

Article

Application of Extended Normal Distribution in Option Price Sensitivities

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Abstract: Empirical evidence indicates that asset returns adhere to an extended normal distribution characterized by excessive kurtosis and non-zero skewness. Consequently, option prices derived from this distribution diverge from those predicted by the Black–Scholes model. Despite the significance of option price sensitivities for risk management in investment portfolios, the existing literature lacks a thorough exploration of these sensitivities within the context of the extended normal distribution. This article addresses this research gap by deriving the Greeks for options based on the extended normal distribution. The Greeks under consideration include Vega, Delta, Theta, Gamma, Rho, Vanna, Charm, and Vera, all of which are crucial for informed financial decision-making. Furthermore, this study provides a detailed analysis of how these option price sensitivities vary with different levels of kurtosis, offering valuable insights for various market applications. This contribution not only enhances the theoretical understanding of option pricing under non-standard distributions but also presents practical implications for portfolio risk management.

Keywords: high-frequency trading; normal CDF; Q-function; option price; option Greeks

MSC: 91G20



Citation: Nayak, G.; Tripathy, S.S.; Imoize, A.L.; Li, C.-T. Application of Extended Normal Distribution in Option Price Sensitivities. *Mathematics* **2024**, *12*, 2346. <https://doi.org/10.3390/math12152346>

Academic Editor: Anatoliy Swishchuk

Received: 19 May 2024

Revised: 12 July 2024

Accepted: 19 July 2024

Published: 27 July 2024



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1. Introduction

Traditionally, the famous Black–Scholes option pricing model assumes a normal distribution of the given asset returns (cf. [1]). But empirically, it has been observed that the probability distribution function (pdf) of returns possesses non-zero skewness and kurtosis as depicted by [2,3] and others. This leads to the deviation of the market option price from the theoretical option price given by Black–Scholes. Several assumptions of the famous Black–Scholes option pricing model are not followed in real life, as noted by [4]. The non-Gaussian nature of the asset returns, and volatility smile from implied volatility have been observed in the financial market (cf. [4] and others [5–10]). This motivated authors [11,12] to propose different option pricing models satisfying the aforementioned observed properties of the asset price generally encountered in high-frequency trading. In this context, studies including [13–15] and others have managed to propose a new option pricing model under stochastic jumps. However, these models are not always acceptable, as the market becomes incomplete due to the presence of various stochastic components. Some authors [16–18] have also used higher moments of the corresponding stock returns to provide different option pricing models. It has also been noticed that different assets are influenced by various information and news, and hence exhibit different properties that cannot be statistically observable. These types of properties across various stocks are

very common and are termed stylized facts [19]. Volume correlation, volatility clustering, leverage effects, and heavy tails are some frequently observed stylized facts in financial time series [19]. Various authors [20,21] have observed the heavy tails of the underlying return distribution through a power law. Such types of behavior of the return distribution give rise to an extended normal distribution. This particular distribution is subject to different values of kurtosis and skewness [4]. Ki et al. [4] have defined the extended normal distribution as a linear combination of two Gaussian distribution functions. Thereafter, it is necessary to derive the option price for high-frequency trading under the said returns distribution. Thus, Ki et al. [4] have managed to provide the European option price under a risk-neutral measure and satisfy the moment restriction condition. They have also examined the validation of the proposed pricing model with the empirical KOSPI 2000 data sets. However, this paper does not deal with the factors associated with option price such as stock price, time left for the option maturity, interest rate, and volatility.

In this paper, we address some of the issues associated with the option price and risks associated with the financial markets. These can be modeled by computing the exact option price sensitivities, which are also referred to as the option Greeks. These option Greeks play a pivotal role in analyzing the uncertainties observed in financial derivatives with respect to different underlying market parameters like the stock price, volatility rate, time of maturity, and interest. Furthermore, several option traders are keen to analyze the option Greeks for hedging the risks associated with the option price. In this context, we provide the applicability of the extended normal distribution for the management of option price sensitivity. We compute some of the well-known options Greeks that play an essential role in determining the decisive measures for financial markets. Thus, the proposed work provides a good understanding to the option traders as well as the investors for the smooth management of asset prices, volatility, and portfolio risks.

This paper consists of five sections. In Section 2, we provide a brief discussion about the generation of extended normal distribution for skewness and kurtosis different from normal distribution. In the next section, we present the corresponding option price for different values of skewness and kurtosis along with its absolute error. Section 3 portrays the matching of the Black–Scholes option price with the option price computed by Ki et al. [4] for kurtosis equal to 3 under the extended normal distribution of the returns. Finally, we derive the higher order (up to two) option Greeks under the extended normal distribution in Section 4. The paper ends with the conclusion and future scope in Section 5.

2. Computation of Option Price under Extended Normal Distribution

This section deals with a unique concept provided by Ki et al. [4] for the underlying stock returns distribution. Suppose $h_{\mu,\sigma}$ represents the Gaussian distribution with mean (μ), standard deviation (σ) and involving other parameters viz., q, λ^2, ω^2 given by $q = 3/2 - 9/\beta^2$

$$\lambda^2 = 1 - \frac{1}{q} \sqrt{q(1-q)(\beta/3-1)}.$$

$$\omega^2 = 1 + \frac{1}{1-q} \sqrt{q(1-q)(\beta/3-1)}.$$

Ki et al. [4] have managed to design a density function $h_{(\mu,\sigma,\alpha,\beta)}$ as given below, for a random variable having non-zero skewness and desired level of kurtosis,

$$h_{(\mu,\sigma,\alpha,\beta)}(z) = \frac{1}{\sigma} \psi_{\alpha,\beta} \left(\frac{z-\mu}{\sigma} \right)$$

where

$$\psi_{\alpha,\beta}(x) = q \left\{ 1 + \frac{\alpha}{6\lambda^6} (x^3 - 3\lambda^2 x) \right\} g_{0,\lambda}(x) + (1-q) \left\{ 1 + \frac{\alpha}{6\omega^6} (x^3 - 3\omega^2 x) \right\} g_{0,\omega}(x)$$

where $g_{0,\lambda}(x)$ is the normal density function with zero mean and λ standard deviation and $g_{0,\omega}(x)$ is the normal density function with zero mean and ω standard deviation. This above-defined density function is treated as an extended normal distribution, with μ, σ, α , and β as the mean, standard deviation, skewness, and kurtosis, respectively. Thus, Ki et al. [4] have provided an option pricing formula whenever the underlying stock price moves according to the above-discussed distribution, and using the moment restriction property:

$$\begin{aligned} C = & S_0 \left\{ \frac{P}{P+Q} N\left(\frac{D+c\lambda^2}{\lambda}\right) + \frac{Q}{P+Q} N\left(\frac{D+c\omega^2}{\omega}\right) \right\} - Ke^{-rt} \left\{ qN\left(\frac{D}{\lambda}\right) \right. \\ & + (1-q)N\left(\frac{D}{\omega}\right) \left. \right\} + \frac{1}{6} \alpha Kce^{-rt} \left\{ \frac{q}{\lambda} \left(\frac{c\alpha^2 - D}{\alpha^2} \right) N'\left(\frac{D}{\lambda}\right) + \left(1 - \frac{q}{\omega}\right) \right. \\ & \left. \left(\frac{c\omega^2 - D}{\omega^2} \right) N'\left(\frac{D}{\omega}\right) \right\} \end{aligned}$$

where S_0 is the current stock price of the underlying stock, K is the strike price of the underlying stock, r is the risk-free interest rate, t is the time to maturity, and N is the standard normal density function.

The work of Ki et al. [4] is pivotal in advancing option pricing models by incorporating higher moments of skewness and kurtosis into the pricing framework. This approach addresses the limitations of the Black–Scholes model, which assumes a normal distribution of returns and fails to capture the observed fat tails and asymmetry in financial markets. By extending the normal distribution, Ki et al. [4] provide a more accurate pricing model that reflects real-world market conditions. This extended model has gained significant attention and validation in the recent literature, such as in the works of Wang et al. [22], who explored similar extensions to account for leptokurtic and skewed distributions, and Bakshi et al. [23], who examined the empirical performance of alternative option pricing models under different market conditions. These studies support the robustness and applicability of incorporating higher moments in option pricing, making the contributions of Ki et al. [4] crucial for both theoretical development and practical implementation in financial markets.

The values of P and Q are given by

$$P = q \exp\left(\lambda^2 c^2 / 2\right) \text{ and } Q = (1 - q) \exp\left(\omega^2 c^2 / 2\right)$$

The value of D is given by

$$D = \left[\ln(S_0 e^{rt} / K) - \ln\left[(P + Q) \left((6 + \alpha c^3) / 6 \right) \right] \right] / \sigma \sqrt{t},$$

where $c = \sigma \sqrt{t}$.

3. Graphical Interpretation

The call option price under extended normal distribution along with Black–Scholes is plotted in Figure 1, with skewness ($\alpha = 0$) and kurtosis ($\beta = 3$).

As stated in Ki et al. [4], the logarithmic of future asset price follows the extended normal distribution with non-zero skewness and kurtosis greater than three, resulting in fat-tailed distribution. For this context, we take kurtosis $\beta = 4$ and skewness $\alpha = 0.6$ for the extended normal distribution in Figure 2. It is revealed from Figure 2 that the option price shows deviation from the Black–Scholes price.

In order to have greater insight, the absolute error of the option price given by Black–Scholes and that of Ki et al. [4] is shown in Figure 3.

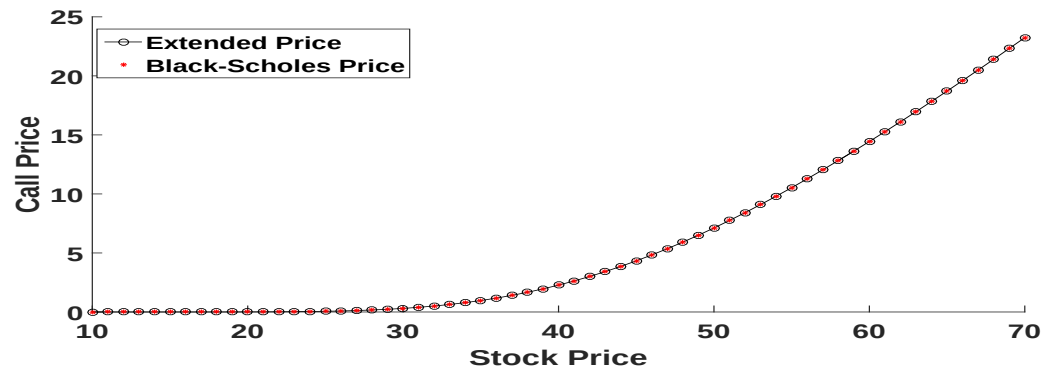


Figure 1. Option price under extended normal distribution for $\alpha = 0$ and $\beta = 3$ along with Black–Scholes price.

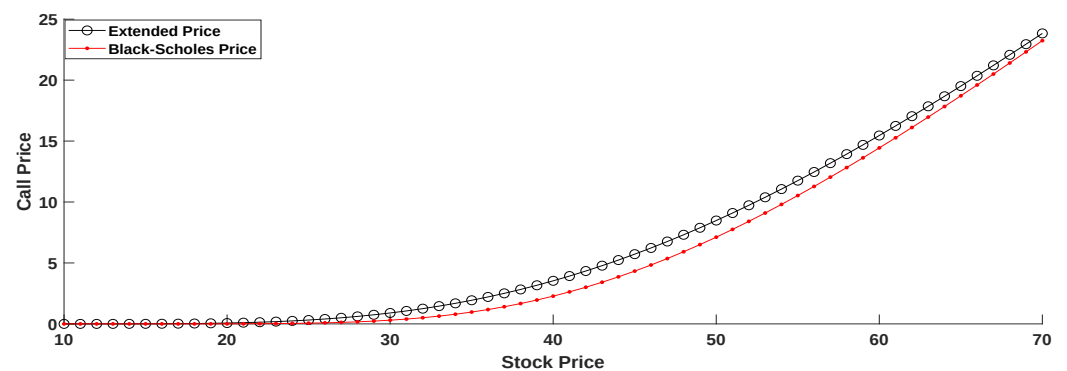


Figure 2. Option price under extended normal distribution for $\alpha = 0.6$ and $\beta = 4$ along with Black–Scholes price.

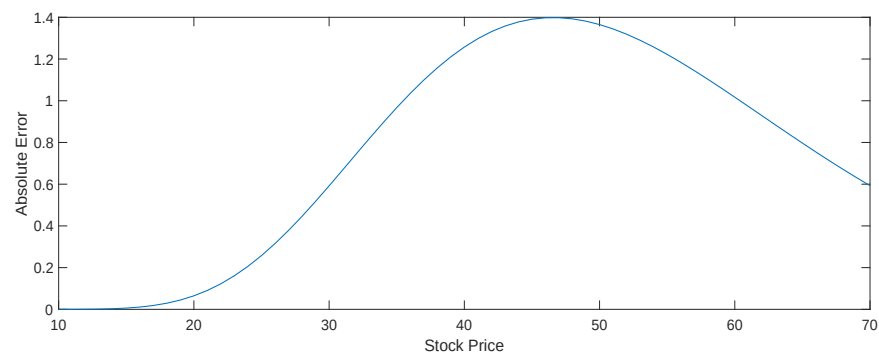


Figure 3. Absolute error between extended normal option price and Black–Scholes option price for $\beta = 4$.

Furthermore, we take kurtosis values of $\beta = 5$ and $\beta = 6$ and a skewness value of $\alpha = 0.6$ for the extended normal distribution in Figures 4 and 5, respectively. It is revealed from Figure 4 that as the kurtosis increases, the option price shows a significant deviation from the conventional Black–Scholes price. Specifically, with $\beta = 5$, the deviation becomes more pronounced, indicating a higher sensitivity to changes in kurtosis. This trend is further accentuated in Figure 5, when $\beta = 6$, where the proposed extended pricing model demonstrates even greater divergence from the Black–Scholes price. This illustrates that the Extended Normal Distribution model captures the impact of higher kurtosis more effectively, highlighting the limitations of the Black–Scholes model in accurately pricing options under conditions of increased kurtosis.

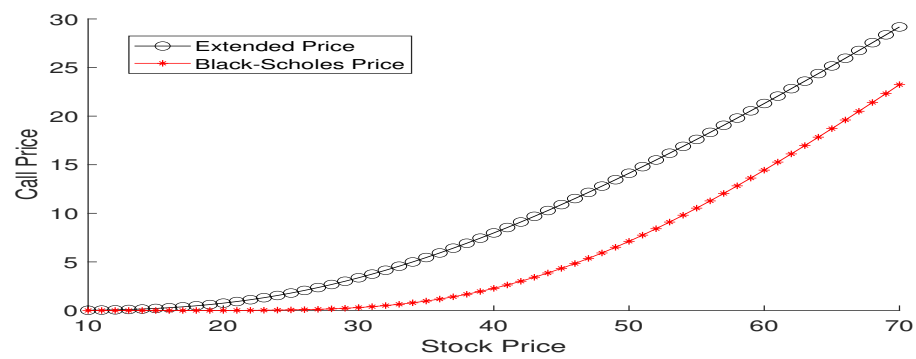


Figure 4. Option price under extended normal distribution for $\alpha = 0.6$ and $\beta = 5$ along with Black-Scholes price.

In order to gain greater insight, the absolute error of the option price given by the Black-Scholes model and that of Ki et al. [4] is shown in Figure 6. For kurtosis values of $\beta = 5$ and $\beta = 6$, the absolute error values exhibit a rapid increase as the stock price rises. Specifically, when $\beta = 5$, the absolute error between the two models starts to increase significantly, demonstrating a clear divergence as the stock price escalates. This trend becomes even more pronounced for $\beta = 6$ in Figure 7, where the absolute error witnesses a sharp rise, indicating a greater discrepancy between the Black-Scholes model and the extended model proposed by Ki et al. [4]. This rapid increase in absolute error with higher kurtosis values underscores the limitations of the Black-Scholes model in accurately pricing options under conditions of increased kurtosis and highlights the improved performance of the extended pricing model in capturing these effects.

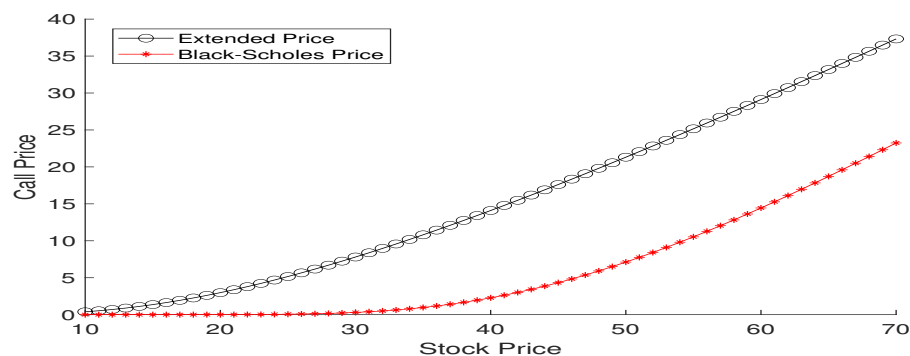


Figure 5. Option price under extended normal distribution for $\alpha = 0.6$ and $\beta = 6$ along with Black-Scholes price.

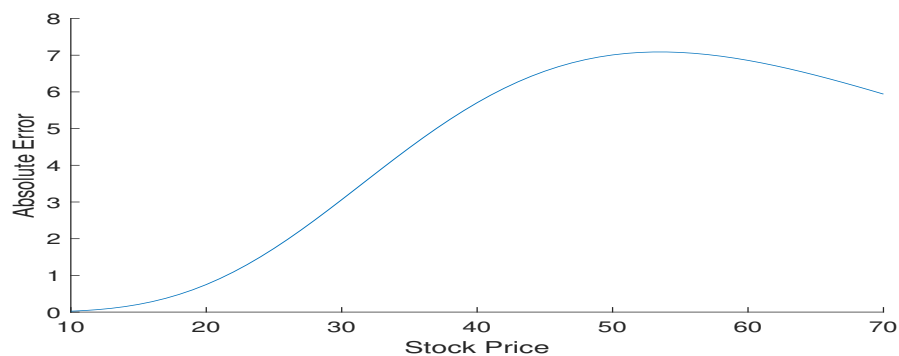


Figure 6. Absolute error between extended normal option price and Black-Scholes option price for $\beta = 5$.

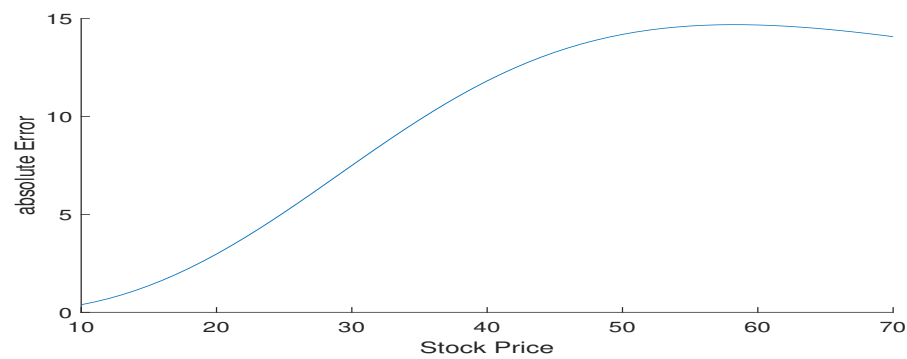


Figure 7. Absolute error between extended normal option price and Black–Scholes option price for $\beta = 6$.

Figure 8 shows the behavior of the option price for a fixed skewness but for different values of kurtosis. It is observed from Figure 8 that the option price goes on increasing with the increase in kurtosis values.

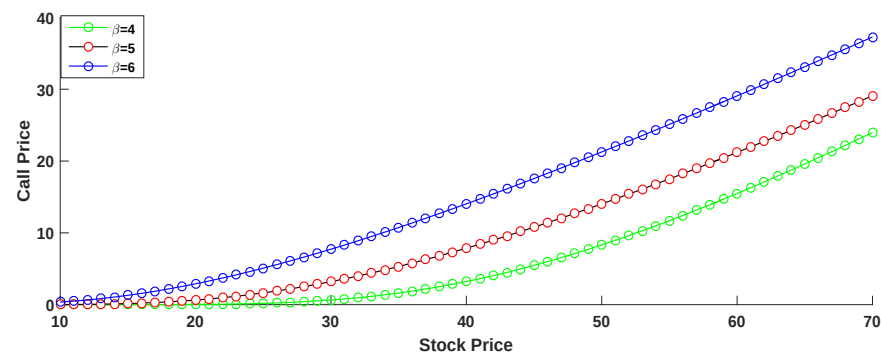


Figure 8. Option price under extended normal distribution for $\beta = 4, 5, 6$.

4. Computation of Option Greeks

Since the market price of various assets seems to fluctuate randomly, it is quite difficult to guess the option price of the underlying asset. Option Greeks are of utmost importance for maintaining a huge portfolio smoothly. These are also known as sensitivity measures, defined as the change in option price corresponding to various underlying parameters like stock price, time to maturity, volatility, and interest rates [24]. It helps the trader to have a brief idea regarding the portfolio's current position. This suggests whether to take more or less of a particular option contract for an underlying asset. To have a better profit with minimum risk, traders must have precise knowledge about the values of each option Greek.

(i) Delta

This sensitivity represents the variation in option premium corresponding to the change in stock price [25]. More clearly, if Delta for a particular stock is 0.35, then each one-unit increase in the stock price results in a 0.35 increase in the option price. It has been observed that Delta always provides positive and negative values for call and put options, respectively. The minimum and maximum values of call delta are 0 and 1, respectively, whereas that of put delta are -1 and 0. The sum of absolute values of call Delta and put Delta always gives one. This shows the minimum and maximum values of call delta are

0 and 1, respectively, whereas that of put delta are -1 and 0 , which satisfies the put-call parity relationship. Thus, under extended normal distribution, Delta is computed as

$$\begin{aligned} \Delta = & \frac{P}{P+Q} N\left(\frac{D}{\lambda} + c\lambda\right) + \frac{Q}{P+Q} N\left(\frac{D}{\omega} + c\omega\right) \\ & + S_0 \left\{ \frac{P}{P+Q} N'\left(\frac{D}{\lambda} + c\lambda\right) \frac{1}{\lambda S_0 \sigma \sqrt{t}} + \frac{Q}{P+Q} N'\left(\frac{D}{\omega} + c\omega\right) \frac{1}{\omega S_0 \sigma \sqrt{t}} \right\} \\ & - K e^{-rt} \left\{ q N'\left(\frac{D}{\lambda}\right) \frac{1}{\lambda S_0 \sigma \sqrt{t}} + (1-q) N'\left(\frac{D}{\omega}\right) \frac{1}{\omega S_0 \sigma \sqrt{t}} \right\} \\ & + \frac{1}{6} \alpha K c e^{-rt} \left\{ \frac{q}{\lambda} \left(c - \frac{D}{\lambda^2}\right) N''\left(\frac{D}{\lambda}\right) \frac{1}{\lambda S_0 \sigma \sqrt{t}} + \frac{q}{\lambda} N'\left(\frac{D}{\lambda}\right) \left(\frac{-1}{\lambda^2 S_0 \sigma \sqrt{t}}\right) \right. \\ & \left. + \frac{1-q}{\omega} N'\left(\frac{D}{\omega}\right) \left(\frac{-1}{\omega^2 S_0 \sigma \sqrt{t}}\right) + \frac{1-q}{\omega} \left(c - \frac{D}{\omega^2}\right) N''\left(\frac{D}{\omega}\right) \frac{1}{\omega S_0 \sigma \sqrt{t}} \right\} \end{aligned} \quad (1)$$

Figure 9 represents call Delta under extended normal distribution with different kurtosis values. The Delta values are quite different from the Black–Scholes Delta. The values of the given Greeks from 51 to 61 and for three different values of the kurtosis are shown in Table 1. The extended Delta for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$ is given in Figure 10.

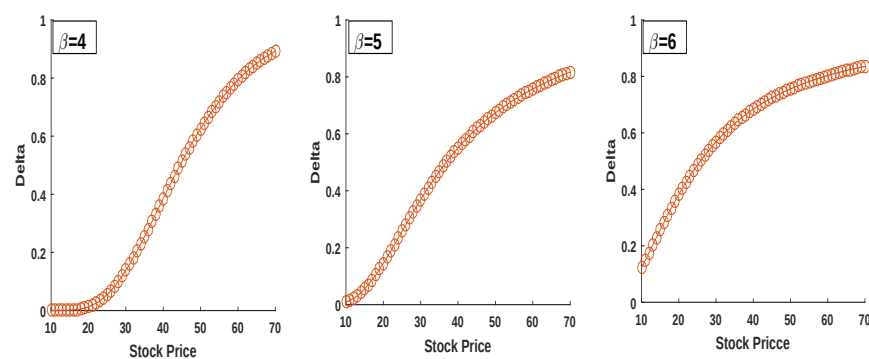


Figure 9. Option Delta under extended normal distribution for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$ and $\beta = 4, 5, 6$.

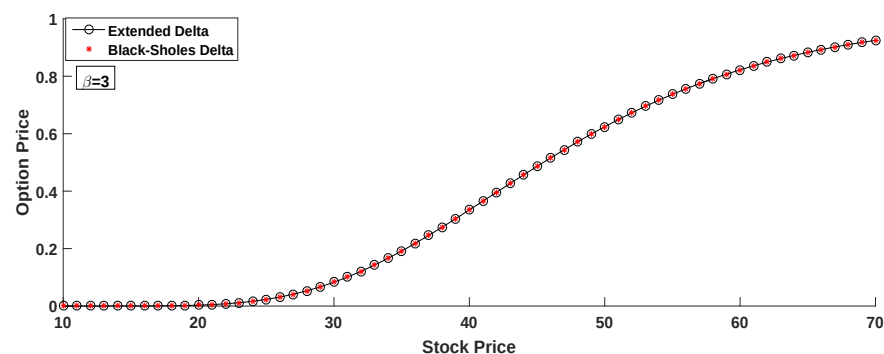


Figure 10. Extended Delta for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$.

(ii) Gamma

This sensitivity is defined as the rate of variation of Delta corresponding to the stock price [26]. It is also obtained by taking the double derivative of the option price under the stock price. Unlike Delta, Gamma always provides positive values for both call and put options. However, both call and put have the same Gamma values. Gamma attains its maximum when the stock price reaches the strike price.

$$\begin{aligned}
\Gamma = & \frac{2P}{P+Q} N' \left(\frac{D}{\lambda} + c\lambda \right) \frac{1}{\lambda S_0 \sigma \sqrt{t}} + \frac{2Q}{P+Q} N' \left(\frac{D}{\omega} + c\omega \right) \frac{1}{\omega S_0 \sigma \sqrt{t}} \\
& + S_0 \left[\frac{P}{P+Q} N'' \left(\frac{D}{\lambda} + c\lambda \right) \frac{1}{\lambda^2 S_0^2 \sigma^2 t} - \frac{P}{P+Q} N' \left(\frac{D}{\lambda} + c\lambda \right) \frac{1}{\lambda S_0^2 \sigma \sqrt{t}} \right. \\
& \left. + \frac{Q}{P+Q} N'' \left(\frac{D}{\omega} + c\omega \right) \frac{1}{\omega^2 S_0^2 \sigma^2 t} - \frac{Q}{P+Q} N' \left(\frac{D}{\omega} + c\omega \right) \frac{1}{\omega S_0^2 \sigma \sqrt{t}} \right] \\
& - K e^{-rt} \left\{ \frac{q}{\lambda^2 S_0^2 \sigma^2 t} N'' \left(\frac{D}{\lambda} \right) - \frac{q}{\lambda S_0^2 \sigma \sqrt{t}} N' \left(\frac{D}{\lambda} \right) \right. \\
& \left. + \frac{(1-q)}{\omega^2 S_0^2 \sigma^2 t} N'' \left(\frac{D}{\omega} \right) - \frac{(1-q)}{\omega S_0^2 \sigma \sqrt{t}} N' \left(\frac{D}{\omega} \right) \right\} \\
& + \frac{1}{6} \alpha K c e^{-rt} \left[\begin{aligned} & \frac{q}{\lambda^3 S_0^2 \sigma \sqrt{t}} N' \left(\frac{D}{\lambda} \right) - \frac{q}{\lambda^4 S_0^2 \sigma^2 t} N'' \left(\frac{D}{\lambda} \right) \\ & + \frac{(1-q)}{\omega^3 S_0^2 \sigma \sqrt{t}} N' \left(\frac{D}{\omega} \right) - \frac{(1-q)}{\omega^4 S_0^2 \sigma^2 t} N'' \left(\frac{D}{\omega} \right) \\ & - \frac{q}{\lambda^4 S_0^2 \sigma^2 t} N'' \left(\frac{D}{\lambda} \right) - \frac{q}{\lambda^2 S_0^2 \sigma \sqrt{t}} \left(c - \frac{D}{\lambda^2} \right) N''' \left(\frac{D}{\lambda} \right) \\ & + \frac{q}{\lambda^3 S_0^2 \sigma^2 t} \left(c - \frac{D}{\lambda^2} \right) N'''' \left(\frac{D}{\lambda} \right) + \frac{q-1}{\omega^4 S_0^2 \sigma^2 t} N'' \left(\frac{D}{\omega} \right) \\ & + \frac{q-1}{\omega^2 S_0^2 \sigma \sqrt{t}} \left(c - \frac{D}{\omega^2} \right) N''' \left(\frac{D}{\omega} \right) + \frac{1-q}{\omega^3 S_0^2 \sigma^2 t} \left(c - \frac{D}{\omega^2} \right) N'''' \left(\frac{D}{\omega} \right) \end{aligned} \right] \quad (2)
\end{aligned}$$

Figure 11 shows the extended Gamma for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$. Figure 12 represents the call Gamma under extended normal distribution with different kurtosis values. We have also given the values of the particular Greeks from 51 to 61 in Table 1 for three different values of the kurtosis.

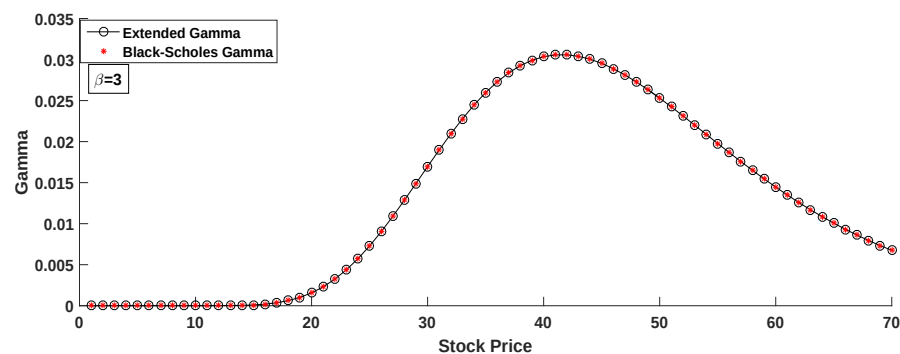


Figure 11. Extended Gamma for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$.

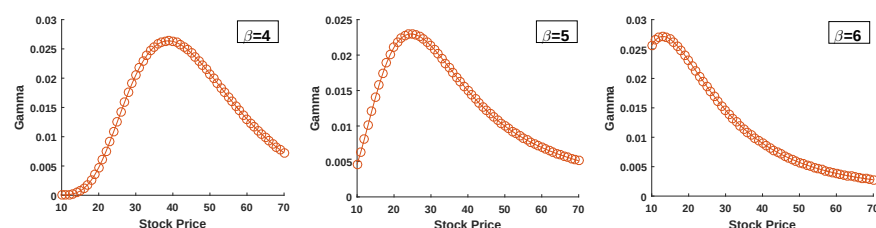


Figure 12. Option Gamma under extended normal distribution for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$ and $\beta = 4, 5, 6$.

(iii) Theta

This sensitivity measure gives an idea about the price change for change in time remaining towards the expiration. It has been noticed that whenever expiration approaches, both call and put Theta decrease. Therefore, this measure always reflects negative values [25]. Keeping the stock price, interest rate, and volatility of underlying assets constant, Theta will show the number of options declined. Option Theta behaves differently at the money, in the money, and out of the money option. If the expiration approaches towards in in-the-money or out-of-the-money option, the option premium increases. However, if the

expiration for out of the out-of-the-money option is far away, the option premium goes on decreasing.

$$\begin{aligned} \theta = -S_0 & \left[\frac{P}{P+Q} N' \left(\frac{D}{\lambda} + \lambda c \right) \left(\frac{D_t}{\lambda} + \frac{\lambda \sigma}{2\sqrt{t}} \right) + \frac{Q}{P+Q} N' \left(\frac{D}{\omega} + \omega c \right) \left(\frac{D_t}{\omega} + \frac{\omega \sigma}{2\sqrt{t}} \right) \right] \\ & + N \left(\frac{D}{\lambda} + \lambda c \right) \frac{PQ\sigma^2(\lambda^2 - \omega^2)}{2(P+Q)^2} + N \left(\frac{D}{\omega} + \omega c \right) \frac{PQ\sigma^2(\omega^2 - \lambda^2)}{2(P+Q)^2} \\ & + Ke^{-rt} \left\{ qN' \left(\frac{D}{\lambda} \right) \frac{D_t}{\lambda} + (1-q)N' \left(\frac{D}{\omega} \right) \frac{D_t}{\omega} \right\} - Kre^{-rt} \left\{ qN \left(\frac{D}{\lambda} \right) + (1-q)N \left(\frac{D}{\omega} \right) \right\} \\ & - \frac{1}{6} \alpha Kce^{-rt} \left\{ \frac{qD_t}{\lambda^2} \left(c - \frac{D}{\lambda^2} \right) N'' \left(\frac{D}{\lambda} \right) + \frac{q}{\lambda} N' \left(\frac{D}{\lambda} \right) \left(\frac{\sigma}{2\sqrt{t}} - \frac{D_t}{\lambda^2} \right) \right. \\ & \left. + \frac{(1-q)D_t}{\omega^2} \left(c - \frac{D}{\omega^2} \right) N'' \left(\frac{D}{\omega} \right) + \frac{1-q}{\omega} N' \left(\frac{D}{\omega} \right) \left(\frac{\sigma}{2\sqrt{t}} - \frac{D_t}{\omega^2} \right) \right\} \\ & + \frac{r}{6} Kc\alpha e^{-rt} \left\{ \frac{q}{\lambda} \left(c - \frac{D}{\lambda^2} \right) N' \left(\frac{D}{\lambda} \right) + \frac{1-q}{\omega} \left(c - \frac{D}{\omega^2} \right) N' \left(\frac{D}{\omega} \right) \right\} \\ & - \frac{K\alpha\sigma e^{-rt}}{12\sqrt{t}} \left\{ \frac{q}{\lambda} \left(c - \frac{D}{\lambda^2} \right) N' \left(\frac{D}{\lambda} \right) + \frac{1-q}{\omega} \left(c - \frac{D}{\omega^2} \right) N' \left(\frac{D}{\omega} \right) \right\} \end{aligned} \quad (3)$$

Figure 13 gives the extended Theta for $T=1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$. Figure 14 depicts the plot of call Theta under extended normal distribution for different kurtosis values. It is quite obvious that the obtained Theta values are different from the Black–Scholes Theta. These values for different kurtosis are shown in Table 1.

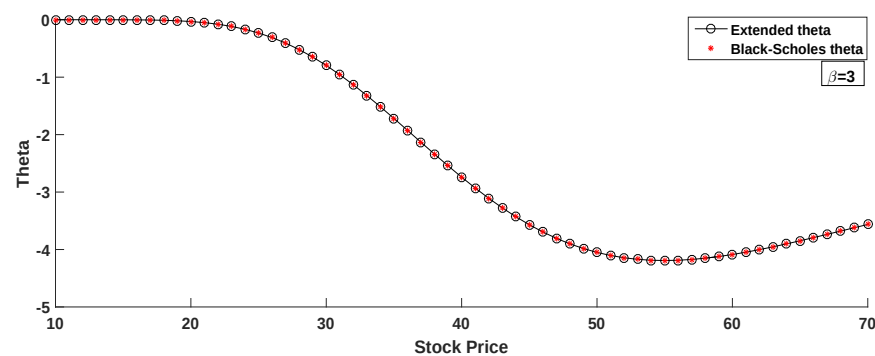


Figure 13. Extended Theta for $T=1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$.

Table 1. Values of Delta, Gamma, and Theta for kurtosis $\beta = 4, 5, 6$.

Stock Price	Delta			Gamma			Theta		
	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 4$	$\beta = 5$	$\beta = 6$
51	0.6460	0.6836	0.7615	0.0199	0.0096	0.0054	−4.6460	−6.9710	−8.8440
52	0.6655	0.6931	0.7699	0.0191	0.0092	0.0052	−4.6900	−7.0410	−8.9300
53	0.6843	0.7021	0.7720	0.0183	0.0089	0.0050	−4.7240	−7.1070	−9.0140
54	0.7022	0.7109	0.7770	0.0175	0.0085	0.0048	−4.7480	−7.1670	−9.0960
55	0.7194	0.7193	0.7817	0.0167	0.0082	0.0046	−4.7620	−7.2230	−9.1750
56	0.7358	0.7275	0.7863	0.0152	0.0077	0.0044	−4.7680	−7.2740	−9.2520
57	0.7514	0.7353	0.7907	0.0144	0.0074	0.0041	−4.7660	−7.3210	−9.3260
58	0.7662	0.7429	0.7949	0.0137	0.0072	0.0040	−4.7560	−7.3640	−9.3980
59	0.7803	0.7502	0.7990	0.0130	0.0069	0.0038	−4.7400	−7.4020	−9.4690
60	0.7937	0.7573	0.8030	0.0123	0.0067	0.0037	−4.7170	−7.4370	−9.5370
61	0.8064	0.7642	0.8067	0.0098	0.0051	0.0032	−4.6890	−7.4680	−9.6030

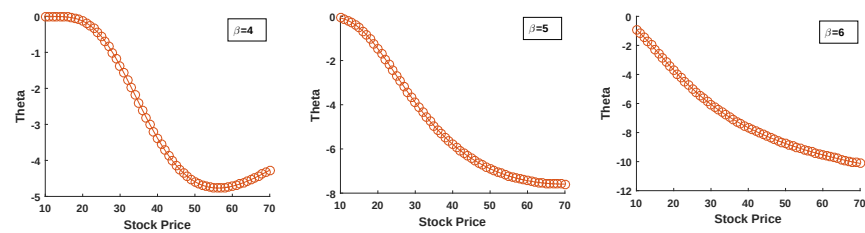


Figure 14. Option Theta under extended normal distribution for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$ and $\beta = 4, 5, 6$.

(iv) Rho

This sensitivity measure is associated with the interest rate, which is defined as the rate of change of options value corresponding to the change in the value of interest rate [27]. It generally does not have any serious impacts for the shorter periods options, whereas it has a significant impacts on the long-term options.

$$\begin{aligned} \rho = S_0 & \left\{ \frac{P}{P+Q} N' \left(\frac{D}{\lambda} + \lambda c \right) \frac{\sqrt{t}}{\lambda \sigma} + \frac{Q}{P+Q} N' \left(\frac{D}{\omega} + \omega c \right) \frac{\sqrt{t}}{\omega \sigma} \right\} \\ & - \frac{t}{6} K \alpha e^{-rt} \left\{ \frac{q}{\lambda} \left(c - \frac{D}{\lambda^2} \right) N' \left(\frac{D}{\lambda} \right) + \frac{1-q}{\omega} \left(c - \frac{D}{\omega^2} \right) N' \left(\frac{D}{\omega} \right) \right\} \\ & + \frac{1}{6} K \alpha e^{-rt} \left\{ \frac{q \sqrt{t}}{\lambda^2 \sigma} \left(c - \frac{D}{\lambda^2} \right) N'' \left(\frac{D}{\lambda} \right) - \frac{(1-q) \sqrt{t}}{\omega^3 \sigma} N'' \left(\frac{D}{\omega} \right) \right. \\ & \left. + \frac{(1-q) \sqrt{t}}{\omega^2 \sigma} \left(c - \frac{D}{\omega^2} \right) N'' \left(\frac{D}{\omega} \right) - \frac{q \sqrt{t}}{\lambda^3 \sigma} N' \left(\frac{D}{\lambda} \right) \right\} \\ & + K e^{-rt} \left\{ (1-q) t N \left(\frac{D}{\omega} \right) - q N' \left(\frac{D}{\lambda} \right) \frac{\sqrt{t}}{\lambda \sigma} \right. \\ & \left. + q t N \left(\frac{D}{\lambda} \right) - (1-q) N' \left(\frac{D}{\omega} \right) \frac{\sqrt{t}}{\omega \sigma} \right\} \end{aligned} \quad (4)$$

Figure 15 is the extended Rho for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$. Figure 16 represents the graph of call Rho under extended normal distribution for different kurtosis values. These Rho values for different kurtosis are shown in Table 2.

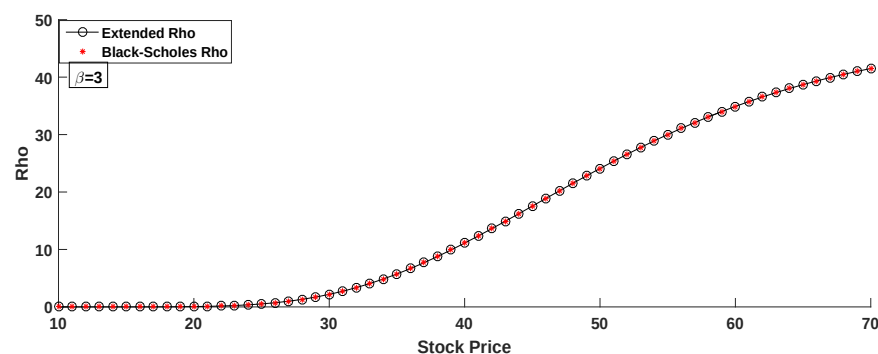


Figure 15. Extended Rho for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$.

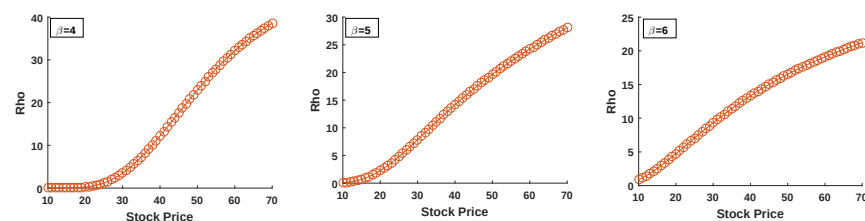


Figure 16. Option Rho under extended normal distribution for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$ and $\beta = 4, 5, 6$.

(v) Vega

This Greek corresponds to the implied volatility of the underlying stock price. It represents the variation in option price corresponding to the variation in the implied volatility of the stock price [25]. The difference between stock price and strike price is independent of this option Greek. However, it has a significant impact on the time value of money.

$$v = S_0 \left[\frac{P}{P+Q} N' \left(\frac{D}{\lambda} + \lambda c \right) \left(\frac{D_\sigma}{\lambda} + \sqrt{t} \lambda \right) + \frac{Q}{P+Q} N' \left(\frac{D}{\omega} + \omega c \right) \left(\frac{D_\sigma}{\omega} + \sqrt{t} \omega \right) \right] \\ + \frac{PQ\sigma t N}{(P+Q)^2} \left\{ (\lambda^2 - \omega^2) N \left(\frac{D}{\lambda} + \lambda c \right) + N \left(\frac{D}{\omega} + \omega c \right) (\omega^2 - \lambda^2) \right\} \\ + \frac{1}{6} K \alpha c e^{-rt} \frac{q D_\sigma}{\lambda^2} \left(c - \frac{D}{\lambda^2} \right) N'' \left(\frac{D}{\lambda} \right) + \frac{(1-q) D_\sigma}{\omega^2} \left(c - \frac{D}{\omega^2} \right) N'' \left(\frac{D}{\omega} \right) \\ - K e^{-rt} \left\{ q N' \left(\frac{D}{\lambda} \right) \frac{D_\sigma}{\lambda} + (1-q) N' \left(\frac{D}{\omega} \right) \frac{D_\sigma}{\omega} \right\} + \frac{1-q}{\omega} \left(\sqrt{t} - \frac{D_\sigma}{\omega^2} \right) N' \left(\frac{D}{\omega} \right) \\ + \frac{1}{6} K \alpha \sqrt{t} e^{-rt} \left\{ \frac{q}{\lambda} \left(c - \frac{D}{\lambda^2} \right) N' \left(\frac{D}{\lambda} \right) + \frac{1-q}{\omega} \left(c - \frac{D}{\omega^2} \right) N' \left(\frac{D}{\omega} \right) \right\} + \frac{q}{\lambda} \left(\sqrt{t} - \frac{D_\sigma}{\lambda^2} \right) N' \left(\frac{D}{\lambda} \right) \quad (5)$$

$$D_\sigma = -\frac{D}{\sigma} - \frac{1}{\sigma \sqrt{t}} \left[\frac{\sigma t (P\lambda^2 + Q\omega^2)}{P+Q} + \frac{3\alpha \sigma^2 t^{3/2}}{(6+\alpha c^3)} \right], \quad D_t = -\frac{D}{2t} - \frac{1}{\sigma \sqrt{t}} \left[r - \frac{\sigma^2 (P\lambda^2 + Q\omega^2)}{2(P+Q)} - \frac{3\alpha \sigma^3 \sqrt{t}}{2(6+\alpha c^3)} \right] \\ D_{\sigma\sigma} = \frac{D}{\sigma^2} + \frac{1}{\sigma^2 t} \left[\frac{\sigma t (P\lambda^2 + Q\omega^2)}{P+Q} - \frac{3\alpha \sigma^2 t^{3/2}}{(6+\alpha c^3)} \right] - \frac{6\alpha \sigma t^{3/2} (6+\alpha c^3) - 9\alpha^2 \sigma^4 t^3}{\sigma \sqrt{t} (6+\alpha c^3)^2} \\ - \sigma t^{3/2} \left[\frac{t(P+Q)(P\lambda^2 + Q\omega^2)(P\lambda^4 + Q\omega^4) - (P\lambda^2 + Q\omega^2)^2}{(P+Q)^2} \right] - \frac{D_\sigma}{\sigma}$$

where D_σ is the derivative of D with respect to σ parameter. Figure 17 extended Vega along with Black–Scholes Vega for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$. Figure 18 represents the graph of call Vega under extended normal distribution for different kurtosis values. One can note that the Vega values are quite different from the Black–Scholes Vega. We have shown the Vega values in Table 2 for different kurtosis.

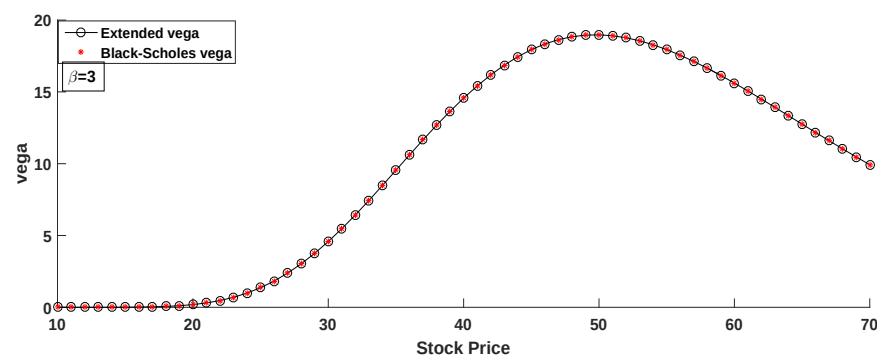


Figure 17. Extended Vega along with Black–Scholes Vega for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$.

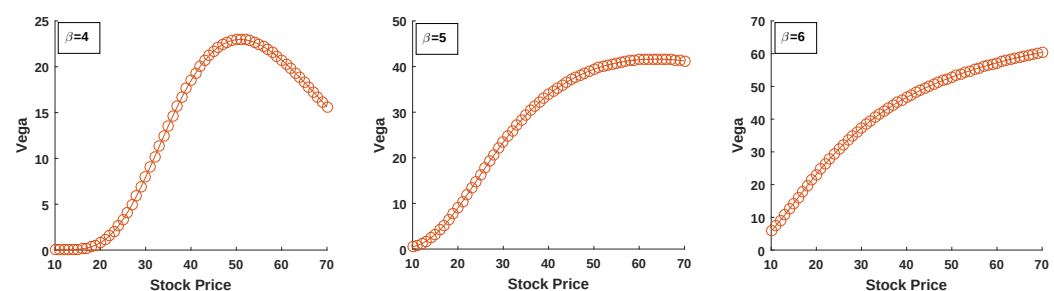


Figure 18. Option Vega under extended normal distribution for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$ and $\beta = 4, 5, 6$.

(vi) Vanna

Vanna may be defined as the rate of change of Delta corresponding to the fluctuation parameter volatility. This Greek is generally applicable for the generation of the Delta-hedge ratio or Vega-hedge ratio. Basically, portfolio holders use such types of option Greeks. This sensitivity measure relies on the affinity between the price change and volatility change. Figure 19 Extended Vanna along with Black–Scholes Vanna for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$. Figure 20 represents the graph of call Vanna under extended normal distribution for different kurtosis values. We have shown the Vanna values in Table 2 for different kurtosis.

Table 2. Values of Rho, Vega, and Vanna for kurtosis $\beta = 4, 5, 6$.

Stock Price	Rho			Vega			Vanna		
	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 4$	$\beta = 5$	$\beta = 6$
51	23.99	20.16	16.81	22.98	39.75	53.35	3.518	1.965	−0.541
52	24.99	20.64	17.09	22.94	40.06	53.84	3.525	2.114	−0.470
53	25.98	21.12	17.36	22.83	40.34	54.31	3.498	2.257	−0.401
54	26.94	21.59	17.62	22.67	40.59	54.77	3.441	2.394	−0.334
55	22.46	40.80	55.21	22.46	40.80	55.21	3.355	2.526	−2.267
56	28.78	22.50	18.14	22.19	41.00	55.63	3.245	2.652	−0.201
57	29.66	22.94	18.39	21.88	41.16	56.05	3.115	2.772	−0.135
58	30.52	23.38	18.63	21.54	41.30	56.46	2.962	2.885	−0.070
59	31.34	23.81	18.87	21.15	41.44	56.83	2.797	2.993	−0.005
60	32.14	24.23	19.10	20.73	41.50	57.21	2.618	3.095	−0.058
61	32.90	24.64	19.33	20.29	41.57	57.58	2.43	3.190	−0.122

$$\begin{aligned}
\frac{\partial \Delta}{\partial \sigma} = & \frac{P}{P+Q} N' \left(\frac{D}{\lambda} + c\lambda \right) \left(\frac{D_\sigma}{\lambda} + \sqrt{t}\lambda \right) + PQ\sigma t N \left(\frac{D}{\lambda} + c\lambda \right) \frac{(\lambda^2 - \omega^2)}{(P+Q)^2} \\
& + \frac{Q}{P+Q} N' \left(\frac{D}{\omega} + c\omega \right) \left(\frac{D_\sigma}{\omega} + \sqrt{t}\omega \right) + PQ\sigma t N \left(\frac{D}{\omega} + c\omega \right) \frac{(\omega^2 - \lambda^2)}{(P+Q)^2} \\
& + S_0 \left[\frac{P}{P+Q} N'' \left(\frac{D}{\lambda} + c\lambda \right) \left(\frac{D_\sigma}{\lambda} + \sqrt{t}\lambda \right) \frac{1}{\lambda S_0 \sigma \sqrt{t}} + N' \left(\frac{D}{\omega} + c\omega \right) \frac{PQ\sqrt{t}}{\omega S_0} \frac{(\omega^2 - \lambda^2)}{(P+Q)^2} \right. \\
& + N' \left(\frac{D}{\lambda} + c\lambda \right) \frac{PQ\sqrt{t}}{\lambda S_0} \frac{(\lambda^2 - \omega^2)}{(P+Q)^2} + \frac{Q}{P+Q} N'' \left(\frac{D}{\omega} + c\omega \right) \left(\frac{D_\sigma}{\omega} + \sqrt{t}\omega \right) \frac{1}{\omega S_0 \sigma \sqrt{t}} \\
& \left. - \frac{P}{P+Q} N' \left(\frac{D}{\lambda} + c\lambda \right) \frac{1}{\lambda S_0 \sigma^2 \sqrt{t}} - \frac{Q}{P+Q} N' \left(\frac{D}{\omega} + c\omega \right) \frac{1}{\omega S_0 \sigma^2 \sqrt{t}} \right] \\
& - Ke^{-rt} \left\{ N'' \left(\frac{D}{\omega} \right) \frac{(1-q)D_\sigma}{\omega^2 S_0 \sigma \sqrt{t}} - N' \left(\frac{D}{\omega} + c\omega \right) \frac{(1-q)}{\omega S_0 \sigma^2 \sqrt{t}} \right. \\
& \left. + qN'' \left(\frac{D}{\lambda} \right) \frac{D_\sigma}{\lambda^2 S_0 \sigma \sqrt{t}} - N' \left(\frac{D}{\lambda} + c\lambda \right) \frac{q}{\lambda S_0 \sigma^2 \sqrt{t}} \right\} \\
& + \frac{1}{6} \alpha K \sqrt{t} e^{-rt} \left\{ \left(c - \frac{D}{\lambda^2} \right) N'' \left(\frac{D}{\lambda} \right) \frac{q}{\lambda^2 S_0 \sigma \sqrt{t}} - N' \left(\frac{D}{\lambda} \right) \left(\frac{q}{\lambda^3 S_0 \sigma \sqrt{t}} \right) + \right. \\
& \left. - N' \left(\frac{D}{\omega} \right) \left(\frac{1-q}{\omega^3 S_0 \sigma \sqrt{t}} \right) + \left(c - \frac{D}{\omega^2} \right) N'' \left(\frac{D}{\omega} \right) \frac{(1-q)}{\omega^2 S_0 \sigma \sqrt{t}} \right\} \\
& + \frac{1}{6} \alpha K c e^{-rt} \left[\frac{qD_\sigma}{\lambda^3 S_0 \sigma \sqrt{t}} \left(c - \frac{D}{\lambda^2} \right) N''' \left(\frac{D}{\lambda} \right) + \frac{q}{\lambda^2 S_0 \sigma \sqrt{t}} \left(\sqrt{t} - \frac{D_\sigma}{\lambda^2} \right) N'' \left(\frac{D}{\lambda} \right) \right. \\
& - \frac{q}{\lambda^2 S_0 \sigma^2 \sqrt{t}} \left(c - \frac{D}{\lambda^2} \right) N'' \left(\frac{D}{\lambda} \right) - \left(\frac{qD_\sigma}{\lambda^4 S_0 \sigma \sqrt{t}} \right) N'' \left(\frac{D}{\lambda} \right) \\
& + \left(\frac{q}{\lambda^3 S_0 \sigma^2 \sqrt{t}} \right) N' \left(\frac{D}{\lambda} \right) + \frac{(1-q)D_\sigma}{\omega^3 S_0 \sigma \sqrt{t}} \left(c - \frac{D}{\omega^2} \right) N''' \left(\frac{D}{\omega} \right) \\
& \left. + \frac{(1-q)}{\omega^2 S_0 \sigma \sqrt{t}} \left(\sqrt{t} - \frac{D_\sigma}{\omega^2} \right) N'' \left(\frac{D}{\omega} \right) - \frac{q}{\omega^2 S_0 \sigma^2 \sqrt{t}} N'' \left(\frac{D}{\omega} \right) \right]
\end{aligned} \tag{6}$$

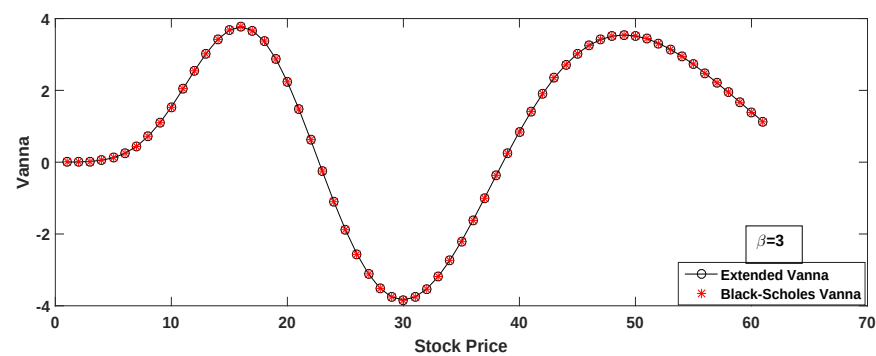


Figure 19. Extended Vanna along with Black–Scholes Vanna for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$.

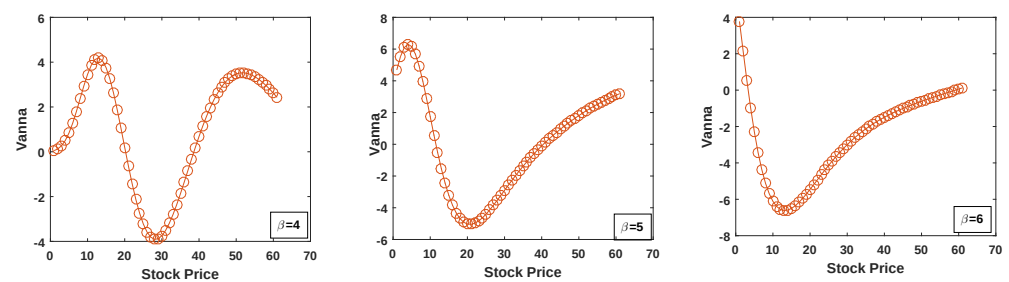


Figure 20. Option Vanna under extended normal distribution for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$ and $\beta = 4, 5, 6$.

(vii) Charm

This sensitivity depends on the underlying stock price and the passage of time for options maturity. For the computation of Charm, Delta is first computed, and then the rate of change of Delta corresponding to the time remaining for options maturity is evaluated. At the end of a weekend, this Greek observes the situation of a Delta-hedging position. Charm may also be defined as the rate of change of Delta corresponding to the time remaining to maturity. This Greek provides a more precise result when days remaining to options maturity are more in number. Figure 21 is the extended Charm along with Black–Scholes Charm for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$. Figure 22 represents the graph of call Charm under extended normal distribution for different kurtosis values. We have shown the Charm values in Table 3 for different kurtosis.

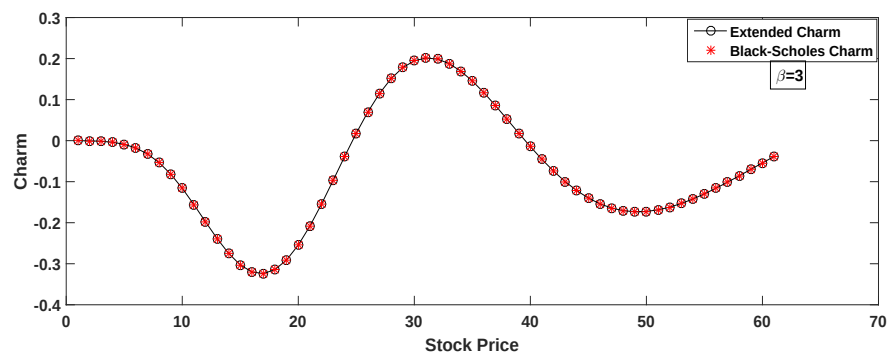


Figure 21. Extended Charm along with Black–Scholes Charm for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$.

$$\begin{aligned}
-\frac{\partial \Delta}{\partial \tau} = & -\frac{P}{P+Q} N' \left(\frac{D}{\lambda} + c\lambda \right) \left(\frac{D_t}{\lambda} + \frac{\sigma\lambda}{2\sqrt{t}} \right) - \frac{Q}{P+Q} N' \left(\frac{D}{\omega} + c\omega \right) \left(\frac{D_t}{\omega} + \frac{\sigma\omega}{2\sqrt{t}} \right) \\
& - \frac{PQ\sigma^2(\lambda^2 - \omega^2)}{2(P+Q)^2} N \left(\frac{D}{\lambda} + c\lambda \right) - \frac{PQ\sigma^2(\omega^2 - \lambda^2)}{2(P+Q)^2} N \left(\frac{D}{\omega} + c\omega \right) \\
-S_0 & \left[\frac{P}{P+Q} N'' \left(\frac{D}{\lambda} + c\lambda \right) \left(\frac{D_t}{\lambda} + \frac{\sigma\lambda}{2\sqrt{t}} \right) \frac{1}{\lambda S_0 \sigma \sqrt{t}} - \frac{P}{P+Q} N' \left(\frac{D}{\lambda} + c\lambda \right) \frac{1}{2\lambda S_0 \sigma \sqrt[3]{t}} \right. \\
& + N' \left(\frac{D}{\lambda} + c\lambda \right) \frac{PQ\sigma}{\lambda S_0 \sqrt{t}} \left\{ \frac{(\lambda^2 - \omega^2)}{2(P+Q)^2} \right\} + N' \left(\frac{D}{\lambda} + c\lambda \right) \frac{PQ\sigma}{\omega S_0 \sqrt{t}} \left\{ \frac{(\omega^2 - \lambda^2)}{2(P+Q)^2} \right\} \\
& \left. + \frac{Q}{P+Q} N'' \left(\frac{D}{\omega} + c\omega \right) \left(\frac{D_t}{\omega} + \frac{\sigma\omega}{2\sqrt{t}} \right) \frac{1}{\omega S_0 \sigma \sqrt{t}} - \frac{Q}{P+Q} N' \left(\frac{D}{\omega} + c\omega \right) \frac{1}{2\omega S_0 \sigma \sqrt[3]{t}} \right] \\
& + Ke^{-rt} \left\{ N'' \left(\frac{D}{\lambda} \right) \frac{qD_t}{\lambda^2 S_0 \sigma \sqrt{t}} - N'' \left(\frac{D}{\omega} + c\omega \right) \frac{(1-q)}{2\omega S_0 \sigma \sqrt[3]{t}} - qN' \left(\frac{D}{\lambda} \right) \frac{r}{\lambda S_0 \sigma \sqrt{t}} \right. \\
& \left. + N' \left(\frac{D}{\omega} \right) \frac{(1-q)D_t}{\omega^2 S_0 \sigma \sqrt{t}} - N' \left(\frac{D}{\lambda} + c\lambda \right) \frac{q}{2\lambda S_0 \sigma \sqrt[3]{t}} - N' \left(\frac{D}{\omega} \right) \frac{r(1-q)}{\omega S_0 \sigma \sqrt{t}} \right\} \\
& + \frac{1}{6} \alpha K cre^{-rt} \left\{ \left(c - \frac{D}{\lambda^2} \right) N'' \left(\frac{D}{\lambda} \right) \frac{q}{\lambda^2 S_0 \sigma \sqrt{t}} - N' \left(\frac{D}{\lambda} \right) \left(\frac{q}{\lambda^3 S_0 \sigma \sqrt{t}} \right) \right. \\
& \left. - N' \left(\frac{D}{\omega} \right) \frac{(1-q)}{\omega^3 S_0 \sigma \sqrt{t}} + \left(c - \frac{D}{\omega^2} \right) N'' \left(\frac{D}{\omega} \right) \frac{(1-q)}{\omega^2 S_0 \sigma \sqrt{t}} \right\} \\
& - \frac{\alpha K \sigma re^{-rt}}{12\sqrt{t}} \left\{ \left(c - \frac{D}{\lambda^2} \right) N'' \left(\frac{D}{\lambda} \right) \frac{q}{\lambda^2 S_0 \sigma \sqrt{t}} - N' \left(\frac{D}{\lambda} \right) \left(\frac{q}{\lambda^3 S_0 \sigma \sqrt{t}} \right) \right. \\
& \left. - N' \left(\frac{D}{\omega} \right) \frac{(1-q)}{\omega^3 S_0 \sigma \sqrt{t}} + \left(c - \frac{D}{\omega^2} \right) N'' \left(\frac{D}{\omega} \right) \frac{(1-q)}{\omega^2 S_0 \sigma \sqrt{t}} \right\} \\
& - \frac{1}{6} \alpha K ce^{-rt} \left\{ \left(c - \frac{D}{\lambda^2} \right) N'' \left(\frac{D}{\lambda} \right) \frac{-q}{2\lambda^2 S_0 \sigma \sqrt[3]{t}} + \left(\frac{\sigma}{\sqrt{t}} - \frac{D_t}{\lambda^2} \right) N'' \left(\frac{D}{\lambda} \right) \frac{q}{\lambda^2 S_0 \sigma \sqrt{t}} \right. \\
& + \left(c - \frac{D}{\lambda^2} \right) N''' \left(\frac{D}{\lambda} \right) \frac{qD_t}{\lambda^3 S_0 \sigma \sqrt{t}} - N'' \left(\frac{D}{\lambda} \right) \left(\frac{qD_t}{\lambda^4 S_0 \sigma \sqrt{t}} \right) \\
& + N' \left(\frac{D}{\lambda} \right) \left(\frac{q}{2\lambda^3 S_0 \sigma \sqrt[3]{t}} \right) - \left(c - \frac{D}{\omega^2} \right) N'' \left(\frac{D}{\omega} \right) \frac{(1-q)D_t}{2\omega^2 S_0 \sigma \sqrt[3]{t}} \\
& + \left(\frac{\sigma}{\sqrt{t}} - \frac{D_t}{\lambda^2} \right) N'' \left(\frac{D}{\omega} \right) \frac{(1-q)}{\omega^2 S_0 \sigma \sqrt{t}} - N'' \left(\frac{D}{\omega} \right) \frac{(1-q)D_t}{\omega^4 S_0 \sigma \sqrt{t}} \\
& \left. + \left(c - \frac{D}{\omega^2} \right) N''' \left(\frac{D}{\omega} \right) \frac{(1-q)D_t}{\omega^3 S_0 \sigma \sqrt{t}} + N' \left(\frac{D}{\omega} \right) \frac{(1-q)}{\omega^3 S_0 \sigma \sqrt[3]{t}} \right\}
\end{aligned} \tag{7}$$

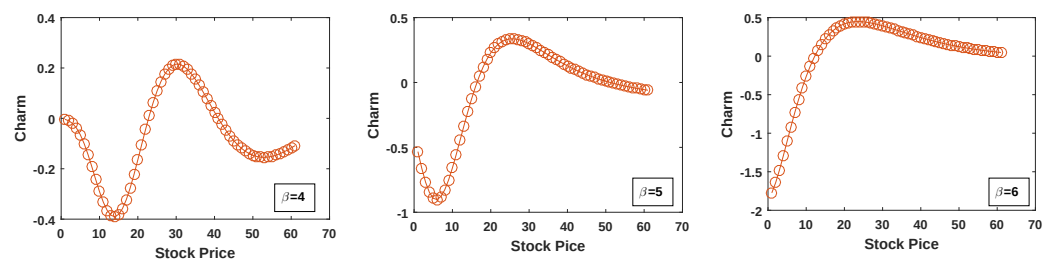


Figure 22. Option Charm under extended normal distribution for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$ and $\beta = 4, 5, 6$.

(viii) Vera

This sensitivity deals with interest rates and volatility associated with the underlying stock price. This may also be defined as the rate of change of Rho corresponding to volatility. This is generally used to know the behavior of Rho hedging as volatility fluctuates.

$$\begin{aligned}
\frac{\partial \rho}{\partial \sigma} = S_0 & \left[\frac{P}{P+Q} N'' \left(\frac{D}{\lambda} + \lambda c \right) \left(\frac{D_\sigma}{\lambda} + \lambda \sqrt{t} \right) \frac{\sqrt{t}}{\lambda \sigma} - \frac{P}{P+Q} N' \left(\frac{D}{\lambda} + \lambda c \right) \frac{\sqrt{t}}{\lambda \sigma^2} \right. \\
& + N' \left(\frac{D}{\lambda} + \lambda c \right) \frac{PQ \sqrt{t} (\lambda^2 - \omega^2)}{\lambda (P+Q)^2} + N' \left(\frac{D}{\omega} + \omega c \right) \frac{Q \sqrt{t} (\omega^2 - \lambda^2)}{\omega (P+Q)^2} \\
& \left. + \frac{P}{P+Q} N'' \left(\frac{D}{\lambda} + \lambda c \right) \left(\frac{D_\sigma}{\lambda} + \lambda \sqrt{t} \right) \frac{\sqrt{t}}{\lambda \sigma} - \frac{P}{P+Q} N' \left(\frac{D}{\lambda} + \lambda c \right) \frac{\sqrt{t}}{\lambda \sigma^2} \right] \\
& - Ke^{-rt} \left\{ N'' \left(\frac{D}{\lambda} \right) \frac{q D_\sigma \sqrt{t}}{\lambda^2 \sigma} - N' \left(\frac{D}{\lambda} \right) \frac{q \sqrt{t}}{\lambda \sigma^2} + N'' \left(\frac{D}{\omega} \right) \frac{(1-q) D_\sigma \sqrt{t}}{\omega^2 \sigma} \right. \\
& \left. + N' \left(\frac{D}{\omega} \right) \frac{(1-q) t D_\sigma}{\omega} - N' \left(\frac{D}{\omega} \right) \frac{(1-q) \sqrt{t}}{\omega \sigma^2} + N' \left(\frac{D}{\lambda} \right) \frac{q t D_\sigma}{\lambda} \right\} \\
& + \frac{K \sqrt{t} \alpha e^{-rt}}{6} \left\{ \frac{q \sqrt{t}}{\lambda^2 \sigma} \left(c - \frac{D}{\lambda^2} \right) N'' \left(\frac{D}{\lambda} \right) - \frac{q \sqrt{t}}{\lambda^3 \sigma} N' \left(\frac{D}{\lambda} \right) - \frac{(1-q) t}{\omega} \left(c - \frac{D}{\omega^2} \right) N' \left(\frac{D}{\omega} \right) \right. \\
& + \frac{q \sqrt{t}}{\omega^2 \sigma} \left(c - \frac{D}{\omega^2} \right) N'' \left(\frac{D}{\omega} \right) - \frac{q t}{\lambda} \left(c - \frac{D}{\lambda^2} \right) N' \left(\frac{D}{\lambda} \right) - \frac{(1-q) \sqrt{t}}{\omega^3 \sigma} N' \left(\frac{D}{\omega} \right) \\
& - \left(c - \frac{D}{\lambda^2} \right) N'' \left(\frac{D}{\lambda} \right) \frac{\sigma t q D_\sigma}{\lambda^2} - \left(c - \frac{D}{\omega^2} \right) N'' \left(\frac{D}{\omega} \right) \frac{\sigma t (1-q) D_\sigma}{\omega^2} \\
& - \frac{q \sigma t}{\lambda} \left(\sqrt{t} - \frac{D_\sigma}{\lambda^2} \right) N' \left(\frac{D}{\lambda} \right) - \frac{(1-q) \sigma t}{\omega} \left(\sqrt{t} - \frac{D_\sigma}{\omega^2} \right) N' \left(\frac{D}{\omega} \right) \left. \right\} \quad (8) \\
& + \frac{K c \alpha e^{-rt}}{6} \left\{ - \frac{q \sqrt{t}}{\lambda^2 \sigma^2} \left(c - \frac{D}{\lambda^2} \right) N'' \left(\frac{D}{\lambda} \right) + \frac{q \sqrt{t}}{\lambda^2 \sigma} \left(\sqrt{t} - \frac{D_\sigma}{\lambda^2} \right) N'' \left(\frac{D}{\lambda} \right) \right. \\
& + \frac{q \sqrt{t} D_\sigma}{\lambda^3 \sigma} \left(c - \frac{D}{\lambda^2} \right) N''' \left(\frac{D}{\lambda} \right) + \frac{q \sqrt{t}}{\lambda^3 \sigma^2} N' \left(\frac{D}{\lambda} \right) - \frac{(1-q) \sqrt{t} D_\sigma}{\omega^4 \sigma} N'' \left(\frac{D}{\omega} \right) \\
& - \frac{q \sqrt{t} D_\sigma}{\lambda^4 \sigma} N'' \left(\frac{D}{\lambda} \right) - \frac{(1-q) \sqrt{t}}{\omega^2 \sigma^2} \left(c - \frac{D}{\omega^2} \right) N'' \left(\frac{D}{\omega} \right) + \frac{(1-q) \sqrt{t}}{\omega^3 \sigma^2} N' \left(\frac{D}{\omega} \right) \\
& \left. + \frac{(1-q) \sqrt{t}}{\omega^2 \sigma} \left(\sqrt{t} - \frac{D_\sigma}{\omega^2} \right) N''' \left(\frac{D}{\omega} \right) + \frac{q \sqrt{t} D_\sigma}{\omega^3 \sigma} \left(c - \frac{D}{\omega^2} \right) N''' \left(\frac{D}{\omega} \right) \right\}
\end{aligned}$$

Figure 23 presents the extended Vera along with Black–Scholes Vera for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$. Figure 24 represents the graph of call Vera under extended normal distribution for different kurtosis values. We have shown the Vera values in Table 3 for different kurtosis.

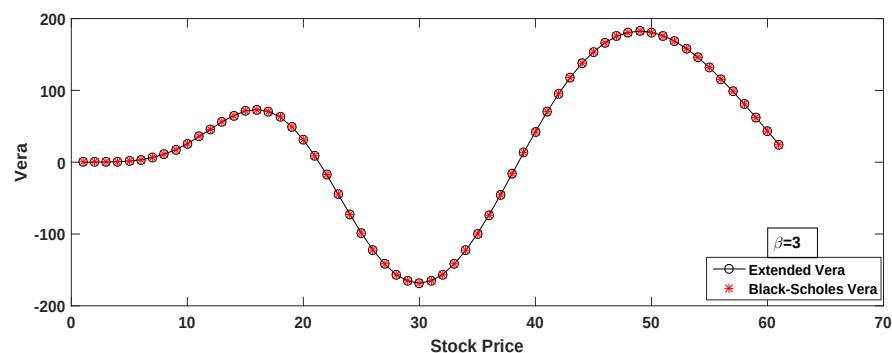


Figure 23. Extended Vera along with Black–Scholes Vera for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$.

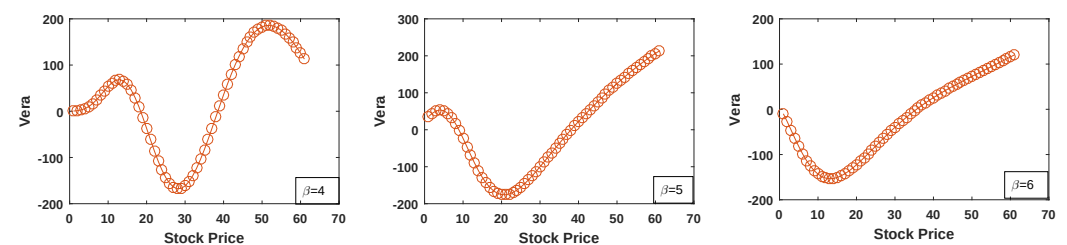


Figure 24. Option Vera under extended normal distribution for $T = 1$, $S_0 = 10$, $\sigma = 0.3$, $r = 0.05$ and $K = 50$ and $\beta = 4, 5, 6$.

Table 3. Values of Charm and Vera for kurtosis $\beta = 4, 5, 6$.

Stock Price	Charm			Vera		
	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 4$	$\beta = 5$	$\beta = 6$
51	−0.151	0.001	0.113	185.4	134.1	77.11
52	−0.154	−0.006	0.105	185.9	143.1	81.36
53	−0.154	−0.013	0.097	184.2	151.9	85.60
54	−0.153	−0.020	0.089	180.6	160.5	89.84
55	−0.150	−0.027	0.082	175.2	168.8	94.09
56	−0.146	−0.033	0.075	168.0	176.9	98.35
57	−0.140	−0.039	0.069	159.4	184.8	102.6
58	−0.133	−0.045	0.063	149.4	192.6	107.0
59	−0.126	−0.050	0.057	138.2	199.6	111.3
60	−0.117	−0.055	0.052	126.0	206.6	115.7
61	−0.108	−0.059	0.047	112.9	213.2	120.2

The plot of Delta, Gamma, Theta, Rho, Vega, Vanna, Charm, and Vera under the extended normal distribution, along with Black–Scholes Delta, Gamma, Theta, Rho, Vega, Vanna, Charm, and Vera, are shown in Figures 10, 11, 13, 15, 17, 19, 21 and 23, respectively, for $\beta = 3$. It is observable that all of the corresponding plots are in close agreement with each other. The results provided in this study are numerical simulations; however, researchers can exploit actual data (which can be expensive) by applying web-harvesting algorithms as deployed by Agrawal [28] to add empirical tests to these results.

5. Conclusions

All of the derived Greeks under the extended normal distribution differ significantly from the Black–Scholes Greeks. It is quite interesting to note that our proposed Greeks coincide with the corresponding Black–Scholes Greeks for zero skewness and a kurtosis value of three. However, as we change the kurtosis values from four to six, it is observed that all the proposed Greeks vary monotonically. In particular, Delta, Vega, and Charm increase with higher kurtosis values, while Gamma, Theta, Rho, Vanna, and Vera decrease as kurtosis increases. The extended normal distribution was chosen over the Black–Scholes model to better capture the effects of skewness and kurtosis in financial data, which are often present in real-world markets, but not accounted for in the Black–Scholes framework. By incorporating these higher moments, our model aims to provide more accurate pricing and risk management tools.

However, there are certain limitations and potential caveats to our approach. One key assumption is the stability of skewness and kurtosis over time, which may not always hold true in dynamic markets. Additionally, the sensitivity of our results to parameter changes needs careful consideration. While the extended normal distribution provides a more flexible framework, it also requires more complex estimation and calibration procedures, which can introduce additional sources of error. Understanding the differences between the derived Greeks and those from the Black–Scholes model is crucial for option pricing and risk management. The increasing Delta, Vega, and Charm with higher kurtosis suggest that options become more sensitive to changes in the underlying asset's price and volatility, potentially leading to higher hedging costs. Conversely, the decreasing Gamma, Theta, Rho, Vanna, and Vera indicate a reduced sensitivity to second-order price movements, time decay, interest rates, and other factors, which may affect portfolio adjustments and risk assessments.

These variations highlight the importance of considering higher moments in financial modeling, especially in markets characterized by fat tails and asymmetry. By understanding these differences, risk managers can better prepare for potential market shocks and adjust their strategies accordingly. In conclusion, while our research provides valuable insights into the behavior of Greeks under the extended normal distribution, it also opens avenues

for further exploration. Future work could focus on the computation of third- and higher-order Greeks, as well as the real-world applicability of our model under varying market conditions. By addressing these aspects, we can continue to refine our understanding and improve the tools available for option pricing and risk management. The computation of third- and higher-order Greeks can be taken up by researchers as future work.

Author Contributions: The manuscript was written through the contributions of all authors. G.N. was responsible for the conceptualization of the topic; article gathering and sorting were carried out by G.N., S.S.T. and A.L.I.; manuscript writing and original drafting and formal analysis were carried out by G.N., A.L.I. and C.-T.L.; writing of reviews and editing were carried out by A.L.I. and S.S.T.; and C.-T.L. led the overall research activity. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported in part by the National Science and Technology Council in Taiwan under contract no: NSTC 113-2410-H-030-077-MY2 and in part by Fu Jen Catholic University in Taiwan under contract no.: A0112014.

Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

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