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Dynamic Modeling of Limit Order Book and Market Maker Strategy Optimization Based on Markov Queue Theory

Fei Xie [1,2,3,†], Yang Liu [4,†], Changlong Hu [5] and Shenbao Liang [6,*]

[1] School of Finance, Shanghai University of Finance and Economics, Shanghai 200090, China; xiefei@mail.shufe.edu.cn
[2] Shanghai Financial Intelligent Engineering Technology Research Center, Shanghai University of Finance and Economics, Shanghai 200090, China
[3] MoE Key Laboratory of Interdisciplinary Research of Computation and Economics, Shanghai University of Finance and Economics, Shanghai 200433, China
[4] School of Mathematics, Shanghai University of Finance and Economics, Shanghai 200090, China; 2022213165@163.sufe.edu.cn
[5] School of Statistics and Data Science, Shanghai University of Finance and Economics, Shanghai 200090, China; hcl2021@163.sufe.edu.cn
[6] School of Law, Shanghai University of Finance and Economics, Shanghai 200433, China
* Correspondence: liang.shenbao@mail.sufe.edu.cn; Tel.: +86-021-65904166
† These authors contributed equally to this work.

Abstract: In recent years, high-frequency trading has become increasingly popular in financial markets, making the dynamic modeling of the limit book and the optimization of market maker strategies become key topics. However, existing studies often lacked detailed descriptions of order books and failed to fully characterize the optimal decisions of market makers in complex market environments, especially in China's A-share market. Based on Markov queue theory, this paper proposes the dynamic model of the limit order and the optimal strategy of the market maker. The model uses a state transition probability matrix to refine the market diffusion state, order generation, and trading process and incorporates indicators such as optimal quote deviation and restricted order trading probability. Then, the optimal control model is constructed and the reference strategy is derived using the Hamilton–Jacobi–Bellman (HJB) equation. Then, the key parameters are estimated using the high-frequency data of Ping An Bank for a single trading day. In the empirical aspect, the six-month high-frequency trading data of 114 representative stocks in different market states such as the bull market and bear market in China's A-share market were selected for strategy verification. The results showed that the proposed strategy had robust returns and stable profits in the bull market and that frequent capture of market fluctuations in the bear market can earn relatively high returns while maintaining 50% of the order coverage rate and 66% of the stable order winning rate. Our study used Markov queuing theory to describe the state and price dynamics of the limit order book in detail and used optimization methods to construct and solve the optimal market maker strategy. The empirical aspect broadens the empirical scope of market maker strategies in the Chinese market and studies the stability and effectiveness of market makers in different market states.

Keywords: limit order book; Markov queuing theory; optimal market-making strategy; finite difference method

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1. Introduction

With the rapid development of electronic and high-frequency trading in financial markets, high-frequency trading and market-making trading have gradually become mainstream, significantly improving market liquidity and playing an important role in price discovery. Market makers, as important participants in financial markets, provide liquidity by continuously quoting buy and sell prices, especially in low-liquidity market environments, where their role is crucial. They primarily profit from the "bid-ask spread" and balance risk and reward amidst market fluctuations.

Since the market maker system was first introduced in the U.S. over-the-counter market in the 1960s, it has gradually matured and is now prevalent in international financial markets. In contrast, China's market maker system began much later, being introduced to the "New Third Board" market only in 2013. In the realms of high-frequency trading and the market maker system, the penetration of the Chinese market is much lower than that of the mature markets in Europe and the United States. To address this, regulatory bodies in recent years have actively promoted the development of high-frequency trading, and domestic financial institutions are seeking technological and model innovations in high-frequency market maker strategies. The limit order book is the core of market maker strategies; it records unexecuted limit buy and sell orders, sorted by price and time priority. The dynamic changes in the limit order book include order submissions, cancellations, and executions, which directly reflect the market's supply and demand status and affect price fluctuations.

To be specific, the behavior of the limit order book is fundamental to understanding and optimizing market maker strategies. The flow of limit orders—particularly the order submission, cancellation, and execution events—provides key insights into market liquidity, volatility, and price discovery. By accurately modeling these dynamic changes, market makers can effectively adapt their strategies to respond to evolving market conditions, thus optimizing their profit margins while managing inventory risk.

The primary objective of market makers is to provide liquidity by maintaining tight bid–ask spreads and managing inventory risk. The dynamics of the limit order book directly impact the market maker's pricing decisions and inventory management strategies. Changes in the order book—such as price movements, order execution rates, and cancellations—inform the market maker's strategy by revealing the market's supply and demand conditions at any given time.

Moreover, the state transitions of the limit order book—such as changes in the best bid and ask prices or order volumes—play a crucial role in shaping optimal market-making strategies. Through the application of Markov queue theory, particularly with the use of state transition probability matrices, we can quantify the probabilities of state changes, which help the market maker forecast potential market movements and adjust strategies accordingly. This method enables market makers to tailor their strategies based on the current market state, thus improving liquidity provision and profitability while minimizing risks.

Therefore, accurately modeling the dynamic behavior of the limit order book is especially important for optimizing market maker strategies. A Markov chain is a stochastic process where the future state depends only on the current state and not on past states. By applying Markov chains to queuing systems, we can build a model that describes the flow of orders, where the arrival, cancellation, and execution of orders constitute the system's inputs and outputs. By constructing a state transition probability matrix, Markov queuing theory can effectively simulate the state changes of the limit order book.

This paper studies the issue of trading strategies within the limit order book in the context of market maker trading. Existing frameworks for market maker research largely rely on the stochastic control model built by Avellaneda and Stoikov (2008) [1], but this model lacks constraints on inventory management. Therefore, this paper first dynamically models the limit order book by combining the properties of the Markov chain and birth–death process, analyzes the randomness of order flow and price jump characteristics, and depicts the dynamic evolution of the order book state. Subsequently, the paper incorporates market makers' inventory management into trading decisions, constructing an optimal trading strategy model based on the Hamilton–Jacobi–Bellman (HJB) equation to address the market maker's trade-off between profit and inventory risk in high-frequency trading. When solving the strategy problem, the principles of dynamic programming transform the stochastic optimal control problem for solving the HJB equation. The paper uses the finite difference method to solve the HJB equation to obtain the optimal strategy. Lastly, the paper conducts empirical analysis using high-frequency trading data from the Chinese market to test the model's effectiveness. The back-testing analysis uses real data, simulating executions strictly according to exchange rules, to assess the strategy's performance and stability under different market conditions and verify its applicability in actual markets.

The main objectives of this paper are threefold. The first is to systematically and dynamically depict the state and price dynamics of the limit order book using the Markov queue model and birth–death process, forming an integrated system of a Markov queue model and limit order book. The second is to incorporate concepts such as the probability of limit order execution and optimal buy–sell quote deviation into existing market-making strategies to reconstruct a utility maximization strategy model based on inventory constraints. The third objective is to verify the effectiveness of market-making strategies in the Chinese A-share market through extensive empirical analysis, thus expanding the empirical scope of existing research. The innovations of this paper are mainly reflected in the following aspects:

Integration of dynamic inventory risk management and quoting strategy: Existing research has mainly focused on either inventory management or quoting strategies. This paper is the first attempt to closely integrate inventory risk with quoting strategies, deriving a dynamic quoting strategy based on the HJB equation, achieving more precise risk control and liquidity optimization.

Incorporating the birth–death process into the depiction of the limit order book: The birth–death process is widely used to describe queuing systems, but its applications in the context of high-frequency trading in limit order books are still limited. By introducing the birth–death process into the modeling of the limit order book, this paper better captures the dynamic changes in order arrivals and cancellations.

Optimization of high-frequency trading strategies specific to the A-share market: Although research on many international markets has applied Markov queuing theory and HJB models for optimization, empirical research targeting the specific high-frequency trading environment and policy constraints of the Chinese market is still lacking. Nearly all domestic research in the empirical realm has focused on a specific stock, which has limitations. This paper fills this gap by verifying the applicability and effectiveness of the model using data from the Chinese A-share market.

Analysis of strategy robustness under complex market conditions: This paper uses a large amount of high-frequency data in empirical research to test, for example, bull markets, bear markets, and fluctuating markets in the A-share market, evaluating strategy effectiveness across different market conditions. It expands the empirical scope of market-making models in the domestic market.

2. Literature Review

High-frequency trading has played a crucial role in modern financial markets, effectively improving market liquidity and price discovery efficiency through the rich short-term price information recorded in limit order books. The financial point process model proposed by Bauwens and Hautsch (2007) [2], along with microstructure research by Abergel and Jedidi (2013) [3], provides effective methods for high-frequency trading modeling. Bacry et al. (2013) [4], based on the Hawkes process price fluctuation model, and the optimal execution model by Almgren and Chriss (2000) [5], further enhance the understanding of order book dynamics. Alfonsi et al. (2010) [6] studied the optimal execution strategies for large orders by incorporating the characteristics of limit order books. Regarding the formation mechanism of limit order books, Sandås P (2001) [7] revealed the impact of order processing costs and information on the bid–ask spread through a quantity–price relationship model. Based on this, Ye Jun (2011) [8], among others, studied the intraday characteristics of limit order books and market liquidity supply. The research shows that market volatility significantly affects liquidity, and limit orders play a key role in price discovery.

Markov queuing theory is widely applied in financial markets for analyzing and optimizing queuing phenomena. Basak and Choudhury (2023) [9] analyzed a Markov system with multiple servers and a single queue, revealing the dynamic system characteristics of queues. The bivariate point process model proposed by Engle and Lunde (2003) [10] offers an effective way for high-frequency trading modeling through the joint analysis of trade and quote arrivals. The dynamic model of the limit order book proposed by Cont et al. (2010) [11] models the dynamics of order books as a multi-class queuing system, efficiently calculating the probabilities of different events.

Research on the market maker model was first proposed by Ho and Stoll (1981) [12]. They used dynamic programming to study the optimal quoting strategy of market makers when facing inventory and transaction risks. Avellaneda and Stoikov (2008) [1] introduced inventory management and transaction probability based on a stochastic control framework, constructing the optimal market-making strategy. Guéant et al. (2013) [13] and Guilbaud et al. (2013) [14] further introduced inventory constraints and market orders, allowing market makers to optimize returns under high inventory risk. Cartea et al. (2015) [15] studied market-making strategies that reduce adverse selection risk through short-term price shifts. Bressan et al. (2020) [16] extended the market-making model to the options market, verifying its robustness. In China, Song Bin et al. (2018) [17] proposed a high-frequency market-making strategy model that combines transaction risk and inventory risk. Li Zheng-wei (2019) [18] introduced algorithmic trading to reduce market impact costs while Wu Hao (2020) [19] proposed an optimization strategy for order placement based on fill probability. Chi Wen-tao (2021) [20] and Zhao Jun-jie (2022) [21] analyzed the microstructure of the Chinese market using high-frequency order book data, providing empirical support for the application of high-frequency market-making strategies. Specifically, the market-making model of Avellaneda and Stoikov assumes that asset prices follow an arithmetic Brownian motion with a standard deviation of σ, and market makers maximize terminal wealth by adjusting bid–ask spreads(Stoikov and Sağlam, 2009) [22]. Order arrival rates follow a Poisson process, and the model derives optimal quoting strategies by solving the HJB equation. However, this model does not set a limit on inventory, which may pose high inventory risks in high-frequency environments; Guéant et al. (2013) [13] incorporated inventory constraints into their model, ensuring unilateral quotes when inventory limits are reached, thus effectively controlling inventory risks. The HJB equation has been further simplified to be applicable in complex high-frequency market environments. Song Bin et al. (2018) [17] constructed a high-frequency market maker strategy model based on the limit order book, introducing the Hawkes process to improve fill intensity estimation and optimize inventory management. This model solves the HJB equation through dynamic programming and differential methods and validates the strategy's effectiveness with data from the A-share market.

Øksendal et al. (2019) [23] discussed solving optimal control problems and Herzog (2005) [24] proposed an iterative approximation algorithm for the HJB equation. Avellaneda and Stoikov (2008) [1] further simplified the solving process through Taylor expansion and Guéantet al. (2013) [13] transformed the HJB equation into a system of linear equations, making the computation of the optimal strategy more straightforward.

In summary, this paper builds a dynamic model of a limit order book based on classic models, incorporating Markov chains and birth–death processes. It adds factors such as limit order fill probability and optimal quote deviation into the market-making strategy, proposing an inventory-constrained utility maximization model. Extensive empirical data from the A-share market are used to verify the robustness and applicability of the strategy.

3. Construction of a Market Maker Model Based on Markov Processes

The limit order book is a collection of unmatched limit buy and sell orders submitted by market participants, reflecting the supply–demand relationship and price levels in the market (Cont, 2001) [25]. In the limit order book, orders at the best bid and ask prices are the most actively flowing, playing a decisive role in market price changes (Cont et al., 2014) [26]. To more accurately depict the dynamic behavior of the limit order book, this paper integrates the state model of the order book into the market-making model, focusing on the state of the first-level order book (i.e., the best bid and ask levels) (Bouchaud et al., 2009) [27].

3.1. Model Assumptions and Basic Settings

3.1.1. Order Book State

The limit order book is represented by the quantity of limit orders at the bid and ask prices (q_t^b, q_t^a), specifically as two interacting queue systems. The remaining level of the order book is considered a reservoir of limit orders, represented by the distribution of queue size at less optimal price levels. Let us define the best bid price s_t^b and the best ask price s_t^a. The queue length of orders q_t^b indicates the total number of limit buy orders at the best bid price s_t^b; q_t^a indicates the total number of limit sell orders at the best ask price s_t^a. Thus, the state of the limit order book can be expressed as a triplet: $D_t = (s_t^b, q_t^b, q_t^a)$, with the state being defined on the discrete state space $\delta\mathbb{Z} \times \mathbb{N}^2$, where $\delta\mathbb{Z}$ represents the discrete set of price levels (in integer multiples of δ) and $\mathbb{N}^2$ denotes the two-dimensional space of order queue sizes at the best bid and ask prices.

The impact of order book events on the order queues is as follows.

When a limit buy order arrives, the number of orders at the best bid price, q_t^b, increases by one unit.

When a limit sell order arrives, the number of orders at the best ask price, q_t^a, increases by one unit.

When a market sell order or the cancellation of a buy order arrives, the number of orders at the best bid price, q_t^b, decreases by one unit.

When a market buy order is placed, or a sell order is canceled, the number of orders at the best ask price, q_t^a, decreases by one unit.

If the number of orders at the best bid price, s_t^b, is exhausted, the price will decrease by one tick; if the number of orders at the best ask price, s_t^a, is exhausted, the price will increase by one tick.

When price levels change, the number of orders at the new best bid and ask prices is drawn from a distribution f defined on $\mathbb{N}^2$, which represents the reservoir of order book depth(Harris and Panchapagesan, 2005) [28]. Therefore, the dynamics of the order book state can be expressed as a Markov process on $\delta\mathbb{Z} \times \mathbb{N}^2$ where transitions depend on the occurrence of order book events.

In state D_t, when an order or cancellation arrives, the state transition follows the rules below:

If an event $T \in \{T_i^a\}_{i\geq 1}$ (seller event) arrives:

$$D_T = \begin{cases} (s_{T^-}^b, q_{T^-}^b, q_{T^-}^a + V_i^a), & \text{if } q_{T^-}^a + V_i^a > 0; \\ (s_{T^-}^b + \delta, R_i^b, R_i^a), & \text{if } q_{T^-}^a + V_i^a = 0. \end{cases} \tag{1}$$

If an event $T \in \{T_i^b\}_{i\geq 1}$ (buyer event) arrives:

$$D_T = \begin{cases} (s_{T^-}^b, q_{T^-}^b + V_i^b, q_{T^-}^a), & \text{if } q_{T^-}^b + V_i^b > 0; \\ (s_{T^-}^b - \delta, \tilde{R}_i^b, \tilde{R}_i^a), & \text{if } q_{T^-}^b + V_i^b = 0. \end{cases} \tag{2}$$

V_i^a and V_i^b are independent and identically distributed random variables (Feller, 1971) [29], taking values of 1 and −1, corresponding to an increase or decrease in orders, respectively. $R_i^b, R_i^a, \tilde{R}_i^b$, and $\tilde{R}_i^a$ are random variables drawn from distribution f, representing the quantity of new pending orders.

3.1.2. Price Dynamics

Let the stock's mid-price process P_t be an exogenously given continuous-time stochastic process, reflecting the overall price level of the market. Assume that P_t satisfies the following stochastic differential equation:

$$dP_t = \mu_p dt + \sigma_p dW_t \tag{3}$$

Here, μ_p is the drift term; σ_p is the volatility; W_t is standard Brownian motion. Assume P_t is independent of other stochastic processes in the model (Cont and De Larrard, 2013) [30]. The simplified representation of the limit order book is shown in Figure 1.

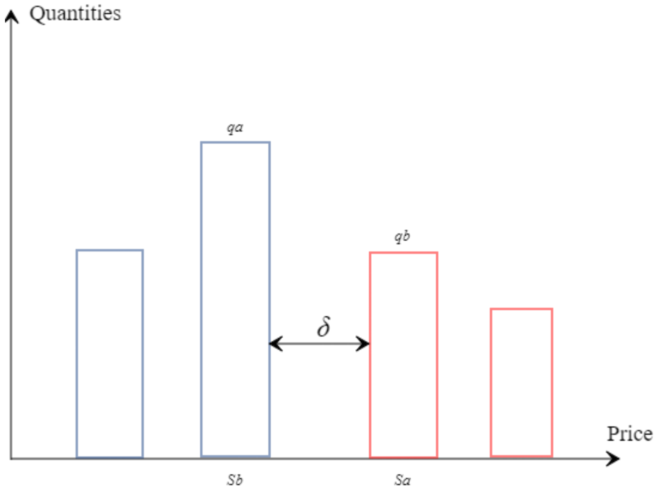

Figure 1. Simplified representation of the limit order book.

The market bid–ask spread S_t is defined as the difference between the best ask price P_t^a and the best bid price P_t^b:

$$S_t = P_t^a - P_t^b \tag{4}$$

The spread S_t is influenced by the behavior of market participants, and we model it as a continuous-time, discrete-state birth–death process, taking values as integer multiples of the minimum quotation unit δ:

$$S_t \in \{\delta, 2\delta, \ldots, m\delta\}, m \in \mathbb{N}^+ \tag{5}$$

To characterize the dynamic behavior of the spread S_t, we model it as a birth–death process. The state space of the spread process S_t is $\mathcal{S} = \{\delta_i = i\delta \mid i = 1,2,\dots,m\}$. The state transitions of S_t include the following:

Birth (spread increases): from δ_i to δ_{i+1}, with rate λ_i.

Death (spread decreases): from δ_i to δ_{i-1}, with rate μ_i.

The infinitesimal generator matrix Q is defined thus:

$$q_{ij} = \begin{cases} \lambda_i, & \text{if } j = i+1 \\ \mu_i, & \text{if } j = i-1 \\ -(\lambda_i + \mu_i), & \text{if } j = i \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

Here, q_{ij} denotes the element in the i^{th} row and j^{th} column of the matrix Q.

Given the mid-price P_t and the spread S_t, the optimal bid price P_t^b and the optimal ask price P_t^a are determined thus:

$$P_t^b = P_t - \frac{S_t}{2}, P_t^a = P_t + \frac{S_t}{2} \tag{7}$$

3.2. Dynamic Modeling of the Limit Order Book and Characterization of Execution Probability

Theorem 1. *The market maker's limit order strategy can be expressed thus:*

$$\alpha_t^{make} = (Q_t^b, Q_t^a, L_t^b, L_t^a) \tag{8}$$

Here, Q_t^b and Q_t^a are the quotes for limit buy and sell orders, respectively; L_t^b and L_t^a are the respective order quantities for limit buy and sell orders.

The optional quote sets *are* as follows:

Limit buy order quote: $Q_t^b \in \{P_t^b, P_t^{b+}\}$, where $P_t^{b+} = P_t^b + \delta$;

Limit sell order quote: $Q_t^a \in \{P_t^a, P_t^{a-}\}$, where $P_t^{a-} = P_t^a - \delta$.

The execution price for limit orders is their quote (Lawler and Limic, 2010) [31].

$$\pi^b(Q_t^b) = Q_t^b,\ \pi^a(Q_t^a) = Q_t^a. \tag{9}$$

According to the study by Chi Wen-tao (2021) [20], the execution probability of a limit order depends on the queue length of the order book and the arrival rate of market orders.

Theorem 2 (Limit Order Execution Probability Theorem)**.** *For a new order placed by a market maker at the optimal bid price, the probability of it being* executed *within time τ is as below:*

$$Probb(\tau) = 1 - exp\left(-\int_t^{t+\tau} \frac{\mu_b(s)}{q_b(s)} ds\right) \tag{10}$$

Among them, $q_b(t)$ is the total order volume at the optimal bid price and $\mu_b(t)$ is the arrival rate of market sell orders.

$q_b(t)$ and $\mu_b(t)$ are functions of time. We assume that within a small time interval Δt, $q_b(t)$ and $\mu_b(t)$ remain constant, i.e., $q_b(t) \approx q_b, \mu_b(t) \approx \mu_b$. Then, the simplified expression for the fill probability of limit orders is as below:

$$\text{Prob} = \begin{cases} 1 - \exp\left(-\frac{\mu_b}{q_b}\Delta t\right), & \text{for limit buy orders} \\ 1 - \exp\left(-\frac{\mu_a}{q_a}\Delta t\right), & \text{for limit sell orders} \end{cases} \tag{11}$$

The market order strategy of the market maker is expressed as follows:

$$\alpha^{\text{take}} = \{(\tau_n, \zeta_n)\}_{n \geq 1} \tag{12}$$

τ_n is the placement time of the nth market order; ζ_n is the order quantity, and $\zeta_n > 0$ indicates a buy order. After the execution of a market order, the changes in cash X_t and inventory Y_t are as follows:

$$\begin{aligned} Y_{\tau_n} &= Y_{\tau_n^-} + \zeta_n \\ X_{\tau_n} &= X_{\tau_n^-} - c\left(\zeta_n, P_{\tau_n}, S_{\tau_n}\right) \end{aligned} \tag{13}$$

Here, the transaction cost $c\left(\zeta_n, P_{\tau_n}, S_{\tau_n}\right) = \zeta_n P_{\tau_n} + |\zeta_n| \frac{S_{\tau_n}}{2}$.

The market order strategy is used for inventory management. When inventory deviates from the target level, it is immediately adjusted through market orders to avoid the price risk associated with excessive inventory (Fodra and Labadie, 2012) [32].

Therefore, the overall strategy of the market maker in this paper includes both a limit order strategy and a market order strategy:

$$A = \left(\alpha^{\text{make}}, \alpha^{\text{take}}\right) \tag{14}$$

The objective of the market maker is to maximize the expected utility of terminal wealth while effectively controlling inventory risk. The optimization problem is formalized thus:

$$\max_{A} \mathbb{E}\left[U(X_T) - \gamma \int_0^T g(Y_t)dt\right] \tag{15}$$

Here, $U(X_T)$ is the utility function related to terminal cash X_T; $\gamma \geq 0$ is the risk aversion coefficient; $g(Y_t)$ is a non-decreasing function related to intraday real-time inventory, typically $g(Y_t) = Y_t^2$; The integral form represents the penalty for fluctuations in intraday inventory. The constraint condition is $Y_T = 0$.

4. Optimal Market-Making Strategies and Model Solutions

4.1. Optimal Market-Making Strategies

In the utility maximization problem, the final utility is influenced by state variables, including the cash process X_t, the inventory process Y_t, the mid-price process P_t, and the spread process S_t. These variables are optimized through the order strategy α of the market maker.

The liquidation function $L(x, y, p, s)$ incorporates the constraint of clearing the end-period inventory into the utility maximization objective function. It is defined thus:

$$L(x, y, p, s) = x + yp - |y|\frac{s}{2} \tag{16}$$

Here, x represents the amount of cash held; y represents the inventory level; p represents the mid-price; s represents the market bid–ask spread. This function indicates the amount of cash a market maker holds after clearing the inventory through market orders under specific market conditions.

Thus, the utility maximization problem is written thus:

$$\max_{\alpha \in \mathcal{A}} \mathbb{E}\left[U\left(L(X_T, Y_T, P_T, S_T)\right) - \gamma \int_0^T g(Y_t)dt\right] \tag{17}$$

Here, γ represents the risk aversion factor and $g(Y_t)$ is the cost of holding inventory at time t.

The value function $v(t, z, s)$ is the maximum expected utility achievable using the optimal strategy at the initial time t and state (z, s), and is defined thus:

$$v(t, z, s) = \sup_{\alpha \in \mathcal{A}} \mathbb{E}_{t,z,s}\left[U\left(L(X_T, Y_T, P_T, S_T)\right) - \gamma \int_t^T g(Y_u)du\right] \tag{18}$$

Given that the spread process S_t only takes finite values $\mathcal{S} = \{\delta, 2\delta, \dots, m\delta\}$, the value function can be decomposed into m lower-dimensional functions $v_i(t,z) = v(t,z,s_i)$, i.e.,

$$v_i(t,z) = v(t,z,s_i),\ s_i = i\delta,\ i = 1,2,\dots,m \tag{19}$$

Thus, the value function $v_i(t,z)$ can be solved in a lower-dimensional space, $[0,T] \times \mathbb{R}^2 \times \mathcal{P}$.

4.2. Establishing the Quasi-Variational Inequality

The mid-price process P_t satisfies the following stochastic differential equation, whose generator is

$$dP_t = \mu_p dt + \sigma_p dW_t \tag{20}$$

Its generator is given thus:

$$\mathcal{L}_p\varphi = \mu_p \frac{\partial \varphi}{\partial p} + \frac{1}{2}\sigma_p^2 \frac{\partial^2 \varphi}{\partial p^2} \tag{21}$$

The spread process S_t is modeled as a birth–death process with the generator Q (Engle and Russell, 1998) [33], and the corresponding operator is as below:

$$\mathcal{R}\varphi(t,x,y,p,s_i) = \sum_{j=1}^{m} q_{ij}\left[\varphi(t,x,y,p,s_j) - \varphi(t,x,y,p,s_i)\right] \tag{22}$$

The state jump functions for limit sell/buy orders are as follows:

$$\begin{gathered}\Gamma^a(x,y,p,s,q^a,l^a) = (x - \pi^a(q^a,p,s)l^a, y + l^a)\\ \Gamma^b(x,y,p,s,q^b,l^b) = (x - \pi^b(q^b,p,s)l^b, y + l^b)\end{gathered} \tag{23}$$

Here, $\pi^a(q^a,p,s)$ and $\pi^b(q^b,p,s)$ are the transaction prices for limit buy and sell orders, respectively, and l^a and l^b represent the quantities of the corresponding limit orders.

For market orders, we define the jump function thus:

$$\Gamma^{\text{take}}(x,y,p,s,\zeta) = (x - c(\zeta,p,s), y + \zeta) \tag{24}$$

Here, ζ is the quantity of market orders (Kharroubi et al., 2010) [34].

For any limit order strategy $q = (q^b, q^a)$ and order quantities $l = (l^b, l^a)$, a second-order operator is defined thus:

$$\begin{gathered}\mathcal{L}^{q,l}\varphi(t,x,y,p,s) = \mathcal{L}_p\varphi + \mathcal{R}\varphi\\ +\text{Prob}_b(q^b,s)l^b[\varphi(\Gamma^b) - \varphi] + \text{Prob}_a(q^a,s)l^a[\varphi(\Gamma^a) - \varphi]\end{gathered} \tag{25}$$

We define the impulse operator $\mathcal{M}$ associated with market order strategies:

$$\mathcal{M}\varphi(t,x,y,p,s) = \sup_{\zeta \in [-\bar{\zeta},\bar{\zeta}]} \varphi\left(t, \Gamma^{\text{take}}(x,y,p,s,\zeta), p, s\right) \tag{26}$$

Based on the definitions above, the quasi-variational(QVI) inequality corresponding to the utility maximization problem can be written thus:

$$\min\left\{-\frac{\partial v_i}{\partial t} - \mathcal{L}_p v_i - \mathcal{R}v_i - H_i(t,x,y,p), v_i(t,x,y,p) - \mathcal{M}v_i(t,x,y,p)\right\} = 0 \tag{27}$$

Here, $H_i(t,x,y,p)$ is the Hamiltonian, defined thus (Cohen and Boxma, 2000) [35]:

$$\begin{gathered}H_i(t,x,y,p) = \sup_{(q^b,l^b)} \text{Prob}_b(q^b,s_i)l^b[v_i(\Gamma^b) - v_i]\\ + \sup_{(q^a,l^a)} \text{Prob}_a(q^a,s_i)l^a[v_i(\Gamma^a) - v_i] - \gamma g(y)\end{gathered} \tag{28}$$

This inequality considers the impact of limit order strategies (HJB part) (Bayraktar and Ludkovski, 2014) [36] and market order strategies (impulse control part). The terminal condition of the value function is as follows:

$$v_i(T, x, y, p) = U(L_i(x, y, p)) \quad (29)$$

Here, $L_i(x, y, p) = x + yp - |y|\frac{s_i}{2}$.

As discussed by Øksendal et al.(2019) [23], the value function v is a viscosity solution of this inequality, and Kharroubi et al. have proven the uniqueness of the viscosity solution. This allows for the use of numerical methods to solve the inequality.

4.3. Numerical Solution—Finite Difference Method

This section employs the finite difference method to numerically solve the quasi-variational inequality established in the previous section. First, we divide the time interval $[0, T]$ into n time steps (Robert and Rosenbaum, 2011) [37], with a time step size of

$$h = \frac{T}{n},\ t_k = kh,\ k = 0,1,2, \ldots, n. \quad (30)$$

We construct a grid for the inventory y within a reasonable inventory range $[Y_{\min}, Y_{\max}]$, with a step size Δy (Ait-Sahalia et al., 2005) [38]:

$$y_j = Y_{\min} + j\Delta y,\ j = 0,1, \ldots, M \quad (31)$$

The spread values s_i are discretized into finite values:

$$s_i = i\delta,\ i = 1,2, \ldots, m \quad (32)$$

Assuming the utility function is linear and market makers do not predict price trends, the value function at each grid point (t_k, y_j, s_i) can be approximated using the finite difference method. The finite difference operators for the first and second derivatives are applied to discretize the differential operators involved in the QVI.

For a detailed description of the numerical implementation, see Appendix A.

5. Parameter Estimation and Empirical Analysis

In this section, we will first select data from one trading day of a stock to estimate key parameters. After completing the parameter estimation, we will conduct empirical and back-test analyses on 114 representative stocks from the Chinese A-share market under different market conditions.

5.1. Data Description and Preprocessing

The period from 1 December 2022 to 1 March 2023 was selected as the bull market and the period from 13 June 2024 to 13 September 2024 was selected as the bear market. In addition, in order to ensure the generality and representativeness of the analysis, we selected stocks from the CSI 300 and CSI 500 indexes, and according to our hypothesis, the one-month sample data of these component stocks in the non-empirical period (13 May to 12 June 2024) were tested for the stability of the mid-price series, the volatility test of the spread process following the birth and death process, and the Markov process test. All the stocks that passed the hypothesis were selected, and 114 stocks were finally obtained as empirical samples according to the results.

(1) Price Dynamics—ADF Test (Stationarity Test): The model assumes a stationary process suitable for Markov modeling. Augmented Dickey–Fuller (ADF) was used to test the existence of unit root in the series to verify the stationarity of the intermediate price series of each stock, where the null hypothesis meant the series had a unit root (non-stationary) and the alternative hypothesis meant the series was stationary. The ADF test statistic t and its p-value indicate stationarity, with a p-value < 0.05 suggesting a rejection of the unit root (stationarity).

(2) Spread Dynamics—Chi-Square Test for Birth–Death Process: This test assesses whether spread changes follow a birth–death process by examining transitions between

spread states (increase, decrease, and no change). The spread is discretized into integer multiples of the tick size (0.01), and transition frequencies for up, down, and no change states are calculated. A chi-square test is applied to test if observed transitions match the expected birth–death transition rates:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} \tag{33}$$

Here, O_i and E_i are observed and expected frequencies. A *p*-value > 0.05 indicates the spread process aligns with birth–death dynamics.

(3) Order Book Transition Counts: This test verifies whether bid and ask volumes change in small, frequent adjustments as expected in a Markov process. The percentage changes in BidVolume1 and AskVolume1 are computed, and the frequency of changes within ±10% is calculated. If over 70% of changes fall within the ±10% range, it suggests that the volume transitions align with a Markov-like behavior of small, frequent moves.

The data in this paper have been obtained from the DolphinDB database, which contains tick-level data, updated every 3 seconds, and stock price information.

The main fields of the data include the following: TradingDay (Trading Date), InstrumentID (Stock Code), LastPrice (Latest Transaction Price), OpenPrice, HighestPrice, LowestPrice, ClosePrice (Opening Price, Highest Price, Lowest Price, and Closing Price of the day), Volume (Trading Volume), Turnover (Transaction Amount), and Timestamp (Timestamp of Data Record accurate to the second). The order book information includes BidPrice1-BidPriceA (bids from one to ten), BidVolume1-BidVolumeA (order volumes for bids one to ten), AskPrice1-AskPriceA (ask prices from one to ten), and AskVolume1-AskVolumeA (order volumes for asks one to ten).

Before analyzing the data and estimating the parameters, the data were cleaned through missing value handling and outlier processing, and non-trading period data was filtered out, retaining only the trading hours from 9:30 to 11:30 and from 13:00 to 15:00. In the parameter estimation part, we selected the trading data of Ping An Bank (000001.SZ) on 1 April 2020.

Figure 2 shows the tick-level data of Ping An Bank on 1 April 2020 after data cleaning. It can be observed that one stock had approximately 4757 rows of data per trading day. For subsequent research, derivative variables such as the midpoint price, market spread, and quote deviation were calculated.

	TradingDay	InstrumentID	LastPrice	PreClosePrice	OpenPrice	HighestPrice	LowestPrice	Volume	Turnover	AskVolume1	AskPrice1	BidVolume1	BidPrice1	Timestamp
0	2020-04-01	1	12.87	12.8	12.86	12.87	12.86	465,800	5.99 × 10^6	39,014	12.87	200	12.86	2020-04-01 09:30:00
1	2020-04-01	1	12.83	12.8	12.86	12.87	12.83	590,100	7.59 × 10^6	1,800	12.85	4,400	12.83	2020-04-01 09:30:03
2	2020-04-01	1	12.85	12.8	12.86	12.87	12.82	656,200	8.44 × 10^6	66,920	12.86	600	12.85	2020-04-01 09:30:06
3	2020-04-01	1	12.86	12.8	12.86	12.87	12.82	682,300	8.77 × 10^6	53,720	12.86	2,400	12.85	2020-04-01 09:30:09
4	2020-04-01	1	12.86	12.8	12.86	12.87	12.82	719,400	9.25 × 10^6	23,420	12.86	100	12.85	2020-04-01 09:30:12
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4752	2020-04-01	1	12.89	12.8	12.86	13.13	12.82	51,637,175	6.70 × 10^8	422,929	12.89	422,929	12.89	2020-04-01 14:59:15
4753	2020-04-01	1	12.89	12.8	12.86	13.13	12.82	51,637,175	6.70 × 10^8	425,029	12.89	425,029	12.89	2020-04-01 14:59:24
4754	2020-04-01	1	12.89	12.8	12.86	13.13	12.82	51,637,175	6.70 × 10^8	426,429	12.89	426,429	12.89	2020-04-01 14:59:33
4755	2020-04-01	1	12.89	12.8	12.86	13.13	12.82	51,637,175	6.70 × 10^8	427,129	12.89	427,129	12.89	2020-04-01 14:59:42
4756	2020-04-01	1	12.89	12.8	12.86	13.13	12.82	51,637,175	6.70 × 10^8	436,229	12.89	436,229	12.89	2020-04-01 14:59:51

4757 rows × 14 columns

Figure 2. Tick data of a single stock on a single trading day.

Figure 3 illustrates the variation curves of stock prices and trading volumes over time for the morning, along with changes in order book depth over time, offering an overview of market trading behavior during this period. It is evident that most spreads clustered around 0.01 CNY. The majority of values oscillated between 0.01 and 0.03 CNY. For subsequent parameter estimation and empirical testing, we will concentrate solely on fluctuations within this spread range, excluding spreads exceeding 0.05 CNY to streamline the calculation process, as this simplification will not materially affect the effectiveness of the strategy.

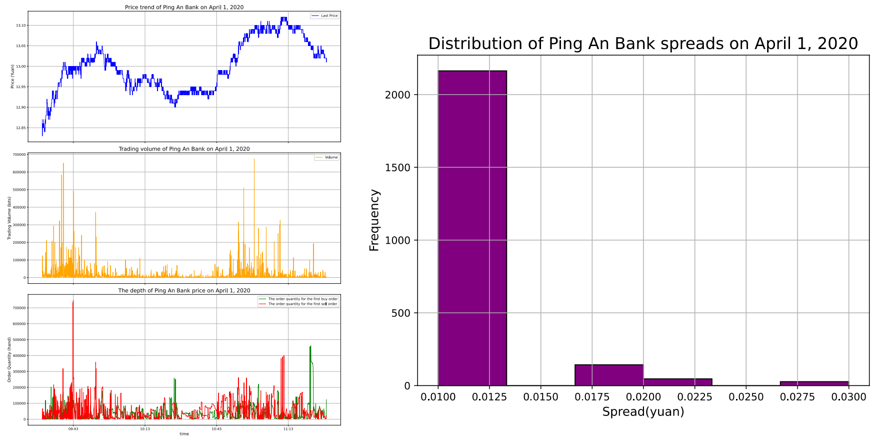

Figure 3. Stock price, volume change, and spread distribution.

5.2. Parameter Estimation

This section estimates parameters based on high-frequency trading data, focusing on the market spread fluctuation process, the probability of limit order execution, and the execution intensity in the model.

5.2.1. Market Spread Process Parameter Estimation

In the actual market, the spread typically changes discretely with a minimum price movement unit δ as the step length. This paper sets the possible values of the market spread as follows:

$$s_i \in \{0.01, 0.02, 0.03, 0.04, 0.05\} \tag{34}$$

We extract the high-frequency spread series and identify the change moments θ_n, recording the spread state s_i at each moment. The number of transitions between spread states is statistically counted, and a count matrix N_{ij} is constructed, where N_{ij} indicates the number of times the spread transitions from state s_i to state s_j. We calculate the transition probability matrix $\hat{\rho}$:

$$\hat{\rho}_{ij} = \frac{N_{ij}}{N_i},\ \forall i, j \tag{35}$$

Here, $N_i = \sum_j N_{ij}$ is the total occurrence count of state s_i. The actual data-derived spread state count N_i is as follows:

$$N_i = \begin{cases} 0, & \text{when } s_i = 0.00 \\ 163, & \text{when } s_i = 0.01 \\ 165, & \text{when } s_i = 0.02 \\ 22, & \text{when } s_i = 0.03 \\ 1, & \text{when } s_i = 0.05 \end{cases} \tag{36}$$

It can be observed from N_i that the market spread mostly remains between 0.01 and 0.02 CNY, indicating an active market and a relatively similar pricing perception between buyers and sellers.

5.2.2. The State Transition Probability Matrix Estimation

The state transition probability matrix ρ_{ij} for the dynamic changes in market price spread represents the probability of the spread transitioning from state $i\delta$ to state δ. The estimation of ρ_{ij} using actual data is as follows:

$$\hat{\rho}_{ij} = \frac{\sum_{n=1}^{K} 1_{\{\hat{S}_{n+1}=j\delta, \hat{S}_n=i\delta\}}}{\sum_{n=1}^{K} 1_{\{\hat{S}_n=i\delta\}}} \quad (37)$$

The spread transition matrix computed from the actual data is as below:

$$\hat{\rho} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.0061 & 0 & 0.9141 & 0.0798 & 0 \\ 0 & 0.9394 & 0 & 0.0545 & 0.0061 \\ 0 & 0.3182 & 0.6818 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 \end{pmatrix} \quad (38)$$

The results show that the market price spread primarily fluctuates between 0.01 and 0.02 CNY. The probability of the spread transitioning from 0.01 to 0.02 CNY is 91.41% while the probability of transitioning back from 0.02 to 0.01 CNY is 93.94%. This reflects the liquidity characteristics of the market and the consistency in price expectations between buyers and sellers.

5.2.3. Intensity of Price Spread Changes and Parameter Estimation

The intensity $\lambda(t)$ of the price spread change process reflects the frequency of spread changes. Assuming that $\lambda(t)$ has a fixed intraday periodicity on trading days, the trading day is divided into different time periods and the intensity for each period is estimated. The number of price spread changes is as below:

$$M_{t_{k+1}} - M_{t_k} \quad (39)$$

The estimated intensity value for each time period is as below:

$$\hat{\lambda}_k = \frac{M_{t_{k+1}} - M_{t_k}}{t_{k+1} - t_k}$$

Taking the execution intensity of limit buy orders λ^b as an example, assume that the execution processes N^a and N^b follow a Poisson process, and the execution intensity $\lambda(t)$ also exhibits intraday periodicity. Different time periods within the trading day are divided (for example, 10:00–11:00, 11:00–11:30, 13:00–14:00, and 14:00–14:57), and the number of executions N and the duration of the state τ in each period are recorded. The estimated execution intensity is as below:

$$\hat{\lambda}_n = \frac{N_{t_{n+1}} - N_{t_n}}{t_{n+1} - t_n} \quad (40)$$

The limit buy order execution intensity estimated from sample data satisfies $\lambda^b(P_t^b, s) < \lambda^b(P_t^{b+}, s)$, indicating that the probability of execution for limit buy orders with higher bid prices is greater than for those with lower bid prices. Figures 4 and 5 show the estimated execution intensities under different spreads and time periods, conforming to the “price priority, time priority” principle and confirming that the execution probability for higher-priced buy orders is higher.

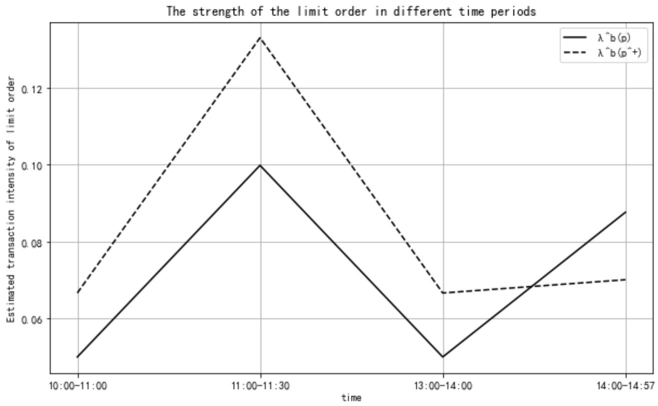

Figure 4. Execution intensity of limit buy orders in different time periods.

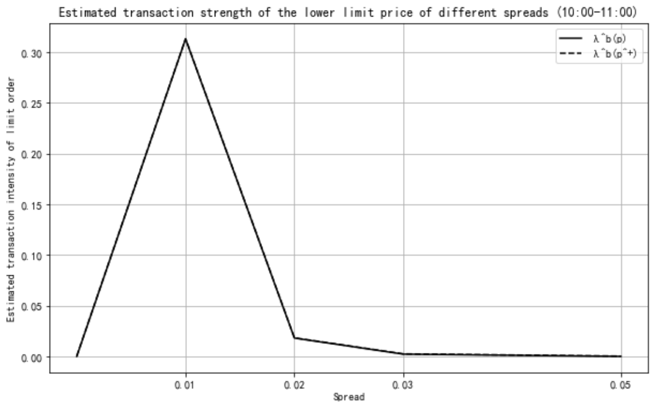

Figure 5. Execution intensity for different spreads in the same time period.

Tables 1 and 2 record the specific values of two intensities. It can be seen from Figure 5 that the transaction intensity λ^b also satisfies the feature of being a non-increasing function with respect to the market spread S; in other words, the larger the spread is, the lower the probability of a limit order being executed will be.

Table 1. Transaction intensity of limit orders in different time periods.

Time Period	$\boldsymbol{\lambda}$^b (p)	$\boldsymbol{\lambda}$^b (p^ +)
09:30–10:30	0.062552	0.062481
10:30–11:30	0.124792	0.124651
13:00–14:00	0.062552	0.062481
14:00–14:57	0.087719	0.065789

Table 2. Execution intensity under different spreads.

Spread	$\boldsymbol{\lambda}$^b(p)	$\boldsymbol{\lambda}$^b(p^ +)
0.01	0.312778	0.313056
0.02	0.180556	0.183333
0.03	0.022222	0.025000
0.05	0.000000	0.002778

5.2.4. One-Dimensional Exponential Hawkes Process Estimation

To accurately describe the impact of historical events on future predictions, this paper uses a one-dimensional exponential Hawkes process to model the execution intensity of limit orders. The key to the Hawkes process is the consideration of the extent of historical events' impacts on the intensity of future occurrences, with its effect decaying exponentially over time. The influence function $h(s)$ in the model is expressed thus:

$$h(s) = \sum_{k=1}^{K} \alpha_k e^{-\beta_k s} \tag{41}$$

The expression for transaction intensity is as below:

$$\lambda^b(t) = \mu + \sum_{k=1}^{K} \int_{u<t} \alpha_k e^{-\beta_k (t-u)} dN_u^b \tag{42}$$

To simplify the estimation process, assume $K = 1$.The model is then simplified to the following:

$$\lambda^b(t) = \mu + \int_{u<t} \alpha e^{-\beta(t-u)} dN_u^b \tag{43}$$

The further processing of the time discretization yields the following:

$$\lambda^b(t) = \mu + \sum_{i<t} \alpha e^{-\beta(t-i)} N_{(i)}^b \tag{44}$$

In this model, within each time interval, the count of limit buy order execution events $N_{(i)}^b$ is a Poisson-distributed random variable with intensity $\lambda^b(i)$. Assuming $N_{(i)}^b$ values are independent of each other, the probability of execution events is as below:

$$P\{N_{(i)}^b = k\} = \frac{\left(\lambda^b(i)\right)^k}{k!} e^{-\lambda^b(i)} \tag{45}$$

We use the maximum likelihood method to estimate the parameters, α, β. The likelihood function L is expressed thus:

$$L = -\sum_{t=1}^{T} \lambda^b(t) + \sum_{t=1}^{T} n_{(t)} \log \lambda^b(t) \tag{46}$$

Thus, the following parameter estimation problem is obtained:

$$\begin{gathered} \max \ L \\ \mu > 0, \alpha > 0 \end{gathered} \tag{47}$$

Based on the observed data, the parameter values estimated using the maximum likelihood method are as follows:

$$\mu = 0.03, \alpha = 0.125, \beta = 0.011 \tag{48}$$

The estimation results Table 3 indicate that during the morning trading session (especially 09:30–10:00), the intensity of bid–ask spread changes is higher, showing a characteristic of an active market with intense competition between buyers and sellers. As the trading session progresses, market activity gradually declines, particularly in the afternoon, with a decrease in the intensity of spread changes. Based on these volatility characteristics, market makers can increase the frequency of order placements during active periods while adopting a more conservative strategy when volatility is lower.

Table 3. Changes at different time periods for Ping An Bank on 1 April 2020.

Time Period	Number of Spread Changes	Intensity of Spread Changes λ
$09:30-10:00$	147	0.082
$10:00-11:00$	104	0.029
$11:00-11:30$	56	0.031
$13:00-14:00$	62	0.017
$14:00-14:57$	72	0.02

Intensity of bid–ask spread. λ Changes at different time periods for Ping An Bank on 1 April 2020.

The estimated parameters $\mu, \alpha, and\ \beta$ reflect the dynamic changes in market trading. The base intensity μ represents the fundamental market fluctuations in the absence of historical event influences while α indicates the impact of historical events on future trading intensity and β controls the decay rate of this influence. The elements of the generator matrix $\hat{Q} = [\hat{q}_{ij}]$, denoted as $\hat{q}_{ij}$, represent the rate at which the price spread transitions from state s_i to state s_j within unit time, where

$$\hat{q}_{ij} = \hat{\lambda}_{\hat{\rho}_{ij}}, i \neq j. \tag{49}$$

$$\hat{q}_{ii} = -\sum_{j \neq i} \hat{q}_{ij} = -\hat{\lambda}(1 - \hat{\rho}_{ii})$$

The calculated generator matrix $\hat{Q}$ is as below:

$$\hat{Q} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.000133 & -0.0217 & 0.019798 & 0.001733 & 0 \\ 0 & 0.020402 & -0.0217 & 0.001183 & 0.000132 \\ 0 & 0.006889 & 0.014811 & -0.0217 & 0 \\ 0 & 0 & 0.0217 & 0 & -0.0217 \end{pmatrix} \tag{50}$$

5.3. Empirical Analysis

5.3.1. Strategy Validation

This section conducts an empirical test of the market-making strategy model using high-frequency trading data from the A-share market. The continuous auction period is discretized into time steps of 15 seconds each, with a total of $n = 828$ steps within a trading day. Assuming short selling is allowed, the inventory limits are set at ± 100 lots and the inventory grid is divided into 201 nodes. We focus on price spreads from 0.01 CNY to 0.05 CNY and perform strategy differentiation under different spread conditions (Biais et al., 1995) [39].

The empirical strategy in this paper is based on a market-making model, aiming to achieve short-term profits by controlling inventory levels and utilizing spread fluctuations. The following provides detailed explanations of key parameter settings and strategy execution.

- Initial Fund: 100,000 CNY.
- Transaction Cost:
  - Buying Cost: Set at 0.0001 (0.01%), representing the stamp duty on buy orders.
  - Selling Cost: Set at 0.0011, including 0.01% stamp duty and 0.1% commission.
- Inventory Threshold: The position limit is determined by the stock's initial price.
- Spread Threshold: Set at 0.2% of the stock price. When inventory exceeds the threshold, the strategy executes market orders based on the spread to manage the position. Additionally, narrow spreads allow larger market orders for more efficient inventory adjustment. For limit orders, if the spread is within the threshold range, the order is placed at the current best bid or ask price. If the spread exceeds the threshold, the limit order price is incrementally adjusted to capture potential profits from a wider spread (Hollifield et al., 2004) [40].
- End-of-Day Clearance: At the end of each trading day, all positions are liquidated using the closing price to avoid overnight risk.
- Return and Performance Calculation:
  - Profit: The total trading profit, calculated as ending capital minus initial capital.
  - Winning Rate: The proportion of profitable trades to total trades.

- Stock Return and Strategy Return: This measures the stock price change and overall strategy return to evaluate the relative performance of the strategy.

A back test was conducted using the high-frequency data of Ping An Bank from the Shenzhen Stock Exchange on 1 April 2020. The model generated 106 buy orders and 102 sell orders, with frequent fluctuations in inventory levels and cash positions during the trading session, reflecting the strategy's activity and position management effectiveness. Despite fluctuations in cash and inventory, the overall profit trend was upward, indicating that the strategy was generally profitable.

Figure 6 presents the back-test results for Ping An Bank, specifically during the morning trading session on 1 April 2020. An analysis was then performed for the time period from 10:00 to 11:00. The performance of strategies under different inventory levels and prices spreads within a period (Table 4). For positive inventory, there are more sell orders when the price spread is smaller, indicating that investors are inclined to lock in profits at smaller spreads; when the spread is larger, the number of orders decreases, and trading becomes more cautious. With negative inventory, the frequency of buy orders is higher, suggesting that investors are more inclined to adjust their positions at that spread.

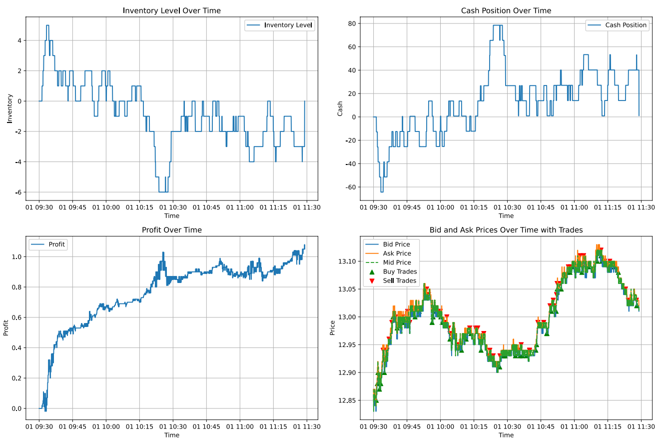

Figure 6. Transaction results of Ping An Bank on 1 April 2020.

From Figure 7, it can be seen that when the inventory was positive and the price spread was small, sell orders were more active and profits gradually increased. This reflected the behavior of investors actively closing positions to capture profits in a narrow spread environment. As the spread widened, both trading frequency and profit volatility declined, indicating that investors adopted a more cautious trading strategy with a larger spread. When the inventory was negative, the frequency of buy orders increased, reflecting investors' tendency to reduce negative positions through buy orders. At this stage, market spreads had a smaller impact on trading, but profits showed a downward trend, indicating the cost associated with adjusting negative positions. Market orders prioritize selling to reduce holdings when inventory is high and prioritize buying to replenish inventory when it is low. For limit orders, when the spread is narrow, orders are placed at the current best bid or ask price; when the spread is wide, the order price is increased by one spread unit, enhancing potential profits but lowering the probability of execution.

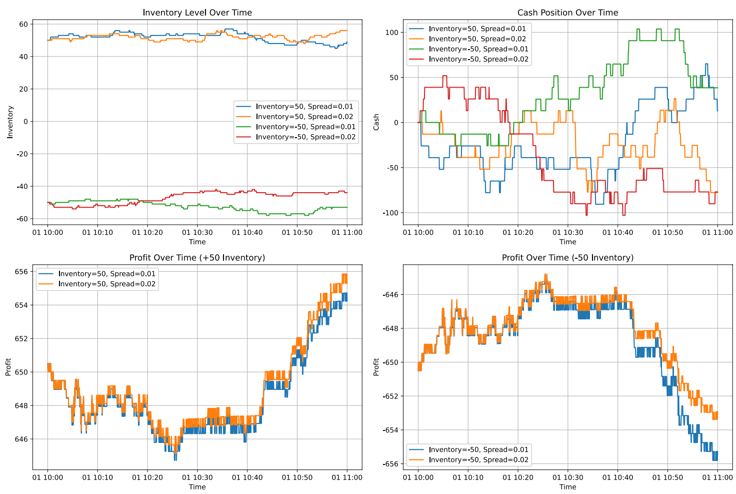

Figure 7. Results under different inventories and spreads (10:00–11:00).

Table 4. Number of order placements under different inventories and spreads (10:00–11:00).

Inventory	Spread	Buy Order Count	Sell Order Count
$y = 50$	$s = 0.01$	24	36
$y = 50$	$s = 0.02$	21	26
$y = -50$	$s = 0.01$	25	28
$y = -50$	$s = 0.02$	25	26

5.3.2. Market-Wide Analysis

This part verifies the performance and stability of the proposed market-making strategy under different market conditions. During the actual order placing operation, the market price check and the real-time inventory holding during the continuous bidding period are observed. Whenever the market price difference changes, the order placing strategy corresponding to the corresponding price difference and inventory volume in the corresponding time node is selected to entrust the order placing. At the closing time of each trading day, all the inventory is entrusted to sell with the market price order.

Let us assume that the market order of each trading day is immediately completed and the market buy and sell are completed at the current buy/sell price. The inventory volume at the end of each trading day is all settled at the middle price and all cleared; the limit order is a unilateral transaction, that is, when the limit buy order is greater than the limit sell order, the limit sell order is completed, and vice versa. Tables A1 and A2 in Appendix B present the trading of the strategy in bull and bear markets.

We chose TCL Technology for analysis (as shown in Figure 8). In these two figures, the upper left corner shows the fluctuation of the inventory, which fluctuated around 0, and the inventory clearing operation was carried out at the end of each trading day to avoid the inventory risk. The upper right corner shows the accumulation of cash and the lower left corner shows the accumulation of income. It can be seen that no matter whether it was the bull market or the bear market, the income was always accumulated, with a small retreat. The lower right corner shows the trend of the stock on the same day. The red arrow and the green arrow represent the put–sell operation and put–buy operation, respectively. It can be seen that the market maker strategy provided liquidity for the market according to the trend of the stock and looked for the opportunity to earn the spread.

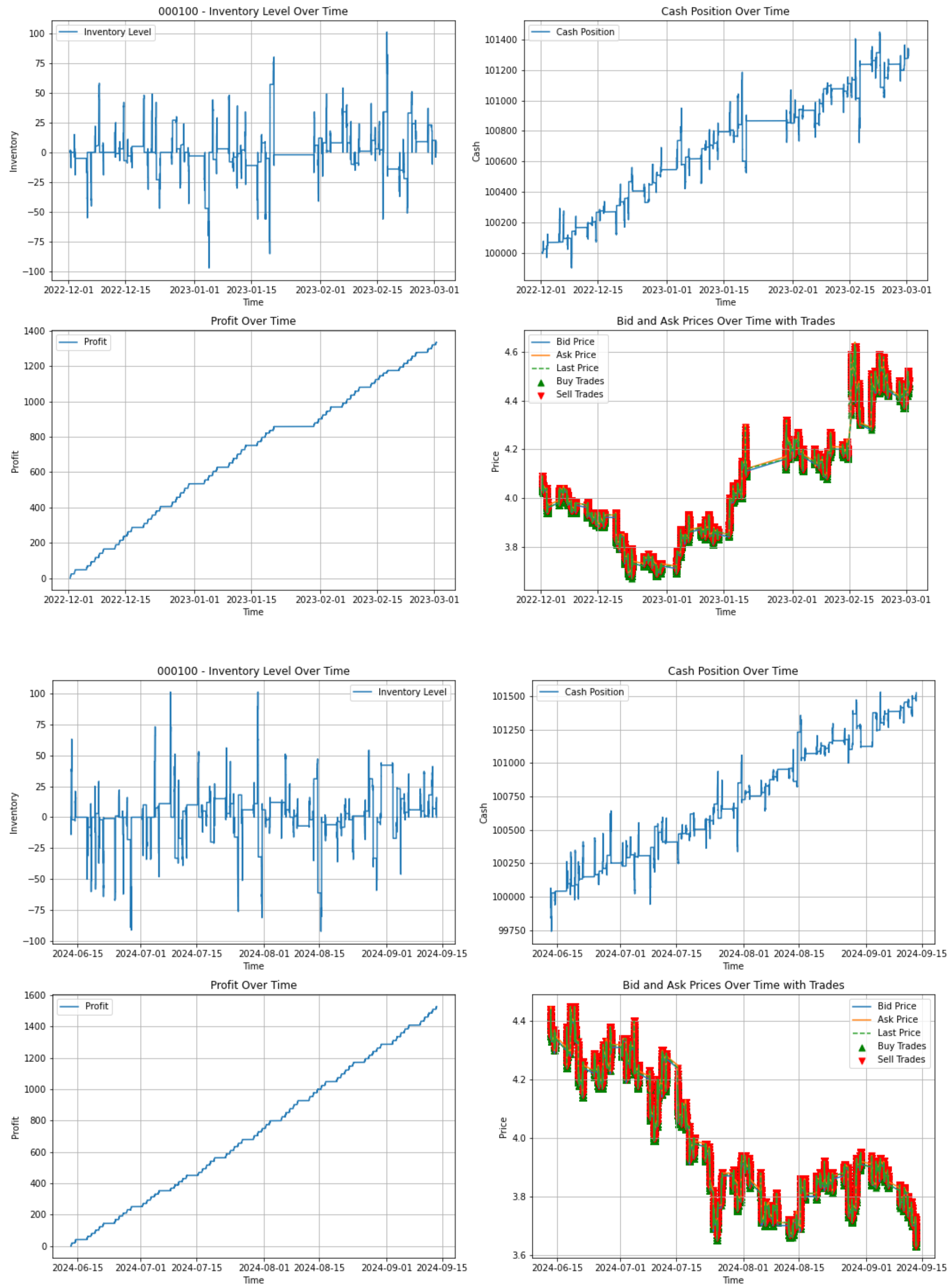

Figure 8. Trading results of TCL Technology under different market conditions (bull market results are in the upper subfigures; bear market results are in the lower subfigures).

The results showed that in bear markets, inventory fluctuations were large, trading frequency was high, and profit volatility was significant, reflecting high market uncertainty. On the other hand, in bull markets, inventory fluctuations were smaller and the changes in trading and profits were smoother. Despite significant differences in market environments, the strategy exhibited a good profit trend in both states, further verifying the robustness and adaptability of the strategy.

Table 5 extracts the strategy results of some samples. It can be seen that the number of limit orders was very frequent, and some market orders were used as supplements to risk control. Such a strategy can achieve stable and effective returns under different market conditions. Especially in the state of the market being down, traders gain a stable income, indicating that the strategy is relatively strong risk resistance. From the winning rate column, you can see that the winning rate of each order can reach about 43%. For other sample stocks (we have shared the strategy output results of these sample stocks in Appendix C and Appendix D, where we have stored the strategy results of the bull and bear markets, respectively), the results showed similar trends, indicating the effectiveness and stability of the strategy under different A-share market conditions. This empirical analysis fills the gap of market-making strategy in China's A-share market.

Table 5. Results of some stock market maker strategies.

Instrument ID	Market State	Buy/Sell Trades	Market Orders	Initial Price	Final Price	Profit	Winning Rate	Stock Return	Strategy Return
000958	Bull	237,321	12	4.44	4.54	917.5	43.82%	2.25%	0.92%
	Bear	262,814	8	3.9	3.26	1362.7	43.75%	−16.41%	1.36%
000166	Bull	269,345	0	4.17	4.25	1306.2	43.06%	1.92%	1.31%
	Bear	310,724	0	4.43	4.23	1384.0	42.39%	−4.51%	1.38%
000598	Bull	234,913	1	5.1	5.15	897.0	43.77%	0.98%	0.90%
	Bear	300,703	35	7.72	6.36	−83.1	41.43%	−17.62%	−0.08%
600008	Bull	278,528	0	2.91	2.98	1792.4	42.75%	2.41%	1.79%
	Bear	302,640	5	2.79	2.79	1944.5	43.17%	0.00%	1.94%
600959	Bull	269,072	2	3.08	3.17	1653.6	44.35%	2.92%	1.65%
	Bear	269,288	10	2.78	2.73	1716.6	44.27%	−1.80%	1.72%

Results of some stock market maker strategies.

We conducted a comparative analysis with existing research, particularly with the study by Song Bin (2018) [17]. Song's research was based on simulated trading for a single stock over one trading day, and it concluded the following similar findings:

- Intraday Inventory Fluctuations: Song's study showed that intraday inventory fluctuates around zero, which helps mitigate the risks brought by market price movements. Our simulated trading also showed similar results, with the market maker's inventory fluctuating around zero, effectively balancing the risks associated with price volatility. This further validates the strategy's effectiveness in managing intraday inventory.
- Profitability of the Strategy: Song pointed out that placing orders using the strategy could yield considerable profits by the end of the trading day. Our empirical analysis also showed that the strategy performed steadily in different market conditions, particularly in the bull and bear markets, achieving stable profits by the end of the trading day.

In contrast to Song's research, our study expanded the stock sample and market states. Our sample consisted of 114 representative stocks, and the market states were divided into bull and bear markets, with detailed comparisons made. The simulation trading intervals for each market state spanned three months. Through these expansions and validations, we have demonstrated the effectiveness of the optimization strategy across a broader stock sample and different market states, further confirming Song Bin's conclusion that the strategy is applicable to the vast majority of A-share stocks.

6. Conclusions

This paper has proposed a dynamic modeling framework for limit order books based on Markov queuing theory and conducted the optimization and empirical analysis of the optimal strategy for market makers. Through rigorous model construction, optimization algorithm design, and multidimensional data validation, it has been proven that the market-making strategy demonstrates robustness and performance across varying market conditions in the Chinese A-share market. The paper's results offer new theoretical perspectives and empirical support for the design of high-frequency market-making strategies.

Specifically, this paper has built a framework for describing the dynamics of market spreads based on Markov queuing theory. By deeply depicting spread states, price dynamics, order generation, and the execution process, a state transition probability matrix that accurately describes the evolution of spreads has been proposed. Subsequently, based on the birth–death process, the intensity of spread changes has been defined. By introducing the price deviation of buy and sell orders and the execution probability of limit orders, the model's granularity and accuracy have been further enhanced. Regarding the optimization problem of the optimal market-making strategy, an optimal control problem aimed at utility maximization has been constructed. In the solution process, the needs of market makers to balance inventory, cash positions, and risk in a dynamic market environment have been comprehensively considered, and the HJB equation has been used to derive a closed-form solution for the optimal quoting strategy.

In terms of sample selection, the constituent stocks of the CSI300 and CSI500 indexes, which are representative of China's A-share market, have been tested, and 114 representative stocks with A-share market have been selected as empirical samples according to the hypothesis of this paper. The results of empirical analysis have shown that the winning rate of each order placed by this strategy is between 43% and 45%, which can have stable returns in the upward market state, and can frequently capture the spread in the downward market state, with higher order times and relatively higher returns. This indicates that the strategy has good adaptability and robustness in different market states, especially when the market fluctuates greatly or the market sentiment is poor. This provides strong empirical support for its practical application in the A-share market and broadens the research scope and empirical scope of high-frequency market maker strategy in China.

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Appendix A. Derivation of Operators and Numerical Methods

For the mid-price, given the assumption that the utility function is linear and that market makers do not predict price trends, the dependence of the value function on p can be simplified to the following:

$$v_i(t,x,y,p) = x + yp + \phi_i(t,y) \tag{A1}$$

Here, $\phi_i(t,y)$ represents the expected return or cost of holding inventory y over the remaining time and $i = 1,2,\ldots,m$ corresponds to different spread states.

According to the quasi-variational inequality (25), due to Equation (31), the time derivative is $\frac{\partial v_i}{\partial t} = \frac{\partial \phi_i}{\partial t}$.The action of the spread generator $\mathcal{R}$ is as below:

$$\mathcal{R}v_i = \sum_{j=1}^{m} q_{ij}\left[\phi_j(t,y) - \phi_i(t,y)\right] \tag{A2}$$

The Hamiltonian function $H_i(t,y)$ involves the choice of the optimal limit order strategy, and its form is as below:

$$H_i(t,y) = \sup_{(q^b,l^b)} \mathrm{Prob}_b(q^b,s_i)[(p-q^b)l^b + \phi_i(t,y+l^b) - \phi_i(t,y)] + \sup_{(q^a,l^a)} \mathrm{Prob}_a(q^a,s_i)[(q^a-p)l^a + \phi_i(t,y-l^a) - \phi_i(t,y)] - \gamma g(y) \quad \text{(A3)}$$

To simplify the expression of $H_i(t,y)$, we introduce the quote deviation $\delta_b = q^b - P_t^b$ and $\delta_a = q^a - P_t^a$. It can be concluded that

$$H_i(t,y) = \sup_{\delta_b,l^b} \mathrm{Prob}_b(\delta_b,s_i)\left[\left(\frac{s_i}{2} - \delta_b\right)l^b + \phi_i(t,y+l^b) - \phi_i(t,y)\right] + \cdots \quad \text{(A4)}$$

We discretize the time derivative thus:

$$\frac{\partial\phi_i}{\partial t} \approx \frac{\phi_i(t_k,y) - \phi_i(t_{k+1},y)}{h} \quad \text{(A5)}$$

Finally, at each time step, the discretized equation is as below:

$$\frac{\phi_i(t_k,y_j) - \phi_i(t_{k+1},y_j)}{h} - \sum_{j'=1}^{m} q_{ij'}\phi_{j'}(t_{k+1},y_j) - H_i(t_k,y_j) = 0 \quad \text{(A6)}$$

After the calculation, the updated formula for the value function is obtained:

$$\phi_i(t_k,y_j) = \max\{\tilde{\phi}_i(t_k,y_j), v_{\text{impulse}}(t_k,y_j)\} \quad \text{(A7)}$$

The value function for the limit order strategy is updated to the following:

$$\tilde{\phi}_i(t_k,y_j) = \phi_i(t_{k+1},y_j) + h\left(\sum_{j'=1}^{m} q_{ij'}\phi_{j'}(t_{k+1},y_j) + H_i(t_k,y_j)\right) \quad \text{(A8)}$$

Consider the value function update for the market order strategy:

$$v_{\text{impulse}}(t_k,y_j) = \sup_{\zeta\in[-\zeta,\bar{\zeta}]} [-c(\zeta,s_i) + \phi_i(t_{k+1},y_j+\zeta)] \quad \text{(A9)}$$

Here, the transaction cost $c(\zeta,s_i) = \frac{s_i}{2}|\zeta|$.

Appendix B. Market Maker Strategy Results

Table A1. Performance of the bull market maker strategies in the A-share market from December 2022 to March 2023.

Instrument ID	Buy/Sell Trades	Market Orders	Initial Price	Final Price	Profit	Winning Rate	Stock Return	Strategy Return
000027	248,995	6	6.16	6.25	568.0	43.23%	1.46%	0.57%
000050	235,933	36	9.2	9.71	−432.0	7.20%	5.54%	−0.43%
000100	278,411	1	4.06	4.48	1334.9	40.86%	10.34%	1.33%
000166	269,345	0	4.17	4.25	1306.2	43.06%	1.92%	1.31%
000559	220,103	3	5.29	5.36	757.3	43.94%	1.32%	0.76%
000591	274,477	1	7.52	7.1	121.9	41.46%	−5.59%	0.12%
000598	234,913	1	5.1	5.15	897.0	43.77%	0.98%	0.90%
000623	238,437	169	15.93	15.7	−2161.6	10.24%	−1.44%	−2.16%
000703	253,526	19	7.3	8.11	54.2	42.44%	11.10%	0.05%
000776	275,615	114	16.5	16.56	−3048.2	11.04%	0.36%	−3.05%
000785	234,094	14	3.96	4.45	1058.3	43.77%	12.37%	1.06%
000883	241,464	2	4.47	4.45	1112.6	44.30%	−0.45%	1.11%
000958	237,321	12	4.44	4.54	917.5	43.82%	2.25%	0.92%
000967	223,810	11	4.94	5.43	777.7	43.32%	9.92%	0.78%

000987	252,023	8	6.51	6.67	362.0	42.72%	2.46%	0.36%
001227	246,551	4	4.11	3.61	1282.7	45.10%	−12.17%	1.28%
002252	268,804	14	5.58	6.21	511.0	42.05%	11.29%	0.51%
002408	222,699	13	7.5	7.19	181.8	44.03%	−4.13%	0.18%
002423	256,468	27	7.07	7.53	−129.0	41.32%	6.51%	−0.13%
002429	253,460	7	3.51	4.58	1235.8	43.39%	30.48%	1.24%
002500	241,403	5	5.65	5.89	629.2	42.84%	4.25%	0.63%
002563	232,004	8	5.1	6.02	649.0	43.04%	18.04%	0.65%
002607	270,816	4	5.01	5.74	599.8	40.37%	14.57%	0.60%
002608	179,394	16	6.43	6.38	322.3	43.30%	−0.78%	0.32%
002736	242,654	7	9.24	9.86	−376.5	8.10%	6.71%	−0.38%
002797	266,437	6	6.07	6.03	616.2	42.92%	−0.66%	0.62%
002926	256,978	23	8.72	8.38	−153.9	37.49%	−3.90%	−0.15%
002939	259,745	12	8.83	8.75	−378.0	14.00%	−0.91%	−0.38%
300253	271,005	26	8.9	10.23	−1174.6	11.38%	14.94%	−1.17%
300296	246,659	4	5.94	6.51	544.4	43.22%	9.60%	0.54%
600000	271,997	4	7.28	7.26	267.7	42.87%	−0.27%	0.27%
600004	276,758	120	15.1	14.31	−2179.3	13.36%	−5.23%	−2.18%
600008	278,528	0	2.91	2.98	1792.4	42.75%	2.41%	1.79%
600015	271,112	2	5.27	5.25	959.6	43.00%	−0.38%	0.96%
600016	278,547	0	3.54	3.43	1595.8	42.83%	−3.11%	1.60%
600018	272,968	3	5.49	5.35	920.4	42.75%	−2.55%	0.92%
600019	278,633	7	5.67	6.62	648.3	41.41%	16.75%	0.65%
600028	279,948	0	4.55	4.74	1245.0	41.24%	4.18%	1.25%
600029	278,132	7	7.68	7.9	−2.9	40.37%	2.86%	0.00%
600039	271,225	77	11.74	13.16	−1106.8	9.99%	12.10%	−1.11%
600050	269,806	9	4.41	5.81	809.3	39.68%	31.75%	0.81%
600056	261,436	100	16.91	15.88	−3485.0	10.93%	−6.09%	−3.48%
600061	271,972	8	6.62	7.06	295.1	42.52%	6.65%	0.30%
600095	265,746	18	8.15	10.43	−844.0	25.02%	27.98%	−0.84%
600098	269,301	9	5.9	5.89	746.8	42.92%	−0.17%	0.75%
600104	276,732	61	15.45	15.19	−2271.6	8.67%	−1.68%	−2.27%
600109	275,736	25	9.17	9.36	−710.5	8.55%	2.07%	−0.71%
600115	276,576	1	5.42	5.47	822.2	41.67%	0.92%	0.82%
600126	252,319	11	4.33	4.88	1043.2	44.39%	12.70%	1.04%
600143	274,199	80	10.17	10.01	−785.0	7.36%	−1.57%	−0.79%
600155	263,266	24	7.4	7.18	147.8	43.44%	−2.97%	0.15%
600166	278,782	7	2.73	3.56	1615.8	41.85%	30.40%	1.62%
600170	276,894	1	2.81	2.71	1845.5	43.12%	−3.56%	1.85%
600177	243,299	1	6.5	6.58	475.5	43.94%	1.23%	0.48%
600219	279,241	0	3.56	3.68	1538.8	42.38%	3.37%	1.54%
600271	259,912	130	11.05	12.37	−1145.4	8.57%	11.95%	−1.15%
600282	268,361	2	3.34	4.05	1453.3	43.34%	21.26%	1.45%
600350	233,793	7	5.77	5.99	623.8	43.53%	3.81%	0.62%
600352	262,242	18	9.85	10.43	−738.4	6.38%	5.89%	−0.74%
600361	212,231	67	6.72	6.44	130.5	43.48%	−4.17%	0.13%
600369	269,516	5	4.05	4.08	1323.0	43.74%	0.74%	1.32%
600390	269,177	13	5.23	5.41	783.8	42.63%	3.44%	0.78%
600399	276,053	133	15.11	14.48	−2278.3	12.75%	−4.17%	−2.28%
600415	272,184	9	4.48	5.31	833.3	41.39%	18.53%	0.83%
600497	277,961	13	5.37	5.58	860.2	42.10%	3.91%	0.86%
600499	276,873	151	16.06	16.25	−2750.6	9.51%	1.18%	−2.75%
600500	263,686	16	7.3	7.03	240.1	42.80%	−3.70%	0.24%

600515	276,845	3	4.35	4.34	986.0	42.12%	−0.23%	0.99%
600516	273,136	12	6.49	6.72	457.2	43.37%	3.54%	0.46%
600517	242,779	14	5.25	5.21	803.4	44.23%	−0.76%	0.80%
600528	255,070	12	8.11	8.97	−149.1	37.76%	10.60%	−0.15%
600578	257,809	6	3.51	3.33	1483.5	43.94%	−5.13%	1.48%
600582	274,622	5	5.45	5.44	934.6	41.82%	−0.18%	0.93%
600583	274,625	11	5.73	6.76	432.4	40.71%	17.98%	0.43%
600637	257,811	16	6.75	7.2	287.5	43.43%	6.67%	0.29%
600655	260,557	10	7.38	7.85	26.2	42.31%	6.37%	0.03%
600704	273,358	6	4.58	4.91	1054.4	43.08%	7.21%	1.05%
600739	243,536	173	13.48	13.57	−1547.3	7.90%	0.67%	−1.55%
600755	270,490	28	7.71	8.84	−50.7	39.39%	14.66%	−0.05%
600808	271,900	2	2.91	3.09	1698.3	43.59%	6.19%	1.70%
600816	185,272	39	3.69	4	778.8	45.02%	8.40%	0.78%
600820	268,422	3	5.55	5.6	867.3	43.06%	0.90%	0.87%
600827	257,906	193	11.46	12.17	−1258.5	9.11%	6.20%	−1.26%
600837	276,244	8	9.08	9.25	−417.7	7.50%	1.87%	−0.42%
600863	279,117	4	3.86	3.53	1521.4	43.19%	−8.55%	1.52%
600871	274,533	1	2.01	2.15	2026.3	43.27%	6.97%	2.03%
600879	275,854	15	6.95	7.77	149.8	42.15%	11.80%	0.15%
600901	271,499	7	5.78	6.07	679.9	41.69%	5.02%	0.68%
600905	279,605	0	5.82	5.65	821.3	40.41%	−2.92%	0.82%
600906	274,690	26	8.94	8.31	−407.3	23.02%	−7.05%	−0.41%
600909	275,763	5	4.99	4.99	1022.3	42.88%	0.00%	1.02%
600919	279,218	5	7.5	7.22	213.6	40.71%	−3.73%	0.21%
600928	250,487	3	3.58	3.55	1383.4	44.67%	−0.84%	1.38%
600956	245,405	171	10.82	10	−661.7	6.01%	−7.58%	−0.66%
600958	277,591	14	9.16	10.37	−921.1	10.43%	13.21%	−0.92%
600959	269,072	2	3.08	3.17	1653.6	44.35%	2.92%	1.65%
600967	262,345	16	8.72	9.73	−339.6	14.72%	11.58%	−0.34%
600968	275,637	5	3.14	3.25	1639.3	43.02%	3.50%	1.64%
600977	271,495	149	13.68	12.38	−1835.1	10.54%	−9.50%	−1.84%
601000	274,504	1	2.95	2.94	1762.9	43.33%	−0.34%	1.76%
601006	276,383	1	6.85	6.89	475.4	42.08%	0.58%	0.48%
601009	276,095	54	10.4	9.93	−641.0	9.47%	−4.52%	−0.64%
601016	279,775	0	4.07	3.96	1421.6	42.32%	−2.70%	1.42%
601077	271,606	0	3.63	3.62	1520.1	43.97%	−0.28%	1.52%
601106	255,082	2	3.05	3.15	1572.1	44.79%	3.28%	1.57%
601108	274,923	6	7.66	7.79	50.4	42.14%	1.70%	0.05%
601111	277,863	60	10.61	11.12	−990.6	11.16%	4.81%	−0.99%
601118	255,435	11	4.46	4.65	1058.7	44.63%	4.26%	1.06%
601139	262,096	21	6.98	7.15	249.7	43.34%	2.44%	0.25%
601158	219,979	4	5.31	5.3	787.3	44.79%	−0.19%	0.79%
601162	278,776	1	3.11	3.03	1713.6	42.93%	−2.57%	1.71%

Table A2. Performance of market maker strategies in the A-share market bear market from June to September 2024.

Instrument ID	Buy/Sell Trades	Market Orders	Initial Price	Final Price	Profit	Winning Rate	Stock Return	Strategy Return
000027	288,338	13	7.33	5.43	381.7	43.11%	−25.92%	0.38%
000050	291,035	23	7.54	6.38	68.7	42.02%	−15.38%	0.07%
000100	316,000	2	4.41	3.64	1526.7	40.72%	−17.46%	1.53%

000166	310,724	0	4.43	4.23	1384.0	42.39%	−4.51%	1.38%
000559	272,036	12	4.79	4.87	1059.7	43.82%	1.67%	1.06%
000591	278,492	9	4.99	4.03	1165.9	44.14%	−19.24%	1.17%
000598	300,703	35	7.72	6.36	−83.1	41.43%	−17.62%	−0.08%
000623	245,520	205	13.93	12.75	−1664.2	6.99%	−8.47%	−1.66%
000703	290,218	26	6.92	5.5	341.3	42.55%	−20.52%	0.34%
000776	302,442	99	12.51	12.34	−1557.6	7.64%	−1.36%	−1.56%
000785	267,054	8	2.72	2.28	1745.5	43.97%	−16.18%	1.75%
000883	297,930	19	6.03	4.53	859.1	42.58%	−24.88%	0.86%
000958	262,814	8	3.9	3.26	1362.7	43.75%	−16.41%	1.36%
000967	253,105	8	4.43	3.79	1139.8	43.71%	−14.45%	1.14%
000987	258,458	24	5.33	4.92	825.8	44.07%	−7.69%	0.83%
001227	266,938	0	2.47	2.11	1893.9	45.29%	−14.57%	1.89%
002252	303,497	13	7.38	6.56	16	41.87%	−11.11%	0.02%
002408	260,904	33	5.85	4.61	722	43.63%	−21.20%	0.72%
002423	288,163	24	7.7	7.22	−105	43.22%	−6.23%	−0.10%
002429	297,095	15	4.87	4.36	1087.5	42.70%	−10.47%	1.09%
002500	278,693	12	4.94	4.9	1052.1	43.54%	−0.81%	1.05%
002563	286,940	17	6.36	4.48	924.3	42.82%	−29.56%	0.92%
002607	291,055	4	1.91	1.85	2113.8	40.89%	−3.14%	2.11%
002608	268,061	17	8.06	6.49	60.2	43.23%	−19.48%	0.06%
002736	253,383	11	9.36	8.66	−290.2	16.14%	−7.48%	−0.29%
002797	297,355	4	5.27	5.27	1009.8	43.10%	0.00%	1.01%
002926	241,404	20	6.8	6.46	304.9	43.35%	−5.00%	0.30%
002939	273,089	20	6.98	6.87	254.3	43.15%	−1.58%	0.25%
300253	287,010	17	6.13	5.22	626.3	42.94%	−14.85%	0.63%
300296	289,871	11	4.74	3.92	1134.70	43.20%	−17.30%	1.13%
600000	313,255	8	8.1	8.34	−293.1	17.59%	2.96%	−0.29%
600004	279,514	142	10.09	8.59	−534.9	5.79%	−14.87%	−0.53%
600008	302,640	5	2.79	2.79	1944.50	43.17%	0.00%	1.94%
600015	310,819	5	6.66	6.14	576.6	41.92%	−7.81%	0.58%
600016	315,637	0	3.8	3.33	1725.00	41.38%	−12.37%	1.73%
600018	308,979	2	5.69	5.67	697.4	42.31%	−0.35%	0.70%
600019	314,803	3	6.75	5.58	548.6	41.07%	−17.33%	0.55%
600028	315,438	2	6.23	6.19	518.1	40.44%	−0.64%	0.52%
600029	302,297	11	5.87	5.4	755.8	43.28%	−8.01%	0.76%
600039	297,390	17	8.39	5.47	328.8	42.38%	−34.80%	0.33%
600050	316,077	1	4.42	4.48	1304.90	40.47%	1.36%	1.30%
600056	264,475	156	10.8	9.71	−772.3	5.71%	−10.09%	−0.77%
600061	279,294	10	5.98	5.73	702	43.79%	−4.18%	0.70%
600095	283,078	24	6.37	5.9	390.5	43.55%	−7.38%	0.39%
600098	283,785	29	6.76	5.46	452.7	43.39%	−19.23%	0.45%
600104	310,638	125	14.05	11.57	−2256.1	8.67%	−17.65%	−2.26%
600109	288,493	15	7.94	7.08	23.3	43.57%	−10.83%	0.02%
600115	296,388	5	3.96	3.6	1479.60	43.53%	−9.09%	1.48%
600126	273,234	13	4.35	3.31	1340.60	43.93%	−23.91%	1.34%
600143	290,984	19	6.97	7.19	287.8	43.08%	3.16%	0.29%
600155	284,195	24	6.45	6.06	459.7	43.59%	−6.05%	0.46%
600166	302,729	2	2.44	2.28	2020.30	43.64%	−6.56%	2.02%
600170	304,148	0	2.24	1.92	2215.70	42.78%	−14.29%	2.22%
600177	299,288	11	7.31	6.61	252.8	42.74%	−9.58%	0.25%
600219	314,864	0	3.76	3.51	1646.20	41.23%	−6.65%	1.65%
600271	277,074	46	7.84	8.14	−325.9	42.23%	3.83%	−0.33%

600282	305,284	7	5.02	4.04	1218.60	42.41%	−19.52%	1.22%
600350	289,753	17	8.93	9.3	−520.6	7.64%	4.14%	−0.52%
600352	277,529	33	8.62	9.03	−369.5	6.76%	4.76%	−0.37%
600361	295,873	20	3.83	3.49	1280.00	42.86%	−8.88%	1.28%
600369	287,723	4	3.7	3.59	1484.80	43.74%	−2.97%	1.48%
600390	281,077	10	4.27	4.01	1307.80	44.14%	−6.09%	1.31%
600399	301,223	28	6	5.09	638.4	43.06%	−15.17%	0.64%
600415	306,867	16	8.5	8.52	−127.6	36.33%	0.24%	−0.13%
600497	310,507	14	5.57	4.59	912.7	42.51%	−17.59%	0.91%
600499	284,677	31	9.4	7.06	−97.1	35.64%	−24.89%	−0.10%
600500	257,907	18	3.78	3.66	1277.30	44.10%	−3.17%	1.28%
600515	278,333	13	3.36	3.07	1560.00	44.22%	−8.63%	1.56%
600516	289,534	19	4.53	3.84	1249.90	43.97%	−15.23%	1.25%
600517	274,913	8	4.53	4.34	1214.60	43.77%	−4.19%	1.21%
600528	258,277	35	7.63	6.67	52.5	43.81%	−12.58%	0.05%
600578	295,427	14	3.35	3.03	1642.30	43.41%	−9.55%	1.64%
600582	308,007	15	6.97	5.22	648.6	42.10%	−25.11%	0.65%
600583	298,321	22	6.09	4.88	779.7	42.86%	−19.87%	0.78%
600637	279,367	22	6.37	6.17	509.6	43.78%	−3.14%	0.51%
600655	265,827	12	5.65	5	814.1	43.83%	−11.50%	0.81%
600704	298,677	7	4.64	4.03	1357.90	43.14%	−13.15%	1.36%
600739	268,962	42	9.29	8.01	−309.7	25.97%	−13.78%	−0.31%
600755	278,353	17	8.11	5.69	360.7	43.12%	−29.84%	0.36%
600808	261,699	3	2.14	1.81	1931.40	44.67%	−15.42%	1.93%
600816	243,533	24	2.94	2.35	1548.10	44.46%	−20.07%	1.55%
600820	303,435	17	6.62	5.33	453.3	42.41%	−19.49%	0.45%
600827	251,365	27	8.08	7.27	−204.4	43.17%	−10.02%	−0.20%
600837	276,452	25	8.12	8.77	−444.7	11.35%	8.00%	−0.44%
600863	311,623	5	4.71	4.07	1336.20	42.48%	−13.59%	1.34%
600871	275,491	1	1.78	1.73	2116.00	43.87%	−2.81%	2.12%
600879	309,224	26	7.83	7.09	−109.5	41.95%	−9.45%	−0.11%
600901	309,595	9	5.22	4.29	1109.40	42.25%	−17.82%	1.11%
600905	315,348	2	4.61	4.07	1338.00	41.15%	−11.71%	1.34%
600906	274,381	22	6.21	5.75	567.6	44.01%	−7.41%	0.57%
600909	294,236	3	4.42	4.33	1272.70	43.31%	−2.04%	1.27%
600919	315,258	7	7.73	7.49	54.7	40.60%	−3.10%	0.05%
600925	258,467	11	5.54	4.86	913.9	44.28%	−12.27%	0.91%
600928	271,478	4	3.34	2.96	1583.40	44.28%	−11.38%	1.58%
600956	262,380	23	9.04	6.59	−39.9	38.50%	−27.10%	−0.04%
600958	306,915	18	8	8.43	−137.7	26.57%	5.37%	−0.14%
600959	269,288	10	2.78	2.73	1716.60	44.27%	−1.80%	1.72%
600967	273,514	17	7.8	6.49	99.3	43.53%	−16.79%	0.10%
600968	304,787	9	4.13	3.92	1440.20	42.53%	−5.08%	1.44%
600977	252,991	232	11.26	9.76	−889.7	6.50%	−13.32%	−0.89%
601000	305,234	13	4.18	4.81	1075.30	42.63%	15.07%	1.08%
601006	314,807	0	7.23	5.93	471	41.26%	−17.98%	0.47%
601009	311,278	108	10.12	9.91	−788	8.70%	−2.08%	−0.79%
601016	298,780	4	3.18	2.65	1874.50	43.95%	−16.67%	1.87%
601077	311,383	2	5.08	4.78	1044.50	41.88%	−5.91%	1.04%
601106	253,672	4	2.4	2.28	1743.50	44.81%	−5.00%	1.74%
601108	297,349	12	6.93	6.42	391.3	43.33%	−7.36%	0.39%
601111	306,898	7	7.49	6.43	275.7	42.81%	−14.15%	0.28%
601118	281,091	13	5.03	4.88	1077.80	43.97%	−2.98%	1.08%

601139	274,328	16	7.11	6.29	329	43.83%	−11.53%	0.33%
601158	255,587	10	5.03	4.26	968.9	44.57%	−15.31%	0.97%
601162	299,987	0	2.37	2.97	2005.80	42.50%	25.32%	2.01%
601169	314,302	5	5.67	4.91	926.7	41.25%	−13.40%	0.93%
601186	310,619	12	8.47	6.84	−132.7	27.77%	−19.24%	−0.13%

Appendix C. Bull Market Full Market Maker Strategy Trends

Figures are accessible on <https://doi.org/10.6084/m9.figshare.28450295.v1>.

Appendix D. Bear Market Full Market Maker Strategy Trends

Figures are accessible on <https://doi.org/10.6084/m9.figshare.28450319.v1>.

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