

Article

Tripartite Evolutionary Game Analysis of Digital Transformation in Sports Equipment Manufacturing Industry

Mingcan Xu ¹  and Jian Yang ^{2,*} ¹ School of Physical Education, Shanxi University of Finance and Economics, Taiyuan 030006, China; 202318010118@stu.sxufe.edu.cn² School of Information, Shanxi University of Finance and Economics, Taiyuan 030006, China

* Correspondence: yangj@sxufe.edu.cn

Abstract

With the widespread application of IoT and AI technologies in the sports equipment manufacturing sector, traditional sports equipment manufacturers are facing challenges such as financial pressures, technological integration barriers, and the gradual reduction in government subsidies during digital transformation. To investigate the intrinsic mechanism of multi-stakeholder collaborative transformation, this paper, based on the bounded rationality assumption, incorporates the government, sports equipment manufacturers, and third-party digital service providers into a unified analytical framework. A tripartite evolutionary game model is constructed, systematically deriving the replication dynamic equations for each stakeholder under different strategies, and analyzing the stability of the system equilibrium point using the Jacobian matrix. This study shows that the sustainability of government subsidies depends on the trade-off between social benefits, administrative costs, and subsidy expenditures; whether manufacturers choose to cooperate is mainly influenced by the cost of self-purchased equipment, digital service fees, operation and maintenance risks, and residual value recovery; and the investment willingness of digital service providers depends on infrastructure construction costs, service revenue, and government support. Further simulation analysis shows that a higher initial willingness to cooperate can accelerate system convergence, a moderate capital interest rate helps balance the incentives of both supply and demand sides, excessively high core equipment prices will inhibit manufacturers' willingness to cooperate, and reasonable digital service fees are key to achieving stable collaboration. This paper provides a theoretical basis for policy optimization, service pricing, and industrial collaborative governance in the digital transformation of the sports manufacturing industry.

Keywords: digital transformation; sports equipment manufacturing; evolutionary game; government subsidies

MSC: 91A80



Academic Editors: Yelin Fu, Wei Wu and Yiji Cai

Received: 3 June 2026

Revised: 29 June 2026

Accepted: 4 July 2026

Published: 7 July 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

1. Introduction

The traditional sports manufacturing industry is undergoing a profound transformation with the rapid development of the Internet of Things (IoT), advanced data analytics models, and AI technologies [1,2]. The development and production of modern sports equipment increasingly take into account the integration of high-precision biomechanical sensors and systems of human response detection [3]. Intelligent production lines and

cloud-based data platforms are enabling customized production in the manufacturing industry [4]. This paradigm not only reconfigures the competitive performance and the user experience of sports products but also drives a comprehensive reorganization of the entire value chain of the sports product industry [5,6]. At the initial stage of such intelligent transformation, facing high technical integration barriers and high upfront capital requirements, governments in different countries have adopted measures such as financial intervention and policy guidance [7]. These macro-level interventions have been crucial to address market failures in the early adoption of new technologies, reduce the risks of information asymmetry for small and medium-sized enterprises (SMEs), and generate positive externalities across the value chain [8,9].

However, as the market for digital hardware matures, the price of general-purpose computing power continues to decline, and smart devices are increasingly adopted, direct fiscal support from the government will inevitably be subject to dynamic adjustments and gradual reductions [10]. The reduction in subsidies is a major challenge for manufacturers of sports equipment, as the purchase of high-end core digital equipment, setting up data centers for smart factories, and hiring cross-disciplinary digital talent all require large upfront capital expenditures [11]. Therefore, enterprises are under great pressure in terms of funds and the potential breakage of their capital chains, which further decelerates the speed of enterprises' upgrades of their digital infrastructure [12]. An increasing number of manufacturing companies are trying to establish long-term collaborative partnerships with specialized third-party digital service providers to overcome the dual predicament of financial limitations and technological barriers [13,14]. These companies convert capital expenditures on heavy assets into flexible operating expenditures via service subscriptions or equipment leasing. This novel approach, while effectively taking advantage of the initial costs of technology acquisition and mitigating the risks of digital iteration, also starts a complicated strategic game among the government, manufacturing enterprises, and digital service providers for the sharing of benefits and the sharing of risks. Most of the existing studies are limited to analyzing the bilateral interaction between the government and enterprises or simply give an empirical evaluation of the static implementation effects of some policies. Few studies have regarded digital service providers as independent core stakeholders in their analytical frameworks, leading to a lack of in-depth exploration of the dynamic evolutionary decision-making mechanisms among the three parties. Significantly, in the dynamic constraint of gradually reducing fiscal subsidies, there is a lack of research on the quantitative analysis of the theoretical boundaries of the interests of all stakeholders' equilibrium.

For this purpose, this study builds a tripartite evolutionary game-theoretic framework of government agencies, sports equipment manufacturers, and third-party digital service providers to analyze the complex interaction logic among multiple agents under bounded rationality constraints. This study makes three specific theoretical contributions.

- First, it provides a rigorous mathematical characterization of a game in which digital service providers (DSPs) are modeled as independent strategic agents making irreversible, asset-specific infrastructure investments—a configuration that yields qualitatively different equilibrium conditions compared to existing tripartite models that treat service providers as passive intermediaries.
- Second, it derives closed-form stability conditions across six equilibrium scenarios that explicitly reveal the economic thresholds at which each stakeholder's strategy flips, providing testable boundary conditions for future empirical research.
- Third, through systematic parameter sensitivity analysis, it identifies a non-trivial "optimal subsidy window" and characterizes critical phase transitions, including the

“paradox of excessive subsidies” and the “service fee bifurcation point”, that carry direct implications for policy design.

Methodologically, this study employs nonlinear dynamic derivations to determine the mathematical boundaries and critical conditions for macro-level government intervention, active enterprise participation in ecological cooperation, and large-scale infrastructure investment by service providers. A distinctive feature of our approach is the explicit incorporation of Equivalent Annual Cost (EAC) conversions, which link the capital interest rate to the strategic decisions of all three agents, a mechanism absent from prior tripartite evolutionary game models. Second, under the assumption of a limited total fiscal subsidy budget, this study discusses how the government can optimize the allocation ratio of fiscal resources between the supply side and the demand side to balance the interests of multiple parties and maximize the overall efficiency of the industrial system. Finally, this study performs an in-depth analysis of the moderating effects of core economic variables, such as market interest rates, initial procurement costs of digital equipment, and digital service fees, on the evolutionary trajectories of market agents’ behaviors.

This paper is organized as follows: Section 2 reviews related literature, identifies gaps, and positions our contributions. Section 3 constructs the tripartite evolutionary game model. Section 4 derives the replicator dynamics and analyzes stability. Section 5.2 conducts Jacobian stability analysis, defines Conditional ESS, and develops propositions with economic interpretations and comparative statics. Section 6 presents simulations and sensitivity analysis focusing on critical thresholds and phase transitions. Section 7 concludes with limitations and future work.

2. Literature Review and Research Positioning

2.1. Digital Transformation in Manufacturing

Digital transformation in manufacturing has emerged as a central topic in industrial economics and operations management in recent years. A rich body of research has accumulated around three interconnected analytical levels: technology adoption barriers, organizational capability reconfiguration, and policy incentive design. We review each level in turn, highlighting at each stage the specific gap that motivates the inclusion of digital service providers as independent strategic agents.

At the technology-adoption level, Maretto et al. [15] conducted a systematic review of 229 real-world cases of digital technology application in manufacturing and found that only 94 studies included performance evaluation and only 30 included economic assessment, revealing a serious absence of cost-benefit analysis frameworks for digital transformation. Yang et al. [16] focused on small, medium, and micro enterprises (SMMEs) and employed multi-criteria decision-making methods to identify key drivers of digital technology adoption in the Industry 4.0 context, pointing out that technology compatibility and organizational readiness are core prerequisites for enterprise digitalization decisions. Gaddekar et al. [17] adopted a barrier perspective and constructed a structured model for evaluating inhibitors of Industry 4.0 implementation in manufacturing, finding that inadequate infrastructure and high initial investment costs are the most significant bottlenecks constraining SME transformation. Krishnan et al. [18] conducted a systematic review of the challenges and benefits of SME transformation toward smart manufacturing, constructing a multi-level analytical framework covering technological, organizational, and environmental dimensions, and emphasizing that management commitment and employee skill retraining are key levers for bridging the “digital divide”.

At the organizational capability and strategy level, Chirumalla et al. [19] distilled 11 critical factors affecting digital transformation in manufacturing firms through a systematic literature review and proposed the “Awareness-Readiness-Technology selection-

Operations” (ARTO) four-stage model, providing an integrative framework for understanding the process mechanisms of enterprise transformation. Cugno et al. [20] used the COVID-19 pandemic as an observation window to empirically examine the impact of Industry 4.0 technologies on production recovery and resilience in manufacturing firms, finding that enterprises with higher digital readiness demonstrated significantly stronger operational adjustment capabilities under exogenous shocks. Ardolino et al. [21] conducted a systematic review of the literature on deploying digital technologies to cope with pandemic disruptions in manufacturing, revealing synergistic mechanisms among IoT, artificial intelligence, and additive manufacturing in supply chain disruption contexts, while noting that most studies remain exploratory cases lacking generalizable quantitative models.

At the digital servitization and supply chain level, Tomaschko et al. [22] systematically reviewed sustainability assessment methods for manufacturing system digitalization, noting significant shortcomings in current research including methodological fragmentation and ambiguous technological boundaries in the quantitative evaluation of environmental performance. Zhang et al. [23] used the CiteSpace bibliometric tool to conduct a knowledge mapping analysis of the manufacturing digital transformation literature, identifying six core research themes: digital technology, dynamic capabilities, circular economy, digital servitization, lean manufacturing, and maturity models. Wang et al. [24] focused on the impact of digital technology on manufacturing supply chain efficiency, reviewing 59 studies and finding that the convergence of IoT and artificial intelligence constitutes the core enabling technology portfolio for smart factories and predictive supply chains.

Overall, the manufacturing digital transformation literature has thoroughly documented adoption barriers, organizational enablers, and servitization trends, yet, it predominantly adheres to a “firm-technology” dyadic paradigm or treats government policy as a unidirectional incentive tool. Few studies position third-party DSPs as strategic agents with independent investment autonomy within a game-theoretic framework. This gap, together with those identified in the evolutionary game literature below, is systematically addressed in Section 2.3.

2.2. Applications of Evolutionary Game Theory

Evolutionary game theory [25] relaxes classical game theory’s stringent assumptions of complete rationality and perfect information, enabling agents to adapt strategies through trial-and-error, imitation, and learning. This framework has emerged as a pivotal analytical tool for examining multi-agent strategic interactions and institutional evolution within industrial systems. Recent years have witnessed growing applications of EGT across manufacturing digital transformation, green supply chain governance, and industrial policy evaluation.

In the field of digital transformation in manufacturing, Dong et al. [26] constructed evolutionary game model among equipment manufacturing enterprises, the government, and consumers, analyzing the impact of government subsidy intensity on the adoption of digital strategies. They found a critical threshold effect in subsidies: moderate subsidies can accelerate enterprise digital transformation, while excessive subsidies may induce subsidy dependence. Zhu et al. [27] focused on the design of digital regulatory policies for SMEs in the manufacturing sector, constructing an evolutionary game model from the perspective of three-way strategies involving the government, third-party demonstration enterprises (TDEs), and SMEs, and proposing a differentiated governance mechanism that balances incentives and constraints. Gao et al. [28] addressed the multi-stakeholder digital collaboration problem in the service-oriented manufacturing value chain, constructing a three-way evolutionary game model among core manufacturing enterprises, service enterprises, and

customers. They used multi-stakeholder simulation to reveal the collaborative path and equilibrium conditions for value co-creation under varying levels of digitalization.

In the field of green manufacturing and low-carbon innovation, Chen et al. [29] constructed an evolutionary game model among manufacturing enterprises and between government and enterprises to analyze the dynamic evolution of green and low-carbon innovation. Without government intervention, enterprises choose to differentiate, with some adhering to traditional innovation while sharing the benefits of green innovation; after the government provides incentives such as transfer payments and tax credits, all enterprises eventually shift towards green and low-carbon innovation. Qiu et al. [30] constructed a three-way evolutionary game model involving commercial banks, digital enterprises, and the manufacturing industry, further subdividing the manufacturing industry into non-polluting and polluting manufacturing industries; and used numerical simulation to analyze the impact of key parameters on evolutionary stable strategies. Wang et al. [31] constructed a tripartite evolutionary game model involving the government, manufacturing, and logistics enterprises to study the cooperative mechanism for low-carbon coordinated development. Through replication dynamic equations and stability analysis, they found that additional costs are a key influencing factor, and that the government can promote cooperation through regulation or fiscal incentives in the early stages of development.

At the level of digital servitization and knowledge sharing, Ma et al. [32] combined evolutionary game models with complex network theory to examine the evolution of knowledge-sharing strategies between manufacturing enterprises and service providers in the process of digital servitization, revealing the formation mechanism of the “prisoner’s dilemma” in knowledge sharing under conditions of information asymmetry. Zhang et al. [33] designed various incentive models and developed corresponding smart contracts based on evolutionary game theory to encourage different enterprises to participate in the “accounting” operation, thereby enhancing the trust of the blockchain shared manufacturing network and maintaining its operational stability. Zhao et al. [34] combined system dynamics with multi-agent evolutionary game theory to simulate the dynamic game process of government, suppliers, and manufacturers in supply chain digitalization decision-making. They discovered the phase transition effect of digital adoption triggered by government reward and punishment thresholds, providing a new perspective for understanding the consequences of policy intervention.

Based on the current application of evolutionary game theory in the manufacturing sector, research exhibits three significant trends: the game structure is expanding from a government-enterprise binary framework to a multi-agent network; policy tools are evolving from static subsidies to dynamic differentiated regulations; and research methods are extending from analytical solutions to system simulation and multi-agent modeling. However, existing tripartite models display a notable pattern: while the number of interacting agents has expanded, the specific identity and strategic calculus of those agents have not been sufficiently differentiated. In particular, digital service providers, who must make irreversible and asset-specific infrastructure commitments, are treated as passive platforms or generic intermediaries rather than as independent strategic investors. This gap is synthesized with the limitations of the manufacturing digital transformation literature in Section 2.3, where we formalize the distinctive features of DSP decision-making and position our contribution.

2.3. Literature Synthesis and Research Positioning

Integrating the two preceding research streams reveals three cross-cutting gaps that motivate this study. First, a fundamental contradiction exists between the two streams. The manufacturing digital transformation literature, particularly at the digital servitiza-

tion level, increasingly recognizes external service providers as indispensable actors that supply specialized infrastructure, absorb technological risk, and enable asset-light business models [9,11,13]. Yet, the evolutionary game literature on manufacturing has almost exclusively treated such providers as passive infrastructure, generic “platforms,” or intermediaries without independent objective functions, investment constraints, or pricing power [12,28]. This contradiction between the recognized empirical importance of service providers and their theoretical invisibility as independent game players constitutes the primary motivation for our study.

Second, the literature exhibits inconsistent findings regarding the effectiveness of government subsidies. Some studies report monotonic positive effects, finding that higher subsidies unambiguously promote digital adoption [26], while others identify threshold or non-monotonic effects, where subsidies beyond a certain level become fiscally unsustainable or induce dependency [27]. Our model helps resolve this inconsistency by showing that the relationship between subsidy intensity and cooperation outcomes is conditional on whether total fiscal expenditure remains below the combined social benefits and avoided stagnation losses. When this fiscal sustainability condition is satisfied, subsidies are effective; when violated, subsidy withdrawal becomes inevitable, and the resulting policy discontinuity can destabilize the entire ecosystem.

Third, existing tripartite models do not capture three distinctive features of DSP strategic decision-making that are essential for analyzing the sports equipment manufacturing context. DSPs make asset-specific infrastructure investments with limited redeployability to other industries. Furthermore, the Equivalent Annual Cost mechanism links capital market conditions directly to all three agents’ strategic payoffs, a feature absent from every prior tripartite evolutionary game model. Additionally, DSPs operate under a dual revenue structure: they can serve cooperative manufacturers through customized solutions at premium rates, while simultaneously serving non-cooperative ones via standardized API services at reduced fees, creating a strategic trade-off between dedicated and general-purpose investment.

Table 1 provides a systematic comparison of our model with six closely related tripartite evolutionary game studies across five dimensions, confirming that our study is the first to (a) model the DSP as an independent agent with asset-specific investment decisions, (b) incorporate the EAC mechanism linking interest rates to strategic payoffs, and (c) focus on the sports equipment manufacturing sector with its distinctive “high-precision sensors, biomechanical data analysis, and customized flexible production” technology stack.

Table 1. Comparison of this study with relevant literature.

Study	Stakeholders	DSP Role	Government Instruments	Key Finding	DSP as Independent Agent?
Tang et al. [12]	Government, Digital platform, Manufacturers	Generic platform (no investment decision)	Not explicitly modeled	Tripartite collaboration mechanism identified	No (passive platform)
Wang et al. [24]	Government, Manufacturers, Logistics	Not modeled (logistics as passive operator)	Regulation, fiscal incentives	Additional costs are key barrier; early-stage govt. intervention needed	No
Dong et al. [26]	Equipment manufacturers, Government, Consumers	Not modeled	Subsidy intensity	Threshold effect of subsidies; excessive subsidies induce dependency	No
Zhu et al. [27]	Government, TDEs, SMEs	Not modeled (TDE as demonstrator)	Differentiated regulatory policies	Differentiated governance balances incentives and constraints	No

Table 1. Cont.

Study	Stakeholders	DSP Role	Government Instruments	Key Finding	DSP as Independent Agent?
Gao et al. [28]	Core manufacturers, Service enterprises, Customers	Generic service enterprise (no asset specificity)	Not explicitly modeled	Multi-agent simulation reveals value co-creation paths	No (passive intermediary)
Qiu et al. [30]	Commercial banks, Digital enterprises, Manufacturing	Not modeled (bank as credit provider)	Green credit policy	Digital technology promotes low-carbon transformation	No
This study	Government, SEMEs, DSPs	Independent agent with asset-specific investment, dual revenue structure, EAC mechanism	Equipment subsidy, provider support, fiscal sustainability constraint	Optimal subsidy, interest rate non-monotonicity, service fee bifurcation	Yes

3. Model Construction

3.1. Model Assumptions

The digital transformation of the sports equipment manufacturing industry involves numerous participants and exhibits significant dynamic interaction characteristics. Before presenting the formal model, it is important to articulate the distinctive features of the sports equipment manufacturing sector that motivate our modeling choices. First, the industry is characterized by high product variety and short product lifecycles (seasonal sports goods, annual model updates), making heavy-asset investment in specialized digital equipment particularly risky compared to sectors with more stable product portfolios. Second, sports equipment manufacturing involves the integration of biomechanical sensors and human-performance-monitoring systems, creating unique requirements for real-time data processing and analytics that generic manufacturing sectors do not face. Third, the industry in China is dominated by SMEs clustered in specific regions, which face common financing constraints and have limited in-house digital capabilities, making the DSP's role structurally more important than in industries dominated by large enterprises with internal R&D resources. Fourth, smart sports products (e.g., connected fitness equipment, wearable performance trackers) require continuous software updates and cloud services, creating an ongoing dependency on DSP infrastructure that distinguishes this sector from one-time equipment purchase models. As shown in Figure 1, this paper constructs a tripartite evolutionary game model covering three core stakeholders: the government, sports equipment manufacturing enterprises (SEMEs), and digital service providers (DSPs). Under the constraints of bounded rationality, these agents are unable to calculate the global optimum in the initial stage. Instead, they continuously learn and evolve by evaluating expected payoffs and eliminating inefficient strategies. Eventually, they gradually tend to adopt high-payoff strategies. Based on the fundamental paradigm of evolutionary game theory, we propose the following core assumptions:

(1) Assumption on Behavioral Strategies

The strategy set for the government is {*Subsidize*, *Not subsidize*}. Choosing “Subsidize” means that the government provides financial support to manufacturers purchasing digital equipment and systems. Additionally, when digital service providers and manufacturers achieve deep cooperation, the government will provide extra subsidies (such as tax incentives or industrial guidance funds) to the service providers. The probabilities of the government choosing “Subsidize” and “Not subsidize” are denoted as x and $1 - x$, respectively.

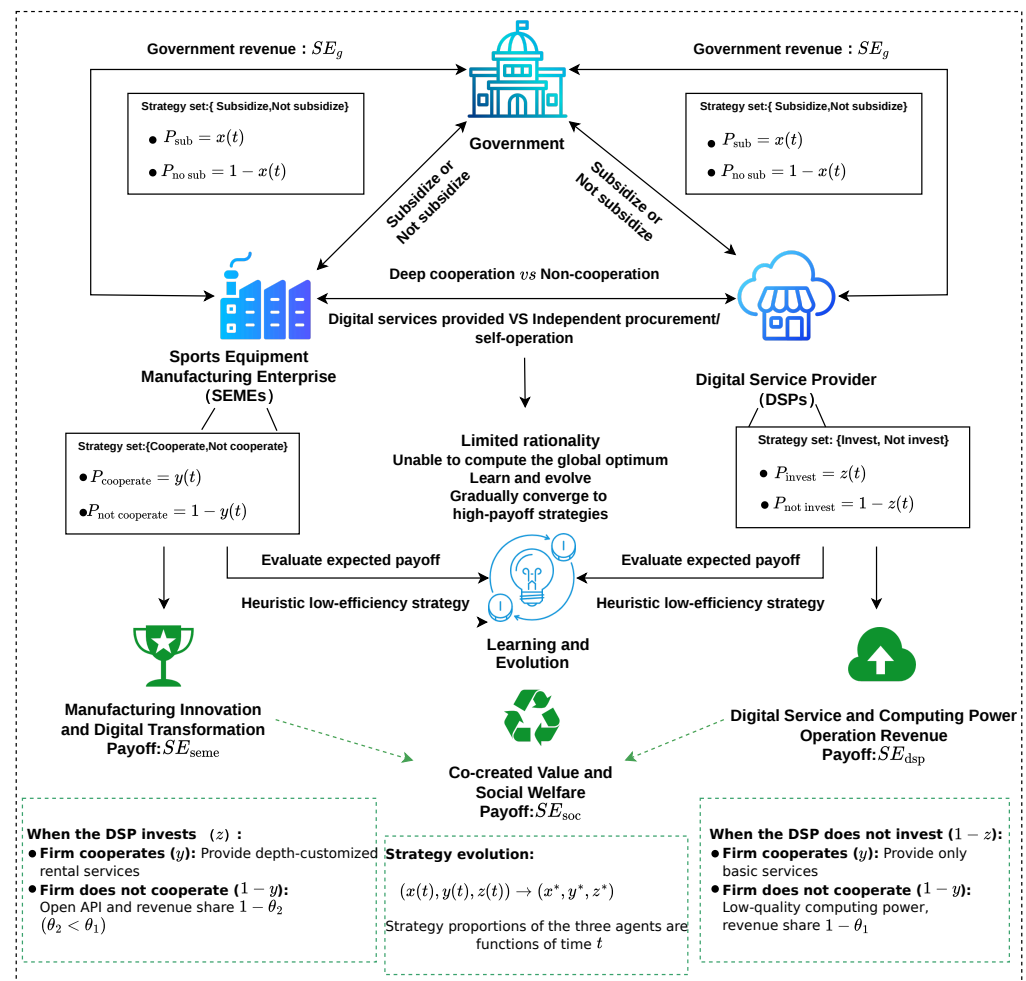


Figure 1. Strategic interaction among the tripartite game players.

The strategy set for sports equipment manufacturing enterprises (SEMEs) is {*Cooperate*, *Not cooperate*}. Choosing “Cooperate” means that the manufacturers sign a lifecycle service agreement with digital service providers. The service providers invest in building the underlying digital infrastructure. Meanwhile, the manufacturers use high-end digital services and equipment through leasing or subscriptions. Choosing “Not cooperate” means that the manufacturers independently purchase digital equipment and build data centers. Consequently, they must independently bear the high risks associated with subsequent operations and maintenance (O&M) and technological iteration. The probabilities of the manufacturers choosing “Cooperate” and “Not cooperate” are denoted as y and $1 - y$, respectively.

The strategy set for digital service providers (DSPs) is {*Invest*, *Not invest*}. Choosing “Invest” means that the service providers invest heavily to build high-standard digital computing centers and professional-grade smart sports manufacturing cloud platforms. Under this strategy, if the manufacturers choose to cooperate, the service providers will offer deeply customized digital leasing services. If the manufacturers do not cooperate, the service providers can still offer basic computing power services to the market through open general API interfaces at a fee ratio of $1 - \theta_2$. Choosing “Not invest” indicates that the service providers refuse to carry out large-scale professional infrastructure upgrades. They only maintain general cloud services with low technological value-added. In this scenario, if the manufacturers do not cooperate, they can only purchase low-quality computing power at a lower fee ratio of $1 - \theta_1$ (where $\theta_2 < \theta_1$). The probability of the digital service

providers choosing “Invest” is denoted as z , and the probability of choosing “Not invest” is $1 - z$. The strategy probabilities (x, y, z) of the three parties are all functions of time t .

The binary strategy formulations adopted above are a deliberate simplification, standard in the evolutionary game literature, that enables closed-form analytical solutions for stability conditions. In practice, government subsidies, manufacturer cooperation, and DSP investment each span a continuous spectrum; the binary choices should be interpreted as the endpoints of these spectra, capturing the critical strategic thresholds that determine whether the cooperative digital ecosystem can take shape.

(2) Parameter Settings and Definitions

To conduct a rigorous mathematical derivation of the system, this paper defines a series of economic and operational parameters. Their specific settings, along with their physical and economic meanings, are presented in Table 2.

Table 2. Definitions of model parameters.

Symbol	Description and Economic Rationale
S_b	Annual fixed operational subsidy to manufacturers; covers baseline operational support for digital transformation activities.
S_e	One-time government subsidy for core digital equipment procurement; reduces the capital expenditure burden on manufacturers adopting digital technologies.
S_p	Government subsidy to DSPs upon cooperation with enterprises; covers land-use incentives, electricity discounts for data centers, and interest-subsidized loans.
I_m	Manufacturers’ operating revenue from conventional sports equipment production; represents the baseline income before digital transformation.
I_d	DSPs’ base digital income without deep cooperation; derived from standardized cloud services such as storage and basic computing.
C_{inf}	DSPs’ cost of investing in high-standard computing infrastructure and cloud platforms; a sunk cost with high asset specificity that limits redeployability to other sectors.
C_{dev}	Annualized procurement cost of core digital equipment and systems; derived from the unit purchase price via the Equivalent Annual Cost (EAC) formula.
V_p	DSPs’ annualized salvage value from centralized recovery of digital assets; higher than V_m due to economies of scale and multi-tenant redeployment capabilities.
V_m	Manufacturers’ annualized salvage value from independent asset recovery; lower than V_p due to limited redeployment options and weaker bargaining power in secondary markets.
τ	Annual digital service fee paid by manufacturers; the core pricing variable that balances DSP cost recovery against manufacturer affordability.
O_p	DSPs’ annual operational cost for computing infrastructure maintenance, including electricity, cooling, and network expenses.
O_m	Annual maintenance and system upgrade cost of digital production equipment; annualized from the total O&M budget via the EAC formula.
U_1	Spot-market computing cost without a contract; higher unit cost than contracted services due to the absence of volume discounts and service-level guarantees.
U_2	Computing cost via the DSP’s platform under contract; lower than U_1 owing to dedicated capacity and long-term volume pricing.
U_3	Computing cost via low-end platform without contract; the lowest unit cost option but with minimal service quality and no customization support.
θ_1	Business loss proportion when switching to low-end platforms without DSP investment; reflects high switching costs, data migration friction, and vendor lock-in effects.

Table 2. Cont.

Symbol	Description and Economic Rationale
θ_2	Business loss proportion when switching to third-party platforms under DSP investment; lower than θ_1 because professional-grade platforms offer better interoperability and data portability.
D	Annual data processing and interaction volume in the sports manufacturing industry; reflects the continuous sensor data streams from smart production lines and connected products.
ω	Computing resource requirement per unit data; higher than generic manufacturing due to the computational intensity of real-time biomechanical analysis.
Ω	Risk cost of technology obsolescence and cybersecurity; a reduced-form parameter capturing expected losses from cybersecurity incidents and AI project failures.
C_0	Conventional operational and labor costs of manufacturing enterprises; excludes digital-transformation-specific expenditures.
C_g	Government's labor and administrative costs for implementing subsidy strategies, including application processing, monitoring, and evaluation.
L_{soc}	Social loss from stagnant industrial upgrading without subsidies; the opportunity cost of foregone industrial competitiveness and diminished innovation spillovers.
B_{dir}	Direct social benefits from government subsidies; aggregate welfare gain including productivity improvements, quality upgrades, and enhanced international competitiveness.
B_{syn}	Synergy benefits from deep DSP–manufacturer cooperation; ecosystem-level gains including data sharing, collaborative innovation, and supply chain integration.

3.2. Parameter Rationale and Thematic Organization

To facilitate the reader's understanding of the model's economic structure, we organize the parameters defined in Table 2 into four thematic categories. **(a) Government fiscal instruments** ($S_b, S_e, S_p, C_g, B_{dir}, B_{syn}, L_{soc}$): These parameters capture the government's dual role as a subsidy provider and a social welfare maximizer. The fiscal sustainability condition $S_e + S_p + C_g < B_{dir} + L_{soc}$ defines the boundary between viable and unsustainable intervention. **(b) Manufacturer cost–benefit structure** ($I_m, C_{dev}, O_m, \tau, V_m, \Omega, C_0$): These parameters determine whether the “Cooperate” (asset-light subscription) or “Not cooperate” (self-build) strategy is financially preferable. The total cost of ownership comparison is central to the manufacturer's strategic calculus. **(c) DSP investment and revenue structure** ($I_d, C_{inf}, C_{dev}, V_p, O_p, \tau, D, \omega$): These parameters govern the DSP's decision to invest in specialized infrastructure. The DSP's dual revenue stream (customized service fees plus standardized API fees) creates a strategic trade-off absent from prior tripartite models. **(d) Market friction and switching-cost parameters** ($\theta_1, \theta_2, U_1, U_2, U_3, \Omega$): These capture the lock-in effects, data portability costs, and technological risks that create path dependencies in digital adoption and influence the stability of cooperative equilibria.

Among the aforementioned parameters, the government subsidy for digital service providers (S_p) mainly covers forms such as land use incentives, electricity price discounts for data centers, and interest-subsidized loans. Since DSPs possess economies of scale and multi-tenant architecture capabilities, they can repurpose obsolete computing nodes or sensor arrays for other industries with lower computing power demands. Therefore, their efficiency in recovering the salvage value of digital assets is inevitably higher than that of independently operating manufacturing enterprises, implying the assumption that $V_p \geq V_m$.

In the long-term financial evaluation of fixed asset investments, it is essential to comprehensively consider the time value of money, capital interest rate discounting, and the effective equipment lifecycle. Therefore, this study adopts the Equivalent Annual Cost (EAC) method. This approach uniformly converts heavy-asset investments into periodic

annualized values. Specifically, the annualized investment costs of core digital equipment, the annualized residual values recovered by both service providers and manufacturers, and the annualized routine operations and maintenance (O&M) costs can be jointly integrated into the following set of EAC formulas:

$$\begin{cases} C_{dev} = c_{dev} \frac{\rho}{1 - (1 + \rho)^{-T}} \\ V_p = \gamma_p c_{dev} \frac{\rho}{1 - (1 + \rho)^{-T}} \\ V_m = \gamma_m c_{dev} \frac{\rho}{1 - (1 + \rho)^{-T}} \\ O_m = c_{maint} \frac{\rho}{1 - (1 + \rho)^{-T}} \end{cases} \quad (1)$$

In this set of formulas, the specific physical meanings of the economic and technical parameters are as follows: c_{dev} is the unit purchase price of the complete smart production line, ρ is the market benchmark capital interest rate, and T is the effective physical lifespan of the equipment. γ_p and γ_m represent the residual value recovery efficiency coefficients of the third-party digital service providers and the equipment manufacturers, respectively. They are used to determine their respective annualized residual values, V_p and V_m . c_{maint} represents the total O&M budget over the equipment's entire physical lifecycle. After conversion, it yields O_m , which is the annualized routine O&M and system upgrade cost of the equipment.

3.3. Payoff Matrix

Based on the above assumptions, the expected payoffs of the government, Sports Equipment Manufacturing Enterprises (SEMEs), and Digital Service Providers (DSPs) under the eight pure strategy combinations are shown in Table 3.

Table 3. Payoff matrix of the tripartite evolutionary game.

Government	SEMEs	DSPs	
		Invest (z)	Not Invest (1 - z)
Subsidize (x)	Cooperate (y)	$B_{dir} + B_{syn} - S_b - S_e - S_p - C_g$ $S_b + I_m - \tau - C_o$ $\tau + S_e + S_p + I_d - C_{inf} - O_p + V_p - D\omega\pi_0 - C_{dev} - O_m - \Omega$	$B_{dir} - S_b - S_e - C_g$ $S_b + S_e + I_m - C_o + V_m - U_1 - C_{dev} - O_m - \Omega$ $I_d - C_{inf} - O_p$
	Not cooperate (1 - y)	$B_{dir} - S_b - S_e - C_g$ $S_b + S_e + I_m - C_o + V_m - \theta_2 U_1 - (1 - \theta_2) U_2 - C_{dev} - O_m - \Omega$ $I_d - C_{inf} - O_p + (1 - \theta_2) D\omega\pi_2$	$B_{dir} - S_b - S_e - C_g$ $S_b + S_e + I_m - C_o + V_m - \theta_1 U_1 - (1 - \theta_1) U_3 - C_{dev} - O_m - \Omega$ $I_d - C_{inf} - O_p + (1 - \theta_1) D\omega\pi_3$
Not subsidize (1 - x)	Cooperate (y)	$B_{syn} - L_{soc} - S_b$ $S_b + I_m - \tau - C_o$ $\tau + I_d - C_{inf} - O_p + V_p - D\omega\pi_0 - C_{dev} - O_m - \Omega$	$-L_{soc} - S_b$ $S_b + I_m - C_o + V_m - U_1 - C_{dev} - O_m - \Omega$ $I_d - C_{inf} - O_p$
	Not cooperate (1 - y)	$-L_{soc} - S_b$ $S_b + I_m - C_o + V_m - \theta_2 U_1 - (1 - \theta_2) U_2 - C_{dev} - O_m - \Omega$ $I_d - C_{inf} - O_p + (1 - \theta_2) D\omega\pi_2$	$-L_{soc} - S_b$ $S_b + I_m - C_o + V_m - \theta_1 U_1 - (1 - \theta_1) U_3 - C_{dev} - O_m - \Omega$ $I_d - C_{inf} - O_p + (1 - \theta_1) D\omega\pi_3$

This matrix details the transfer paths of benefit distribution. For example, under the (Subsidize, Cooperate, Invest) strategy profile, the service providers bear the purchase cost (C_{dev}) and receive the subsidies (S_e and S_p). Meanwhile, the manufacturers avoid massive capital expenditures, needing only to pay the fixed annual service fee (τ).

4. Replicator Dynamics Equation and Strategy Stability Analysis

4.1. Strategy Stability Analysis of the Government

The expected payoffs of the government choosing to “Subsidize” and “Not subsidize” are defined as E_{11} and E_{12} , respectively:

$$E_{11} = yz(B_{dir} + B_{syn} - S_b - S_e - S_p - C_g) + y(1-z)(B_{dir} - S_b - S_e - C_g) + (1-y)z(B_{dir} - S_b - S_e - C_g) + (1-y)(1-z)(B_{dir} - S_b - S_e - C_g) \quad (2)$$

which simplifies to:

$$E_{11} = yzB_{syn} - yzS_p + B_{dir} - S_b - S_e - C_g \quad (3)$$

$$E_{12} = yz(B_{syn} - L_{soc} - S_b) + y(1-z)(-L_{soc} - S_b) + (1-y)z(-L_{soc} - S_b) + (1-y)(1-z)(-L_{soc} - S_b) \quad (4)$$

which simplifies to:

$$E_{12} = yzB_{syn} - L_{soc} - S_b \quad (5)$$

According to the Malthusian equation, the average expected utility of the government’s strategy is $\bar{E}_1 = xE_{11} + (1-x)E_{12}$. From this, the replicator dynamics equation for the government choosing the “Subsidize” strategy is derived:

$$F(x) = \frac{dx}{dt} = x(E_{11} - \bar{E}_1) = x(1-x)(E_{11} - E_{12}) = x(1-x)(-yzS_p + B_{dir} - S_e - C_g + L_{soc}) \quad (6)$$

According to the stability theorem of differential equations, a replicator dynamics system requires $F(x) = 0$ and the first-order derivative $dF(x)/dx \leq 0$ for x to be an Evolutionarily Stable Strategy (ESS). Taking the derivative of $F(x)$ yields:

$$\frac{dF(x)}{dx} = (1-2x)(-yzS_p + B_{dir} - S_e - C_g + L_{soc}) \quad (7)$$

To simplify the analysis of the boundary conditions, an auxiliary function is defined:

$$H_1(y) = -yzS_p + B_{dir} - S_e - C_g + L_{soc} \quad (8)$$

If $y = y^* = (B_{dir} - S_e - C_g + L_{soc})/zS_p$, then $H_1(y) = 0$, which leads to $F(x) \equiv 0$. This implies that the government’s strategy choice remains constant over time. Since $\partial H_1(y)/\partial y = -zS_p < 0$, $H_1(y)$ is a strictly monotonically decreasing function with respect to y .

When $y \neq y^*$ and lies in the interval $y^* < y < 1$, it follows that $H_1(y) < 0$. Substituting $x = 0$ and $x = 1$ into the derivative equation yields $dF(x)/dx|_{x=0} < 0$ and $dF(x)/dx|_{x=1} > 0$. Therefore, $x = 0$ is the evolutionary stable point. This indicates that in the context of high cooperation from manufacturers, the government tends to evolve towards the “Not subsidize” steady state to reduce its fiscal burden. Conversely, when $0 < y < y^*$, $x = 1$ is the evolutionary stable point. This demonstrates that when the manufacturers’ willingness to cooperate is low, the government must adopt the “Subsidize” strategy to drive the market.

4.2. Strategy Stability Analysis of SEMEs

The expected payoffs of the manufacturers choosing to “Cooperate” and “Not cooperate” are represented by E_{21} and E_{22} , respectively:

$$\begin{aligned} E_{21} = & xz(S_b + I_m - \tau - C_o) \\ & + x(1-z)(S_b + S_e + I_m - C_o + V_m - U_1 - C_{dev} - O_m - \Omega) \\ & + (1-x)z(S_b + I_m - \tau - C_o) \\ & + (1-x)(1-z)(S_b + I_m - C_o + V_m - U_1 - C_{dev} - O_m - \Omega) \end{aligned} \quad (9)$$

which simplifies to:

$$\begin{aligned} E_{21} = & -xzS_e + xS_e + z(-V_m + U_1 - \tau + C_{dev} + O_m + \Omega) \\ & + S_b + I_m - C_o + V_m - U_1 - C_{dev} - O_m - \Omega \end{aligned} \quad (10)$$

$$\begin{aligned} E_{22} = & xz(S_b + S_e + I_m - C_o + V_m - \theta_2 U_1 - (1 - \theta_2)U_2 - C_{dev} - O_m - \Omega) \\ & + x(1-z)(S_b + S_e + I_m - C_o + V_m - \theta_1 U_1 - (1 - \theta_1)U_3 - C_{dev} - O_m - \Omega) \\ & + (1-x)z(S_b + I_m - C_o + V_m - \theta_2 U_1 - (1 - \theta_2)U_2 - C_{dev} - O_m - \Omega) \\ & + (1-x)(1-z)(S_b + I_m - C_o + V_m - \theta_1 U_1 - (1 - \theta_1)U_3 - C_{dev} - O_m - \Omega) \end{aligned} \quad (11)$$

which simplifies to:

$$\begin{aligned} E_{22} = & xS_e + z[(\theta_1 - \theta_2)U_1 - (1 - \theta_2)U_2 + (1 - \theta_1)U_3] \\ & + S_b + I_m - C_o + V_m - \theta_1 U_1 - (1 - \theta_1)U_3 - C_{dev} - O_m - \Omega \end{aligned} \quad (12)$$

The average expected payoffs of the manufacturers is $\bar{E}_2 = yE_{21} + (1-y)E_{22}$. Based on this, its replicator dynamics equation is derived as follows:

$$\begin{aligned} F(y) = & \frac{dy}{dt} = y(E_{21} - \bar{E}_2) \\ = & y(1-y) \left[-xzS_e + z(-V_m - \tau + C_{dev} + O_m + \Omega \right. \\ & \left. + (1 - \theta_1 + \theta_2)U_1 + (1 - \theta_2)U_2 - (1 - \theta_1)U_3) + (1 - \theta_1)(U_3 - U_1) \right] \end{aligned} \quad (13)$$

Taking the partial derivative of $F(y)$ with respect to y yields:

$$\begin{aligned} \frac{dF(y)}{dy} = & (1-2y) \left[-xzS_e + z(-V_m - \tau + C_{dev} + O_m + \Omega \right. \\ & \left. + (1 - \theta_1 + \theta_2)U_1 + (1 - \theta_2)U_2 - (1 - \theta_1)U_3) + (1 - \theta_1)(U_3 - U_1) \right] \end{aligned} \quad (14)$$

To facilitate the analysis, we introduce an auxiliary function:

$$\begin{aligned} H_2(x) = & -xzS_e + z(-V_m - \tau + C_{dev} + O_m + \Omega + (1 - \theta_1 + \theta_2)U_1 \\ & + (1 - \theta_2)U_2 - (1 - \theta_1)U_3) + (1 - \theta_1)(U_3 - U_1) \end{aligned} \quad (15)$$

We define its threshold as follows:

$$z^* = \frac{-(1 - \theta_1)(U_3 - U_1)}{-xS_e - V_m - \tau + C_{dev} + O_m + \Omega + (1 - \theta_1 + \theta_2)U_1 + (1 - \theta_2)U_2 - (1 - \theta_1)U_3} \quad (16)$$

When $z = z^*$, $F(y) \equiv 0$, meaning that the manufacturers are in a state of strategic neutrality. Since the partial derivative satisfies $\partial H_2(z)/\partial z > 0$, $H_2(z)$ is a strictly monotonically increasing function. When the service providers' willingness to invest strengthens,

i.e., $z^* < z < 1$, it follows that $H_2(z) > 0$. At this point, $dF(y)/dy|_{y=1} < 0$; therefore, $y = 1$ (“Cooperate”) is the evolutionary stable point. This indicates that the digital service infrastructure will generate a strong siphoning effect. It drives manufacturers to abandon independent development and evolve towards an asset-light subscription cooperation model. Conversely, when $0 < z < z^*$, $y = 0$ is the stable point, indicating “Not cooperate”.

4.3. Strategy Stability Analysis of DSPs

The expected payoffs of the digital service providers choosing to “Invest” and “Not invest” are represented by E_{31} and E_{32} , respectively:

$$\begin{aligned} E_{31} = & xy(\tau + S_e + S_p + I_d - C_{inf} - O_p + V_p - D\omega\pi_0 - C_{dev} - O_m - \Omega) \\ & + x(1-y)(I_d - C_{inf} - O_p + (1-\theta_2)D\omega\pi_2) \\ & + (1-x)y(\tau + I_d - C_{inf} - O_p + V_p - D\omega\pi_0 - C_{dev} - O_m - \Omega) \\ & + (1-x)(1-y)(I_d - C_{inf} - O_p + (1-\theta_2)D\omega\pi_2) \end{aligned} \quad (17)$$

which simplifies to:

$$E_{31} = xy(S_e + S_p) + y(\tau + V_p - D\omega\pi_0 - C_{dev} - O_m - \Omega) + I_d - C_{inf} - O_p + (1-\theta_2)D\omega\pi_2 \quad (18)$$

$$\begin{aligned} E_{32} = & xy(I_d - C_{inf} - O_p) + x(1-y)(I_d - C_{inf} - O_p + (1-\theta_1)D\omega\pi_3) \\ & + (1-x)y(I_d - C_{inf} - O_p) + (1-x)(1-y)(I_d - C_{inf} - O_p + (1-\theta_1)D\omega\pi_3) \end{aligned} \quad (19)$$

which simplifies to:

$$E_{32} = -y(1-\theta_1)D\omega\pi_3 + I_d - C_{inf} - O_p + (1-\theta_1)D\omega\pi_3 \quad (20)$$

The average expected utility of the digital service providers is $\bar{E}_3 = zE_{31} + (1-z)E_{32}$. Based on this, its replicator dynamics equation is derived as follows:

$$\begin{aligned} F(z) = & \frac{dz}{dt} = z(E_{31} - \bar{E}_3) \\ = & z(1-z) \left[xy(S_e + S_p) + y(\tau + V_p - D\omega\pi_0 - C_{dev} - O_m - \Omega) \right. \\ & \left. + (1-\theta_1)D\omega\pi_3 - (1-\theta_2)D\omega\pi_2 + (1-\theta_2)D\omega\pi_2 - (1-\theta_1)D\omega\pi_3 \right] \end{aligned} \quad (21)$$

Taking the partial derivative of $F(z)$ with respect to z yields:

$$\begin{aligned} \frac{dF(z)}{dz} = & (1-2z) \left[xy(S_e + S_p) + y(\tau + V_p - D\omega\pi_0 - C_{dev} - O_m - \Omega) \right. \\ & \left. + (1-\theta_1)D\omega\pi_3 - (1-\theta_2)D\omega\pi_2 + (1-\theta_2)D\omega\pi_2 - (1-\theta_1)D\omega\pi_3 \right] \end{aligned} \quad (22)$$

To facilitate the analysis, we define an auxiliary function:

$$\begin{aligned} H_3(x) = & xy(S_e + S_p) + y[\tau + V_p - D\omega\pi_0 - C_{dev} - O_m - \Omega \\ & + (1-\theta_1)D\omega\pi_3 - (1-\theta_2)D\omega\pi_2] + (1-\theta_2)D\omega\pi_2 - (1-\theta_1)D\omega\pi_3 \end{aligned} \quad (23)$$

When the probability of government subsidies reaches the critical threshold x^* , $F(z) \equiv 0$. Since $\partial H_3(x)/\partial x > 0$, $H_3(x)$ is a strictly monotonically increasing function. Therefore, if $x^* < x < 1$, the service providers, with generous subsidies offsetting their costs, will inevitably choose to evolve towards the $z = 1$ (“Invest”) strategy. Conversely, if government subsidies are insufficient ($0 < x < x^*$), the exorbitant construction costs of the computing infrastructure will force the service providers to retreat to the $z = 0$ (“Not invest”) strategy.

5. Jacobian Matrix and Stability Analysis of Equilibrium Points

5.1. Jacobian Matrix

By coupling the replicator dynamics equations of the three parties, we construct a three-dimensional differential dynamical system that describes the evolution of the system's state. The necessary condition for the system to reach an equilibrium state is that the evolutionary rates of all strategy proportions are zero, i.e., $F(x) = F(y) = F(z) = 0$. Solving this non-linear system yields eight local equilibrium points in pure strategies: $(0, 0, 0)$, $(0, 0, 1)$, $(0, 1, 0)$, $(1, 0, 0)$, $(1, 1, 0)$, $(1, 0, 1)$, $(0, 1, 1)$, and $(1, 1, 1)$.

To evaluate the asymptotic stability of these local equilibrium points, we construct the Jacobian matrix (J):

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \quad (24)$$

The derivations of the core elements within the matrix are as follows:

$$\left\{ \begin{array}{l} J_{11} = \frac{\partial F(x)}{\partial x} = (1-2x)(-yzS_p + B_{dir} - S_e - C_g + L_{soc}) \\ J_{12} = \frac{\partial F(x)}{\partial y} = -x(1-x)zS_p \\ J_{13} = \frac{\partial F(x)}{\partial z} = -x(1-x)yS_p \\ J_{21} = \frac{\partial F(y)}{\partial x} = -y(1-y)zS_e \\ J_{22} = \frac{\partial F(y)}{\partial y} = (1-2y) \left[-xzS_e + z(-V_m - \tau + C_{dev} + O_m + \Omega \right. \\ \quad \left. + (1-\theta_1 + \theta_2)U_1 + (1-\theta_2)U_2 - (1-\theta_1)U_3) + (1-\theta_1)(U_3 - U_1) \right] \\ J_{23} = \frac{\partial F(y)}{\partial z} = y(1-y) \left[-xS_e - V_m - \tau + C_{dev} + O_m + \Omega \right. \\ \quad \left. + (1-\theta_1 + \theta_2)U_1 + (1-\theta_2)U_2 - (1-\theta_1)U_3 \right] \\ J_{31} = \frac{\partial F(z)}{\partial x} = z(1-z)y(S_e + S_p) \\ J_{32} = \frac{\partial F(z)}{\partial y} = z(1-z) \left[x(S_e + S_p) + \tau + V_p - D\omega\pi_0 - C_{dev} - O_m - \Omega \right. \\ \quad \left. + (1-\theta_1)D\omega\pi_3 - (1-\theta_2)D\omega\pi_2 \right] \\ J_{33} = \frac{\partial F(z)}{\partial z} = (1-2z) \left[xy(S_e + S_p) + y(\tau + V_p - D\omega\pi_0 - C_{dev} - O_m - \Omega \right. \\ \quad \left. + (1-\theta_1)D\omega\pi_3 - (1-\theta_2)D\omega\pi_2) + (1-\theta_2)D\omega\pi_2 - (1-\theta_1)D\omega\pi_3 \right] \end{array} \right. \quad (25)$$

According to the Lyapunov stability criterion, an equilibrium point is an evolutionarily stable strategy (ESS) of the system if and only if the real parts of all three eigenvalues ($\lambda_1, \lambda_2, \lambda_3$) of the Jacobian matrix are strictly less than zero. Points with any eigenvalue possessing a positive real part are unstable, acting as either saddle points or unstable nodes. By substituting the coordinates of each point into the Jacobian matrix to solve for the eigenvalues, the distribution of the system's eigenvalues is presented in Table 4.

From a mathematical structure perspective, the two boundary points, $(0, 0, 0)$ and $(1, 0, 0)$, are unstable. The core reason lies in the fact that, in scenarios where digital service providers and manufacturers engage in cloud data interactions without formal cooperation, they are more inclined to sign standardized computing power agreements to avoid the higher costs associated with sporadic orders. Mathematically, the inequality $D\omega[(1-\theta_2)\pi_2 - (1-\theta_1)\pi_3] > 0$ holds consistently, resulting in positive eigenvalues

for these points. Consequently, the system cannot maintain a stable state at these boundary coordinates.

Table 4. Eigenvalues and stability analysis of the system.

	λ_1	λ_2	λ_3	Stability
$E_1(0,0,0)$	$B_{dir} - S_e - C_g + L_{soc}$	$(1 - \theta_1)(U_3 - U_1)$	$D\omega[(1 - \theta_2)\pi_2 - (1 - \theta_1)\pi_3]$	Unstable
$E_2(0,0,1)$	$B_{dir} - S_e - C_g + L_{soc}$	$D\omega[(1 - \theta_1)\pi_3 - (1 - \theta_2)\pi_2]$	$U_2 - \tau + C_{dev} + O_m + \Omega - V_m + \theta_2(U_1 - U_2)$	Conditional ESS
$E_3(0,1,0)$	$\tau - C_{dev} - O_m - \Omega + V_p - D\omega\pi_0$	$B_{dir} - S_e - C_g + L_{soc}$	$(1 - \theta_1)(U_1 - U_3)$	Conditional ESS
$E_4(0,1,1)$	$-\tau + C_{dev} + O_m + \Omega - V_p + D\omega\pi_0$	$B_{dir} - S_e - S_p - C_g + L_{soc}$	$\tau - C_{dev} - O_m - \Omega + V_m - \theta_2 U_1 + (1 - \theta_2)U_2$	Conditional ESS
$E_5(1,0,0)$	$S_e - B_{dir} + C_g - L_{soc}$	$(1 - \theta_1)(U_3 - U_1)$	$D\omega[(1 - \theta_2)\pi_2 - (1 - \theta_1)\pi_3]$	Unstable
$E_6(1,0,1)$	$S_e - B_{dir} + C_g - L_{soc}$	$D\omega[(1 - \theta_1)\pi_3 - (1 - \theta_2)\pi_2]$	$U_2 - \tau + C_{dev} + O_m + \Omega - V_m - S_e + \theta_2(U_1 - U_2)$	Conditional ESS
$E_7(1,1,0)$	$S_e - B_{dir} + C_g - L_{soc}$	$(1 - \theta_1)(U_1 - U_3)$	$\tau - C_{dev} - O_m - \Omega + V_p + S_e + S_p - D\omega\pi_0$	Conditional ESS
$E_8(1,1,1)$	$S_e + S_p - B_{dir} + C_g - L_{soc}$	$D\omega\pi_0 - \tau + C_{dev} + O_m + \Omega - V_p - S_e - S_p$	$V_m + \tau - C_{dev} - O_m - \Omega - U_2 + \theta_2(U_2 - U_1)$	Conditional ESS

5.2. Stability Point Analysis

Before proceeding with the scenario-by-scenario analysis, we formally define the stability concepts employed in this section. A Conditional Evolutionarily Stable Strategy (Conditional ESS) is an equilibrium point that is asymptotically stable only when specific inequality constraints on the model parameters are satisfied. Asymptotic stability here requires that all eigenvalues of the Jacobian matrix have strictly negative real parts. Unlike an unconditional ESS, which remains stable across all admissible parameter values, a Conditional ESS represents a state attainable only under identifiable economic conditions. This distinction makes Conditional ESS analysis particularly valuable for policy design because it delineates the precise parameter thresholds required to achieve or avoid specific outcomes. Stability is evaluated via the Lyapunov indirect method. An equilibrium is asymptotically stable if and only if all corresponding eigenvalues have strictly negative real parts, and it is unstable otherwise. Building on this framework, we translate the six locally asymptotically stable equilibria into evolutionary game propositions that offer intuitive economic interpretations.

5.2.1. Scenario 1: Equilibrium Point $E_2(0,0,1)$

In the tripartite evolutionary game model, the necessary and sufficient condition for the system to converge to the local equilibrium point $E_2(0,0,1)$ (Not subsidize, Not cooperate, Invest) is that the strategic interests of the game agents satisfy the following set of economic incentive compatibility constraint formulas:

$$\begin{cases} B_{dir} - S_e - C_g + L_{soc} < 0 \\ C_{dev} + O_m + \Omega + \theta_2 U_1 + (1 - \theta_2)U_2 - V_m < \tau \\ D\omega[(1 - \theta_1)\pi_3 - (1 - \theta_2)\pi_2] < 0 \end{cases} \quad (26)$$

As shown in Figure 2, this equilibrium point reveals a deep-seated economic mechanism formed by a severe price gap between supply and demand under a market envi-

ronment characterized by the “complete withdrawal of government regulation”: manufacturing enterprises comprehensively abandon digital transformation, while digital service providers maintain infrastructure investments by relying on the long-tail dividends of the general market.

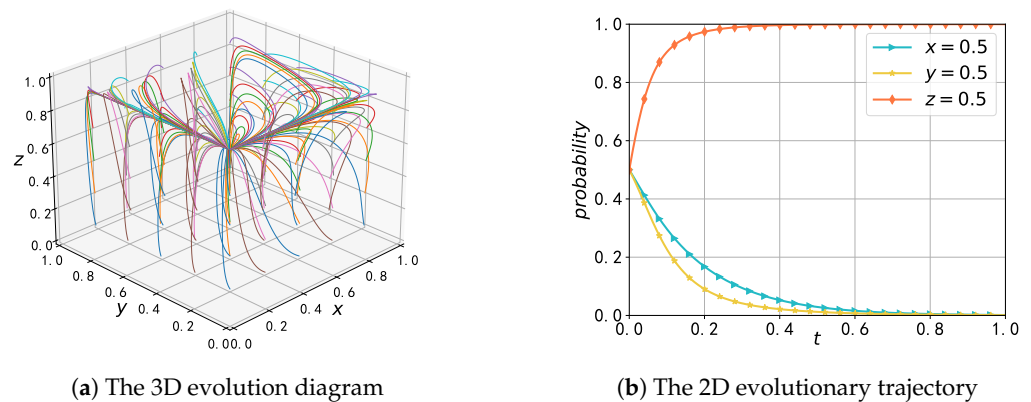


Figure 2. The evolutionary trajectory towards the equilibrium point $E_2(0,0,1)$.

Specifically, first, if the government continues to provide financial subsidies in this scenario, its administrative costs will significantly exceed the direct social benefits generated, causing fiscal intervention to lose its economic justification at the macroeconomic level; the rational government thus chooses to “Not subsidize,” allowing the market to return to autonomous regulation. Second, under the condition of no subsidies, the annual leasing service fee charged by the digital service providers is already higher than the Total Cost of Ownership (TCO) over the entire lifecycle for manufacturing enterprises, which includes independent hardware procurement, depreciation, and O&M costs, minus the residual value. This creates a distinct cost inversion that breaches the financial tolerance boundary of the manufacturing enterprises, prompting them to rationally choose to “Not cooperate.” Finally, even without a collaborative contract with specific manufacturing enterprises, the premium returns obtained by the service providers by offering fragmented computing services to the general market are still strictly higher than the baseline returns of maintaining traditional low-end computing services. This ensures that they consistently adopt the “Invest” strategy under a minimum return guarantee.

5.2.2. Scenario 2: Equilibrium Point $E_3(0,1,0)$

In the tripartite evolutionary game model, the necessary and sufficient condition for the system to converge to the local equilibrium point $E_3(0,1,0)$ (Not subsidize, Cooperate, Not invest) is that the strategic interests of the game agents satisfy the following set of economic incentive compatibility constraint formulas:

$$\begin{cases} B_{dir} - S_e - C_g + L_{soc} < 0 \\ (1 - \theta_1)(U_1 - U_3) < 0 \\ \tau + V_p < C_{dev} + O_m + \Omega + D\omega\pi_0 \end{cases} \quad (27)$$

As shown in Figure 3, this equilibrium point reveals that in a market environment characterized by the “complete withdrawal of government subsidies,” despite manufacturing enterprises possessing a strong willingness for digital cooperation, the third-party service providers face soaring technological operating costs and negative return expectations. This ultimately leads to the stagnation of digital infrastructure construction and the interruption of technology supply.

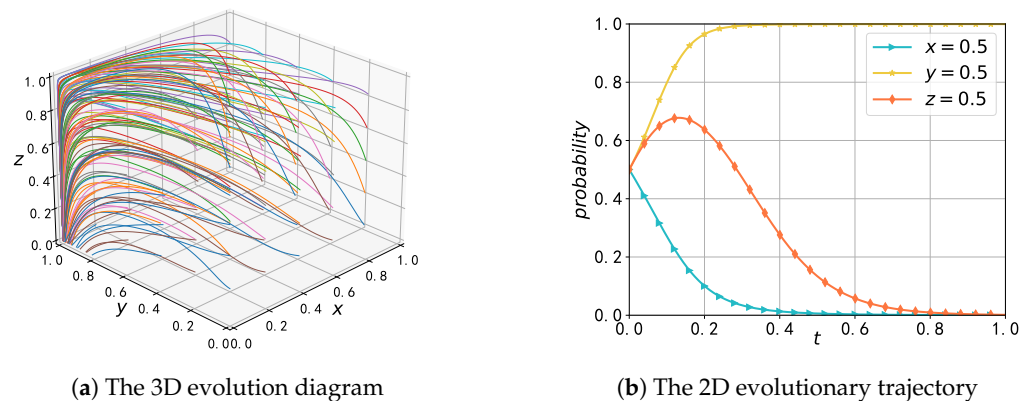


Figure 3. The evolutionary trajectory towards the equilibrium point $E_3(0, 1, 0)$.

From a macroeconomic perspective, if the government continues to implement direct financial subsidies in this scenario, its public budget expenditures and administrative costs will significantly exceed the direct social benefits generated, causing fiscal intervention to lose its economic justification at the macroeconomic level. The rational government thus chooses to “Not subsidize,” driving the market back to autonomous regulatory mechanisms. From the demand-side perspective, manufacturing enterprises exhibit a high degree of reliance on “Cooperative” subscription services when facing data security risks or the threat of losing digital capabilities. When routine computing expenditures are strictly lower than the external substitution costs induced by data breaches or technological barriers (a condition that holds consistently under lifecycle security evaluations), enterprises rationally maintain a preference to “Cooperate,” even in the absence of government equipment purchase subsidies. However, on the supply side, following the withdrawal of government subsidies, the sum of the market-oriented annual leasing revenue and the annualized residual value recovery obtained by the service providers through the “Invest” strategy remains insufficient to cover their comprehensive lifecycle costs. The persistent losses under the heavy-asset operating model compel them to ultimately abandon the “Invest” decision, resulting in a rupture in the technology supply.

5.2.3. Scenario 3: Equilibrium Point $E_4(0, 1, 1)$

In the tripartite evolutionary game model, the necessary and sufficient condition for the system to converge to the local equilibrium point $E_4(0, 1, 1)$ (Not subsidize, Cooperate, Invest) is that the strategic interests of the game agents satisfy the following set of economic incentive compatibility constraint formulas:

$$\begin{cases} S_p + S_e + C_g > B_{dir} + L_{soc} \\ \tau < C_{dev} + O_m + \Omega - V_m + \theta_2 U_1 + (1 - \theta_2) U_2 \\ \tau > C_{dev} + O_m + \Omega - V_p + D\omega\pi_0 \end{cases} \quad (28)$$

As shown in Figure 4, this equilibrium point embodies the typical characteristics of industrial digital transformation entering a mature stage, transitioning smoothly from being “policy-dividend-driven” to “market-mechanism-led.”

On the one hand, the marginal social net benefit of continued public policy intervention has fallen below its fiscal and administrative costs. When the total expenditure induced by the government subsidizing both the supply and demand sides strictly exceeds the direct social benefits generated, fiscal stimulus loses its economic justification at the macroeconomic level. The rational government thus chooses to “Not subsidize,” driving public resources to return to a standard market allocation. On the other hand, in a market-oriented

environment without subsidies, the micro-mechanism can still effectively maintain the collaborative incentives for manufacturing enterprises. At this point, the annual leasing service fee paid by enterprises adopting the “Cooperative” subscription model is strictly lower than their total cost of ownership (TCO) over the entire lifecycle under the “Not cooperate” path, which includes independent hardware procurement, depreciation, O&M costs, and substitute computing costs. This ensures that their rational preference remains stably anchored to “Cooperate.” Simultaneously, the annual service revenue obtained by the third-party service providers is sufficient to cover the comprehensive net lifecycle costs, such as infrastructure depreciation and system O&M. Relying solely on market-oriented returns, they can sustain positive investment incentives, guaranteeing the sustainability of the technology supply.

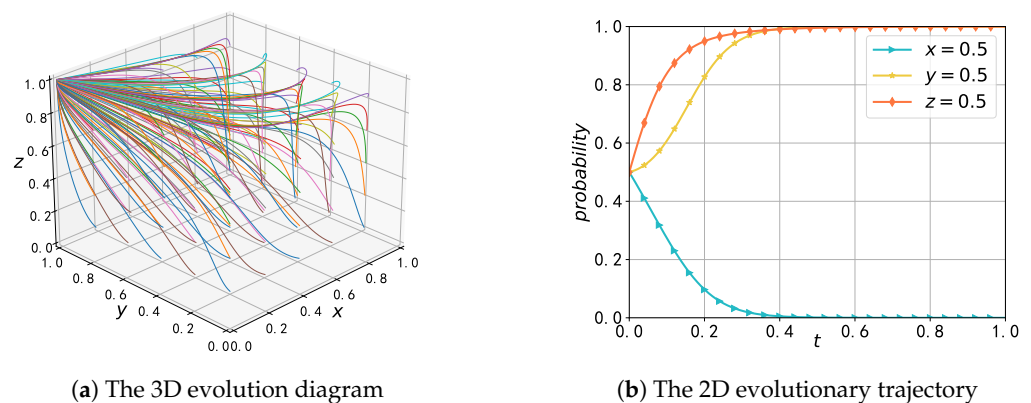


Figure 4. The evolutionary trajectory towards the equilibrium point $E_4(0, 1, 1)$.

5.3. Scenario 4: Equilibrium Point $E_6(1, 0, 1)$

In the tripartite evolutionary game model, the necessary and sufficient condition for the system to converge to the local equilibrium point $E_6(1, 0, 1)$ (Subsidize, Not cooperate, Invest) is that the strategic interests of the game agents satisfy the following set of economic incentive compatibility constraint formulas:

$$\begin{cases} S_e - B_{dir} + C_g - L_{soc} < 0 \\ C_{dev} + O_m + \Omega + \theta_2 U_1 + (1 - \theta_2) U_2 - V_m - S_e < \tau \\ D\omega[(1 - \theta_1)\pi_3 - (1 - \theta_2)\pi_2] < 0 \end{cases} \quad (29)$$

As shown in Figure 5, this equilibrium point reveals an industrial upgrade supply-demand mismatch mechanism triggered by price transmission friction, under the background of “strong government intervention” and “service providers actively deploying infrastructure”.

First, from a macroeconomic perspective, the total cost of the government implementing digital upgrade subsidies and related administrative expenditures remains lower than the sum of the direct social benefits generated and the industrial losses avoided. This establishes the social welfare justification for fiscal intervention, prompting the rational government to choose the “Subsidize” strategy. However, a significant supply-demand mismatch emerges at the micro-level: despite the existence of targeted government subsidies, manufacturing enterprises still face an undesirable financial predicament. The reason lies in the fact that the annual service fee set by the service providers remains significantly higher than the enterprises’ Total Cost of Ownership (TCO) over the entire lifecycle under the “Not cooperate” path, which includes independent hardware procurement, assuming depreciation and O&M costs, deducting residual equipment value, and factoring in substitute computing costs. This exorbitant heavy-asset transfer cost breaches the budget constraints

of the enterprises, forcing them to rationally choose to “Not cooperate.” Simultaneously, the digital service providers exhibit strong “Invest” incentives on the supply side. After investing in high-standard computing infrastructure, the business profits they obtain by offering fragmented computing services to the general market are strictly higher than the baseline returns of maintaining traditional computing services. The premium dividends generated by the standardized platform constitute the market-driven momentum for the service providers’ continuous investment.

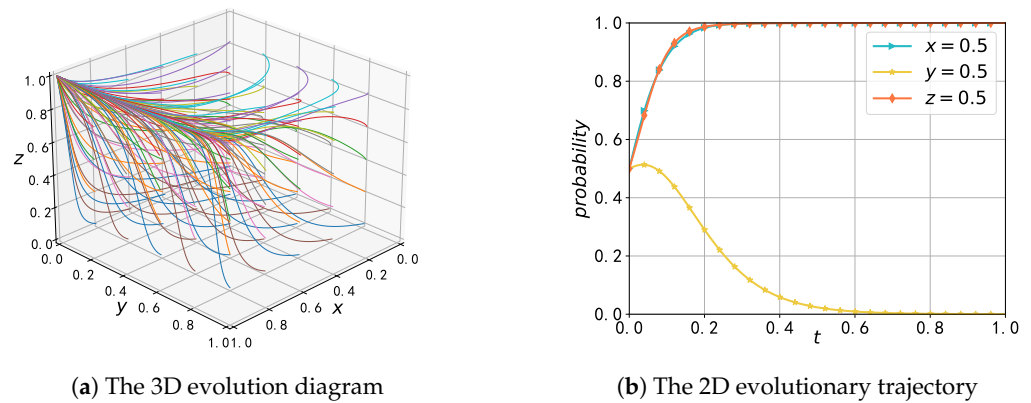


Figure 5. The evolutionary trajectory towards the equilibrium point $E_6(1, 0, 1)$.

5.4. Scenario 5: Equilibrium Point $E_7(1, 1, 0)$

In the tripartite evolutionary game model, the necessary and sufficient condition for the system to converge to the local equilibrium point $E_7(1, 1, 0)$ (Subsidize, Cooperate, Not invest) is that the strategic interests of the game agents satisfy the following set of economic incentive compatibility constraint formulas:

$$\begin{cases} S_e - B_{dir} + C_g - L_{soc} < 0 \\ (1 - \theta_1)(U_1 - U_3) < 0 \\ \tau + V_p + S_e + S_p < C_{dev} + O_m + \Omega + D\omega\pi_0 \end{cases} \quad (30)$$

As shown in Figure 6, this equilibrium point reflects a deep-seated mechanism where, under the background of “the government regulation being in a strong guidance phase” and “manufacturing enterprises possessing a strong willingness to cooperate,” the digital service providers face exorbitant heavy-asset operating costs on the supply side, ultimately leading to insufficient capital-driven momentum and a rupture in the technology supply. Specifically, the sum of the government’s digital upgrade subsidies and administrative expenditures remains strictly less than the sum of the direct social benefits generated and the losses avoided from upgrade stagnation. This establishes the social welfare justification for the government’s macroeconomic intervention, prompting it to firmly choose the “Subsidize” strategy to overcome initial market failures. Simultaneously, manufacturing enterprises exhibit a strong rigid demand for the “Cooperative” subscription model when facing potential security frictions or the risk of digital capability loss. When routine computing expenditures are strictly lower than the external substitution costs induced by information breaches or capability loss risks, this incentive constraint holds consistently. Manufacturing enterprises become highly reliant on the computing power hosting of service providers to hedge against market technological barriers. However, the digital service providers are deeply mired in a financial loss predicament. The sum of the operational revenue and the dual fiscal incentives they can obtain by choosing the “Invest” strategy still cannot cover the comprehensive lifecycle costs, such as heavy-asset procurement, de-

preciation, and O&M. This severe capital inversion leads the service providers to ultimately abandon the “Invest” decision.

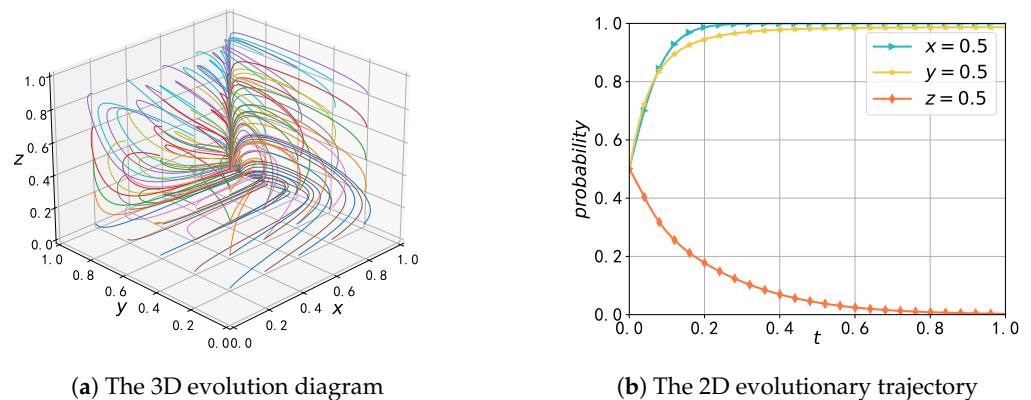


Figure 6. The evolutionary trajectory towards the equilibrium point $E_7(1, 1, 0)$.

5.5. Scenario 6: Equilibrium Point $E_8(1, 1, 1)$

As shown in Figure 7, the necessary and sufficient condition for the system to converge to the global optimal collaborative steady state $E_8(1, 1, 1)$ (Subsidize, Cooperate, Invest) is that the strategic interests of the game agents satisfy the following set of economic incentive compatibility constraint formulas:

$$\begin{cases} S_p + S_e + C_g < B_{dir} + L_{soc} \\ \tau + S_e < C_{dev} + O_m + \Omega - V_m + \theta_2 U_1 + (1 - \theta_2) U_2 \\ D\omega\pi_0 + C_{dev} + O_m + \Omega - V_p < \tau + S_e + S_p \end{cases} \quad (31)$$

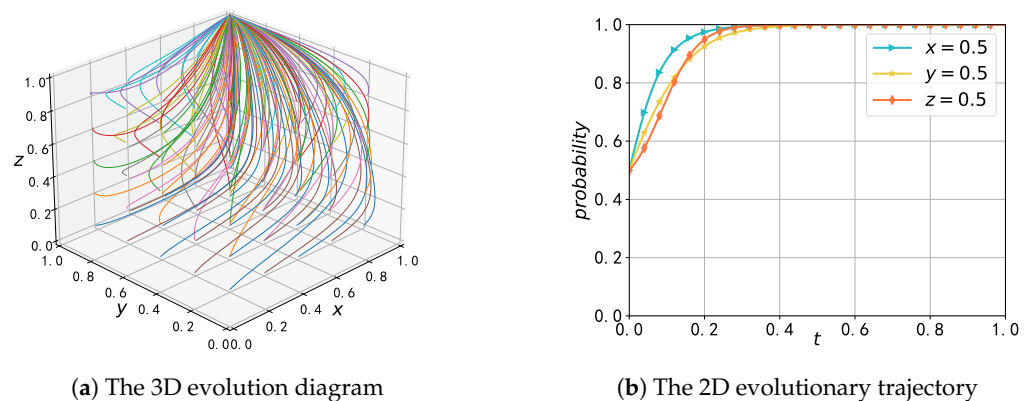


Figure 7. The evolutionary trajectory towards the equilibrium point $E_8(1, 1, 1)$.

First, this steady state establishes the social welfare boundary of public policy intervention. When the sum of the government’s total fiscal expenditure for subsidizing both the supply and demand sides and the administrative costs is strictly less than the sum of the direct social benefits generated by the policy and the industrial stagnation losses avoided, fiscal intervention can achieve a net efficiency gain in total social welfare. Building upon this, the desirable cost boundary for manufacturing enterprises is established. The net actual service fee paid by enterprises adopting the “cooperative” subscription model must be lower than their total cost of ownership (TCO) under the “Not cooperate” option, which involves independent procurement, independent O&M, and bearing the costs of technological upgrades and substitute computing. This price advantage drives the manufacturing sector to evolve towards an asset-light subscription ecosystem. Finally, the capital profit-seeking boundary of the digital service providers determines the behavior on the

supply side. The net lifecycle costs of the service providers investing in the computing infrastructure must be fully covered by the service fee revenue and government subsidies, thereby ensuring the momentum for the service providers to sustain the “Invest” strategy.

6. Simulation Analysis

To study the dynamic evolution trajectory of the tripartite game system and verify the robustness of related propositions under complex parameter interventions, we conducted numerical simulation experiments. Given the situation in practical scenarios where multiple variables and parameters are difficult to measure precisely, in order to ensure the wide applicability and scientificity of the evolutionary laws, the setting of simulation parameters follows the following principles. First, the assignment of benchmark parameters draws on multiple sources: (a) published industry data on smart sports equipment manufacturing costs and market structures (Li et al. [35]; Sui et al. [36]); (b) interviews conducted with specialists in relevant fields; the interviewees included two senior managers from sporting goods manufacturers, the technical director of a digital service provider focusing on the Industrial Internet of Things, a government official from the Provincial Department of Industry and Information Technology, and an academic researcher specializing in industrial economics, and their professional insights lent support to the identification of reasonable parameter ranges; and (c) publicly available policy documents specifying subsidy rates and tax incentives for smart manufacturing pilot programs in China (MIIT, 2023–2024). Second, to verify the convergence of the system to the ideal optimal state of the tripartite cooperation, the initial parameter values are strictly limited within the threshold conditions of (1, 1, 1) in Scenario 6. Third, the principle of introducing control variables is adopted to explore the sensitivity of the evolutionary stable strategy (ESS), that is, to minimize the linkage interference of other variables as much as possible when specific parameter changes occur.

Through the above design, we can effectively explore the dynamic evolution trend of the tripartite strategy equilibrium, and the specific parameter settings are detailed in Table 5.

Table 5. Source of simulation parameters.

Parameter	Baseline Value	Source and Calibration Rationale
S_e	30	Expert interview data; consistent with China’s smart manufacturing pilot subsidy rates (MIIT, 2023–2024), approximately 10% of equipment cost.
S_p	10	Expert interview data; reflects typical land-use and electricity price incentives for technology service providers in Chinese industrial parks.
C_{dev}	320	Annualized from unit equipment cost ($c_{dev} \approx 2000$) using EAC formula with $\rho = 3\%$, $T = 7$ years.
V_p	15	Annualized residual value; DSP recovery efficiency $\gamma_p = 0.10$, consistent with IT asset disposition industry benchmarks.
V_m	14	Annualized residual value; manufacturer recovery efficiency $\gamma_m = 0.093$, lower than V_p due to limited redeployment.
O_m	12	Annualized from total O&M budget over equipment lifecycle; approximately 8% of c_{dev} .
U_1, U_2, U_3	4×10^{-5}	Normalized unit computing costs; baseline equality reflects neutral starting point for comparative statics.
θ_1	0.9	High switching cost for low-end platforms; upper-range calibration reflecting specialized sports data processing requirements.

Table 5. Cont.

Parameter	Baseline Value	Source and Calibration Rationale
θ_2	0.5	Moderate switching cost for professional-grade platforms; mid-range from cloud computing switching-cost literature.
D	6000	Data throughput estimate for a mid-sized smart sports equipment factory; expert interview validated.
ω	100	Computational intensity coefficient for sports biomechanical data; higher than generic manufacturing.
Ω	1.5	Reduced-form risk parameter; conservative estimate of annual expected losses. Sensitivity analysis confirms robustness.
C_g	1.0	Administrative cost; approximately 2–3% of total subsidy disbursement.
L_{soc}	5.0	Conservative estimate of annual social loss from industrial stagnation; approximately 10% of B_{dir} .
B_{dir}	50	Aggregate social benefit from digital adoption; ratio $B_{dir}/S_e \approx 1.56$ is the critical parameter.

6.1. Impact of Initial Probabilities of Participating Agents

First, we examine the impact of stakeholders' initial willingness to participate (i.e., the initial state of the system) on the evolutionary path and convergence speed. By setting five groups of incrementally increasing initial willingness combinations, ranging from (0.1, 0.1, 0.1) to (0.9, 0.9, 0.9), we observe the dynamic evolutionary process of the three parties' strategies over time.

As shown in Figure 8, when the initial willingness is relatively high (e.g., 0.7 or 0.9), the system rapidly converges to the global optimal equilibrium point (1, 1, 1), indicating that a higher starting point for cooperation can significantly shorten the friction period of collaborative transformation. Although the initial willingness is extremely low [0.1, 0.1, 0.1], the system is still capable of achieving effective synergy, and its evolutionary path exhibits distinct phased characteristics. Specifically, although the government demonstrates a low propensity to subsidize in the early stage, once the comprehensive social benefits exceed the fiscal expenditure threshold, its probability of providing subsidies rises sharply and approaches 1 first, highlighting the guiding role of public policy in addressing market failures. Subsequently, through special equipment subsidies (S_e) and support policies targeted at service providers (S_p), the government effectively mitigates the high risks faced by digital service providers due to heavy-asset investments, causing their investment willingness (z) to increase rapidly. Although the manufacturing enterprises' willingness to cooperate (y) experiences a lag period of approximately $t = 0.2$, their probability of cooperation accelerates as the digital infrastructure gradually improves. Ultimately, the strategies of all three parties jointly converge to a stable cooperative equilibrium at around $t \approx 0.6$.

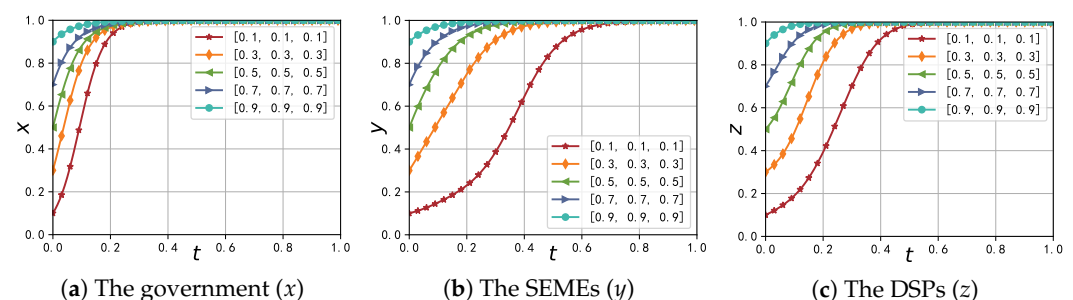


Figure 8. Impact of initial probabilities on the evolutionary trajectories.

6.2. Impact of the Capital Interest Rate (ρ) on the System

The capital interest rate directly affects the time value of digital assets, the depreciation costs of heavy assets, and the financing pressure on enterprises. Combining actual interest rate levels in financial markets in recent years, we set five scenarios with $\rho = 1.0\%$, 2.0% , 3.0% , 4.0% , and 5.0% to conduct sensitivity simulations and analyze their impact on the strategic evolution of the government, manufacturing enterprises, and digital service providers. As shown in Figure 9, manufacturing enterprises and digital service providers respond to interest rate changes in diametrically opposite directions, making it difficult for the system to reach a collaborative equilibrium under extreme interest rate conditions.

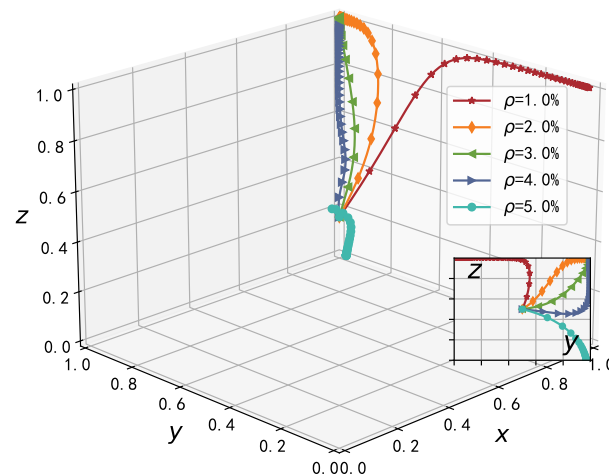


Figure 9. Impact of the capital interest rate (ρ) on the evolutionary trajectories.

In a low-interest-rate environment ($\rho = 1.0\%$), financing costs are extremely low, prompting manufacturing enterprises to tend to take out loans to independently purchase intelligent production lines. Since the total cost of ownership over the full life cycle is significantly reduced, the self-build model becomes financially more attractive. Consequently, their willingness to cooperate (y) weakens sharply, approaching a non-cooperative state. Meanwhile, service providers accelerate investment due to financing convenience, causing their investment willingness (z) to rise rapidly to 1. Therefore, the system fails to converge to the ideal $(1, 1, 1)$ steady state. Conversely, in a high-interest-rate scenario ($\rho = 5.0\%$), manufacturing enterprises prefer an asset-light operation model due to prohibitively high financing costs, which theoretically should enhance their willingness to cooperate. However, exorbitant interest expenses substantially increase the input costs for service providers regarding infrastructure. With the annual service fee τ remaining unchanged, their profit margins are eroded, ultimately leading to a decline in the service providers' investment willingness (z). Although there is a demand for cooperation from manufacturing enterprises, the probability of cooperation (y) struggles to increase sufficiently due to the lack of effective supply, causing the system to deviate from the optimal path once again. Notably, when the interest rate is within the range of 2.0% to 4.0% , the system exhibits the best synergy. In this interval, moderate financing costs not only curb the impulse of manufacturing enterprises towards self-construction but also ensure that service providers obtain reasonable investment returns. The strategic evolution of all three parties remains stable, eventually converging to the deep cooperation equilibrium of $(1, 1, 1)$.

6.3. Impact of Core Digital Equipment Price (C_{dev}) on the System

This section examines the impact of the price of core digital equipment, C_{dev} (such as smart sensors, high-precision robotic arms, and industrial internet hardware), on the cooperation dynamics between sports manufacturing enterprises and digital service providers. While keeping the annual service fee rate and other parameters constant, we set C_{dev} at 1000, 1500, 2000, 2500, and 3000 to simulate the evolutionary paths of the tripartite strategies.

As shown in Figure 10, C_{dev} exerts a significant inhibitory effect on the manufacturers' willingness to cooperate while presenting a positive incentive effect on the service providers' investment tendency (z). This reflects the structural contradiction between the premium pricing power of upstream technology suppliers and the cost sensitivity of downstream applications during the process of digital-intelligent transformation. Specifically, as C_{dev} rises, the time required for manufacturing enterprises to reach a cooperative equilibrium is significantly prolonged. Although they adopt an asset-light operation model, the increase in core equipment costs is often indirectly transmitted downstream through service pricing, thereby raising the overall threshold for transformation. Meanwhile, the substantial equipment investment lengthens the entire investment recovery cycle, weakening the short-term financial appeal of cost reduction and efficiency improvement, which leads manufacturing enterprises to become more conservative in their cooperation decisions. Conversely, the service providers' willingness to invest strengthens as C_{dev} increases. Under the constraint of fixed service fee rates, higher equipment prices usually correspond to higher project unit prices and more substantial government subsidies. This expectation of "high investment, high return" effectively incentivizes service providers to accelerate their market layout, causing their strategies to converge more rapidly to an active investment state. However, if C_{dev} exceeds the cost tolerance of manufacturing enterprises, even with active investment from service providers, the rhythm of supply and demand will still be severely mismatched, making it difficult for the system to converge to the ideal (1, 1, 1) collaborative steady state.

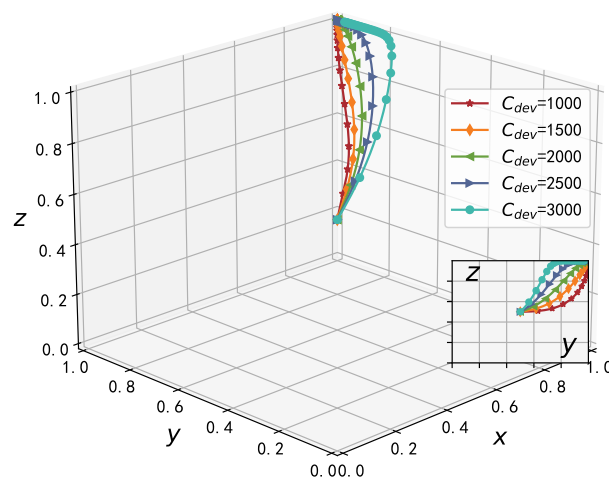


Figure 10. Impact of the core digital equipment price (C_{dev}) on the evolutionary trajectories.

6.4. Impact of the Annual Digital Service Fee (τ) on the System

The annual service fee is the core price variable that sustains the cooperative contract between manufacturing enterprises and digital service providers. To identify a reasonable pricing interval for stable system operation, we vary τ within the range of $0.95C_{dev}$ to $1.20C_{dev}$ while keeping other parameters constant to simulate the evolutionary dynamics of the tripartite strategies.

The simulation results in Figure 11 indicate that system stability is highly sensitive to the service fee level, with supply-demand synergy achieved only within a specific interval. When $\tau = 0.95C_{dev}$, the service providers' revenue cannot cover equipment depreciation, O&M expenses, and computing power costs. Even with government subsidies, maintaining basic profitability is impossible, causing their investment willingness (z) to continuously decline toward zero and leading to a substantive market exit from the supply side. At the critical point of $\tau = 1.0C_{dev}$, the system exhibits significant oscillations. Initially, because government subsidies have not fully taken effect, some service providers hesitate due to heavy-asset investment pressure; however, as policy signals clear and subsidy funds arrive, expected risks decrease, enabling the investment strategy to bottom out and eventually converge to a cooperative equilibrium. The system shows the best synergy when τ is between $1.05C_{dev}$ and $1.10C_{dev}$. Within this interval, service providers secure technology iteration and capital recovery through a moderate 5%–10% premium, while manufacturing enterprises control operating costs via an asset-light model. This alignment of financial incentives drives the tripartite strategies to rapidly converge to a deep-cooperation steady state of (1, 1, 1). However, once $\tau \geq 1.15C_{dev}$, the present value of the annual service fee exceeds the total lifecycle cost of ownership that manufacturing enterprises would incur through self-purchase, making the cooperation economically non-viable. Consequently, the manufacturing enterprises' willingness to cooperate (y) drops sharply as they revert to independent procurement or traditional production, leaving a shrunken demand side, which throws the system out of balance again.

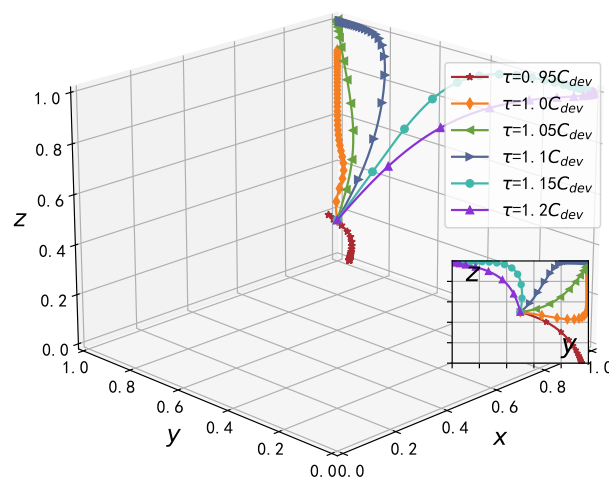


Figure 11. Impact of the annual digital service fee (τ) on the evolutionary trajectories.

In summary, the annual service fee is not a case of “the higher, the better” but contains a critical equilibrium interval. Policymakers should implement guided pricing mechanisms or moderate supervision to prevent excessive technology premiums from eroding downstream profit margins, thereby strengthening the industrial foundation that supports the digital and intelligent transformation of the sports manufacturing industry.

6.5. Impact of Government Special Equipment Subsidies (S_e) on the System

This section focuses on government subsidies for the procurement of core digital equipment and analyzes their impact on the dynamics of the tripartite game. Under the premise that other parameters remain unchanged, numerical simulations are conducted with subsidy amounts ranging from 25 to 50. The simulation results are shown in Figure 12.

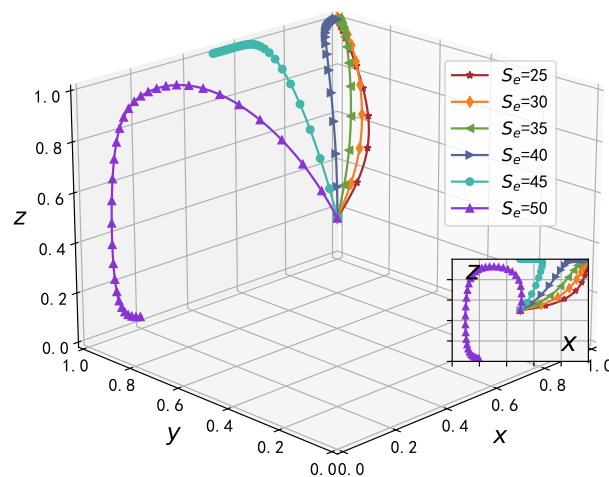


Figure 12. Impact of the government special equipment procurement subsidy (S_e) on the evolutionary trajectories.

As the figure shows, higher subsidy intensity is not necessarily better; excessive incentives may actually disrupt system stability. When the subsidy is $S_e = 50$, although it initially lowers the investment threshold for digital service providers and drives z to rise rapidly, high fiscal expenditure becomes unsustainable. The government quickly withdraws the subsidy due to budgetary pressures ($x \rightarrow 0$), which severely impacts market expectations, causing service providers' investment intentions to decline rapidly after their peak, eventually evolving into a state of no investment ($z \rightarrow 0$). At the critical level of $S_e = 45$, although the system can converge to a cooperative steady state, the process is volatile. Because the subsidy is still close to the fiscal sustainability boundary, the government's support weakens over time, causing a slowdown in service provider investment, a significant slowdown in convergence speed, and a substantial increase in transformation costs. In contrast, adopting low-to-medium intensity subsidies ($S_e = 25$ to 40) results in a highly robust system. On the one hand, the government's fiscal burden is controllable, and administrative costs are within a reasonable range, enabling a smooth transition to a normalized guidance mechanism and accelerating policy convergence. On the other hand, supply and demand sides gradually coordinate their behavior within a controllable risk framework. The willingness of manufacturing enterprises to cooperate (y) and the investment inclination of service providers (z) both steadily increase, eventually efficiently approaching a steady state of $(1, 1, 1)$.

In summary, government subsidies exhibit non-linear effects. Aggressive high subsidies easily lead to policy unsustainability, while moderate, stable, low-to-medium intensity subsidies are more likely to achieve a collaborative mechanism among all parties with relatively small fiscal input.

6.6. Impact of Government Subsidies for Digital Service Providers (S_p)

Finally, we examined the impact of government-specific support policies for digital service providers (such as tax breaks for computing power, preferential land use for data centers, and R&D interest subsidies) on the system's evolution. Under the premise that other conditions remain unchanged, we conducted multiple experiments with S_p ranging from 0 to 50, and the results are shown in Figure 13.

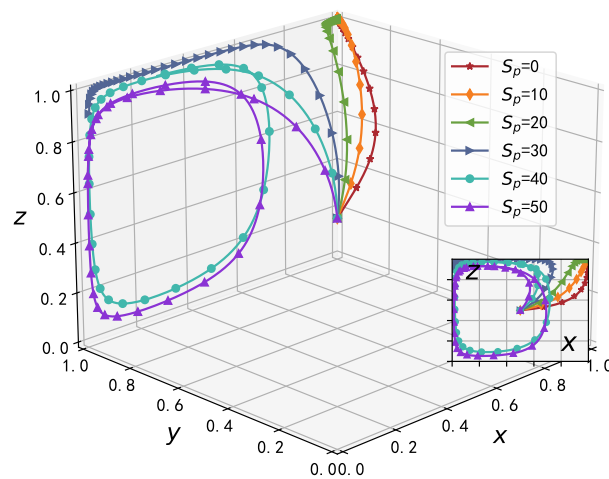


Figure 13. Impact of the government subsidy for digital service providers (S_p) on the evolutionary trajectories.

When $S_p = 0$, service providers must bear the high costs of cloud platform construction and computing power investment alone, and the system convergence process is slow. However, when S_p increases to 10 or 20, moderate fiscal support effectively offsets the initial sunk costs, significantly shortens the decision-making process, and causes their investment intentions to converge towards a steady state. At this point, subsidies have a strong marginal incentive effect, and the government's spending is within its means, showing that the program is sustainable. However, as S_p continues to increase, high subsidies may initially encourage service providers to enter the market quickly, but sustained high fiscal expenditure will gradually approach the government's fiscal affordability threshold. Once the cost of subsidies exceeds the total social welfare gains they bring, the government's willingness to maintain the policy begins to weaken. This situation causes service providers to invest less quickly in the later stages of evolution, which greatly increases the time it takes for the system to converge. When $S_p = 50$, the probability of government subsidies drops sharply ($x \rightarrow 0$). Service providers' cost advantage disappears instantly, and their investment intention z rapidly declines after reaching its peak, eventually evolving into a state of no investment.

Simulation results show that $S_p = 10\text{--}30$ constitutes a steady-state range that balances incentive effectiveness and fiscal sustainability. Within this range, policies can effectively activate service providers' investment while maintaining sufficient continuity and predictability, thereby ensuring stable access to digital services for manufacturing enterprises and driving the convergence of the three-party strategies to a synergistic state of (1, 1, 1).

6.7. Critical Thresholds and Phase Transition Analysis

The simulation results presented in Sections 6.2–6.6 reveal several critical thresholds at which the system undergoes qualitative behavioral changes. We synthesize these findings here to provide a unified picture of the system's non-linear dynamics.

Subsidy withdrawal boundary. The simulations in Sections 6.5 and 6.6 consistently identify a fiscal sustainability boundary at approximately $S_e + S_p \approx B_{dir} + L_{soc} - C_g$. When total subsidy expenditure approaches this boundary, the government's willingness to subsidize (x) begins to decline, creating a self-reinforcing cycle: lower x reduces the effective subsidy received by DSPs, weakening their investment incentive (z), which in turn reduces the social synergy benefits (B_{syn}) that justify the subsidy. This feedback loop produces a sharp phase transition rather than a smooth degradation, once the fiscal boundary is crossed, all three strategies collapse from (1, 1, 1) to a non-cooperative state within a narrow

parameter window. This finding has direct policy relevance, subsidy programs should include automatic triggers for gradual phase-down well before this boundary is reached, rather than maintaining high subsidy levels until fiscal pressure forces abrupt termination.

Optimal interest rate window. The capital interest rate ρ exhibits a non-monotonic relationship with system stability. For $\rho < 2\%$, cheap financing incentivizes manufacturers to self-build rather than cooperate, while for $\rho > 4\%$, high financing costs erode DSP profitability despite strong manufacturer demand for cooperation. The cooperative equilibrium is achievable only within $\rho \in [2\%, 4\%]$. This “optimal window” arises from the EAC mechanism, the interest rate simultaneously affects manufacturers’ self-build cost and DSPs’ investment cost, but in opposite directions, creating the non-monotonicity.

Service fee bifurcation point. The annual service fee τ exhibits a bifurcation at $\tau \approx C_{dev}$. For $\tau < C_{dev}$, DSP revenues are insufficient to cover equipment depreciation and O&M, causing investment collapse ($z \rightarrow 0$). For $\tau \in [1.05C_{dev}, 1.10C_{dev}]$, both manufacturers and DSPs find cooperation financially viable, and the system converges to $(1, 1, 1)$. For $\tau > 1.15C_{dev}$, the present value of service fees exceeds manufacturers’ TCO under self-build, causing cooperation collapse ($y \rightarrow 0$). The sharpness of these transitions, particularly at the upper boundary, suggests that service pricing policy requires particular caution near these thresholds.

The paradox of excessive subsidies. A counterintuitive finding emerges from the comparison of S_e and S_p sensitivity analyses. While moderate subsidies ($S_e \in [25, 40]$, $S_p \in [10, 30]$) effectively promote cooperation, high subsidies ($S_e \geq 45$, $S_p \geq 40$) trigger a self-defeating dynamic: initial rapid convergence toward cooperation is followed by government fiscal withdrawal as the sustainability boundary is breached, leading to abrupt market collapse. This “paradox of excessive subsidies” challenges the common intuition that higher subsidies uniformly produce better outcomes, and underscores the importance of calibrating subsidy intensity within sustainable bounds.

In summary, these threshold analyses demonstrate that the tripartite system’s macro-behavior is characterized by abrupt phase transitions rather than continuous, predictable responses to parametric variations. Consequently, policymakers must remain highly attentive to the system’s proximity to these critical boundaries, given that minor parameter perturbations near these thresholds can induce disproportionately large shifts in systemic outcomes.

7. Conclusions, Limitations, and Future Work

7.1. Conclusions

This paper developed a tripartite evolutionary game model to investigate the strategic interactions among the government, sports equipment manufacturers (SEMEs), and digital service providers (DSPs) in the digital transformation of smart sports equipment manufacturing. Drawing on evolutionary game theory and replicator dynamics, we derived the conditions under which the tripartite system converges to the ideal cooperative equilibrium in which the government actively subsidizes digital adoption, manufacturers fully embrace cooperation with DSPs, and DSPs undertake deep, asset-specific investments in specialized computing infrastructure.

The theoretical analysis yields two principal contributions. First, we formally characterized the eight equilibrium points of the tripartite system and derived the Jacobian-based stability conditions that determine convergence to each equilibrium. Among these, the unique globally stable equilibrium $(1, 1, 1)$ is achievable only under a stringent set of parameter constraints that we identify in closed form. These stability conditions provide a rigorous analytical basis for understanding when and why tripartite cooperation collapses and advance existing evolutionary game-theoretic treatments of digital transformation that have predominantly focused on bilateral interactions. Second, by embedding the Equiva-

lent Annual Cost (EAC) methodology into the payoff structure, the model explicitly links the capital interest rate to the strategic decisions of both manufacturers and DSPs, revealing a non-monotonic relationship that cannot be captured by models treating financing costs as simple linear cost drivers.

The simulation analysis systematically demonstrates that the tripartite system exhibits sharp phase transitions rather than smooth, continuous responses to parametric changes. These phase transitions, synthesized in Section 6.7, carry direct implications for policy design by identifying the boundaries, in terms of fiscal expenditure, service pricing, and financing conditions, within which the cooperative equilibrium remains viable. A notable finding is that moderate, fiscally sustainable subsidies can effectively promote cooperation, whereas excessive subsidies can breach the government's fiscal sustainability limits and trigger a sudden systemic collapse. These findings underscore the importance of calibrating policy interventions within sustainable bounds rather than pursuing maximal subsidy intensity, and of designing policy frameworks with built-in mechanisms for gradual phase-down as the digital ecosystem matures.

In summary, this study provides a formal analytical framework for understanding tripartite coordination in smart manufacturing ecosystems and demonstrates that the DSP is not merely a passive intermediary but a transformative agent whose independent investment calculus qualitatively reshapes the equilibrium structure.

7.2. Limitations

Despite the theoretical contributions discussed above, several limitations of the present study should be acknowledged, each of which points to productive directions for future research.

Agent homogeneity and deterministic dynamics: The current model assumes homogeneous agents within each stakeholder group and deterministic replicator dynamics. In reality, sports equipment manufacturers differ substantially in size (from small workshops to publicly listed firms), digital maturity, and access to financing, implying heterogeneous adoption thresholds. Similarly, DSPs vary in their technological specialization and geographic coverage.

Binary strategy simplifications: As acknowledged in Section 3.1, the binary strategy assumptions for all three agents represent deliberate simplifications that enable closed-form analytical solutions. In practice, government subsidy intensity, manufacturer cooperation depth, and DSP investment scale are inherently continuous or multi-level decisions.

Uncertainty and risk: The current model treats all costs and benefits as deterministic and known in advance, abstracting from fundamental uncertainties that characterize real-world digital transformation, including AI project failures, cybersecurity incidents, rapid technological obsolescence, and market demand volatility. While the risk cost parameter Ω partially captures these factors in reduced form, the model does not explicitly account for the stochastic nature of these disruptions.

Empirical validation: The current study is primarily theoretical and simulation-based. While we conducted preliminary expert interviews to validate the model structure and parameter ranges, systematic empirical validation using firm-level panel data, structured surveys of manufacturers and DSPs, or in-depth comparative case studies of firms at different stages of digital transformation would substantially strengthen the evidence base for the model's predictions.

7.3. Future Work

Building on the limitations identified above, we outline several promising directions for future research.

First, future work could relax the homogeneous-agent assumption by employing agent-based modeling with heterogeneous agent populations characterized by empirical distributions of cost structures, risk preferences, and digital maturity levels. Incorporating stochastic perturbations into the replicator dynamics equations would further enable analysis of the system's resilience to random shocks arising from technology breakthroughs, policy changes, or macroeconomic fluctuations. Integrating prospect theory to capture asymmetric risk attitudes, specifically loss aversion, where firms weigh potential losses from failed digital investments more heavily than equivalent gains, could yield more behaviorally realistic predictions than the expected utility framework.

Second, the binary strategy assumptions could be generalized in several ways: (a) modeling government subsidy intensity as a continuous decision variable, potentially using optimal control theory to characterize the optimal subsidy trajectory over time, including the conditions for gradual phase-out; (b) introducing multi-level cooperation strategies for manufacturers (e.g., {No cooperation, Partial outsourcing, Hybrid model, Full cooperation}); and (c) modeling DSP investment as a continuous capacity variable or as a multi-stage decision (pilot → regional deployment → full-scale implementation).

Third, to address the deterministic abstraction, future research could incorporate stochastic evolutionary game theory with explicit noise processes in the payoff functions, apply fuzzy set theory to model the ambiguity surrounding key parameters subject to rapid technological change, or employ robust optimization approaches to identify strategies that perform well across a range of possible future states.

Fourth, systematic empirical validation constitutes a critical next step. Specific approaches include: (a) a Delphi study with a larger panel of industry experts for systematic parameter calibration; and (b) comparative case studies of two or three sports equipment manufacturing firms at different stages of digital transformation to empirically trace their strategic evolution paths.

Finally, comparative studies applying the tripartite framework to other manufacturing sectors with similar characteristics (e.g., medical devices, consumer electronics, automotive components) would help establish the boundary conditions of our findings and assess their generalizability beyond the sports equipment industry.

Author Contributions: Conceptualization, M.X.; methodology, M.X. and J.Y.; validation, M.X.; formal analysis, M.X. and J.Y.; investigation, J.Y.; writing—original draft preparation, M.X.; writing—review and editing, J.Y.; supervision, J.Y.; funding acquisition, J.Y. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported in part by the National Social Science Fund of China (Grant No. 23BJY205).

Data Availability Statement: No data was used for the research described in this article.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Qi, Q.; Xu, Z.; Rani, P. Big data analytics challenges to implementing the intelligent Industrial Internet of Things (IIoT) systems in sustainable manufacturing operations. *Technol. Forecast. Soc. Change* **2023**, *190*, 122401. [\[CrossRef\]](#)
2. Yang, G.; Nie, Y.; Li, H.; Wang, H. Digital transformation and low-carbon technology innovation in manufacturing firms: The mediating role of dynamic capabilities. *Int. J. Prod. Econ.* **2023**, *263*, 108969. [\[CrossRef\]](#)
3. Mangla, S.K.; Luthra, S.; Garza-Reyes, J.A.; Jabbour, C.J.C.; Brem, A. Sustainability and Industry 4.0: The role of social, environmental and technological factors in the development of digital manufacturing. *Technol. Forecast. Soc. Change* **2024**, *201*, 123223. [\[CrossRef\]](#)
4. Xu, J.; Yu, Y.; Zhang, M.; Zhang, J.Z. Impacts of digital transformation on eco-innovation and sustainable performance: Evidence from Chinese manufacturing companies. *J. Clean. Prod.* **2023**, *393*, 136278. [\[CrossRef\]](#)

5. Fan, X.; Zhao, S.; Shao, D.; Wang, S.; Zhang, B. Talking and walking: Corporate digital transformation and government subsidies. *Financ. Res. Lett.* **2024**, *64*, 105444. [\[CrossRef\]](#)
6. Chen, B.; Lin, H.; Shan, B.; Xiao, Y. Government investment in science and technology, digital transformation, and innovation in manufacturing enterprises. *Financ. Res. Lett.* **2024**, *69*, 106299. [\[CrossRef\]](#)
7. Sankar, M.K.; Gupta, S.; Luthra, S.; Jagtap, S.; Singh, S.; Kumar, A. The emergence of digitalization to the manufacturing sector in the sustainability context: A multi-stakeholder perspective analysis. *J. Clean. Prod.* **2024**, *468*, 142983. [\[CrossRef\]](#)
8. Lin, B.; Xu, C. The effects of industrial robots on firm energy intensity: From the perspective of technological innovation and electrification. *Technol. Forecast. Soc. Change* **2024**, *203*, 123373. [\[CrossRef\]](#)
9. Chirumalla, K.; Leoni, L.; Oghazi, P. Moving from servitization to digital servitization: Identifying the required dynamic capabilities and related microfoundations to facilitate the transition. *J. Bus. Res.* **2023**, *158*, 113668. [\[CrossRef\]](#)
10. Dalenogare, L.S.; Le Dain, M.A.; Ayala, N.F.; Pezzotta, G.; Frank, A.G. Building digital servitization ecosystems: An analysis of inter-firm collaboration types and social exchange mechanisms among actors. *Technovation* **2023**, *124*, 102756. [\[CrossRef\]](#)
11. Bustinza, O.F.; Molina, L.M.; Vendrell-Herrero, F.; Opazo-Basaez, M. AI-enabled smart manufacturing boosts ecosystem value capture: The importance of servitization pathways within digital-intensive industries. *Int. J. Prod. Econ.* **2024**, *277*, 109411.
12. Tang, D.; Li, J.; Qu, S.; Boamah, V. Tripartite collaboration among government, digital technology platform, and manufacturing enterprises: Evolutionary game model. *Sustainability* **2023**, *15*, 7946. [\[CrossRef\]](#)
13. Struwe, S.; Slepnirov, D. Unlocking digital servitization: A conceptualization of value co-creation capabilities. *J. Bus. Res.* **2023**, *160*, 113825. [\[CrossRef\]](#)
14. Yang, Y.; Zhang, C.; Liu, B.; Huang, Y.; Tai, Y. Mystery of special government subsidies: How does digital transformation promote enterprise innovation and development? *Econ. Anal. Policy* **2024**, *83*, 1–16. [\[CrossRef\]](#)
15. Maretto, L.; Faccio, M.; Battini, D. The adoption of digital technologies in the manufacturing world and their evaluation: A systematic review of real-life case studies and future research agenda. *J. Manuf. Syst.* **2023**, *68*, 576–600. [\[CrossRef\]](#)
16. Yang, L.; Zou, H.; Shang, C.; Ye, X.; Rani, P. Adoption of information and digital technologies for sustainable smart manufacturing systems for industry 4.0 in small, medium, and micro enterprises (SMMEs). *Technol. Forecast. Soc. Change* **2023**, *188*, 122308. [\[CrossRef\]](#)
17. Gadekar, R.; Sarkar, B.; Gadekar, A. Model development for assessing inhibitors impacting Industry 4.0 implementation in Indian manufacturing industries: An integrated ISM-Fuzzy MICMAC approach. *Int. J. Syst. Assur. Eng. Manag.* **2024**, *15*, 646–671.
18. Krishnan, R. Challenges and benefits for small and medium enterprises in the transformation to smart manufacturing: A systematic literature review and framework. *J. Manuf. Technol. Manag.* **2024**, *35*, 918–938. [\[CrossRef\]](#)
19. Chirumalla, K.; Oghazi, P.; Nnewuku, R.E.; Tuncay, H.; Yahyapour, N. Critical factors affecting digital transformation in manufacturing companies. *Int. Entrep. Manag. J.* **2025**, *21*, 54. [\[CrossRef\]](#)
20. Cugno, M.; Castagnoli, R.; Büchi, G.; Pini, M. Industry 4.0 and production recovery in the covid era. *Technovation* **2022**, *114*, 102443.
21. Ardolino, M.; Bacchetti, A.; Dolgui, A.; Franchini, G.; Ivanov, D.; Nair, A. The impacts of digital technologies on coping with the COVID-19 pandemic in the manufacturing industry: A systematic literature review. *Int. J. Prod. Res.* **2024**, *62*, 1953–1976.
22. Tomaschko, F.; Reichelt, L.; Krommes, S. Digitalisation of manufacturing systems: A literature review of approaches to assess the sustainability of digitalisation technologies in production systems. *Sustainability* **2024**, *16*, 6275. [\[CrossRef\]](#)
23. Zhang, Z.; Zhang, N.; Gu, J. A systematic literature review of digital transformation of manufacturing enterprises: Bibliometric analysis and knowledge framework. In *Proceedings of the Wuhan International Conference on E-Business*; Springer: Berlin/Heidelberg, Germany, 2023; pp. 144–155.
24. Wang, X.; Kumar, V.; Kumari, A.; Kuzmin, E. Impact of digital technology on supply chain efficiency in manufacturing industry. In *Digital transformation in Industry: Digital Twins and New Business Models*; Springer: Berlin/Heidelberg, Germany, 2022; pp. 347–371.
25. Yang, J.; Jia, W.; Guo, Q.; Pu, P.; Zhang, J. Modeling and optimization of data trading supply chain regulatory strategies based on system dynamics. *J. Ind. Manag. Optim.* **2026**, *22*, 1034–1062. [\[CrossRef\]](#)
26. Dong, Z.; Dai, P. Dynamic mechanism of digital transformation in equipment manufacturing enterprises based on evolutionary game theory: Evidence from China. *Systems* **2023**, *11*, 493. [\[CrossRef\]](#)
27. Zhu, J.; Baker, J.S.; Song, Z.; Yue, X.G.; Li, W. Government regulatory policies for digital transformation in small and medium-sized manufacturing enterprises: An evolutionary game analysis. *Humanit. Soc. Sci. Commun.* **2023**, *10*, 1–18. [\[CrossRef\]](#)
28. Gao, J.; Zhang, W.; Guan, T.; Feng, Q. Evolutionary game study on multi-agent collaboration of digital transformation in service-oriented manufacturing value chain. *Electron. Commer. Res.* **2023**, *23*, 2217–2238.
29. Chen, H.; Wang, J.; Miao, Y. Evolutionary game analysis on the selection of green and low carbon innovation between manufacturing enterprises. *Alex. Eng. J.* **2021**, *60*, 2139–2147. [\[CrossRef\]](#)
30. Qiu, Z.; Chen, Y.; Han, H.; Wang, T. Research on Digital Technology to Promote Low-Carbon Transformation of Manufacturing Industries Under the Perspective of Green Credit: An Evolutionary Game Theory Approach. *Sustainability* **2024**, *16*, 11203. [\[CrossRef\]](#)

31. Wang, H.; Chen, L.; Liu, J. An evolutionary game theory analysis linking manufacturing, logistics, and the government in low-carbon development. *J. Oper. Res. Soc.* **2022**, *73*, 1014–1032.
32. Ma, R.; Jiang, L.; Wang, T.; Wang, X.; Ruan, J. How do manufacturing companies and service providers share knowledge in the context of servitization? An evolutionary-game model of complex networks. *Int. J. Prod. Res.* **2023**, *61*, 4279–4301.
33. Zhang, F.; Wu, L.; Liu, W.; Ding, K.; Hui, J.; Leng, J.; Zhou, X. Evolutionary game-based incentive models for sustainable trust enhancement in a blockchained shared manufacturing network. *Adv. Eng. Inform.* **2022**, *54*, 101791. [[CrossRef](#)]
34. Zhao, D.; Wang, X. Evolutionary game of digital decision-making in supply chains based on system dynamics. *RAIRO-Oper. Res.* **2024**, *58*, 475–510. [[CrossRef](#)]
35. Li, H.; Lu, L.; Yang, S. Servitization level, digital transformation, and enterprise performance of sporting goods manufacturing enterprises in China. *Heliyon* **2024**, *10*, e33520. [[CrossRef](#)] [[PubMed](#)]
36. Sui, X.; Hu, H.; Wang, R. The impact of digital transformation on the servitization transformation of manufacturing firms. *Res. Int. Bus. Financ.* **2025**, *73*, 102588.

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.