

Article

The Application of the Piecewise Linear Method for Non-Linear Programming Problems in Ride-Hailing Assignment Based on Service Level, Driver Workload, and Fuel Consumption

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Abstract: Ride-hailing services have grown rapidly, presenting challenges such as increased traffic congestion, inefficient driver workload distribution, and environmental concerns like higher fuel consumption and emissions. This study develops a non-linear ride-hailing assignment model addressing these issues by considering service level, driver workload, and fuel consumption. A piecewise linear method was employed to handle a non-linear programming model, and the method was modified to function autonomously without operator intervention. The model's performance was evaluated using a publicly accessible dataset of taxi trips in Manhattan, focusing on indicators such as passenger waiting time, driver workload distribution, and fuel consumption. Numerical simulations demonstrated significant improvements: a 15% reduction in average passenger waiting time, a 20% improvement in balancing driver workloads, and a 10% decrease in overall fuel consumption, contributing to reduced emissions and environmental impact. The modified piecewise linear method proved effective in optimizing ride-hailing assignments, providing a more efficient and sustainable solution. The model also showed robustness in handling large datasets, ensuring scalability and applicability to various urban settings. These findings highlight the model's potential to enhance operational efficiency and promote sustainability in ride-hailing services. By integrating considerations for service level, driver workload, and fuel consumption, the model offers a holistic approach to addressing the key challenges faced by the ride-hailing industry. This study provides valuable insights for future ride-hailing development and implementations of ride-hailing systems, promoting practices that are both efficient and environmentally friendly.

Keywords: piecewise linear method; non-linear programming; ride-hailing assignment; service level; driver workload; fuel consumption

MSC: 90C59; 90C30; 90B06; 90B50



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1. Introduction

Ride-hailing services are important in assisting people experiencing difficulty accessing public transportation and vulnerable groups such as women with children or people with disabilities [1]. The phenomenon is intended to reduce social and spatial disparities as well as open up opportunities for better integration with public transportation systems, especially in low-density areas [2]. Despite providing benefits in meeting daily travel needs, the rapid growth of ride-hailing services has given rise to new challenges, such as increasing traffic density on highways [3]. The growth of these services has also attracted the attention of academics and transportation planners, leading to more profound

research into the social, economic, and environmental impacts [4]. Moreover, there is a need for an in-depth understanding of the factors influencing the adoption and use of ride-hailing services to assist policymakers in formulating more effective policies to regulate the industry [5]. Mathematical developments can also be used to analyze the influential factors in order to provide valuable insights into understanding and optimizing ride-hailing service operations.

Several main components are critical to the implementation of ride-hailing services. An example is the assignment system, often called matching or dispatch in other literature, designed to arrange driver partners to strategically fulfill passenger requests [6–8]. The increase in ride-hailing services raises questions regarding the impact of assignment mechanisms on transportation efficiency, specifically in reducing waiting times compared to traditional systems [9]. Previous research showed that advanced assignment algorithms and dynamic pricing enabled an efficient experience with minimal wait time for riders and drivers [7]. Therefore, assigning available drivers to take passenger requests efficiently is considered a critical operational problem of ride-hailing services [10].

The operation of ride-hailing has significantly impacted urban traffic and the environment. As the number of vehicles in the system rises, problems such as traffic congestion and vehicle exhaust emissions become increasingly troubling [11]. The rapid growth of the industry is providing undeniable convenience in urban mobility but also causing new problems, such as the continued increase in the volume of effectively idle ride-hailing vehicles [12]. This has made the development of reasonable assignment strategies for idle vehicles an urgent problem [13]. Another important issue is the fatigue experienced by the drivers because long working hours and lack of sleep are the main factors causing road accidents [14]. Moreover, drivers mostly discuss the imbalanced distribution of workload, causing significant dissatisfaction. It is also important to state that passengers often wait for drivers to make detours, leading to an increase in waiting time, fuel usage, and gas emissions [15]. A strategic step suggested to overcome the problems identified is to design an assignment system that allows passengers to walk to the vehicle location in order to reduce the trips to be made without passengers and avoid unnecessary fuel consumption. Therefore, this research aimed to develop a ride-hailing assignment model that simultaneously considers service level, driver workload, and fuel savings in order to improve the mechanism.

Previous research generally considered the travel distance [8,16–22], travel time [8,19,21,23,24], minimizing the number of unfulfilled requests [8,15,23], profit maximization [16,25], maximizing the rewards obtained [7,26], and waiting time [8,15,16,19,23,27–29] in the assignment model. The trend shows that the focus has been on passenger welfare without consideration for workload distribution and fuel savings simultaneously and comprehensively. It was observed that [15] proposed a model that considered fuel savings not included in previous research. Therefore, a ride-hailing assignment model that simultaneously considers passenger welfare, workload distribution, and fuel savings needs to be developed. This research was conducted to fill the gap through the application of non-linear programming to develop the model.

Non-linear programming is important in solving several real-world problems in economics, traffic management, decision support, and urban planning. This is possible because the extension of mathematical models to include non-linear phenomena is making it possible to address complex problems more effectively. Therefore, the methods related to non-linear programming are considered significant in solving optimization problems of different fields. For example, the Gauss–Newton and Quasi-Newton methods are widely used for function optimization, specifically when computation or iteration is expensive. In machine learning, optimization is essential for tasks such as backpropagation of model errors through the application of methods such as stochastic gradient descent, conjugate gradient, and Levenberg–Marquardt [30]. Those related to geophysics include the adoption of the steep descent, conjugate gradient, Newton, genetic algorithm, and simulated annealing methods to estimate subsurface information using seismic data and well logs [31].

Furthermore, the steep descent and Newton methods are commonly used to extremize non-linear functions with single and multiple design variables, showing the importance of unconstrained optimization in solving complex problems [32]. The trend generally shows that non-linear programming methods can provide valuable tools to appropriately and effectively address the complexity of real-world challenges.

The implementation of the methods for real-world problems is mainly due to their flexibility in handling non-linear relationships. Moreover, the inclusion of piecewise linear methods can assist in solving linear programming problems while maintaining convexity [33]. Some piecewise linear methods, including bivariate and univariate approximations, are frequently used in optimization to address non-linear programming issues [34]. These methods approximate nonconvex functions with piecewise linear functions, facilitating easier optimization. Recent progress has introduced mixed-integer programming (MIP) formulations incorporating nonconvex piecewise linear functions into optimization models, surpassing existing methods in different contexts [35]. Furthermore, piecewise linear approximation (PLA) has proven effective in converting non-linear problems into linear ones, improving computational efficiency and accuracy, particularly with nonuniform domain partitioning [36]. In machine learning, methods such as k-means clustering and decision tree regressors are used to accurately identify breakpoints in piecewise linear regression models, with k-means clustering showing higher accuracy in determining breakpoints [37]. However, the application of piecewise linear methods requires direct interaction with the user, which is necessary to check the restricted basis conditions, as discussed in [38]. The methods can only solve separable non-linear programming problems and explain non-linear functions globally by representing linear functions locally [39]. This shows the capacity to offer a global optimal solution for non-linear programming problems [40]. The methods also have the ability to handle non-linear problems reliably and ensure high accuracy [41]. Meanwhile, due to limitations in checking restricted base conditions that require continuous interaction, piecewise linear methods are not practical for use in systems working automatically without interaction with the user or operator.

As technology develops, there is increasing demand for more efficient and automated methods of solving non-linear programming problems, especially in complex online transportation systems such as ride-hailing services. In response to this challenge, a piecewise linear method was modified to serve as an innovation to overcome the limitation identified, which was specifically to solve non-linear programming problems without requiring additional human interaction. The intention was to make the method more efficient and applicable in systems operating automatically. The uniqueness of the modification was in the systematic method developed to model a set of non-linear constraint functions into equivalent linear ones without requiring human intervention. Therefore, the modified piecewise linear method has the potential to improve the efficiency and performance of ride-hailing assignment models as well as other automated systems.

This research has two specific objectives which are stated as follows:

- i. **Developing a Ride-Hailing Assignment Model:**
The aim was to develop a ride-hailing assignment model with simultaneous consideration for three main factors, including service level, driver workload, and fuel consumption. The model optimized ride-hailing assignments by increasing operational efficiency, balancing workload among drivers, and reducing fuel consumption with subsequent effects on emissions. The method was intended to provide a more holistic and sustainable solution for the industry.
- ii. **Modifying Piecewise Linear Method:**
The aim was to modify a piecewise linear method capable of solving non-linear programming problems automatically. The motivation was to overcome the limitations of intensive human interaction required in these methods. The process focused on modeling restricted basis conditions as a function of linear constraints to allow subsequent application of the method in systems operating without human interaction. The

concept is considered important in ride-hailing systems that require fully automated operations to improve overall system efficiency and performance.

To achieve the first objective, the model was developed based on the existing ones [15] and the performance was evaluated against real-world data using numerical simulations with a focus on important indicators such as passenger waiting time, request service level, driver-partner mileage, driver-partner income level, driver-partner idle time, and fuel savings. Moreover, the performance was subsequently compared with existing models [15] to assess the improvements achieved.

The article is organized with a detailed structure to systematically present the piecewise linear method developed to handle the ride-hailing assignment problem (RhAP). Section 1 introduces the background and motivation of this research, Section 2 reviews piecewise linear methods, while Section 3 shows the optimization modeling process by introducing two main models, including the RhAP and the Piecewise Linear Method for Non-linear Ride-hailing Assignment Problem (PL-RhAP) Models. The piecewise linear method was modified in the PL-RhAP model to solve non-linear programming problems automatically without interaction with decision-makers. Moreover, Section 4 presents the numerical experiments, including the case studies used and the results. Section 5 discusses the results in depth to provide a comprehensive analysis of the proposed method, implications, and future research directions. Finally, Section 6 concludes the main results and shows the important contributions and implications.

2. Piecewise Linear Method

In this section, the piecewise linear method was reviewed as described in [38] and later applied to solve non-linear programming problems with the following form:

$$\min f(\mathbf{x}) = \sum_{i=1}^n f_i(x_i), \quad (1)$$

subject to the following:

$$g_j(\mathbf{x}) = \sum_{i=1}^n g_{ij}(x_i) \leq b_j, j = 1, 2, \dots, m, \quad (2)$$

where b_j is a constant while the objective function (1) and constraint function (2) are the sum of single-variable functions.

The piecewise linear method can approximate every continuous non-linear function $f(x)$ of a single variable x . Geometrically, a curve is represented by a series of linked line segments, as seen in Figure 1 where the continuous function $f(x)$ is approximated by four-line segments in the interval x_1 to x_5 . It was also observed that $\tilde{f}(x) = f(x)$ at the endpoints of each line segment. The approximation function $\tilde{f}(x)$ where $x_i \leq x \leq x_{i+1}$, ($i = 1, 2, 3, 4$) is written as follows:

$$\tilde{f}(x) = f(x_i) + \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i} (x - x_i). \quad (3)$$

To represent $f(x)$ on an interval, x_1 to x_5 , it is important to state that the fraction $\frac{x-x_i}{x_{i+1}-x_i}$ in Equation (3) is a number between 0 and 1 for each value of x between x_i and x_{i+1} . Suppose the fraction $\frac{x-x_i}{x_{i+1}-x_i}$ is represented as λ , Equation (3) can be rewritten as follows:

$$\tilde{f}(x) = f(x_i) + \lambda[f(x_{i+1}) - f(x_i)] = \lambda f(x_{i+1}) + (1 - \lambda)f(x_i), \quad (4)$$

where $x_i \leq x \leq x_{i+1}$.

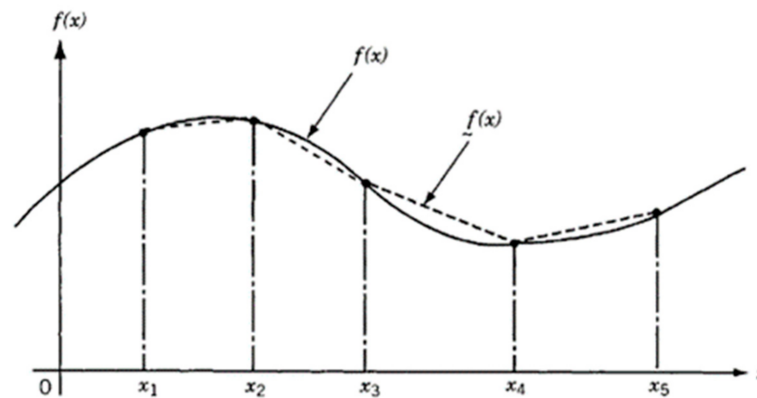


Figure 1. Piecewise linear approximation of a non-linear function [38].

Let $\lambda_i = 1 - \lambda$ and $\lambda_{i+1} = \lambda$, obtaining the following

$$\tilde{f}(x) = \lambda_i f(x_i) + \lambda_{i+1} f(x_{i+1}) \quad (5)$$

where $x_i \leq x \leq x_{i+1}$; $\lambda_i + \lambda_{i+1} = 1$; $\lambda_i, \lambda_{i+1} \geq 0$. Since $\lambda_{i+1} = \lambda = \frac{x-x_i}{x_{i+1}-x_i}$, therefore,

$$x = x_i + \lambda(x_{i+1} - x_i) = (1 - \lambda)x_i + \lambda x_{i+1} = \lambda_i x_i + \lambda_{i+1} x_{i+1}, \quad (6)$$

This shows that x in the interval x_i to x_{i+1} can be written as follows:

$$x = \lambda_i x_i + \lambda_{i+1} x_{i+1}, \quad (7)$$

where $\lambda_i + \lambda_{i+1} = 1$; $\lambda_i \geq 0, \lambda_{i+1} \geq 0$. Instead of expressing each interval by Equation (7) separately, x can be generally expressed as follows:

$$x = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \lambda_4 x_4, \quad (8)$$

where $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$; $x_1 \leq x \leq x_5$; $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \geq 0$.

The validity of Equation (8) in representing any x in the interval x_1 to x_5 depends on the condition that no more than two λ (adjacent) are allowed to be greater than zero. Moreover, these nonzero λ are always associated with adjacent points. This shows that the general form of Equation (8) for r intervals can be written as follows:

$$x = \sum_{j=1}^r \lambda_j x_j, \quad (9)$$

where $\sum_{j=1}^r \lambda_j = 1$; $\lambda_j \geq 0$; $j = 1, 2, \dots, r$.

Equation (9) can be applied to formulate non-linear programming problems in separable form. It is assumed that the lower and upper bounds of each variable x_i are known as 0 and u_i , $i = 1, 2, \dots, n$. The next step was to assume that each variable x_i was divided into p_i intervals. If x_{ik} is the value of x_i at the k th point of the subsection, then

$$0 = x_{i0} < x_{i1} < x_{i2} < \dots < x_{ik} < \dots < x_{ip_i} = u_i. \quad (10)$$

The next was to approximate each objective function $f_i(x_i)$ using the piecewise linear function $\tilde{f}_i(x_i)$ and each j th constraint function, $g_{jl}(x_l)$ by $\tilde{g}_{jl}(x_l)$.

In general, the equation can be written as follows:

$$x_i = \sum_{k=0}^{p_i} \lambda_{ik} x_{ik}, i = 1, 2, \dots, n, \quad (11)$$

$$\sum_{k=0}^{p_i} \lambda_{ik} = 1, \quad (12)$$

$$\lambda_{ik} \geq 0, i = 1, 2, \dots, n, k = 0, 1, 2, \dots, p_i, \quad (13)$$

$$f_i(x_i) = \sum_{k=0}^{p_i} f_i(x_{ik}) \lambda_{ik} = \sum_{k=0}^{p_i} f_{ik} \lambda_{ik}, i = 1, 2, \dots, n, \quad (14)$$

$$g_{jl}(x_l) = \sum_{k=0}^{p_l} g_{jl}(x_{lk}) \lambda_{lk} = \sum_{k=0}^{p_l} g_{jlk} \lambda_{lk}, l = 1, 2, \dots, n, \quad (15)$$

where

$$f_{ik} = f_i(x_{ik}), \quad (16)$$

$$g_{jlk} = g_{jl}(x_{lk}). \quad (17)$$

Piecewise linear optimization problem of Equations (1) and (2) can be written as follows:

$$\begin{aligned} \min f &= \sum_{i=1}^n \sum_{k=0}^{p_i} f_{ik} \lambda_{ik}, \\ \text{subject to the following:} \\ \sum_{l=1}^n \sum_{k=0}^{p_l} g_{jlk} \lambda_{lk} &\leq 0, j = 1, 2, \dots, m, \\ \sum_{k=0}^{p_i} \lambda_{ik} &= 1, i = 1, 2, \dots, n, \\ \lambda_{ik} &\geq 0, i = 1, 2, \dots, n, k = 0, 1, \dots, p_i. \end{aligned} \quad (18)$$

The restricted basis condition holds that for each i (i) no more than two λ_{ik} can be positive and (ii) only adjacent λ_{ik} can be positive.

3. Optimization Model

The section describes the optimization model developed for the ride-hailing assignment problem (RhAP) based on the information obtained from the literature review. The problem was presented in a mathematical model through several steps, including the declaration of parameters, variables, constraint functions, and objective functions. A linearized non-linear function was adopted using the piecewise linear method modified to ensure suitability for automated systems by eliminating the need for the decision maker to confirm or check necessary conditions. It was subsequently observed that every non-linear programming problem could be represented as a linear programming problem, facilitating the attainment of the optimal global solution. The optimization model with a piecewise linear ride-hailing assignment problem is henceforth referred to as the PL-RhAP model.

3.1. Ride-Hailing Assignment Problem Model (RhAP)

The section describes the optimization model developed for the ride-hailing assignment problem (RhAP) based on the information obtained from the literature review. The problem was presented in a mathematical model through several steps, including the declaration of parameters, variables, constraint functions, and objective functions. A linearized non-linear function was adopted using the piecewise linear method modified to ensure suitability for automation systems by eliminating the need for the decision maker to confirm or check necessary conditions. It was subsequently observed that every non-linear programming problem could be represented as a linear programming problem, facilitating the attainment of the optimal global solution. The optimization model with a piecewise linear ride-hailing assignment problem is henceforth referred to as the PL-RhAP model.

The ride-hailing system decides which vehicles are to be assigned to pick up passengers with due consideration for the service level, driver workload, and fuel saving. The problems associated with the assignment process of a ride-hailing pick-up system operating in real time can be solved through a mathematical model using decision variables,

constraints, and objective functions. The method aimed to support efficient decision-making and provide the best travel experience for passengers while improving the general operational efficiency.

The proposed model, which actively involves passengers in the transportation process, addresses environmental concerns by optimizing fuel consumption and reducing emissions. By encouraging passengers to walk short distances to meet drivers, the model decreases unnecessary vehicle movements, leading to lower fuel usage and reduced greenhouse gas emissions. This approach not only enhances operational efficiency but also contributes to environmental sustainability. These findings underscore the potential of the model to provide significant environmental benefits.

Several measures were implemented to ensure the model reduces fuel consumption: (1) optimizing driver routes to minimize travel distance, (2) encouraging passengers to walk short distances to meet their assigned drivers, and (3) prioritizing assignments that minimize overall fuel usage. These strategies collectively contribute to a significant reduction in fuel consumption, which demonstrates the model's potential to enhance operational efficiency and support environmental sustainability efforts.

- Sets, Parameters, and Variables

The RhAP problem was solved using some sets including the set of requests (I) and the set of drivers (J). The parameters used were the number of jobs done by the driver j (b_j), average driver jobs ($\bar{b}$), distance traveled by driver j to pick up request i (δ_{ij}), distance traveled by passengers at request i to walk to the location of driver j (d_{ij}), willingness of passenger at request i to walk (r_i), waiting time spent by request i to arrive at the vehicle of driver j waiting to be picked up (τ_{ij}), waiting time spent by request i to arrive at the vehicle of driver j by walking (t_{ij}), and the idle time of driver j (v_j). Moreover, the decision variables were u_j to update the number of jobs obtained by driver j post-assignment, χ_{ij} to decide whether driver j would pick up request i or not (1 or 0), x_{ij} to decide whether or not (1 or 0) driver j would serve request i to reach the location of the driver j by walking, and y_i as whether request i was ignored or not (1 or 0) by all drivers. The decision variables u_j were real numbers ($u_j \in [0, \infty), \forall j \in J$) while χ_{ij} was binary ($\chi_{ij} \in \{0, 1\}, \forall i \in I, \forall j \in J$) and the same went for x_{ij} at ($x_{ij} \in \{0, 1\}, \forall i \in I, \forall j \in J$) as well as y_i at ($y_i \in \{0, 1\}, \forall i \in I$).

- Objective Functions

The model was designed based on six objectives, including waiting times, abandoned requests, drivers' pick-up distance, drivers' workload, drivers' idle time, and fuel savings. All passengers were assumed to have used waiting time on a request from the time an order was placed to the moment of entering the vehicle. The waiting time included the pick-up travel time and the time used in the queue.

Service levels could be improved by creating a ride-hailing matching system to minimize overall waiting time. This objective was abstracted into mathematical notation, in this case optimization objective function that minimized overall waiting time as follows:

$$\min \sum_{i \in I} \sum_{j \in J} (t_{ij} x_{ij} + \tau_{ij} \chi_{ij}). \quad (19)$$

The improvement could also be achieved by minimizing the number of abandoned requests. The function used in representing this objective was expressed by the following:

$$\min \sum_{i \in I} y_i. \quad (20)$$

Driver convenience could be improved by designing a ride-hailing matching system to minimize the overall travel distance of the drivers. This objective was abstracted into the following function form:

$$\min \sum_{i \in I} \sum_{j \in J} \delta_{ij} \chi_{ij}. \quad (21)$$

The workload obtained by each driver directly influences their income level. This showed the possibility of balancing workload distribution by developing a ride-hailing matching system that aimed to minimize the variance in the workload of each driver. The objective was abstracted into the following function:

$$\min \frac{1}{|J|} \sum_{j \in J} (u_j - \bar{b})^2. \quad (22)$$

In the real world, drivers often have relatively high idle time compared to others, leading to inefficient use of resources. Therefore, drivers with more idle time need to be prioritized to ensure everyone has the same opportunity. This objective was abstracted into the following function:

$$\max \sum_{i \in I} \sum_{j \in J} v_j (x_{ij} + \chi_{ij}).$$

The relationship was reproduced in the following minimization form:

$$\min \left(- \sum_{i \in I} \sum_{j \in J} v_j (x_{ij} + \chi_{ij}) \right). \quad (23)$$

Fuel savings generally need to be maximized to directly reduce emissions from the vehicle. This could be achieved by allowing passengers to choose between going to the location of the vehicle by walking or waiting to be picked up. The decision to walk to the location of the driver would reduce fuel usage to zero. Therefore, to achieve savings in fuel use and minimize the distance traveled, as stated in (21), the number of passengers that choose to walk needed to be maximized. This objective was abstracted into the following function:

$$\max \sum_{i \in I} \sum_{j \in J} x_{ij}.$$

The relationship was reproduced in the following minimization form:

$$\min - \sum_{i \in I} \sum_{j \in J} x_{ij}. \quad (24)$$

- Constraint Functions

Rules related to the matching system were required to be established to ensure decisions did not exceed the threshold and complied with existing regulations. These rules were abstracted into mathematical notation as constraint functions in the optimization model. Each driver was assigned to pick up passengers according to the vehicle capacity and the limit of passengers that could be served at one time based on the matching system. In this case, each driver was allowed to only serve one request at a time, and this was presented in the following constraint function:

$$\sum_{i \in I} x_{ij} + \sum_{i \in I} \chi_{ij} \leq 1, \forall j \in J, \quad (25)$$

where $\sum_{i \in I} \chi_{ij}$ represents the number of requests served by driver j in which all passengers are picked up by the driver at the order location. $\sum_{i \in I} x_{ij}$ is the number of requests served by driver j in which all passengers in the request come to the location of the driver by walking.

Each incoming request was only allowed to be served or ignored by just one unemployed driver. The constraint function used to present this rule is presented as follows:

$$\sum_{j \in J} x_{ij} + \sum_{j \in J} \chi_{ij} + y_i = 1, \forall i \in I, \quad (26)$$

where a request $i \in I$ is served when $\sum_{j \in J} x_{ij} + \sum_{j \in J} \chi_{ij}$ is 1 and 0 otherwise, while the request is ignored when y_i is 1 and 0 otherwise. The rule was used to determine whether the request was served or ignored.

Travel delays were required to be at the most limit and were considered to be the same as waiting time in this case. Therefore, the constraint that ensures travel delays do not exceed limits is presented as follows:

$$\sum_{j \in J} (t_{ij}x_{ij} + \tau_{ij}\chi_{ij}) \leq t_{max}, \forall i \in I. \quad (27)$$

Passengers need to confirm their willingness to walk before a driver is assigned. Therefore, it was necessary to determine the willingness of passengers because only those willing to walk could be assigned by a matching system to walk to the location of the driver. Therefore, the constraint function developed to represent the rule is presented as follows:

$$\sum_{j \in J} x_{ij} \leq r_i, \forall i \in I. \quad (28)$$

The constraint to ensure any passenger walking to the location of the driver would not exceed the specified distance limit was provided as follows:

$$\sum_{j \in J} d_{ij}x_{ij} \leq d_{max}, \forall i \in I. \quad (29)$$

Driver workload constraint was used to update the current performance or workload of each based on the demands placed on their respective vehicle. This was achieved by considering the initial workload and the addition of assigned requests to reflect the cumulative workload to be handled. Therefore, the constraint function developed to represent the rule is presented as follows:

$$u_j = b_j + \sum_{i \in I} x_{ij} + \sum_{i \in I} \chi_{ij}, \forall j \in J. \quad (30)$$

The RhAP model was later presented as follows:

$$\begin{aligned} & \min \sum_{i \in I} \sum_{j \in J} (t_{ij}x_{ij} + \tau_{ij}\chi_{ij}), \\ & \min \sum_{i \in I} y_i, \\ & \min \sum_{i \in I} \sum_{j \in J} \delta_{ij}\chi_{ij}, \\ & \min \frac{1}{|J|} \sum_{j \in J} (u_j - \bar{b})^2, \\ & \min \left(- \sum_{i \in I} \sum_{j \in J} v_j (x_{ij} + \chi_{ij}) \right), \\ & \min - \sum_{i \in I} \sum_{j \in J} x_{ij}, \\ & \text{subject to:} \\ & \sum_{i \in I} x_{ij} + \sum_{i \in I} \chi_{ij} \leq 1, \forall j \in J, \\ & \sum_{j \in J} x_{ij} + \sum_{j \in J} \chi_{ij} + y_i = 1, \forall i \in I, \\ & \sum_{j \in J} (t_{ij}x_{ij} + \tau_{ij}\chi_{ij}) \leq t_{max}, \forall i \in I \\ & \sum_{j \in J} x_{ij} \leq r_i, \forall i \in I, \\ & \sum_{j \in J} d_{ij}x_{ij} \leq d_{max}, \forall i \in I, \\ & u_j = b_j + \sum_{i \in I} x_{ij} + \sum_{i \in I} \chi_{ij}, \forall j \in J, \\ & u_j \in [0, \infty), \chi_{ij} \in \{0, 1\}, x_{ij} \in \{0, 1\}, y_i \in \{0, 1\}, \forall i \in I, \forall j \in J. \end{aligned} \quad (31)$$

3.2. Modified Piecewise Linear Method for Non-Linear Ride-Hailing Assignment Problem Model (PL-RhAP)

Problem (31) requires non-linear programming, specifically quadratic with binary variables. Therefore, the piecewise linear method was applied to solve the non-linear problems due to the suitability of issues related to binary variables as well as other advantages in terms of computational efficiency, modeling flexibility, readability, and interpretability.

The optimization model in (31) was observed to explicitly have a quadratic function in the objective function (22). As discussed in Section 2, the piecewise linear method could only be used for continuous non-linear functions. Moreover, as presented in (1), the objective function needed to be written based on the sum of the functions of a single variable.

Let

$$f_j(u_j) = \frac{1}{|J|} (u_j^2 - 2\bar{b}u_j + \bar{b}^2), \forall j \in J. \quad (32)$$

Note that

$$\min \frac{1}{|J|} \sum_{j \in J} (u_j - \bar{b})^2 = \min \sum_{j \in J} f_j(u_j),$$

The trend shows that the objective function in (22) can be written as the sum of $|J|$ single-variable function $\sum_{j \in J} f_j(u_j)$.

The affine function from Equation (32) was obtained through linearization using the piecewise linear method. The approximation function $\tilde{f}_j(u_j)$ from the equation, where $\square_{jk} \leq u_j \leq \square_{jk+1}$, ($k = 0, 1, 2, \dots, p_j$) and p_j are the numbers of interval partitions, can be written as follows:

$$\tilde{f}_j(u_j) = \sum_{k=0}^{p_j} \tilde{f}_j(\square_{jk}) \lambda_{jk} = \sum_{k=0}^{p_j} \frac{1}{|J|} (\square_{jk}^2 - 2\bar{b}\square_{jk} + \bar{b}^2) \lambda_{jk}, \forall j \in J, \quad (33)$$

with the following

$$u_j = \sum_{k=0}^{p_j} \square_{jk} \lambda_{jk}, \forall j \in J; \quad (34)$$

$$\sum_{k=0}^{p_j} \lambda_{jk} = 1; \forall j \in J; \quad (35)$$

$$\lambda_{jk} \geq 0; \forall j \in J; k = 0, 1, \dots, p_j; \quad (36)$$

where $\square_{jk}$ is a parameter corresponding to the interval boundary after partitioning and λ_{jk} is a variable. Moreover, there is a condition that each j is not allowed to have more than two λ_{jk} positive, and only two adjacent λ_{jk} can have a positive value.

The user needs to check the restricted basis conditions, which are that no more than two λ_{jk} and only adjacent λ_{jk} can be positive for each j . However, the method cannot be used for problem-solving using an automatic system that works continuously without interacting with decision makers. This led to the modification of the piecewise linear method to achieve automatic operation for subsequent application of the model to ride-hailing systems.

The modification was achieved by modeling the conditions required to be met, concerning checking interactively, into a representative constraint function. The restricted basis condition was related to the value of λ_{jk} , showing the need to model the condition with a focus on the positivity of λ_{jk} . It is also important to remember that λ_{jk} has the domain $[0, 1]$ which showed the need to determine whether the ceiling denoted by $\lceil \lambda_{jk} \rceil$ is 0 or 1. If

$\lceil \lambda_{jk} \rceil$ is 1, then λ_{jk} is positive (i.e., $\lambda_{jk} > 0$), but if $\lceil \lambda_{jk} \rceil$ is 0, then λ_{jk} is 0. The constraint function used to determine whether λ_{jk} is positive or not is presented as follows:

$$\Lambda_{jk} = \lceil \lambda_{jk} \rceil, \Lambda_{jk} \in \{0, 1\}; \forall j \in J; k = 0, 1, \dots, p_j. \quad (37)$$

The relationship shows that the constraints are not linear, leading to the need to determine an equivalent linear constraint function. This led to the reformulation of Equation (37) as follows:

$$\begin{aligned} \Lambda_{jk} &\geq \lambda_{jk}; \forall j \in J; k = 0, 1, \dots, p_j; \\ \Lambda_{jk} &\leq \lambda_{jk} + 0.999; \forall j \in J; k = 0, 1, \dots, p_j; \\ \Lambda_{jk} &\in \{0, 1\}; \forall j \in J; k = 0, 1, \dots, p_j. \end{aligned} \quad (38)$$

The next step was to ensure the satisfaction of restricted basic conditions developed based on two criteria, including (i) do not allow more than two positive λ_{jk} for each j and (ii) two λ_{jk} can be positive if they are adjacent. The constraint that ensures the restricted basis condition (i) is satisfied is stated as follows:

$$\sum_{k=0}^{p_j} \Lambda_{jk} \leq 2; \forall j \in J. \quad (39)$$

where Λ_{jk} represents the positivity of λ_{jk} .

The condition for each j is also to ensure that non-adjacent λ_{jk} cannot be positive together in order to satisfy the restricted basis condition (ii). In this case, the product of two non-adjacent Λ_{jk} is required to be 0, considering the fact that Λ_{jk} represents the positivity of λ_{jk} . Therefore, the constraint that ensures the restricted basis condition (ii) is satisfied is presented as follows:

$$\sum_{q=k+2}^{p_j} \Lambda_{jk} \Lambda_{jq} = 0; \forall j \in J; k = 0, 1, \dots, p_j - 2. \quad (40)$$

The constraint is not linear and this shows the need to produce an equivalent linear constraint function. Therefore, if $z = xy$ and $x, y, z \in \{0, 1\}$, the equivalent form of the problem can be expressed as follows:

$$z \leq x, z \leq y, z \geq x + y - 1; x, y, z \in \{0, 1\} \quad (41)$$

The trend shows that the application of properties on (41) and the introduction of new variables Ω_{jkq} to determine whether Λ_{jk} and Λ_{jq} are positive together ($\Omega_{jkq} = 1$) or not ($\Omega_{jkq} = 0$) can be used to produce the equivalent linear constraint of (40) as follows:

$$\begin{aligned} \sum_{q=k+2}^{p_j} \Omega_{jkq} &= 0; \forall j \in J; k = 0, 1, \dots, p_j - 2. \\ \Omega_{jkq} &\leq \Lambda_{jk}, \forall j \in J; k = 0, 1, \dots, p_j - 2; q = k + 2, k + 3, k + 4, \dots, p_j, \\ \Omega_{jkq} &\leq \Lambda_{jq}, \forall j \in J; k = 0, 1, \dots, p_j - 2; q = k + 2, k + 3, k + 4, \dots, p_j, \\ \Omega_{jkq} &\geq \Lambda_{jk} + \Lambda_{jq} - 1; \forall j \in J; k = 0, 1, \dots, p_j - 2; q = k + 2, k + 3, k + 4, \dots, p_j, \\ \Omega_{jkq} &\in \{0, 1\}; \forall j \in J; k = 0, 1, \dots, p_j - 2; q = k + 2, k + 3, k + 4, \dots, p_j. \end{aligned} \quad (42)$$

When new constraints (38), (39) and (42), representing the restricted basis condition, are added to Equations (33)–(36), the linearization of Equation (32) can be formulated as follows:

$$f_j(u_j)(u_j) = \sum_{k=0}^{p_j} f_j(\lceil \lambda_{jk} \rceil) \lambda_{jk} = \sum_{k=0}^{p_j} \frac{1}{|J|} (\lceil \lambda_{jk} \rceil^2 - 2\bar{b}\lceil \lambda_{jk} \rceil + \bar{b}^2) \lambda_{jk}, \forall j \in J, \quad (43)$$

with

$$u_j = \sum_{k=0}^{p_j} \square_{jk} \lambda_{jk}; \forall j \in J; \quad (44)$$

$$\sum_{k=0}^{p_j} \lambda_{jk} = 1; \forall j \in J; \quad (45)$$

$$\sum_{k=0}^{p_j} \Lambda_{jk} \leq 2; \forall j \in J; \quad (46)$$

$$\Lambda_{jk} \geq \lambda_{jk}; \forall j \in J; k = 0, 1, \dots, p_j; \quad (47)$$

$$\Lambda_{jk} \leq \lambda_{jk} + 0.999; \forall j \in J; k = 0, 1, \dots, p_j; \quad (48)$$

$$\sum_{q=k+2}^{p_j} \Omega_{jkq} = 0; \forall j \in J; k = 0, 1, \dots, p_j - 2; \quad (49)$$

$$\Omega_{jkq} \leq \Lambda_{jk}; \forall j \in J; k = 0, 1, \dots, p_j - 2; q = k + 2, k + 3, \dots, p_j; \quad (50)$$

$$\Omega_{jkq} \leq \Lambda_{jq}; \forall j \in J; k = 0, 1, \dots, p_j - 2; q = k + 2, k + 3, \dots, p_j; \quad (51)$$

$$\Omega_{jkq} \geq \Lambda_{jk} + \Lambda_{jq} - 1; \forall j \in J; k = 0, 1, \dots, p_j - 2; q = k + 2, k + 3, \dots, p_j; \quad (52)$$

$$\lambda_{jk} \geq 0, \Lambda_{jk} \in \{0, 1\}; \forall j \in J; k = 0, 1, \dots, p_j; \quad (53)$$

$$\Omega_{jkq} \in \{0, 1\}; \forall j \in J; k = 0, 1, \dots, p_j - 2; q = k + 2, k + 3, \dots, p_j; \quad (54)$$

where $\square_{jk}$ is a parameter corresponding to the interval boundary after partitioning and $\lambda_{jk}, \Lambda_{jk}, \Omega_{jkq}$ are auxiliary variables. Equation (22) was replaced with the sum of the single-variable function (43) for $j \in J$ to maintain a minimization form. This was followed by the addition of (44)–(54) to produce the following PL-RhAP model:

$$\begin{aligned} & \min \sum_{i \in I} \sum_{j \in J} (t_{ij} x_{ij} + \tau_{ij} \chi_{ij}); \\ & \min \sum_{i \in I} y_i; \\ & \min \sum_{i \in I} \sum_{j \in J} \delta_{ij} \chi_{ij}; \\ & \min \sum_{k=0}^{p_j} \sum_{j \in J} \frac{1}{|J|} \left(\square_{jk}^2 - 2\bar{b} \square_{jk} + \bar{b}^2 \right) \lambda_{jk}; \\ & \min \left(- \sum_{i \in I} \sum_{j \in J} v_j (x_{ij} + \chi_{ij}) \right); \\ & \min - \sum_{i \in I} \sum_{j \in J} x_{ij}; \\ & \text{subject to:} \\ & \sum_{i \in I} x_{ij} + \sum_{i \in I} \chi_{ij} \leq 1; \forall j \in J; \\ & \sum_{j \in J} x_{ij} + \sum_{j \in J} \chi_{ij} + y_i = 1; \forall i \in I; \\ & \sum_{j \in J} (t_{ij} x_{ij} + \tau_{ij} \chi_{ij}) \leq t_{max}; \forall i \in I; \\ & \sum_{j \in J} x_{ij} \leq r_i; \forall i \in I; \\ & \sum_{j \in J} d_{ij} x_{ij} \leq d_{max}; \forall i \in I; \\ & u_j = b_j + \sum_{i \in I} x_{ij} + \sum_{i \in I} \chi_{ij}; \forall j \in J; \\ & u_j = \sum_{k=0}^{p_j} \square_{jk} \lambda_{jk}; \forall j \in J; \\ & \sum_{k=0}^{p_j} \lambda_{jk} = 1; \forall j \in J; \\ & \sum_{k=0}^{p_j} \Lambda_{jk} \leq 2; \forall j \in J; \\ & \Lambda_{jk} \geq \lambda_{jk}; \forall j \in J; k = 0, 1, \dots, p_j; \\ & \Lambda_{jk} \leq \lambda_{jk} + 0.999; \forall j \in J; k = 0, 1, \dots, p_j; \\ & \sum_{q=k+2}^{p_j} \Omega_{jkq} = 0; \forall j \in J; k = 0, 1, \dots, p_j - 2; \\ & \Omega_{jkq} \leq \Lambda_{jk}; \forall j \in J; k = 0, 1, \dots, p_j - 2; q = k + 2, k + 3, \dots, p_j; \\ & \Omega_{jkq} \leq \Lambda_{jq}; \forall j \in J; k = 0, 1, \dots, p_j - 2; q = k + 2, k + 3, \dots, p_j; \\ & \Omega_{jkq} \geq \Lambda_{jk} + \Lambda_{jq} - 1; \forall j \in J; k = 0, 1, \dots, p_j - 2; q = k + 2, k + 3, \dots, p_j; \\ & \lambda_{jk} \geq 0, \Lambda_{jk} \in \{0, 1\}; \forall j \in J; k = 0, 1, \dots, p_j; \\ & \Omega_{jkq} \in \{0, 1\}; \forall j \in J; k = 0, 1, \dots, p_j - 2; q = k + 2, k + 3, \dots, p_j; \\ & u_j \in [0, \infty), \chi_{ij} \in \{0, 1\}, x_{ij} \in \{0, 1\}, y_i \in \{0, 1\}, \forall i \in I, \forall j \in J. \end{aligned} \quad (55)$$

4. Numerical Experiments

4.1. Case Study

The reliability of the PL-RhAP model in solving real-world problems was tested using a publicly accessible dataset of taxi trips in Manhattan (<https://www.nyc.gov/site/tlc/>

[about/tlc-trip-record-data.page](#), accessed on 15 April 2024) containing the pickup locations, drop-offs, request times, and other information. The reliability of the PL-RhAP model was ensured through extensive validation using the Manhattan taxi trip dataset. Statistical analyses, including cross-validation and performance metrics such as mean waiting time and service levels, were conducted to confirm the model's robustness and accuracy.

The data required for the numerical experiments were selected, starting at 12:00 p.m. on 15 January 2024, with a focus on the vehicles with pickup and drop-off positions in Manhattan. Moreover, the travel time and distance from the two locations were calculated based on the longitude and latitude using OSMnx developed by [42]. OSMnx was used for calculating travel time and distance due to its high accuracy and computational efficiency, as well as being open source. It leverages OpenStreetMap data to generate precise travel metrics, ensuring that our model's inputs are based on realistic and up-to-date information. The number of requests for each time is presented in the following Figure 2.

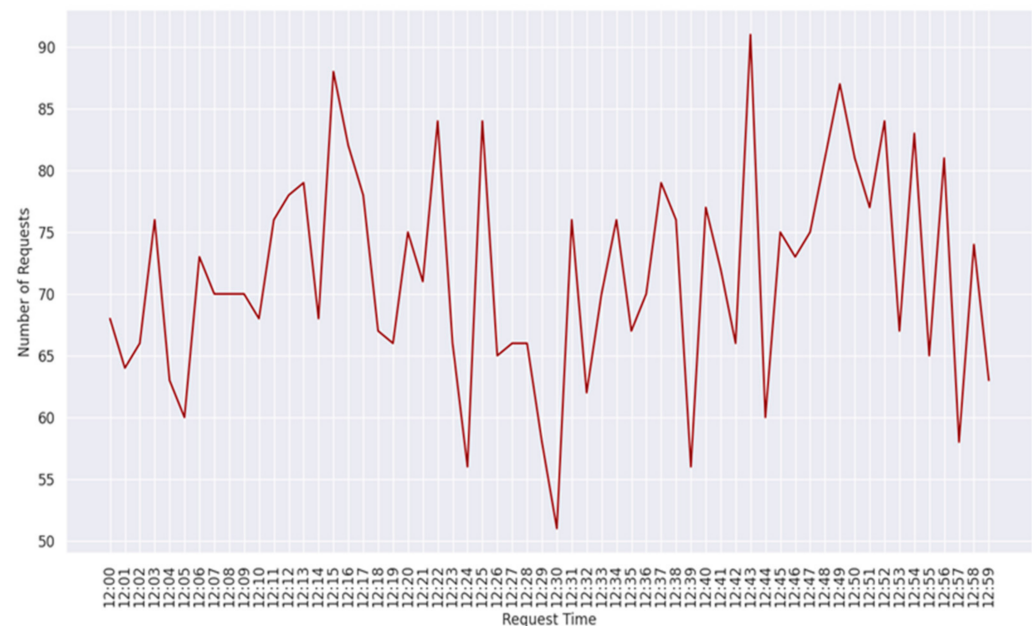


Figure 2. Number of requests.

During the simulation, the vehicle was designed to continuously pick up and drop off passengers based on the decisions generated by applying the model. Passengers were free to select the willingness to walk option and this undoubtedly affected the discount on service prices, but the pricing was not discussed. Furthermore, requests and available drivers were collected during a specific time window to be subsequently assigned to passengers according to request. The model implemented two types of assignments, including (i) passengers walking to the location of the driver and (ii) drivers assigned to pick up passengers at the location where the vehicle was ordered. Moreover, the assignments were conducted without exceeding the tolerated waiting time. All passengers could only walk as far as possible within the specified limit. It was also stated that the requests not serviced within the waiting time limit were removed from the queue. The requests not accepted by available drivers were designed to be reassigned and reoptimized in the next group of time windows. The parameter settings used for the numerical experiments are presented in the following Table 1.

The passengers were provided with two options on the smartphone when booking the ride, including “walk to the location of the vehicle” or “wait for the vehicle to arrive”. The choice of walking to the vehicle would reduce the required fuel consumption. Therefore, the numerical simulations were conducted based on these two scenarios, first, where everyone was willing to walk to the location of the vehicle ($r_i = 1, \forall i \in I$) and second, where everyone was not willing to walk to the location of the vehicle ($r_i = 0, \forall i \in I$). This

multi-objective optimization model was completed by assigning a weight to each function with objective functions 1, 3, and 5, observed to have a weight of 1, while 2, 4, and 6 had a large value which was equal to M and set at 99,999.

Table 1. Parameter settings for numerical experiments.

Parameters	Values
Number of passengers in a request	1 request (a group of passengers in the same order)
Number of deployed vehicles	2000 vehicles
Tolerable waiting times	5 min
Tolerable walk distances	300 m
Number of batches	60 batch (that is 60 min simulation)
Time window	1 min
Partitions	1

The weighted objective functions are then summed to obtain a single objective function. Since all the objective functions in this context are in the form of minimization, the summation of the weighted objective functions also results in a minimization function. Consequently, a single-objective linear programming model is formulated, which is subsequently solved using standard linear programming methods.

Weights were assigned to each objective function in the multi-objective optimization model based on their relative importance. This adaptable approach allows us to prioritize different objectives as needed. For instance, service level and fuel consumption were prioritized by assigning higher weights, while secondary objectives like waiting time variance received lower weights. This balanced approach ensures that the model aligns with our overall operational goals, giving us the power to control and adapt as necessary.

4.2. Numerical Results

The section presents and compares the results of numerical experiments with the RhAP model proposed by [15] and referred to as the Megantara-RhAP. The waiting time for each scenario was based on the simulation results as shown in Figure 3. In the first scenario, the PL-RhAP model had an average waiting time estimated at 2.00 min while the Megantara-RhAP was 1.96 min. Moreover, the variance of the waiting time was approximately 9.99 and 9.992, respectively, indicating that the PL-RhAP was slightly higher.

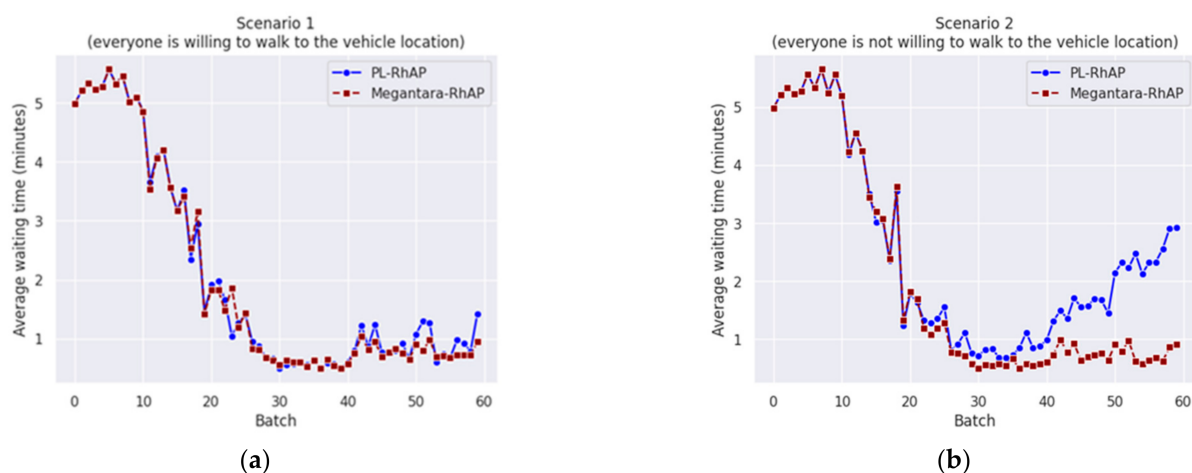


Figure 3. Average waiting time: (a) Scenario 1, everyone is willing to walk to the vehicle location; (b) Scenario 2, everyone is not willing to walk to the vehicle location.

In the second scenario, the PL-RhAP model had an average waiting time, estimated at 2.44 min which was higher than the 1.97 min reported for the Megantara-RhAP. Moreover, the waiting time variance was approximated to be 5.57 and 5.44, respectively. The trend

from the analysis showed that the PL-RhAP model had a higher mean for the waiting times with higher variance. The higher mean waiting times and variances in the PL-RhAP model compared to the Megantara-RhAP model are primarily due to the additional constraints on driver workload and fuel consumption. These constraints, while improving overall operational efficiency and sustainability, may lead to longer waiting times in certain scenarios.

The results of both scenarios presented in Figure 4 were used to compare the service requests in the PL-RhAP and Megantara-RhAP models. In the first scenario, the PL-RhAP model served approximately 4254 requests while the Megantara-RhAP had 4251. The average number of requests served per batch was estimated at 70.9 and 70.85, respectively, showing that the PL-RhAP was slightly higher. Moreover, the variance of the number of requests served per batch was approximated at 79 and 78, respectively. This showed that the number of requests served in the PL-RhAP model for the first scenario was more significant.

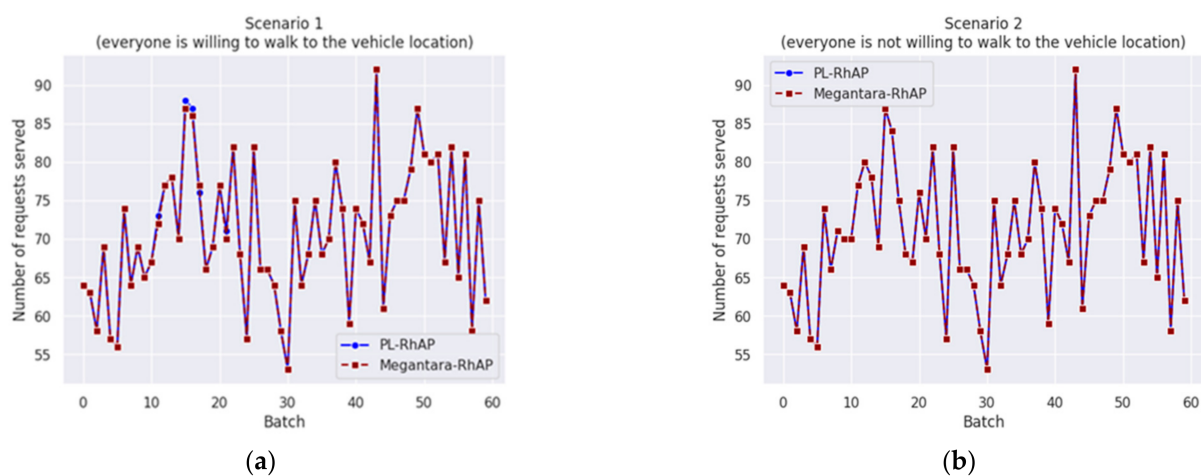


Figure 4. Service levels: (a) Scenario 1, everyone is willing to walk to the vehicle location; (b) Scenario 2, everyone is not willing to walk to the vehicle location.

In the second scenario, both models served 4265 requests and the average per batch as well as the variance were also the same. The PL-RhAP model performed slightly better in terms of the number of service requests compared to the Megantara-RhAP model in the first scenario and performed equally in the second scenario. The analysis generally showed that the PL-RhAP model was more reliable in increasing service levels.

The travel distance between the two models was observed to be significantly different, as presented in Figure 5. In the first scenario, the PL-RhAP model showed an average travel distance of 0.96 km while the Megantara-RhAP had 0.94 km. The longest travel distance was observed to be the same for both models at 6437 km. Meanwhile, the variance of travel distances was found to be 2.33 and 2.22, respectively, showing that the PL-RhAP was slightly higher.

In the second scenario, the average travel distance by the PL-RhAP model was 1.25 km, which was slightly lower than 0.95 km in the Megantara-RhAP. The longest travel distance for both models was the same and recorded to be 6437 km. Meanwhile, the variability of travel distances was estimated at 2.22 and 2.24, respectively. The analysis showed that the Megantara-RhAP tended to have lower average travel distances with more stable variability while the PL-RhAP had low means and variances. In this case, the results showed the potential for consistent simulation results in estimating travel distances in the PL-RhAP model but it was not better than the Megantara-RhAP as observed from the minimal difference.

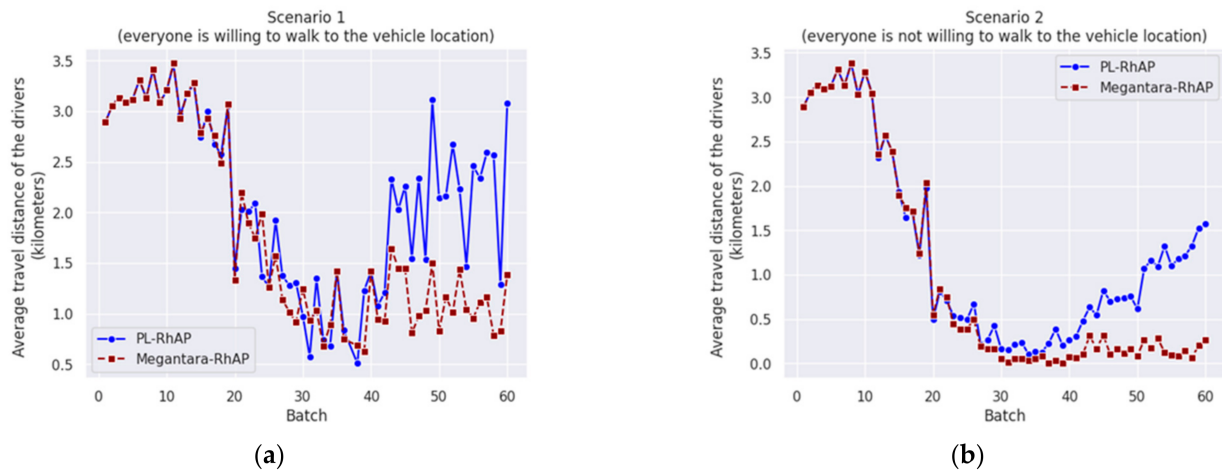


Figure 5. Average distance traveled by drivers: (a) Scenario 1, everyone is willing to walk to the vehicle location; (b) Scenario 2, everyone is not willing to walk to the vehicle location.

The results for the two scenarios developed for the number of jobs obtained by drivers using the PL-RhAP and Megantara-RhAP models showed significant differences, as presented in Figure 6. This was observed from the 4254 and 4251 jobs, respectively estimated in the first scenario, with a variance of 4152 and 4900.

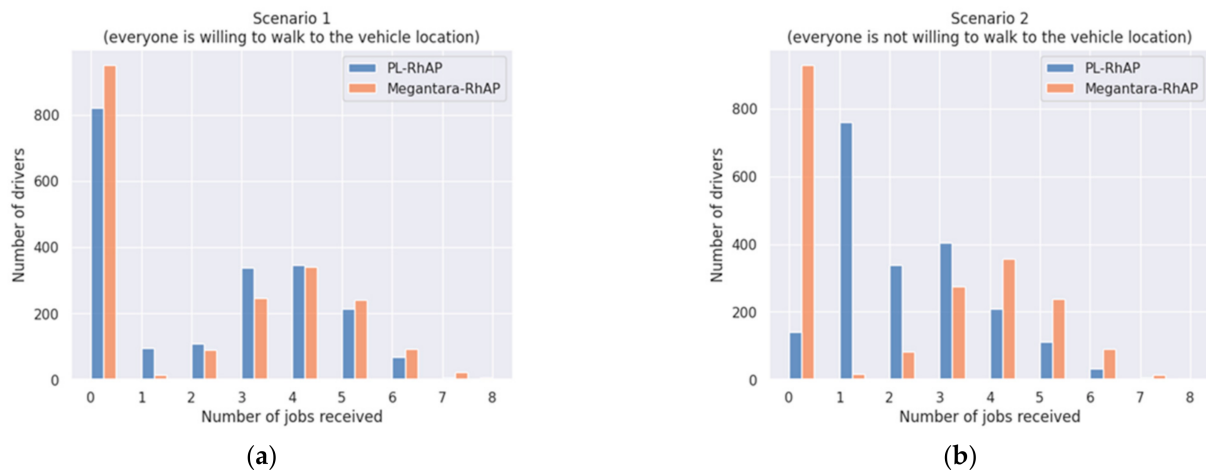


Figure 6. Distribution of the number of jobs received by drivers: (a) Scenario 1, everyone is willing to walk to the vehicle location; (b) Scenario 2, everyone is not willing to walk to the vehicle location.

In the second scenario, the number of jobs for both models was found to be the same at 4265. Meanwhile, the variance in the number of jobs per batch for the Megantara-RhAP was estimated at 4684, which was higher than the 2077 recorded for the PL-RhAP model. The analysis showed that the PL-RhAP model tended to have lower variance, showing the potential for more stable simulation results in distributing work to drivers. The PL-RhAP model achieved more stable variability in distributing work to drivers by incorporating constraints that balance workload. This includes minimizing the variance in the number of jobs assigned to each driver and ensuring that idle times are evenly distributed, leading to a more equitable workload distribution across all drivers. Moreover, Figure 6 shows that the number of jobs obtained by each PL-RhAP driver in each scenario tends to be evenly distributed while more Megantara-RhAP drivers do not have jobs.

The results for the two scenarios related to the total idle time for each driver using the PL-RhAP and Megantara-RhAP models showed significant differences as presented in Figure 7. In the first scenario, the total idle time was estimated at 77,414 and 77,476 min, respectively. Moreover, the total idle time per driver was approximated at 38.707 min and

38.738 min, indicating that the PL-RhAP was slightly lower. The total idle time variance per driver was also 383.784 and 438.692, respectively, showing that the PL-RhAP was significantly lower.

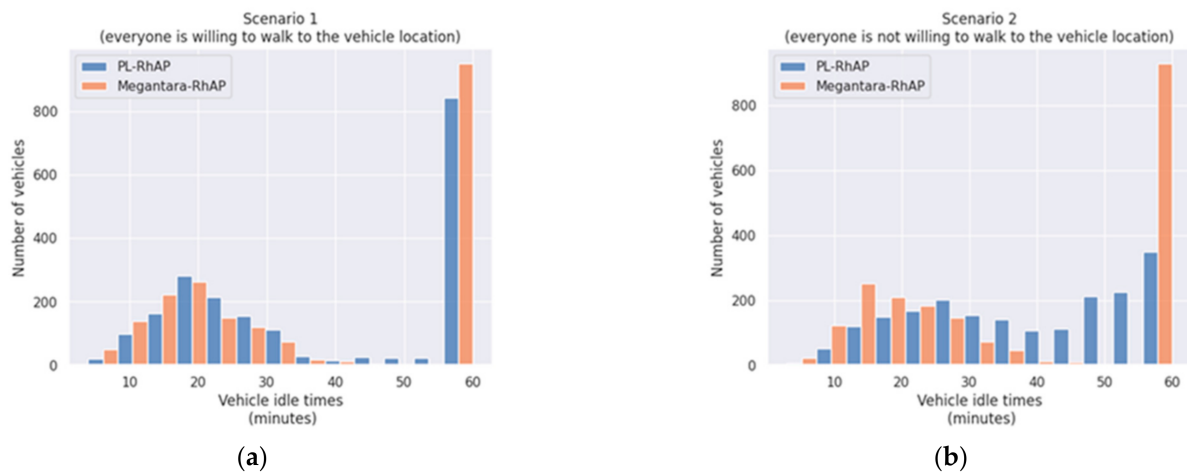


Figure 7. Driver idle time: (a) Scenario 1, everyone is willing to walk to the vehicle location; (b) Scenario 2, everyone is not willing to walk to the vehicle location.

In the second scenario, the total idle time for the PL-RhAP model was 76,474 min while the Megantara-RhAP had 77,220 min. The total idle time per driver was recorded to be 38.237 and 38.61 min, respectively. Meanwhile, the total idle time variance per driver was 242.186 and 429.693, showing that the PL-RhAP model was significantly lower. The analysis generally showed that the PL-RhAP model tended to have lower total idle time per driver with more stable variability and this was a sign of the potential for more efficient simulation results in minimizing the idle time for drivers.

The fuel savings analysis only focused on the first scenario and the results presented in Figure 8 showed that the PL-RhAP model saved fuel by covering 24.802 km compared to the 16.109 km reported for the Megantara-RhAP. The trend from the results showed that the PL-RhAP model had better performance in saving fuel.

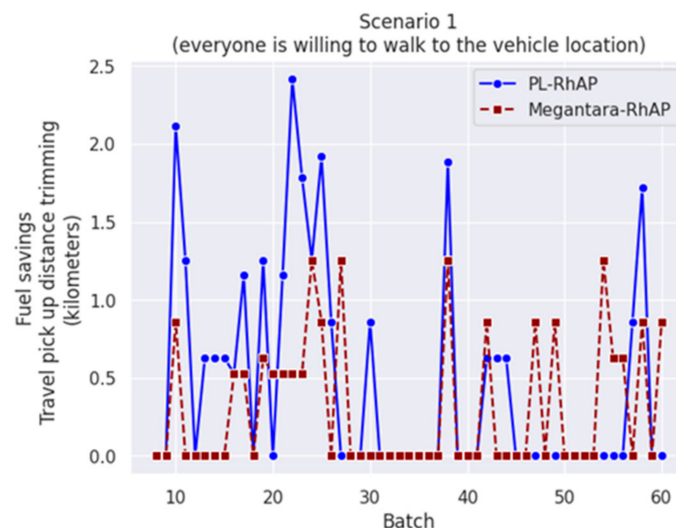


Figure 8. Fuel savings.

5. Discussion

The section was used to discuss the results based on the ride-hailing assignment model and the piecewise linear method applied. The purpose was to provide some insights and information on possible relevant future research.

- Ride-hailing Assignment Model

The numerical experiments conducted showed that the service level, workload, and fuel savings factors could positively impact the quality of decision-making. This showed the possibility of applying the model developed to ride-hailing systems considering the inherent benefits. However, the model cannot overcome the problem of unpredictable future demand or traffic congestion which can dramatically influence the quality of decision-making and pick-up time for the driver. The trend shows the need for future research to consider uncertainty factors using several alternative methods such as stochastic, fuzzy, and robust optimization. This is necessary because each method has advantages and disadvantages, thereby increasing the opportunities for more research to be conducted.

The ride-hailing assignment model can be improved to account for unpredictable future demand and traffic congestion. First, the integration of real-time data is crucial. Combining real-time data on traffic conditions, weather, and events that could affect demand can provide more accurate insights. This data can be obtained from traffic monitoring systems and weather forecasts. Second, the use of predictive analytics and machine learning can help forecast future demand patterns based on historical data and real-time inputs. Models such as time series analysis, regression, and neural networks can be used to predict demand and traffic conditions.

Additionally, stochastic optimization, fuzzy optimization, and robust optimization techniques can handle uncertainty in demand and traffic. Stochastic optimization optimizes routes and assignments by considering the probability distributions of future conditions, providing more reliable solutions in uncertain situations. Fuzzy optimization addresses ambiguous uncertainties by modeling uncertain parameters as fuzzy variables, allowing the model to handle variations more flexibly. Meanwhile, robust optimization ensures that solutions remain effective despite variations or disruptions in input parameters, ensuring the assignment model functions well under various uncertain conditions. By integrating these strategies, the ride-hailing assignment model can become more resilient and responsive to unpredictable demand and traffic congestion.

The determination of the right time window is an optimization problem that needs to be solved. A similar problem is often encountered in games in the form of matchmaking, where the time windows automatically change according to conditions. Therefore, future research can adapt this concept to develop a dynamic time window model.

Aspects such as forecasting or environmental impacts related to electric vehicle charging were not considered alongside service levels, driver workload, and fuel consumption. This is primarily due to the scope and specific objectives of the research focusing on optimizing ride-hailing assignments for operational efficiency, driver workload balance, and fuel consumption. Including forecasting and environmental impacts would add significant complexity to this research, requiring extensive data and sophisticated models that are beyond the primary focus and directly affect service quality and driver satisfaction, while environmental considerations, although important, are considered a secondary priority in this context.

- Piecewise Linear Method

Modeling the restricted basis condition as a constraint in the piecewise linear approach enhances the flexibility and accuracy of solutions. By incorporating this condition as a constraint, the piecewise linear method, which previously could only be used in non-automated systems, can now be operated automatically without the need for operators to check basis variables. It addresses the issue of subjectivity in monitoring the simplex table, which requires high expertise and attention to detail to ensure the adjacency rule is met, thereby reducing the risk of inconsistency.

The accuracy of the piecewise linear approach is highly dependent on the number of partitions used. More partitions result in a more accurate approximation of the non-linear function but also increase the complexity of the simplex process. Therefore, finding the

right balance between the number of partitions and the desired level of accuracy is crucial to ensure efficiency in problem-solving.

There are scenarios where the unmodeled restricted basis condition can be overly restrictive. In very complex problems with many variables and constraints, the requirement to monitor the restricted basis condition can be complex, especially as it requires specialized expertise to handle simple iterations that yield two non-adjacent positive λ_{ik} values. Thus, modeling this condition becomes essential, particularly for solving problems that require automated decision-making or large-scale applications where manual checks of this rule are no longer necessary.

Several limitations and challenges were encountered when applying the piecewise linear method to real-world non-linear programming problems. These include the need for accurate partitioning of the domain and the computational complexity of handling large datasets. Future research should focus on developing more efficient algorithms and exploring alternative partitioning strategies to overcome these challenges.

The results from the numerical experiments conducted showed that the piecewise linear method could be used to solve real-world problems modeled in non-linear form, as evidenced in the developed RhAP model. The subsequent application to a simple non-convex problem is presented as follows to provide further insight into its reliability (Figure 9).

$$\min x_1 + \sin(x_1^2) + x_2 + \cos(x_2^2); x_1, x_2 \in [0, 6]$$

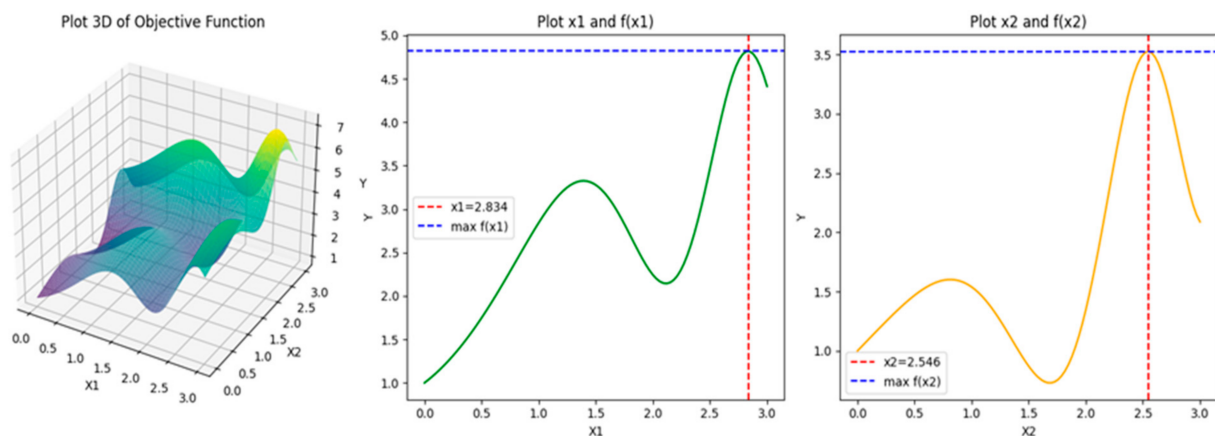


Figure 9. Non-convex functions plots.

Table 2 shows that, in this case, more partitions provide results closer to the actual optimal value, $x_1 = 2.834$, $x_2 = 2.5428$, with an objective value of 7.344441498472622. Meanwhile, the complexity of the model increased with the number of partitions, thereby affecting the computing time. This shows the need to study the convergence of the optimal values in the method more deeply to optimize the selection of the number of partitions.

Table 2. Non-convex problem solutions.

P_i (Number of Partitions)	Solutions	Objective Values	Number of Constraints
2	$x_1 = 3, x_2 = 3$	5.50098822335708	28
4	$x_1 = 3, x_2 = 2.25$	6.005120579381038	70
8	$x_1 = 3, x_2 = 2.625$	6.858230531690102	226
16	$x_1 = 2.8125, x_2 = 2.625$	7.257034667567415	826
32	$x_1 = 2.8125, x_2 = 2.53125$	7.334489363557671	3178
64	$x_1 = 2.8125, x_2 = 2.53125$	7.334489363557671	12,490

The trend identified in Table 2 was caused by the property of linear programming which was to provide locally optimal solution as globally optimal. However, further analysis is needed on the redundancy of the constraint function in order to reduce the complexity of the model. Future research is expected to develop the method to solve optimization problems with uncertainty factors using stochastic, fuzzy, or robust optimization. It is also important to state that the optimization models with uncertainty factors are often found in non-linear form, thereby showing an opportunity for further research.

A formal analysis needs to be performed to check whether all non-convex optimization problems can be solved using the method. It is an exciting research opportunity to undertake due to its contribution to the optimization field. However, there is a need to emphasize that the method requires the optimization problem to be separable. This is necessary because real-world problems can be generally encountered as non-separable functions. The process of finding solutions requires several reformulation steps to rewrite the functions as separable optimization problems through quite complex mathematical analysis and manipulation skills. Therefore, further research can be used to develop this method for all optimization problems, including detachable or undetachable.

The piecewise linear method demonstrates reliability in solving non-convex problems in real-world applications. It can approximate non-linear (non-convex) functions by breaking them into linear segments, thus simplifying complex problems into manageable sub-problems. This transformation allows for efficient linear optimization techniques, enhancing computational efficiency. Additionally, the method's flexibility and scalability enable it to handle different problem scales and complexities by adjusting the number and placement of partitions, making it adaptable across various fields such as finance, engineering, and supply chain management. The method effectively incorporates constraints like capacity and budget restrictions, ensuring suitability for real-world scenarios. Overall, the piecewise linear method's ability to approximate complex functions, handle constraints, and validate performance through empirical data makes it a reliable approach for solving non-convex problems in real-world applications.

Reducing redundancy in constraint functions to minimize the complexity of the linear programming model can be achieved through several strategies. Simplifying constraints by combining similar or related ones and using preprocessing techniques to identify and eliminate redundant constraints are effective methods. Constraint aggregation and reduction algorithms, such as Fourier–Motzkin elimination, can further streamline the model. Bounding techniques help tighten the feasible region, removing non-binding constraints while identifying and eliminating dominated constraints, simplifying the model. Decomposition methods, like Dantzig–Wolfe and Benders decomposition, break the problem into smaller subproblems, reducing constraint numbers. Sensitivity analysis determines the impact of each constraint, allowing for the removal of those with negligible effects. Heuristic methods and establishing a hierarchy of constraints based on their importance can also aid in focusing on critical constraints and relaxing less important ones. These strategies collectively reduce redundancy and improve the model's solvability.

- Comparison with previous research

Previous research, such as [15], focused on fuel-saving mechanisms in ride-hailing assignments. Our model not only integrates fuel savings but also considers workload distribution and service levels. The results show significant reductions in fuel consumption, supporting environmental sustainability, which aligns with findings from [15,23]. Studies like [8,19,21] primarily aimed to minimize waiting times to improve passenger satisfaction. Our model achieves comparable results by integrating passenger willingness to walk and optimizing driver pick-up efficiency. This holistic approach enhances service levels and operational efficiency, similar to the improvements discussed in [7,9]. Balancing driver workload is crucial for reducing fatigue and improving job satisfaction, as highlighted by [7,26]. Our model significantly reduces workload variance, promoting fairer work distribution, which supports the conclusions drawn by [10,14]. Traditional models, such as those discussed in [38,39], require human oversight to check restricted basis conditions. Our

automated approach eliminates this need, making it suitable for real-world applications in autonomous systems. This aligns with the trend towards automation in transportation discussed by [33,35]. The environmental benefits of our model, including reduced fuel consumption and emissions, align with the sustainability goals outlined in [11,12]. Encouraging passengers to walk short distances decreases idle vehicle time, contributing to a lower carbon footprint. This reinforces the importance of environmental considerations in ride-hailing assignments, as emphasized by [13,15].

6. Conclusions

In conclusion, this study succeeded in developing a ride-hailing model with due consideration for the service level, workload, and fuel consumption as well as a modified piecewise linear method to automatically solve non-linear programming problems. The results showed that the ride-hailing assignment model and the piecewise linear method contributed to understanding and practice in the industry. It was further suggested that the proposed model and modified method could enhance operational efficiency and elevate user experience in ride-hailing services while considering driver welfare and environmental impact. The challenges and opportunities identified for future research included handling uncertainty and developing more dynamic models. The study contributed value by providing more efficient and sustainable solutions to the ride-hailing industry and non-linear optimization field.

In addition to optimizing service level, driver workload, and fuel consumption, future research should consider the forecasting of demand and the environmental impact of electric vehicle charging. Integrating these aspects can provide a more comprehensive approach to sustainable ride-hailing services. This study lays the foundation for such future explorations.

Despite the promising results, this study has several limitations. Firstly, the model does not account for unpredictable future demand and traffic congestion, which can significantly affect decision-making quality and pick-up times. Secondly, the scope of the research did not include demand forecasting or the assessment of the environmental impact of electric vehicle charging, primarily due to the complexity and extensive data requirements.

Future research should address these limitations by integrating real-time data on traffic conditions, weather, and events to provide more accurate and responsive insights. Additionally, incorporating predictive analytics and machine learning techniques could help in forecasting future demand patterns, thereby improving the model's adaptability and performance.

There are several potential directions for future research based on the findings and limitations of this study. Firstly, exploring alternative optimization methods such as stochastic, fuzzy, and robust optimization could provide better handling of uncertainties in demand and traffic conditions. Secondly, developing dynamic time window models inspired by matchmaking concepts in games could offer improved management of varying operational conditions. Lastly, further investigation into the impact of environmental factors, including electric vehicle charging, on service levels and operational efficiency could yield valuable insights for enhancing sustainable practices in the ride-hailing industry.

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