STAR-RIS-Assisted Millimeter-Wave Secure Communication with Multiple Eavesdroppers

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Abstract: Aiming to address the limited coverage of conventional reconfigurable intelligent surfaces (RISs), this study proposes a millimeter-wave secure communication scheme based on the simultaneously transmitting and reflecting reconfigurable intelligent surface (STAR-RIS). It uses the transmission and reflection functions of the STAR-RIS to achieve multiple users’ full-area communication coverage and meet the security communication needs of different users. To maximize the system sum rate under the security communication requirements of users in the transmission and reflection regions, this study proposes a joint optimization design scheme consisting of transmit beamforming at the base station (BS) and transmitting and reflecting coefficients at the STAR-RIS based on the energy-splitting protocol, and it models the rate optimization problem with information leakage constraints under imperfect eavesdroppers’ channel state information (ECSI). First, a series of transformations is proposed to solve the coupling between the optimization variables, and then, an efficient iterative algorithm based on successive convex approximation (SCA) and semi-definite relaxation (SDR) is proposed. Aiming to address the amplitude and phase constraints of the STAR-RIS, an optimization method comprising a penalty concave–convex procedure is adopted. The simulation results show that, compared with the conventional RIS, the proposed STAR-RIS assistance scheme can achieve the full coverage of the communication system and effectively improve the system sum rate while ensuring the safe transmission of information. The combination of STAR-RIS and millimeter-wave can promote the efficient and safe transmission of information in dense cities.

Keywords: millimeter-wave; STAR-RIS; multiple users; multiple eavesdroppers; imperfect ECSI

MSC: 94-10

1. Introduction

Millimeter-wave communication is considered a promising technology with the potential to support multi-gigabit wireless applications [1]. Millimeter-wave technology adopts a line-of-sight (LOS) transmission mode. It transmits in space in the form of direct beamforming, with a narrow beam and strong directivity. Because the frequency band is relatively high and there are not many interference sources, the reliability and stability of its propagation are excellent. However, millimeter-wave signals are easily blocked by obstacles [2], especially in indoor or densely populated urban environments [3]. Regarding this problem, the reconfigurable intelligent surface (RIS) has become a promising and cost-effective solution, as it can establish a reliable connection even when the LOS link is blocked by obstacles [4]. In recent years, the RIS has emerged as a promising technology to realize the propagation link in the next-generation configured wireless network [5]. RISs can dynamically change the wireless channel by using many low-cost passive reflective components integrated into the plane to subtly change the reflected signal, thus improving the system’s performance [6]. The conventional RIS only has the reflection function. The
transmitter communicates with the user through the RIS, and the eavesdroppers try to intercept confidential information in a non-colluding way. Ref. [7] focuses on the secure transmission of millimeter-wave and terahertz systems assisted by the RIS, in which the base station (BS) communicates with the destination through the RIS in the presence of passive eavesdroppers. Ref. [8] focuses on the security issues of confidential communication in a multiple RIS-assisted terahertz system, where potential eavesdroppers can intercept the communication link.

However, conventional RISs can only reflect the incident wireless signals. In this case, the transmitter and receiver must be on the same side of the RIS [9], resulting in smart radio environments (SREs) in half space. However, this location restriction may not always be satisfied in practice and severely limits the flexibility and effectiveness of RISs, because the users may be on both sides of the RIS. To overcome this limitation, Ref. [10] proposed the new concept of a simultaneous transmitting and reflecting reconfigurable intelligent surface (STAR-RIS). This technology can achieve full space coverage because the STAR-RIS can transmit and reflect incident waves at the same time.

On the other hand, security is also an important aspect of wireless networks [11]. The authors of [12] studied the secure beamforming and phase-shifting design of an RIS-assisted multiple-input single-output (MISO) downlink channel and proposed an algorithm based on an alternating-direction multiplier. Meanwhile, Ref. [13] studied the secure precoding and phase shifting design of RIS-assisted multiple-input multiple-output (MIMO) channels and developed a maximization–minimization (MM) optimization method. The authors of [14] studied secure communication assisted by a STAR-RIS to maximize the secrecy rate of the system under perfect channels. Moreover, Ref. [15] studied a penalty-based secure beamforming algorithm developed to solve the resulting non-convex optimization problem. The authors of [16] studied secure transmission in a STAR-RIS-assisted uplink non-orthogonal multiple access system. According to the availability of eavesdropping channel state information (CSI), the minimum security capacity is maximized by considering the eavesdroppers’ complete CSI and statistical CSI.

The above security communication scheme achieves superior security performance. However, the introduction of STAR-RIS also brings additional eavesdropping challenges. The most recent studies [17,18] have focused on STAR-RIS-assisted communication with perfect CSI. In general, it is difficult to obtain perfect CSI due to the unreliable behavior of eavesdroppers and the feedback delay or channel estimation error, so eavesdroppers in millimeter-wave systems can intercept the transmission of confidential information, causing serious security issues. However, in the case of multiple eavesdroppers, imperfect channels’ impact on millimeter-wave communication has not yet been explored.

Based on the above problems, this study evaluates the STAR-RIS-assisted secure transmission in millimeter-wave systems, where eavesdroppers can eavesdrop on the confidential information transmitted and reflected by the STAR-RIS. Under the assumption of imperfect ECSI and information leakage constraints, the system sum rate of the legitimate receivers is defined to measure the transmission performance. The main purpose of this work is to study the security transmission problem in the millimeter-wave system, define the transmitting and reflecting coefficients of the STAR-RIS, and examine the transmit beamforming of the BS, seeking to reduce the information leakage caused by imperfect eavesdroppers’ channel state information (ECSI) and improve the security performance of millimeter-wave communication. Specifically, the main contributions of this study can be summarized as follows.

By jointly designing the transmitting and reflecting coefficients at the STAR-RIS and the transmit beamforming at the BS, the sum rate of legitimate users is maximized under the information leakage constraints. Owing to the coupled variables and non-convexity of the objective function, the formulated problem is non-convex and difficult to solve. To overcome these obstacles, the problem is transformed into a solvable formula.

To solve the current design problems, this study proposes an iterative suboptimal algorithm with high computational efficiency. We first propose a series of transformations
to solve the coupling between the optimization variables, which paves the way for the development of efficient iterative algorithms based on successive convex approximation (SCA) and semi-definite relaxation (SDR). For the amplitude and phase shift constraints at the STAR-RIS, an optimization method based on the penalty concave-convex procedure is adopted. Unlike the traditional alternative optimization method, which is prone to falling into a low-efficiency stationary point during the iterative process in practice, our algorithm can jointly optimize all optimization variables at each iteration and has a higher system sum rate.

**Notations:** The scalars, vectors, and matrices are represented by lowercase letter \( x \), boldface lowercase letter \( \mathbf{x} \), and boldface uppercase letter \( \mathbf{X} \), respectively. \( \mathbb{C}^{N \times M} \) denotes the space of \( N \times M \) matrices with complex entries. The modulus of a complex-valued scalar is denoted by \( | \cdot | \). The Euclidean norm, spectral norm, nuclear norm, and Frobenius norm of a matrix are denoted by \( || \cdot ||, || \cdot ||_2, || \cdot ||_1, \) and \( || \cdot ||_F \), respectively. The conjugate transpose, conjugate, expectation, rank, and trace of a matrix are denoted as \( ( \cdot )^H, ( \cdot )^*, \mathbb{E}\{ \cdot \}, \text{Rank}(\cdot), \) and \( \text{Tr}(\cdot) \), respectively. The maximum eigenvalue of a matrix is denoted by \( \lambda_{\text{max}}(\cdot) \), and the eigenvector associated with the maximum eigenvalue is denoted by \( \Phi_{\text{max}}(\cdot) \). \( X \succeq 0 \) mean that the matrix is positive semi-definite. \( \Re \) stands for the real part of a complex number. \( \text{diag}(x) \) denotes a diagonal matrix with its diagonal elements given by the vector. \( \text{Diag}(X) \) denotes a vector whose elements are extracted from the main diagonal elements of the matrix. \( \nabla f(\cdot) \) represents the gradient. \( I_N \) denotes an \( N \times N \) identity matrix.

2. System Model

2.1. Signal Model

This study considers secure communication under multiple users. Since multiple users exist in different locations, the transmitter needs to send information to these multiple users at the same time. Therefore, this study introduces STAR-RIS to ensure secure communication. A STAR-RIS is capable of independently controlling the transmitted and reflected signals, which introduces additional degrees of freedom. To characterize this unique feature, for a STAR-RIS with \( M \) elements, let \( s_m \) represent the signal incident on the \( m \)-th element, where \( m \in \mathcal{M} \equiv \{1, 2, \ldots, M\} \). After reconfiguration with the corresponding transmission and reflection coefficients, the signal transmitted and reflected by the \( m \)-th element is given as \( t_m = (\sqrt{\alpha_m^t e^{j\phi_m^t}})s_m, r_m = (\sqrt{\alpha_m^r e^{j\phi_m^r}})s_m \), where \( \alpha_m^t, \alpha_m^r \in [0, 1], \phi_m^t, \phi_m^r \in [0, 2\pi) \) describes the amplitude and phase shift adjustment of the incident signal by the \( m \)-th element during transmission and reflection, respectively.

The adjustments of the phase shifts of transmission and reflection can usually be implemented independently of one another. However, the transmission and reflection amplitude coefficients must comply with the law of the conservation of energy: the sum of the energy of the transmission and reflection signals must be equal to the energy of the incident signal. Therefore, the sum of \( \alpha_m^t \) and \( \alpha_m^r \) should be equal to 1. Thus, after adjusting the amplitude coefficients for transmission and reflection, each element can operate in full transmission mode, full reflection mode, or a simultaneous transmission and reflection mode. Based on the above basic signal model, an energy-splitting protocol for a STAR-RIS in wireless networks is proposed.

All elements of the STAR-RIS that transmit and reflect simultaneously work in both modes. Therefore, the transmission and reflection coefficients are modeled as \( \Phi_t = \text{diag}(\sqrt{\alpha_1^t e^{j\phi_1^t}}, \ldots, \sqrt{\alpha_M^t e^{j\phi_M^t}}), \Phi_r = \text{diag}(\sqrt{\alpha_1^r e^{j\phi_1^r}}, \ldots, \sqrt{\alpha_M^r e^{j\phi_M^r}}) \), where \( \alpha_m^t, \alpha_m^r \in [0, 1], \phi_m^t, \phi_m^r \in [0, 2\pi), \forall m \in \mathcal{M} \). For the energy splitting protocol, since the transmission coefficient and reflection coefficient of each element can be optimized, higher system design flexibility can be achieved.

However, the unique ability of the STAR-RIS to reconfigure the whole space’s radio propagation environment will inevitably lead to eavesdropping. In other words, eavesdroppers near the STAR-RIS can access the confidential information transmitted through the STAR-RIS, which poses a serious security challenge from the perspective of information theory. Fortunately, physical-layer security technology is expected to protect STAR-RIS-
assisted communication by utilizing the inherent characteristics of wireless channels (such as fading, noise, and interference).

Figure 1 shows a MISO system consisting of a BS, a STAR-RIS, two users, and 2K eavesdroppers. The BS and the STAR-RIS are, respectively, equipped with N antennas and M elements; Bob\(_1\) and Bob\(_2\) are legitimate receivers; Eve\(_{t,k}\) and Eve\(_{r,k}\) are eavesdroppers, where \(k \in K \triangleq \{1, 2, \cdots, K\}\), and each of them is a single-antenna node. In Figure 1, it is assumed that the STAR-RIS serves both the transmission and reflection regions. Bob\(_1\) and Bob\(_2\) are located in the transmission and reflection regions, respectively. Eve\(_{t,k}\) and Eve\(_{r,k}\) are located in the transmission and reflection regions, respectively. Due to a serious blockage, the direct links between the BS and the users can be ignored. The distances from the BS to the STAR-RIS, from the STAR-RIS to Bob\(_1\), from the STAR-RIS to Eve\(_{t,k}\), from the STAR-RIS to Bob\(_2\), and from the STAR-RIS to Eve\(_{r,k}\) are denoted by \(H \in \mathbb{C}^{M \times N}\), \(h_t \in \mathbb{C}^{M \times 1}\), \(g_{t,k} \in \mathbb{C}^{M \times 1}\), \(h_r \in \mathbb{C}^{M \times 1}\), and \(g_{r,k} \in \mathbb{C}^{M \times 1}\), respectively.

![Figure 1. The STAR-RIS-aided system model.](image)

The BS simultaneously sends independent signals to each user at the same frequency. \(s_t\) and \(s_r\) are represented as the signals sent to Bob\(_1\) and Bob\(_2\), where \(\mathbb{E}\{|s_t|^2\} = \mathbb{E}\{|s_r|^2\} = 1\). Then, the signals received by Bob\(_1\), Bob\(_2\), Eve\(_{t,k}\), and Eve\(_{r,k}\) are

\[
y_{b,t} = h_t^H \Phi_t H (w_t s_t + w_r s_r) + n_{b,t} \tag{1}
\]

\[
y_{b,r} = h_r^H \Phi_r H (w_t s_t + w_r s_r) + n_{b,r} \tag{2}
\]

\[
y_{e_{t,k}} = g_{t,k}^H \Phi_t H (w_t s_t + w_r s_r) + n_{e_{t,k}} \tag{3}
\]

\[
y_{e_{r,k}} = g_{r,k}^H \Phi_r H (w_t s_t + w_r s_r) + n_{e_{r,k}} \tag{4}
\]

where \(w_t \in \mathbb{C}^{N \times 1}\) and \(w_r \in \mathbb{C}^{N \times 1}\) represent the beamforming of Bob\(_1\) and Bob\(_2\); \(\Phi_t \in \mathbb{C}^{M \times M}\) and \(\Phi_r \in \mathbb{C}^{M \times M}\) represent the transmission and reflection coefficients of Bob\(_1\) and Bob\(_2\); \(n_{b,t}, n_{b,r}\) are the Gaussian noise at Bob\(_1\) and Bob\(_2\), both with a mean of zero and variance \(\sigma^2_n\); \(n_{e_{t,k}}, n_{e_{r,k}}\) are the Gaussian noise at Eve\(_{t,k}\) and Eve\(_{r,k}\), both with a mean of zero and variance \(\sigma^2_n\). According to equation (1) for Bob\(_1\) and Bob\(_2\), the following signal-to-interference-plus-noise ratios (SINR) at Bob\(_1\) and Bob\(_2\) are given by

\[
\Gamma_{b,t} = \frac{|h_t^H \Phi_t H w_t|^2}{|h_t^H \Phi_t H w_t|^2 + \sigma^2_n} \tag{5}
\]

\[
\Gamma_{b,r} = \frac{|h_r^H \Phi_r H w_r|^2}{|h_r^H \Phi_r H w_r|^2 + \sigma^2_n} \tag{6}
\]
The achievable rates (bits/s/Hz) of Bob$_t$ and Bob$_r$ are given by $R_{b,t} = \log_2(1 + \Gamma_{b,t}),$ $R_{b,r} = \log_2(1 + \Gamma_{b,r})$. Since multiple eavesdroppers are arbitrarily distributed to nearby users, non-colluding eavesdropping is considered. In non-colluding eavesdropping, the eavesdroppers are independent and individually overhear private information. Here, we assume that the eavesdroppers only eavesdrop on the information sent to users who are located in the same region. As a result, the received SINR of Eve$_{t,k}$ and Eve$_{r,k}$ can be expressed as

$$\Gamma_{e,t,k} = \frac{|g_{b,t}^H \Phi_t Hw_t|^2}{|g_{r,t}^H \Phi_r Hw_r|^2 + \sigma_c^2}$$  \hspace{1cm} (7)

$$\Gamma_{e,r,k} = \frac{|g_{b,r}^H \Phi_r Hw_r|^2}{|g_{r,r}^H \Phi_r Hw_r|^2 + \sigma_c^2}$$  \hspace{1cm} (8)

The achievable rates of Eve$_{t,k}$ and Eve$_{r,k}$ are given by $R_{e,t,k} = \log_2(1 + \Gamma_{e,t,k}), R_{e,r,k} = \log_2(1 + \Gamma_{e,r,k})$. Hence, the secrecy rate between the BS and Bob$_t$ is $R_{s,t} = R_{b,t} - \max_{k \in K} R_{e,t,k}$, the secrecy rate between the BS and Bob$_r$ is $R_{s,r} = R_{b,r} - \max_{k \in K} R_{e,r,k}$, and the system sum secrecy rate is given by $R_s = R_{s,t} + R_{s,r}$.

### 2.2. Channel State Information

We consider a scenario where the BS adopts a uniform linear array (ULA) with $N$ antennas, and the STAR-RIS consists of a uniform rectangular array (URA) with $M = M_1 M_2$ elements, where $M_1$ and $M_2$ represent the number of elements along the horizontal and vertical axes, respectively. In our simulation, the STAR-RIS-user and STAR-RIS-eavesdropper channels are both generated based on the following geometric channel model [19]:

$$h = \sqrt{M \sum_{l=1}^L a_l \lambda_t \lambda_t (\phi_{al}, \phi_{el})}$$  \hspace{1cm} (9)

where $L$ is the number of paths, $a_l$ is the complex gain associated with the $l$-th path, $\phi_{al}(\phi_{el})$ is the relevant azimuth (elevation) deviation angle, and $a_l \in \mathbb{C}^{M \times 1}$ is the normalized emission array response vector, while $\lambda_t$ and $\lambda_l$ represent the gains of the receiving and transmitting antenna units. According to [20,21], for the STAR-RIS-user and STAR-RIS-eavesdropper links, $\lambda_t$ and $\lambda_l$ are set to 0 dBi and 9.82 dBi. The complex gain $a_l$ is generated based on the complex Gaussian distribution $a_l \sim CN(0, 10^{-0.1f}),$ where $\kappa$ is given as $e = e + 10 f \log_{10}(d) + \zeta$ and $d$ represents the distance between the transmitter and receiver, $\zeta \sim N(0, \sigma_c^2)$. We set the values of $e = 72$, $f = 2.92$, and $\sigma_c^2 = 8.7$ dB, as shown in real channel measurements in a 28 GHz Non-Line-of-Sight (NLOS) scenario. The BS-STAR-RIS channel has the characteristics of the rank-one geometric channel model, which provides the following:

$$H = \sqrt{N M} \lambda_t \lambda_t (\phi_{al}, \phi_{el}) a_l^H (\phi)$$  \hspace{1cm} (10)

Here, $\theta_{al}(\theta_{el})$ represents the azimuth (elevation) arrival angle related to the BS-STAR-RIS path. $\phi$ is the relevant departure angle, where $a_l \in \mathbb{C}^{M \times 1}$ and $\phi \in \mathbb{C}^{N \times 1}$ represent the normalized receiving and transmitting array response vectors, respectively. Both $\lambda_t$ and $\lambda_l$ are the antenna gains.

In considering the impact of the imperfect eavesdropper channel on secure communication, the bounded CSI error models are used for the cascaded BS-STAR-RIS-Eve$_{t,k}$ channel, the cascaded BS-STAR-RIS-Eve$_{r,k}$ channel, the direct STAR-RIS-Eve$_{t,k}$ channel, and the direct STAR-RIS-Eve$_{r,k}$ channel. Specifically, we denote the cascaded BS-STAR-RIS-Eve$_{t,k}$ channel as

$$G_{t,k} = \text{diag}(g_{t,k}^H) H = G_{t,k} + \Delta G_{t,k}, \forall k$$  \hspace{1cm} (11)

with $Y_{t,k} = \{\Delta G_{t,k} \in \mathbb{C}^{M \times N}: ||\Delta G_{t,k}||_F \leq \rho_{t,k}, \forall k\}$, where $G_{t,k}, \forall k$, is the estimation of the corresponding channel $G_{t,k}$. The channel estimation error of $G_{t,k}, \forall k$, is denoted by $\Delta G_{t,k}$. 
The continuous set $Y_{t,k}, \forall k$, collects all possible channel estimation errors, while constant $\rho_{t,k}, \forall k$, denotes the maximum value of the norm of the CSI estimation error $\Delta G_{t,k}, \forall k$.

Since passive eavesdroppers typically hide their presence from the BS, they interact less frequently with the desired communication system. It is more practical to assume that only partial eavesdroppers’ CSI is available to legitimate users. The direct STAR-RIS-Eve$_{t,k}$ channel can be estimated by exploiting conventional uplink pilot transmission. We denote the direct STAR-RIS-Eve$_{t,k}$ channel as

$$g_{t,k} = \mathbb{E}_{\Delta G_{t,k}} + \Delta g_{t,k}, \forall k$$

with $\Omega_{t,k} = \{ \Delta g_{t,k} \in \mathbb{C}^{M \times 1} : ||\Delta g_{t,k}||_2 \leq \mu_{t,k}, \forall k \}$, where $\mathbb{E}_{\Delta G_{t,k}}$ is the estimate of the corresponding channel $g_{t,k}$ and the corresponding channel estimation error is $\Delta g_{t,k}$. The continuous set $\Omega_{t,k}$ collects all possible channel estimation errors, while constant $\mu_{t,k}$ is the maximum value of the norm of the CSI estimation error. Similarly, the BS-STAR-RIS-Eve$_{r,k}$ channel and the STAR-RIS-Eve$_{r,k}$ channel are denoted as $G_{r,k} = \text{diag}(g_{r,k}^H)H = \mathbb{G}_{r,k} + \Delta G_{r,k}$ with $Y_{r,k} = \{ \Delta G_{r,k} \in \mathbb{C}^{M \times N} : ||\Delta G_{r,k}||_F \leq \rho_{r,k}, \forall k \}$ and $g_{r,k} = \mathbb{G}_{r,k} + \Delta g_{r,k}$ with $\Omega_{r,k} = \{ \Delta G_{r,k} \in \mathbb{C}^{M \times 1} : ||\Delta G_{r,k}||_2 \leq \mu_{r,k}, \forall k \}$, respectively.

3. Problem Formulation

Here, we aim to maximize legitimate users’ sum rate by jointly designing beamforming at the BS and transmission and reflection coefficients at the STAR-RIS. Specifically, for the energy-splitting protocol, the problem can be expressed as

$$\max_{w_t, w_r, \Phi_t, \Phi_r} R_{b,t} + R_{b,r}$$

s.t. C1: $||w_t||^2 + ||w_r||^2 \leq P$
C2: $\Phi_m, \Phi'_m \in [0, 2\pi), \forall m \in M$
C3: $\alpha_m + \alpha'_m = 1, \alpha_m, \alpha'_m \in [0, 1], \forall m$
C4: $\max_{\Delta G_{r,k}} R_{e,t,k} \leq \tau_{t,k}, \max_{\Delta G_{r,k}} R_{e,r,k} \leq \tau_{r,k}, \forall k$

The objective function in (13) maximizes the system sum rate. Constraint C1 ensures that the transmit power at the BS does not exceed its maximum transmit power budget $P$. Constraint C2 represents the phase shift response of the $m$-th element, while constraint C3 represents the amplitude response of the $m$-th element. Constants $\tau_{t,k}$ and $\tau_{r,k}$ in C4 are the maximum tolerable information leakage to Eve$_{t,k}$ and Eve$_{r,k}$ for the wiretapping of the signal transmitted to Bob$_t$ and Bob$_r$, considering the CSI error. In particular, C4 can guarantee that the system secrecy rate $R_s$ is bounded from below, i.e., $R_s \geq R_{b,t} - \tau_{t,k} + R_{b,r} - \tau_{r,k}$.

Remark 1. The formula in (13) for the maximization of the system sum rate considering the information leakage constraints proposed is more flexible in resource allocation than the formula for the direct maximization of the system secrecy rate (such as [23,24]). In particular, the proposed problem formula can be adjusted with $\tau_{t,k}, \tau_{r,k}, \forall k$, to control the secrecy performance level of a single eavesdropper, offering a more flexible formula to determine the communication security level in heterogeneous practical applications. This is widely used in the literature [25–28].

4. System Sum Rate Maximization

4.1. Problem Transformation

Before solving the formulation problem, we first rewrite the problem in (13). Defining $\theta_t = [\sqrt{\alpha_1 e^{j\theta_1}}, \ldots, \sqrt{\alpha_M e^{j\theta_M}}]^H$ and $\theta_r = [\sqrt{\alpha_1 e^{j\theta_1}}, \ldots, \sqrt{\alpha_M e^{j\theta_M}}]^H$, we have $h_t^H \Phi_t H = \theta_t^H F_t, g_{t,k}^H \Phi_t H = \theta_t^H G_{t,k}, h_r^H \Phi_r H = \theta_r^H F_r$, and $g_{r,k}^H \Phi_r H = \theta_r^H G_{r,k}$, where the matrices
\[ F_t = \text{diag}(h_t^H)H, \quad G_{tk} = \text{diag}(g_{tk}^H)H, \quad F_r = \text{diag}(h_r^H)H, \quad \text{and} \quad G_{rk} = \text{diag}(g_{rk}^H)H. \] Hence, the received rates \((R_{b,t}, R_{b,r})\) and information leakages at eavesdroppers \((R_{e,t,k}, R_{e,r,k})\) in the constraint C4 can be equivalently rewritten as

\[
R_{b,t} = \log_2 (1 + \frac{\text{Tr}(F_t W_t F_t^H \Theta_t)}{\text{Tr}(F_t W_t F_t^H \Theta_t) + \sigma_r^2})
\]

\[
R_{b,r} = \log_2 (1 + \frac{\text{Tr}(F_t W_t F_t^H \Theta_r)}{\text{Tr}(F_t W_t F_t^H \Theta_r) + \sigma_r^2})
\]

\[
R_{e,t,k} = \log_2 (1 + \frac{\text{Tr}(G_{tk} W_t G_{tk}^H \Theta_t)}{\text{Tr}(G_{tk} W_t G_{tk}^H \Theta_t) + \sigma_r^2})
\]

\[
R_{e,r,k} = \log_2 (1 + \frac{\text{Tr}(G_{rk} W_r G_{rk}^H \Theta_r)}{\text{Tr}(G_{rk} W_r G_{rk}^H \Theta_r) + \sigma_r^2})
\]

in which we define the beamforming matrices as \(W_t = w_t w_t^H\) and \(W_r = w_r w_r^H\), denote \(\Theta_t = \theta_t \theta_t^H\) satisfying \(\Theta_t \succeq 0\) and \(\text{Rank}(\Theta_t) = 1\), and denote \(\Theta_r = \theta_r \theta_r^H\) satisfying \(\Theta_r \succeq 0\) and \(\text{Rank}(\Theta_r) = 1\). \(W_t, W_r, \Theta_t, \Theta_r\) are Hermitian matrices.

By solving the following optimization problem, the performance lower bound of the problem can be obtained:

\[
\max_{W_t, W_r, \Theta_t, \Theta_r} \quad \log_2 (1 + \frac{\xi_t}{\xi_t + \sigma_b^2}) + \log_2 (1 + \frac{\xi_r}{\xi_r + \sigma_b^2}) 
\]

s.t. \(C1: \text{Tr}(W_t) + \text{Tr}(W_r) \leq P\)

\(C2, C3, C4\)

\(C5: W_t \succeq 0, W_r \succeq 0\)

\(C6: \text{Rank}(W_t) \leq 1, \text{Rank}(W_r) \leq 1\)

\(C7: \xi_t - \text{Tr}(F_t W_t F_t^H \Theta_t) \leq 0, \quad \xi_r - \text{Tr}(F_t W_t F_t^H \Theta_r) \leq 0\)

\(C8: \text{Tr}(F_t W_t F_t^H \Theta_t) \leq \xi_t, \quad \text{Tr}(F_t W_t F_t^H \Theta_r) \leq \xi_r\)

\(C9: \Theta_t = \theta_t \theta_t^H, \quad \Theta_r = \theta_r \theta_r^H\)

where \(\xi_t, \xi_r, \xi_t, \) and \(\xi_r\) are slack variables. Moreover, the non-convex constraint in C9 in (14) can be replaced with the following constraints:

\[
C9a : Y_t = \begin{bmatrix} \Theta_t & \theta_t^H \\ \theta_t & 1 \end{bmatrix} \succeq 0, \quad Y_r = \begin{bmatrix} \Theta_r & \theta_r^H \\ \theta_r & 1 \end{bmatrix} \succeq 0
\]

\(C9b : \text{Rank}(Y_t) = 1, \text{Rank}(Y_r) = 1\)

Therefore, the optimization problem in (14) is equivalent to the following problem:

\[
\max_{W_t, W_r, \Theta_t, \Theta_r} \quad \log_2 (1 + \frac{\xi_t}{\xi_t + \sigma_b^2}) + \log_2 (1 + \frac{\xi_r}{\xi_r + \sigma_b^2}) 
\]

s.t. \(C1, C2, C3, C4, C5, C6, C7, C8, C9a, C9b\)
4.2. Optimization Variable Decoupling

For the optimization problem in (17), the non-convexity arises from constraints C4, C7, and C8 and the objective function due to severely coupled variables. In particular, there are two types of variable coupling, i.e., the fractional form of two continuous variables in the SINR and the multiplication between $W_i$, $W_r$, $\Theta_i$, $\Theta_r$, e.g., $\text{Tr}(F_i W_i F_i^H \Theta_i)$ and $\text{Tr}(F_r W_r F_r^H \Theta_r)$. To address the first type of coupling, we rewrite the SINR in (14) in its equivalent form, namely the difference of convex (D.C.) form:

$$
\max_{W_i, W_r, \Theta_i, \Theta_r} \quad \text{obj}(\xi, \zeta, t_i, t_r) - D_{obj}(t_i, t_r)
$$

where $\text{obj}(\xi, \zeta, t_i, t_r) = \log_2(\xi + t_i + \sigma^2_i) + \log_2(\zeta + t_r + \sigma^2_r)$ and $D_{obj}(t_i, t_r) = \log_2(t_i + \sigma^2_i) + \log_2(t_r + \sigma^2_r)$ are two functions that are both concave with respect to $\xi, \zeta, t_i, t_r$.

We take the term $-\text{Tr}(F_i W_i F_i^H \Theta_i)$ as an example to explain the construction of a convex constraint approximating the non-convex quality-of-service constraint C7. The term is rewritten as

$$
-\text{Tr}(F_i W_i F_i^H \Theta_i) = \frac{1}{2} || W_i - F_i^H \Theta_i F_i ||_F^2 - \frac{1}{2} \text{Tr}(W_i^H W_i) - \frac{1}{2} \text{Tr}(F_i^H \Theta_i F_i^H F_i^H \Theta_i F_i)
$$

Now, the last two terms in (19) are non-convex with respect to $W_i$ and $\Theta_i$, respectively. We construct a global underestimator for the non-convex terms via first-order Taylor approximation. Specifically, we have

$$
\text{Tr}(W_i^H W_i) \geq -|| W_i^{(i)} ||_F^2 + 2 \text{Tr}((W_i^{(i)})^H W_i)
$$

$$
\text{Tr}(F_i^H \Theta_i F_i^H F_i^H \Theta_i F_i) \geq -|| F_i^{(i)} \Theta_i^{(i)} F_i^{(i)} ||_F^2 + 2 \text{Tr}((F_i F_i^H \Theta_i F_i) F_i^H \Theta_i F_i)
$$

where $i$ denotes the iteration index. Using the above transformation, C4, C7, and C8 can be written in the following form:

$$
C4_1 : \min_{\Delta G_{i,k}} \frac{1}{2} || W_i + G_{i,k}^H \Theta_i G_{i,k} \Theta_i ||_F^2 - \frac{1}{2} \text{Tr}(W_i^H W_i) - \frac{1}{2} \text{Tr}(G_{i,k}^H \Theta_i G_{i,k}^H \Theta_i G_{i,k}^H G_{i,k})
$$

$$
+ (2^\tau_i - 1) \frac{1}{2} || W_i - G_{i,k}^H \Theta_i G_{i,k} ||_F^2 - \frac{1}{2} \text{Tr}(W_i^H W_i) - \frac{1}{2} \text{Tr}(G_{i,k}^H \Theta_i G_{i,k}^H \Theta_i G_{i,k}^H \Theta_i G_{i,k})
$$

$$
- (2^\tau_i - 1) \sigma_i^2 \leq 0, \forall k
$$

$$
C4_2 : \min_{\Delta G_{i,k}} \frac{1}{2} || W_i + G_{i,k}^H \Theta_i G_{i,k} \Theta_i ||_F^2 - \frac{1}{2} \text{Tr}(W_i^H W_i) - \frac{1}{2} \text{Tr}(G_{i,k}^H \Theta_i G_{i,k}^H \Theta_i G_{i,k}^H \Theta_i G_{i,k})
$$

$$
+ (2^\tau_i - 1) \frac{1}{2} || W_i - G_{i,k}^H \Theta_i G_{i,k} ||_F^2 - \frac{1}{2} \text{Tr}(W_i^H W_i) - \frac{1}{2} \text{Tr}(G_{i,k}^H \Theta_i G_{i,k}^H \Theta_i G_{i,k}^H \Theta_i G_{i,k})
$$

$$
- (2^\tau_i - 1) \sigma_i^2 \leq 0, \forall k
$$

$$
C7_1 : \xi_i + \frac{1}{2} || W_i - F_i^H \Theta_i F_i ||_F^2 - \text{Tr}((W_i^{(i)})^H W_i) + \frac{1}{2} || W_i^{(i)} ||_F^2 + \frac{1}{2} || F_i^H \Theta_i^{(i)} F_i ||_F^2
$$

$$
- \text{Tr}((F_i F_i^H \Theta_i^{(i)} F_i F_i^H)^H \Theta_i) \leq 0
$$

$$
C7_2 : \xi_r + \frac{1}{2} || W_r - F_r^H \Theta_r F_r ||_F^2 - \text{Tr}((W_r^{(i)})^H W_r) + \frac{1}{2} || W_r^{(i)} ||_F^2 + \frac{1}{2} || F_r^H \Theta_r^{(i)} F_r ||_F^2
$$

$$
- \text{Tr}((F_r F_r^H \Theta_r^{(i)} F_r F_r^H)^H \Theta_r) \leq 0
$$
\[ C_{81} : -\nu + \frac{1}{2} ||W_r + F^H_l \Theta_l F_l||^2 - \text{Tr}((W_r^{(i)})^H W_r) + \frac{1}{2} ||W_i^{(i)}||^2 + \frac{1}{2} ||F^H_l \Theta_l F_l||^2 \\
-\text{Tr}((F^H_l \Theta_l F_l)(F^H_l \Theta_l F_l)) \leq 0 \] (26)
\[ C_{82} : -\nu + \frac{1}{2} ||W_r + F^H_l \Theta_l F_l||^2 - \text{Tr}((W_r^{(i)})^H W_r) + \frac{1}{2} ||W_i^{(i)}||^2 + \frac{1}{2} ||F^H_l \Theta_l F_l||^2 \\
-\text{Tr}((F^H_l \Theta_l F_l)(F^H_l \Theta_l F_l)) \leq 0 \] (27)

4.3. S-Procedure

On the other hand, due to the CSI uncertainty, there are infinite possibilities for constraint C4. As a result, we adopt the S-procedure to tackle these issues. Then, for constraint C4, there is an infinite number of quadratic matrix inequalities (QMIs) due to the CSI uncertainty set. To overcome this problem, we first introduce slack optimization variables \( \Psi_{i,k} \) and \( \Psi_{r,k} \) to replace \( G^H_{i,k} \Theta_l G_{r,k} \) and \( G^H_{r,k} \Theta_l G_{r,k} \). Hence, constraint C4 can be equivalently rewritten as
\[
C_{4a1} : \frac{1}{2} ||W_r + \Psi_{i,k}||^2 - \frac{1}{2} \text{Tr}(W_r^H W_r) - 2\tau^{-1}_{i,k} \text{Tr}(\Psi_{i,k}^H \Psi_{i,k}) + (2\tau^{-1}_{i,k} - 1) \{ \frac{1}{2} ||W_r - \Psi_{i,k}||^2 - \frac{1}{2} \text{Tr}(W_r^H W_r) \} - (2\tau^{-1}_{i,k} - 1)\sigma_{r,k}^2 \leq 0, \forall k
\] (28)
\[
C_{4a2} : \frac{1}{2} ||W_r + \Psi_{r,k}||^2 - \frac{1}{2} \text{Tr}(W_r^H W_r) - 2\tau^{-1}_{r,k} \text{Tr}(\Psi_{r,k}^H \Psi_{r,k}) + (2\tau^{-1}_{r,k} - 1) \{ \frac{1}{2} ||W_r - \Psi_{r,k}||^2 - \frac{1}{2} \text{Tr}(W_r^H W_r) \} - (2\tau^{-1}_{r,k} - 1)\sigma_{r,k}^2 \leq 0, \forall k
\] (29)
\[
C_{4b} : \Psi_{i,k} \geq \max_{\Delta G_{i,k}} G^H_{i,k} \Theta_l G_{r,k}, \forall k, \Psi_{r,k} \geq \max_{\Delta G_{r,k}} G^H_{r,k} \Theta_l G_{r,k}, \forall k
\] (30)

Note that constraint C4b still involves an infinite number of inequality constraints. To circumvent this difficulty, we use the Generalized S-procedure to convert the infinite constraint C4b to an equivalent form with a finite number of constraints by applying Lemma 1.

**Lemma 1 (Generalized S-Procedure [29]).** Let \( f(X) = X^H DX + X^H B + B^H X + E \) and \( J \succeq 0 \). For some \( \epsilon \geq 0 \), \( f(X) \succeq 0, \forall X \in \{ X | |XJX^H| \leq 1 \} \) is equivalent to
\[
\begin{bmatrix} E & B^H \\ B & D \end{bmatrix} - \epsilon \begin{bmatrix} I & 0 \\ 0 & -J \end{bmatrix} \succeq 0
\] (31)

On the basis of the above Lemma 1, we can express the first term of the constraint C4b as
\[
C_{4b1} : -\Delta G^H_{i,k} \Theta_l \Delta G_{i,k} - \Delta G^H_{i,k} \Theta_l \Delta G_{r,k} - \Delta G^H_{r,k} \Theta_l \Delta G_{i,k} - \Delta G^H_{r,k} \Theta_l \Delta G_{r,k} + \Psi_{i,k} \succeq 0, \forall k
\] (32)

Then, applying Lemma 1, the first term of constraint C4b is equivalently transformed as
\[
\overline{C_{4b1}} : S_{C_{4b}}(\Theta_l, e_{i,k}) = \begin{bmatrix} -G^H_{i,k} \Theta_l G_{i,k} + \Psi_{i,k} - \epsilon_{i,k} I_N \\ -G^H_{r,k} \Theta_l G_{i,k} \end{bmatrix} \succeq 0, \forall k
\] (33)
\[
\overline{C_{4c1}} : e_{i,k} \geq 0, \forall k
\] (34)
Similarly, the second term of constraint C4b can be equivalently represented as

$$\mathcal{C}_{4b}^{2} : S_{C4b}^{2}(\Theta, e_{r,k}) = \begin{bmatrix} -\mathcal{G}_{r,k}^{H} \Theta \mathcal{G}_{r,k} + \mathcal{Y}_{r,k} - e_{r,k}^{I} \mathcal{I}_{N} & -e_{r,k}^{I} \mathcal{I}_{M} - \Theta \\ -\Theta \mathcal{G}_{r,k} & e_{r,k}^{I} \mathcal{I}_{M} - \Theta \end{bmatrix} \succeq 0, \forall k$$  \hspace{1cm} (35)$$

$$\mathcal{C}_{4c}^{2} : e_{r,k}^{I} \geq 0, \forall k$$  \hspace{1cm} (36)

4.4. SCA- and SDR-Based Iterative Algorithm

Next, to tackle the non-convexity of constraints C4a and C9b and the objective function in (17), we first transform them into the D.C. form such that SCA can be applied to obtain a suboptimal solution. Firstly, the rank-one constraint C9b is rewritten using Lemma 2:

**Lemma 2.** The rank-one constraint in the first term of C9b is equivalent to the following D.C. form constraint:

$$\mathcal{C}_{9b}^{1} : ||Y_{i}||_{*} - ||Y_{i}||_{2} \leq 0$$  \hspace{1cm} (37)

**Proof.** Since $Y_{i}$ is a Hermitian matrix, the inequality $||Y_{i}||_{*} = \sum_{i} \rho_{i} \geq ||Y_{i}||_{2} = \max_{i} \{\rho_{i}\}$ holds, where the $i$-th singular value of $Y_{i}$ is denoted by $\rho_{i}$. Thus, the inequality in (37) holds if and only if $Y_{i}$ has a unit rank. ☐

Aiming to tackle the D.C. form constraint $\mathcal{C}_{9b}^{1}$, a lower bound of $||Y_{i}||_{2}$ is given by the first-order Taylor approximation for any feasible point $Y_{i}^{(t)}$ during the $i$-th iteration

$$||Y_{i}||_{2} \geq ||Y_{i}^{(t)}||_{2} + \text{Tr}((\lambda_{\max}(Y_{i}^{(t)})\times \lambda_{\max}^{H}(Y_{i}^{(t)}))(Y_{i} - Y_{i}^{(t)}))$$  \hspace{1cm} (38)

On the basis of the SCA method, C9b can be obtained, which is given by

$$\mathcal{C}_{9b}^{1} : ||Y_{i}||_{*} - ||Y_{i}^{(t)}||_{2} - \text{Tr}((\lambda_{\max}(Y_{i}^{(t)})\lambda_{\max}^{H}(Y_{i}^{(t)}))(Y_{i} - Y_{i}^{(t)})) \leq 0$$

$$\mathcal{C}_{9b}^{2} : ||Y_{i}||_{*} - ||Y_{i}^{(t)}||_{2} - \text{Tr}((\lambda_{\max}(Y_{i}^{(t)})\lambda_{\max}^{H}(Y_{i}^{(t)}))(Y_{i} - Y_{i}^{(t)})) \leq 0$$  \hspace{1cm} (39)

Since $\mathcal{C}_{9b} \Rightarrow \mathcal{C}_{9b}^{1}$, replacing C9b with $\mathcal{C}_{9b}^{1}$ can ensure that the former is satisfied when the proposed algorithm converges. On the other hand, it can be observed that, for the non-convex objective function in (18), the non-convex constraint is in a D.C.-differentiable form. Similarly, after deriving the first-order Taylor expansions corresponding to the non-convex component, for any feasible point $t_{i}^{(t)}, t_{r}^{(t)}$, we have the following inequalities:

$$D_{obj}(t_{i}, t_{r}) \leq \nabla_{t_{i}}^{H} D_{obj}(t_{i}^{(t)})(t_{i} - t_{i}^{(t)}) + \nabla_{t_{r}}^{H} D_{obj}(t_{r}^{(t)})(t_{r} - t_{r}^{(t)}) + D_{obj}(t_{i}^{(t)}, t_{r}^{(t)})$$  \hspace{1cm} (40)

where $\nabla_{t_{i}}^{H} D_{obj}(t_{i}) = \frac{1}{(ln2)(c_{0}^{2} + u)} \nabla_{t_{r}}^{H} D_{obj}(t_{r}) = \frac{1}{(ln2)(c_{0}^{2} + u)}$. As such, the lower bound of the objective function (18) is given by

$$N_{obj}(t_{i}, t_{r}, t_{i}, t_{r}) - D_{obj}(t_{i}^{(t)})(t_{i} - t_{i}^{(t)}) - \nabla_{t_{i}}^{H} D_{obj}(t_{i}^{(t)})(t_{i} - t_{i}^{(t)}) - D_{obj}(t_{i}^{(t)}, t_{r}^{(t)})$$  \hspace{1cm} (41)
Similarly, to overcome the non-convexity of $C_{4a}$, we can use the inequality in (20) to obtain

\begin{align*}
C_{4a1} : & \|W_t + \Psi_{t,k}\|^2_2 + \|W^{(i)}_t\|^2_2 - 2\text{Tr}((W^{(i)}_t)^H W_t) + 2\tau_{t,k} \|\Psi^{(i)}_{t,k}\|^2_2 - 2\tau_{t,k+1} \text{Tr}((\Psi^{(i)}_{t,k})^H \Psi_{t,k}) \\
& - (2\tau_{t,k+1} - 2)s^2 + (2\tau_{t,k} - 1)\{\|W_t - \Psi_{t,k}\|^2_2 + \|W_t^{(i)}\|^2_2 - 2\text{Tr}((W_t^{(i)})^H W_t)\} \leq 0
\end{align*}

\begin{align*}
C_{4a2} : & \|W_t + \Psi_{r,k}\|^2_2 + \|W_t^{(i)}\|^2_2 - 2\text{Tr}((W_t^{(i)})^H W_t) + 2\tau_{r,k} \|\Psi_{r,k}\|^2_2 - 2\tau_{r,k+1} \text{Tr}((\Psi_{r,k})^H \Psi_{r,k}) \\
& - (2\tau_{r,k+1} - 2)s^2 + (2\tau_{r,k} - 1)\{\|W_t - \Psi_{r,k}\|^2_2 + \|W_t^{(i)}\|^2_2 - 2\text{Tr}((W_t^{(i)})^H W_t)\} \leq 0
\end{align*}

Next, we focus on the optimization of $\theta, \theta'$. We rewrite constraints $C2$ and $C3$ as $C10 : \text{Diag}(\theta, \theta') = 1$. It is obvious that the above constraint $C10$ is non-convex. To cope with this, we introduce auxiliary vectors $x_k = [x_{1,m}, \ldots, x_{L,M}]^T, x_r = [x_{r,1}, \ldots, x_{r,M}]^T$ to linearize $C10$, and $x_{1,m} = [\theta_m]^*[\theta_m]_m, x_{r,m} = [\theta_r]^*[\theta_r]_m$.

Following the principles of the penalty concave-convex procedure, we relax $x_{1,m}, x_{r,m}$ by $x_{1,m} \leq [\theta_m]^*[\theta_m]_m \leq x_{1,m}, x_{r,m} \leq [\theta_r]^*[\theta_r]_m \leq x_{r,m}$, and $x_{1,m} \leq [\theta_m]^*[\theta_m]_m, x_{r,m} \leq [\theta_r]^*[\theta_r]_m$ are approximated by $x_{l,m} \leq 2\text{R}([\theta_m]^*[\theta_m]_m) - [\theta_m]^*[\theta_m]_m, x_{r,m} \leq 2\text{R}([\theta_r]^*[\theta_r]_m) - [\theta_r]^*[\theta_r]_m$.

Thus, we obtain problem (42), as shown at the bottom of the page, where $y_{l,m} \geq 0, y_{r,m} \geq 0$ are the slack variables for the modulus constraints; $\sum_{m=1}^{2M} y_{l,m}, \sum_{m=1}^{2M} y_{r,m}$ are the penalty terms added into the objective function, which is scaled by the multiplier $\eta^{(i)}$ in the $i$-th iteration. In addition, $\eta^{(i)}$ is updated with $\eta^{(i)} = \min(\gamma\eta^{(i-1)}, \eta_{\max})$, where the upper bound $\eta_{\max}$ is used to avoid numerical problems. The original problem can be obtained from the following optimization problem:

\begin{align}
& \max_{W_t, W_r, Y_t, Y_r, \Theta, \Theta'} N_{\text{obj}}(\zeta_t, \zeta_r, \zeta_t, \zeta_r) - \gamma H_{\text{obj}}(l_t^{(i)}(\eta_t^{(i)} - l_t^{(i)})) - \gamma H_{\text{obj}}(l_r^{(i)}(\eta_r^{(i)} - l_r^{(i)})) - \gamma H_{\text{obj}}(l_t^{(i)}(\eta_r^{(i)} - l_r^{(i)})) - \gamma H_{\text{obj}}(l_t^{(i)}(\eta_t^{(i)} - l_t^{(i)})) - \eta^{(i)} \left( \sum_{m=1}^{2M} y_{l,m} + \sum_{m=1}^{2M} y_{r,m} \right) \\
& \text{s.t.} \quad C1, C4a, C4b, C4c, C5, C6, C7, C8, C9a, C9b
\end{align}

\begin{align*}
C10a : & \{[\theta_m]^*[\theta_m]_m \leq x_{l,m} + y_{l,m}, [\theta_m]^*[\theta_m]_m \leq x_{r,m} + y_{r,m} \}
\end{align*}

\begin{align*}
C10b : & \{[\theta_m]^*[\theta_m]_m - 2\text{R}([\theta_m]^*[\theta_m]_m) \leq y_{l,m} + y_{r,m} - x_{l,m}, [\theta_m]^*[\theta_m]_m - 2\text{R}([\theta_m]^*[\theta_m]_m) \leq y_{r,m} + y_{r,m} - x_{r,m} \}
\end{align*}

\begin{align*}
C10c : & \{x_{l,m} + y_{l,m} = 1, x_{l,m} \geq 0, x_{r,m} \geq 0 \}
\end{align*}

Since the rank-one constraint in $C6$ is the only non-convex part of the optimization problem (42), we apply the SDR technique to drop the rank constraint in $C6$, such that the relaxed rank constraint version of (42) can be solved by using a standard numerical solver for convex programming. In the following theorem, the tightness of the adopted SDR is revealed.

**Theorem 1.** For $P > 0$ and if (42) is feasible, the rank-one constraint of $C6$ in (42) can always be obtained.

**Proof.** Please refer to Appendix A for the proof of Theorem 1. \(\square\)

Due to the use of SCA, solving the problem in (42) provides a lower bound. To tighten the lower bound of performance, we iteratively update the feasible solution by solving the problem in (42). The proposed algorithm is shown in Algorithm 1.
Algorithm 1: Main loop: SCA

1. **Input** the maximum number of iterations $i_{\text{max}}$, the initial iteration index $i = 0$, and variables $W^0_t$, $W^0_r$, $Y^0_t$, $Y^0_r$, $\mathbf{\Theta}^0_t$, $\mathbf{\Theta}^0_r$, $\mathbf{\Psi}^0_{t,k}$, $\mathbf{\Psi}^0_{r,k}$, $t^0_r$, $t^0_r$, $\eta^0$.

2. **Output** the optimal variables $W_t$, $W_r$, $Y_t$, $Y_r$, $\mathbf{\Theta}_t$, $\mathbf{\Theta}_r$, $\mathbf{\Psi}_{t,k}$, $\mathbf{\Psi}_{r,k}$, $x_t, x_r, y_t, y_r, e_{t,k}^{\text{CR}}, e_{r,k}^{\text{CR}}$, $\xi_t, \xi_r, t_r, t_r$.

3. **repeat**

4. Solve the problem (42) with $W_t^i$, $W_r^i$, $Y_t^i$, $Y_r^i$, $\mathbf{\Theta}_t^i$, $\mathbf{\Theta}_r^i$, $\mathbf{\Psi}_{t,k}^i$, $\mathbf{\Psi}_{r,k}^i$, $x_t, x_r, y_t, y_r, e_{t,k}^{\text{CR}}, e_{r,k}^{\text{CR}}$, $\xi_t, \xi_r, t_r, t_r$ to obtain $W_t, W_r, Y_t, Y_r, \mathbf{\Theta}_t, \mathbf{\Theta}_r, \mathbf{\Psi}_{t,k}, \mathbf{\Psi}_{r,k}, x_t, x_r, y_t, y_r, e_{t,k}^{\text{CR}}, e_{r,k}^{\text{CR}}$, $\xi_t, \xi_r, t_r, t_r$.

5. Set $i = i + 1$, and $\eta^{(i)}$ is updated with $\eta^{(i)} = \min\{\eta^{(i-1)}, \eta_{\text{max}}\}$.

6. Replace $W_t^{i-1}, W_r^{i-1}, Y_t^{i-1}, Y_r^{i-1}, \mathbf{\Theta}_t^{i-1}, \mathbf{\Theta}_r^{i-1}, \mathbf{\Psi}_{t,k}^{i-1}, \mathbf{\Psi}_{r,k}^{i-1}$ with $W_t, W_r, Y_t, Y_r, \mathbf{\Theta}_t, \mathbf{\Theta}_r, \mathbf{\Psi}_{t,k}, \mathbf{\Psi}_{r,k}$.

7. **until** convergence or $i = i_{\text{max}}$.

The computational complexity of each iteration of the proposed algorithm is given by $O((l_1 V_1^2 + l_2 V_2^2 + V_3^2) \times \sqrt{T_1 \log \frac{1}{\epsilon}})$, where $l_1 = 11 + 6K + 6M$ and $V_1 = 16 + 4K$ are the number of inequalities and the number of variables of the proposed scheme, respectively. Besides, $\epsilon$ is the threshold of convergence tolerance. The computational complexity of each iteration of the conventional RIS algorithm is given by $O((V_2^2 + V_3^2 + V_3^2) \times \sqrt{T_1 \log \frac{1}{\epsilon}})$, where $l_2 = 10 + 6K + 6M$ and $V_2 = 14 + 4K$ are the number of inequalities and the number of variables of the conventional RIS scheme, respectively. Although the complexity of conventional RIS is lower, it needs to use multiple RISs to achieve full space coverage, so it is not as flexible and cost-effective as the deployment of STAR-RIS.

5. Simulation Results and Analysis

We assume that the BS and the STAR-RIS are deployed at (0, 0, 10) and (5, 20, 10) meters, and Bob, and Bob, are deployed at (6, 21, 10) and (4, 19, 10) meters. The number of antennas at the BS is 36, and the maximum transmit power is 10mW. Eve$_{t,k}$ and Eve$_{r,k}$ are deployed on half-circles centered at the STAR-RIS with a radius of 5 m. We assume that the STAR-RIS is composed of $M = M_t M_r$ elements, and we define $v_{G_{t,k}}, v_{G_{r,k}}, v_{G_{t,k}}, v_{G_{r,k}}$ as the maximum normalized estimation error for the channel $G_{t,k}, G_{r,k}, G_{t,k}, G_{r,k}$, i.e., $v_{G_{t,k}} = \frac{\rho_{t,k}}{|G_{t,k}|^2}, v_{G_{r,k}} = \frac{\rho_{t,k}}{|G_{t,k}|^2}, v_{G_{t,k}} = \frac{\rho_{r,k}}{|G_{r,k}|^2}, v_{G_{r,k}} = \frac{\rho_{r,k}}{|G_{r,k}|^2}$. Unless further specified, we fix $v = v_{G_{t,k}} = v_{G_{r,k}} = v_{G_{t,k}} = v_{G_{r,k}}$ and $v^2 = 0.1$, which is the normalized mean square error based on the channel estimation method in [30]. The maximum tolerable channel capacity of potential eavesdroppers is $\tau_{t,k} = \tau_{r,k} = 2.1 \text{ b/s/Hz}$. The noise power for users and eavesdroppers is $\sigma_n^2 = \sigma_e^2 = -80 \text{ dBm}$. For the parameters of the penalty concave–convex procedure method, we set $\eta_{\text{max}} = 10$ and $\gamma = 0.8$.

For the millimeter-wave secure communication system assisted by a STAR-RIS, this study proposes a joint optimization design scheme considering the transmission beamforming vector of the BS and the transmission and reflection coefficients of the STAR-RIS to maximize the system sum rate. The power budget of the BS is an important factor affecting the system sum rate. As shown in Figure 2, with an increase in the power budget, the system sum rate also increases. By increasing the power, the signal strength of the receiving user can be improved, but at this time, the signal strength of the eavesdropper will also be increased. Therefore, the power of the BS should be set reasonably. When the transmission power is small, such as 0 dBm or 5 dBm, the performance improvement of STAR-RIS is not obvious. When the transmitting power is large, such as 20 dBm or 25 dBm, the performance improvement of STAR-RIS is more obvious. It can also be seen from the figure that the sum rate of STAR-RIS is greater than the sum rate of the conventional RIS under the same power of the BS.
The power budget $P$ (dBm) 

Figure 2. System sum rate versus the power budget.

Figure 3 shows the impact of the number of elements of the two RISs on the system sum rate. It can be seen from the figure that increasing the number of elements of the RIS can increase the system sum rate. As the number of RIS elements increases, for the STAR-RIS, the signal strength transmitted to the transmission region increases, and the signal strength reflected in the reflection region also increases. For a conventional RIS, the signal strength reflected in the reflection region increases. Therefore, for both RISs, the system sum rate increases. It can also be seen from the figure that, with the same number of elements, the sum rate of STAR-RIS is greater than the sum rate of the conventional RIS. With the increase of the number of elements, the relative value of the performance increase of STAR-RIS is almost unchanged.

Figure 3. System sum rate versus number of elements.

Figure 4 shows the impact of the number of antennas of the BS on the system sum rate. By increasing the number of antennas of the BS, the system sum rate can be improved. As the number of antennas in the BS increases, the strength of the transmitted signal increases, as do the signal transmitted to the RIS and the strength of the received signal at the user side. With the same number of BS antennas, the sum rate of STAR-RIS is greater than the sum rate of the conventional RIS. As the number of antennas increases, the relative value of the performance increase of STAR-RIS is almost similar.
6. Conclusions

This study considers a millimeter-wave secure communication system assisted by a STAR-RIS, in which the BS realizes the requirements of secure communication with users in the transmission region and reflection region through the STAR-RIS. By jointly optimizing the transmit beamforming of the BS and the transmitting and reflecting coefficients of the STAR-RIS, the system sum rate is maximized. For the non-convex optimization problem, this study uses a series of transformations to decouple the optimization variables, uses the $S$-Procedure algorithm to solve the imperfect ECSI, and then, uses the SCA and SDR to design an iterative optimization algorithm. At this time, the problem is transformed into a convex problem, which can be solved using the standard convex programming numerical solver. The simulation results show that the millimeter-wave secure communication scheme assisted by a STAR-RIS proposed in this study can significantly improve the system sum rate and ensure the safe transmission of information. STAR-RIS can be installed in all parts of the city to achieve full coverage of the communication range. Besides, our proposed approach has the potential to improve data transmission speeds and security in various fields like autonomous vehicles, smart cities, and remote healthcare services. The combination of millimeter-wave and RIS can also minimize signal interference and reduce power consumption, making it more sustainable for long-term use.

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Appendix A. Proof of Theorem 1

Proof of Theorem 1. We first introduce the slack optimization variables $C_{4a,j,k}$ and $C_{4b,j,k}$ and the constraint C11 to facilitate the proof. Then, the relaxed rank constraint version of the problem (42) can be transformed into its equivalent form, i.e.,
\[
\max_{W_i, W_r, Y_i, Y_r, \Theta_i, \Theta_r, Y_{i,t}, Y_{r,t}, s_{i}, s_{r}, y_{i}, y_{r}, t_i, t_r, c_{i,r}, c_{r,i}} N_{\text{obj}}(\xi_i, \xi_r, t_i, t_r) - \nabla^H_i D_{\text{obj}}(t_i) (t_i - t_i(i)) - \nabla^H_r D_{\text{obj}}(t_r) (t_r - t_r(i)) - D_{\text{obj}}(t_i, t_r) - \eta(i) \left( \sum_{m=1}^{2M} y_{t,m} + \sum_{m=1}^{2M} y_{r,m} \right)
\tag{A1}
\]

s.t.
\[
\begin{align*}
& C_{4a_1} : \|C_{4a_{i,j,k}}\|^2 + \|W_{i,j}^{(i)}\|^2 - 2\text{Tr}((W_{i,j}^{(i)})^H W_i) + 2\|\Psi_{i,j,k}\|^2 - 2c_{i,j,k}^2 - 2(\|W_i - \Psi_{i,j,k}\|^2 + \|W_{r} - \Psi_{i,j,k}\|^2 - 2\|W_{i,j}^{(i)}\|^2) \leq 0 \\
& C_{4a_2} : \|C_{4a_{i,j,k}}\|^2 + \|W_{i,j}^{(i)}\|^2 - 2\text{Tr}((W_{i,j}^{(i)})^H W_i) + 2\|\Psi_{i,j,k}\|^2 - 2(\|W_i - \Psi_{i,j,k}\|^2 + \|W_{r} - \Psi_{i,j,k}\|^2 - 2\|W_{i,j}^{(i)}\|^2) \leq 0 \\
& C_{11} : C_{4a_{i,j,k}} \succeq W_i + \Psi_{i,j,k} C_{4a_{i,j,k}} \succeq W_r + \Psi_{i,j,k}
\end{align*}
\]

\[
\mathcal{L} = -\delta_{C_{1},t} W_i - \delta_{C_{1},r} W_r + \sum_{k \in K} (2 \delta_{C_{4a_{i,j,k}}} \text{Tr}((W_{i,j}^{(i)})^H W_i) + 2\delta_{C_{4a_{i,j,k}}} \text{Tr}((W_{i,j}^{(i)})^H W_r))
\]
\[
+ \text{Tr}(M_{C_{5},t} W_i + M_{C_{5},r} W_r) - \sum_{k \in K} (\text{Tr}(M_{C{11},t,k} W_i) + \text{Tr}(M_{C{11},r,k} W_r)) + \Delta
\tag{A2}
\]

Since (A1) is jointly convex with the optimization variables and satisfies Slater’s constraint qualification, its strong duality holds. The Lagrangian function of problem (A1) in terms of $W_i$ and $W_r$ is given in (A2). $\Delta$ in (A2) denotes the collection of terms that are irrelevant to $W_i$ and $W_r$. In (A2), $\delta_{C_{1},t}$, $\delta_{C_{1},r}$ are the Lagrange multipliers for the constraint $C_1$; $\delta_{C_{4a_{i,j,k}}}$, $\delta_{C_{4a_{i,j,k}}}^r$ are the Lagrange multipliers for the constraint $C_{4a_{i,j,k}}$ respectively. $M_{C_{5},t}$, $M_{C_{5},r}$ are the Lagrange multiplier matrices corresponding to constraint $C5$. $M_{C{11},t,k}$, $M_{C{11},r,k}$ are the Lagrange multiplier matrices corresponding to constraint $C11$. Then, examining the Karush–Kuhn–Tucker (KKT) conditions for problem (A1) yields

\[
K1 : M_{C{11},t,k} \succeq 0, M_{C{11},r,k} \succeq 0, \delta_{C_{1},t} \succeq 0, \delta_{C_{1},r} \succeq 0, \delta_{C_{4a_{i,j,k}}} \succeq 0, \delta_{C_{4a_{i,j,k}}}^r \succeq 0
\tag{A3}
\]

\[
K2 : M_{C_{5},t}^* W_i^* = 0, M_{C_{5},r}^* W_r^* = 0, M_{C_{10},t,k}^* W_i^* = 0, M_{C_{10},r,k}^* W_r^* = 0
\tag{A4}
\]

\[
K3 : \forall \mathbf{w} \mathcal{L} = 0, \forall \mathbf{w} \mathcal{L} = 0
\]

For the ease of presentation, K3 in (A5) can be recast as

\[
\delta_{C_{1},t}^* \mathbf{I}_N - Y_i^* = M_{C_{5},t}^* \delta_{C_{1},t}^* \mathbf{I}_N = Y_i^* = M_{C_{5},r}^*
\tag{A6}
\]

where $Y_i^* = Y_r^* = 2(\sum_{k \in K} \delta_{C_{4a_{i,j,k}}}^t W_{i,j}^{(i)} + \sum_{k \in K} \delta_{C_{4a_{i,j,k}}}^r W_{r,j}^{(i)}) - (\sum_{k \in K} M_{C{11},t,k}^* + \sum_{k \in K} M_{C{11},r,k}^*)$.

From (A4), we know that matrix $W_i^*$ lies in the null space of $M_{C_{5},t}^*$. To reveal the rank of $W_i^*$, we first investigate the structure of $M_{C_{5},t}^*$. When $\delta_{C_{1},t}^* = 0$, $Y_i^* \succeq 0$, which leads to the dual problem of the unbounded problem (A1). Thus, $\delta_{C_{1},t}^* > 0$ holds. According to (A6), if $\lambda_{\max}(Y_i^*) > \delta_{C_{1},t}^*$, then $\lambda_{\max}(M_{C{5},t}^*) < 0$, which contradicts K1 in (A3). If $\lambda_{\max}(Y_i^*) < \delta_{C_{1},t}^*$, $M_{C_{5},t}^*$ is a positive definite matrix with full rank, which yields the solution $W_i^* = 0$ and $\text{Rank}(W_i^*) = 0$. If $\lambda_{\max}(Y_i^*) = \delta_{C_{1},t}^*$ holds at the optimal solution, in order to have a bounded optimal dual solution, the null space of $M_{C_{5},t}^*$ is spanned by vector $r_{\max} \in \mathbb{C}^{N+1}$, which is a unit-norm eigenvector of $Y_i^*$ associated with eigenvalue $\lambda_{\max}(Y_i^*)$. Therefore, $W_i^* = v_{\max} r_{\max}^H$ is the optimal beamforming matrix, where $v$ is a parameter that enables
the power of the transmitter to satisfy constraint C1. Similarly, the same analysis can be performed on $W_r$. □

References


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