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Cap-and-Trade Policy Design for Production and Abatement Decisions in a Closed-Loop Supply Chain

Zhaolong Bian, Fangting Zhong and Jian Cao *

College of Management, Zhejiang University of Technology, Hangzhou 310023, China;
2112004002@zjut.edu.cn (Z.B.); 211123040011@zjut.edu.cn (F.Z.)

* Correspondence: jcao@zjut.edu.cn

Abstract

Within closed-loop supply chains (CLSCs), limited attention has been given to firms' production and abatement decisions involving carbon permit transfer between an original equipment manufacturer (OEM) and an independent remanufacturer (IR) under a cap-and-trade policy (CTP). Several questions remain unresolved: How does CTP reshape production and abatement decisions through carbon permit transfer under binding emission constraints? Can such regulation reallocate abatement responsibilities between firms and generate environmental and economic benefits? To address these questions, this paper develops a constrained Cournot game model capturing competitive interactions between an OEM and an IR under regulation. The results show that CTP reallocates abatement responsibilities toward the firm with a lower abatement difficulty, inducing full abatement as a corner solution. When remanufactured products exhibit a high low-carbon level, a moderate increase in cap stringency promotes remanufacturing output and market share. By contrast, once full abatement is reached, stricter regulation expands output. Moreover, when remanufacturing features substantial cost savings and a high low-carbon level, CTP consistently improves social welfare. This study provides insights into how cap-and-trade policies shape production and abatement decisions in CLSCs.

Keywords: carbon permit transfer; Cournot game; remanufacturing; abatement; cap-and-trade policy

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1. Introduction

As global warming accelerates, carbon reduction has become an international priority. The manufacturing sector, accounting for approximately 24.4% of global carbon emissions [1], faces particularly stringent decarbonization pressures. In response, firms have increasingly relied on two major technological pathways to reduce emissions: abatement technologies, such as carbon capture, energy efficiency upgrades, and renewable energy integration, and remanufacturing technologies, which reduce emissions indirectly by extending product life cycles and recovering value from end-of-life products.

Remanufacturing has attracted growing attention as a cornerstone of circular and low-carbon economies. By prolonging product use and reducing the demand for virgin materials, remanufacturing can lower energy consumption, material use, and carbon

emissions by up to 60%, 70%, and 80%, respectively [2]. Leading firms such as Caterpillar and Hewlett-Packard, along with rapidly developing remanufacturing clusters in China, highlight its increasing strategic importance. Together, abatement and remanufacturing constitute two complementary yet distinct technological approaches toward deep decarbonization in manufacturing.

However, the large-scale adoption of these technologies cannot rely solely on firms' internal incentives. Government intervention is essential to stimulate abatement investments and align private decisions with social objectives [3]. Among various policy instruments, cap-and-trade policy (CTP) has emerged as one of the most widely implemented market-based mechanisms for controlling industrial emissions. As of 2024, CTP has been adopted in 36 countries or regions, covering about 19% of global greenhouse gas emissions [4]. By allocating emission permits under an aggregate cap and permitting trading among firms, CTP creates both scarcity and flexibility, thereby incentivizing cost-effective emissions reduction [5].

Despite its prevalence, existing CTP schemes largely overlook remanufacturing activities. In most carbon markets, remanufacturers are excluded from permit allocation and trading, with only a few exceptions, such as New Zealand and South Korea [6]. This exclusion is particularly problematic in closed-loop supply chains (CLSCs), where manufacturing and remanufacturing are jointly organized. CLSCs provide a natural setting for integrating abatement and circular economy objectives by linking forward and reverse flows, improving resource efficiency, and reducing environmental burdens [7]. Yet, when a CLSC comprises an original equipment manufacturer (OEM) and an independent remanufacturer (IR), the lack of tailored carbon regulation becomes more pronounced. In such systems, the OEM and the IR compete in overlapping product markets, face asymmetric technologies and costs, and may both undertake abatement investments. Product competition, demand cannibalization, and heterogeneous abatement capabilities jointly complicate firms' incentives, potentially weakening abatement efforts and undermining the effectiveness of circular practices [8].

These challenges point to a critical gap in existing policy design: CTP has not been systematically adapted to the characteristics of CLSCs, particularly those involving OEM–IR competition. Rather than replicating the full institutional complexity of real-world carbon markets, this study draws on the core regulatory logic of CTP and embeds it into a stylized CLSC framework. Specifically, emission permits are allocated to both the OEM and the IR, and the two firms are allowed to engage in inter-firm carbon permit transfer, generating a shadow price that reflects the marginal value and scarcity of carbon permits under the binding emissions cap. Within this setting, firms simultaneously make production, abatement investment, and permit transfer decisions under a binding emissions cap.

Against this background, this study seeks to answer two fundamental questions:

- (1) How do OEMs and IRs jointly determine their abatement investment and carbon permit transfer decisions in a competitive CLSC?
- (2) Can such a CTP design improve environmental outcomes and economic performance, including firms' profits, generalized consumer surplus, and social welfare?

To address these questions, we employ Cournot competition and optimization theory to develop an integrated model of production, abatement, and permit transfer in a CLSC. The analysis characterizes equilibrium outcomes, derives the boundary conditions under which different strategies emerge, and examines how firms' decisions adjust with changes in cap stringency.

This study contributes to the literature in three main ways:

- (1) Within a CLSC framework, we use a shadow price to characterize the marginal value and scarcity of carbon permits arising from inter-firm permit transfer between the OEM and the IR.
- (2) We explicitly allow for full abatement to arise as a corner solution, rather than restricting the analysis to interior solutions with partial abatement.
- (3) We jointly account for the cost-saving and low-carbon attributes of remanufactured products and evaluate the effectiveness of CTP from multiple perspectives, including firms' profits, generalized consumer surplus that internalizes environmental costs, and social welfare. An extended numerical analysis based on the Steyr 615-67 engine further enhances the practical relevance of the results.

The remainder of this paper is organized as follows. Section 2 presents the related literature review and explains our contributions. Section 3 presents the modeling framework, including the background, assumptions, and construction of the baseline and CTP-regulated models. Section 4 derives the equilibrium outcomes and analyzes the policy-induced adjustments in firms' decisions, profits, generalized consumer surplus, and social welfare under CTP. Section 5 calibrates the model using a real-world remanufacturing case to illustrate the numerical implications of the theoretical results. Section 6 provides managerial and policy insights based on the analytical findings. Section 7 concludes the paper and outlines directions for future research.

2. Literature Review

The literature pertinent to this paper primarily focuses on three aspects: (1) CLSC management under CTP, (2) interactions between abatement and remanufacturing, and (3) competitive strategies of firms in low-carbon contexts.

2.1. CLSC Management Under CTP

As CTP has been increasingly implemented worldwide, empirical evidence indicates that the EU ETS enhances productivity primarily among technologically advanced firms [9] and accelerates the substitution of renewable energy for fossil fuels, particularly as carbon prices rise [10]. Recent studies further reveal that China's national carbon market improves firms' ESG performance through governance and financing channels [11], while design-oriented evidence suggests that expanding permit auctioning increases market activity without offsetting efficiency gains [12]. Although existing studies provide valuable ex post evidence on carbon market effectiveness, they offer limited guidance on how CTP should be designed to support circular operations. Against this backdrop, recent research has shifted from low-carbon outcomes toward circularity, adopting a forward-looking perspective to examine the design and implementation of CTP in CLSCs.

Huang and Lu [13] analyzed multiple contractual mechanisms in CLSCs under CTP constraints using a Stackelberg game framework and showed that, although CTP facilitates recycling and reduces carbon emissions, it comes at the expense of consumer surplus. Focusing on regulatory enforcement, Yang et al. [14] examined firms' non-compliance behavior in CLSCs under CTP through a network game model, highlighting the critical role of penalty design in ensuring emissions compliance and sustaining the effectiveness of carbon markets. Extending the policy toolbox, Sun and Liu [15] investigated the joint effects of CTP, government subsidies, and consumer education on OEM–IR remanufacturing competition and demonstrated that carbon pricing and subsidies can effectively stimulate OEM participation in remanufacturing, while demand-side interventions generate heterogeneous impacts across firms. Recently, Fang et al. [16] explored alternative carbon permit allocation schemes in an OEM–IR setting; however, their analysis regulated only the OEM and left the IR outside the regulatory scope.

Overall, existing studies on CLSCs under CTP have primarily focused on regulatory design, examining how alternative policy instruments affect firms' decisions and evaluating regulatory effectiveness. However, as carbon markets continue to expand their coverage, CLSCs are increasingly subject to binding permit constraints, under which inter-firm permit transfer and the scarcity-driven shadow valuation of permits become relevant decision dimensions that remain underexplored in the literature.

2.2. Abatement Investment

As an increasing number of firms have committed to ambitious abatement targets (e.g., Apple [17] and Lenovo [18]), the question of how to effectively incentivize corporate abatement investments—such as carbon capture and storage technologies [3]—has become a growing research focus. In this context, Fan et al. [19] compared carbon taxes and CTP under correlated demand and carbon price uncertainty, showing that the flexibility inherent in carbon trading may weaken incentives for abatement investment, whereas carbon taxes can simultaneously enhance abatement investment, firms' profits, and consumer surplus, and reduce total emissions when the correlation lies within a moderate range.

Driven by the broader transition toward a circular economy, subsequent research has extended this line of inquiry to explore the interaction between abatement investment and remanufacturing. Some studies have adopted financing and technological perspectives, showing that the effectiveness of abatement investments in promoting remanufacturing depends critically on the specific remanufacturing strategy [20] and the relative carbon advantages of remanufactured products [21]. However, these studies have typically assumed that new and remanufactured products are sold in separate markets, thereby neglecting product competition and abatement efforts for remanufactured products. Building on this framework, Ding et al. [22] and Luo et al. [23] explicitly incorporated product cannibalization and the impact of abatement investments on remanufactured emissions, finding that abatement investments are more effective in competitive settings [22] and become essential when emission savings from remanufacturing are insufficient [23].

Although existing studies have jointly examined abatement investment and remanufacturing, few have considered the possibility of full abatement, as most have focused on partial abatement. From a modeling perspective, it is therefore important to explicitly consider full abatement as a corner solution, because production and remanufacturing decisions under the full-abatement regime have not been systematically examined in the existing CLSC literature. This boundary case thus warrants separate analysis.

2.3. Competitive Strategies of Firms in Low-Carbon Contexts

A growing body of literature examines how low-carbon regulation reshapes firms' strategic interactions by embedding environmental objectives into competitive decision-making.

Under a CTP framework, Luo et al. [24] showed that co-opetition can improve short-term profitability and reduce emissions, although the environmental gains are achieved mainly through demand suppression rather than green technology investment. Focusing on technology diffusion, Chen et al. [25] further demonstrated that the optimal choice of low-carbon licensing schemes hinges on the trade-off between technology transfer benefits and intensified competition. Building on this line of research, Anand and Giraud-Carrier [26] and Huang et al. [27] explicitly incorporated inter-firm carbon permit transfer decisions. However, these studies primarily concentrated on competition among OEMs. In contrast, studies on OEM–IR competition have mainly examined the impacts of government regulations on firms' decisions, such as government subsidies [28], carbon taxes [29], and CTP [30], or have focused on inter-firm contractual and strategic interaction

mechanisms, including licensing schemes [31], contract design [32], and abatement investments [33].

Although the above studies have examined inter-firm competition, they have paid limited attention to the intrinsic attributes of remanufactured products. In particular, remanufacturing technologies should be evaluated not only for their economic advantages in cost savings but also for their low-carbon benefits in emissions reduction. Considering these two dimensions jointly can help clarify firms' low-carbon management incentives and inform the analysis of regulatory design.

2.4. Contributions

Taken together, the review in Sections 2.1–2.3 indicates that few studies have examined carbon permit transfer decisions between the OEM and IR within a closed-loop supply chain, nor have they differentiated firms' abatement strategies to explore the interaction between abatement and remanufacturing. To fill this gap, this study designs a CTP framework within a CLSC that allows an OEM and an IR to engage in inter-firm carbon permit transfer, giving rise to a shadow price that reflects the scarcity of permits under an aggregate emissions cap. We investigate the conditions under which firms' equilibrium decisions emerge and analyze how these decisions adjust with changes in cap stringency. The policy impacts are then evaluated with respect to firms' profits, generalized consumer surplus, and social welfare, thereby providing a systematic assessment of policy-induced outcome changes.

Distinct from existing cap-and-trade CLSC models, this study further considers the reallocation of abatement responsibilities within the CLSC and explicitly examines firms' decision mechanisms when full abatement arises as a corner solution. In particular, policy effectiveness is assessed by accounting jointly for the cost-saving and emission-reduction attributes of remanufactured products, thereby clarifying how regulatory design shapes firms' outcomes. For clarity, Table 1 summarizes the main contributions of this study.

Table 1. Position of this research in the literature.

Studies	CTP	CLSC	Full Abatement	Inter-Firm Permit Transfer	IR Competition
Perdiguero and Sanz [10]	√				
Huang and Lu [13]	√	√			
Fan et al. [19]	√		√		
Anand and Giraud-Carrier [26]	√		√	√	
Cao and Shao [32]		√			√
Sun et al. [33]	√	√			√
This study	√	√	√	√	√

3. Methodology

3.1. Background

To clarify the analytical setting, Figure 1 depicts the regulatory structure considered in this study. The government imposes an aggregate emissions cap and allocates carbon permits to both the OEM and the IR, thereby constraining their production and remanufacturing decisions. The OEM produces new products, while the IR engages in remanufacturing; both firms compete in the downstream market, thereby generating demand cannibalization. Permit transfer between the OEM and the IR is permitted as a compliance mechanism, with permit exchanges giving rise to a shadow price that reflects the marginal value and scarcity of carbon permits under the binding emissions cap. This framework links emission constraints, abatement decisions, and product market competition, and

provides a tractable setting for analyzing firms' equilibrium behavior under different regulatory configurations. Table 2 summarizes the key variables and parameters used in the model.

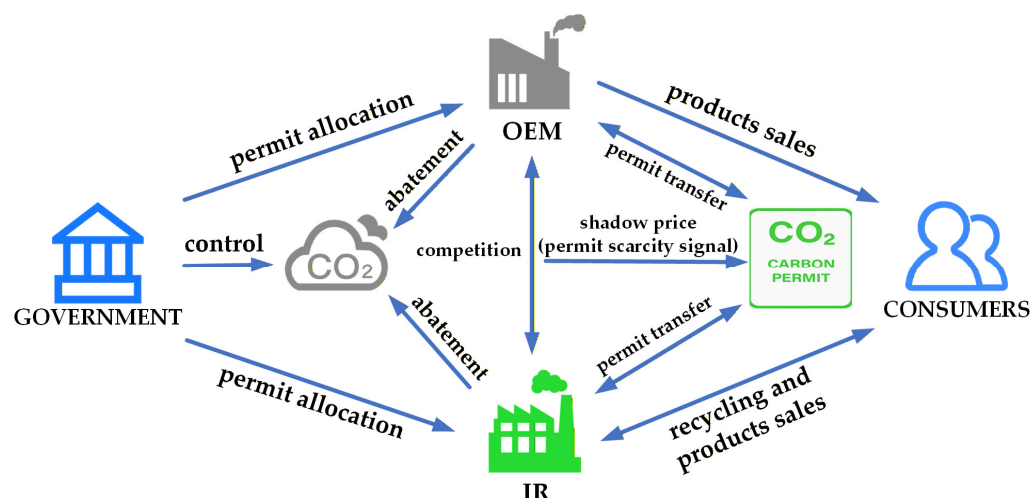


Figure 1. Analytical framework of the CLSC under CTP.

Table 2. Variables and parameters.

Symbol	Definition
Variables	
$\lambda_n (\lambda_r)$	OEM (IR) abatement rate
$q_n (q_r)$	New product (remanned product) output
$p_n (p_r)$	New product (remanned product) price
$v_n (v_r)$	Carbon permit transfer volume of the OEM (IR)
π_M, π_R, π_T	Profits of the OEM, IR, and CLSC
p_e	Shadow price of carbon permits
Parameters	
Δ	Cost advantage of remanufacturing
δ	Consumer acceptance of remanned products
θ	Low-carbon level of remanned products
G	Total permits allocated by the government to the CLSC
$G_M (G_R)$	Initial carbon permits allocated by the government to the OEM (IR)
$\eta_n (\eta_r)$	Abatement difficulty of the OEM (IR)

3.2. Assumptions

To facilitate model development and reflect key characteristics of the carbon-regulated CLSC, the following assumptions are established.

Assumption 1. To capture the cannibalization effect between new and remanned products, and to reflect horizontal competition between the OEM and the IR in the downstream market, the following inverse demand function is derived, following Ferrer and Swaminathan [34].

$$p_n = 1 - q_n - \delta q_r \quad (1)$$

$$p_r = \delta (1 - q_n - q_r) \quad (2)$$

Assumption 2. This study employs a single-period steady-state model to analyze interactions under CTP, thereby avoiding initial and terminal time effects for improved real-world applicability [8]. This assumption is further supported by empirical practice. For example, the Hejian remanufacturing cluster in Cangzhou, China—designated as a national demonstration base—has established a nationwide logistics network, full-process traceability, and mature remanufacturing technologies, enabling stable and large-scale remanufacturing of used generators, starters, and drill bits. In such mature settings, production and remanufacturing activities operate in a stable and repetitive manner. Consequently, a mature production horizon can be viewed as a sequence of identical steady-state periods, implying that optimizing profit in a representative single period is equivalent to optimizing aggregate profit over the entire mature production cycle [35].

Assumption 3. Let the unit production costs of new and remanufactured products be $\Delta + c_r$ and c_r , where Δ denotes the cost savings from remanufacturing (i.e., the “cost advantage of remanufacturing”). A larger Δ indicates a more pronounced remanufacturing cost advantage. Since this study focuses on firms’ competition, to prevent the OEM or the IR from exiting the market, the following condition must be satisfied: $(1-\delta)c_r/\delta < \Delta < (2-\delta-c_r)/2$.

Assumption 4. Let the initial per-unit emissions of new and remanufactured products be e_n and $(1-\theta)e_n$, where $\theta \in (0,1)$ denotes “the low-carbon level of remanufactured products”. A higher θ indicates that remanufactured products are inherently less carbon-intensive than new products due to material reuse and process characteristics. Under CTP, the OEM and IR further invest in technologies to reduce emissions beyond these intrinsic levels. The achieved reductions are $e_n\lambda_n$ and $e_n(1-\theta)\lambda_r$, yielding actual emissions of $e_n(1-\lambda_n)q_n$ and $e_n(1-\theta)(1-\lambda_r)q_r$, where q_n and q_r are output levels. Abatement costs are modeled as $\eta_n(e_n\lambda_nq_n)^2$ and $\eta_r(e_n(1-\theta)\lambda_rq_r)^2$ [26], where η_n and η_r are carbon reduction cost coefficients for the OEM and IR, respectively. These coefficients reflect the difficulty of abatement, with larger values indicating a higher difficulty. Accordingly, η_n and η_r are hereafter referred to as the abatement difficulty of the OEM and IR.

3.3. Models

Before formulating the specific models, it is necessary to clarify the choice of the competitive framework. This study adopts a Cournot competition model to characterize the strategic interaction between the OEM and the IR. The rationale is twofold.

First, Cournot competition explicitly captures quantity-based rivalry under decentralized decision-making, where each firm determines its production and abatement strategies without observing the rival’s specific choices *ex ante*. This information structure is well-suited to CLSCs in which the OEM and the IR compete in overlapping product markets and make decisions independently based on common market fundamentals. In contrast, Stackelberg models typically assume a clear leader–follower hierarchy with observable prior moves, which is more representative of market environments with pronounced asymmetries in decision power, where OEMs hold dominant market positions and remanufacturing activities are relatively immature.

Second, this study is forward-looking in that it focuses on a more mature stage of the remanufacturing industry, where IRs have stable access to cores, accumulated technological capabilities, and relatively stable production conditions. Under such circumstances, the OEM and the IR are better characterized as strategically comparable players, rather than as leader and follower. This modeling choice is also consistent with Assumption 2, which ensures sufficient core availability for the IR and rules out supply-side instability.

Therefore, employing a Cournot framework allows us to more accurately reflect horizontal competition between the OEM and the IR and to examine how CTP, abatement investment, and carbon permit transfer under a binding emissions cap jointly determine firms' abatement responsibilities and production decisions. Within this framework, Sections 3.3.1 and 3.3.2 respectively formulate the benchmark model and the CTP-regulated model.

3.3.1. Benchmark Model Without CTP

Without regulation, the OEM and the IR are not subject to CTP, and the following optimization problems define their objectives:

$$\begin{aligned} \max_{\lambda_n, q_n} \pi_M &= (p_n - c_r - \Delta)q_n - \eta_n (\lambda_n q_n)^2 \\ \text{s.t. } q_n &> 0, 0 \leq \lambda_n \leq 1 \end{aligned} \quad (3)$$

$$\begin{aligned} \max_{\lambda_r, q_r} \pi_R &= (p_r - c_r)q_r - \eta_r ((1-\theta)\lambda_r q_r)^2 \\ \text{s.t. } q_r &> 0, 0 \leq \lambda_r \leq 1 \end{aligned} \quad (4)$$

In this model, the OEM and IR earn revenues of $(p_n - c_r - \Delta)q_n$ and $(p_r - c_r)q_r$, where p_n and p_r are defined in Equations (1) and (2). Their abatement costs are $\eta_n (\lambda_n q_n)^2$ and $\eta_r ((1-\theta)\lambda_r q_r)^2$. To ensure meaningful results, production and reduction rates must satisfy $q_n > 0$, $q_r > 0$ and $0 \leq \lambda_n \leq 1$, $0 \leq \lambda_r \leq 1$. Solving the first-order conditions in Equations (3) and (4) yields the following unregulated equilibrium: $q_n^* = (2 - \delta - 2\Delta - c_r)/(4 - \delta)$, $\lambda_n^* = 0$, $q_r^* = (\delta + \Delta\delta + c_r\delta - 2c_r)/(4\delta - \delta^2)$, $\lambda_r^* = 0$. To keep $q_n^* > 0$ and $q_r^* > 0$, $(1 - \delta)c_r/\delta < \Delta < (2 - \delta - c_r)/2$ must hold (see Assumption 3). These results show that, in the absence of carbon regulation, firms optimally choose zero abatement and rely solely on production decisions, leading to no emission reduction incentives, which motivates the introduction of CTP.

Based on the equilibrium, the OEM and IR's profits are $\pi_M^* = (2\Delta + c_r + \delta - 2)^2/(\delta - 4)^2$, $\pi_R^* = (\delta + \Delta\delta - 2c_r + \delta c_r)^2/(\delta(\delta - 4)^2)$, $\pi_T^* = \pi_M^* + \pi_R^*$. Total carbon emissions are $q_n^* + (1 - \theta)q_r^* = (\delta(3 - \Delta - \delta - \theta - \Delta\theta) + (2\theta - \delta\theta - 2)c_r)/(\delta(4 - \delta)) := G_{\max}$. For analysis, this unregulated total emission level, denoted $G_{\max}$, represents the maximum emission level that arises in the absence of carbon regulation and serves as a reference point for evaluating how CTP constrains emissions and reallocates abatement responsibilities in subsequent analysis.

3.3.2. Model with CTP and Endogenous Permit Transfer

This section analyzes the OEM and IR's equilibrium decisions under CTP, where they must optimize abatement, production, and carbon permit transfer decisions. The following optimization problems define their objectives:

$$\begin{aligned} \max_{\lambda_n, q_n, v_n} \pi_M &= (p_n - c_r - \Delta)q_n - \eta_n (q_n \lambda_n)^2 + p_e v_n \\ \text{s.t. } &\begin{cases} 0 \leq \lambda_n \leq 1 \\ q_n(1 - \lambda_n) \leq G_M - v_n \end{cases} \end{aligned} \quad (5)$$

$$\begin{aligned} \max_{\lambda_r, q_r, v_r} \pi_R &= (p_r - c_r)q_r - \eta_r \left((1-\theta)q_r \lambda_r \right)^2 + p_e v_r \\ \text{s.t.} \quad &\begin{cases} 0 \leq \lambda_r \leq 1 \\ (1-\theta)q_r (1-\lambda_r) \leq G_R - v_r \end{cases} \end{aligned} \quad (6)$$

Here, p_e is the shadow price. The shadow price arises endogenously from the Nash equilibrium of the OEM–IR game as the Lagrange multiplier associated with the binding emission constraint, reflecting the marginal economic value of carbon permits under a limited aggregate supply. It captures the scarcity of carbon permits within the CLSC, rather than a market-clearing price generated by an explicit emissions trading system. v_n and v_r are the OEM's and IR's permit transfer volumes, respectively. The terms $p_e v_n$ and $p_e v_r$ represent their respective revenues or costs from permit transfer. A firm sells surplus permits if emissions are below its allocation ($v_n > 0$ or $v_r > 0$), or buys additional permits if exceeding it ($v_n < 0$, $v_r < 0$). The constraints $q_n(1-\lambda_n) \leq G_M - v_n$ and $(1-\theta)q_r(1-\lambda_r) \leq G_R - v_r$ ensure that firms' emissions remain within their post-transfer permit holdings. Let $\hat{G}_M = G_M - v_n$ and $\hat{G}_R = G_R - v_r$ denote the final permit holdings. It satisfies $\hat{G}_M + \hat{G}_R = G$, where G denotes the total carbon permit (emissions cap) allocated by the government. For explanations of the remaining components in Equations (5) and (6), please refer to Section 3.3.1.

As noted in Section 3.3.1, the total permit must satisfy $G \in (0, G_{\max})$ for the emissions constraint to be binding; otherwise, firms behave as in the unregulated scenario and do not choose permit transfer. The maximum unregulated emissions are $G_{\max} = (\delta(3-\Delta-\delta-\theta-\Delta\theta) + (2\theta-\delta\theta-2)c_r) / (\delta(4-\delta))$ with corresponding equilibria summarized in the Proposition 1 in Section 4.1. When the total permit G is below this benchmark level, carbon permits become scarce, giving rise to a positive shadow price that governs firms' abatement and permit transfer decisions under CTP. All threshold expressions are provided in Appendix A, and all proofs of the Proposition 1 and the corollaries are presented in Appendix B.

4. Results

4.1. Equilibrium Outcomes and Strategic Responses

This subsection analyzes the equilibrium outcomes induced by the CTP framework in the CLSC. By characterizing firms' optimal responses to binding emission constraints and carbon permit transfer, we derive the resulting equilibrium decisions. These analytical results, summarized in the following Proposition 1, illustrate how abatement responsibilities, production decisions, and permit transfer patterns are endogenously determined under CTP and provide the basis for the subsequent analysis of firms' economic performance and social outcomes.

Proposition 1. *Under CTP, the OEM and the IR exhibit three distinct equilibria (I–III). The equilibrium expressions are presented in Table 3, and the firms' decision conditions are reported in Table 4.*

Table 3. Equilibrium expressions under CTP.

Equilibrium I

$$\lambda_n^* = \frac{\eta_r Z_2^1}{Z_3^1}, \quad \lambda_r^* = \frac{\eta_n Z_2^1}{(1-\theta)Z_4^1}, \quad q_n^* = \frac{Z_3^1}{Z_1^1}, \quad q_r^* = \frac{Z_4^1}{Z_1^1}, \quad \hat{G}_M^* = \frac{Z_3^1 - \eta_r Z_2^1}{Z_1^1}, \quad \hat{G}_R^* = G - \hat{G}_M^*, \quad p_e^* = \frac{2\eta_n \eta_r Z_2^1}{Z_1^1}$$

Equilibrium II

$$\lambda_n^{II*} = \frac{Z_2^{II} - GZ_3^{II}}{Z_2^{II}}, \quad \lambda_r^{II*} = 1, \quad q_n^{II*} = \frac{Z_2^{II}}{Z_3^{II}}, \quad q_r^{II*} = \frac{(\delta - c_r)Z_3^{II} - \delta Z_2^{II}}{2Z_1^{II}Z_3^{II}}, \quad \widehat{G}_M^{II*} = G, \quad \widehat{G}_R^{II*} = 0, \quad p_e^{II*} = \frac{2\eta_n(Z_2^{II} - GZ_3^{II})}{Z_3^{II}}$$

Equilibrium III

$$\lambda_n^{III*} = 1, \quad \lambda_r^{III*} = \frac{(1-\theta)Z_1^{III} - GZ_3^{III}}{(1-\theta)Z_1^{III}}, \quad q_n^{III*} = \frac{(1-\Delta - c_r)Z_3^{III} - \delta Z_1^{III}}{2(1+\eta_n)Z_3^{III}}, \quad q_r^{III*} = \frac{Z_1^{III}}{Z_3^{III}}, \quad \widehat{G}_M^{III*} = 0, \quad \widehat{G}_R^{III*} = G, \quad p_e^{III*} = \frac{2\eta_r((1-\theta)Z_1^{III} - GZ_3^{III})}{Z_3^{III}}$$

$$\begin{aligned} Z_1^I &= \eta_n(\delta(4-\delta) + 4\eta_r(\theta^2 + (\delta-2)\theta + 1)) + \eta_r\delta(4-\delta) & (Z_1^I > 0), \\ Z_2^I &= -G\delta(4-\delta) - \delta^2 + \delta(3-\Delta) - 2c_r - \theta(\delta(1+\Delta+c_r) - 2c_r) \\ Z_3^I &= 2\eta_r\eta_n G\delta(\theta+1) + \eta_n(\delta(2-2\Delta-c_r-\delta) - 2\eta_r(1-\Delta-c_r)(1-\theta)(\theta-\theta_1)) + \eta_r\delta(2-2\Delta-c_r-\delta) \\ Z_4^I &= \eta_n(\delta(1+\Delta+c_r) - 2c_r - 2\eta_r(1-\Delta-c_r)(\theta_1-\theta)) + \eta_r(\delta(1+\Delta+c_r) - 2c_r) - 4\eta_n\eta_r G(\theta-\theta_3) \\ Z_1^{II} &= \delta + \eta_r(1-\theta)^2 \quad (Z_1^{II} > 0), \quad Z_2^{II} = 4GZ_1^{II}\eta_n + 2(1-\Delta-c_r)(1-\theta)^2\eta_r + \delta(2-2\Delta-c_r-\delta) \quad (Z_2^{II} > 0), \\ Z_3^{II} &= (4\delta + 4\eta_r(1-\theta)^2)\eta_n + 4\eta_r(1-\theta)^2 + (4\delta - \delta^2) \quad (Z_3^{II} > 0), \\ Z_1^{III} &= \delta(1+\Delta+c_r) - 2c_r + 2\eta_n(\delta-c_r) + 4G(1+\eta_n)(1-\theta)\eta_r \quad (Z_1^{III} > 0). \end{aligned}$$

Depending on the low-carbon level θ of remanufactured products, abatement difficulty η_j ($j = n, r$), and the total permit G allocated by the government, the CLSC exhibits different equilibrium regimes, as shown in Table 4.

Table 4. Equilibrium selection under different product, technology, and policy conditions.

Product and Technology Dimension	Policy Dimension	Equilibrium Type	Decision Characteristics
Case 1: $0 < \theta \leq \max\{\theta_1, 0\}$ or $\max\{\theta_1, 0\} < \theta < 1$ and $0 < \eta_r < \eta_{r1}$	Case 1-i $0 < \eta_n < \eta_{n1}$	$0 < G < G_1$ Equilibrium III	$\lambda_n^{III*} = 1, \lambda_r^{III*} < 1;$ $\widehat{G}_M^{III*} = 0, \widehat{G}_R^{III*} = G$
		$G_1 \leq G < G_{\max}$ Equilibrium I	$0 < \lambda_n^{I*}, \lambda_r^{I*} < 1; 0 < \widehat{G}_M^{I*}, \widehat{G}_R^{I*} < G$
	Case 1-ii $\eta_n \geq \eta_{n1}$	$0 < G < G_2$ Equilibrium II	$\lambda_n^{II*} < 1, \lambda_r^{II*} = 1;$ $\widehat{G}_M^{II*} = G, \widehat{G}_R^{II*} = 0$
		$G_2 \leq G < G_{\max}$ Equilibrium I	$0 < \lambda_n^{I*}, \lambda_r^{I*} < 1; 0 < \widehat{G}_M^{I*}, \widehat{G}_R^{I*} < G$
Case 2: $\max\{\theta_1, 0\} < \theta < 1$ and $\eta_r \geq \eta_{r1}$	$\eta_n > 0$	$0 < G < G_1$ Equilibrium III	$\lambda_n^{III*} = 1, \lambda_r^{III*} < 1;$ $\widehat{G}_M^{III*} = 0, \widehat{G}_R^{III*} = G$
		$G_1 \leq G < G_{\max}$ Equilibrium I	$0 < \lambda_n^{I*}, \lambda_r^{I*} < 1; 0 < \widehat{G}_M^{I*}, \widehat{G}_R^{I*} < G$

Table 4 reports equilibrium abatement and carbon permit transfer decisions under the CTP. The equilibrium patterns can be understood through three interrelated dimensions: the product dimension, the technology dimension, and the policy dimension.

(1) Product dimension: Role of the low-carbon level of remanufactured products

From the product dimension, the low-carbon level of remanufactured products, measured by θ , affects how remanufactured products compare with new products in terms of emissions intensity, which in turn influences firms' abatement and production decisions under a binding emissions cap. A higher θ implies that remanufactured products are intrinsically less carbon-intensive than new products. However, differences in product carbon characteristics alone do not determine firms' abatement responsibilities or production outcomes, which also depend on the abatement difficulty (η_n, η_r).

When the IR either faces a strong need to actively reduce emissions to enhance competitiveness, or possesses both a high low-carbon product advantage and sufficient

capacity to further reduce emissions (corresponding to Case 1), the remanufacturing side plays an active role in abatement and carbon permit supply. By contrast, when remanufactured products are intrinsically low-carbon, and the IR faces a high abatement difficulty (corresponding to Case 2), the IR's ability to contribute to additional emission cuts is limited. In such industries, abatement efforts are primarily undertaken by the OEM, while the IR competes in the market by leveraging the stronger low-carbon advantage of remanufactured products.

(2) Technology dimension: Reallocation of abatement responsibilities

From the technology dimension, firms' equilibrium strategies are primarily shaped by their relative abatement difficulty, captured by η_n and η_r . When one firm has a lower abatement difficulty, abatement responsibilities are reallocated toward that firm through permit transfer and the associated shadow price.

If the OEM faces a relatively low abatement difficulty (corresponding to Case 1-i or Case 2), stringent regulation induces the OEM to undertake deeper or even full abatement, while the IR adjusts mainly through partial abatement and carbon permit purchases (Equilibrium III). In contrast, when the OEM's abatement difficulty is high (Case 1-ii), the IR becomes the primary emissions reducer under a tight permit regime, supplying carbon permits to the OEM through permit transfer (Equilibrium II). These outcomes reflect how differences in the abatement difficulty give rise to a reallocation of abatement responsibilities within the CLSC, rather than an ex ante assignment of abatement tasks.

It is worth noting that full abatement arises as a corner solution in the equilibrium analysis. In this case, the firm reduces its emissions to zero at the production stage, implying that it undertakes the highest level of abatement effort under the given emissions cap.

(3) Policy dimension: Impact of total carbon permit allocation

From the policy dimension, the total carbon permit G allocated by the government determines the stringency of the binding emissions constraint, which in turn shapes firms' equilibrium responses under CTP. Under a tight permit regime ($0 < G < G_j$, $j = 1, 2$), abatement responsibilities are reallocated toward the firm with lower marginal abatement costs, while the other firm adjusts primarily through permit transfer to satisfy the emissions constraint. When the permit is relatively relaxed ($G_j \leq G < G_{\max}$, $j = 1, 2$), both firms engage in partial abatement, and permit transfer plays a limited role in equilibrium.

Overall, the equilibrium outcomes in Table 4 reflect a structured interaction between (i) the low-carbon level of remanufactured products, (ii) the abatement difficulty, and (iii) the policy stringency. This interpretation provides a mechanism-based explanation of how a binding emissions cap leads to a reallocation of abatement responsibilities and permit transfer positions within the CLSC across different industrial settings.

While Table 4 identifies how equilibrium abatement and permit transfer positions are determined by product-level low-carbon attributes and firms' technological capability, these outcomes depend critically on the stringency of carbon regulation. To examine how firms' strategies adjust as regulatory pressure changes, we next conduct sensitivity analysis with respect to the total carbon permit G . Corollary 1 and Table 5 summarize how firms' technological, production, and permit transfer responses evolve as the cap becomes more stringent.

Corollary 1. *As cap stringency increases, equilibrium abatement rates, production decisions, and permit transfer positions adjust accordingly, as summarized in Table 5.*

Table 5. Equilibrium technological, production, and permit transfer responses to cap stringency.

Scenario	Cap Stringency	Technology Responses	Production Responses	Permit Transfer Responses
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		λ_n^*	λ_r^*	q_n^*	q_r^*	$q_r^* - q_n^*$	$\hat{G}_M^{j*}$	$\hat{G}_R^{j*}$	p_e^{j*}
Case 1-i	Low	↑	↑	↓	Situation A	Situation B	↓	↓	↑
	High	→	↑	↑	↓	↓	→	↓	↑
Case 1-ii	Low	↑	↑	↓	Situation A	Situation B	↓	↓	↑
	High	↑	→	↓	↑	↑	↓	→	↑
Case 2	Low	↑	↑	↓	Situation A	Situation B	↓	Situation C	↑
	High	→	↑	↑	↓	↓	→	↓	↑

1. “Low” and “High” refer to the stringency of the emissions cap, where “Low” corresponds to a more relaxed cap (i.e., a higher total permit G), and “High” indicates a more stringent cap (i.e., a lower total permit G). ↑, ↓, and → respectively indicate that the variable is positively, negatively, or not correlated with the level of cap stringency. 2. The column $(q_r^* - q_n^*)$ is to analyze the relative market share of remanufactured products compared to new products. A larger value of $(q_r^* - q_n^*)$ suggests a higher proportion of remanufactured products in the market. 3. Situation A: $\begin{cases} \downarrow, \theta < \theta_3 \\ \rightarrow, \theta = \theta_3 \\ \uparrow, \theta > \theta_3 \end{cases}$; Situation B: $\begin{cases} \downarrow, \theta < \theta_2 \\ \rightarrow, \theta = \theta_2 \\ \uparrow, \theta > \theta_2 \end{cases}$; Situation C: $\begin{cases} \uparrow, \theta > \theta_3 \text{ and } \eta_r > \eta_{r,2} \\ \rightarrow, \theta > \theta_3 \text{ and } \eta_r = \eta_{r,2} \\ \downarrow, \text{otherwise} \end{cases}$.

Table 5 reports the sensitivity of equilibrium outcomes to changes in cap stringency, grouped into technology, production, and permit transfer dimensions.

(1) Technology responses: Adjustment of abatement rates

Under “Low Cap Stringency”, both firms adopt interior abatement levels in equilibrium. As cap stringency increases within this range, equilibrium abatement rates adjust smoothly rather than exhibiting immediate transitions to corner solutions.

Under “High Cap Stringency”, equilibrium outcomes differ depending on firms’ relative abatement difficulty. When the OEM exhibits a relatively low abatement difficulty (Case 1-i or Case 2), the equilibrium is characterized by the OEM attaining full abatement as a corner solution, so its abatement rate remains fixed and does not vary with further increases in cap stringency. In contrast, the IR’s equilibrium abatement rate continues to rise along the interior solution as the cap becomes more stringent. By contrast, when the OEM’s abatement difficulty is relatively high (Case 1-ii), equilibrium outcomes are characterized by the IR reaching full abatement as a corner solution, meaning that the IR’s abatement rate no longer varies with further increases in cap stringency.

(2) Production responses: Reallocation between new and remanufactured products

Under “Low Cap Stringency”, the OEM consistently reduces its output, whereas the IR adjusts its remanufacturing output flexibly according to the low-carbon level of remanufactured products. Specifically, when the low-carbon level is relatively high, the IR expands remanufacturing output; when it is relatively low, the IR contracts output (as illustrated by Situation A in Table 5). Correspondingly, a higher low-carbon level of remanufactured products also contributes to an increase in their market share (as illustrated by Situation B in Table 5).

Under “High Cap Stringency”, when the OEM’s abatement difficulty is relatively low (Case 1-i or Case 2), equilibrium production allocations shift toward a higher output of new products, while remanufacturing output is reduced in equilibrium. As a result, the relative market share of remanufactured products declines. In contrast, when the OEM faces a relatively high abatement difficulty (Case 1-ii), equilibrium production allocations shift away from new products and toward remanufactured products. In this regime, the equilibrium output of new products decreases, while remanufacturing output increases, leading to a higher relative market share of remanufactured products.

(3) Permit transfer responses: Post-transfer holdings and shadow price

Under “Low Cap Stringency”, equilibrium permit transfer outcomes differ systematically between Case 1 and Case 2, while the shadow price increases monotonically as cap stringency rises. In Case 1, as cap stringency increases, the equilibrium post-transfer permit holdings of both the OEM and the IR decline continuously. In Case 2, the adjustment of post-transfer permit holdings becomes asymmetric. While the OEM’s equilibrium permit holdings continue to decline as cap stringency increases, the IR’s permit holdings do not necessarily decrease (see Situation C in Table 5). In particular, when the low-carbon level of remanufactured products is relatively high and the IR faces a high abatement difficulty, equilibrium outcomes may involve an increase in the IR’s post-transfer permit demand despite a tighter cap.

Under “High Cap Stringency”, when the OEM’s abatement difficulty is relatively low (Case 1-i or Case 2), the OEM attains full abatement as a corner solution and therefore transfers all permits to the IR. As a result, the OEM’s post-transfer permit holdings become insensitive to further increases in cap stringency, while the IR’s permit holdings continue to decline. By contrast, when the OEM faces a relatively high abatement difficulty (Case 1-ii), the opposite pattern arises: the IR’s permit holdings remain unchanged once full abatement is reached, whereas the OEM’s permit holdings continue to decrease as the cap becomes more stringent. In all cases, the equilibrium shadow price increases monotonically with cap stringency, reflecting growing permit scarcity.

4.2. Economic and Welfare Impacts of CTP in a CLSC

This section examines whether the implementation of CTP generates economic and social value in the CLSC. Section 4.2.1 examines the resulting profit implications for both firms, while Section 4.2.2 evaluates the effects on generalized consumer surplus and social welfare.

4.2.1. Effects of CTP on Firms’ Profits

As shown in Section 4.1, the total carbon permit allocated by the government and the low-carbon level of remanufactured products largely determine the relative competitive positions between the OEM and the IR. These asymmetries in abatement efficiency and competitive position imply that the two firms may value carbon permits differently. This, in turn, creates scope for permit transfer between the OEM and the IR. Accordingly, a natural question arises as to whether permit transfer under a binding emissions cap can increase the profits of both parties, and under what conditions this outcome can be sustained. To address this issue, this study proposes Corollary 2.

Corollary 2. *Under a relatively relaxed carbon permit scenario, the impact of carbon permit allocation on firms’ profits is as follows:*

- (a) *The impact of carbon permit allocation on OEM and IR profits: Given the conditions $0 < \theta \leq \theta_3$, $G_{M1} < G_M < G_{\max}$, $G_{R1} < G_R < G_{\max}$, or alternatively $\theta_3 < \theta < \theta_4$ and $G_{M1} < G_M < G_{\max}$, there exists a critical value $\bar{G} < G_{\max}$, such that if $G \in (\bar{G}, G_{\max})$, then both the OEM and the IR obtain higher equilibrium profits under CTP compared to the baseline model ($\pi_M^* > \underline{\pi}_M^*$ and $\pi_R^* > \underline{\pi}_R^*$), where $G_{M1} + G_{R1} < G_{\max}$.*
- (b) *The impact of carbon permit allocation on total profits of the CLSC: There exists a critical value $\bar{G} < G_{\max}$, such that if $G \in (\bar{G}, G_{\max})$, the total profits of the CLSC under CTP exceed those in the baseline model ($\pi_T^* > \underline{\pi}_T^*$).*

- (c) The impact of the low-carbon level of remanufactured products θ on critical thresholds: The critical thresholds vary with θ such that $\frac{\partial G_{M1}}{\partial \theta} > 0$ and $\frac{\partial G_{R1}}{\partial \theta} < 0$.

The three sub-conclusions of Corollary 2 are as follows:

Corollary 2(a). To simultaneously enhance the profits of both the OEM and IR, the government should set the initial and total permit levels based on the low-carbon level θ of remanufactured products. When θ is low ($0 < \theta \leq \theta_3$), both firms require relatively high initial permit (G_M, G_R). When θ is moderate ($\theta_3 < \theta < \theta_4$), the IR's permit can be reduced, while the OEM still requires a high initial allocation. In this case, the government needs to maintain a relatively large total permit level ($G \in (\bar{G}, G_{\max})$) to ensure that the profits of both firms increase. When θ is high ($\theta_4 \leq \theta < 1$), the IR's profit increases, whereas the OEM's profit decreases, as the IR's strengthened competitive edge erodes the OEM's margin. In such scenarios, profit improvements for both firms arise only when $G_{M1} + G_{R1} < G_{\max}$.

Corollary 2(b). Regardless of how the initial permit is allocated, as long as the government provides a sufficiently large total permit ($G \in (\bar{G}, G_{\max})$), the overall profit of the CLSC will increase when the CTP mechanism is introduced.

Corollary 2(c). As θ increases, the permit threshold G_{M1} required to improve OEM profits rises, while the threshold G_{R1} for enhancing IR profits falls. This reflects how the low-carbon level of remanufactured products reshapes the relative competitive advantage between the OEM and the IR, thereby altering the permit allocation conditions under which each firm's profit can be improved.

Figure 2 visually illustrates Corollary 2. The parameter values used in Figure 2 are calibrated based on the Steyr 615-67 engine case; detailed data sources and calibration procedures are provided in Section 5. The solid blue line denotes the OEM's profit-enhancing permit threshold G_{M1} , the dashed red line indicates the IR's threshold G_{R1} , and the orange dot-dashed line ($G_{M1} + G_{R1}$) marks the minimum total permit required to improve both firms' profits simultaneously. Within the shaded region, permit transfer real-locates compliance costs across firms in a way that leads to higher profits for both the OEM and the IR, a result that can be described as a "win-win outcome" in terms of firm-level profits. Importantly, this outcome does not rely on cooperation, coordination, or joint decision-making, but arises endogenously from decentralized responses to permit scarcity under the CTP.

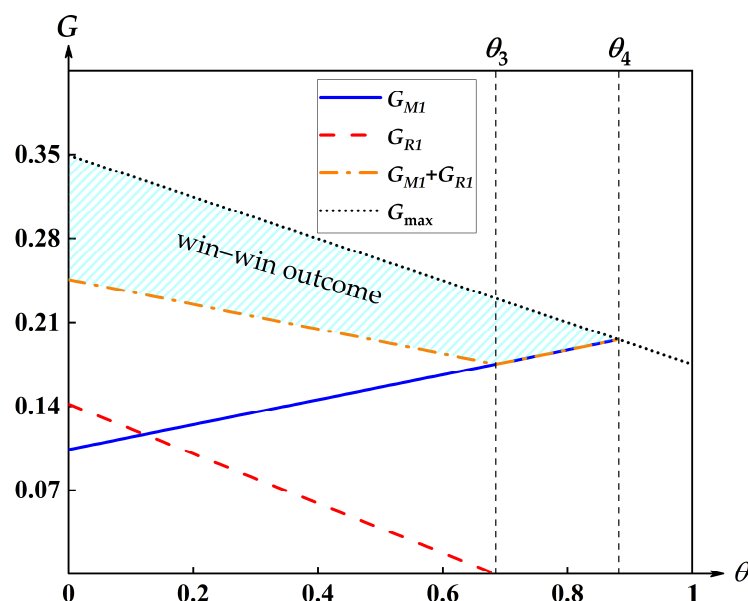


Figure 2. Impacts of carbon permit allocation on firms' profits ($\Delta = 0.24$, $\delta = 0.63$, $c_r = 0.3$).

In summary, as long as the low-carbon level of remanufactured products does not exceed a certain threshold, carbon permit transfer between the OEM and the IR can simultaneously improve the profits of both parties, yielding a win–win outcome. In contrast, an excessively high level of low-carbon content in remanufactured products may undermine the OEM's profitability. This result is economically intuitive. Under extended producer responsibility regulations, the OEM is required to assume responsibility for end-of-life product recovery, while the IR effectively undertakes part of this responsibility on the OEM's behalf through remanufacturing activities. Accordingly, an implicit compensation mechanism is required to reallocate abatement and recovery responsibilities between the two parties. Carbon permit transfer provides such a mechanism by allowing the OEM to internalize part of the recovery and abatement responsibility through permit transfer, thereby promoting emissions reduction while simultaneously facilitating recycling and remanufacturing activities within the CLSC. Therefore, by embedding a carbon permit transfer mechanism within the CLSC, this study identifies a new pathway through which the OEM can fulfill its extended producer responsibility, distinct from the mechanisms examined in existing studies [36,37].

4.2.2. Effects of CTP on Generalized Consumer Surplus and Social Welfare

Section 4.2.1 demonstrates that carbon permit transfer within the CLSC can reduce emissions while increasing firms' profits, thereby generating a win–win outcome at the firm level. However, improvements in a firm's profitability do not necessarily translate into gains in consumer welfare or overall social welfare. To provide a comprehensive evaluation of the CTP design, this section examines its effects on generalized consumer surplus and social welfare. Drawing on the study by Anand and Giraud-Carrier [26], to better capture consumers' perception of carbon emissions, we introduce the concept of generalized consumer surplus and define the expressions for generalized consumer surplus GCS and social welfare SW:

$$GCS = (q_n)^2/2 + \delta q_n q_r + \delta (q_r)^2/2 - \varepsilon G^2 \quad (7)$$

$$SW = \pi_T + GCS \quad (8)$$

The first half of Equation (7), $(q_n)^2/2 + \delta q_n q_r + \delta(q_r)^2/2$, represents standard consumer surplus, whereas the second half captures the social carbon cost εG^2 . Here, G denotes the aggregate emissions cap set by the government. Under the binding permit balance condition imposed in this study, aggregate emissions equal G at equilibrium, so εG^2 can be interpreted as the social cost associated with the equilibrium emission level. The parameter ε is the social carbon cost coefficient; a larger ε implies higher marginal social damages from emissions. The quadratic form εG^2 reflects increasing marginal social carbon costs. Integrating these two components yields the generalized consumer surplus, as presented in Equation (7). Subsequently, social welfare can be derived according to Equation (8).

For deeper analysis, this paper defines the expression $a_3 = (\delta^2 + 4\theta\delta)\Delta + 4c_r + \theta(\delta^2 + 3c_r\delta - 4c_r) - \delta(3c_r + 4) + \delta^2(c_r + 2)$ ($\frac{\partial a_3}{\partial \Delta} > 0$, $\frac{\partial a_3}{\partial \theta} > 0$) as the “comprehensive advantage” of remanufacturing. Unlike previous studies, this paper considers not only the cost advantage of remanufacturing (Δ) but also the low-carbon level of remanufactured products (θ), together constituting the “comprehensive advantage” of remanufacturing. A higher value of a_3 corresponds to a higher “comprehensive advantage” of remanufacturing.

Corollary 3. Under a relatively relaxed carbon permit scenario, the impacts of carbon permit transfer on generalized consumer surplus GCS and social welfare SW are as follows:

- (a) Impact on GCS : when $\varepsilon > \varepsilon_1$, there exists $\bar{G} < G_{\max}$, such that if $G \in (\bar{G}, G_{\max})$, then $GCS^I > GCS^*$;
- (b) Impact on SW : when $a_3 < 0$ and $\varepsilon > \varepsilon_2$ ($\varepsilon_2 > 0$), or when $a_3 \geq 0$, there exists $\bar{G} < G_{\max}$, such that if $G \in (\bar{G}, G_{\max})$, then $SW^I > SW^*$, where $a_3 = (\delta^2 + 4\theta\delta)\Delta + 4c_r + \theta(\delta^2 + 3c_r\delta - 4c_r) - \delta(3c_r + 4) + \delta^2(c_r + 2)$;
- (c) Effect of abatement difficulty on critical thresholds: $\frac{\partial \varepsilon_1}{\partial \eta_n} > 0$, $\frac{\partial \varepsilon_1}{\partial \eta_r} > 0$; when $a_3 < 0$, $\frac{\partial \varepsilon_2}{\partial \eta_n} > 0$, $\frac{\partial \varepsilon_2}{\partial \eta_r} > 0$.
- (d) Comparison of thresholds related to GCS and SW : $\varepsilon_1 > \varepsilon_2$.

To facilitate the exposition of Corollary 3, we set $\Delta \in \{0.24, 0.50\}$, $\delta = 0.63$, $c_r = 0.3$, and $\theta = 0.8$, and plotted Figure 3 accordingly. The parameter value $\Delta = 0.24$ was calibrated based on the Steyr 615-67 engine case, as detailed in Section 5. In contrast, $\Delta = 0.50$ represents a forward-looking scenario in which remanufacturing costs further decline due to technological progress and scale effects. When $\Delta = 0.24$, the comprehensive advantage of remanufacturing is relatively low, leading to $a_3 < 0$; when $\Delta = 0.50$, the comprehensive advantage of remanufacturing is relatively high, and $a_3 > 0$.

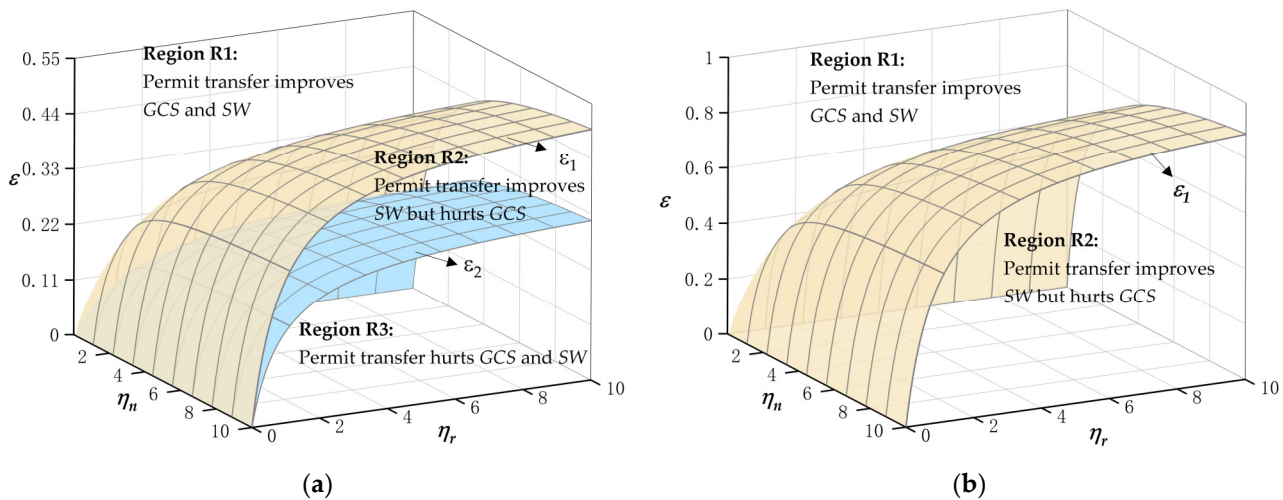


Figure 3. Impact of CTP on GCS and SW. (a) illustrates the case of a low comprehensive advantage of remanufacturing ($a_3 < 0$); (b) illustrates the case of a high comprehensive advantage of remanufacturing ($a_3 \geq 0$). The yellow surface represents the threshold surface for increasing GCS, while the blue surface represents the threshold surface for increasing SW.

In particular, Figure 3a,b depict how CTP affects GCS and SW under scenarios featuring a low ($a_3 < 0$) and high ($a_3 \geq 0$) comprehensive advantage of remanufacturing, respectively. Corollary 3(a) indicates that, when the social carbon cost coefficient is sufficiently high ($\varepsilon > \varepsilon_1$), CTP increases GCS when the government allocates a relatively large total carbon permit ($G \in (\bar{G}, G_{\max})$). According to Corollary 3(b), when the comprehensive advantage of remanufacturing is low and the social carbon cost coefficient is high ($a_3 < 0$, $\varepsilon > \varepsilon_2$, Figure 3a), or when the comprehensive advantage is high ($a_3 \geq 0$, Figure 3b), CTP improves SW when total permit is abundant. This result highlights that, when the comprehensive advantage of remanufacturing is high, CTP consistently enhances SW under relatively abundant total permit. Corollary 3(c) reveals that, as abatement becomes more challenging (η_n and η_r increase), the threshold social carbon cost coefficients required to improve GCS and SW also rise. Therefore, reducing firms' abatement difficulty is more conducive to enhancing generalized consumer surplus and social welfare in the process of carbon mitigation. Finally, Corollary 3(d) indicates that, if CTP enhances GCS, it will necessarily improve SW.

Moreover, it can be readily shown that $\theta > \theta_3$ is a necessary condition for $a_3 > 0$. Under this condition, the implementation of the CTP is more conducive to improving social welfare. Further combined with Figure 2, we find that, when $\theta_3 < \theta < \theta_4$, the CTP simultaneously increases the profits of both the OEM and the IR. This indicates that, within a certain range of the low-carbon level of remanufactured products, regulation is more effective in enhancing both firms' profitability and social welfare.

In practice, governments can conduct field investigations to collect cost and emission data from the CLSC, thereby identifying the empirical values of the thresholds θ_3 and θ_4 . For example, in the Steyr 615-67 engine industry ($\Delta = 0.24$, $c_r = 0.3$, $\delta = 0.63$, data sources are provided in Section 5), we obtain $\theta_3 = 0.6850$ and $\theta_4 = 0.8824$. This implies that, when the low-carbon level of remanufactured products (θ) lies within the interval $(0.6850, 0.8824)$, governments may prioritize regulatory pilot programs, thereby balancing economic performance and environmental sustainability.

5. Numerical Analysis of Policy Impacts Under the CTP

Section 4 examines only scenarios with relatively abundant carbon permit. This section employs numerical simulations to systematically examine the effects of CTP under both scarce and abundant permit regimes on firms' profits and social performance within the CLSC. To contextualize the model and enhance its empirical relevance, we conducted a case study based on the Steyr 615-67 engine. The Steyr 615-67 engine was chosen for the following reasons. First, it is a widely used diesel engine in the construction and agricultural machinery sectors, which are typically carbon-intensive and have high remanufacturing potential. Second, the engine's production, remanufacturing cost, and emission data are relatively accessible, allowing for accurate parameter estimation based on field evidence.

According to Cao et al. [29], the production cost of a new Steyr 615-67 engine is USD 5359, with carbon emissions of approximately 1.5 t CO₂ per unit, whereas the remanufacturing cost is USD 3001, generating about 0.3 t CO₂ per unit (80% reduction). Consumers' acceptance of remanufactured products is set to $\delta = 0.63$. Based on these data, the remanufacturing cost saving was calculated as USD 2358. To ensure that all cost parameters were within the interval $(0,1)$, all monetary values are scaled down by a factor of 10,000. Accordingly, the parameter values were set as follows: $\Delta = 0.24$, $\theta = 0.8$, $\delta = 0.63$, and $c_r = 0.3$.

Given that Case 1 and Case 2 yielded qualitatively similar results, we focused on Case 1 and set $\eta_r = 2$. Accordingly, the total carbon permit cap was determined to be $G_{\max} = 0.2099$. To illustrate how a firm's profits vary under different initial carbon permit allocations, we considered $G_M = \{0.15, 0.18, 0.19\}$.

5.1. Abatement Rate and Shadow Price

How do firms' abatement rate $\lambda_j^* (j = n, r)$ and shadow price p_e^* evolve along the pathway to carbon reduction and climate mitigation? Accordingly, Figure 4 is presented below, where η_n denotes the OEM's abatement difficulty and G represents the total carbon permit, which reflects the total emissions of the CLSC.

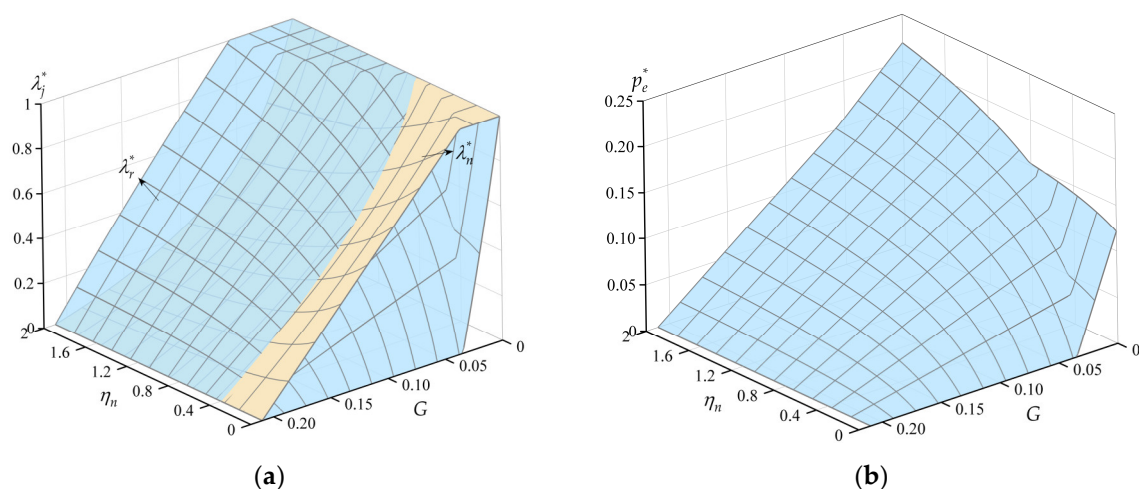


Figure 4. Impacts of η_n and G on $\lambda_j^* (j = n, r)$ and p_e^* . (a) illustrates the abatement rate of OEM (λ_n^*) and IR (λ_r^*); (b) illustrates the optimal shadow price.

As shown in Figure 4a, a smaller total permit G reflects stricter regulation, prompting firms to increase their abatement rate. However, as the OEM's reduction difficulty η_n

increases, its rate λ_n^* declines to contain abatement costs. In turn, the IR raises its reduction rate λ_r^* to strengthen its competitive position. Figure 4b shows that the shadow price p_e^* rises with higher η_n or lower G , due to increased scarcity of permit. The price response is especially sharp when G is low and η_n increases.

5.2. Firms' Profits

How are firms' profits affected by CTP, and how do they vary with the total carbon permit allocation (G) and the OEM's abatement difficulty (η_n)? Accordingly, Figures 5 and Figure 6 are presented below to illustrate firm-level and CLSC-level outcomes, respectively. Specifically, Figure 5a–c show the OEM's profit π_M^* , and Figure 5d–f depict the IR's profit π_R^* , representing the profit performance of individual firms. In contrast, Figure 6 presents the total profit π_T^* of the CLSC, which remains unaffected by the OEM's initial permit allocation; therefore, this distinction is omitted in Figure 6.

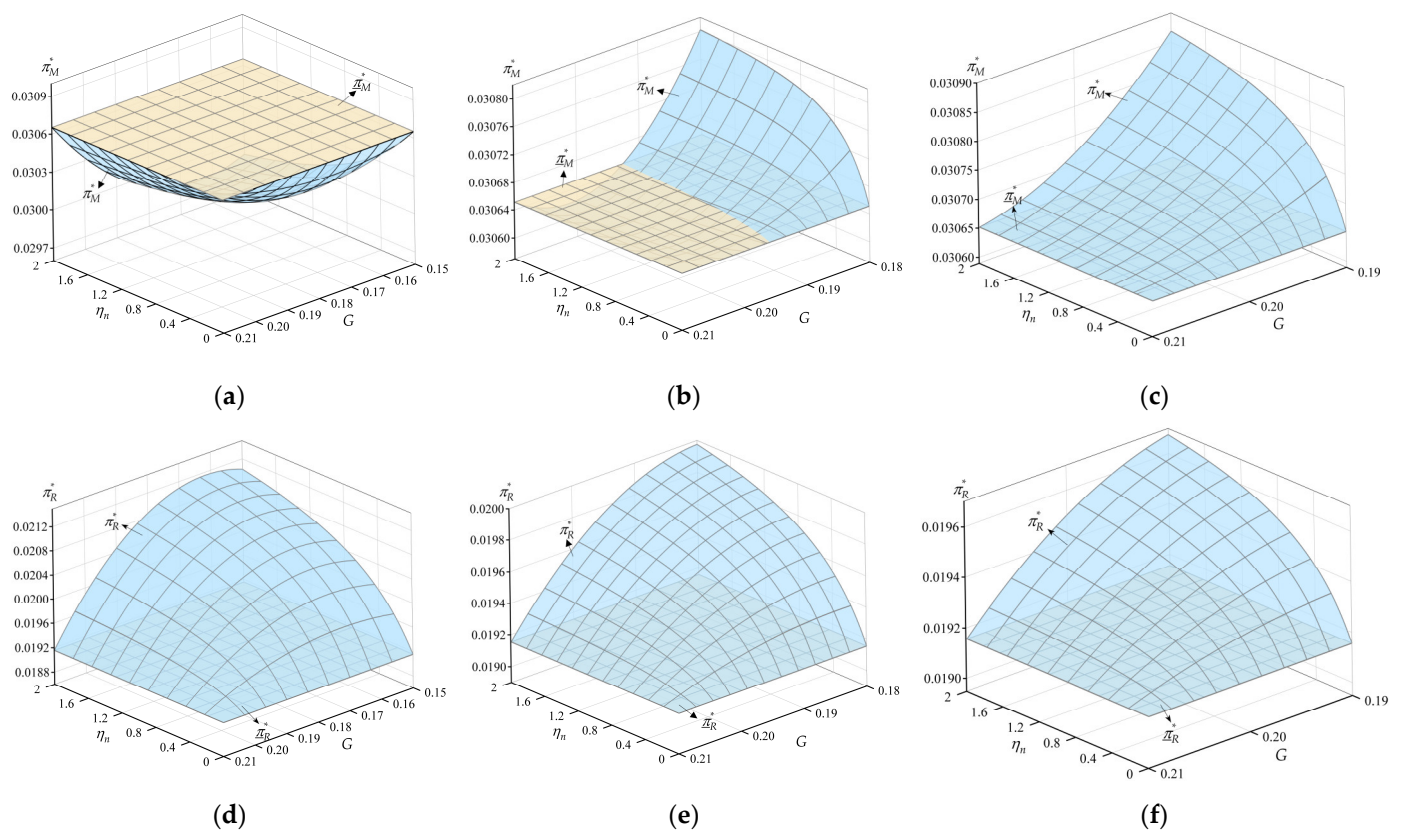


Figure 5. Impacts of η_n and G on individual firms' profit. (a–c) show the OEM profit surface diagrams for $G_M = 0.15$, $G_M = 0.18$, and $G_M = 0.19$, respectively; (d–f) display the IR profit surface diagrams for $G_M = 0.15$, $G_M = 0.18$, and $G_M = 0.19$, respectively. The yellow surface indicates firms' profits under the baseline model (denoted by an underlined symbol), while the blue surface reflects profits under the CTP.

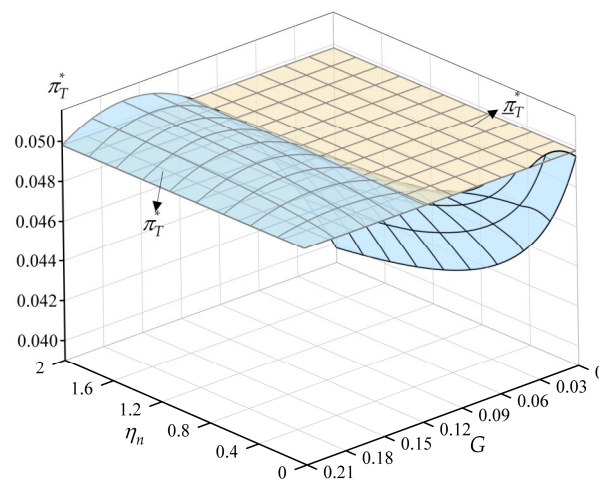


Figure 6. Impacts of η_n and G on the profit of CLSC. The yellow surface indicates the total profit of the CLSC under the baseline model (denoted by an underlined symbol), while the blue surface reflects the total profit of the CLSC under CTP.

As shown in Figure 5a–c, the OEM's initial carbon permit G_M significantly affects its profit. With a small permit (e.g., $G_M = 0.15$), CTP reduces profit. When the permit is moderate (e.g., $G_M = 0.18$), CTP increases profit only if total permit G is low; otherwise, profit declines. A large permit (e.g., $G_M = 0.19$) consistently improves the OEM's profit under CTP. As shown in Figure 5d–f, CTP increases the IR's profit across all OEM initial permit scenarios. The IR benefits more when the OEM's abatement difficulty η_n is higher and the total permit G is lower. To maximize this effect, the government should avoid setting excessively high total permit.

Moreover, as the government allocates more initial carbon permit (G_M) to the OEM, the initial permit (G_R) assigned to the IR decreases correspondingly, leading to a reduction in the IR's profit (see Figure 5d–f). This indicates that the initial allocation of carbon permit plays a moderating role in firms' profitability.

As shown in Figure 6, CTP does not always enhance the profit of the CLSC along the pathway to carbon reduction; improvements occur only under a relatively high total permit, which is consistent to Corollary 2(b). Combined with the findings above, this suggests that the government can balance carbon reduction goals and economic outcomes by appropriately increasing both the OEM's initial permit and the total permit, thereby alleviating firms' compliance pressure under the permit constraint.

5.3. Generalized Consumer Surplus and Social Welfare

How does CTP affect generalized consumer surplus and social welfare? Accordingly, Figure 7 is presented below. Figure 7a,b illustrate generalized consumer surplus, and Figure 7c,d depict social welfare.

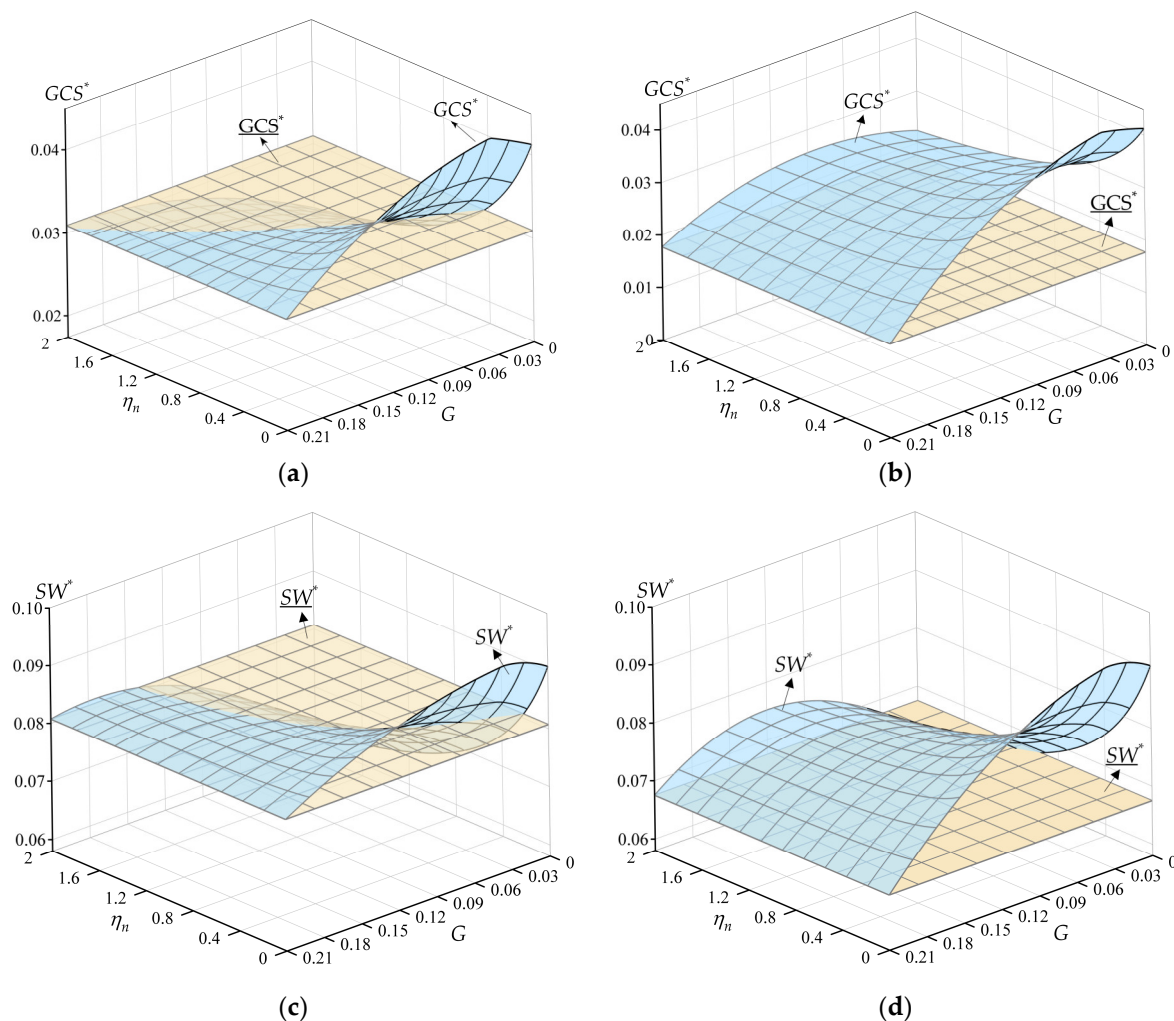


Figure 7. Impacts of η_n and G on GCS^* and SW^* . (a,b) show the generalized consumer surplus for $\varepsilon = 0.3$ and $\varepsilon = 0.6$, respectively; (c,d) present the social welfare for $\varepsilon = 0.3$ and $\varepsilon = 0.6$, respectively. The yellow surface shows outcomes under the baseline model (denoted by an underlined symbol), while the blue surface reflects outcomes under the CTP.

As shown in Figure 7a,b, the impact of CTP on generalized consumer surplus GCS^* depends on the social carbon cost ε . When $\varepsilon = 0.3$, CTP increases GCS^* only if η_n is low. At a higher social carbon cost $\varepsilon = 0.6$, CTP consistently improves GCS^* . A smaller value of η_n indicates a lower abatement difficulty for the OEM. Moreover, it is evident that a smaller η_n leads to a higher level of GCS^* . Therefore, reducing the OEM's abatement difficulty enhances the positive effect of CTP on GCS^* . As shown in Figure 7c,d, unlike GCS^* , when the total permit G is large, CTP consistently enhances social welfare. Notably, at a high social carbon cost $\varepsilon = 0.6$, CTP improves social welfare in most cases. This highlights that the higher the social carbon cost, the greater the value of implementing the CTP. Overall, when the OEM's abatement difficulty is low and the social carbon cost is high, environmental performance and social welfare are jointly improved in most cases. However, when the total carbon permit is small, GCS^* and social welfare may increase, while the OEM's profit tends to decline.

The numerical analysis serves not only as an illustration but also as a verification of the analytical results derived in the Proposition 1 and Corollaries 1–3. Specifically, the simulation outcomes reproduce the threshold structures and monotonicity properties established theoretically. (1) Figure 4 confirms the Proposition 1 by showing that abatement

responsibilities shift toward the firm with the lower abatement difficulty as cap stringency increases. (2) Figures 5 and 6 validate Corollary 2, demonstrating that profit-improving regions exist under relatively large total permit and that initial allocation plays a moderating role. (3) Figure 7 confirms Corollary 3 by illustrating the critical role of the social carbon cost coefficient ε and the comprehensive advantage captured by a_3 . More specifically, under the calibrated Steyr 615-67 engine case, we obtained $a_3 = -0.5839 < 0$, indicating that the comprehensive advantage of remanufacturing is relatively low. In this setting, CTP does not universally improve social welfare, even when the total permit is relatively relaxed. This pattern is clearly illustrated in Figure 7, where the social welfare surface under CTP exceeds the baseline only within certain parameter regions.

Overall, the numerical illustrations are fully consistent with the analytical results, thereby strengthening the robustness of the theoretical findings.

6. Policy Implications

This section translates the theoretical findings into concrete guidance for carbon policy design. Table 6 summarizes how cap stringency, permit allocation, and products' attributes jointly inform regulatory strategies under the CTP.

Table 6. Analytical results and policy implications.

Results	What it Means	Policy Implications
Abatement responsibilities are endogenously assigned based on relative abatement difficulty (Proposition 1).	Under CTP, the firm with relatively lower abatement difficulty naturally undertakes deeper or even full abatement, while the other firm relies more on permit transfer.	Policymakers may allow and facilitate permit transfer within the CLSC, enabling abatement responsibilities to be endogenously allocated to the more efficient firm rather than imposed administratively.
Production shifts between new and remanufactured products in a non-monotonic way (Corollary 1).	Cap tightening reshapes production incentives depending on firms' abatement difficulty and the low-carbon level of remanufactured products, rather than uniformly favoring remanufacturing.	Policymakers should avoid assuming that stricter caps always expand remanufacturing; policy outcomes depend on both technological and product-level characteristics.
As θ increases, the OEM's profit-improving threshold rises, while the IR's threshold declines (Corollary 2).	A higher remanufacturing carbon advantage strengthens IR's competitiveness and shifts profit balance. Extremely high θ may crowd out OEM profitability.	Permit allocation should account for product-level carbon advantages. In sectors with highly carbon-efficient remanufacturing, differentiated allocation can help mitigate excessive competitive pressure.
a_3 captures the cost advantage Δ and low-carbon level θ of remanufacturing (Corollary 3).	Remanufacturing competitiveness is jointly driven by cost savings and carbon performance, not cost alone.	Policymakers may assess both cost and low-carbon attributes when designing differentiated policies for remanufactured products.
When $\theta_3 < \theta < \theta_4$, CTP is more conducive to higher firm profits and social welfare (Corollary 2 + Corollary 3).	This interval represents a range in which economic incentives and environmental objectives are more closely aligned, making win-win outcomes easier to sustain.	Policymakers may consider regulatory pilots and gradual cap tightening within this θ range, where profitability and social welfare improvements are more likely to occur simultaneously.
When the η_n is low, CTP generates stronger improvements in GCS^* and social welfare (numerical analysis).	When abatement is relatively easy for the OEM, firms can comply with the cap mainly by reducing emissions rather than cutting production, so CTP improves generalized consumer surplus and social welfare more effectively.	CTP is more effective in sectors with mature abatement technologies; in high-difficulty sectors, permit-based regulation may be complemented by technology support policies.

Overall, the policy implications highlight that effective carbon regulation can rely on firms' endogenous responses to permit constraints rather than uniform administrative mandates. Cap stringency reallocates abatement responsibilities toward firms with a lower abatement difficulty, while differentiated permit allocation helps moderate competitive dynamics within the CLSC. Moreover, considering both cost advantages and low-carbon product attributes is essential for understanding how profitability and social welfare respond under different parameter conditions. These insights suggest that a well-calibrated and flexible CTP framework can better accommodate the trade-offs between economic outcomes and environmental objectives.

7. Conclusions

This study develops a constrained Cournot game model to analyze the strategic interactions between an OEM and an IR regarding production, abatement, and carbon permit transfer within a CLSC. It examines how the low-carbon level of remanufactured products, firms' abatement difficulty, and the allocation of total and initial carbon permits determine equilibrium outcomes. The analysis further evaluates the impacts of CTP-induced permit constraints and allocation rules on a firm's profits, generalized consumer surplus, and social welfare under a stylized regulatory framework.

- (1) Equilibrium abatement strategies are jointly shaped by product-level low-carbon attributes, relative abatement difficulty, and total permit stringency. Under stringent caps, abatement responsibilities shift toward the firm with lower marginal abatement cost. When the low-carbon level of remanufactured products is low—or high with strong IR abatement capability—the OEM adopts full or partial abatement depending on its own difficulty, with the IR adjusting accordingly. When the IR faces a high abatement difficulty with a high low-carbon level, the OEM fully abates while the IR partially reduces. Under lenient caps, both firms typically adopt partial abatement.
- (2) Increasing cap stringency systematically reshapes production and permit transfer decisions. When one firm fully abates, it expands output while the partially abating firm raises its reduction rate but reduces output and permit demand. When both firms partially abate, the OEM contracts new-product output and permit demand, whereas the IR expands remanufacturing output as the low-carbon level rises. In all scenarios, tighter caps increase the shadow price, reflecting a greater scarcity of carbon permits.
- (3) The profit effects of CTP depend critically on both the total permit cap and the initial permit allocation. Both theoretical and numerical results indicate that, when the low-carbon level of remanufactured products remains below a threshold, an appropriately calibrated combination of the total permit and initial allocation can enhance profits for both the OEM and the IR.
- (4) The welfare implications of CTP hinge on the comprehensive advantage of remanufacturing and the social carbon cost. CTP more readily enhances generalized consumer surplus and social welfare when the social carbon cost is high. When remanufacturing exhibits strong comprehensive advantages, a relatively relaxed cap can always improve social welfare. The Steyr 615-67 case further shows that, under a low social carbon cost, generalized consumer surplus increases when the OEM's abatement difficulty is low, while social welfare improves when either the OEM's abatement difficulty is low or total permit is sufficiently large.

This study is subject to several limitations. First, the analysis relies on a single-period modeling framework, which abstracts from dynamic considerations such as permit banking, investment dynamics, and technology evolution. Second, the framework focuses on a stylized CLSC with one OEM and one IR, whereas many industries involve multiple

competing firms. Extending the model to dynamic and multi-firm settings constitutes a promising direction for future research.

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Appendix A. (Threshold Expressions)

1. Regarding the low-carbon level of remanufactured products

$$\theta_1 = \frac{1-\Delta-\delta}{1-\Delta-c_r}, \quad \theta_2 = \frac{2(1-\delta)}{\delta+2}, \quad \theta_3 = \frac{2-\delta}{2}, \quad \theta_4 = \frac{8\delta-8c_r+\Delta\delta^2+\delta^3-5\delta^2+4c_r\delta}{\delta(8-3\delta)-\delta^2\Delta-c_r(\delta^2-4\delta+8)}.$$

2. Regarding firms' abatement difficulty

$$\eta_{m1} = \frac{\eta_r(\delta(\Delta+c_r+1)-2c_r)(1-\theta)}{-2(1-\theta)(1-\Delta-c_r)(\theta-\theta_1)\eta_r+\delta(2-2\Delta-\delta-c_r)}, \quad \eta_{r1} = \frac{\delta(2-\delta-2\Delta-c_r)}{2(1-\theta)(1-\Delta-c_r)(\theta-\theta_1)}, \quad \eta_{r2} = \frac{\delta(4-\delta)}{4(1-\theta)(\theta-\theta_3)}.$$

3. Regarding the initial carbon permit allocation between the OEM and the IR

$$G_{M1} = \frac{2(1+\theta)(2-\delta-2\Delta-c_r)}{(4-\delta)^2}, \quad G_{R1} = \frac{-4(\Delta\delta+\delta+\delta c_r-2c_r)(\theta-\theta_3)}{\delta(4-\delta)^2}.$$

4. Regarding the total permit allocation of the competitive system

$$G_1 = \frac{(2(1-\theta)(1-\Delta-c_r)(\theta-\theta_1)\eta_r+\delta(2\Delta+c_r+\delta-2))\eta_n+\eta_r(1-\theta)(\delta(\Delta+c_r+1)-2c_r)}{\delta\eta_r(2\eta_n(1+\theta)+4-\delta)},$$

$$G_2 = \frac{(2(1-\theta)(1-c_r-\Delta)(\theta-\theta_1)\eta_r+\delta(2\Delta+c_r+\delta-2))\eta_n+\eta_r(1-\theta)(\delta(\Delta+c_r+1)-2c_r)}{\eta_n(4(1-\theta)(\theta-\theta_3)\eta_r-\delta(4-\delta))}.$$

5. Regarding the social carbon cost coefficient

$$a_1 = (4c_r-3c_r\delta+c_r\delta^2-4\delta+\Delta\delta^2+2\delta^2)\theta+8\delta-4\Delta\delta+2\Delta\delta^2-5\delta^2+\delta^3+c_r\delta^2-4c_r-c_r\delta \quad (a_1 > 0),$$

$$a_2 = -(\delta+\Delta\delta+c_r\delta-2c_r)\theta+3\delta-\Delta\delta-\delta^2-2c_r \quad (a_2 > 0),$$

$$a_3 = (\delta^2+4\theta\delta)\Delta+4c_r+\theta(\delta^2+3c_r\delta-4c_r)-\delta(3c_r+4)+\delta^2(c_r+2),$$

$$Z_1^l = \eta_n(\delta(4-\delta)+4\eta_r(\theta^2+(\delta-2)\theta+1))+\eta_r\delta(4-\delta) \quad (Z_1^l > 0),$$

$$\varepsilon_1 = \frac{\delta \eta_n \eta_r a_1}{a_2 Z_1^1}, \quad \varepsilon_2 = \frac{-\delta \eta_n \eta_r a_3}{a_2 Z_1^1}.$$

Appendix B. (Proofs of the Proposition 1 and Corollaries)

Proof of Proposition 1 (equilibrium solutions under cap-and-trade system).

Step 1: Formulation of the Lagrangian function

The OEM's optimization problem:

$$\begin{aligned} \max_{\lambda_n, q_n, v_n} \pi_M &= (1 - q_n - \delta q_r - c_r - \Delta) q_n - \eta_n (q_n \lambda_n)^2 + p_e v_n \\ \text{s.t.} \quad &\begin{cases} 0 \leq \lambda_n \leq 1 \\ q_n (1 - \lambda_n) \leq G_M - v_n \end{cases} \end{aligned}$$

Construct the Lagrangian function for the OEM's optimization problem:

$$L_M = (1 - q_n - \delta q_r - c_r - \Delta) q_n - \eta_n (q_n \lambda_n)^2 + p_e v_n + \alpha_n \lambda_n + \beta_n (1 - \lambda_n) + \gamma_n (G_M - v_n - q_n (1 - \lambda_n))$$

The Kuhn–Tucker necessary conditions are as follows:

$$\frac{\partial L_M}{\partial \lambda_n} = -2 \lambda_n \eta_n q_n^2 + \gamma_n q_n + \alpha_n - \beta_n = 0 \quad (\text{A1})$$

$$\frac{\partial L_M}{\partial q_n} = \gamma_n (\lambda_n - 1) - c_r - 2 q_n - \delta q_r - \Delta - 2 \lambda_n^2 q_n \eta_n + 1 = 0 \quad (\text{A2})$$

$$\frac{\partial L_M}{\partial v_n} = p_e - \gamma_n = 0 \quad (\text{A3})$$

The complementary slackness conditions are as follows: $\alpha_n \lambda_n = 0$; $\beta_n (1 - \lambda_n) = 0$; $\gamma_n (G_M - v_n - q_n (1 - \lambda_n)) = 0$.

Feasibility conditions: $q_n > 0$, $\alpha_n \geq 0$, $\beta_n \geq 0$, $\gamma_n \geq 0$, $0 \leq \lambda_n \leq 1$, $q_n (1 - \lambda_n) \leq G_M - v_n$.

- (1) First, we show that $\alpha_n = 0$. Assume $\alpha_n > 0$; it follows that $\lambda_n = \beta_n = 0$. Substituting these into Equation (A1), we obtain $\alpha_n = -\gamma_n q_n \leq 0$, which is a contradiction. Hence, it always holds that $\alpha_n = 0$.
- (2) From Equation (A3), we have $\gamma_n = p_e > 0$. Further applying the complementary slackness condition $\gamma_n (G_M - v_n - q_n (1 - \lambda_n)) = 0$, we derive:

$$G_M - v_n - q_n (1 - \lambda_n) = 0 \quad (\text{A4})$$

- (3) For the analysis of β_n , we distinguish two cases:

(a) $\beta_n = 0$.

From Equation (A1), we find $\gamma_n = 2 \lambda_n \eta_n q_n$. Substituting this into Equation (A2), we have: $q_n = \frac{-p_e - c_r - \delta q_r - \Delta + 1}{2}$. Considering the constraint $0 \leq \lambda_n \leq 1$, we thus obtain: $p_e \leq 2 \eta_n q_n$.

(b) $\beta_n > 0$.

From the complementary slackness condition $\beta_n(1-\lambda_n)=0$, it immediately follows that $\lambda_n=1$. Substituting this result into Equations (A2) and (A4), we get: $q_n = \frac{1-\Delta-\delta q_r - c_r}{2(1+\eta_n)}$, $v_n = G_M$. Further, from Equation (A1), we obtain: $p_e > 2\eta_n q_n$.

The IR's optimization problem:

$$\begin{aligned} \max_{\lambda_r, q_r, v_r} \pi_R &= (\delta(1-q_n-q_r)-c_r)q_r - \eta_r((1-\theta)q_r\lambda_r)^2 + p_e v_r \\ \text{s.t.} \quad &\begin{cases} 0 \leq \lambda_r \leq 1 \\ (1-\theta)q_r(1-\lambda_r) \leq G_R - v_r \end{cases} \end{aligned}$$

Construct the Lagrangian function for the IR's optimization problem:

$$L_R = (\delta(1-q_n-q_r)-c_r)q_r - \eta_r((1-\theta)q_r\lambda_r)^2 + p_e v_r + \alpha_r \lambda_r + \beta_r(1-\lambda_r) + \gamma_r(G_R - v_r - (1-\theta)q_r(1-\lambda_r))$$

The Kuhn–Tucker necessary conditions are as follows:

$$\frac{\partial L_R}{\partial \lambda_r} = \alpha_r - \beta_r + \gamma_r q_r (1-\theta) - 2\lambda_r q_r^2 \eta_r (1-\theta)^2 = 0 \quad (\text{A5})$$

$$\frac{\partial L_R}{\partial q_r} = -c_r - \delta(q_n + q_r - 1) - \delta q_r - \gamma_r(1-\lambda_r)(1-\theta) - 2\lambda_r^2 q_r \eta_r (1-\theta)^2 = 0 \quad (\text{A6})$$

$$\frac{\partial L_R}{\partial v_r} = p_e - \gamma_r = 0 \quad (\text{A7})$$

The complementary slackness conditions are as follows: $\alpha_r \lambda_r = 0$; $\beta_r(1-\lambda_r) = 0$; $\gamma_r(G_R - v_r - (1-\theta)q_r(1-\lambda_r)) = 0$.

Feasibility conditions: $q_r > 0$, $\alpha_r \geq 0$, $\beta_r \geq 0$, $\gamma_r \geq 0$, $0 \leq \lambda_r \leq 1$, $(1-\theta)q_r(1-\lambda_r) \leq G_R - v_r$.

- (1) $\alpha_r = 0$. Suppose $\alpha_r > 0$. It follows that $\lambda_r = \beta_r = 0$. Substituting these values into Equation (A5), we obtain $\alpha_r = -\gamma_r q_r (1-\theta) < 0$, which leads to a contradiction. Therefore, $\alpha_r = 0$ always holds.
- (2) From Equation (A7), we derive: $\gamma_r = p_e > 0$. Furthermore, applying the complementary slackness condition $\gamma_r(G_R - v_r - (1-\theta)q_r(1-\lambda_r)) = 0$, we obtain:

$$G_R - v_r - (1-\theta)q_r(1-\lambda_r) = 0 \quad (\text{A8})$$

- (3) Considering the variable β_r , we analyze two cases:

(a) $\beta_r = 0$.

From Equation (A5), we obtain: $\gamma_r = 2\lambda_r q_r \eta_r (1-\theta)$. Substituting this result into Equation (A6), we derive the expression for q_r : $q_r = \frac{-c_r - \delta q_n + \delta - p_e(1-\theta)}{2\delta}$. Given that $0 \leq \lambda_r \leq 1$, we obtain the following inequality: $p_e \leq 2q_r \eta_r (1-\theta)$.

(b) $\beta_r > 0$.

From the complementary slackness condition $\beta_r(1-\lambda_r)=0$, it follows that $\lambda_r=1$. Substituting this into Equations (A6) and (A8), we derive: $q_r = \frac{-c_r + \delta(1-q_n)}{2\delta + 2\eta_r(1-\theta)^2}$, and we establish the relationship: $v_r = G_R$. Furthermore, from Equation (A5), we obtain: $p_e > 2q_r\eta_r(1-\theta)$.

Step 2: Equilibrium analysis

To simplify calculations, we define a new variable, $\hat{G}_i = G_i - v_i$, for $i = M, R$, $j = n, r$. Given the permit balance condition, $v_n + v_r = 0$, it follows that: $\hat{G}_M + \hat{G}_R = G_M - v_n + G_R - v_r = G$.

Scenario 1. $0 < p_e \leq \min\{2\eta_n q_n, 2q_r\eta_r(1-\theta)\}$.

Under this scenario, the reaction functions for the OEM and IR are given by: $q_n = \frac{-c_r - p_e - \delta q_r - \Delta + 1}{2}$, $p_e = 2\lambda_n \eta_n q_n$, $\hat{G}_M = q_n(1-\lambda_n)$; $q_r = \frac{-c_r - \delta q_n + \delta - p_e(1-\theta)}{2\delta}$, $p_e = 2\lambda_r q_r \eta_r(1-\theta)$, $\hat{G}_R = (1-\theta)q_r(1-\lambda_r)$. By incorporating the condition $(\hat{G}_M + \hat{G}_R = G)$ into the above system of equations and solving them simultaneously, we arrive at the final expressions for the relevant variables.

$$Z_1^I = \eta_n \left(\delta(4-\delta) + 4\eta_r(\theta^2 + (\delta-2)\theta + 1) \right) + \eta_r \delta(4-\delta) \quad (Z_1^I > 0),$$

$$Z_2^I = -G\delta(4-\delta) - \delta^2 + \delta(3-\Delta) - 2c_r - \theta(\delta(1+\Delta+c_r) - 2c_r),$$

$$Z_3^I = 2\eta_r \eta_n G\delta(\theta+1) + \eta_n \left(\delta(2-2\Delta-c_r-\delta) - 2\eta_r(1-\Delta-c_r)(1-\theta)(\theta-\theta_1) \right) + \eta_r \delta(2-2\Delta-c_r-\delta),$$

$$Z_4^I = \eta_n \left(\delta(1+\Delta+c_r) - 2c_r - 2\eta_r(1-\Delta-c_r)(\theta_1-\theta) \right) + \eta_r \left(\delta(1+\Delta+c_r) - 2c_r \right) - 4\eta_n \eta_r G(\theta-\theta_3),$$

$$q_n^{I*} = \frac{Z_3^I}{Z_1^I}, \quad q_r^{I*} = \frac{Z_4^I}{Z_1^I}, \quad p_e^{I*} = \frac{2\eta_n \eta_r Z_2^I}{Z_1^I}, \quad \lambda_n^{I*} = \frac{\eta_r Z_2^I}{Z_3^I}, \quad \lambda_r^{I*} = \frac{\eta_n Z_2^I}{(1-\theta)Z_4^I}, \quad \hat{G}_M^{I*} = \frac{Z_3^I - \eta_r Z_2^I}{Z_1^I}, \quad \hat{G}_R^{I*} = G - \hat{G}_M^{I*}.$$

Denote the equilibrium solution described above as Equilibrium I. It can be straightforwardly demonstrated that the following derivative is positive:

$$\frac{\partial(2\eta_n q_n^{I*} - p_e^{I*})}{\partial G} = \frac{2\delta\eta_n \eta_r [2(1+\theta)\eta_n + (4-\delta)]}{\delta(4-\delta)(\eta_n + \eta_r) + 4\eta_n \eta_r [\delta\theta + (\theta-1)^2]} > 0. \quad \text{Given the constraint}$$

$$p_e^{I*} \leq 2\eta_n q_n^{I*}, \quad \text{we have the condition } \max\{G_1, 0\} \leq G < G_{\max}, \quad \text{where } \theta_1 = \frac{1-\Delta-\delta}{1-c_r-\Delta},$$

$$G_1 = \frac{(2(1-\theta)(1-\Delta-c_r)(\theta-\theta_1)\eta_r - \delta(2-\delta-2\Delta-c_r))\eta_n + \eta_r(1-\theta)(\delta(\Delta+c_r+1) - 2c_r)}{\delta\eta_r(2\eta_n(1+\theta) + 4-\delta)}. \quad \text{It is}$$

$$\text{easy to verify that } G_{\max} - G_1 = \frac{(2-2\Delta-c_r-\delta)Z_1^I}{\delta\eta_r(4-\delta)(2\eta_n(1+\theta) + 4-\delta)} > 0.$$

Furthermore, evaluating the derivative below, we have $\frac{\partial(2\eta_r q_r^{I*}(1-\theta) - p_e^{I*})}{\partial G} = 2\eta_n \eta_r (-4(1-\theta)(\theta-\theta_3)\eta_r + \delta(4-\delta))/Z_1^I$, where $\theta_3 = \frac{2-\delta}{2}$, which satisfies the conditions: it is negative when either $\theta \leq \theta_3$ or $\theta > \theta_3$ and $\eta_r < \eta_{r2}$; and it

is positive when $\theta > \theta_3$ and $\eta_r > \eta_{r2}$, where $\eta_{r2} = \frac{\delta(4-\delta)}{4(1-\theta)(\theta-\theta_3)}$. To satisfy the inequality $2\eta_r q_r^*(1-\theta) - p_e^* \geq 0$, the conditions required are:

$$\textcircled{1} \text{ For } \theta \leq \theta_3 \text{ or } \theta > \theta_3 \text{ and } \eta_r < \eta_{r2}, \text{ we require } \max\{G_2, 0\} \leq G < G_{\max}, \text{ where}$$

$$G_2 = \frac{(2(1-\theta)(1-\Delta-c_r)(\theta-\theta_1)\eta_r - \delta(2-\delta-2\Delta-c_r))\eta_n + \eta_r(1-\theta)(\delta(\Delta+c_r+1)-2c_r)}{\eta_n(\delta(\delta-4)+4\eta_r(1-\theta)(\theta-\theta_3))},$$

$$\eta_{r1} = \frac{\delta(2-\delta-2\Delta-c_r)}{2(1-\theta)(1-\Delta-c_r)(\theta-\theta_1)}, \quad G_{\max} - G_2 = \frac{(1-\theta)(\delta(\Delta+c_r+1)-2c_r)Z_1^l}{\delta\eta_n(4-\delta)(\delta(4-\delta)+4(1-\theta)(\theta_3-\theta)\eta_r)} > 0.$$

$\textcircled{2}$ For $\theta > \theta_3$ and $\eta_r > \eta_{r2}$, $G_{\max} < G_2$, $2\eta_r q_r^*(1-\theta) - p_e^* > 0$ can be easily verified. $\textcircled{3}$ For $\theta > \theta_3$ and $\eta_r = \eta_{r2}$, $2\eta_r q_r^*(1-\theta) - p_e^* > 0$ always holds.

For the convenience of subsequent analysis, we first compare the following key thresholds: $\theta_3 - \theta_1 = \frac{\delta-2c_r+\Delta\delta+c_r\delta}{2-2\Delta-2c_r} > 0$, $\eta_{r1} - \eta_{r2} = \frac{\delta(\theta+1)(\delta-2c_r+\Delta\delta+c_r\delta)}{4(\theta-1)(\theta-\theta_3)(\theta-\theta_1)(1-c_r-\Delta)}$,

$$G_1 - G_2 = \frac{Z_1^l \left[\eta_n \left((2(1-\theta)(1-\Delta-c_r)(\theta-\theta_1))\eta_r - \delta(2-\delta-2\Delta-c_r) \right) + \eta_r(1-\theta)(\delta(\Delta+c_r+1)-2c_r) \right]}{\delta\eta_n\eta_r(2\eta_n(1+\theta)+4-\delta)(-4\eta_r(1-\theta)(\theta-\theta_3)+\delta(4-\delta))}$$

. Integrating the above conditions, we proceed with the analysis as follows:

Case 1: When $0 < \theta \leq \max\{\theta_1, 0\}$, or when $\max\{\theta_1, 0\} < \theta < 1$ and $0 < \eta_r < \eta_{r1}$, if $0 < \eta_n < \eta_{n1}$, then $G_1 > G_2$ and we can get $G_1 \leq G < G_{\max}$ ($G_1 > 0$); if $\eta_n \geq \eta_{n1}$, then $G_1 \leq G_2$, and we can get $G_2 \leq G < G_{\max}$ ($G_2 \geq 0$), where

$$\eta_{n1} = \frac{\eta_r(\delta(\Delta+c_r+1)-2c_r)(1-\theta)}{-2(1-\theta)(1-\Delta-c_r)(\theta-\theta_1)\eta_r + \delta(2-2\Delta-\delta-c_r)}.$$

Case 2: When $\max\{\theta_1, 0\} < \theta < 1$ and $\eta_r \geq \eta_{r1}$, then we can get $G_1 \leq G < G_{\max}$ ($G_1 > 0$).

Based on the above analysis and the **proof of Corollary 1**, it can be straightforwardly demonstrated that the following equilibrium conditions hold: $q_n^* > 0$, $q_r^* \geq 0$, $0 \leq \lambda_n^* \leq 1$, $0 \leq \lambda_r^* \leq 1$, $p_e^* > 0$, $\hat{G}_M^* \geq 0$, $\hat{G}_R^* \geq 0$.

The following result can be readily verified to confirm that the above inequality holds: $q_n^*(G=0) > 0$ (Case 1), $q_n^*(G=G_1) > 0$; $q_r^*(G=G_1) > 0$ ($\max\{\theta_1, 0\} < \theta < 1$), $q_r^*(G=G_2) > 0$ ($0 < \theta \leq \max\{\theta_1, 0\}$), $q_r^*(G=G_{\max}) > 0$; $\lambda_n^*(G=G_{\max}) = 0$, $\lambda_n^*(G=G_1) = 1$; $\lambda_r^*(G=G_{\max}) = 0$, $\lambda_r^*(G=G_2) = 1$, $(1-\lambda_r^*)(G=G_1) > 0$ (Case 2); $p_e^*(G=G_{\max}) = 0$; $\hat{G}_M^*(G=G_1) = 0$; $\hat{G}_R^*(G=G_2) = 0$, $\hat{G}_R^*(G=G_{\max}) > 0$; $\hat{G}_R^*(G=G_1) > 0$ (Case 1 ($0 < \eta_n < \eta_{n1}$) and Case 2).

Scenario 2. $\min\{2\eta_n q_n, 2q_r \eta_r(1-\theta)\} < p_e \leq \max\{2\eta_n q_n, 2q_r \eta_r(1-\theta)\}$.

$$(2.1) \quad 2q_r \eta_r(1-\theta) < p_e \leq 2\eta_n q_n$$

In this scenario, the OEM's and the IR's reaction functions are given by:

$$q_n = \frac{-p_e - c_r - \delta q_r - \Delta + 1}{2}, \quad p_e = 2\lambda_n \eta_n q_n, \quad \hat{G}_M = q_n(1-\lambda_n), \quad q_r = \frac{-c_r + \delta(1-q_n)}{2\delta + 2\eta_r(1-\theta)}, \quad \lambda_r = 1,$$

$\hat{G}_R = (1-\theta)q_r(1-\lambda_r)$. By incorporating the condition ($\hat{G}_M + \hat{G}_R = G$) into the above system

of equations and solving them simultaneously, we arrive at the final expressions for the relevant variables.

$$Z_1^{\text{II}} = \delta + \eta_r (1 - \theta)^2 \quad (Z_1^{\text{II}} > 0),$$

$$Z_2^{\text{II}} = 4GZ_1^{\text{II}}\eta_n + 2(1 - \Delta - c_r)(1 - \theta)^2 \eta_r + \delta(2 - 2\Delta - c_r - \delta) \quad (Z_2^{\text{II}} > 0),$$

$$Z_3^{\text{II}} = (4\delta + 4\eta_r(1 - \theta)^2)\eta_n + 4\eta_r(1 - \theta)^2 + (4\delta - \delta^2) \quad (Z_3^{\text{II}} > 0),$$

$$q_n^{\text{II}*} = \frac{Z_2^{\text{II}}}{Z_3^{\text{II}}}, \quad q_r^{\text{II}*} = \frac{(\delta - c_r)Z_3^{\text{II}} - \delta Z_2^{\text{II}}}{2Z_1^{\text{II}}Z_3^{\text{II}}}, \quad p_e^{\text{II}*} = \frac{2\eta_n(Z_2^{\text{II}} - GZ_3^{\text{II}})}{Z_3^{\text{II}}}, \quad \lambda_n^{\text{II}*} = \frac{Z_2^{\text{II}} - GZ_3^{\text{II}}}{Z_2^{\text{II}}}, \quad \lambda_r^{\text{II}*} = 1, \quad \hat{G}_M^{\text{II}*} = G, \quad \hat{G}_R^{\text{II}*} = 0.$$

Label the equilibrium solution above as Equilibrium II. One can verify directly that:

$$2\eta_n q_n^{\text{II}*} - p_e^{\text{II}*} = 2G\eta_n > 0 \quad \text{.} \quad \text{Moreover,}$$

$$\frac{\partial(2\eta_r q_r^{\text{II}*}(1 - \theta) - p_e^{\text{II}*})}{\partial G} = \frac{2\eta_n(-4(1 - \theta)(\theta - \theta_3)\eta_r + \delta(4 - \delta))}{Z_3^{\text{II}}}, \quad \text{where } \theta_3 = \frac{2 - \delta}{2}, \quad \text{which satisfies the conditions: it is positive when either } \theta \leq \theta_3 \text{ or } \theta > \theta_3 \text{ and } \eta_r < \eta_{r2}; \text{ and it is}$$

negative when $\theta > \theta_3$ and $\eta_r > \eta_{r2}$, where $\eta_{r2} = \frac{\delta(4 - \delta)}{4(1 - \theta)(\theta - \theta_3)}$. To ensure that

$$2\eta_r q_r^{\text{II}*}(1 - \theta) - p_e^{\text{II}*} < 0, \quad \text{the following conditions need to be satisfied: ① } \theta \leq \theta_3 \text{ or } \theta > \theta_3$$

and $\eta_r < \eta_{r2}$ ($\eta_{r2} > \eta_{r1}$) with $0 < G < \max\{G_2, 0\}$, where G_2 is the solution to equation

$$2\eta_r q_r^{\text{II}*}(1 - \theta) - p_e^{\text{II}*} = 0 \quad \text{.} \quad \text{② } \theta > \theta_3 \text{ and } \eta_r > \eta_{r2} \quad (\eta_{r2} > \eta_{r1}) \text{ with}$$

$\min\{\max\{G_2, 0\}, G_{\max}\} < G < G_{\max}$ ($G_2 > G_{\max}$), which mean that this condition cannot be

satisfied. ③ $\theta > \theta_3$ and $\eta_r = \eta_{r2}$, $2\eta_r q_r^{\text{II}*}(1 - \theta) - p_e^{\text{II}*} = \frac{\delta - 2c_r + \Delta\delta + c_r\delta}{\delta} > 0$, which means

that $2\eta_r q_r^{\text{II}*}(1 - \theta) - p_e^{\text{II}*} < 0$ cannot be satisfied.

For condition ①, by further comparing the relationship between G_2 and 0 as well as $G_{\max}$, the conditions for the existence of Equilibrium II can be derived: (i) $0 < \theta \leq \max\{\theta_1, 0\}$ and $\eta_n \geq \eta_{n1}$ with $0 < G < G_2$; (ii) $\max\{\theta_1, 0\} < \theta < 1$ and $0 < \eta_r < \eta_{r1}$ and $\eta_n \geq \eta_{n1}$, with $0 < G < G_2$.

Under these assumptions, we can readily confirm: $q_n^{\text{II}*} \geq 0$, $q_r^{\text{II}*} \geq 0$, $0 \leq \lambda_n^{\text{II}*} \leq 1$, $0 \leq \lambda_r^{\text{II}*} \leq 1$, $p_e^{\text{II}*} > 0$, $\hat{G}_M^{\text{II}*} \geq 0$, $\hat{G}_R^{\text{II}*} \geq 0$. These findings affirm that Equilibrium II satisfies all non-negativity and feasibility constraints, while ensuring a positive shadow price.

The following result can be readily verified to confirm that the above inequality holds: $q_r^{\text{II}*}(G = 0) > 0$, $q_r^{\text{II}*}(G = G_2) > 0$ ($\theta \leq \theta_3$ or $\theta > \theta_3$ and $0 < \eta_r < \eta_{r1}$ where $\theta_3 > \theta_1$), $\lambda_n^{\text{II}*}(G = 0) = 1$, $\lambda_n^{\text{II}*}(G = G_2) > 0$ ($0 < \theta \leq \max\{\theta_1, 0\}$ or $\max\{\theta_1, 0\} < \theta < 1$ and $0 < \eta_r < \eta_{r1}$); when $\lambda_n^{\text{II}*} > 0$ ($0 < G < G_2$), it can be readily shown that $p_e^{\text{II}*} > 0$.

$$(2.2) \quad 2\eta_n q_n < p_e \leq 2q_r \eta_r (1 - \theta)$$

In this scenario, the OEM's and the IR's reaction functions are given by: $\lambda_n = 1$,

$$q_n = \frac{1 - \Delta - \delta q_r - c_r}{2(1 + \eta_n)}, \quad p_e = 2\lambda_r q_r \eta_r (1 - \theta), \quad \hat{G}_M = q_n(1 - \lambda_n); \quad q_r = \frac{-c_r - \delta q_n + \delta - p_e(1 - \theta)}{2\delta},$$

$\hat{G}_R = (1 - \theta)q_r(1 - \lambda_r)$. By incorporating the condition ($\hat{G}_M + \hat{G}_R = G$) into the above system

of equations and solving them simultaneously, we arrive at the final expressions for the relevant variables.

$$Z_1^{\text{III}} = \delta(1 + \Delta + c_r) - 2c_r + 2\eta_n(\delta - c_r) + 4G(1 + \eta_n)(1 - \theta)\eta_r \quad (Z_1^{\text{III}} > 0),$$

$$q_n^{\text{III}*} = \frac{(1 - \Delta - c_r)Z_3^{\text{II}} - \delta Z_1^{\text{III}}}{2(1 + \eta_n)Z_3^{\text{II}}}, \quad \lambda_n^{\text{III}*} = 1, \quad \widehat{G}_M^{\text{III}*} = 0, \quad q_r^{\text{III}*} = \frac{Z_1^{\text{III}}}{Z_3^{\text{II}}}, \quad \lambda_r^{\text{III}*} = \frac{(1 - \theta)Z_1^{\text{III}} - GZ_3^{\text{II}}}{(1 - \theta)Z_1^{\text{III}}}, \quad \widehat{G}_R^{\text{III}*} = G,$$

$$p_e^{\text{III}*} = \frac{2\eta_r((1 - \theta)Z_1^{\text{III}} - GZ_3^{\text{II}})}{Z_3^{\text{II}}}.$$

Denote the solution above as Equilibrium III. It can be verified that

$$2q_r^{\text{III}*}\eta_r(1 - \theta) - p_e^{\text{III}*} = 2G\eta_r > 0, \quad \text{and} \quad \frac{\partial(p_e^{\text{III}*} - 2\eta_n q_n^{\text{III}*})}{\partial G} = -\frac{4\delta\eta_r(1 + \theta)\eta_n + 2\delta(4 - \delta)\eta_r}{Z_3^{\text{II}}} < 0.$$

Because $p_e^{\text{III}*} - 2\eta_n q_n^{\text{III}*} > 0$, one must have $0 < G < \max\{G_1, 0\}$ ($G_1 < G_{\max}$). To ensure that $G_1 > 0$, the following conditions must be satisfied: (i) $0 < \theta \leq \max\{\theta_1, 0\}$ and $0 < \eta_n < \eta_{n1}$; (ii) $\max\{\theta_1, 0\} < \theta < 1$ and $0 < \eta_r < \eta_{r1}$ and $0 < \eta_n < \eta_{n1}$; (iii) $\max\{\theta_1, 0\} < \theta < 1$ and $\eta_r \geq \eta_{r1}$. Under these conditions, we can confirm: $q_n^{\text{III}*} \geq 0$, $q_r^{\text{III}*} \geq 0$, $0 \leq \lambda_n^{\text{III}*} \leq 1$, $0 \leq \lambda_r^{\text{III}*} \leq 1$, $p_e^{\text{III}*} > 0$, $\widehat{G}_M^{\text{III}*} \geq 0$, $\widehat{G}_R^{\text{III}*} \geq 0$. Hence, Equilibrium III satisfies all non-negativity and feasibility requirements, guaranteeing a positive shadow price under the given constraints.

The following result can be readily verified to confirm that the above inequality holds: $q_n^{\text{III}*}(G = G_1) > 0$, $\lambda_r^{\text{III}*}(G = G_1) > 0$.

Scenario 3. $p_e > \max\{2\eta_n q_n, 2q_r \eta_r(1 - \theta)\}$.

In this scenario, the OEM's and the IR's reaction functions are: $\lambda_n = 1$, $q_n = \frac{1 - \Delta - \delta q_r - c_r}{2(1 + \eta_n)}$, $\widehat{G}_M = 0$; $\lambda_r = 1$, $q_r = \frac{-c_r + \delta(1 - q_n)}{2\delta + 2\eta_r(1 - \theta)}$, $\widehat{G}_R = 0$. It follows that $\widehat{G}_M + \widehat{G}_R = 0$, which is different from G . Therefore, we exclude this case from our analysis. In conclusion, the resulting strategies for all enterprises are summarized in Table 3 and Table 4.

Step 3: Verifying the second-order sufficient conditions

(1) Interior solutions

Next, we separately check the optimality of interior solutions and corner (boundary) solutions. For the OEM and IR interior solutions, the Hessian matrices of π_M with respect to λ_n , q_n and v_n , and of π_R with respect to λ_r , q_r and v_r , are respectively

given by:

$$H_M = \begin{pmatrix} -2\eta_n q_n^2 & -4\eta_n \lambda_n q_n & 0 \\ -4\eta_n \lambda_n q_n & -2\eta_n \lambda_n^2 - 2 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$H_R = \begin{pmatrix} -2\eta_r q_r^2 (1 - \theta)^2 & -4\eta_r \lambda_r q_r (1 - \theta)^2 & 0 \\ -4\eta_r \lambda_r q_r (1 - \theta)^2 & -2\delta - 2\eta_r \lambda_r^2 (1 - \theta)^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

In practical settings, both H_M and H_R

are negative semidefinite Hessian matrices, which guarantees that the interior solutions are optimal.

(2) OEM corner solution ($p_e > 2\eta_n q_n$)

Under this scenario, $\lambda_n = 1$ and $q_n = \frac{1-\Delta-\delta q_r - c_r}{2(1+\eta_n)}$, $v_n = G_M$. From the constraints $\lambda_n \leq 1$ and $q_n(1-\lambda_n) \leq G_M - v_n$, we define $\bar{g}_{n1} = \lambda_n - 1$, $\bar{g}_{n2} = q_n(1-\lambda_n) - G_M + v_n$. At this corner solution, the gradient of the OEM's profit function is $\nabla \pi_M = (-2q_n^2 \eta_n \quad 0 \quad p_e)^T$. Consider a nonzero feasible direction $\underline{d}_M = (d_{M1} \quad d_{M2} \quad d_{M3})^T$. From $(\nabla \bar{g}_{n1})^T \underline{d}_M \leq 0$ and $(\nabla \bar{g}_{n2})^T \underline{d}_M \leq 0$, we derive $d_{M1} \leq 0$, $-q_n d_{M1} + d_{M3} \leq 0$. If $\underline{d}_M$ is indeed a feasible direction, then $(\underline{d}_M)^T \nabla \pi_M = -2q_n^2 \eta_n d_{M1} + p_e d_{M3} \leq p_e (-q_n d_{M1} + d_{M3}) \leq 0$. This shows that the directional derivative in any feasible direction is non-positive, implying local optimality at the corner solution.

(3) IR corner solution ($p_e > 2q_r \eta_r (1-\theta)$)

In this case, $\lambda_r = 1$ and $q_r = \frac{-c_r + \delta(1-q_n)}{2\delta + 2\eta_r(1-\theta)^2}$, $v_r = G_R$. From the constraints $\lambda_r \leq 1$ and $(1-\theta)q_r(1-\lambda_r) \leq G_R - v_r$, we define $\bar{g}_{r1} = \lambda_r - 1$, $\bar{g}_{r2} = (1-\theta)q_r(1-\lambda_r) - G_R + v_r$. The gradient of the IR's profit function at this corner solution is $\nabla \pi_R = (-2q_r^2 \eta_r (1-\theta)^2 \quad 0 \quad p_e)^T$. Let a nonzero feasible direction be $\underline{d}_R = (d_{R1} \quad d_{R2} \quad d_{R3})^T$. From $(\nabla \bar{g}_{r1})^T \underline{d}_R \leq 0$ and $(\nabla \bar{g}_{r2})^T \underline{d}_R \leq 0$, we get $d_{R1} \leq 0$, $-(1-\theta)q_r d_{R1} + d_{R3} \leq 0$. Consequently, $(\underline{d}_R)^T \nabla \pi_R = -2q_r^2 \eta_r (1-\theta)^2 d_{R1} + p_e d_{R3} \leq p_e (-(1-\theta)q_r d_{R1} + d_{R3}) \leq 0$. Again, the directional derivative in every feasible direction is non-positive, confirming that the corner solution is locally optimal.

By conducting these verifications for both the interior and corner cases, we ensure the second-order conditions hold and thus establish the local optimality of the candidate solutions. $\square$

Proof of Corollary 1 (impact of G on the variables).

To simplify the exposition, the following notation is introduced:

$$Z_1^I = \eta_n (\delta(4-\delta) + 4\eta_r (\theta^2 + (\delta-2)\theta + 1)) + \eta_r \delta(4-\delta) \quad (Z_1^I > 0),$$

$$Z_3^I = 2\eta_r \eta_n G \delta (\theta + 1) + \eta_n (\delta(2-2\Delta - c_r - \delta) - 2\eta_r (1-\Delta - c_r)(1-\theta)(\theta - \theta_1)) + \eta_r \delta(2-2\Delta - c_r - \delta),$$

$$Z_4^I = \eta_n (\delta(1+\Delta + c_r) - 2c_r - 2\eta_r (1-\Delta - c_r)(\theta_1 - \theta)) + \eta_r (\delta(1+\Delta + c_r) - 2c_r) - 4\eta_n \eta_r G (\theta - \theta_3),$$

$$Z_1^{II} = \delta + \eta_r (1-\theta)^2 \quad (Z_1^{II} > 0),$$

$$Z_2^{II} = 4GZ_1^{II} \eta_n + 2(1-\Delta - c_r)(1-\theta)^2 \eta_r + \delta(2-2\Delta - c_r - \delta) \quad (Z_2^{II} > 0),$$

$$Z_3^{II} = (4\delta + 4\eta_r (1-\theta)^2) \eta_n + 4\eta_r (1-\theta)^2 + (4\delta - \delta^2) \quad (Z_3^{II} > 0),$$

$$Z_1^{III} = \delta(1+\Delta + c_r) - 2c_r + 2\eta_n (\delta - c_r) + 4G(1+\eta_n)(1-\theta) \eta_r \quad (Z_1^{III} > 0)$$

$$\frac{\partial q_n^{j*}}{\partial G} = \begin{cases} 2\delta\eta_n\eta_r(\theta+1)/Z_1^I > 0 & , j = I \\ 4\eta_n(\delta+\eta_r(1-\theta))^2/Z_3^{\text{II}} > 0 & , j = \text{II} \\ -2\delta\eta_r(1-\theta)/Z_3^{\text{II}} < 0 & , j = \text{III} \end{cases}, \quad \frac{\partial q_r^{j*}}{\partial G} = \begin{cases} -4\eta_n\eta_r(\theta-\theta_3)/Z_1^I < 0 \Leftrightarrow \theta > \theta_3 & , j = I \\ -2\delta\eta_n/Z_3^{\text{II}} < 0 & , j = \text{II} \\ 4\eta_r(\eta_n+1)(1-\theta)/Z_3^{\text{II}} > 0 & , j = \text{III} \end{cases}$$

$$\frac{\partial \lambda_n^{j*}}{\partial G} = \begin{cases} -\delta\eta_r(2-2\Delta-c_r-\delta)Z_1^I/(Z_3^I)^2 < 0 & , j = I \\ -Z_3^{\text{II}}(2(1-\Delta-c_r)\eta_r(1-\theta)^2+\delta(2-2\Delta-c_r-\delta))/(Z_2^{\text{II}})^2 < 0 & , j = \text{II} \\ 0 & , j = \text{III} \end{cases}$$

$$\frac{\partial \lambda_r^{j*}}{\partial G} = \begin{cases} -\eta_n(\Delta\delta+c_r\delta+\delta-2c_r)Z_1^I/((1-\theta)(Z_4^I)^2) < 0 & , j = I \\ 0 & , j = \text{II} \\ -(\delta(1+\Delta+c_r)-2c_r+2\eta_n(\delta-c_r))Z_3^{\text{II}}/((1-\theta)(Z_1^{\text{III}})^2) < 0 & , j = \text{III} \end{cases}$$

$$\frac{\partial(q_r^{j*}-q_n^{j*})}{\partial G} = \begin{cases} -2\eta_n\eta_r(\theta-\theta_2)(\delta+2)/Z_1^I < 0 \Leftrightarrow \theta > \theta_2 & , j = I \\ -2\eta_n(2Z_1^{\text{II}}+\delta)/Z_3^{\text{II}} < 0 & , j = \text{II} \\ (4\eta_n\eta_r(1-\theta)+\eta_r(4+2\delta)(1-\theta))/Z_3^{\text{II}} > 0 & , j = \text{III} \end{cases}$$

$$\frac{\partial p_e^{j*}}{\partial G} = \begin{cases} -2\delta\eta_n\eta_r(4-\delta)/Z_1^I < 0 & , j = I \\ -2\eta_n(\delta(4-\delta)+4\eta_r(1-\theta)^2)/Z_3^{\text{II}} < 0 & , j = \text{II} \\ -2\delta\eta_r(4+4\eta_n-\delta)/Z_3^{\text{II}} < 0 & , j = \text{III} \end{cases}$$

$$\frac{\partial \hat{G}_M^{j*}}{\partial G} = \begin{cases} \delta\eta_r(2\eta_n-\delta+2\eta_n\theta+4)/Z_1^I > 0 & , j = I \\ 1 > 0 & , j = \text{II} \\ 0 & , j = \text{III} \end{cases}$$

$$\frac{\partial \hat{G}_R^{j*}}{\partial G} = \begin{cases} \eta_n(\delta(4-\delta)-4\eta_r(1-\theta)(\theta-\theta_3))/Z_1^I < 0 \Leftrightarrow \theta > \theta_3 \ \& \ \eta_r > \eta_{r2} & , j = I \\ 0 & , j = \text{II} \\ 1 > 0 & , j = \text{III} \end{cases}$$

□

Proof of Corollary 2 (impact of CTP on firms' profits).

Under a relatively relaxed carbon permit scenario, where $G \in (\max\{G_1, G_2\}, G_{\max})$, firms adopt Equilibrium I. It can be verified that the profit functions π_M^* , π_R^* , and π_T^* are continuous in G . Furthermore, the partial derivatives of each profit function with respect to G , evaluated at $G = G_{\max}$, are given by:

$$\left. \frac{\partial \pi_M^*}{\partial G} \right|_{G=G_{\max}} = -2(4-\delta)\delta\eta_n\eta_r(G_M-G_{M1})/Z_1^I < 0 \Leftrightarrow G_M > G_{M1},$$

$$\left. \frac{\partial \pi_R^*}{\partial G} \right|_{G=G_{\max}} = -2\delta(4-\delta)\eta_n\eta_r(G_R-G_{R1})/Z_1^I < 0 \Leftrightarrow G_R > G_{R1},$$

$$\frac{\partial \pi_T^*}{\partial G} \Big|_{G=G_{\max}} = -2\delta\eta_n\eta_r \left((4\Delta + 3\delta + \Delta\delta + c_r\delta - 4)\theta - (4 - 3\delta)\Delta + 2c_r\delta - 3\delta - 4c_r + \delta^2 + 4 \right) / ((4 - \delta)Z_1^1) < 0$$

, where $G_{M1} = \frac{2(1+\theta)(2-\delta-2\Delta-c_r)}{(4-\delta)^2}$, $G_{R1} = \frac{-4(\Delta\delta + \delta + \delta c_r - 2c_r)(\theta - \theta_3)}{\delta(4-\delta)^2}$.

By exploiting the continuity of these profit functions and these derivative conditions, we find: If $G_M > G_{M1}$, then there exists some $\bar{G} < G_{\max}$ such that for all $G \in (\bar{G}, G_{\max})$, the OEM's profit π_M^* under CTP exceeds its profit π_M^* without regulation, i.e., $\pi_M^* \Big|_{G \in (\bar{G}, G_{\max})} > \pi_M^* \Big|_{G=G_{\max}} = \pi_M^*$. If $G_R > G_{R1}$, there exists some $\bar{G} < G_{\max}$ ensuring $\pi_R^* \Big|_{G \in (\bar{G}, G_{\max})} > \pi_R^* \Big|_{G=G_{\max}} = \pi_R^*$. By the same argument, there exists $\bar{G} < G_{\max}$ ensuring $\pi_T^* > \pi_T^*$ for G in $(\bar{G}, G_{\max})$. Further combining the positions of G_{M1} and G_{R1} relative to 0 and $G_{\max}$ yields the specific conditions under which the OEM's and IR's profits both increase in the presence of regulation. In particular, when $0 < \theta \leq \theta_3$ and $G_{M1} < G_M < G_{\max}$ and $G_{R1} < G_R < G_{\max}$, or when $\theta_3 < \theta < \theta_4$ and $G_{M1} < G_M < G_{\max}$, there exists $\bar{G} < G_{\max}$ such that for any $G \in (\bar{G}, G_{\max})$, we have $\pi_M^* > \pi_M^*$ and $\pi_R^* > \pi_R^*$. The relevant parameters satisfy:

$$G_{\max} - G_{M1} = - \frac{(\delta(8-3\delta) - \delta^2\Delta - c_r(\delta^2 - 4\delta + 8))(\theta - \theta_4)}{\delta(4-\delta)^2} \begin{cases} > 0, & 0 < \theta < \theta_4 \\ \leq 0, & \text{otherwise} \end{cases},$$

$$G_{\max} - G_{R1} = \frac{(3\delta + \delta\theta - 8)\Delta - 5\delta + \delta\theta + \delta^2 + 8 + 2c_r\delta + c_r\delta\theta - 6c_r - 2c_r\theta}{(\delta-4)^2} > 0,$$

$$G_{\max} - (G_{M1} + G_{R1}) = \frac{(3\delta + \Delta(\delta+4) - 4 + c_r\delta)\theta - 3\delta + \delta^2 + \Delta(3\delta-4) + 4 + 2c_r\delta - 4c_r}{(\delta-4)^2} > 0, \text{ where}$$

$$\theta_4 = \frac{8\delta - 8c_r + \Delta\delta^2 + \delta^3 - 5\delta^2 + 4c_r\delta}{\delta(8-3\delta) - \delta^2\Delta - c_r(\delta^2 - 4\delta + 8)} \quad (0 < \theta_4 < 1),$$

$$\theta_3 - \theta_4 = - \frac{\delta(4-\delta)(\delta + \Delta\delta - 2c_r + c_r\delta)}{2(8\delta - 3\delta^2 - \Delta\delta^2 - 8c_r + 4c_r\delta - c_r\delta^2)} < 0. \quad \square$$

Proof of Corollary 3 (impact of CTP on GCS and SW).

Under relatively moderate CTP ($G \in (\max\{G_1, G_2\}, G_{\max})$), the firms operate according to Equilibrium I. One can show that GCS^* is continuous in G . In addition, its partial derivative with respect to G , evaluated at $G_{\max}$, satisfies:

$$\frac{\partial GCS^*}{\partial G} \Big|_{G=G_{\max}} = - \frac{2a_2(\varepsilon - \varepsilon_1)}{\delta(4-\delta)} \begin{cases} < 0, & \varepsilon > \varepsilon_1 \\ \geq 0, & \text{otherwise} \end{cases}, \quad \text{where}$$

$$a_1 = (4c_r - 3c_r\delta + c_r\delta^2 - 4\delta + \Delta\delta^2 + 2\delta^2)\theta + 8\delta - 4\Delta\delta + 2\Delta\delta^2 - 5\delta^2 + \delta^3 + c_r\delta^2 - 4c_r - c_r\delta \quad (a_1 > 0),$$

$$a_2 = -(\delta + \Delta\delta + c_r\delta - 2c_r)\theta + 3\delta - \Delta\delta - \delta^2 - 2c_r \quad (a_2 > 0), \quad \varepsilon_1 = \frac{\delta\eta_n\eta_r a_1}{a_2 Z_1^1} \quad (\varepsilon_1 > 0). \text{ Similarly,}$$

$$SW^* \text{ is also continuous in } G, \text{ with } \frac{\partial SW^*}{\partial G} \Big|_{G=G_{\max}} = - \frac{2a_2(\varepsilon - \varepsilon_2)}{\delta(4-\delta)} \begin{cases} < 0, & \varepsilon > \varepsilon_2 \\ \geq 0, & \text{otherwise} \end{cases}, \text{ where}$$

$$\varepsilon_2 = \frac{-\delta\eta_n\eta_r a_3}{a_2 Z_1^1}, \quad a_3 = (\delta^2 + 4\theta\delta)\Delta + 4c_r + \theta(\delta^2 + 3c_r\delta - 4c_r) - \delta(3c_r + 4) + \delta^2(c_r + 2),$$

$$a_{3\min} = a_3 \Big|_{\Delta = \frac{(1-\delta)c_r}{\delta}} = -(\delta - c_r)(4 - 2\delta - \delta\theta) < 0 \quad \left(\quad c_r < \delta < 1 \quad \right),$$

$$a_{3\max} = a_3 \Big|_{\Delta = \frac{2-c_r-\delta}{2}} = (\delta - c_r)(4 - \delta)(\theta - \theta_3) > 0 \Leftrightarrow \theta > \theta_3.$$

Using the continuity of these functions along with the conditions for these derivatives, we conclude: If $\varepsilon > \varepsilon_1$, then there exists $\bar{G} < G_{\max}$ such that $GCS^*|_{G \in (\bar{G}, G_{\max})} > \underline{GCS}^*$. Likewise, if $a_3 < 0$ and $\varepsilon > \varepsilon_2$, or if $a_3 \geq 0$, there exists $\bar{G} < G_{\max}$ such that $SW^*|_{G \in (\bar{G}, G_{\max})} > \underline{SW}^*$. Moreover, the effects of η_n and η_r on the critical values ε_1 and ε_2 (when $a_3 < 0$) are given by: $\frac{\partial \varepsilon_1}{\partial \eta_n} = \frac{\delta^2 \eta_r^2 (4 - \delta) a_1}{a_2 (Z_1^1)^2} > 0$, $\frac{\partial \varepsilon_1}{\partial \eta_r} = \frac{\delta^2 \eta_n^2 (4 - \delta) a_1}{a_2 (Z_1^1)^2} > 0$, $\frac{\partial \varepsilon_2}{\partial \eta_n} = -\frac{\delta^2 \eta_r^2 (4 - \delta) a_3}{a_2 (Z_1^1)^2} > 0$, $\frac{\partial \varepsilon_2}{\partial \eta_r} = -\frac{\delta^2 \eta_n^2 (4 - \delta) a_3}{a_2 (Z_1^1)^2} > 0$. $\varepsilon_1 - \varepsilon_2 = \frac{\delta^2 \eta_n \eta_r ((3\delta + \theta(\delta + 4) - 4)\Delta + \theta(3\delta + c_r \delta - 4) - 3\delta - 4c_r + 2c_r \delta + \delta^2 + 4)}{a_2 Z_1^1} > 0$ $((1 - \delta)c_r / \delta < \Delta < (2 - \delta - c_r) / 2)$. $\square$

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