

Article

Nonlinear Magnetoconvection of a Power-Law Fluid Saturated Porous Layer

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Abstract

This article examines the thermohaline stability of a power-law fluid saturating a porous layer in the presence of a magnetic field. The system stability is analyzed using both linear and weakly nonlinear instability theories. Within the linear framework, the Galerkin method is employed to derive analytical expressions for the Rayleigh number corresponding to steady and oscillatory modes of instability. Takens–Bogdanov and Hopf bifurcation points are identified, highlighting the transition mechanisms between different instability regimes. An increase in the Hartmann number delays the onset of convection. The critical Rayleigh number is a monotonic increasing function of the solute Rayleigh number, whereas it is a non-monotonic function of the Peclet number. To investigate heat and mass transport characteristics, an amplitude equation is derived in the weakly nonlinear regime. The results reveal that increasing the Hartmann, Lewis, and Peclet numbers enhances both heat and mass transport, whereas an opposite trend is observed with increasing the solute Rayleigh number.

Keywords: nonlinear phenomena; porous media; analytical approach; power-law fluid; magnetoconvection

MSC: 76E06

1. Introduction

The study of Darcy–Benard instability (DBI) refers to the key role of this behavior in several applied fields of science and engineering. Many research papers have dealt with DBI by using Darcy’s equation to represent Newtonian fluid flow in a porous medium [1]. The study of the rheological behavior and convective flow of dilatant and pseudoplastic liquids is commonly described by a power-law mathematical model. Several models exist in the literature for describing non-Newtonian liquids; however, the Ostwald–de Waele power-law model has been the most widely employed. Shenoy [2] provides a concise review of heat transfer in non-Newtonian fluids within porous layers. In addition, numerous experimental and theoretical studies have examined the convection of power-law fluids in porous media.

In hydrodynamic stability, Waters [3] conducted the first study on the thermal stability of a power-law fluid. Later, Barletta and Nield [4] conducted a linear stability analysis of a power-law fluid in a porous layer with throughflow, assuming impermeable and isothermal boundary planes. Alves and Barletta [5] further investigated the convective instability of



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a power-law fluid with throughflow, extending the earlier study by incorporating more general Robin thermal boundary conditions. In another work, Alves and Barletta [6] examined convective and absolute instabilities of a power-law fluid in Prats flow. Their results revealed that pseudoplastic and Newtonian fluids exhibit similar behavior, while dilatant fluids display distinct characteristics.

Kumari and Murthy [7] studied the problem of convective flow of a power-law fluid with upward throughflow. Recently, Reddy et al. [8] investigated the effect of chemical reaction on a power-law fluid in a porous slab. Fakiri et al. [9] modeled a power-law fluid using the Darcy–Forchheimer law, in which they solved the system of equations using an asymptotic method of analysis.

The motion of an electrically conducting fluid through a porous medium in the presence of a magnetic field, known as hydromagnetic convection, is an important area of research. When such a fluid flows under a magnetic field, electric currents are induced within the medium. Despite its practical relevance, the convective behavior of electrically conducting fluids in porous media has not been extensively studied. However, geophysical applications highlight its significance, particularly in geothermal regions where the Earth's crust acts as a porous medium and is subjected to the geomagnetic field. In metallurgical processes, such as continuous casting, electromagnetic stirring is employed to control the solidification of molten metals. This technique improves mechanical properties and promotes the formation of a uniform and fine-grained microstructure in the final product [10].

However, existing studies on convection in porous layers saturated with power-law fluids have typically examined stationary instability only using numerical methods. Analytical studies of the thermal convection of a power-law fluid in a Darcy porous layer with different external fields are comparatively limited. In the present work, we extend the analysis of Kumari and Murthy [11] by explicitly incorporating a transverse magnetic field and by developing a fully analytical framework for both stationary and oscillatory convection. Using a Galerkin method, we derive closed-form expressions for the critical thresholds and obtain an amplitude equation governing the weakly nonlinear evolution near onset. Such an analytical weakly nonlinear treatment of oscillatory convection in a Darcy porous layer saturated with a power-law fluid under magnetic effects has not been reported previously.

The paper is organized as follows. In Section 2, we discuss the mathematical formulation of the flow configuration. In Section 3, we discuss the linear stability analysis. A detailed description of the derivation of the amplitude equation is presented in Section 4. Section 5 deals with the discussion of results. The conclusions obtained from the linear stability theory are stated in Section 6.

2. Basic Equations

Let us consider a plane horizontal homogeneous isotropic porous layer with thickness d , saturated by an Ostwald–de Waele (power-law) fluid. Let the vertical axis z be parallel to the gravitational acceleration $\mathbf{g} = -g\hat{e}_z$. The Oberbeck–Boussinesq approximation is considered valid. The equations governing this system are the continuity equation, Darcy's law and the convection–diffusion equation:

$$\nabla \cdot \mathbf{u} = 0, \quad (1)$$

$$\frac{\mu^*}{K} |\mathbf{u}|^{n-1} \mathbf{u} = -\nabla P - \rho_0 \mathbf{g} (\alpha_T (T - T_0) - \alpha_C (C - C_0)) + \mathbf{j} \times B, \quad (2)$$

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f (\mathbf{u} \cdot \nabla) T = (\rho c)_m k_T \nabla^2 T, \quad (3)$$

$$\phi \frac{\partial C}{\partial t} + (\mathbf{u} \cdot \nabla) C = \phi D_c \nabla^2 C, \quad (4)$$

with

$$\begin{aligned} w &= w_0, \quad T = T + \Delta T, \quad C = C + \Delta C \text{ on } z = 0, \\ w &= w_0, \quad T = T, \quad C = C \text{ on } z = 1. \end{aligned} \quad (5)$$

For Newtonian fluid ($n = 1$), $\mu^*/K = \mu/K$ and it becomes

$$\frac{\mu^*}{K} = 2\zeta \mu \left(\frac{3\phi}{50K} \right)^{(n+1)/2} \left(\frac{3n+1}{n\phi} \right)^n. \quad (6)$$

where

$$\left\{ \begin{array}{l} \zeta = 25/12 - \text{Christopher and Middleman,} \\ \mathbf{u} = (u, v, w) - \text{velocity, } \mu - \text{dynamic viscosity,} \\ P - \text{pressure, } T - \text{temperature, } C - \text{concentration,} \\ \phi - \text{porosity, } K = \text{permeability, } \rho_0 - \text{reference density,} \\ \alpha_T - \text{thermal expansion coefficient, } \alpha_C - \text{density coefficient for salinity,} \\ \mathbf{g} - \text{gravity, } t - \text{time, } B_0 - \text{magnetic field, } \mathbf{j} - \text{electrical current,} \\ w_0 - \text{prescribed vertical throughflow velocity,} \\ c - \text{specific heat capacity per unit mass of the fluid,} \\ k_T - \text{thermal diffusivity, } D_c - \text{mass diffusivity.} \end{array} \right.$$

Moreover, the term $\mathbf{j} \times B$ represents Lorentz force. On neglecting the displacement currents, the Maxwell equations which govern the electrodynamic field are

$$\nabla^* \times \mathbf{H} = \mathbf{j}, \quad \nabla^* \times \mathbf{E} = -\frac{\partial^* B}{\partial t^*}, \quad \nabla^* \cdot B = 0, \quad \nabla^* \cdot \mathbf{j} = 0.$$

where $\mathbf{E}$ is the electric field, $\mathbf{H} = \frac{B}{\mu_m}$ is the magnetized force, and μ_m is the magnetic permeability. The electric field $\mathbf{E}$ can be represented in terms of a potential, i.e., $\mathbf{E} = -\nabla^* \phi$. Therefore, $\nabla^* \times \mathbf{E} = 0$ implies $\frac{\partial^* B}{\partial t^*} = 0$. Hence, B is the function of the spatial variable only. And further, we assume $\mathbf{B} = B_0 \hat{e}_z$. Therefore, the current is given by

$$\mathbf{j} = \sigma (-\nabla^* \phi + \mathbf{u}^* \times B),$$

where σ is the electric conductivity. Since $\nabla^* \cdot \mathbf{j} = 0$, the above equation becomes

$$\nabla^* \cdot \mathbf{j} = \sigma \left(\nabla^{*2} \phi + B \cdot (\nabla^* \times \mathbf{u}^*) - \mathbf{u}^* \cdot (\nabla^* \times B) \right) = 0.$$

It is assumed that the third component of vorticity $\nabla^* \times \mathbf{u}^* = 0$. Since $B = B_0 \hat{e}_z$ it follows that $B \cdot (\nabla^* \times \mathbf{u}^*) = 0$. Hence, we get

$$\nabla^* \times B = \frac{1}{\frac{1}{\mu_m \sigma}} (-\nabla^* \phi + \mathbf{u}^* \times B).$$

Here, $\eta = \frac{1}{\frac{1}{\mu_m \sigma}}$ is the resistivity. Let the magnetic Prandtl number be $Pr = \frac{\nu}{\eta} \rightarrow 0$, and it follows that $\nabla^* \times B = 0$ which implies $\nabla^* \phi = 0$. Therefore,

$$\mathbf{j} \times B = \sigma B_0^2 (\mathbf{u} \times \hat{e}_z) \times \hat{e}_z.$$

The dimensionless quantities defined as

$$\begin{aligned} x &= x^* d, & y &= y^* d, & z &= z^* d, \\ u &= \frac{\phi k_T}{d} u^*, & v &= \frac{\phi k_T}{d} v^*, & w &= \frac{\phi k_T}{d} w^*, \\ t &= \frac{d^2}{k_T} t^*, & p &= \frac{\mu \phi k_T}{\kappa} p^*, & T &= \Delta T T^*, \\ C &= \Delta C C^*. \end{aligned} \quad (7)$$

as well as

$$\begin{cases} Ra = \frac{g \rho_0 \alpha_T \Delta T K d^n}{\phi^n k_T^n \mu} - \text{thermal Rayleigh number}, \\ Rs = \frac{g \rho_0 \alpha_C \Delta C K d^n}{\phi^n k_T^n \mu} - \text{solutal Rayleigh number}, \\ Le = \frac{k_T}{D_c} - \text{Lewis number}, \\ Ha^2 = \frac{\sigma_1 B_0^2 K}{\mu} - \text{Hartmann number}. \end{cases}$$

By eliminating the pressure by doing some algebraic operations, we obtain Equations (1)–(5) as

$$\nabla \cdot \mathbf{u} = 0, \quad (8)$$

$$\nabla \times (|\mathbf{u}|^{n-1} \mathbf{u}) = \nabla \times (RaT - RsC) \hat{e}_z + Ha^2 [(\mathbf{u} \times \hat{e}_z) \times \hat{e}_z], \quad (9)$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \nabla^2 T, \quad (10)$$

$$\frac{\partial C}{\partial t} + (\mathbf{u} \cdot \nabla) C = \frac{1}{Le} \nabla^2 C, \quad (11)$$

$$w = Pe, \quad T = 1, \quad C = 1 \quad \text{on } z = 0,$$

$$w = Pe, \quad T = 0, \quad C = 0 \quad \text{on } z = 1. \quad (12)$$

3. Basic State

The basic flow is given by

$$u_b = 0, \quad v_b = 0, \quad w_b = Pe. \quad (13)$$

where $Pe = \frac{w_0 d}{\phi D_c}$ is the Péclet number. On putting Equation (13) into Equations (10) and (11), we obtain

$$T_b = \frac{e^{Pe} - e^{Pe z}}{e^{Pe} - 1}, \quad C_b = \frac{e^{Pe Le} - e^{Pe Le z}}{e^{Pe Le} - 1}. \quad (14)$$

4. Linear Stability Analysis

The perturbation of the basic state can be written as

$$\mathbf{u} = \mathbf{u}_b + \epsilon \mathbf{u}', \quad T = T_b + \epsilon T', \quad C = C_b + \epsilon C'. \quad (15)$$

where $\epsilon \ll 1$. By substituting Equation (15) into Equations (8)–(12) and by neglecting terms $O(\epsilon^2)$ or higher, we get

$$\begin{aligned} n \left(\frac{\partial^2 w'}{\partial x^2} + \frac{\partial^2 w'}{\partial y^2} \right) + \frac{\partial^2 w'}{\partial z^2} &= \frac{Ra}{|Pe|^{n-1}} \left(\frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial y^2} \right) \\ &\quad - \frac{Rs}{|Pe|^{n-1}} \left(\frac{\partial^2 C'}{\partial x^2} + \frac{\partial^2 C'}{\partial y^2} \right) - \frac{Ha^2}{|Pe|^{n-1}} \frac{\partial^2 w'}{\partial z^2}, \\ \frac{\partial T'}{\partial t} + \frac{dT_b}{dz} w' + Pe \frac{\partial T'}{\partial z} &= \frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial y^2} + \frac{\partial^2 T'}{\partial z^2}, \end{aligned} \quad (16)$$

$$\frac{\partial C'}{\partial t} + w' \frac{dC_b}{dz} + Pe \frac{\partial C'}{\partial z} = \frac{1}{Le} \left(\frac{\partial^2 C'}{\partial x^2} + \frac{\partial^2 C'}{\partial y^2} + \frac{\partial^2 C'}{\partial z^2} \right), \quad (17)$$

$$z = 0, 1 \quad : \quad w' = 0, \quad T' = 0, \quad C' = 0. \quad (18)$$

Consider the normal mode solution as

$$(w', T', C') = (W(z), \theta(z), C(z)) e^{i(mx+ly-\omega t)}. \quad (19)$$

where

$$\begin{cases} q = \sqrt{l^2 + m^2} - \text{dimensionless wave number,} \\ \omega - \text{angular frequency.} \end{cases}$$

Using Equation (19), the above Equations become

$$\left(D^2 - nq^2 + \frac{Ha^2}{|Pe|^{n-1}} D^2 \right) W + \frac{Ra}{|Pe|^{n-1}} q^2 \theta - \frac{Rs}{|Pe|^{n-1}} q^2 C = 0, \quad (20)$$

$$\left(D^2 - q^2 - PeD - i\omega \right) \theta + WPe \frac{dT_b}{dz} = 0, \quad (21)$$

$$\left(\frac{1}{Le} \left(D^2 - q^2 \right) - PeD - i\omega \right) C + \frac{Pe}{\tau_1} W \frac{dC_b}{dz} = 0, \quad (22)$$

$$z = 0, 1 \quad : \quad W = \theta = C = 0. \quad (23)$$

By taking account of the boundary conditions, we can choose the analytical solution as

$$(W, \theta, C) = (W_0, \theta_0, C_0) \sin \pi z. \quad (24)$$

By substituting Equation (24) in (20)–(23), we obtain

$$\begin{pmatrix} \pi^2 + nq^2 + \frac{Ha^2 \pi^2}{|Pe|^{n-1}} & -\frac{Ra}{|Pe|^{n-1}} q^2 & \frac{Rs}{|Pe|^{n-1}} q^2 \\ -\frac{4\pi^2}{4\pi^2 + Pe^2} & \sigma + \delta^2 & 0 \\ -\frac{4\pi^2}{4\pi^2 + Le^2 Pe^2} & 0 & \sigma + \frac{\delta^2}{Le} \end{pmatrix} \begin{pmatrix} W_0 \\ \theta_0 \\ C_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad (25)$$

where $\delta^2 = \pi^2 + q^2$.

The solvability condition for non-trivial solutions requires the determinant of the resulting 3×3 coefficient matrix to vanish, leading to a complex dispersion relation linking the Rayleigh number Ra and the oscillation frequency ω . The determinant is expanded and rearranged to express Ra explicitly as a function of ω . Owing to the use of a single-mode Galerkin truncation and the assumption of sinusoidal vertical eigenfunctions, the resulting instability criterion represents an approximate analytical threshold, valid near the onset of convection. After algebraic simplification, the Rayleigh number takes the form

$$Ra = \frac{A_1 + A_2\omega^2 + i\omega(A_3 + A_4\omega^2)}{4Pe\pi^2q^2(iLe\omega + \delta^2)}, \quad (26)$$

where

$$\begin{cases} A_1 = LePe^n(Pe^2 + 4\pi^2)(Le^2Pe^2 + 4\pi^2)(\pi^2 + nq^2)\delta^6 \\ \quad + LePe\pi^2\delta^4(Pe^2 + 4\pi^2)(4Leq^2Rs + Ha^2(Le^2Pe^2 + 4\pi^2)\delta^2), \\ A_2 = LePe^n(Pe^2 + 4\pi^2)(Le^2Pe^2 + 4\pi^2)(\pi^2 + nq^2)\delta^6 \\ \quad + LePe\pi^2\delta^4(4q^2Rs + Ha^2(Le^2Pe^2 + 4\pi^2)\delta^2), \\ A_3 = LePe^n(Pe^2 + 4\pi^2)(Le^2Pe^2 + 4\pi^2)(\pi^2 + nq^2)\delta^6 \\ \quad + LePe\pi^2\delta^4(4(Le - 1)q^2Rs + Ha^2(Le^2Pe^2 + 4\pi^2)\delta^2), \\ A_4 = Le^3(Pe^2 + 4\pi^2)(Le^2Pe^2 + 4\pi^2)(Ha^2Pe\pi^2 + Pe^n(\pi^2 + nq^2)). \end{cases}$$

4.1. Stationary Convection

Stationary instability corresponds to the onset of convection through steady flow patterns, which occurs when the growth rate of disturbances is purely real. This is mathematically represented by setting the oscillation frequency $\omega = 0$ leading to the stationary Rayleigh number Ra_{sc} . This results in

$$Ra_{sc} = \frac{(Pe^2 + 4\pi^2)}{4q^2} \left(\frac{4LeRs q^2}{Le^2Pe^2 + 4\pi^2} + \delta^2 \left(Ha^2 + \frac{1}{\pi^2} Pe^{-1+n} (\pi^2 + nq^2) \right) \right). \quad (27)$$

For Newtonian fluid, in the absence of magnetic field and concentration ($Ha^2 = 0$, $Rs = 0$), the above stationary Rayleigh number reduces to $Ra_{sc} = \frac{\delta_{sc}^4}{q_{sc}^2}$ with the critical values $Ra_{sc}^c = 4\pi^2$ and $q_{sc}^c = \pi$, which agrees with the results of Horton and Rogers [12] and Lapwood [13] for the onset of convection in a porous layer. Also, we depict a neutral curve for the limiting case (see Figure 1). We obtain the critical Rayleigh number as $4\pi^2$ and the corresponding critical wave number π which agrees with the results of Horton and Rogers [12] and Lapwood [13].

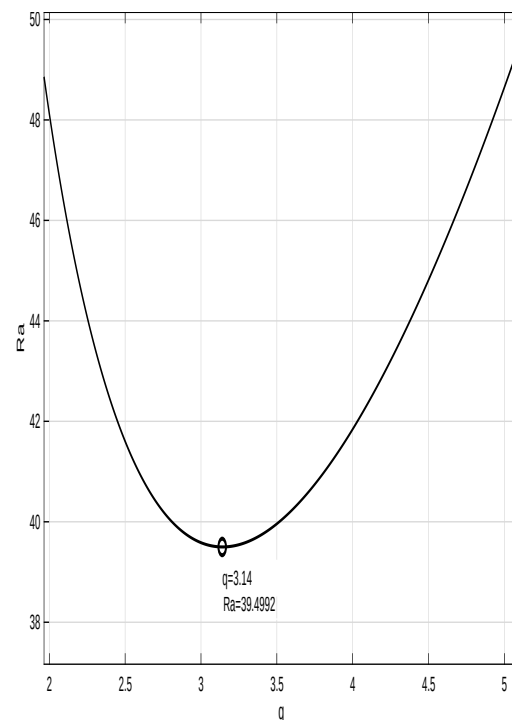


Figure 1. Comparison of present results with Horton and Rogers [12] and Lapwood [13].

4.2. Oscillatory Convection

We examine the real and imaginary parts of the Rayleigh number Ra and impose the condition that the imaginary part vanishes. Substituting the resulting expression for ω^2 into the real part then yields the thermal Rayleigh number Ra_{oc} , corresponding to the onset of oscillatory convection:

$$Ra_{oc} = \frac{(Pe^2 + 4\pi^2)}{4LePeq^2\pi^2(Le^2Pe^2 + 4\pi^2)} \left(\delta^2 Pe^n (1 + Le)(\pi^2 + nq^2) (Le^2Pe^2 + 4\pi^2) + Pe\pi^2 (4LeRs q^2 + Ha^2 \delta^2 (Le^2Pe^2 + 4\pi^2)) \right). \quad (28)$$

5. Weakly Nonlinear Analysis

In this section we derive the two-dimensional amplitude equation for stationary instability in order to study the heat and mass transport. Now, we consider the nonlinear terms in the governing equations. There are many ways to derive the amplitude equation [14]. We follow the procedure performed by [15]:

$$n \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} = \frac{Ra}{|Pe|^{n-1}} \frac{\partial^2 T}{\partial x^2} - \frac{Rs}{|Pe|^{n-1}} \frac{\partial^2 C}{\partial x^2} - \frac{Ha^2}{|Pe|^{n-1}} \frac{\partial^2 w}{\partial z^2}, \quad (29)$$

$$\frac{\partial T}{\partial t} + \frac{dT_b}{dz} w + Pe \frac{\partial T}{\partial z} + (\mathbf{u} \cdot \nabla) T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}, \quad (30)$$

$$\frac{\partial C}{\partial t} + w \frac{dC_b}{dz} + Pe \frac{\partial C}{\partial z} + (\mathbf{u} \cdot \nabla) C = \frac{1}{Le} \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial z^2} \right), \quad (31)$$

$$z = 0, 1 : w = 0, T = 0, C = 0. \quad (32)$$

A power series expansion in terms of ϵ introduced for the above system of equations as follows:

$$Ra = Ra_0 + \epsilon^2 Ra_2 + \epsilon^4 Ra_4 + \epsilon^6 Ra_6 + \dots, \quad (33)$$

$$w = w_1 \epsilon + \epsilon^2 w_2 + \epsilon^3 w_3 + \epsilon^4 w_4 + \dots, \quad (34)$$

$$T = T_1 \epsilon + \epsilon^2 T_2 + \epsilon^3 T_3 + \epsilon^4 T_4 + \dots, \quad (35)$$

$$C = C_1 \epsilon + \epsilon^2 C_2 + \epsilon^3 C_3 + \epsilon^4 C_4 + \dots. \quad (36)$$

Also, we rescale time ' t ' as $\tau = \epsilon^2 t$. Employing (33)–(36) in Equations (29)–(32) and equating the coefficients of corresponding powers of ϵ on either side, we get the matrix representation as

$$LW = N. \quad (37)$$

where

$$L = \begin{pmatrix} n \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \frac{Ha^2}{|Pe|^{n-1}} \frac{\partial^2}{\partial z^2} & -\frac{Ra}{|Pe|^{n-1}} \frac{\partial^2}{\partial x^2} & \frac{Rs}{|Pe|^{n-1}} \frac{\partial^2}{\partial x^2} \\ -\frac{dT_b}{dz} & \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} & 0 \\ -\frac{dC_b}{dz} & 0 & \frac{1}{Le} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \end{pmatrix} \quad (38)$$

and $W = (w_i \ T_i \ C_i)^T$, $i = 1, 2, 3$. N contains nonlinear terms and will be according to the order of ϵ . N contains nonlinear terms and will be according to the order of ϵ .

First-order system of ϵ :

The solutions for the first-order system (37) are obtained by utilizing the given boundary conditions as follows:

$$\begin{aligned} u_1 &= \pi A(\tau) \sin qx \cos \pi z, \\ w_1 &= -qA(\tau) \cos qx \sin \pi z, \\ T_1 &= \frac{-4q\pi^2}{\delta^2(4\pi^2 + Pe^2)} A(\tau) \cos qx \sin \pi z, \\ C_1 &= \frac{-4qLe\pi^2}{\delta^2(4\pi^2 + Le^2Pe^2)} A(\tau) \cos qx \sin \pi z. \end{aligned} \quad (39)$$

Second-order system of ϵ :

We consider the second-order of the system (37) as

$$LW_2 = (R_{21} \ R_{22} \ R_{23})^T, \quad (40)$$

where

$$R_{21} = 0, \ R_{22} = u_1 \frac{\partial T_1}{\partial x} + w_1 \frac{\partial T_1}{\partial z}, \ R_{23} = u_1 \frac{\partial C_1}{\partial x} + w_1 \frac{\partial C_1}{\partial z},$$

The solutions for the second-order system are obtained by utilizing the given boundary conditions as follows:

$$\begin{aligned} u_2 &= 0, \\ w_2 &= 0, \\ T_2 &= \frac{q^2 \pi^2}{\delta^2(4\pi^2 + Pe^2)^2} A^2(\tau) (Pe \cos 2\pi z - 2\pi \sin 2\pi z), \\ C_2 &= \frac{Le^2 q^2 \pi^2}{\delta^2(4\pi^2 + Pe^2 Le^2)^2} A^2(\tau) (Pe Le \cos 2\pi z - 2\pi \sin 2\pi z). \end{aligned} \quad (41)$$

Third-order system of ϵ :

We consider the third-order of the system (37) as

$$LW_2 = (R_{31} \ R_{32} \ R_{33})^T, \quad (42)$$

where

$$R_{31} = \frac{Ra_2}{|Pe|^{n-1}} \frac{\partial^2 T_1}{\partial x^2}, \quad R_{32} = u_1 \frac{\partial T_2}{\partial x} + w_1 \frac{\partial T_2}{\partial z} + \frac{\partial T_1}{\partial \tau}, \quad R_{33} = u_1 \frac{\partial C_2}{\partial x} + w_1 \frac{\partial C_2}{\partial z} + \frac{\partial C_1}{\partial \tau},$$

Although we are not interested in solving the third-order system, for the purpose of finding the amplitude equation, it is sufficient to apply the Fredholm solvability condition. To implement the Fredholm solvability condition, the solution of the adjoint of the first-order system should be orthogonal to the right-hand side of Equations (29)–(32). Thus, the condition can be expressed as

$$\int_0^1 \int_0^{\frac{2\pi}{q_c}} (R_{31} \hat{w} + R_{32} \hat{T} + R_{33} \hat{C}) dx dz = 0, \quad (43)$$

yields three distinct contributions: (i) a term proportional to $\frac{dA}{d\tau}$, arising from $\frac{dT_1}{d\tau}$ and $\frac{dC_1}{d\tau}$; (ii) a linear term proportional to A , originating from the Ra_2 -dependent forcing; and (iii) a cubic nonlinear term proportional to A^3 , generated by quadratic interactions between the first- and second-order fields.

We find the adjoint of Equations (29)–(32) as

$$L = \begin{pmatrix} n \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \frac{Ha^2}{|Pe|^{n-1}} \frac{\partial^2}{\partial z^2} & -\frac{dT_b}{dz} & -\frac{dC_b}{dz} \\ -\frac{Ra}{|Pe|^{n-1}} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + Pe \frac{\partial}{\partial z} & 0 \\ \frac{Rs}{|Pe|^{n-1}} \frac{\partial^2}{\partial x^2} & 0 & \frac{1}{Le} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + Pe \frac{\partial}{\partial z} \right) \end{pmatrix} \quad (44)$$

and $W = (w_i \ T_i \ C_i)^T$, $i = 1, 2, 3$.

Solving the system yields the adjoint solution as

$$\begin{aligned} \hat{u}_1 &= \pi A(\tau) \sin qx \cos \pi z, \\ \hat{w}_1 &= -q A(\tau) \cos qx \sin \pi z, \\ \hat{T}_1 &= \frac{q^3 Ra}{\delta^2 |Pe|^{n-1}} A(\tau) \cos qx \sin \pi z, \\ \hat{C}_1 &= \frac{q^3 Le Rs}{\delta^2 |Pe|^{n-1}} A(\tau) \cos qx \sin \pi z. \end{aligned} \quad (45)$$

After performing the integration, we obtain a Ginzburg–Landau equation for stationary instability as

$$\lambda_1 \frac{dA(\tau)}{d\tau} = \lambda_2 A(\tau) - \lambda_3 A^3(\tau), \quad (46)$$

where

$$\begin{aligned} \lambda_1 &= \frac{2q^3 \pi^3 (4Ra\pi^2 + Le^2 (4Rs\pi^2 + RaPe^2 + RsPe^2))}{\delta^4 |Pe|^{n-1} (4\pi^2 + Pe^2) (4\pi^2 + Le^2 Pe^2)}, \\ \lambda_2 &= \frac{2Ra q^3 \pi^3}{\delta^2 |Pe|^{n-1} (4\pi^2 + Pe^2)}, \end{aligned}$$

$$\lambda_3 = \frac{Ra q^5 \pi^3}{4\delta^4 |Pe|^{n-2} (4\pi^2 + Pe^2)^2}.$$

The above equation is known as the Bernoulli equation. Solving Equation (46) analytically is challenging; hence, it is solved numerically using Mathematica 11.0. The built-in function NDSolve is employed with the initial condition $A(0) = c_0$, where c_0 denotes the amplitude of convection. Additionally, both Ra_2 and Ra_c are assumed to be the same to minimize the number of parameters.

6. Heat and Mass Transfer

6.1. Nusselt Number

By the definition of the horizontally average Nusselt number, the Nusselt number is

$$\begin{aligned} Nu &= 1 + \frac{\frac{q_c}{2\pi} \int_0^{\frac{2\pi}{q_c}} \frac{d\theta_2}{dz} dz}{\frac{q_c}{2\pi} \int_0^{\frac{2\pi}{q_c}} \frac{d\theta_b}{dz} dz}, \\ &= 1 + \frac{4A^2 q^2 \pi^4 n^2 (e^{Pe} - 1)}{Pe \delta^2 (4\pi^2 + Pe^2)^2}. \end{aligned} \quad (47)$$

6.2. Sherwood Number

By the definition of horizontally average Sherwood number. The Sherwood number is

$$\begin{aligned} Sh &= 1 + \frac{\frac{q_c}{2\pi Le} \int_0^{\frac{2\pi}{q_c}} \frac{dC_2}{dz} dz}{\frac{q_c}{2\pi} \int_0^{\frac{2\pi}{q_c}} \frac{dC_b}{dz} dz}, \\ &= 1 + \frac{4A^2 q^3 \pi^4 Le (e^{LePe} - 1)}{Pe \delta^2 (4\pi^2 + Le^2 Pe^2)^2}. \end{aligned} \quad (48)$$

6.3. Theoretical Scaling of the Nusselt Number near the Onset of Convection

In this section, we theoretically establish the dependence of the Nusselt number on the supercriticality parameter using the derived weakly nonlinear amplitude equation.

From the weakly nonlinear analysis, the evolution of the convection amplitude $A(\tau)$ is governed by the Ginzburg–Landau equation (Equation (46)):

$$\lambda_1 \frac{dA}{d\tau} = \lambda_2 A - \lambda_3 A^3. \quad (49)$$

At a steady state, the amplitude reaches equilibrium, and hence $\frac{dA}{d\tau} = 0$. Therefore, the steady-state amplitude satisfies

$$\lambda_2 A - \lambda_3 A^3 = 0. \quad (50)$$

For non-trivial convection ($A \neq 0$), we obtain

$$A^2 = \frac{\lambda_2}{\lambda_3}. \quad (51)$$

Near the onset of convection, the Rayleigh number is expressed as

$$Ra = Ra_c (1 + \epsilon), \quad (52)$$

where $\epsilon = \frac{Ra - Ra_c}{Ra_c}$ is the supercriticality parameter and $\epsilon \ll 1$. Since the linear growth coefficient λ_2 is proportional to the deviation from criticality, we have

$$\lambda_2 \propto (Ra - Ra_c) \propto \epsilon. \quad (53)$$

Thus, the steady-state amplitude satisfies

$$A^2 \propto \epsilon. \quad (54)$$

The Nusselt number, defined as the ratio of total heat transport to conductive heat transport, is given by Equation (47):

$$Nu = 1 + KA^2, \quad (55)$$

where

$$K = \frac{4q_c^2 \pi^4 n^2 (e^{Pe} - 1)}{Pe \delta^2 (4\pi^2 + Pe^2)^2} \quad (56)$$

is a constant depending on system parameters. Substituting the amplitude scaling into the Nusselt number expression yields

$$Nu = 1 + K\epsilon. \quad (57)$$

Therefore,

$$Nu - 1 \propto \epsilon. \quad (58)$$

This result proves that the Nusselt number increases linearly with the supercriticality parameter near the onset of convection.

This linear scaling is a fundamental result of weakly nonlinear convection theory and confirms that the convective heat transport grows proportionally to the square of the convection amplitude near the instability threshold.

6.4. Numerical Verification of the Nusselt Number Scaling

To verify the weakly nonlinear prediction, the amplitude Equation (46) is solved numerically using Mathematica 11.0 for a representative parameter set near the onset of convection. The steady-state amplitude is obtained by integrating the evolution equation until convergence to a constant value is achieved, with a tolerance of 10^{-8} . The corresponding Nusselt number is then computed using Equation (47).

Figure 2 shows the variation of the Nusselt number with the supercriticality parameter $\epsilon = (Ra - Ra_c)/Ra_c$. It is observed that the Nusselt number increases linearly with ϵ near the onset of convection. This confirms the theoretical prediction of the weakly nonlinear analysis that $Nu - 1 \propto \epsilon$ since the steady-state amplitude satisfies $A^2 \propto \epsilon$.

This linear dependence is in excellent agreement with classical weakly nonlinear convection theory reported by Horton and Rogers [12], Schluter et al. [16], and Nield and Bejan [1], thereby validating the correctness of the derived amplitude equation and the associated heat transport coefficient.

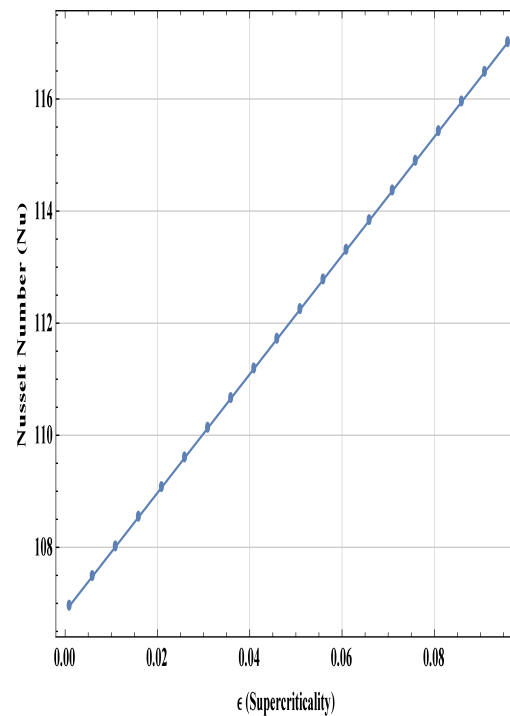


Figure 2. Variation of the Nusselt number with supercriticality parameter ϵ . Here, we fix $Pe = 1$, $n = 1$, $Le = 1$, $Rs = 50$ and $Ha^2 = 0.1$. The linear dependence near the onset of convection confirms the weakly nonlinear theoretical prediction and agrees with the classical results.

7. Discussion

The numerical results and conclusions are presented in this section. The thermosolutal stability of a power-law fluid with magnetic effect has been studied. The linear and weakly nonlinear theories have been discussed. The present analysis is restricted to weakly nonlinear convection near the instability threshold and employs a single-mode Galerkin approximation. This approach captures the dominant mode accurately for small supercriticality but may not fully represent strongly nonlinear regimes where higher harmonics become significant. In addition, the Darcy model assumes negligible inertial effects and a homogeneous porous layer, which is appropriate for low-permeability porous media. Therefore, the present results are most reliable near the onset of convection and within the parameter ranges considered. In the linear theory, we employ the first-order Galerkin method to obtain algebraic expressions for the Rayleigh number at stationary and oscillatory instabilities. The present results are compared with the existing results in the literature.

The Hartmann number typically ranges from $Ha^2 \ll 1$ to about 10^2 in laboratory magnetoconvection experiments, whereas in liquid-metal flows, it can reach values of 10^3 – 10^5 . In this regime, strong magnetic damping suppresses velocity fluctuations and stabilizes convection [17]. The Peclet number varies over a wide range. It is typically $Pe \ll 1$ in microscale flows, porous media diffusion, and slow groundwater transport, where diffusion dominates. In contrast, $Pe \approx 10$ – 10^3 is common in heat and mass transfer equipment, while $Pe \gg 10^4$ occurs in high-speed flows, industrial heat exchangers, and geophysical transport processes, where advection is dominant [18]. The Lewis number is usually in the range $Le \approx 0.01$ – 0.1 for gases and $Le \approx 1$ for many liquid mixtures, and can reach values of 10^2 – 10^4 in saltwater, polymeric fluids, and geophysical systems, indicating that heat diffuses much faster than solute. The solute Rayleigh number, which quantifies buoyancy driven by concentration gradients, ranges from $Rs \approx 10$ – 10^4 in laboratory porous-layer experiments to 10^6 – 10^9 or higher in oceanic thermohaline circulation, groundwater contamination, chemical reactors, and crystal growth processes, where strong solutal buoyancy drives convection and pattern formation [19].

In Tables 1 and 2, we present some examples where steady or oscillatory instabilities set in. In Table 1, for $n = 0.5$, there exists a threshold $Le^* \in (0, 1)$ such that for $Le < Le^*$, oscillatory convection sets in, while for $Le^* \in (0, 1)$, there is a switch from stationary to oscillatory convection. Similar behavior can be observed for the other values of n in both tables. The effect of Le is also evident from these tables. For low Pe , the critical Ra_{sc} (critical Ra_{oc}) is an increasing (decreasing) function of Le . A non-monotonic dependence of n on the critical Ra_{sc} and critical Ra_{oc} is also observed. Furthermore, Newtonian fluids are more stable than pseudoplastic and dilatant fluids. For $Pe = 5$, the critical Ra_{sc} is a non-monotonic function of Le , whereas the critical Ra_{oc} is a decreasing function of Le . Both the critical Ra_{sc} and critical Ra_{oc} are non-monotonic functions of n . Newtonian fluids are more unstable than pseudoplastic and dilatant fluids.

Table 1. Critical Ra with Lewis number for $Ha^2 = 0.5$, $Rs = 100$ and $Pe = 0.5$.

	n = 0.5		n = 1		n = 1.5	
Le	Rs_{sc}^c	Rs_{oc}^c	Rs_{sc}^c	Rs_{oc}^c	Rs_{sc}^c	Rs_{oc}^c
0	49.1457	—	49.1588	—	49.1457	—
1	149.1458	198.2915	149.1588	198.3177	149.1458	190.0008
2	245.4401	171.8658	245.4531	171.8854	245.4401	165.6478
3	334.7671	160.7347	334.7801	160.7522	334.7671	155.2076
4	414.6458	152.8072	414.6589	152.8236	414.6458	147.6255
5	483.5411	145.8539	483.5541	145.8697	483.5411	140.8795
6	540.8501	139.2874	540.8632	139.3027	540.8501	134.4512
7	586.7592	132.9685	586.7723	132.9834	586.7592	128.2309
8	622.0304	126.8995	622.0435	126.9143	622.0304	122.2361
9	647.7816	121.1215	647.7947	121.1361	647.7816	116.5155
10	665.2963	115.6754	665.3095	115.6898	665.2963	111.1155

Table 2. Critical Ra with Lewis number for the fixed values of $Ha^2 = 0.5$, $Rs = 100$ and $Pe = 5$.

	n = 0.5		n = 1		n = 1.5	
Le	Rs_{sc}^c	Rs_{oc}^c	Rs_{sc}^c	Rs_{oc}^c	Rs_{sc}^c	Rs_{oc}^c
0	195.8355	—	79.7838	—	195.8355	—
1	295.8355	491.6711	179.7838	259.5677	295.8355	491.6711
2	288.2921	339.9815	172.2403	165.9041	288.2921	339.9815
3	268.9739	285.4935	152.9222	130.7579	268.9739	285.4935
4	254.5218	259.4659	138.4701	114.4014	254.5218	259.4659
5	244.3535	244.7062	128.3019	105.4442	244.3535	244.7062
6	237.0148	235.3379	120.9631	99.9443	237.0148	235.3379
7	231.5301	228.9112	115.4783	96.2807	231.5301	228.9112
8	227.2984	224.2478	111.2467	93.6897	227.2984	224.2478
9	223.9446	220.7182	107.8929	91.7719	223.9446	220.7182
10	221.2259	217.9581	105.1742	90.3012	221.2259	217.9581

In Figure 3, we identify the bifurcation points namely, the Takens–Bogdanov bifurcation point. The Takens–Bogdanov bifurcation point is the point at which the oscillatory neutral curve intersects the stationary neutral curve, where the oscillation frequency becomes zero. At the Takens–Bogdanov bifurcation point, we have

$$Ra_s(q_{as}) = Ra_o(q_{oc}) \text{ and } q_{as} = q_{oc}. \quad (59)$$

The presence of a Takens–Bogdanov bifurcation indicates parameter regimes in which stationary and oscillatory convection interact strongly. This interaction can lead to multistability, increased sensitivity to parameter variations, and complex transport behavior, which are important for the safe design and efficient operation of thermofluid systems.

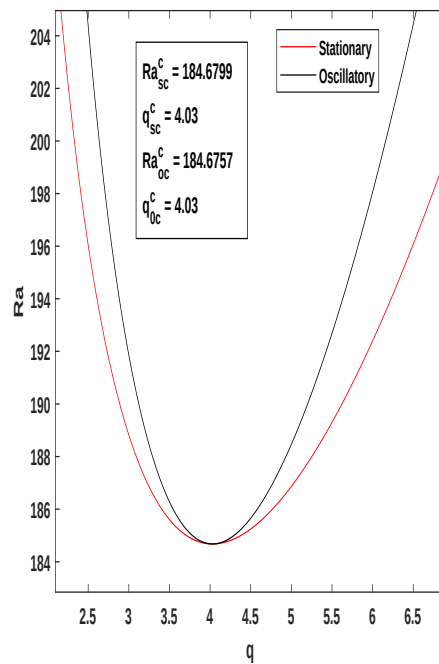


Figure 3. Identification of Takens–Bogdanov bifurcation point for the fixed values of $Ha^2 = 1.5$, $Rs = 100$, $n = 0.5$, $Le = 1.28$, and $Pe = 1.14$.

The effect of the Hartmann number on linear instability is shown in Figures 4 and 5. From these two figures, it is clear that the critical Ra_{sc} and critical Ra_{oc} are monotonically increasing functions of the Hartmann number for pseudoplastic, dilatant, and Newtonian fluids. Also, both the critical Ra_{sc} and critical Ra_{oc} are non-monotonic functions of n for a low Peclet number. On the other hand, they increase as n increases. Hence, pseudoplastic fluids are more unstable than dilatant fluids.

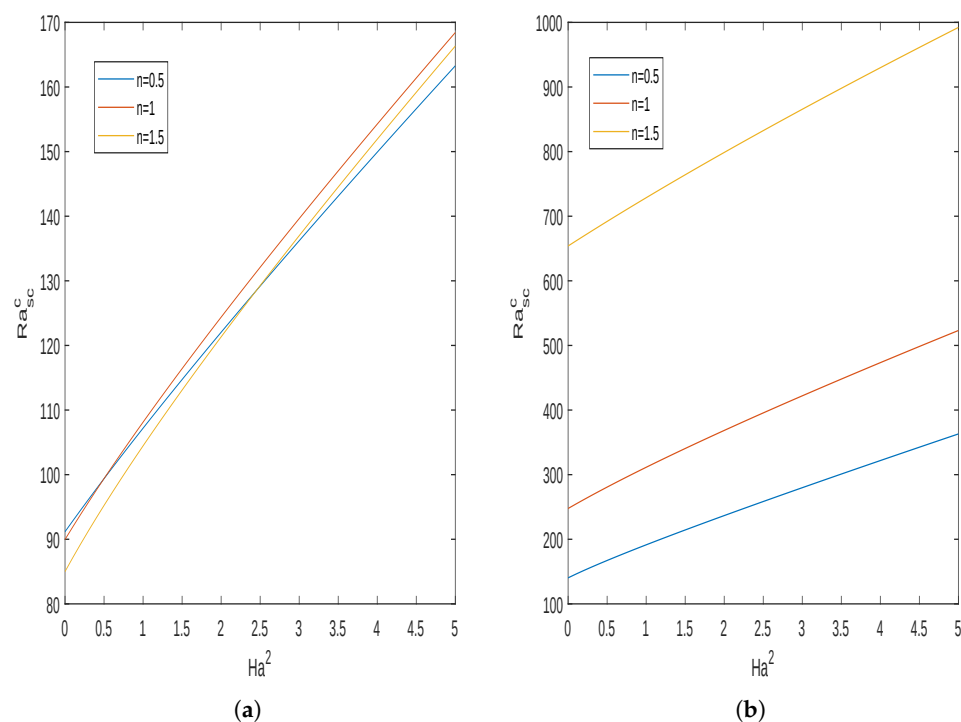


Figure 4. Increasing trend of critical Ra_{sc} with Hartmann number for the fixed values of $Le = 0.5$, $Rs = 100$ and (a) $Pe = 0.5$, (b) $Pe = 10$.

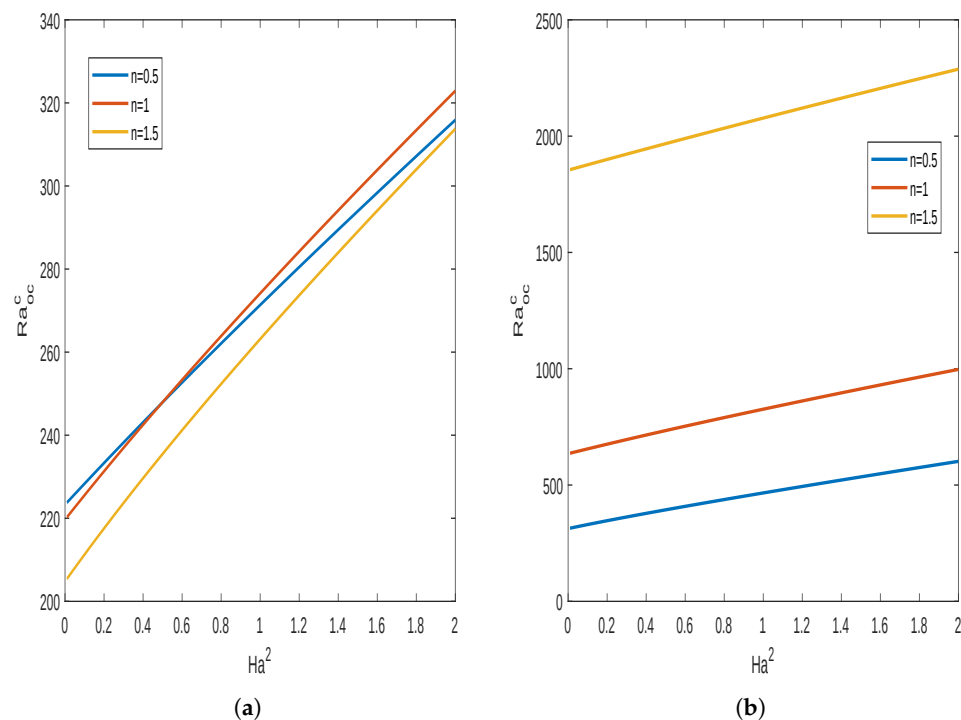


Figure 5. Increasing trend of critical Ra_{oc} with Hartmann number for the fixed values of $Le = 0.5$, $Rs = 100$ and (a) $Pe = 0.5$, (b) $Pe = 10$.

The variation of critical Ra with a solute Rayleigh number is shown in Figures 6 and 7. From these two figures, we see an increase in critical Ra with an increase in the solute Rayleigh number. We know that an increase in the solute Rayleigh number causes a decrease in the concentration of the solute and hence stabilizing influence on the flow. For a small Peclet number, dilatant fluids are most unstable than the pseudoplastic and Newtonian fluids. On the other hand, for a large Peclet number, pseudoplastic fluids are more unstable than the dilatant and Newtonian fluids. Figure 8 displays the variation of critical Ra with a Peclet number. For minimal values of Peclet numbers, the critical values of Ra versus n tend to be uniformly zero for $n > 1$, with a sudden jump to infinity when approaching $n = 1$ from above. For very large values of Peclet numbers, the critical values of Ra versus n tend to be uniformly zero for $n < 1$, with a sudden jump to infinity when approaching $n = 1$ from below. For a pseudoplastic fluid ($n < 1$), this effective viscosity is infinite when the shear rate tends to zero, and zero when the shear rate tends to infinity. The reverse occurs for a dilatant fluid ($n > 1$). If the effective viscosity of the fluid tends to infinity, the onset of the convective instability is completely inhibited, resulting in an infinite value of Ra_c . If the effective viscosity of the fluid becomes vanishingly small, the onset of the convective instability takes place, though small, with the boundary temperature difference being ΔT , thus resulting in a vanishing Ra_c .

Figures 9–11 show the effect of different parameters on the Nusselt number for pseudoplastic, Newtonian, and dilatant fluids respectively. As the Hartmann number increases, the Nusselt number also increases, leading to increased heat transfer and system stability for all cases. Also, as the Lewis number increases, the Nusselt number decreases. The Lewis number is the ratio of thermal diffusivity to the solutal diffusivity. As the Lewis number increases, heat diffuses faster than the mass, enhancing the heat transfer. We observe from these figures that the Nusselt number increases as the Peclet number increases. When the Peclet number increases, the imposed throughflow becomes stronger. This enhances the advection of heat from the lower to the upper boundary, making convection dominant over conduction. Hence, the Nusselt number increases with an increasing Peclet

number. Furthermore, the value of the Nusselt number decreases as the solute Rayleigh number increases. The increase in the solute Rayleigh number leads to stronger solutal buoyancy forces, which delays the heat transfer in the system and results in a diminishing Nusselt number.

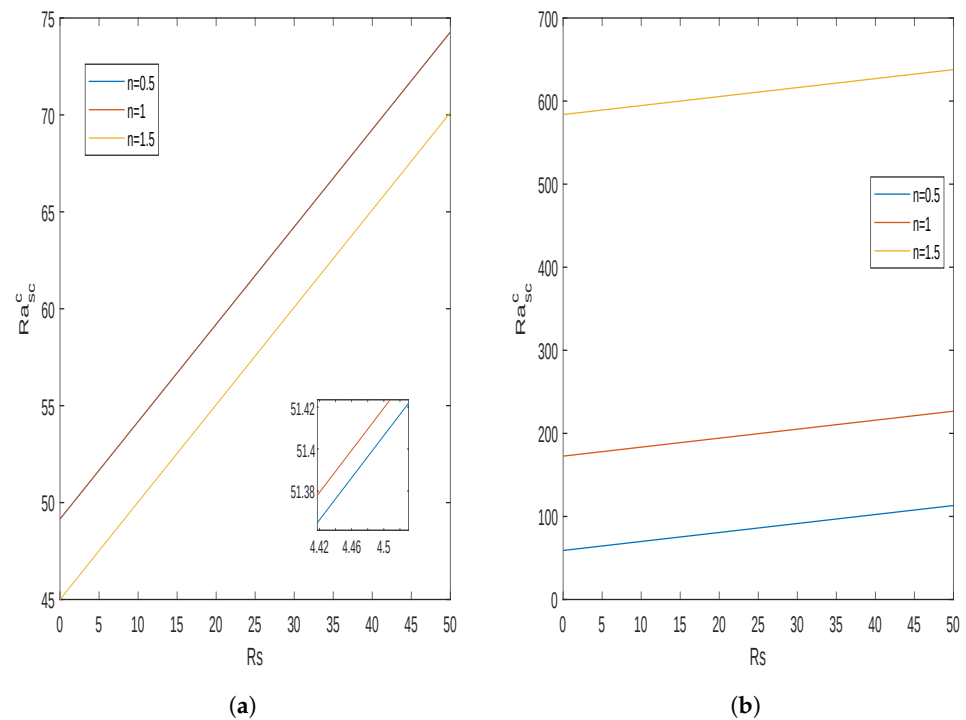


Figure 6. An increase in the solute Rayleigh number raises the critical Ra_{sc} for the fixed values of $Le = 0.5$, $Ha^2 = 1.5$ and (a) $Pe = 0.5$, (b) $Pe = 10$.

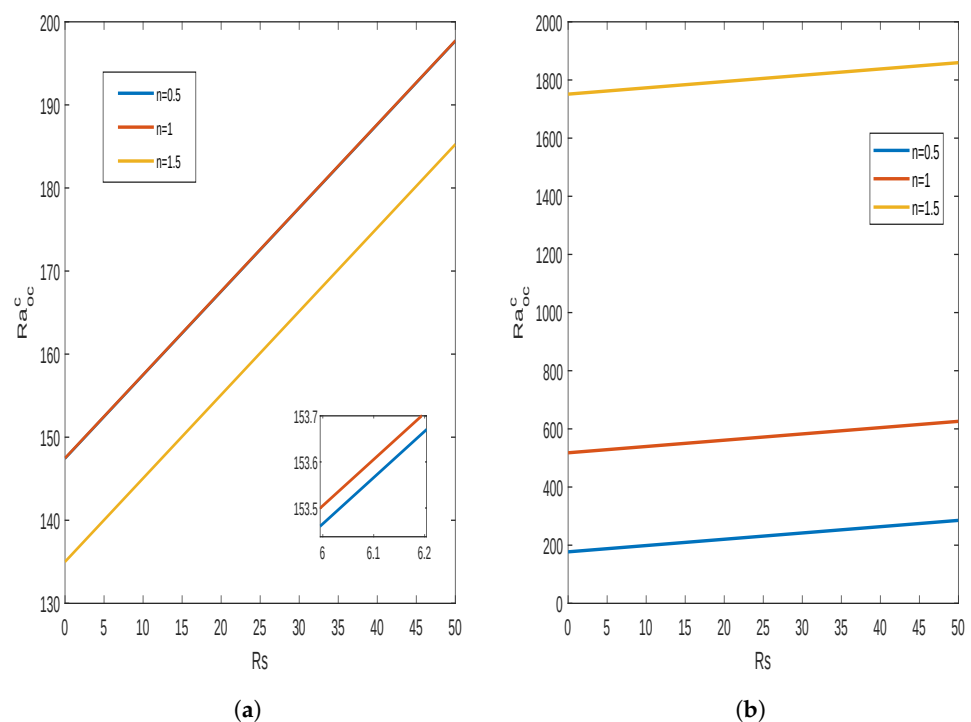


Figure 7. The critical Ra_{oc} increases monotonically with the solute Rayleigh number for the fixed values of $Le = 0.5$, $Ha^2 = 1.5$ and (a) $Pe = 0.5$, (b) $Pe = 10$.

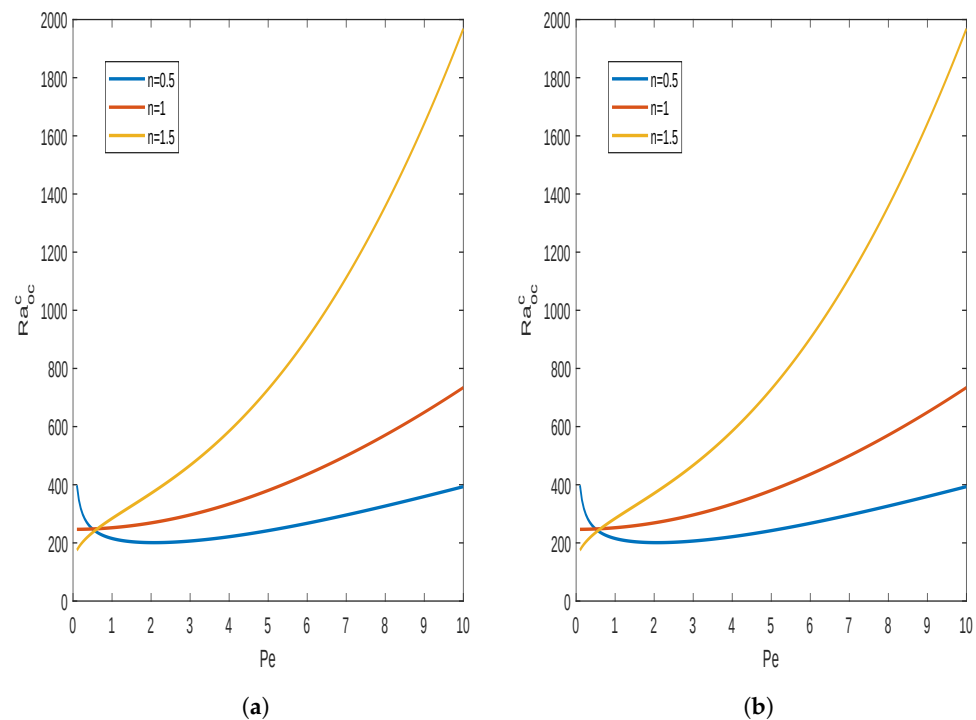


Figure 8. Change of critical Ra_{oc} with Peclet number for the fixed values of $Ha^2 = 0.5$, $Le = 0.5$, $Rs = 50$ and (a) Stationary, (b) Oscillatory.

Figures 12–14 show the effect of different parameters on the Sherwood number for pseudoplastic, Newtonian, and dilatant fluids respectively. As the Hartmann number increases, the Sherwood number also increases, leading to increased mass transfer for all cases. Although the Lorentz force is generally known to suppress fluid motion, the present results show that increasing the Hartmann number enhances heat and mass transfer. This apparent anomaly arises from the coupled effects of magnetic field, porous resistance, and nonlinear convection. While the magnetic field increases the critical Rayleigh number, it also alters the flow structure in the post-critical regime, leading to more efficient vertical transport. Moreover, in a power-law fluid, magnetic damping modifies the effective viscosity, which can enhance convective heat and mass fluxes. We observe from these figures that the Sherwood number increases as the Peclet number increases. An increase in throughflow intensifies advective solute transport and thins the concentration boundary layer, resulting in enhanced mass transfer. The Sherwood number also increases with the Lewis number since a larger Lewis number corresponds to lower solute diffusivity relative to thermal diffusivity; this maintains sharper concentration gradients and intensifies solutal buoyancy-driven convection, thereby enhancing mass transfer. In contrast, an increase in the solute Rayleigh number leads to a decrease in the Sherwood number. This is because concentration-induced buoyancy produces a stabilizing stratification that opposes fluid motion, weakens convective mixing, and consequently reduces the effectiveness of mass transfer.

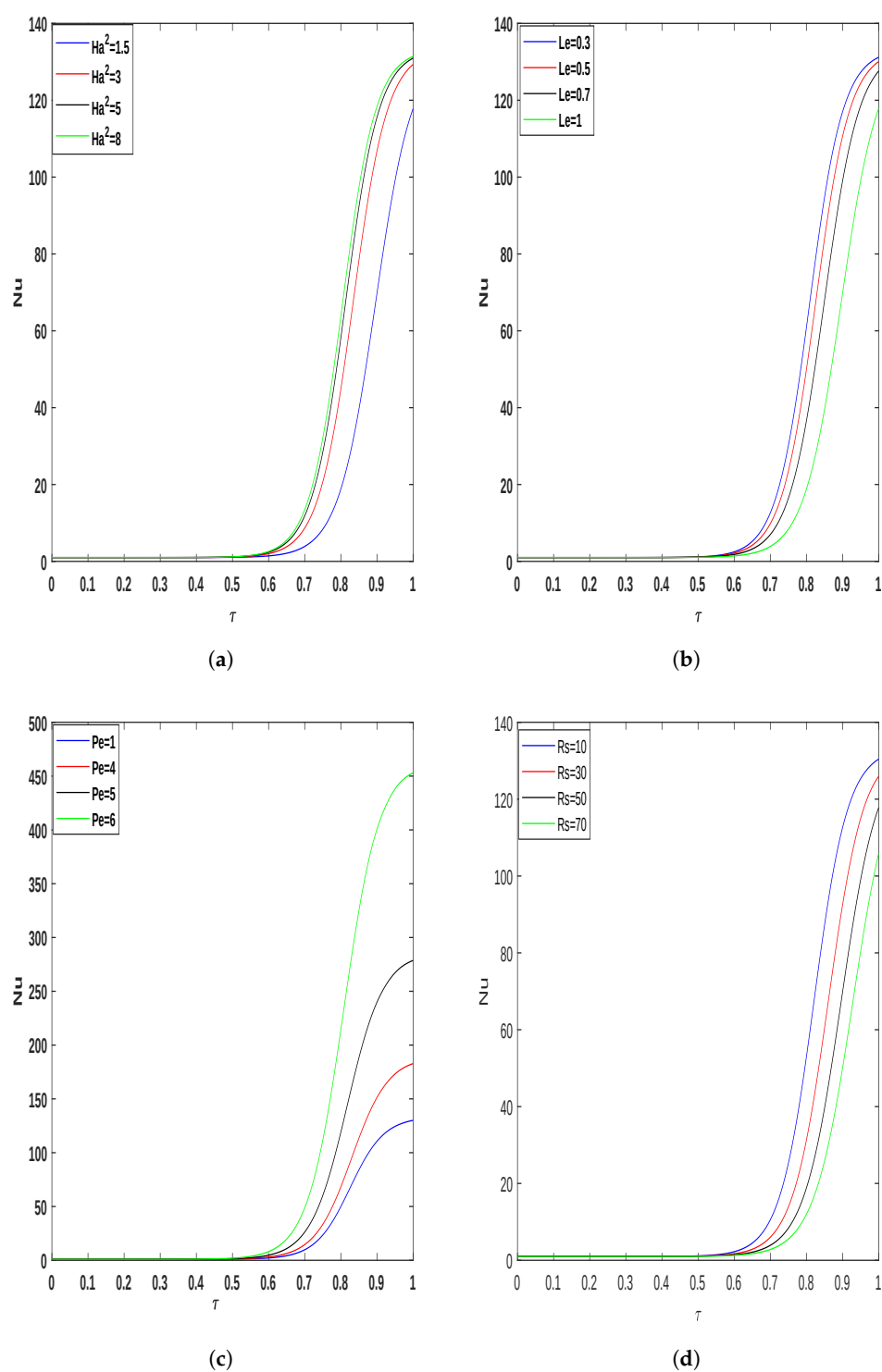


Figure 9. Plot of Nusselt number with τ for pseudoplastic fluids for the fixed values of (a) $Le = 1$, $Pe = 1$, $Rs = 50$, (b) $Pe = 1$, $Ha^2 = 1.5$, $Rs = 50$, (c) $Le = 1$, $Rs = 50$, $Ha^2 = 1.5$ and (d) $Le = 1$, $Pe = 1$, $Ha^2 = 1.5$.

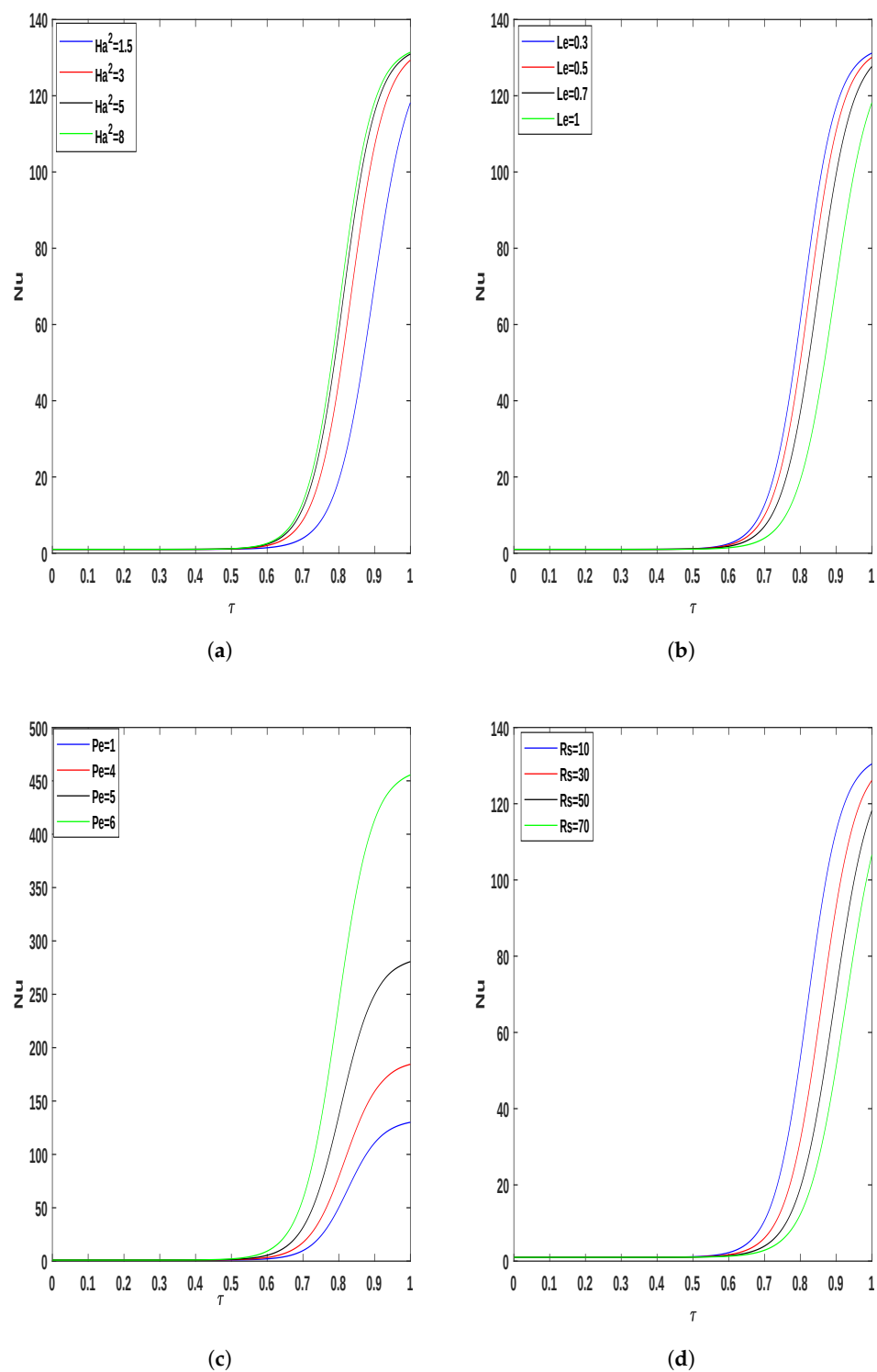


Figure 10. Plot of Nusselt number with τ for Newtonian fluids for the fixed values of (a) $Le = 1$, $Pe = 1$, $Rs = 50$, (b) $Pe = 1$, $Ha^2 = 1.5$, $Rs = 50$, (c) $Le = 1$, $Rs = 50$, $Ha^2 = 1.5$ and (d) $Le = 1$, $Pe = 1$, $Ha^2 = 1.5$.

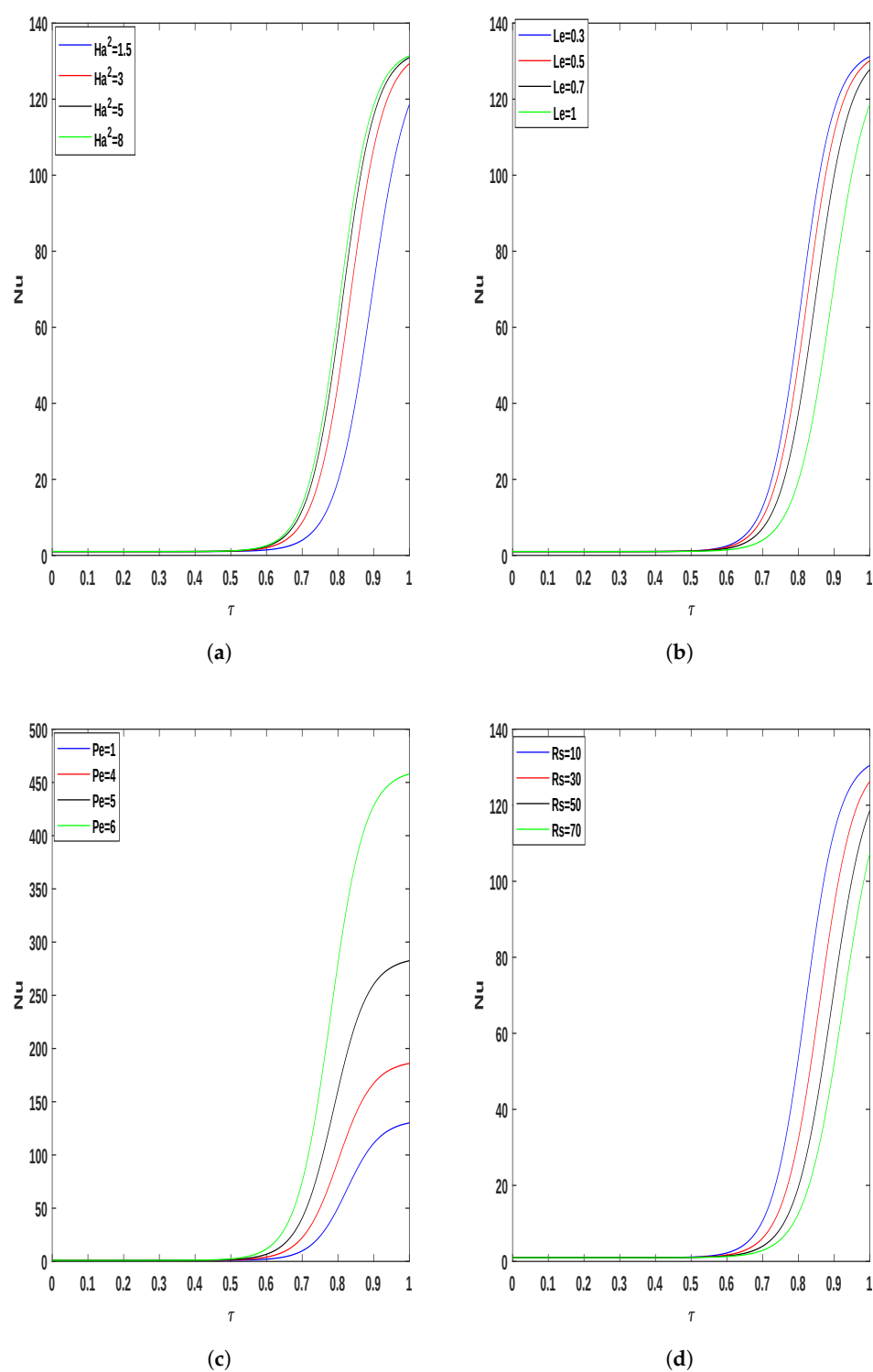


Figure 11. Plot of Nusselt number with τ for dilatant fluids for the fixed values of (a) $Le = 1$, $Pe = 1$, $Rs = 50$, (b) $Pe = 1$, $Ha^2 = 1.5$, $Rs = 50$, (c) $Le = 1$, $Rs = 50$, $Ha^2 = 1.5$ and (d) $Le = 1$, $Pe = 1$, $Ha^2 = 1.5$.

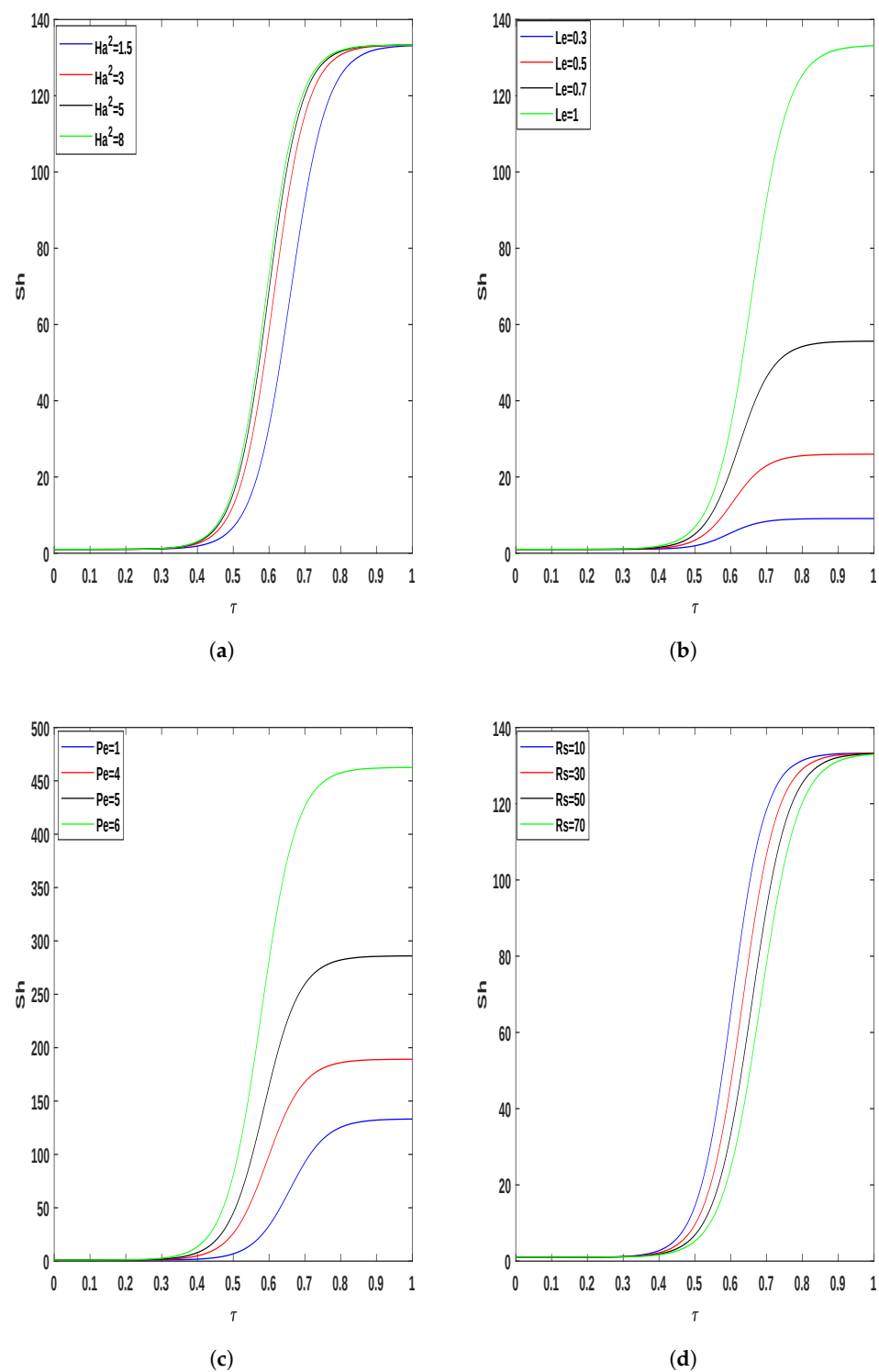


Figure 12. Plot of Sherwood number with τ for pseudoplastic fluids for the fixed values of (a) $Le = 1$, $Pe = 1$, $Rs = 50$, (b) $Pe = 1$, $Ha^2 = 1.5$, $Rs = 50$, (c) $Le = 1$, $Rs = 50$, $Ha^2 = 1.5$ and (d) $Le = 1$, $Pe = 1$, $Ha^2 = 1.5$.

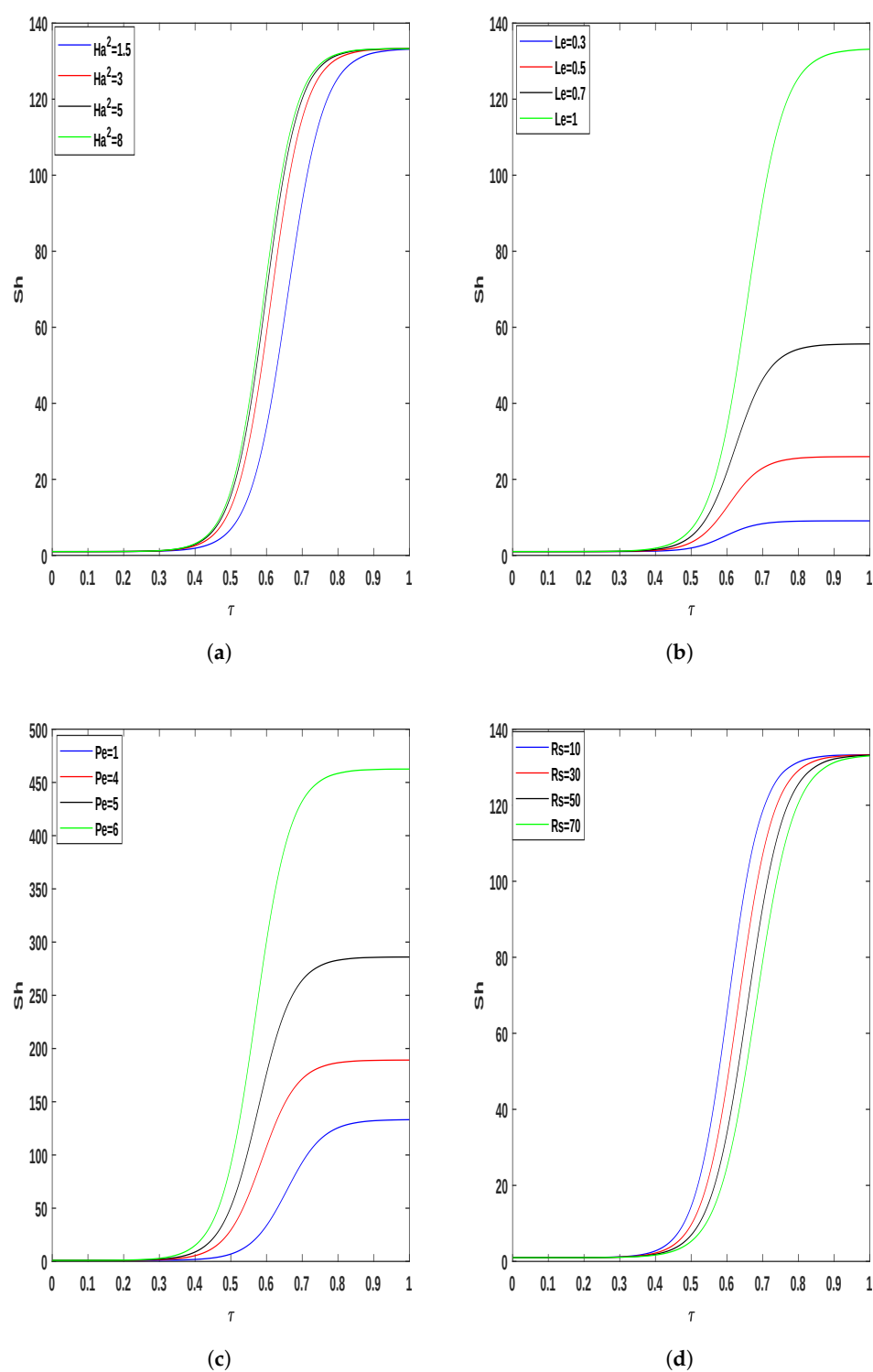


Figure 13. Plot of Sherwood number with τ for Newtonian fluids for the fixed values of (a) $Le = 1$, $Pe = 1$, $Rs = 50$, (b) $Pe = 1$, $Ha^2 = 1.5$, $Rs = 50$, (c) $Le = 1$, $Rs = 50$, $Ha^2 = 1.5$ and (d) $Le = 1$, $Pe = 1$, $Ha^2 = 1.5$.

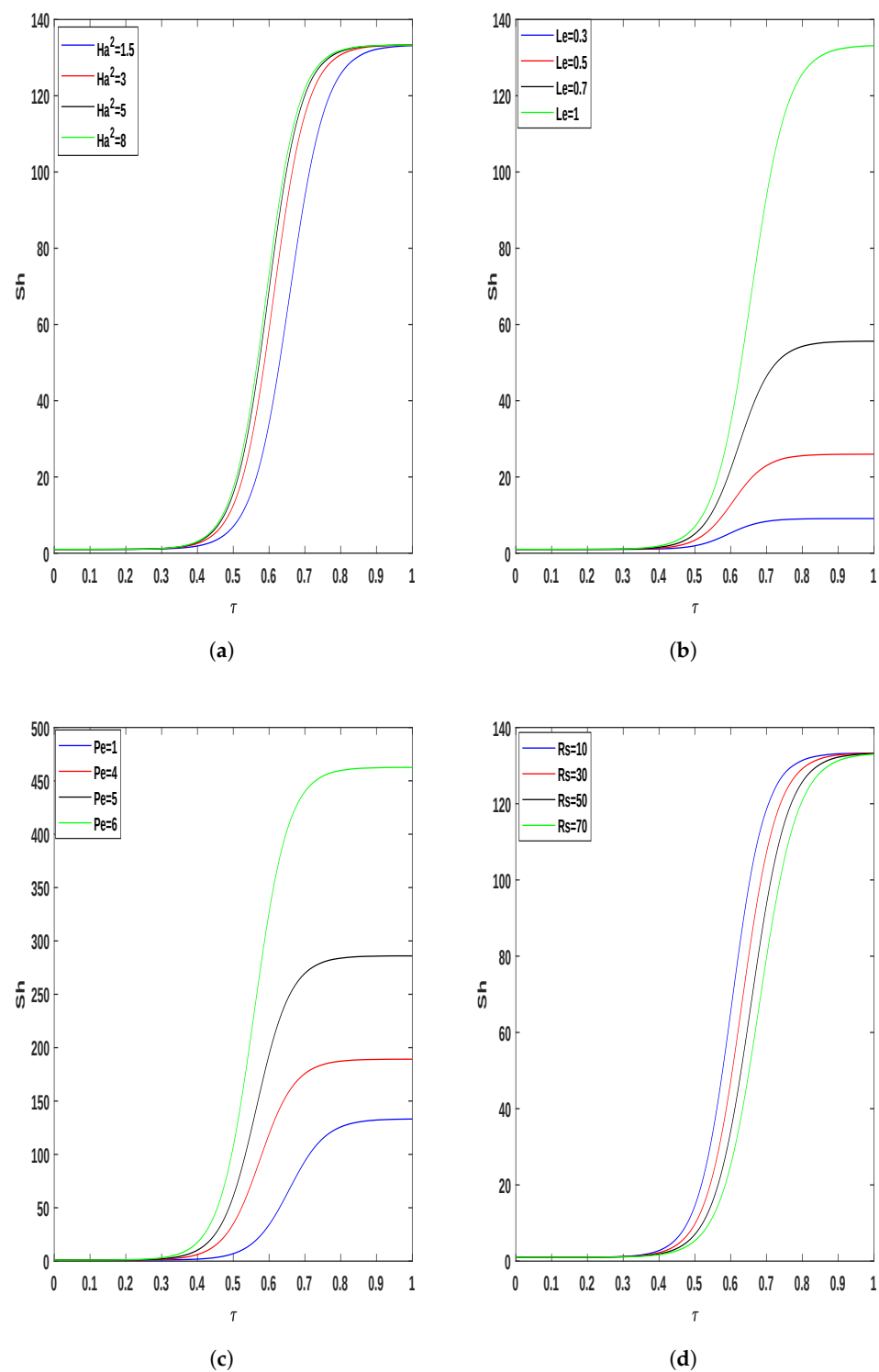


Figure 14. Plot of Sherwood number with τ for dilatant fluids for the fixed values of (a) $Le = 1$, $Pe = 1$, $Rs = 50$, (b) $Pe = 1$, $Ha^2 = 1.5$, $Rs = 50$, (c) $Le = 1$, $Rs = 50$, $Ha^2 = 1.5$ and (d) $Le = 1$, $Pe = 1$, $Ha^2 = 1.5$.

8. Conclusions

Thermohaline stability of a power-law fluid in a porous layer with magnetic field is studied. The system stability is investigated using linear instability and weakly non-linear instability. In linear stability analysis, a Galerkin method is employed to obtain

the analytical expression for Rayleigh number for stationary and oscillatory instability. Takens–Bogdanov bifurcation points are identified. The regions where stationary or oscillatory instability sets in are identified. From the linear stability results, we obtain that the Hartmann number and solute Rayleigh number have a stabilizing effect on the system. Furthermore, the critical Rayleigh number is a non-monotonic function of the Peclet number and power-law index. In weakly nonlinear stability, we derive the amplitude equation using multiple scale analysis. In order to study the heat and mass transport, we calculate the Nusselt and Sherwood number. The Hartmann number, Lewis number, and Peclet number enhance the heat and mass transport, whereas an opposite behavior is seen for the solute Rayleigh number.

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