)\dot{\zeta}_j + \sum_{j=1}^{i} \dot{\zeta}_j \\ &+ \frac{1}{2}\sum_{j=1}^{i}\left(a_j^2 + \varepsilon_j^2 + 3\xi_{j+1}^2 + \frac{\sigma_j \theta_j^2}{\Gamma_j}\right) + \sum_{j=1}^{i} {x_{j+1}}^2.\end{aligned} \tag{35}$$

Step n: Based on (1), (4), (11) and (13), one has

$$\begin{aligned}\dot{\xi}_n &= \dot{e}_n - \dot{\beta}_n = \dot{x}_n - \dot{\omega}_n - \dot{\beta}_n = f_n + \lambda_n g_n u - \dot{\omega}_n - \dot{\beta}_n \\ &= f_n + \lambda_n g_n[\pi(v)v + \vartheta(v)] - \dot{\omega}_n - \dot{\beta}_n.\end{aligned} \tag{36}$$

Take a Lyapunov function V_n in the following form

$$V_n = V_{n-1} + \frac{1}{2}\xi_n^2 + \frac{1}{2\Gamma_n}\tilde{\theta}_n^2, \tag{37}$$

where $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$.

Differentiating V_n, one can obtain

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + \xi_n \dot{\xi}_n - \frac{1}{\Gamma_n}\tilde{\theta}_n \dot{\hat{\theta}}_n \\ &= \dot{V}_{n-1} + \xi_n\big(f_n + \lambda_n g_n(\pi(v)v + \vartheta(v)) - \dot{\omega}_n - \dot{\beta}_n\big) - \frac{1}{\Gamma_n}\tilde{\theta}_n\dot{\hat{\theta}}_n \\ &= \dot{V}_{n-1} + \xi_n f_n + \xi_n \lambda_n g_n \pi(v)v + \xi_n \lambda_n g_n \vartheta(v) - \xi_n \dot{\omega}_n - \xi_n \dot{\beta}_n - \frac{1}{\Gamma}\tilde{\theta}_n \dot{\hat{\theta}}_n.\end{aligned} \tag{38}$$

Similarly, according to Lemma 1 and Young's inequality, one can obtain

$$\xi_n f_n \leq \frac{\xi_n^2 \theta_n \psi_n^T \psi_n}{2a_n^2} + \frac{1}{2}a_n^2 + \frac{1}{2}\xi_n^2 + \frac{1}{2}\varepsilon_n^2, \tag{39}$$

where a_n is a given positive scalar.

$$\lambda_n g_n \xi_n \vartheta(v) \leq \lambda_n(\frac{1}{2}\xi_n^2 + \frac{1}{2}\Re^2), \tag{40}$$

where $|\vartheta(v)| < L_p$, $|g_i| < \Theta_i$, and $|g_n \vartheta(v)| < \Re$, with $\Re = \Theta_n L_p$.

Then, the compensation signal $\dot{r}_n$ is designed to be

$$\dot{\beta}_n = -k_n \beta_n. \tag{41}$$

Combined with the above formula, (38) is rewritten as

$$\begin{aligned}\dot{V}_n \leq& \dot{V}_{n-1} + \frac{\xi_n^2 \theta_n \psi_n^T \psi_n}{2a_n^2} + \frac{1}{2}a_n^2 + \frac{1}{2}\xi_n^2 + \frac{1}{2}\varepsilon_n^2 + \lambda_n g_n \xi_n \pi(v)v \\ &+ \lambda_n\left(\frac{1}{2}\xi_n^2 + \frac{1}{2}\Re_n^2\right) + \xi_n k_n \beta_n - \frac{1}{\lambda_n}\tilde{\theta}_n \dot{\hat{\theta}} - \xi_n \dot{\omega}_n.\end{aligned} \tag{42}$$

The virtual control signal v and the Nussbaum-type gain ζ_n are designed as

$$\begin{cases} v = \frac{1}{\Im}N(\zeta_n)\left(\frac{\xi_n \hat{\theta}_n \psi_n^T \psi_n}{2a_n^2} + k_n e_n + \xi_n - \dot{\omega}_n\right), \\ \dot{\zeta}_n = \xi_n\left(\frac{\xi_n \hat{\theta}_n \psi_n^T \psi_n}{2a_n^2} + k_n e_n + \xi_n - \dot{\omega}_n\right). \end{cases} \tag{43}$$

Combining the above equation, (42) can be rewritten as

$$\dot{V}_n \leq \dot{V}_{n-1} - k_n \xi_n^2 + \lambda_n g_n N(\zeta_n)\dot{\zeta}_n + \dot{\zeta}_n + \frac{1}{2}\Re_n^2 + \frac{\tilde{\theta}_n}{\Gamma_n}\left(\frac{\Gamma_n \xi_n^2 \psi_n^T \psi_n}{2a_n^2} - \dot{\hat{\theta}}_n\right) + \frac{1}{2}a_n^2 + \frac{1}{2}\varepsilon_n^2. \tag{44}$$

Next, the adaptive law $\hat{\theta}_n$ is designed as $\dot{\hat{\theta}}_n = \frac{\Gamma_n \xi_n^2 \psi_n^T \psi_n}{2a_n^2} - \sigma_n \hat{\theta}_n$, and with the help of Young's inequality $\frac{\sigma_n \tilde{\theta}_n \hat{\theta}_n}{\Gamma_n} \leq \frac{\sigma_n \theta_n^2}{2\Gamma_n} - \frac{\sigma_n \tilde{\theta}_n^2}{2\Gamma_n}$, one obtain

$$\begin{aligned}\dot{V}_n \leq& \dot{V}_{n-1} - k_n \xi_n^2 + \lambda_n g_n N(\zeta_n)\dot{\zeta}_n + \dot{\zeta}_n - \frac{\sigma_n \tilde{\theta}_n^2}{2\Gamma_n} + \frac{1}{2}a_n^2 + \frac{1}{2}\varepsilon_n^2 + \frac{\sigma_n \theta_n^2}{2\Gamma_n} + \frac{1}{2}\Re_n^2 \\ \leq& -\sum_{j=1}^{n} k_j \xi_j^2 - \sum_{j=1}^{n} \frac{1}{2}\frac{\sigma_j \tilde{\theta}_j^2}{\Gamma_j} + \sum_{j=1}^{n-1} g_j N(\zeta_j)\dot{\zeta}_j + \lambda_n g_n N(\zeta_n)\dot{\zeta}_n + \sum_{j=1}^{n}\dot{\zeta}_j \\ &+ \frac{1}{2}\sum_{j=1}^{n}\left(a_j^2 + \varepsilon_j^2 + \frac{\sigma_j \theta_j^2}{\Gamma_j}\right) + \sum_{j=1}^{n-1} x_{j+1}^2 + \frac{3}{2}\sum_{j=1}^{n-1}\xi_{j+1}^2 + \frac{1}{2}\Re_n^2.\end{aligned} \tag{45}$$

3.2. Stability Analysis

Theorem 1. *Consider the nonlinear system (1) under Assumptions 1–3, utilizing the error compensation signals (18), (29), (41), virtual controllers (22) and (33), as well as the actual controller (43) designed in this study, and combining the constructed parameter adaptive law along with the provided signal xd, it is assured that all closed-loop signals remain bounded, and the tracking error is driven to the vicinity of the origin.*

Proof. Denote $D = \{2k_j\sigma_j, \ \ \forall j = 1, \dots, n\}$, (45) can be rewritten as

$$\dot{V}_n \leq -DV_n + C + \sum_{i=1}^{n-1}(g_j N(\zeta_j) + 1)\dot{\zeta}_j + \dot{\zeta}_n(\lambda_n g_n N(\zeta_n) + 1), \tag{46}$$

where $C = \frac{1}{2}\sum_{j=1}^{n}\left(a_j^2 + \varepsilon_j^2 + \frac{\sigma_j \theta_j^2}{\Gamma_j}\right) + \sum_{j=1}^{n-1} x_{j+1}^2 + \frac{3}{2}\sum_{j=1}^{n-1}\xi_{j+1}^2 + \frac{1}{2}\Re_n^2$.

Thus, multiplying (46) by e^{Dt} results in

$$\frac{d}{dt}\left(V_n e^{Dt}\right) \leq e^{Dt}\sum_{j=1}^{n-1}\left(g_j N(\zeta_j) + 1\right)\dot{\zeta}_j + e^{Dt}\left(\lambda_n g_n N(\zeta_n) + 1\right)\dot{\zeta}_n + e^{Dt}C. \tag{47}$$

Integrating the above equation to the interval $[0, t)$, one can obtain

$$\begin{aligned}V_n(t) \leq& e^{-Dt}\int_0^t \sum_{j=1}^{n-1}(g_j N(\zeta_j) + 1)\dot{\zeta}_j e^{D\tau} d\tau + e^{-Dt}\int_0^t (\lambda_n g_n N(\zeta_n) + 1)\dot{\zeta}_n e^{D\tau} d\tau \\ &+ \frac{C}{D} + e^{-Dt}V_n(0) - \frac{C}{D}e^{-Dt}.\end{aligned} \tag{48}$$

According to Lemma 2, it can be inferred that V_n, ζ_n and $\int_0^t(\lambda_n g_n N(\zeta_n) + 1)\dot{\zeta}_n d\tau$ are bounded. Thus, ξ_n and $\tilde{\theta}_n$ are bounded. In addition, similar to the previous derivation, it can be derived that V_{n-1}, ζ_{n-1}, ξ_{n-1} ,$\tilde{\theta}_{n-1}$ and V_i, ζ_i, ξ_i ,$\tilde{\theta}_i$ are all bounded, which derive the boundedness of $\int_0^t \sum_{i=1}^{n-1}(g_j N(\zeta_j) + 1)\zeta_j e^{D\tau} d\tau$ and $\int_0^t(\lambda_n g_n N(\zeta_n) + 1)\dot{\zeta}_n e^{D\tau} d\tau$. Thus, the following formula holds

$$0 \leq V_n \leq \left[\Lambda_1 + \Lambda_2 + V_n(0) - \frac{C}{D}\right]e^{-Dt} + \frac{C}{D}, \tag{49}$$

where Λ_1 represents $\int_0^t \sum_{i=1}^{n-1}(g_j N(\zeta_j) + 1)\zeta_j e^{D\tau} d\tau$, and Λ_2 denotes $\int_0^t(\lambda_n g_n N(\zeta_n) + 1)\dot{\zeta}_n e^{D\tau} d\tau$.

According to (49), one can obtain

$$\lim_{t\to\infty} V_n(t) \leq \frac{C}{D}. \tag{50}$$

Connecting Equations (37) and (49), the following equation is established

$$|\xi_n| \leq \sqrt{2\left[(\Lambda_1 + \Lambda_2 + V(0) - \frac{C}{D})e^{-Dt} + \frac{C}{D}\right]}, \tag{51}$$

which implies that

$$\lim_{t\to\infty} |\xi_n| \leq \sqrt{\frac{2C}{D}}. \tag{52}$$

this means ξ_n is bounded.

According to (13), it is evident that the boundedness of e_n correlates with β_n. According to the results in [31], it can be obtained that β_n is bounded. Then, the following formula holds

$$\lim_{t\to\infty} |e_n| \leq \lim_{t\to\infty}(|\xi_n| + |\beta_n|) \leq \sqrt{\frac{2C}{D}} + \Delta, \tag{53}$$

where Δ represents a positive constant that satisfies $|\beta_n| \leq \Delta$.

This proves that e_n and ξ_n are bounded. Finally, all of the signals in (1) are all bounded. This completes the proof. □

Remark 3. *Even though the control strategy presented in this paper shows an outstanding control performance, it still has its limitations. For instance, the equivalent transformation technique depends on precise system models and parameters; large errors in parameter estimation might impact the control effectiveness. Furthermore, for extreme nonlinear effects, our method may need further refinement or combination with other techniques.*

Remark 4. *Refs. [27–30] investigated nonlinear systems with unknown control directions. However, these studies did not account for the error induced by the filter or the effects of an input dead zone and saturation. Unlike these studies, this paper employs command-filtering technology to address the complexity explosion issue and proposes a transformation method to mitigate the impact of an input dead zone and saturation on the system.*

4. Simulation Results

This section provides two illustrative examples to demonstrate the feasibility of the proposed approach.

Example 1. *The following second-order nonlinear system are considered*

$$\begin{cases} \dot{x}_1 = 0.1{x_1}^2 + \lambda_1 g_1(\bar{x}_1)x_2 \\ \dot{x}_2 = 0.2x_1x_2 + x_1 + \lambda_2 g_2(\bar{x}_2)u \\ y = x_1, \end{cases} \tag{54}$$

where $g_1(\bar{x}_1) = 4$, $g_2(\bar{x}_2) = 1$, $\lambda_1 = -1$, $\lambda_2 = 1$, *and* u *is defined as*

$$u = \begin{cases} 5, & v > 5 \\ 0.6(v - 0.6), & 0.6 < v < 5 \\ 0, & -0.6 < v < 0.6 \\ 0.6(v + 0.6), & -5 < v < 0.6 \\ -5, & v < -5. \end{cases} \tag{55}$$

The virtual controller α_1 is designed as

$$\begin{cases} \alpha_1 = N(\zeta_1)\left(\frac{\xi_1\hat{\theta}_1\psi_1^T\psi_1}{2a_1^2} + k_1e_1 + \frac{1}{2}\xi_1 + \frac{5}{2}g_1^2\xi_1 - \dot{y}_d\right), \\ \dot{\zeta}_1 = \xi_1\left(\frac{\xi_1\hat{\theta}_1\psi_1^T\psi_1}{2a_1^2} + k_1e_1 + \frac{1}{2}\xi_1 + \frac{5}{2}g_1^2\xi_1 - \dot{y}_d\right). \end{cases} \tag{56}$$

The controller v is designed as

$$\begin{cases} v = \frac{1}{\Im}N(\zeta_2)\left(\frac{\xi_2\hat{\theta}_2\psi_2^T\psi_2}{2a_2^2} + k_2e_2 + \xi_2 - \dot{\omega}_2\right), \\ \dot{\zeta}_2 = \xi_2\left(\frac{\xi_2\hat{\theta}_2\psi_2^T\psi_2}{2a_2^2} + k_2e_2 + \xi_2 - \dot{\omega}_2\right), \end{cases} \tag{57}$$

where the initial state variables of the system are $x_1(0) = 0.2$, $x_2(0) = -0.1$, $\hat{\theta}_1(0) = 0.5$, $\hat{\theta}_1(0) = 0.2$, and the desired trajectory $y_d = 0.5\sin(t)$. The design parameters are given as $k_1 = 2$, $k_2 = 1$, $\Im = 1$, $\sigma_1 = 0.08$, $\sigma_2 = 0.08$, $a_1 = 2$, $a_2 = 7$, $\Gamma_1 = 0.3$, $\Gamma_2 = 0.3$, $\varpi = 50$, and $\varphi = 1$ and select $N(\zeta_1) = \zeta_1^2\cos(\zeta_1)$ and $N(\zeta_2) = \zeta_2^2\cos(\zeta_2)$ with $\zeta_1(0) = 0$ and $\zeta_2(0) = 0$. In addition, to handle nonlinear terms, one might choose the following fuzzy membership function $\mu_{P_m^1} = e^{-\frac{\left(x_1+x_j^0\right)^2}{2}}$, $\mu_{P_m^2} = e^{-\frac{\left(x_2+x_j^0\right)^2}{2}}$, ${x_j}^0 = 3,\ 2,\ 1,\ 0\ ,-1,-2,-3,\ j = 1,\ldots 7.$

The simulation results are illustrated in Figures 1–6. The trajectories of the system output y and the reference signal y_d, using the control strategy proposed in this paper and the control strategy with the same design parameters from reference [27], are shown in Figure 1. According to Figure 1, we can see that the system output y can effectively track the reference signal y_d, and the control method proposed in this paper, which accounts for input dead zones and saturation, achieves a higher tracking accuracy compared with the control method proposed in reference [27]. Figures 2 and 3 show states x_1 and x_2 of the system and the trajectories of the adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$, respectively. Figure 4 shows the evolution of signals u and v. Figures 5 and 6 describe the Nussbaum function signals ζ_1, ζ_2, $N(\zeta_1)$, and $N(\zeta_2)$. Based on the above simulation results, it is evident that the proposed scheme achieves an excellent tracking performance, and all of the signals within the closed-loop system are bounded. This demonstrates the effectiveness of the proposed control scheme.

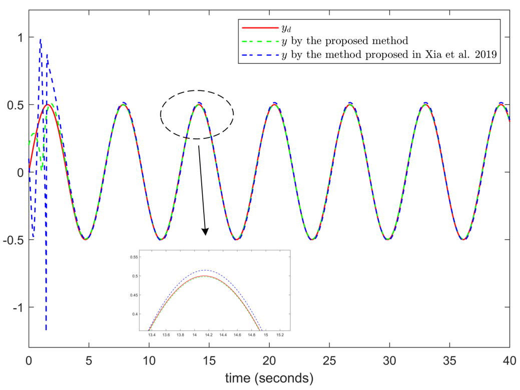

Figure 1. Trajectories of y_d and y [27].

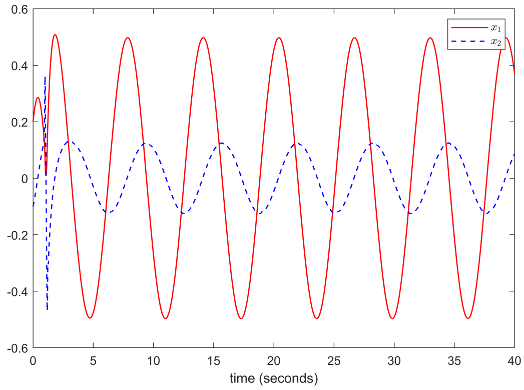

Figure 2. The trajectories of x_1 and x_2.

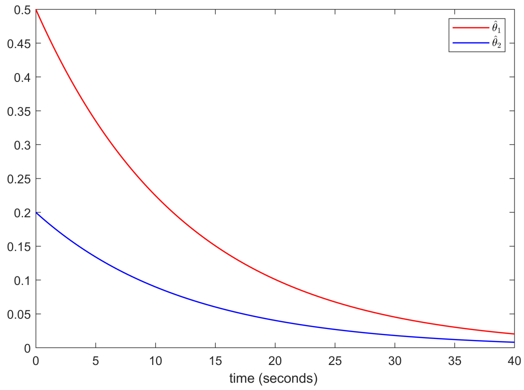

Figure 3. Adaptive parameters.

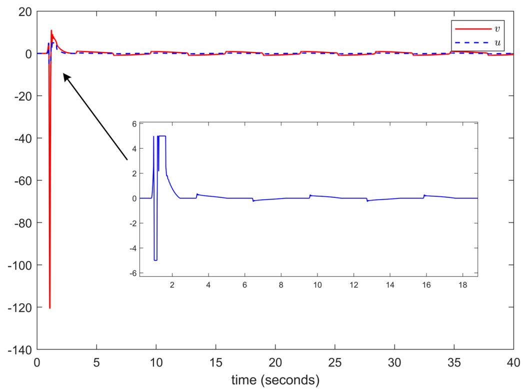

Figure 4. Trajectories of the control input.

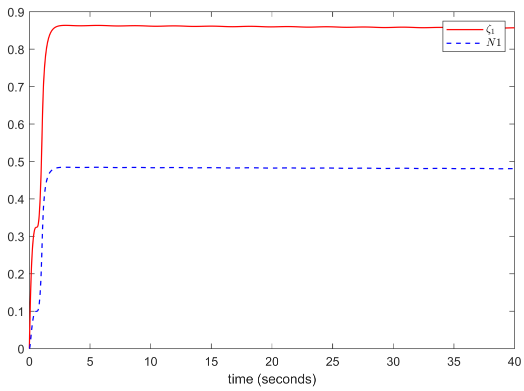

Figure 5. The trajectories of ζ_1 and $N(\zeta_1)$.

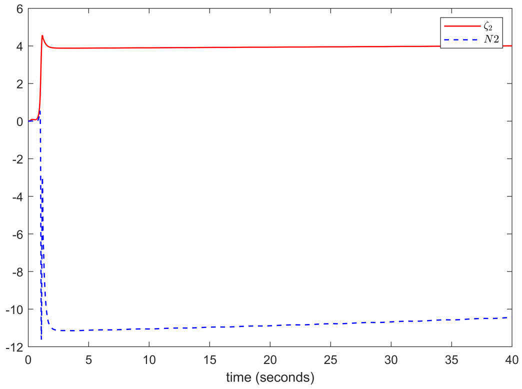

Figure 6. The trajectories of ζ_2 and $N(\zeta_2)$.

Example 2. *Consider the pendulum model shown in Figure 7.*

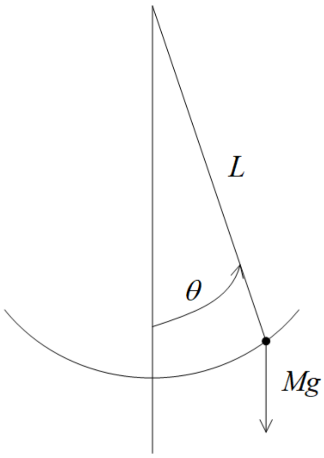

Figure 7. Pendulum.

Its equation of motion in the tangential direction can be written as

$$ML\ddot{\theta} + kL\dot{\theta} + Mg\sin\theta = u, \tag{58}$$

where $M = 1$ denotes the mass of the dot; θ is the angle subtended by the rod and the vertical axis through the pivot point; $k = 2$ represents the friction coefficient; $L = 1$ is the length of the rod; $g = 9.8$ represents the acceleration due to gravity; $\dot{\theta}$ and $\ddot{\theta}$ are angular velocity and angular acceleration, respectively.

Define $x_1 = \theta(t)$ and $x_2 = \dot{\theta}(t)$. Then, the state equations are

$$\begin{cases} \dot{x}_1 = \lambda_1 g_1(\bar{x}_1)x_2 \\ \dot{x}_2 = -2x_2 - 10\sin(x_1) + \lambda_2 g_2(\bar{x}_1)u \\ y = x_1, \end{cases} \tag{59}$$

where $\lambda_1 = 1$, $\lambda_2 = -1$, $g_1(\bar{x}_1) = 1$, $g_2(\bar{x}_1) = 1$, and u is defined as

$$u = \begin{cases} 20, & v > 20 \\ 0.5(v - 0.5), & 0.5 < v < 20 \\ 0, & -0.5 < v < 0.5 \\ 0.5(v + 0.5), & -20 < v < 0.5 \\ -20, & v < -20 \end{cases} \tag{60}$$

The virtual controller, controller, desired signal, Nussbaunm functions, and fuzzy membership function designs are similar to those in Example 1. The initial parameters are chosen as $x_1(0) = 0.2$, $x_2(0) = -0.1$, $\hat{\theta}_1(0) = 0.3$, and $\hat{\theta}_1(0) = 0.5$, and the desired trajectory is $y_d = 0.5\sin(t)$. The design parameters are $k_1 = 4$, $k_2 = 3$, $\Im = 1$, $\sigma_1 = 1$, $\sigma_2 = 1$, $a_1 = 10$, $a_2 = 10$, $\Gamma_1 = 0.7$, $\Gamma_2 = 0.7$, $\varpi = 50$, and $\varphi = 1$.

The simulation results are depicted in Figures 8–13. The above simulation results show that the developed adaptive command-filtered fuzzy control scheme achieves a satisfactory tracking performance, with all of the signals in the control system remaining bounded.

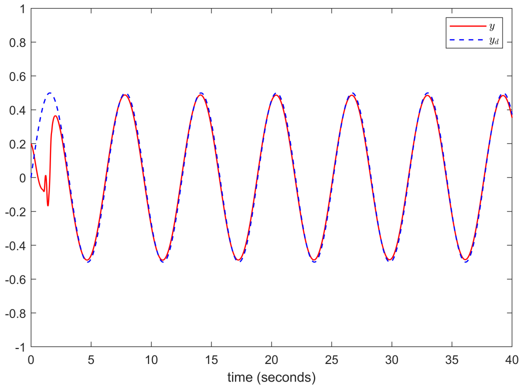

Figure 8. The trajectories of y_d and y.

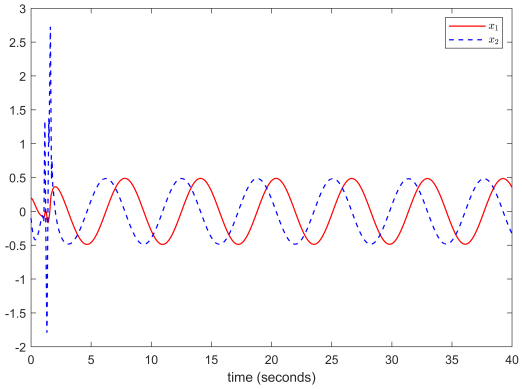

Figure 9. The trajectories of x_1 and x_2.

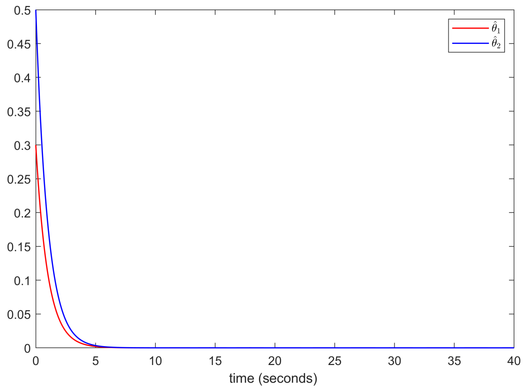

Figure 10. Adaptive parameters.

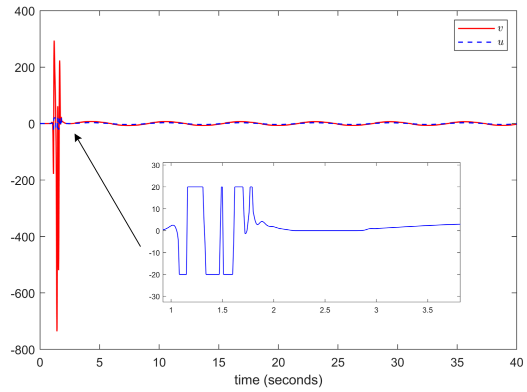

Figure 11. The trajectories of the control input.

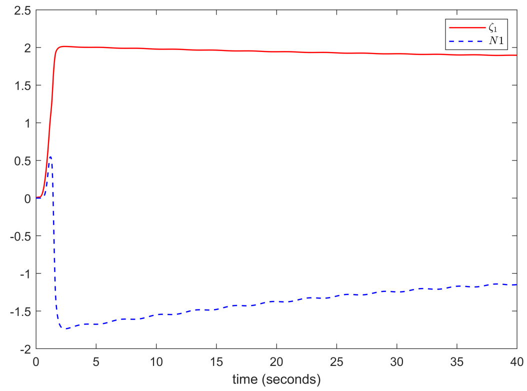

Figure 12. The trajectories of ζ_1 and $N(\zeta_1)$.

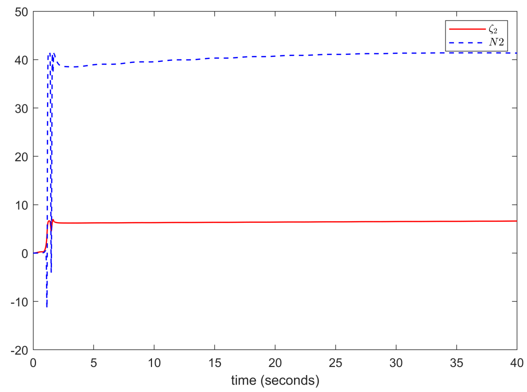

Figure 13. The trajectories of ζ_2 and $N(\zeta_2)$.

5. Conclusions

This paper proposes a command-filtering adaptive fuzzy tracking control strategy for nonlinear systems with unknown control directions, input dead zones, and saturation. A novel approach is applied to analyze the effects of input dead zone and saturation. By combining the fuzzy logic system and the command filter, an adaptive fuzzy logic controller is constructed to ensure that the error signal converges to a bounded compact set around the origin. The combination of the Nussbaum function and the backstepping method solves the difficulty caused by the unknown system control direction. Based on the adaptive tracking controller proposed in this article, the boundedness of all signals in the closed-loop system is guaranteed.

Funding: This research received no external funding.

Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

Conflicts of Interest: The author declares no conflicts of interest.

References

1. Xu, H.; Zhu, Q.; Zheng, W.X. Exponential stability of stochastic nonlinear delay systems subject to multiple periodic impulses. *IEEE Trans. Autom. Control* **2024**, *69*, 2621–2628. [CrossRef]
2. Fan, L.; Zhu, Q.; Zheng, W.X. Stability analysis of switched stochastic nonlinear systems with state-dependent delay. *IEEE Trans. Autom. Control* **2024**, *69*, 2567–2574. [CrossRef]
3. Zhao, Y.; Zhu, Q. Stabilization of stochastic highly nonlinear delay systems with neutral term. *IEEE Trans. Autom. Control* **2023**, *68*, 2544–2551. [CrossRef]
4. Do, K.D.; Pan, J. Nonlinear formation control of unicycle-type mobile robots. *Robot. Auton. Syst.* **2007**, *55*, 191–204. [CrossRef]
5. Merabti, H.; Bouchachi, I.; Belarbi, K. Nonlinear model predictive control of quadcopter. In Proceedings of the 2015 16th International Conference on Sciences and Techniques of Automatic Control and Computer Engineering (STA), Monastir, Tunisia, 21–23 December 2015; IEEE: Piscataway, NJ, USA, 2015; pp. 208–211.
6. Geng, X.; Zhan, D.; Zhou, Z. Supervised nonlinear dimensionality reduction for visualization and classification. *IEEE Trans. Syst. Man Cybern. Part B Cybern.* **2005**, *35*, 1098–1107. [CrossRef]
7. Goege, D.; Fuellekrug, U.; Sinapius, M.; Link, M.; Gaul, L. Advanced test strategy for identification and characterization of nonlinearities of aerospace structures. *AIAA J.* **2005**, *43*, 974–986. [CrossRef]
8. Liu, Y.; Zhao, W.; Liu, L.; Li, D.; Tong, S.; Chen, C.P. Adaptive neural network control for a class of nonlinear systems with function constraints on states. *IEEE Trans. Neural Netw. Learn. Syst.* **2021**, *34*, 2732–2741. [CrossRef]
9. Zhang, J.; Niu, B.; Wang, D.; Wang, H.; Duan, P.; Zong, G. Adaptive neural control of nonlinear nonstrict feedback systems with full-state constraints: A novel nonlinear mapping method. *IEEE Trans. Neural Netw. Learn. Syst.* **2021**, *34*, 999–1007. [CrossRef]
10. Al Mahturi, A.; Santoso, F.; Garratt, M.A.; Anavatti, S.G. A Robust Self-Adaptive Interval Type-2 TS Fuzzy Logic for Controlling Multi-Input–Multi-Output Nonlinear Uncertain Dynamical Systems. *IEEE Trans. Syst. Man Cybern. Syst.* **2020**, *52*, 655–666. [CrossRef]
11. Park, J.H.; Kim, S.H.; Park, T.S. Output-feedback adaptive neural controller for uncertain pure-feedback nonlinear systems using a high-order sliding mode observer. *IEEE Trans. Neural Netw. Learn. Syst.* **2018**, *30*, 1596–1601. [CrossRef]
12. Wu, J.; Wu, Z.; Li, J.; Wang, G.; Zhao, H.; Chen, W. Practical adaptive fuzzy control of nonlinear pure-feedback systems with quantized nonlinearity input. *IEEE Trans. Syst. Man Cybern. Syst.* **2018**, *49*, 638–648. [CrossRef]
13. Li, Y.; Yang, G. Adaptive neural control of pure-feedback nonlinear systems with event-triggered communications. *IEEE Trans. Neural Netw. Learn. Syst.* **2018**, *29*, 6242–6251. [CrossRef] [PubMed]
14. Swaroop, D.; Hedrick, J.K.; Yip, P.P.; Gerdes, J.C. Dynamic surface control for a class of nonlinear systems. *IEEE Trans. Autom. Control* **2000**, *45*, 1893–1899. [CrossRef]
15. Zhang, Z.; Wen, C.; Xing, L.; Song, Y. Event-triggered adaptive control for a class of nonlinear systems with mismatched uncertainties via intermittent and faulty output feedback. *IEEE Trans. Autom. Control* **2023**, *68*, 8142–8149. [CrossRef]
16. Huang, S.; Zong, G.; Wang, H.; Zhao, X.; Alharbi, K.H. Command filter-based adaptive fuzzy self-triggered control for MIMO nonlinear systems with time-varying full-state constraints. *Int. J. Fuzzy Syst.* **2023**, *25*, 3144–3161. [CrossRef]
17. Yang, Y.; Tang, L.; Zou, W.; Ding, D. Robust adaptive control of uncertain nonlinear systems with unmodeled dynamics using command filter. *Int. J. Robust Nonlinear Control* **2021**, *31*, 7764–7784. [CrossRef]
18. Wang, L.; Wang, H.; Liu, P.X.; Ling, S.; Liu, S. Fuzzy finite-time command filtering output feedback control of nonlinear systems. *IEEE Trans. Fuzzy Syst.* **2020**, *30*, 97–107. [CrossRef]
19. Yu, H.; Yu, J.; Wang, Q.; Lin, C. Time-varying BLFs-based adaptive neural network finite-time command-filtered control for nonlinear systems. *IEEE Trans. Syst. Man Cybern. Syst.* **2023**, *53*, 4696–4704. [CrossRef]
20. Tian, M.; Tao, G. Adaptive dead-zone compensation for output-feedback canonical systems. *Int. J. Control* **1997**, *67*, 791–812. [CrossRef]
21. Chen, M.; Tao, G. Adaptive fault-tolerant control of uncertain nonlinear large-scale systems with unknown dead zone. *IEEE Trans. Cybern.* **2015**, *46*, 1851–1862. [CrossRef]
22. Kumar, R.; Singh, U.P.; Bali, A.; Raj, K. Hybrid neural network controller for uncertain nonlinear discrete-time systems with non-symmetric dead zone and unknown disturbances. *Int. J. Control* **2023**, *96*, 2003–2011. [CrossRef]
23. Tang, F.; Wang, H.; Zhang, L.; Xu, N.; Ahmad, A.M. Adaptive optimized consensus control for a class of nonlinear multi-agent systems with asymmetric input saturation constraints and hybrid faults. *Commun. Nonlinear Sci. Numer. Simul.* **2023**, *126*, 107446. [CrossRef]
24. Zhao, N.; Tian, Y.; Zhang, H.; Herrera-Viedma, E. Learning-Based Adaptive Fuzzy Output Feedback Control for MIMO Nonlinear Systems With Deception Attacks and Input Saturation. *IEEE Trans. Fuzzy Syst.* **2024**, *32*, 2850–2862. [CrossRef]
25. Wang, H.; Kang, S.; Zhao, X.; Xu, N.; Li, T. Command filter-based adaptive neural control design for nonstrict-feedback nonlinear systems with multiple actuator constraints. *IEEE Trans. Cybern.* **2021**, *52*, 12561–12570. [CrossRef]

26. Lu, Y.; Liu, W.; Ma, B. Finite-time command filtered tracking control for time-varying full state-constrained nonlinear systems with unknown input delay. *IEEE Trans. Circuits Syst. II Express Briefs* **2022**, *69*, 4954–4958. [CrossRef]
27. Xia, J.; Zhang, J.; Feng, J.; Wang, Z.; Zhuang, G. Command filter-based adaptive fuzzy control for nonlinear systems with unknown control directions. *IEEE Trans. Syst. Man Cybern. Syst.* **2019**, *51*, 1945–1953. [CrossRef]
28. Yang, Y.; Tang, L.; Zou, W.; Ahn, C.K. Novel command-filtered Nussbaum design for continuous-time nonlinear dynamical systems with multiple unknown high-frequency gains. *Nonlinear Dyn.* **2023**, *111*, 4313–4323. [CrossRef]
29. Yang, Y.; Liu, G.; Li, Q.; Ahn, C.K. Multiple adaptive fuzzy Nussbaum-type functions design for stochastic nonlinear systems with fixed-time performance. *Fuzzy Sets Syst.* **2024**, *476*, 108767. [CrossRef]
30. Ye, H.; Zhao, K.; Wu, H.; Song, Y. Adaptive control with global exponential stability for parameter-varying nonlinear systems under unknown control gains. *IEEE Trans. Cybern.* **2023**, *53*, 7858–7867. [CrossRef]
31. Hu, J.; Zhang, H. Immersion and invariance based command-filtered adaptive backstepping control of VTOL vehicles. *Automatica* **2013**, *49*, 2160–2167. [CrossRef]