

Article

Mechanical Properties and Constitutive Model for Artificially Structured Soils under Undrained Conditions

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Abstract: Triaxial tests are performed for remolded, artificially isotropic, and anisotropic structured samples under undrained conditions at confining pressures of 25, 100, and 200 kPa. Based on these test results, a binary-medium constitutive model is formulated based on homogenization theory and a breakage mechanism to describe the behaviors of structured soils. In this model, the binary-medium material is idealized as a representative volume element (RVE) composed of bonded elements, whose mechanical behaviors are expressed by the linearly elastic model, and frictional elements, whose mechanical behaviors are described by the double-yield surfaces constitutive model. The parameters of the bonded and frictional elements are determined from the test results of structured and remolded samples, respectively. The expressions for the breakage ratio and local stress coefficient matrix are introduced, and their parameters are provided. The computed results are compared with the test results, demonstrating that the model can reflect the main deformation features of structured soil relatively well, including the influence of anisotropy, gradual damage to particle bonding, and pore development.

Keywords: artificially structured soils; undrained triaxial tests; binary-medium constitutive model; initially stress-induced anisotropy

MSC: 74Q15; 74Q10



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1. Introduction

During sedimentation, a ‘structure’ forms in natural soils. The deformation and strength characteristics of natural soils are influenced by this structure, which is bound to evolve with the deformation of natural soils. The mechanical properties of natural soils differ from those of reconstituted or remolded soils as a result of the restraint imposed by this structure [1], which consists of ‘fabric’—the arrangement of soil particles—and ‘bonding’, reflected in interparticle forces [2–4]. In the process of loading on natural soils, the fabric changes and bonding gradually breaks. Therefore, investigating the effect of this structure on the mechanical properties of natural soils is necessary. In addition, a constitutive model that considers the evolution of the structure can objectively reflect the deformation and failure of natural soils.

With the aim of investigating the influence of this structure, many compression tests on natural and remolded soils have been conducted by scholars. The rules of nonlinear compression curves and the influential intrinsic parameters were summarized in [5–8]. It has been concluded that the compressibility of remolded soils is determined by the liquid limit, the remolded yield stress, and the void ratio at the remolded yield stress [1,7]. The compression characteristics of natural clays can be divided into three periods: elastic deformation, gradual damage to soil structure, and ultimately mechanical behaviors similar to those of remolded soils. However, the factors influencing the compression characteristics

of natural soils are still more diverse and complex, such as cementation, initial void ratio, and initial water content. In order to avoid being influenced by sample disturbances, artificially structured soils were prepared, where cement and salt particles form bonds and voids in the soils, respectively [9–12]. It was observed that the adhesive properties of cement and voids caused by washed salt particles significantly change the strength and fabric of structured soils compared to remolded soils. To better understand the yielding, strength, anisotropy, and deformation features of structured and remolded soils, numerous triaxial and true triaxial tests under drained and undrained conditions have been performed [13–16], and the effects of the loading rate [17] and stress path on stress–strain behaviors in natural soils have also been discussed [18–20].

In studying the macroscopic stress–strain behaviors of structured soils, several constitutive models have been used, including the elastoplastic model [21–23], modified Cam-clay model [24–26], and kinematic hardening or bounding-surface model [27–29]. Structured soils exhibit breakage of interparticle bonding compared with remolded soils, and the damage to the structure extends gradually under continuous loading. The higher the degree of structural damage, the closer the macroscopic stress–strain properties of structured soils resemble those of remolded soils. Recently, researchers have paid more attention to constitutive models that consider the influence of soil or rock structure, such as the disturbed state concept (DSC) model [30,31], damage mechanical model [32–34], and micromechanical model [35,36].

Strain softening, volumetric contraction followed by dilatancy, and the appearance of shear bands have been found in structured soils subjected to relatively lower stress under triaxial and biaxial conditions. However, strain hardening and volumetric contraction occur under relatively higher stress. Disturbances and structural damage promote a nonuniform distribution of stress and strain in soil elements, where higher or equal local stress relative to the strength of the bonds causes the rupture of the bonds. Because of the deposition or pre-compaction process, the void ratio of structured soils with initial stress-induced anisotropy is much lower than that of isotropic structured soils, which also affects the initial stiffness of the soils [37,38]. Therefore, formulating a constitutive model for structured soils that considers the nonuniform stress and strain of soil elements and uses parameters from microscopic mechanisms to reflect strain softening is significant.

To provide some guidance in engineering design, such as rapid drawdown, rapid flood, and earthquake loading, soils are consolidated under long-term effective stress and pore water pressure. The mechanical characteristics of soil under undrained conditions are affected by pore water pressure, and thus the results of undrained triaxial tests on consolidated soils could provide evidence for design and research [39–41]. In this paper, undrained triaxial tests are performed on artificially isotropic and anisotropic structured soils subjected to static loading at three different confining pressures of 25 kPa, 100 kPa, and 200 kPa, and the mechanical behaviors of these soils are presented and analyzed. Referring to an existing binary-medium constitutive model [42], a new constitutive model is formulated for artificially isotropic and initially anisotropic structured soils under undrained triaxial tests. Based on the test results, the constitutive model parameters are determined and the model is verified. The proposed constitutive model describes the evolution of stress and deformation in structured soils under different consolidated stress and stress ratios throughout the loading process. Compared with previous studies, the constitutive model has good applicability and can be applied in geotechnical engineering under similar conditions.

2. Test Results of Structured Soils

The samples of remolded, isotropic, and initially anisotropic artificially structured soils are prepared following the method of Luo and Liu [38,43], and undrained triaxial tests for consolidated samples are completed using a GCTS (Tempe, AZ, USA) triaxial system.

2.1. Sample Preparation

The material used to prepare artificially structured soils here is a mixture of silty clay, kaolin clay, cement, and highly purified salt particles. In the structured soil sample, silty clay is the main matrix, kaolin clay increases the content of fine particles, cement provides interparticle bonding, and salt particles generate large pores through dissolution. The silty clay is sourced from about 5 m below the ground surface of a construction site in Chengdu, with a specific gravity, G_s , of 2.72, and a liquid limit, w_L , and plastic limit, w_P , of 29.11% and 17.06%, respectively. After drying, the silty clay is sieved through a 0.5 mm screen, and the characteristic particle sizes are $d_{50} = 0.032$ mm, $C_u = d_{60}/d_{10} = 39.17 > 5$, and $C_c = d_{30}^2/(d_{60}d_{10}) = 2.3 \in (1, 3)$, according to the gradation determined through sieving, where C_u is the coefficient of uniformity and C_c is the coefficient of curvature. From this, it can be determined that the main matrix material is fine-grained soil of a well-graded type. M1 industrial kaolin clay and 32.5R composite Portland cement are used. The mass fractions of silty clay, kaolin clay, cement, and salt particles are 65, 20, 5, and 10%, respectively. To form the samples, the uniform dry mixture is compacted in a mold in four layers to achieve a dry bulk density of 1.49 g/cm^3 . The compacted samples are vacuum-sealed in a vacuum chamber, and about 1 h later, distilled water is slowly added to fully soak the samples. Then, the samples are cured in running water, where the salt dissolves and the cement generates some materials, which results in the formation of large pores and bonding within the samples. When the measured salt content of the surrounding water reaches the maximum value of flowing water, the salt particles are considered thoroughly dissolved. After a 7-day cure, the samples, with a dry bulk density of 1.34 g/cm^3 and a void ratio of 1.253, are taken out and tested in a triaxial apparatus. The soil grading curves of the structured soils and test apparatus are shown in Figure 1.

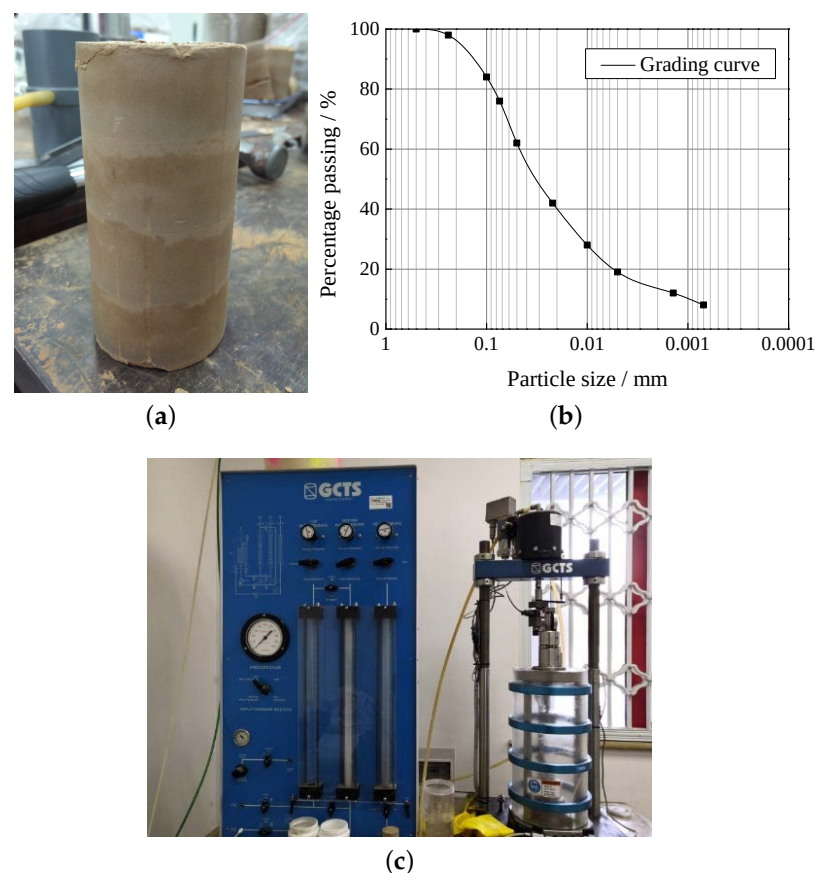


Figure 1. (a) Sample of structured soils. (b) Grading curve. (c) The GCTS triaxial system.

The natural soils are deposited in geological environments for a long time, exhibiting properties similar to anisotropic materials, which exhibit different horizontal and vertical properties. The method of applying an external load of 70 kPa to the upper end of the samples during the curing process is adopted to simulate the stress state of natural soils [38], resulting in different lateral confinements. This way, the properties of the samples are different in the vertical and horizontal directions, forming transversely isotropic samples, also called initially stress-induced anisotropic structured soils, after hydration. To investigate the structural effects on the mechanical characteristics of soils, the remolded samples are prepared using the structured soil samples tested, which are dried and sieved through a 0.5 mm screen.

2.2. Results and Analysis

The samples were loaded under undrained conditions and confining pressures of 25, 100, and 200 kPa, with a loading rate of 0.06 mm/min after consolidation. The results of the triaxial compression tests on remolded, isotropic, and initially stress-induced anisotropic structured soils are presented in Figure 1, showing the stress–strain curves and the pore water pressure versus axial strain curves. In Figure 2, x kPa (RS) means that the remolded sample was tested under a confining pressure of x kPa, while ISS and ASS denote the isotropic and anisotropic structured samples, respectively.

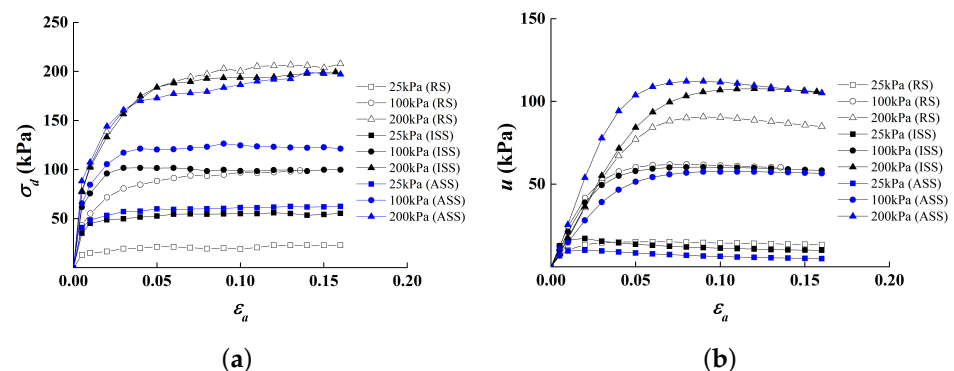


Figure 2. Curves of the tested results (RS, ISS, and ASS represent the remolded, isotropic, and anisotropic structured soils, respectively) under undrained triaxial tests at confining pressures of 25, 100, and 200 kPa: (a) Curves of deviatoric stress versus axial strain. (b) Curves of pore water pressure versus axial strain (σ_d is the deviatoric stress, ϵ_a is the axial strain, and u is the pore water pressure).

We found that the mechanical properties of the remolded samples and the two kinds of structured soils were significantly different. For remolded soils, strain-hardening behaviors were observed from the stress–strain curves at confining pressures ranging from 25 to 200 kPa, and all curves of the pore water pressure had a peak value. The isotropic and anisotropic structured soils exhibited slight strain-softening at lower confining pressures (25 and 100 kPa) and strain-hardening at a higher confining pressure (200 kPa), where the curves were closer to those of remolded soils. Because of the existence of bonding among soil particles in the two types of structured soils, their strengths were obviously greater, and the pore water pressure was smaller than in remolded soils at lower confining pressures, but the remolded soils showed the highest strength and smallest pore water pressure at the relatively higher confining pressure.

Differences were also found between the behaviors of isotropic structured soils and initially stress-induced anisotropic structured soils. Interparticle bonding was stronger in anisotropic structured soils due to preloading during the curing process; thus, the strength and stiffness were higher, and the produced pore water pressure was smaller at lower confining pressures.

3. Constitutive Model for Artificially Structured Soils

From previous studies by researchers and the test results in this paper, it is evident that structure has a great influence on the mechanical properties of soils. The cohesive and frictional resistances together enhance the bearing capacity of structured soil elements, where the cohesive resistance reaches a peak value when the strain is relatively smaller, while frictional resistance plays a crucial role when the strain is relatively larger.

In structured soils, the cohesive component and frictional complement exhibit brittle and nonlinear elastic behaviors, respectively, and cohesion is primarily provided by cementation bonding between particles. Bonding blocks form where the cementation strength is high; otherwise, weakened bands form. These bonded blocks and weakened bands distribute inhomogeneously, and during the loading process, the bonded blocks may break up gradually and turn into weakened bands, which exhibit elastoplastic behavior.

As shown in Figure 3, structured soils can be conceptualized as an element of heterogeneous materials consisting of bonded blocks, referred to as bonded elements, and weakened bands, referred to as frictional elements. Based on homogenization theory for heterogeneous materials [33,43,44], a representative volume element (RVE) of artificially structured soils is presented as a binary-medium material composed of bonded elements and frictional elements. Therefore, the progressive breakage of a structured soil sample can be considered as a process where bonded elements break up and transform into frictional elements, leading to strain-softening or strain-hardening behaviors depending on the decrease in the bearing capacity of bonded elements and the increase in the bearing capacity of frictional elements. The mechanical behaviors of bonded and frictional elements are schematically depicted in Figure 4. Under external loading, bonded elements undergo elastic deformation when the stress is less than the bond strength (q), and fail and break up when the stress reaches q ; elastoplastic behavior is generated in the frictional elements formed by the broken bonded elements.

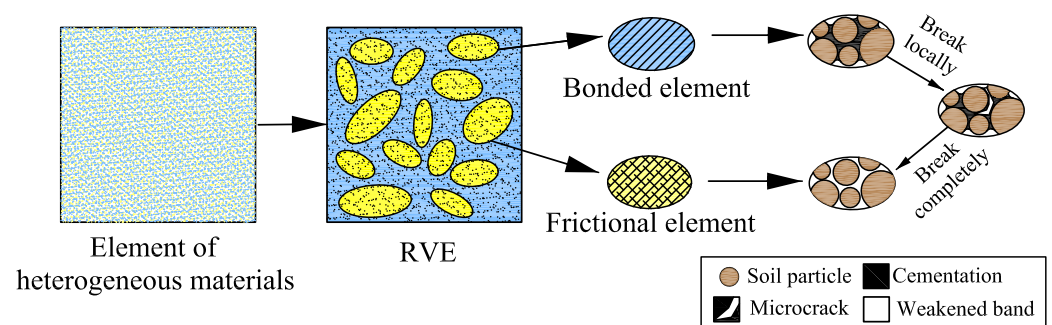


Figure 3. Schematic diagram of the heterogeneous materials and microscopic distribution of structured soils.

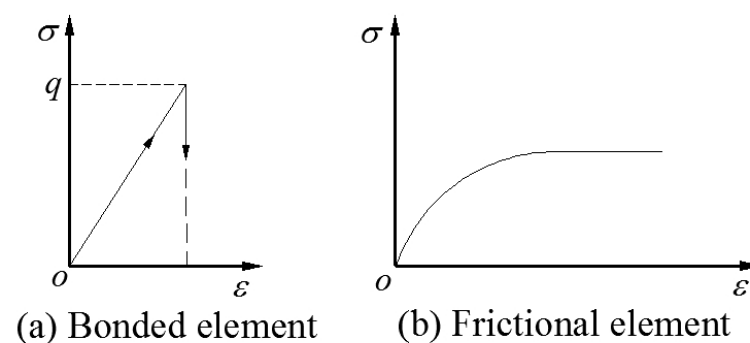


Figure 4. Schematic diagram of the mechanical behaviors of (a) bonded elements and (b) frictional elements.

3.1. Formulation of Binary-Medium Constitutive Model

According to the theory of the binary-medium constitutive model, for an RVE, the local stress and local strain are, respectively, denoted as σ_{ij}^{local} and ϵ_{ij}^{local} . The average stress, σ_{ij} , and the average strain, ϵ_{ij} , can be expressed as follows:

$$\sigma_{ij} = \frac{1}{V} \int \sigma_{ij}^{local} dV \quad (1a)$$

$$\epsilon_{ij} = \frac{1}{V} \int \epsilon_{ij}^{local} dV \quad (1b)$$

where V is the volume of the RVE.

By denoting σ_{ij}^b and σ_{ij}^f (ϵ_{ij}^b and ϵ_{ij}^f) as the stresses (strains) of bonded and frictional elements, respectively, the average stress and average strain can be calculated as follows:

$$\sigma_{ij} = (1 - \lambda)\sigma_{ij}^b + \lambda\sigma_{ij}^f \quad (2a)$$

$$\epsilon_{ij} = (1 - \lambda)\epsilon_{ij}^b + \lambda\epsilon_{ij}^f \quad (2b)$$

where λ is the breakage ratio, defined as the volume ratio of frictional elements to the RVE volume, and is assumed to be a function of the stress:

$$\lambda = \frac{v_f}{v} = f(\sigma_{ij}) \quad (3)$$

By derivation of Equations (2a) and (2b), the following incremental expressions for the stress and strain can be obtained:

$$d\sigma_{ij} = (1 - \lambda^0)d\sigma_{ij}^b + \lambda^0 d\sigma_{ij}^f + d\lambda(\sigma_{ij}^{f0} - \sigma_{ij}^{b0}) \quad (4a)$$

$$d\epsilon_{ij} = (1 - \lambda^0)d\epsilon_{ij}^b + \lambda^0 d\epsilon_{ij}^f + d\lambda(\epsilon_{ij}^{f0} - \epsilon_{ij}^{b0}) \quad (4b)$$

where $d\sigma_{ij}$ and $d\epsilon_{ij}$ are the increments of the average stress and strain of the RVE; λ^0 is the current breakage ratio; $d\lambda$ is the increment of the breakage ratio; σ_{ij}^{b0} and σ_{ij}^{f0} (ϵ_{ij}^{b0} and ϵ_{ij}^{f0}) are the current stress (strain) of bonded and frictional elements, respectively; and $d\sigma_{ij}^{f0}$ ($d\epsilon_{ij}^{b0}$ and $d\epsilon_{ij}^{f0}$) are the increments of the stress (strain) of bonded and frictional elements, respectively. Using D_{ijkl}^b and D_{ijkl}^f to denote the tangential stiffness matrices of bonded and frictional elements, the relationships for the incremental stress and incremental strain are given by:

$$d\sigma_{ij}^b = D_{ijkl}^b d\epsilon_{kl}^b \quad (5a)$$

$$d\sigma_{ij}^f = D_{ijkl}^f d\epsilon_{kl}^f \quad (5b)$$

According to Equations (4a) and (5a), the following equation can be obtained:

$$d\epsilon_{ij}^b = (D_{ijkl}^b)^{-1} d\sigma_{ij}^b = \frac{1}{1 - \lambda^0} (D_{ijkl}^b)^{-1} [d\sigma_{kl} - \lambda^0 d\sigma_{kl}^f - d\lambda(\sigma_{kl}^{f0} - \sigma_{kl}^{b0})] \quad (6)$$

where $(D_{ijkl}^b)^{-1}$ is the inverse matrix of the tangential stiffness matrix of bonded elements.

The local stress matrix, C_{ijkl} , represents the relationship between the stress of frictional elements and the average stress of the RVE as follows:

$$\sigma_{ij}^f = C_{ijkl} \sigma_{kl} \quad (7)$$

The incremental expression of Equation (7) is:

$$d\sigma_{ij}^f = C_{ijkl}^0 d\sigma_{kl} + dC_{ijkl} \sigma_{kl}^0 \quad (8)$$

where C_{ijkl}^0 is the current local stress matrix.

Substituting Equations (6)–(8) into Equation (4b) with some manipulations, the relationship between the incremental average strain and incremental average stress of the RVE is given as follows:

$$\begin{aligned} d\varepsilon_{ij} = & \left[\lambda^0 \left(\left(D_{ijts}^f \right)^{-1} - \left(D_{ijts}^b \right)^{-1} \right) C_{tskl}^0 + \left(D_{ijkl}^b \right)^{-1} \right] d\sigma_{kl} \\ & + \lambda^0 \left(\left(D_{ijts}^f \right)^{-1} - \left(D_{ijts}^b \right)^{-1} \right) dC_{tskl} \sigma_{kl}^0 \\ & - d\lambda \left(D_{ijkl}^b \right)^{-1} (\sigma_{kl}^{f0} - \sigma_{kl}^{b0}) + d\lambda (\varepsilon_{ij}^{f0} - \varepsilon_{ij}^{b0}) \end{aligned} \quad (9)$$

where $(D_{ijts}^f)^{-1}$ is the inverse matrix of the tangential stiffness matrix of frictional elements and dC_{tskl} is the incremental local stress matrix. These parameters must be determined, and they can be divided into four parts, including the constitutive relationships of bonded and frictional elements, the breakage ratio, and the local stress matrix.

It is considered that $\sigma_{kl}^{f0} = \sigma_{kl}^{b0}$, $\varepsilon_{ij} = 0$, $\varepsilon_{ij}^b = 0$, and $\varepsilon_{ij}^f = 0$ at the beginning. Thus, the average strain of the RVE at initial loading is expressed as:

$$\begin{aligned} d\varepsilon_{ij} = & \left[\lambda^0 \left(\left(D_{ijts}^f \right)^{-1} - \left(D_{ijts}^b \right)^{-1} \right) C_{tskl}^0 + \left(D_{ijkl}^b \right)^{-1} \right] d\sigma_{kl} \\ & + \lambda^0 \left(\left(D_{ijts}^f \right)^{-1} - \left(D_{ijts}^b \right)^{-1} \right) dC_{tskl} \sigma_{kl}^0 \end{aligned} \quad (10)$$

3.2. Constitutive Relationship of Bonded Elements

Due to sedimentation, the mechanical properties of natural soils show horizontal isotropy and differences in the horizontal and vertical directions. It is assumed here that bonded elements behave as linearly elastic-brittle and cross-anisotropic materials in artificially structured soils [43]. Here, the vertical direction (z-axis) is taken as the symmetry axis, with the x-axis and y-axis in the horizontal plane. The stress–strain relationship of bonded elements can be derived from Equation (5a):

$$\begin{Bmatrix} d\varepsilon_x \\ d\varepsilon_y \\ d\varepsilon_z \\ d\varepsilon_{yz} \\ d\varepsilon_{zx} \\ d\varepsilon_{xy} \end{Bmatrix}_b = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 \\ D_{12} & D_{11} & D_{13} & 0 & 0 & 0 \\ D_{13} & D_{13} & D_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{D_{11}-D_{12}}{2} \end{bmatrix}_b^{-1} \begin{Bmatrix} d\sigma_x \\ d\sigma_y \\ d\sigma_z \\ d\sigma_{yz} \\ d\sigma_{zx} \\ d\sigma_{xy} \end{Bmatrix}_b \quad (11)$$

where D_{11} , D_{12} , D_{13} , D_{33} , and D_{44} are material constants, and they can be determined from the stress–strain curves during the initial loading stage of structured samples because the structured soils are regarded as elastic initially.

3.3. Constitutive Relationship of Frictional Elements

The frictional elements are formed from the broken bonded elements generated during the loading process, and the mechanical properties of the frictional elements can be assumed to be those of isotropic remolded soils. Owing to the complexity of the mechanical behaviors of soils, a single yield surface is insufficient to explain the changes in the deformation or pore water pressure, so two or more yield surfaces are adopted and used by some researchers [45–47]. The calculated value of the deformation in the elastoplastic constitutive

model with double-yield surfaces lies between the nonlinear elastic constitutive model and the Cam-clay model, making it closer to reality and more adaptive for soils subjected to different stress paths [45]. Therefore, the constitutive relationship of friction elements is described by the constitutive model with double-yield surfaces.

The two yield surfaces corresponding to the volumetric compression deformation, f_1 , and shear deformation, f_2 , adopted in this paper, are determined following Shen [45]:

$$f_1 = (p^f)^2 + r^2 (\tau^f)^2 \quad (12a)$$

$$f_2 = \frac{(\tau^f)^s}{(p^f)^{s-1}} \quad (12b)$$

where s and r are the parameters of the yield surfaces, and p and τ are the octahedral compressive stress and octahedral shear stress, respectively, which are defined as follows:

$$p^f = \frac{1}{3}(\sigma_1^f + \sigma_2^f + \sigma_3^f) \quad (13a)$$

$$\tau^f = \frac{1}{3} \sqrt{(\sigma_1^f - \sigma_2^f)^2 + (\sigma_2^f - \sigma_3^f)^2 + (\sigma_3^f - \sigma_1^f)^2} \quad (13b)$$

where σ_1^f , σ_2^f , and σ_3^f are the principal stresses of frictional elements.

With respect to p^f and τ^f , the derivatives of f_1 and f_2 are:

$$\frac{\partial f_1}{\partial p^f} = 2p^f \quad (14a)$$

$$\frac{\partial f_2}{\partial p^f} = -\frac{(s-1)(\tau^f)^s}{(p^f)^s} \quad (14b)$$

$$\frac{\partial f_1}{\partial \tau^f} = 2r^2 \tau^f \quad (14c)$$

$$\frac{\partial f_2}{\partial \tau^f} = \frac{s(\tau^f)^{s-1}}{(p^f)^{s-1}} \quad (14d)$$

Therefore, the incremental forms of f_1 and f_2 , df_1 and df_2 , can be expressed as follows:

$$df_1 = 2p^f dp^f + 2r^2 \tau^f d\tau^f \quad (15a)$$

$$df_2 = -\frac{(s-1)(\tau^f)^s}{(p^f)^s} dp^f + \frac{s(\tau^f)^{s-1}}{(p^f)^{s-1}} d\tau^f \quad (15b)$$

where dp^f and $d\tau^f$ denote the incremental octahedral compressive stress and incremental octahedral shear stress of frictional elements, respectively.

During the loading process, plastic volumetric strain, plastic shear strain, or both, are generated in the frictional elements. The strain of the frictional elements can be divided into two parts: the volumetric strain, ε_v^f , caused by yielding at the compression yield surface, and the octahedral shear strain, ε_s^f , caused by yielding at the shear yield surface. The relative increments of the volumetric strain and shear strain, $d\varepsilon_v^f$ and $d\varepsilon_s^f$, are expressed as follows:

$$d\varepsilon_v^f = d\varepsilon_1^f + 2d\varepsilon_3^f = \frac{dp^f}{K} + A_1 \frac{\partial f_1}{\partial p^f} df_1 + A_2 \frac{\partial f_2}{\partial p^f} df_2 \quad (16a)$$

$$d\varepsilon_s^f = \frac{2\sqrt{2}}{3} (d\varepsilon_1^f - d\varepsilon_3^f) = \frac{d\tau^f}{G} + \frac{2}{3} \left(A_1 \frac{\partial f_1}{\partial \tau^f} df_1 + A_2 \frac{\partial f_2}{\partial \tau^f} df_2 \right) \quad (16b)$$

where K and G are, respectively, the elastic volumetric modulus and elastic shear modulus; A_1 and A_2 are the plastic coefficients related to the yield surfaces, f_1 and f_2 , respectively; and $d\varepsilon_1^f$ and $d\varepsilon_3^f$ are the incremental principle strains of frictional elements in conventional

triaxial tests. The tangential deformation modulus, $E_f = (d\sigma_1^f)/(d\varepsilon_1^f)$, and tangential volumetric modulus, $\mu_f = (d\varepsilon_v^f)/(d\varepsilon_1^f)$, are obtained according to the Duncan–Chang model [45]. a , b , and η are set as follows:

$$a = 4(p^f)^2 r^2 A_1 \quad (17a)$$

$$b = \frac{(\tau^f)^{2s-2}}{(p^f)^{2s-2}} A_2 \quad (17b)$$

$$\eta = \frac{\tau^f}{p^f} \quad (17c)$$

where a , b , and η are assumed to be variates correlated to the stress state only and independent of stress paths.

By substituting Equations (13a)–(15b) and Equations (17a)–(17c) into Equations (16a) and (16b) with some manipulations, the relationship of the modulus of frictional elements can be derived:

$$\frac{3\mu_f}{E_f} = \frac{1}{K} + \frac{1}{r} \left(\frac{1}{r} + \sqrt{2}\eta r \right) a + \eta(s-1) [\eta(s-1) - \sqrt{2}s] b \quad (18a)$$

$$\frac{9}{E_f} = \frac{1}{K} + \frac{3}{G} + \left(\sqrt{2}\eta r + \frac{1}{r} \right)^2 a + [\eta(s-1) - \sqrt{2}s]^2 b \quad (18b)$$

The expressions of a , b , and η can be deduced from Equations (18a) and (18b):

$$a = r^2 \frac{\sqrt{2}s \left(\frac{3\mu_f}{E_f} - \frac{1}{K} \right) + \eta(s-1) \left(\frac{9}{E_f} - \frac{3\mu_f}{E_f} - \frac{3}{G} \right)}{\sqrt{2}[s + r^2\eta^2(s-1)](1 + \sqrt{2}r^2\eta)} \quad (19a)$$

$$b = \frac{\frac{9}{E_f} - \frac{3\mu_f}{E_f} - \frac{3}{G} - \sqrt{2}r^2\eta \left(\frac{3\mu_f}{E_f} - \frac{1}{K} \right)}{\sqrt{2}[s + r^2\eta^2(s-1)][\sqrt{2}s - \eta(s-1)]} \quad (19b)$$

3.4. Structural Parameters of Breakage Ratio and Local Stress Coefficient Matrix

With the changing stress and strain of the RVE, the breakage ratio (λ) and local stress coefficient matrix (C) vary. In this paper, λ is assumed to correlate only to the stress levels of the RVE, and it is given an initial value, λ_0 , which is determined by the confining pressure after consolidation. λ_0 is smaller than 1.0 and increases with increasing confining pressure at the initial loading state. λ increases gradually with loading and eventually approaches 1.0, at which point the bonded elements transform into frictional elements. The determination of λ_0 , λ , and the increment of λ , $d\lambda$, are expressed as follows:

$$\kappa_0 = \beta \zeta_1 \left(\frac{p}{\gamma p_a} \right)^{\zeta_2} \quad (20a)$$

$$\lambda_0 = 1 - \exp(-\kappa_0) \quad (20b)$$

$$d\kappa = \left[\zeta_1 \left(\frac{p}{\gamma p_a} \right)^{\zeta_2} + \zeta_1 \left(\frac{\tau}{\gamma p_a} \right)^{\zeta_2} \right] d\varepsilon_1 \quad (20c)$$

$$\kappa = \kappa_0 + \int_0^{\varepsilon_1} \left[\zeta_1 \left(\frac{p}{\gamma p_a} \right)^{\zeta_2} + \zeta_1 \left(\frac{\tau}{\gamma p_a} \right)^{\zeta_2} \right] d\varepsilon_1 \quad (20d)$$

$$\lambda = 1 - \exp(-\kappa) \quad (20e)$$

$$d\lambda = d\kappa \exp(-\kappa) \quad (20f)$$

where p and τ are the octahedral compressive stress and octahedral shear stress of the RVE; κ_0 , κ , and $d\kappa$ are the intermediate parameters used to calculate λ_0 and λ ; β , γ , ζ_1 , and ζ_2

are the material constants determined by the magnitude of the confining pressure and the anisotropic property of structured soils; and p_a is the atmospheric pressure of 101.325 kPa.

The local stress coefficient matrix, which bridges the stresses of the frictional elements and the RVE, varies gradually during the loading process and is influenced by the stress levels and stress loading history. Here, the local stress coefficients are assumed to be calculated by λ , and their increment relationships can also be obtained as follows:

$$C = \alpha - \psi\lambda \quad (21a)$$

$$dC = -\psi d\lambda \quad (21b)$$

where α and ψ are the model parameters.

Hence, the local stress coefficient matrix is:

$$C_{ijkl} = \begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix} \quad (22)$$

Because of the limitations in observing the mesoscopic and microscopic mechanisms of soils under loading, the breakage ratio and local stress coefficient, which are internal variables, are difficult to determine by microscopic or mesoscopic parameters. Thus, the evolution laws of the breakage ratio and local stress coefficient are similar to the determination methods of hardening parameters in plasticity or damage factors in damage mechanics [43]. The expressions are formulated based on the breakage mechanism of artificially structured soils, and the parameters are adjusted and determined according to the test results.

4. Determination of Model Parameters and Model Verification

The remolded soils, isotropic structured soils, and anisotropic structured soils are tested under conventional undrained triaxial stress conditions. The maximum principal stress is applied in the vertical direction, which is set as the z-axis direction, and the other two principal stresses are applied in the horizontal plane.

For undrained tests, the effective stress–strain relationships can be described by Equation (10), whose controlling condition of undrained tests is $d\varepsilon_v = d\varepsilon_1 + 2d\varepsilon_3 = 0$, that is, the volumetric deformation is regarded as 0, and the confining pressure is constant, i.e., $d\sigma_3 = 0$. Therefore, the incremental pore water pressure, du , can be calculated:

$$du = -d\sigma'_3 \quad (23)$$

where $d\sigma'_3$ is the increment of the effective confining pressure.

The determination methods of the parameters of the breakage ratio and local stress matrix, as well as those of the bonded and frictional elements, are introduced in the following sections.

4.1. Determination of Model Parameters for Bonded Elements

Under conventional triaxial stress conditions, the effective stress–strain relationship of bonded elements can be rewritten as follows:

$$\begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_3 \end{Bmatrix}_b = \begin{bmatrix} \frac{1}{E_{vb}} & -\frac{2\nu_{vb}}{E_{vb}} \\ -\frac{\nu_{vb}}{E_{vb}} & \frac{(1-\nu_{hb})}{E_{hb}} \end{bmatrix} \begin{Bmatrix} d\sigma_1 \\ d\sigma_3 \end{Bmatrix}_b \quad (24)$$

where four material parameters of bonded elements, E_{vb} , E_{hb} , ν_{vb} , and ν_{hb} , are contained. E_{vb} and E_{hb} , respectively, denote the tangential elastic deformation modulus in the vertical and horizontal directions, and ν_{vb} and ν_{hb} , respectively, denote the tangential Poisson ratios in the vertical and horizontal planes. For isotropic structured soils, the mechanical properties in all directions are supposed to be equal, that is, $E_{vb} = E_{hb}$ and $\nu_{vb} = \nu_{hb}$. In the initial loading stage, the mechanical properties of the RVE are similar to elastic materials,

and the external loading is mainly borne by the bonded elements. Accordingly, E_{vb} and v_{vb} are determined by the triaxial test results for 0.3% strain of artificially structured soil samples with initially stress-induced anisotropy, and E_{hb} and v_{hb} are determined by the strain of 0.3% of the triaxial test results of artificially isotropic structured samples.

4.2. Determination of Model Parameters for Frictional Elements

The frictional elements, which are formed from the broken bonded elements, behave similarly to remolded soils, exhibiting mechanical properties similar to elastoplastic materials. The stress–strain relationship of frictional elements can be simplified as follows:

$$\begin{Bmatrix} d\varepsilon_1^f \\ d\varepsilon_3^f \end{Bmatrix} = \begin{bmatrix} \frac{1}{3}(F_1 + F_3) & \frac{1}{3}(F_2 + F_4) \\ \frac{1}{6}(2F_1 - F_3) & \frac{1}{6}(2F_2 - F_4) \end{bmatrix} \begin{Bmatrix} d\sigma_1^f \\ d\sigma_3^f \end{Bmatrix} \quad (25)$$

where the expressions for F_1 – F_4 are:

$$F_1 = \frac{1}{3K} + \frac{1}{3} \left[\frac{a}{r^2} + \eta^2(s-1)^2b \right] + \frac{\sqrt{2}}{3} \eta[a - s(s-1)b] \quad (26a)$$

$$F_2 = \frac{2}{3K} + \frac{2}{3} \left[\frac{a}{r^2} + \eta^2(s-1)^2b \right] - \frac{\sqrt{2}}{3} \eta[a - s(s-1)b] \quad (26b)$$

$$F_3 = \frac{1}{G} + \frac{\sqrt{2}}{3} \eta[a - s(s-1)b] + \frac{2}{3}(r^2\eta^2a + s^2b) \quad (26c)$$

$$F_4 = -\frac{1}{G} + \frac{2\sqrt{2}}{3} \eta[a - s(s-1)b] - \frac{2}{3}(r^2\eta^2a + s^2b) \quad (26d)$$

In the double-yield surfaces constitutive model, the parameters are obtained from the results of remolded samples under drained triaxial tests [48]. The tangential deformation modulus, E_f , and tangential Poisson ratio, v_f can both be determined using the nonlinear elastic model of the Duncan–Chang hyperbolic model [49,50] as follows:

$$E_f = E_0(1 - R_s)^2 \quad (27a)$$

$$E_0 = K_e p_a \left(\frac{\sigma_3}{p_a} \right)^z \quad (27b)$$

$$R_s = \frac{R_f (\sigma_1 - \sigma_3)(1 - \sin \varphi)}{2c \cos \varphi + \sigma_3 \sin \varphi} \quad (27c)$$

$$v_0 = J_1 - J_2 \lg \left(\frac{\sigma_3}{p_a} \right) \quad (27d)$$

$$v_f = \frac{J_1 - J_2 \lg \left(\frac{\sigma_3}{p_a} \right)}{\left[1 - \frac{Q(\sigma_1 - \sigma_3)}{K_e p_a \left(\frac{\sigma_3}{p_a} \right)^z \left(1 - \frac{R_f(\sigma_1 - \sigma_3)(1 - \sin \varphi)}{2} \right)} \right]^2} \quad (27e)$$

where E_0 is the initial tangential deformation modulus of remolded soil, with its two constants, K_e and z , determined from 0.3% of the triaxial test results of remolded soils. c and φ are the cohesion and internal friction angles, respectively. The failure stress ratio, R_f , is calculated as $R_f = (\sigma_1 - \sigma_3)_f / (\sigma_1 - \sigma_3)_{ult}$. J_1 and J_2 are the test-specific constants correlating the initial Poisson ratio with the confining pressure, while Q is the constant of the Poisson ratio determined by the correlative curves of ε_1 and ε_3 .

s and r are the constants used in the double-yield surfaces constitutive model, and they are adjusted based on comparison with the test results. K and G , which are the elastic volumetric modulus and shear modulus of frictional elements, are obtained from loading–unloading–reloading triaxial tests conducted on remolded soils. E_{ur} denotes the average

slope of the hysteresis loops observed in the stress–strain curves in the loading–unloading–reloading tests of remolded soils, and its determination is as follows:

$$E_{ur} = K_{ur} p_a \left(\frac{\sigma_3}{p_a} \right)^z \quad (28)$$

where $\lg(K_{ur})$ and z are, respectively, the intercept and slope of the line, $\lg\left(\frac{E_{ur}}{p_a}\right)$ vs $\lg\left(\frac{\sigma_3}{p_a}\right)$, where z is the same as the value in Equation (27b).

Therefore, K and G are calculated as follows:

$$K = \frac{E_{ur}}{3(1 - 2v_f)} \quad (29a)$$

$$G = \frac{E_{ur}}{2(1 + v_f)} \quad (29b)$$

The volumetric modulus, μ_f , is expressed as follows:

$$\mu_f = 2c_d \left(\frac{\sigma_3^f}{p_a} \right)^d \frac{E_0 R_s}{\sigma_1^f - \sigma_3^f} \frac{1 - R_d}{R_d} \left(1 - \frac{R_s}{1 - R_s} \frac{1 - R_d}{R_d} \right) \quad (30)$$

where c_d is the maximum volumetric strain under $\sigma_3 = p_a$; d is the power of the relationship between the volumetric strain and σ_3 ; and R_d is the stress ratio at the maximum volumetric strain.

4.3. Determination of Structural Parameters

From the test results, it can be seen that compared with isotropic structured soils, the strength and stiffness of anisotropic structured soils with initially stress-induced conditions are higher under relatively lower confining pressures. This can be explained by the fact that there are fewer broken bonded elements in the RVE of anisotropic structured soils, where the assumed breakage ratio is smaller. In Equations (20a)–(20f), which describe the breakage ratio of structured soils, β is constant, γ varies with confining pressure, and ζ_1 and ζ_2 are influenced by anisotropy and confining pressure.

The parameters of the local stress coefficient in Equation (21a) are α and ψ , and they are related to the confining pressure.

4.4. Model Verification

The effective stress–strain relationship is expressed as follows:

$$\begin{aligned} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_3 \end{Bmatrix} &= \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix} \begin{Bmatrix} d\sigma_1^f \\ d\sigma_3^f \end{Bmatrix} + \\ &\begin{Bmatrix} dC(n_{13}\sigma_1^f + n_{14}\sigma_3^f) + \frac{d\lambda(C-1)}{1-\lambda}(n_{15}\sigma_1^f + n_{16}\sigma_3^f) + d\lambda(\varepsilon_1^f - \varepsilon_1^b) \\ dC(n_{23}\sigma_1^f + n_{24}\sigma_3^f) + \frac{d\lambda(C-1)}{1-\lambda}(n_{25}\sigma_1^f + n_{26}\sigma_3^f) + d\lambda(\varepsilon_3^f - \varepsilon_3^b) \end{Bmatrix} \end{aligned} \quad (31)$$

where $n_{11} \sim n_{16}$ and $n_{21} \sim n_{26}$ are expressed as follows:

$$n_{11} = \lambda^0 \left[\frac{1}{3}(F_1 + F_3) - \frac{1}{E_{vb}} \right] C^0 + \frac{1}{E_{vb}} \quad (32a)$$

$$n_{12} = \lambda^0 \left[\frac{1}{3}(F_2 + F_4) + \frac{2v_{vb}}{E_{vb}} \right] C^0 - \frac{2v_{vb}}{E_{vb}} \quad (32b)$$

$$n_{13} = \lambda^0 \left[\frac{1}{3}(F_1 + F_3) - \frac{1}{E_{vb}} \right] \quad (32c)$$

$$n_{14} = \lambda^0 \left[\frac{1}{3}(F_2 + F_4) + \frac{2v_{vb}}{E_{vb}} \right] \quad (32d)$$

$$n_{15} = -\frac{1}{E_{vb}} \quad (32e)$$

$$n_{16} = \frac{2v_{vb}}{E_{vb}} \quad (32f)$$

$$n_{21} = \lambda^0 \left[\frac{1}{6}(2F_1 - F_3) + \frac{v_{vb}}{E_{vb}} \right] C^0 - \frac{v_{vb}}{E_{vb}} \quad (32g)$$

$$n_{22} = \lambda^0 \left[\frac{1}{6}(2F_2 - F_4) - \frac{1 - v_{hb}}{E_{hb}} \right] C^0 + \frac{1 - v_{hb}}{E_{hb}} \quad (32h)$$

$$n_{23} = \lambda^0 \left[\frac{1}{6}(2F_1 - F_3) + \frac{v_{vb}}{E_{vb}} \right] \quad (32i)$$

$$n_{24} = \lambda^0 \left[\frac{1}{6}(2F_2 - F_4) - \frac{1 - v_{hb}}{E_{hb}} \right] \quad (32j)$$

$$n_{25} = \frac{v_{vb}}{E_{vb}} \quad (32k)$$

$$n_{26} = -\frac{1 - v_{hb}}{E_{hb}} \quad (32l)$$

where $d\sigma'_{kl}$ and σ'_{kl} represent the increment and initial value of the effective stress, respectively, while σ_{kl}^{b0} and σ_{kl}^{f0} represent the effective stress carried by bonded and frictional elements, respectively.

All the parameters of the bonded elements, frictional elements, breakage ratio, and local stress coefficient in the binary-medium constitutive model are provided in Tables 1 and 2. The determination method of the parameters displayed in Tables 1 and 2 is described in Sections 4.1–4.3. Firstly, E_{vb} , v_{vb} , E_{hb} , and v_{hb} for bonded elements; and K_e , z , v_0 , c , φ , K_{ur} , R_f , μ_{f0} , c_d , d , and R_d for frictional elements, are determined based on the results of the triaxial experiments on structured and remolded soils. Model parameters, Q , s , and r for frictional elements; β_i , β_a , γ_i , γ_a , ζ_{1i} , ζ_{1a} , and ζ_2 for the breakage ratio; and ψ , α_i , and α_a for the local stress coefficient matrix are all modified during calculations. Referring to the research of Liu [43] and He [48], some parameters depend on the magnitude of the confining pressure, and thus they are given as computational formulas vs. σ_3 .

Table 1. Parameters of bonded and frictional elements.

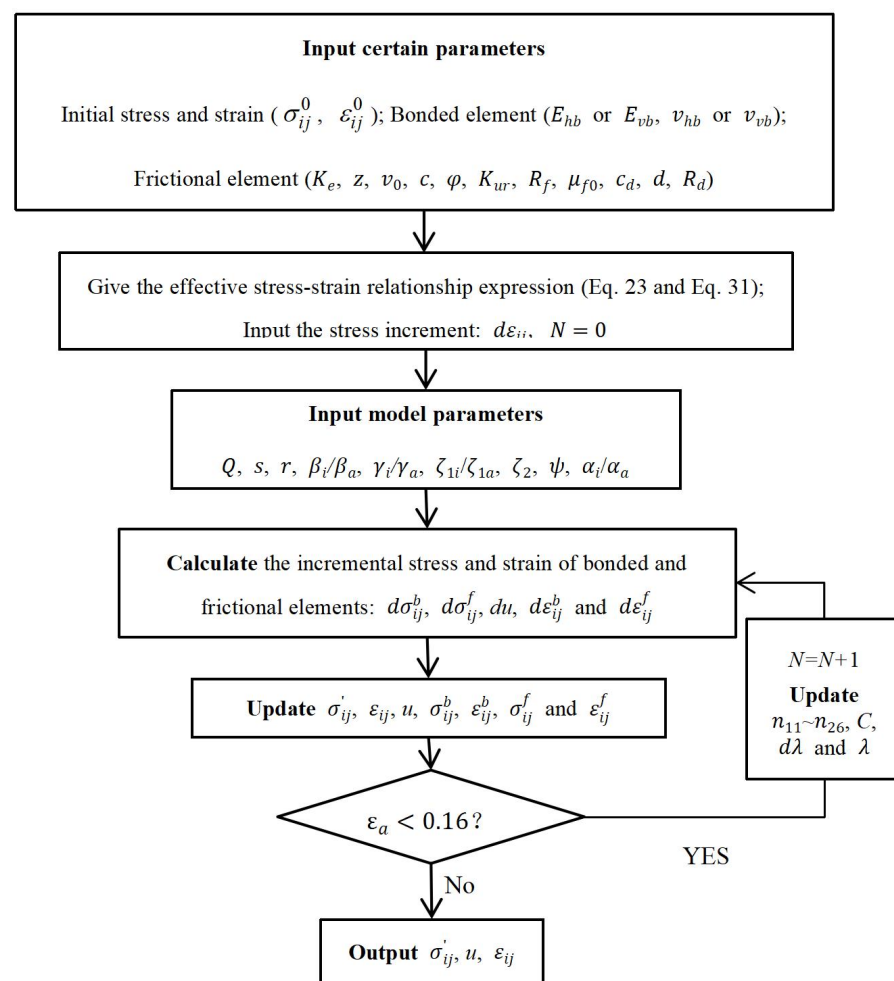
Bonded Elements	
E_{hb}	$10^{2.4992} p_a (\sigma_3 / p_a)^{0.3188}$
v_{hb}	$0.2213 - 0.3115 \lg(\sigma_3 / p_a)$
E_{vb}	$10^{2.5536} p_a (\sigma_3 / p_a)^{0.2713}$
v_{vb}	$0.28 - 0.1983 \lg(\sigma_3 / p_a)$
Frictional elements	
K_e	88.8
z	0.3425
v_0	$0.1913 - 0.2979 \lg(\sigma_3 / p_a)$
Q	0.0085
R_f	0.95
c	0
φ	27°
K_{ur}	332.9
μ_{f0}	$0.6328 + 0.5955 \lg(\sigma_3 / p_a)$
c_d	0.048
d	2.0
R_d	$0.9081 + 0.0451 \lg(\sigma_3 / p_a)$
s	3.0
r	$2.981 - 3.3219 \lg(\sigma_3 / p_a)$

Table 2. Structural parameters of breakage ratio and local strain coefficient matrix.

Breakage Ratio	
β_i	0.02
β_a	0.016
γ_i	$1.1189 + 0.8068 \lg(\sigma_3/p_a)$
γ_a	$1.0368 + 0.8186 \lg(\sigma_3/p_a)$
ζ_{1i}	$27.635 + 0.1757\sigma_3$
ζ_{1a}	$18.041 + 0.203\sigma_3$
ζ_2	$1.3245 + 0.5457 \lg(\sigma_3/p_a)$
Local stress coefficient matrix	
ψ	0.5
α_i	$1.8661 + 0.937 \lg(\sigma_3/p_a)$
α_a	$1.7870 + 0.978 \lg(\sigma_3/p_a)$

Note: The subscripts i and a represent the modeling parameters of isotropic structured soils and anisotropic structured soils, respectively.

These parameters are used in the programs (as shown in Figure 5), and with the increment in the strain, the stress and pore water pressure of artificially isotropic and anisotropic structured soils are computed.

**Figure 5.** The calculation flowchart of the constitutive model.

The stress–strain curves of deviatoric stress, σ_d , and pore water pressure, u , versus axial strain, ε_a , for isotropic and anisotropic structured samples are shown in Figures 6 and 7, where the test and computed results of isotropic (anisotropic) structured samples are

denoted as ISS and CISS (ASS and CASS), respectively. It can be observed that there are some slight differences between the computed and test results, but the mechanical properties are reflected in the constitutive model: structured soils exhibit slight strain-softening behavior at relatively lower confining pressures (25 and 100 kPa) and strain-hardening behavior at a higher confining pressure (200 kPa). The pore water pressures of all tests peak, which indicates a tendency toward volumetric contraction turning into volumetric dilatancy during the loading process. The value of the computed pore water pressure at 200 kPa closely matches the test results, with slight differences in the later stages. With an increase in confining pressure, the stress–strain curves of artificially isotropic and anisotropic structured soils become closer because more bonded elements break up and transform into frictional elements. This behavior of the RVE approximates the frictional elements more after higher confining pressure consolidation, which is also reflected in the variation, λ , which approaches 1.0 in Figure 8. In Figure 8, it can be seen that the breakage ratios of isotropic structured soils are larger than the others, explained by the higher strength and stiffness of anisotropic structured samples caused by external loading during curing.

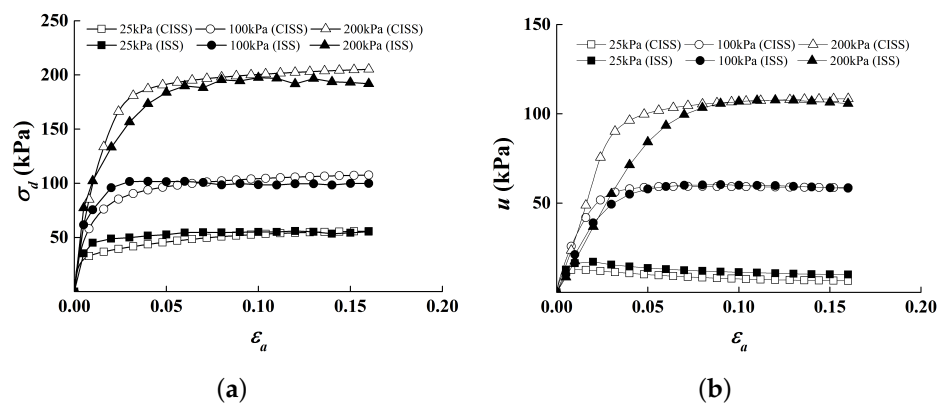


Figure 6. Comparison of test results (ISS) and computed results (CISS) of isotropic structured samples under undrained triaxial tests at confining pressures of 25, 100, and 200 kPa: (a) Curves of deviatoric stress versus axial strain. (b) Curves of pore water pressure versus axial strain.

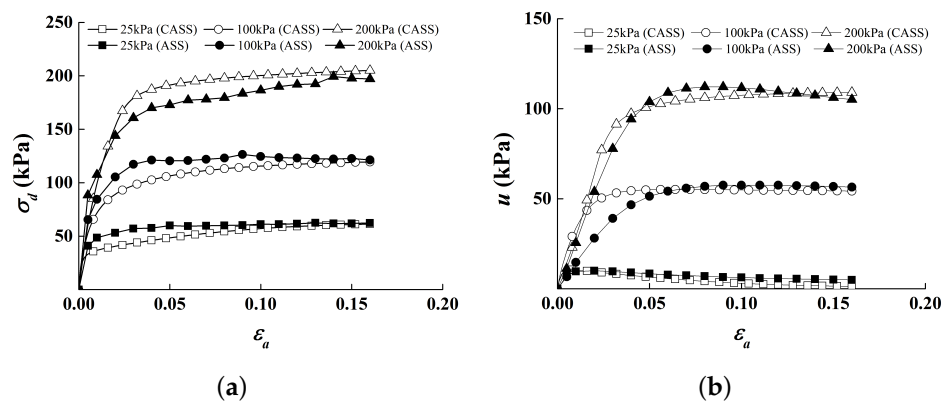


Figure 7. Comparison of test results (ASS) and computed results (CASS) of anisotropic structured samples under undrained triaxial tests at confining pressures of 25, 100, and 200 kPa: (a) Curves of deviatoric stress versus axial strain. (b) Curves of pore water pressure versus axial strain.

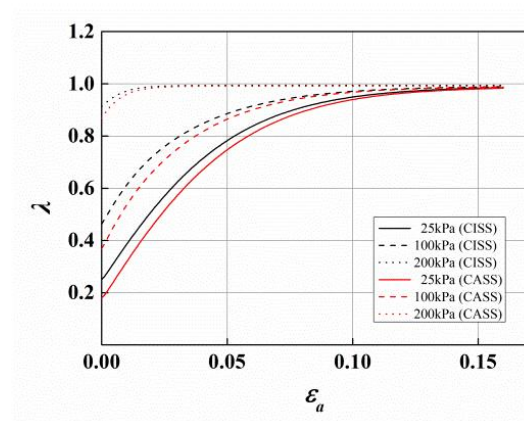


Figure 8. Evolution of breakage ratio of isotropic and anisotropic structured soils with axial strain.

5. Conclusions

Samples of remolded soils, artificially isotropic structured soils, and artificially structured soils with initially stress-induced anisotropy are tested under undrained triaxial conditions at three confining pressures of 25, 100, and 200 kPa. At relatively lower confining pressures (25 and 100 kPa), the structured samples exhibit slightly strain-softening behavior, and their strengths are higher than those of the remolded samples. Under the relatively higher confining pressure (200 kPa), the structured samples exhibit strain-hardening behavior similar to the remolded samples. The pore water pressures of all samples initially increase and then exhibit a modest decrease, which indicates a tendency toward volumetric contraction turning into volumetric dilatancy during the loading process. The lower the confining pressure, the more obvious this tendency.

A binary-medium constitutive model based on homogenization theory and a breakage mechanism is formulated in this paper. The RVE consists of bonded elements exhibiting elastic behavior and frictional elements exhibiting elastoplastic behavior. Thus, a linear elastic constitutive model and an elastoplastic constitutive model with double-yield surfaces are adopted to describe the stress–strain relationship of the bonded and frictional elements, respectively. The test results are compared with the computed results, clearly verifying the constitutive model. It replicates the mechanical properties of structured soils relatively well, including strain softening at lower confining pressures and a transition to strain hardening with increasing confining pressure, as well as an initial increase in pore pressure followed by a slight decrease during the initial loading stage.

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