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Adaptive Iterative Learning Control for Uncertain MIMO Systems with Application on Underwater Robot Trajectory Tracking

Yaqiong Ding ¹, Yingxian Zhong ¹, Dan Xiang ¹, Junliu Zhong ¹ , Jing Ling ¹ and Hanguang Jia ^{2,3,*}¹ School of Artificial Intelligence, Guangzhou Maritime University, Guangzhou 510725, China² School of Automation Science and Engineering, South China University of Technology, Guangzhou 510641, China³ State Key Laboratory of Reliability Technology for Electronic Components, China Electronic Product Reliability and Environmental Testing Research Institute, Guangzhou 510610, China

* Correspondence: jiahanguang@ceprei.com

Abstract

This paper addresses the trajectory tracking control problem of underwater robots operating in uncertain underwater environments. An adaptive iterative learning control algorithm is proposed to handle external disturbances caused by environmental uncertainties, as well as system challenges including non-uniform trial lengths and unknown control gain matrices. To improve trajectory tracking accuracy, an external disturbance compensation term is introduced, and the controller parameters are updated online using the tracking error between the desired and actual trajectories. Furthermore, an adaptive learning rate strategy is developed to compensate in real time for errors induced by external disturbances and other uncertainties, enabling dynamic optimization of the controller parameters based on the actual system trajectory. The proposed method relaxes the stringent traditional assumptions that require the control gain matrix to be known and strictly a symmetric positive definite (or negative definite). Based on Lyapunov stability theory and finite-time control techniques, a composite energy function is constructed to rigorously prove the convergence of the closed-loop system. Simulation results demonstrate that the proposed algorithm ensures satisfactory trajectory tracking accuracy and robustness for the underwater robot in the presence of external disturbances and various uncertainties, effectively solving the stabilization control problem in uncertain underwater environments.

Keywords: underwater robots; adaptive iterative learning control; Uneven test length; control gain matrix

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1. Introduction

In recent years, rapid advancements in intelligent technologies for marine equipment have led to significant progress in the fields of underwater robotics and ship motion control. At the level of advanced control methods, model predictive control (MPC), reinforcement learning (RL), and iterative learning control (ILC) have emerged as three mainstream technical routes. MPC explicitly handles system constraints and nonlinear dynamics in the time domain through a receding horizon optimization mechanism. For example, Shen et al. adopted a Lyapunov-constrained MPC framework to address actuator saturation and thrust

allocation issues in AUV trajectory tracking [1]. Reinforcement learning, being data-driven, avoids reliance on accurate mathematical models. For instance, Gao et al. proposed an event-triggered reinforcement learning framework for ship dynamic positioning control, effectively reducing communication overhead and enhancing disturbance rejection performance [2]. Despite these advancements, standard model predictive control (MPC) and reinforcement learning (RL) techniques still encounter distinct bottlenecks when deployed on marine equipment. On one hand, MPC-based schemes are fundamentally constrained by high computational complexity, especially when balancing real-time performance and input constraints on embedded platforms, a critical limitation thoroughly investigated in [3]. On the other hand, while RL exhibits massive potential for model-free optimization, it notoriously suffers from low sample efficiency and a deficiency in rigorous theoretical stability and safety guarantees. Although recent hybrid paradigms have attempted to leverage robust MPC to provide safety blankets for data-driven RL algorithms [4], these methodologies remain confined to the time domain. More critically, none of the aforementioned feedback, predictive, or reinforcement learning control frameworks fully exploit the historical error trajectories accumulated over multiple repeated trials to systematically suppress periodic or repetitive disturbances. Consequently, under repetitive operational scenarios, their tracking performance relies heavily on instantaneous real-time corrections rather than achieving the asymptotic convergence characteristic of iterative learning control. Furthermore, the hydrodynamic challenges escalate under extreme sea conditions; as experimentally demonstrated by Youn et al., ship capsizing under irregular waves exhibits severe nonlinear roll evolution [5], indicating that marine environmental disturbances possess profound uncertainty and complex spectral redistribution characteristics. Such volatile, yet repetitive, operational environments impose significantly higher demands on the learning ability and stability of periodic control strategies, which powerfully motivates the development of our proposed scheme.

In fact, repetitive operations of underwater robots are extremely common in engineering practice: during seabed oil and gas exploration, AUVs need to repeatedly traverse designated areas to complete full-coverage seismic data acquisition; in coral reef ecological monitoring, biomimetic robots must periodically shuttle through observation areas; port and waterway maintenance relies on robots to repeatedly perform sweeping and reef detection tasks; and deep-sea sampling operations require robots to travel back and forth between target points and the mother ship multiple times. Such tasks inherently feature “repetitive operation,” providing a physical basis for leveraging historical iterative information to improve control performance—this is precisely the application scenario of ILC.

ILC is a typical control scheme for repetitive motion systems [6–11]. Its core idea is to use the tracking error from the previous iteration to correct the current control input, gradually approaching the ideal trajectory over multiple iterations. It has low requirements for model accuracy and offers significant advantages in nonlinear systems. Sakagami et al. combined ILC with time-scale transformation and successfully applied it to the dynamic control of underwater robotic manipulators [12–14], verifying the feasibility of this method in underwater repetitive operations. However, traditional ILC has limited adaptability to system uncertainties and external disturbances; its convergence performance degrades significantly under complex conditions such as ocean currents and model parameter perturbations, making it difficult to directly meet the demands of deep-sea operations.

To enhance the robustness of control systems, adaptive iterative learning control (AILC) integrates adaptive laws with iterative learning mechanisms, enabling online identification of uncertain system parameters and compensation for unknown external disturbances during the iteration process [15,16]. However, traditional ILC and most existing AILC methods

generally impose restrictive assumptions that the control gain matrix must be symmetric and invertible. Among them, recent research has relaxed the constraint on the control gain matrix from “symmetric” to “asymmetric” [17], effectively breaking through the limitation of traditional AILC methods that require the gain matrix to be symmetric positive definite. Nevertheless, most existing AILC results target general nonlinear systems or land-based operation platforms [17,18], and they have not yet been specifically designed to account for the special operational characteristics of underwater robots, such as strong coupling, severe external disturbances, and flow field uncertainty. Underwater manipulators are affected by hydrodynamic coupling effects, asymmetric thruster layouts, and dynamic load disturbances, causing their system control gain matrices to exhibit significant asymmetry. If the assumption of a symmetric positive definite gain matrix is still adopted in controller design, it may lead to degraded control accuracy, poor system stability, or even instability. Meanwhile, MPC heavily relies on accurate system dynamics models, and the complex, variable underwater environment substantially reduces model accuracy and reliability; reinforcement learning suffers from long training convergence cycles, insufficient real-time response capability, and difficulty adapting to dynamic underwater conditions. Therefore, designing a robust and effective AILC algorithm under the general condition that the control gain matrix is only required to be invertible (without symmetry or positive definiteness) is key to breaking through the bottleneck of high-precision trajectory tracking control for underwater robots.

In addition to gaining asymmetry, non-uniform iteration lengths [19] constitute another critical bottleneck restricting the engineering application of ILC. Traditional ILC is mostly based on the assumption of fixed iteration cycles [20,21]. During actual operation, underwater robots are susceptible to sudden ocean currents, obstacle avoidance maneuvers, thruster performance degradation, and sensor faults, leading to dynamically changing trajectory lengths. As a result, the error from incomplete intervals cannot be properly utilized, directly slowing down algorithm convergence, reducing tracking accuracy, and even causing system instability. Currently, research on variable iteration lengths has made preliminary progress in areas such as robotic manipulators [22], but existing methods do not consider the fluid dynamics characteristics of underwater robots or the complex strong disturbance environment. Their control structures and convergence analysis theories cannot be directly transferred to underwater scenarios. Therefore, there is an urgent need to conduct research on iterative learning control with variable trajectory lengths for underwater robots by designing novel error handling and compensation methods to relax the constraint of fixed iteration lengths and effectively address this key bottleneck in practical applications.

While contemporary iterative learning control frameworks have successfully resolved various system uncertainties, distinct theoretical gaps remain regarding multi-constrained maritime applications. Reference [15] presents an adaptive learning paradigm for nonlinear systems under randomly varying trial lengths, yet it requires a priori knowledge of control directions and structural dynamics. For non-parametric systems with asymmetric gains, a unified adaptive controller was developed in [17], but its validity is constrained by the assumption of strictly uniform iteration horizons. To handle non-uniform lengths, a difference-based ILC scheme was proposed in [22] for visual servoing manipulators. However, since its algorithmic core relies on discrete-type difference updates across successive batches, its execution is structurally restricted to the iteration domain. In fundamental contrast, the robust adaptive mechanism proposed in this work is derived from continuous differential operations. Operating as a time-domain continuous adaptive iterative learning control strategy, it enables real-time tracking along the continuous-time axis while concurrently managing parameter adaptation across the trial domain. This dual-domain learning capability establishes a fundamentally superior computational logic, bypasses the

dependency on known control directions, and offers exceptional resilience against volatile, non-uniform maritime operational profiles without relying on structural approximations.

In summary, none of the aforementioned studies can simultaneously address the compound challenges of unknown control directions, system uncertainties, non-uniform trajectory lengths and time-varying trajectory durations. Targeting this research gap, this paper investigates adaptive iterative learning tracking control for underwater vehicles with variable trajectory lengths under uncertain flow fields. The core contributions are outlined as follows:

1. Uncertain systems with asymmetric control gain matrices: We relax the traditional ILC assumption on gain matrix symmetry and effectively adapt to unknown parameters and unknown control directions through an adaptive online compensation mechanism, significantly improving system robustness against asymmetric gains.
2. Consideration of non-uniform iteration lengths and corresponding error compensation mechanisms: We break through the traditional constraint of fixed trajectory length, enabling full utilization of error information from incomplete intervals and ensuring convergence speed and tracking accuracy under variable-length conditions.
3. Design of an adaptive iterative learning control law and a differential-form parameter update law with only two parameters: The adaptive parameter update law implicitly incorporates historical iteration error information, enabling more efficient extraction of error features in the iteration domain and accelerating the learning process.

These three contributions are organically combined to effectively adapt to complex repetitive operation scenarios of underwater robots, filling gaps in existing research while enriching the theoretical framework of variable-length iterative learning control, and providing new technical ideas for intelligent repetitive operation control of underwater robots.

To clearly present the research ideas and technical roadmap, the paper is structured as follows: Section 1 describes the current state of underwater robot control, clarifying the research background, objectives, and main contributions; Section 2 establishes the dynamic model of the underwater robot and defines the control objectives and constraints; Section 3 designs the adaptive iterative learning control algorithm, elaborates the variable trajectory length error processing method, and completes the algorithm construction; Section 4 proves the algorithm convergence using Lyapunov stability theory; Section 5 verifies the algorithm performance through simulation experiments and compares it with traditional algorithms; Section 6 summarizes the research findings. Table 1 displays the definitions of the symbols used in the paper.

Table 1. List used in the paper. of Symbols.

No.	Symbol	Description
1	k	Iteration number
2	t	Time variable
3	T	Upper bound of the iteration time interval
4	ε_k	Piecewise time instant of the k -th iteration
5	$v_k(t)$	System state vector of the k -th iteration
6	$r_d(t)$	Desired reference trajectory
7	$E_k(t)$	Tracking error vector
8	$L_k(t)$	Improved tracking error vector
9	$W_k(t)$	Composite energy function
10	$Z_k(t)$	Iterative energy sequence
11	$f_k(t)$	Piecewise improved error function

Table 1. Cont.

No.	Symbol	Description
12	n	Dimension of the system state
13	M	System inertia matrix
14	$\hat{M}$	Estimated inertia matrix
15	$f_{CD}(\cdot)$	Nonlinear dynamic term of the system
16	$d_k(t)$	Lumped disturbance of the k -th iteration
17	$\tau_k(t)$	Iterative learning control input
18	$\xi(E_k(t))$	Regression basis function vector
19	$\hat{\theta}_k(t)$	Adaptive parameter estimation vector
20	θ	Ideal unknown constant parameter vector
21	$\tilde{\theta}_k(t)$	Parameter estimation error vector
22	Γ	Adaptive learning rate matrix
23	Q	Symmetric positive-definite matrix in Lyapunov equation
24	$(\cdot)^T$	Transpose of a vector or matrix
25	$\ \cdot\ _2^2$	Square of the 2-norm
26	I_n	n -dimensional identity matrix
27	C^-	Left half of the complex plane
28	$\sigma(\cdot)$	Set of eigenvalues of a matrix
29	$\text{sign}(\cdot)$	Sign function
30	U	Uniform distribution

2. Problem Formulation

The motion control system of underwater robots inherently possesses multi-input multi-output (MIMO), nonlinear, and nonparametric characteristics, determined by its operational nature and environmental demands. This system is difficult to precisely characterize using explicit parametric models, featuring multidimensional input and output channels. Inputs include propeller thrust, while outputs encompass position and attitude, making it a typical complex control system. In the practice of trajectory tracking control for underwater robots, existing literature indicates that traditional adaptive control methods and adaptive iterative learning control typically require the controlled system to satisfy the assumption of linear parameterization of nonlinear functions. However, such structural constraints not only severely limit the practical application of adaptive control techniques in underwater robotics but also hinder the full realization of adaptive iterative learning control's core advantage—low dependency on prior system models—in underwater robot control. Notably, the complex and variable operational environment of underwater robots, coupled with uncertain hydrodynamic disturbances, further exacerbates their nonparametric nonlinear characteristics. These systems are more prevalent than linearly parameterized counterparts and suffer from a greater scarcity of prior modeling knowledge. Consequently, theoretical research on adaptive control and iterative learning control for MIMO nonlinear nonparametric dynamic systems in underwater robots holds significant value and importance.

Consider the following underwater robotic MIMO nonlinear dynamic system with nonparametric characteristics:

$$\dot{v}_k(t) = f_{CD}[v_k(t)] + M^{-1}\tau_k(t) + d_k(t), \quad (1)$$

Among them, k represents the number of iterations; $M^{-1} \in R^{n \times n}$ denotes the inverse matrix of the inertia matrix M of the underwater robot; $f_{CD}[v_k(t)] \in R^n$ stands for the known nonlinear hydrodynamic force function composed of Coriolis force, centripetal force and damping force, which is fully expressed as $f_{CD}[v_k(t)] = -M^{-1}[C(v_k(t))v_k(t) + D(v_k(t))v_k(t)]$; $v_k(t) \in R^n$ is the velocity vector of the underwater robot while $\tau_k(t) \in \mathbb{R}^n$ is the control

input of the underwater robot; $d_k(t) \in R^n$ represents the external disturbance of the system; the desired trajectory of the system is $r_d(t)$; and the ideal time length of the system is T .

Assumption 1. Let the nonlinear mapping $f_{CD}[v_k(t)]$ of the underwater robot satisfy the sublinear growth constraint:

$$\|f_{CD}[v_k(t)]\|_\infty \leq \alpha_1 \|v_k(t)\|_\infty + \alpha_2,$$

where α_1 and α_2 denote positive-definite parameters.

Assumption 2. Suppose the iterative learning process of the underwater robot satisfies the following consistent boundedness conditions: all iterative initial values $v_k(0) \in R^n$, expected trajectories $r_k(t)$ of iterative transformations, and external disturbances $d_k(t)$, and $v_k(0) = r_d(0)$.

Assumption 3. When the inverse matrix of the inertia matrix M of the underwater robot satisfies the invertibility condition, there exist parameter uncertainties in its matrix elements.

Remark 1. Compared to the global Lipschitz condition for nonlinear functions, which is widely used in ILC designs, the linear growth condition is a little relaxed since it can be derived from the global Lipschitz condition [17].

Remark 2. It is worth emphasizing that the standard assumptions on the invertibility of the inertia matrix and bounded operational conditions are well-grounded in practical maritime engineering. For a physical rigid-body underwater manipulator, the inertia matrix is inherently symmetric and positive-definite, which structurally precludes the occurrence of representation singularities. Moreover, the assumption of bounded initial conditions is highly aligned with real-world scenarios, where the robot always initiates its trajectory tracking tasks from a physically constrained and localized deployment zone, structurally guaranteeing that initial state errors cannot grow infinitely.

3. Design of Adaptive Iterative Learning Controller for Non-Uniform Test Length

3.1. Uneven Test Length

In traditional iterative learning control algorithms for underwater robot trajectory tracking, the task execution interval is constrained to a fixed value to achieve an ideal iterative learning process. However, during actual underwater operations, unpredictable factors such as unknown disturbances in complex river basin environments, hydrodynamic uncertainties, and underwater constraints often make it difficult to maintain this fixed time interval. This practical challenge has spurred research addressing the issue of non-uniform test lengths in underwater robotic systems. Below, we illustrate scenarios where non-uniform test lengths arise, drawing from real-world operational contexts of underwater robots.

The first scenario involves underwater robots conducting near-shore observation tasks. Due to environmental disturbances such as swift coastal currents and obstructions from submerged reefs, the system may prematurely terminate a single iteration to prevent collisions or deviation from safe zones, resulting in actual operation times shorter than the pre-set ideal duration. The second scenario involves underwater robots conducting seabed resource exploration. The preset iteration duration is based on ideal hydrodynamic conditions, but actual underwater currents constantly vary in speed and direction. This causes fluctuations in the time required to complete a single exploration task, preventing consistent test durations. The third scenario involves periodic inspection tasks performed by multi-input multi-output nonlinear nonparametric underwater robots. Factors such as interference from marine life and temporary changes in equipment load often prevent

the completion of a single iteration task within the predetermined duration. This study focuses on addressing the issue of non-uniform test lengths in more general multi-input multi-output nonlinear nonparametric underwater robotic systems.

As a fundamental step in the adaptive iterative learning control framework, the tracking error is defined as

$$E_k(t) = r_d(t) - v_k(t), \quad (2)$$

where $E_k(t) = \begin{bmatrix} E_k^{(1)}(t) & E_k^{(2)}(t) & \cdots & E_k^{(n)}(t) \end{bmatrix}$.

Under non-uniform trial length conditions, let ε_k denote the duration of each iterative operation. The modified tracking error is formulated as follows:

$$f_k(t) = \begin{cases} E_k(t), & 0 \leq t \leq \varepsilon_k \\ E_k(\varepsilon_k), & \varepsilon_k < t \leq T \end{cases}$$

It can be concluded that $f_k(t) = \lambda_k E_k(t) + (1 - \lambda_k) E_k(\varepsilon_k)$, where $\lambda_k \in \{0, 1\}$. When $\lambda_k = 1$, the system stops halfway during task execution and fails to reach the desired trial duration; conversely, when $\lambda_k = 0$, the system reaches the desired duration.

3.2. Underwater Robot Controller Design

Building upon Assumptions 1–3 as the theoretical foundation, this study develops an adaptive iterative learning control scheme with the following architectural configuration:

$$\tau_k(t) = \frac{\hat{M}^{-1} E_k(t) E_k^T(t) \zeta(E_k(t)) \hat{\theta}_k(t)}{\|E_k(t)\|_2^2} \quad (3)$$

$$\begin{cases} \dot{\hat{\theta}}_k(t) = \Gamma \zeta^T(E_k(t)) E_k(t) \in \mathbb{R}^2, & 0 \leq t \leq \varepsilon_k \\ \hat{\theta}_k(t) = \hat{\theta}_{k-1}(t) \in \mathbb{R}^2, & \varepsilon_k < t \leq T \end{cases} \quad (4)$$

$$\zeta(E_k(t)) = \begin{bmatrix} E_k(t) & \text{sign}(E_k(t)) \end{bmatrix} \in \mathbb{R}^{n \times 2}, \quad (5)$$

where $\hat{\theta}_{-1}(t) = 0 \in \mathbb{R}^2$; $\Gamma \in \mathbb{R}^n$ is the learning gain to be designed. Equation (3) formulates the control input that constitutes the core controller architecture, while Equation (4) defines the adaptive parameter updating mechanism. The algorithm's innovation resides in its synergistic optimization process integrating cross-iteration parameter transfer with error feedback, which dynamically calibrates control parameters through historical iteration data and real-time error signals to achieve progressive input refinement.

Remark 3. In practical applications, non-parameterized nonlinear systems are more universal than linearly parameterized counterparts and require less prior knowledge of system models, which justifies the significance of developing adaptive ILC techniques for such systems. While existing schemes introduce approximators like fuzzy systems and neural networks [23,24] to handle nonlinear uncertainties, they generally suffer from an excessive number of tunable adaptive parameters, resulting in heavy computational loads and large storage consumption. To address the above limitations, the algorithm proposed in this paper is characterized by low computational complexity and modest memory usage, rendering it highly suitable for resource-constrained embedded platforms. It employs only two adaptive variables $\vartheta = \begin{bmatrix} p & q \end{bmatrix}^T \in \mathbb{R}^2$, and the update law is structurally simple and easy to implement in the time domain.

Figure 1 delineates the architectural diagram of the adaptive iterative learning control system, whose operational principle can be characterized by three pivotal phases: synergis-

tic parameter updating through cross-iteration data fusion and error-driven compensation. Table 2 displays the control steps of the ILC law.

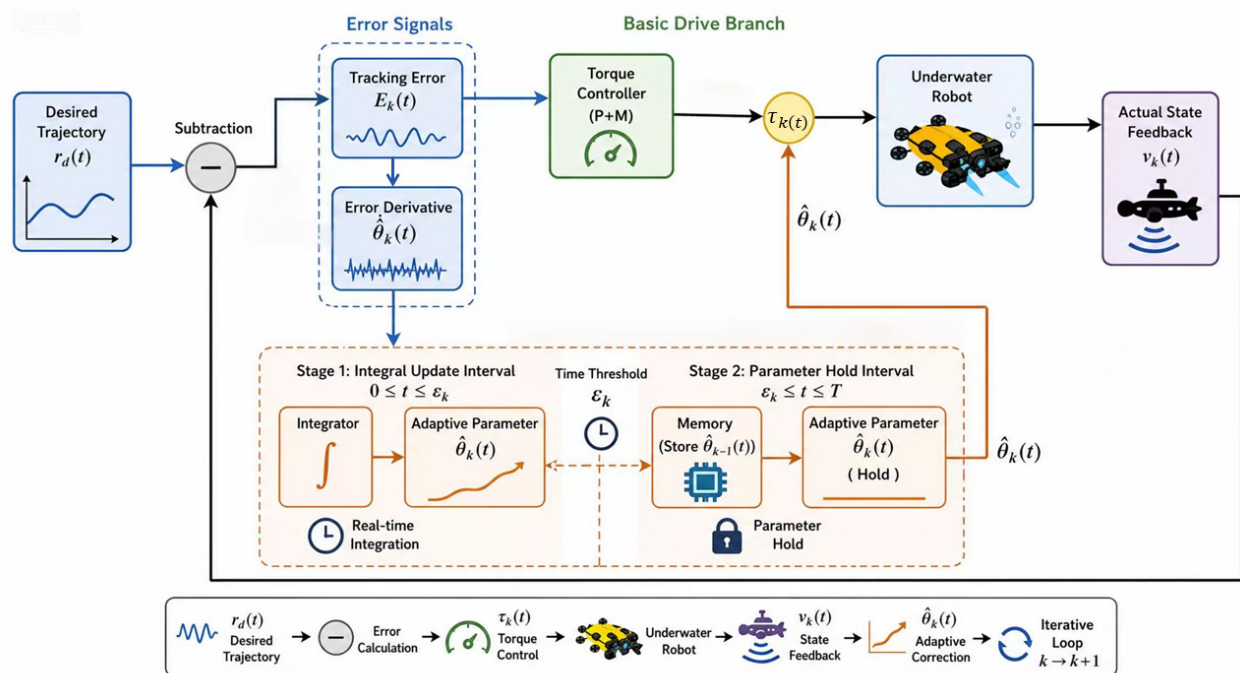


Figure 1. Architectural diagram of the adaptive iterative learning control system.

Table 2. Iterative Learning Control Algorithm Steps.

No.	Operation Steps	Remarks
1	Set the desired system trajectory $r_d(t)$, time interval $t \in [0, T]$, and provide initial state information $\tau_0(t)$ and $v_k(0)$ ($k = 0, 1, 2, \dots$)	Initialization: define target trajectory, time horizon, and initial values
2	Store the results of the k -th iteration (system input $\tau_k(t)$ and measured output $v_k(t)$)	Record input–output data of the current iteration
3	Calculate the tracking error $E_k(t) = r_d(t) - v_k(t)$, and verify the convergence condition: check if $\lim_{k \rightarrow +\infty} E_k(t) \leq \delta$ (where δ is the prescribed error tolerance). If satisfied, proceed to Step 6; otherwise, execute Step 4	Judge if accuracy is achieved, decide to continue or stop
4	Select the learning gain Γ , feed back the system input and errors ($\tau_k(t)$ and $E_k(t)$) to the learning law, and compute the system input $\tau_{k+1}(t)$ for the next iteration	Update control input via the learning law
5	Set $k = k + 1$ and go to Step 2	Increment iteration counter and enter next loop
6	Stop the iteration	Algorithm terminates, output final results

4. Stability Analysis

Theorem 1. Based on Assumptions 1–3, under the action of the adaptive control law (3) and adaptive parameter update laws (4) and (5), when the control gain matrix M^{-1} of the multi-input multi-output nonlinear system (1) satisfies $\sigma(-M^{-1}\hat{M}^{-1}) \subset C^-$, and $\Gamma > 0$ is satisfied, the tracking error of the system is $\lim_{k \rightarrow \infty} E_k(t) = 0$.

Proof. Because of $\sigma(-M^{-1}\hat{M}^{-1}) \subset C^-$, there exists a positive definite symmetric matrix $Q = Q^T \in R^{n \times n}$, satisfying

$$\left(M^{-1}\hat{M}^{-1}\right)^T Q + Q\left(M^{-1}\hat{M}^{-1}\right) = 2I_n. \quad (6)$$

Building upon the theoretical foundation of Assumptions 1–2, we establish the following formal definition:

$$\sup_{0 \leq t \leq T} \|Q\dot{r}_d(t) - Qd_k(t)\|_\infty \leq \beta \quad (7)$$

$$\sup_{0 \leq t \leq T} \|Q\|_{m_\infty} \leq c, \quad (8)$$

and

$$\begin{cases} p = nc\alpha_1 \\ q = n\beta + nc\alpha_1\|r_d(t)\|_\infty + nc\alpha_2 \end{cases} \quad (9)$$

$$\theta = \begin{bmatrix} p & q \end{bmatrix}^T \in \mathbb{R}^2, \quad (10)$$

where α_1 and α_2 are the unknown parameters presented in Assumption 1, and $\hat{\theta}_k(t)$ denotes the estimate of θ at time t for the k -th iteration, with the definition: $\tilde{\theta}_k(t) = \hat{\theta}_k(t) - \theta$. The subsequent analysis is divided into three parts.

Part 1: Introduce a positive-definite energy function $L_k(t)$

$$L_k(t) = \frac{1}{2} E_k^T Q E_k \quad (11)$$

Therefore, from Equations (1) and (2), it follows that

$$\begin{aligned} \dot{L}_k(t) &= E_k^T Q \dot{E}_k \\ &= E_k^T (Q\dot{r}_d - Q\dot{v}_k) \\ &= E_k^T [Q\dot{r}_d - Q(f_{CD}[v_k(t)] + M^{-1}\tau_k + d_k)] \\ &= E_k^T [Q\dot{r}_d - Qf_{CD}[v_k(t)] - Qd_k] - E_k^T QM^{-1}\tau_k \end{aligned} \quad (12)$$

Performing algebraic manipulations on the term $E_k^T [Q\dot{r}_d - Qf_{CD}[v_k(t)] - Qd_k]$ in (14) with the aid of (2), Assumption 1, and (7)–(8), we arrive at

$$\begin{aligned} &E_k^T [Q\dot{r}_d - Qf_{CD}[v_k(t)] - Qd_k] \\ &\leq \|E_k^T [Q\dot{r}_d - Qf_{CD}[v_k(t)] - Qd_k]\|_\infty \\ &\leq n\|E_k\|_\infty \|Q\dot{r}_d - Qf_{CD}[v_k(t)] - Qd_k\|_\infty \\ &\leq n\|E_k\|_\infty (\|Q\dot{r}_d - Qd_k\|_\infty + \|Q\|_{m_\infty} \|f_{CD}[v_k(t)]\|_\infty) \\ &\leq n\|E_k\|_\infty (\beta + c\alpha_1\|v_k\|_\infty + c\alpha_2) \\ &= \|E_k^T\|_\infty (n\beta + nc\alpha_1\|r_d\|_\infty + nc\alpha_2 + nc\alpha_1\|E_k\|_\infty) \\ &= \|E_k^T\|_\infty (q + p\|E_k\|_\infty) \\ &= E_k^T (q \cdot \text{sign}(E_k) + pE_k) \end{aligned} \quad (13)$$

Moreover, substituting (13) into (10) and (5) leads to

$$E_k^T [Q\dot{r}_d - Qf_{CD}[v_k(t)] - Qd_k] \leq E_k^T \zeta(E_k) \theta \quad (14)$$

For the term $-E_k^T QM^{-1}\tau_k$ in (12), under the condition $E_k \neq 0$, substituting the control law (3) and the identity $(M^{-1}\hat{M}^{-1})^T Q + Q(M^{-1}\hat{M}^{-1}) = 2I_n$ from (6), we obtain

$$-E_k^T QM^{-1}\tau_k = -\frac{1}{2} [E_k^T QM^{-1}\tau_k + \tau_k^T (M^{-1})^T Q E_k]$$

$$\begin{aligned}
&= -\frac{1}{2} E_k^T Q M^{-1} \cdot \frac{\hat{M}^{-1} E_k E_k^T \zeta(E_k) \hat{\theta}_k}{\|E_k\|_2^2} \\
&\quad - \frac{1}{2} \cdot \left(\frac{\hat{M}^{-1} E_k E_k^T \zeta(E_k) \hat{\theta}_k}{\|E_k\|_2^2} \right)^T (M^{-1})^T Q E_k \\
&= -\frac{1}{2} E_k^T \left(Q M^{-1} \hat{M}^{-1} + \hat{M}^{-T} (M^{-1})^T Q \right) E_k \cdot \frac{E_k^T \zeta(E_k) \hat{\theta}_k}{\|E_k\|_2^2} \\
&= -E_k^T E_k \cdot \frac{E_k^T \zeta(E_k) \hat{\theta}_k}{\|E_k\|_2^2} \\
&= -E_k^T \zeta(E_k) \hat{\theta}_k
\end{aligned}$$

So

$$-E_k^T Q M^{-1} \tau_k = -E_k^T \zeta(E_k) \hat{\theta}_k \quad (15)$$

We note that when $E_k(t) = 0$, Equation (14) remains valid regardless of the value of $\tau_k(t)$. Therefore, in the control law (3), we design $\tau_k(t) = \tau_k(t^-)$ with $\tau_k(0) = 0$ for the case $E_k(t) = 0$.

Substituting relations (13) and (15) into expression (12), and $\tilde{\theta}_k(t) = \hat{\theta}_k(t) - \theta$, we derive

$$\begin{aligned}
\dot{L}_k(t) &\leq E_k^T \zeta(E_k) \theta - E_k^T \zeta(E_k) \hat{\theta}_k \\
&= -E_k^T \zeta(E_k) \tilde{\theta}_k
\end{aligned} \quad (16)$$

Part 2: In light of (11), a Lyapunov-like composite energy function is introduced as follows

$$W_k(t) = L_k(t) + \frac{1}{2\Gamma} \tilde{\theta}_k^T \tilde{\theta}_k \geq 0 \quad (17)$$

Considering (16) and $\dot{\tilde{\theta}}_k = \dot{\hat{\theta}}_k$, it follows that

$$\begin{aligned}
\dot{W}_k(t) &= \dot{L}_k(t) + \frac{1}{\Gamma} \tilde{\theta}_k^T \dot{\tilde{\theta}}_k \\
&\leq -E_k^T \zeta(E_k) \tilde{\theta}_k + \frac{1}{\Gamma} \tilde{\theta}_k^T \dot{\hat{\theta}}_k
\end{aligned} \quad (18)$$

When $0 \leq t \leq \varepsilon_k$, the adaptive law (4) takes effect as $\dot{\hat{\theta}}_k(t) = \Gamma \zeta^T(E_k) E_k$. Given that θ is constant, $\dot{\tilde{\theta}}_k(t) = \dot{\hat{\theta}}_k(t)$, substituting gives

$$\dot{W}_k(t) \leq -E_k^T \zeta(E_k) \tilde{\theta}_k + \frac{1}{\Gamma} \tilde{\theta}_k^T [\Gamma \zeta^T(E_k) E_k] = 0 \quad (19)$$

When $\varepsilon_k < t \leq T$, the adaptive law stops updating, so $\dot{\hat{\theta}}_k(t) = 0$, and thus $\dot{\tilde{\theta}}_k(t) = 0$. At this time,

$$\dot{W}_k(t) = \dot{L}_k(t) \leq -E_k^T \zeta(E_k) \tilde{\theta}_k < 0 \quad (20)$$

To sum up, throughout the entire iteration interval $[0, T]$, we have

$$\dot{W}_k(t) \leq 0 \quad (21)$$

Therefore, we conclude that $E(t)$, $\tilde{\theta}(t)$, and $\hat{\theta}(t)$ are bounded, which in turn implies the boundedness of $\tau(t)$ according to control law (3).

Part 3: Introduce a sequence $\{Z_k(T)\}$ as follows

$$Z_k(T) = \frac{1}{2\Gamma} \tilde{\theta}_k^T(T) \tilde{\theta}_k(T) \geq 0 \quad (22)$$

It then follows that

$$\begin{aligned} \Delta Z_k(T) &= Z_k(T) - Z_{k-1}(T) \\ &= \frac{1}{2\Gamma} \tilde{\theta}_k^T(T) \tilde{\theta}_k(T) - \frac{1}{2\Gamma} \tilde{\theta}_{k-1}^T(T) \tilde{\theta}_{k-1}(T) \\ &= \frac{1}{\Gamma} \int_0^T \dot{\tilde{\theta}}_k^T \dot{\tilde{\theta}}_k dt \end{aligned} \quad (23)$$

For $0 \leq t \leq \varepsilon_k$, the parameter adaptive update law is activated, i.e.,

$$\dot{\tilde{\theta}}_k(t) = \Gamma \zeta^T(E_k) E_k.$$

Substituting it into the integral expression and canceling the positive definite gain Γ , we obtain

$$\frac{1}{\Gamma} \int_0^{\varepsilon_k} \tilde{\theta}_k^T(t) [\Gamma \zeta^T(E_k) E_k] dt = \int_0^{\varepsilon_k} \tilde{\theta}_k^T(t) \zeta^T(E_k) E_k dt.$$

For $\varepsilon_k < t \leq T$, the parameter update is suspended, i.e.,

$$\dot{\tilde{\theta}}_k(t) = 0,$$

so the contribution of this interval to the integral is 0. In summary, combining the integral results over the two intervals, the complete expression for the iterative energy increment is obtained as

$$\begin{aligned} \Delta Z_k(T) &= \int_0^{\varepsilon_k} \tilde{\theta}_k^T(t) \zeta^T(E_k) E_k dt + \int_{\varepsilon_k}^T 0 dt \\ &= \int_0^T \tilde{\theta}_k^T(t) \zeta^T(E_k) E_k dt. \end{aligned} \quad (24)$$

On the other hand, by integrating Equation (16) over the interval $[0, T]$, we obtain

$$L_k(T) - L_k(0) \leq - \int_0^T E_k^T \zeta(E_k) \tilde{\theta}_k dt \quad (25)$$

According to Assumption 2, we have $E_k(0) = 0$, and then $L_k(0) = 0$. Substituting into (25) yields

$$\begin{aligned} \int_0^T \tilde{\theta}_k^T \zeta^T(E_k) E_k dt &= \int_0^T E_k^T \zeta(E_k) \tilde{\theta}_k dt \\ &\leq -L_k(T) \end{aligned} \quad (26)$$

Substituting Equation (26) into Equation (24), we obtain

$$\Delta Z_k(T) \leq -L_k(T) \leq 0 \quad (27)$$

Thus, $\{Z_k(T)\}$ constitutes a non-increasing sequence bounded below by 0. In addition, Equation (27) indicates that

$$\begin{aligned} L_k(T) &\leq Z_{k-1}(T) - Z_k(T) \\ &\leq Z_{k-1}(T) \leq Z_1(T) \end{aligned} \quad (28)$$

which ensures that $L_k(T)$ is bounded.

$$\sum_{j=2}^k L_j(T) \leq -\sum_{j=2}^k \Delta Z_j(T) = Z_1(T) - Z_k(T) \leq Z_1(T) \quad (29)$$

Finally, we note that (29) yields

$$\sum_{j=2}^k L_j(T) \leq Z_1(T). \quad (30)$$

In the limit as $k \rightarrow \infty$, this gives

$$\sum_{j=2}^{+\infty} L_j(T) \leq Z_1(T). \quad (31)$$

Hence, by the Barbalat-like lemma [25], we obtain $\lim_{k \rightarrow +\infty} L_k(T) = 0$ and

$$\lim_{k \rightarrow +\infty} \int_0^T E_k^T E_k dt = 0,$$

which indicates that $E_k(t)$ tends to zero in the L^2 -norm over $[0, T]$. This concludes the proof of Theorem 1. $\square$

5. Experimental Results and Discussion

Example: For the variable trial length adaptive learning control problem of a 2-degree-of-freedom (2-DOF) underwater robot with uncertain nonlinear systems, the core lies in designing a control strategy with dynamic adjustment capability to address unknown system dynamic parameters, external disturbances, and nonlinear characteristics of the model.

The system inertia matrix $\mathbf{M}$ is composed of rigid-body inertial terms and hydrodynamic added mass contributions: $\mathbf{M} = \begin{bmatrix} m - X_{\dot{u}} & 0 \\ 0 & I_z - N_{\dot{r}} \end{bmatrix}$. The system parameters are defined as follows: m represents the total mass of the vehicle; I_z denotes the moment of inertia about the vertical z-axis; and $X_{\dot{u}}$ and $N_{\dot{r}}$ correspond to the hydrodynamic added mass coefficients in the surge and yaw directions, respectively.

The Coriolis-centripetal matrix $\mathbf{C}(\mathbf{v})$ accounts for rigid-body and added mass coupling effects: $\mathbf{C}(\mathbf{v}) = \begin{bmatrix} 0 & -m v \\ m v & 0 \end{bmatrix}$. The lateral velocity component is denoted by v . To ensure hydrodynamic modeling accuracy, the Coriolis-centripetal matrix must be augmented with added mass effects, yielding the complete equation: $\mathbf{C}(\mathbf{v}) = \mathbf{C}_{\text{rigid}} + \mathbf{C}_{\text{added}}$.

The damping matrix $\mathbf{D}(\mathbf{v})$ captures both linear and quadratic damping phenomena: $\mathbf{D}(\mathbf{v}) = \begin{bmatrix} d_{11} + d_{u|u}|u| & 0 \\ 0 & d_{66} + d_{r|r}|r| \end{bmatrix}$ with the parameters defined as follows: d_{11} representing the linear damping coefficient in surge motion; d_{66} representing the linear damping coefficient in yaw rotation; and $d_{u|u}$, $d_{r|r}$ denoting the quadratic damping coefficients associated with the velocity-squared damping effects.

To ensure that the simulation scenario is highly consistent with real-world embedded control deployment, this paper employs a rigorous disturbance model and fixed-step solver in the simulation. The core simulation and computational parameters are comprehensively presented in Table 3, and the detailed characteristics of the disturbance signals are summarized in Table 4.

Figure 2 shows the trial lengths corresponding to different iteration times, which are $\tau_1 = 1.5$, $\tau_2 = 1.8$, $\tau_3 = 1$, $\tau_{10} = 1.3$, and $\tau_{60} = 4\pi$, where $\tau_k = 4\pi$ is the desired trial length, so as to ensure the authenticity and practicality of the simulation.

To comprehensively verify the trajectory tracking performance of the proposed adaptive iterative learning algorithm under complex interference environments, comparative experiments are conducted in a MATLAB 2026a simulation environment. The experiments adopt a circular trajectory as the tracking target and systematically evaluate the convergence characteristics, tracking accuracy, and control input response of the algorithm under two typical external disturbance conditions: random perturbations and sinusoidal periodic perturbations. The maximum absolute error is uniformly adopted as the quantitative evaluation metric. In the simulation, the mass parameter of the underwater robot is set to

$m = 50$ kg. The inverse of the system inertia matrix is $M^{-1} = \begin{bmatrix} \frac{1}{450} & 0 \\ 0 & \frac{1}{225} \end{bmatrix}$ and the system estimation matrix is selected as $\hat{M}^{-1} = 50 \begin{bmatrix} \cos(\frac{\pi}{3}) & -\sin(\frac{\pi}{3}) \\ \sin(\frac{\pi}{3}) & \cos(\frac{\pi}{3}) \end{bmatrix}$.

$$\begin{aligned} \text{spec}(-M^{-1}\hat{M}^{-1}) &= \text{spec}\left(-\begin{bmatrix} \frac{1}{450} & 0 \\ 0 & \frac{1}{225} \end{bmatrix} \times 50 \begin{bmatrix} \cos(\frac{\pi}{3}) & -\sin(\frac{\pi}{3}) \\ \sin(\frac{\pi}{3}) & \cos(\frac{\pi}{3}) \end{bmatrix}\right) \\ &= \{-0.0833 + 0.1332i, -0.0833 - 0.1332i\} \subset C^-, \end{aligned}$$

This parameter satisfies the stability condition of the theorem proposed in this paper. Meanwhile, the learning gain in the parameter update rule is set to $\Gamma = 1$, and the maximum number of iterations is set to 60. Figure 3 presents, through three-dimensional surfaces, the two components $d_k^{(1)}$ and $d_k^{(2)}$ of random perturbations and sinusoidal periodic perturbations, showing their variation with time and iteration number. It clearly illustrates the spatiotemporal distribution differences between the two types of disturbances: random perturbations exhibit irregular spatiotemporal fluctuation characteristics, while sinusoidal periodic perturbations show obvious periodicity and regularity, laying the foundation for subsequent comparison of algorithm robustness under the two disturbances.

Table 3. Simulation Environment and Solver Configurations.

Configuration Item	Specific Value/Setting
Sampling Time (T_s)	0.001 s
Controller Execution Frequency	1000 Hz
Maximum Iterations (k)	60
Iteration Stopping Criterion	Reaching maximum trial number $k = 60$
Per-step Computational Complexity	$\mathcal{O}(1)$ (Explicit algebraic updates)

Table 4. Summary of disturbance signals used in the simulation.

Type	Component	Amplitude	Angular Freq. (rad/s)	Freq. (Hz)	Phase (rad)	Math. Expression
Periodic	X-axis	$A_1 = 0.5$	$\omega_1 = 4\pi$	2	$\phi_1 = 0$	$d_k(t) = 0.5 \sin(4\pi t)$
	Y-axis	$A_2 = 0.3$	$\omega_2 = 6\pi$	3	$\phi_2 = \pi/4$	$d_k(t) = 0.3 \cos(6\pi t + \pi/4)$
Random	X-axis	$[-0.01, 0.01]$	—	—	—	$d_k(t) \sim \mathcal{U}[-0.01, 0.01]$
	Y-axis	$[-0.01, 0.01]$	—	—	—	$d_k(t) \sim \mathcal{U}[-0.01, 0.01]$

Figure 4, plotted in a semi-logarithmic coordinate system, shows the decay process of the two tracking error components $EE_k^{(1)}$ and $EE_k^{(2)}$ with the iteration number k under the

two disturbance environments. This paper adopts the maximum absolute error as the core evaluation metric, and its calculation formula for the k -th iteration is

$$EE_k^{(i)} = \max_{t \in [0, T_k]} |e_k^{(i)}(t)|, \quad i = 1, 2$$

where $e_k^{(i)}(t)$ is the tracking error of the i -th degree of freedom at time t during the k -th iteration, and T_k is the operation duration of the k -th iteration. To quantitatively evaluate the convergence performance and steady-state accuracy of the algorithm, the following definitions are clearly made: the convergence iteration number is the smallest iteration number at which the tracking error first enters a stable fluctuation range with no obvious decay trend; the steady-state error is the average stable amplitude of the maximum absolute error of the system over the last 10 iterations after convergence, which accurately reflects the steady-state tracking accuracy of the algorithm.

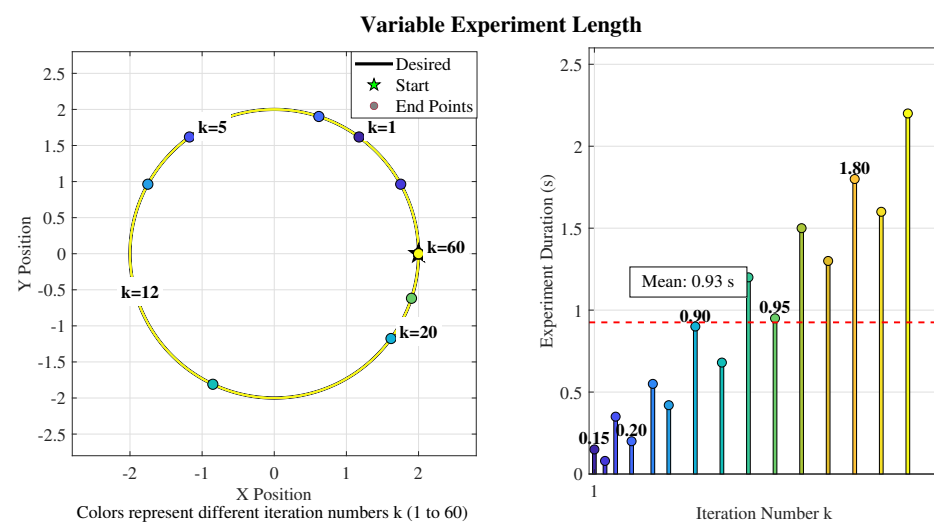


Figure 2. Varying trial lengths along iteration axis.

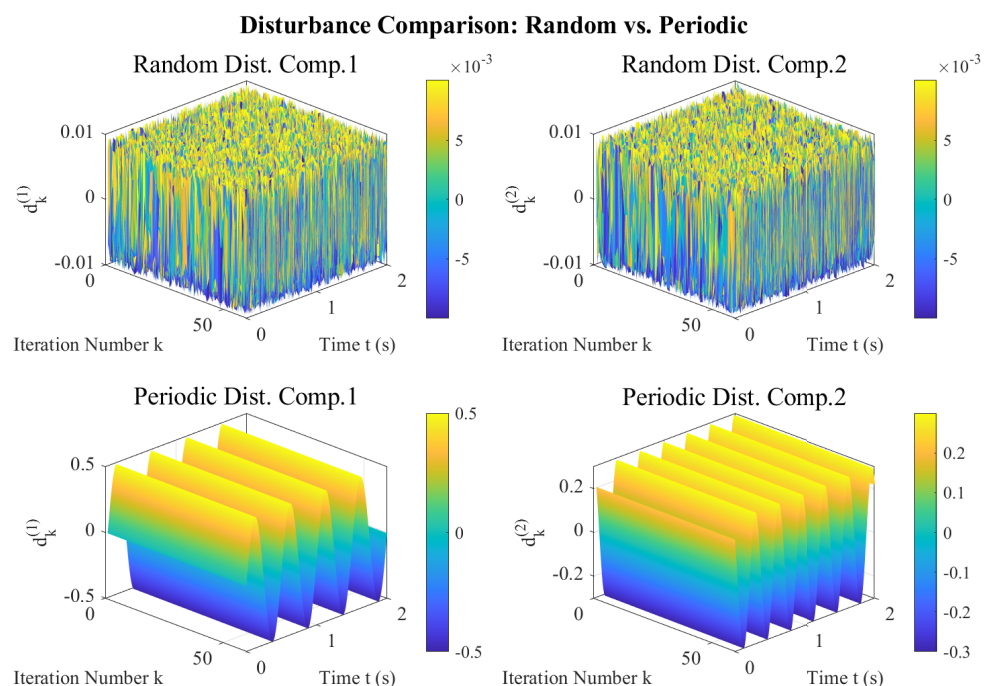


Figure 3. Spatiotemporal distribution comparison of random and periodic disturbances.

As illustrated in Figure 4, the tracking error components of the system exhibit a monotonic decay trend with increasing iteration number under both disturbance conditions, and the errors approach zero at the 60th iteration, demonstrating favorable convergence characteristics of the algorithm in complex disturbance environments. To quantify the error decay magnitude during iteration, key data at representative iteration steps are extracted from Figure 4 and compiled into Table 5, titled 'Comparison of Maximum Absolute Trajectory Tracking Errors under Different Iteration Counts for Figure 4'. As shown in the table, under both random and periodic disturbances, the maximum absolute tracking errors of $EE_k^{(1)}$ and $EE_k^{(2)}$ both present a remarkable downward trend as the iteration count rises from 3 to 30. Specifically, after 30 iterations, the error of $EE_k^{(1)}$ decreases to 0.0529 under random disturbance and 0.0399 under periodic disturbance, while the errors of $EE_k^{(2)}$ stabilize at 0.0512 and 0.0531 for the two disturbances, respectively, which intuitively reflects the convergence effect and tracking accuracy improvement in the early phase of iterative learning. Further quantitative analysis on the full-cycle simulation data indicates that under random disturbance, the two error components reach steady state at the 25th and 30th iterations with steady-state errors of 0.0429 and 0.0515, respectively. Under sinusoidal periodic disturbance, they converge to steady state at the 24th and 30th iterations, corresponding to steady-state errors of 0.0434 and 0.0507. Comprehensive comparison shows that the convergence speed and steady-state accuracy of the first-degree-of-freedom error are generally consistent under the two disturbances. For the second-degree-of-freedom error, it requires fewer convergence iterations and achieves faster convergence under random disturbance, whereas it yields smaller steady-state error and higher tracking accuracy under sinusoidal periodic disturbance. These results fully validate that the proposed adaptive iterative learning algorithm possesses outstanding adaptive learning capability and strong robust tracking performance against both random and periodic disturbances.

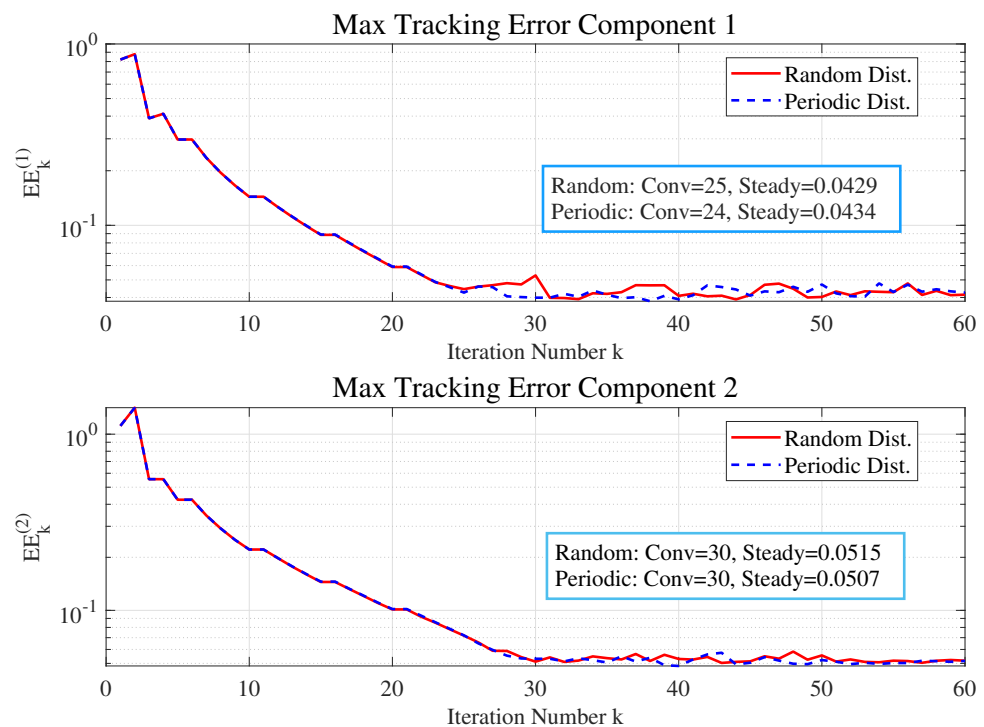


Figure 4. Convergence curves of tracking errors under two different disturbances.

Figures 5 and 6 show the tracking effects of the actual system output on the desired circular trajectory at different iteration numbers under random and sinusoidal pe-

riodic disturbances, respectively. Both figures select the tracking results at iterations $k = [3, 5, 10, 15, 20, 30]$, from which it can be intuitively observed that as the iteration number increases, the actual trajectory of the robot gradually approaches the desired circular trajectory, and the tracking error continuously decreases. Under both disturbances, the algorithm achieves asymptotic tracking of the desired trajectory. When the iteration number reaches 60, the actual system output completely tracks the desired circular trajectory, fully verifying the effectiveness of the algorithm in trajectory tracking accuracy.

Table 5. Comparison of Maximum Absolute Trajectory Tracking Errors under Different Iteration Counts for Figure 4.

Tracking Error	Disturbance Type	Iteration Count					
		3	5	10	15	20	30
$EE_k^{(1)}$	Random	0.3890	0.2973	0.1437	0.0886	0.0589	0.0529
	Periodic	0.3890	0.2973	0.1438	0.0890	0.0592	0.0399
$EE_k^{(2)}$	Random	0.5554	0.4246	0.2215	0.1449	0.1012	0.0512
	Periodic	0.5554	0.4246	0.2215	0.1456	0.1017	0.0531

Figures 7 and 8 respectively capture the control input curves for the first 0.2 s of the last iteration, showing the time-varying characteristics of the two controller output components u_1 and u_2 under the two disturbances. Under the random disturbance environment, the control input exhibits certain random fluctuation characteristics, with its amplitude range reflecting the control energy required by the controller to suppress random disturbances. Under the periodic disturbance, the control input shows a periodic variation pattern consistent with the disturbance frequency, and the overall fluctuation is more stable. Simulation results indicate that the output amplitudes of the controller under the two disturbance conditions are basically similar, and the overall control energy consumption is essentially equivalent, demonstrating that the algorithm can adaptively adjust the control input according to the disturbance characteristics, achieving reasonable energy allocation while ensuring tracking accuracy.

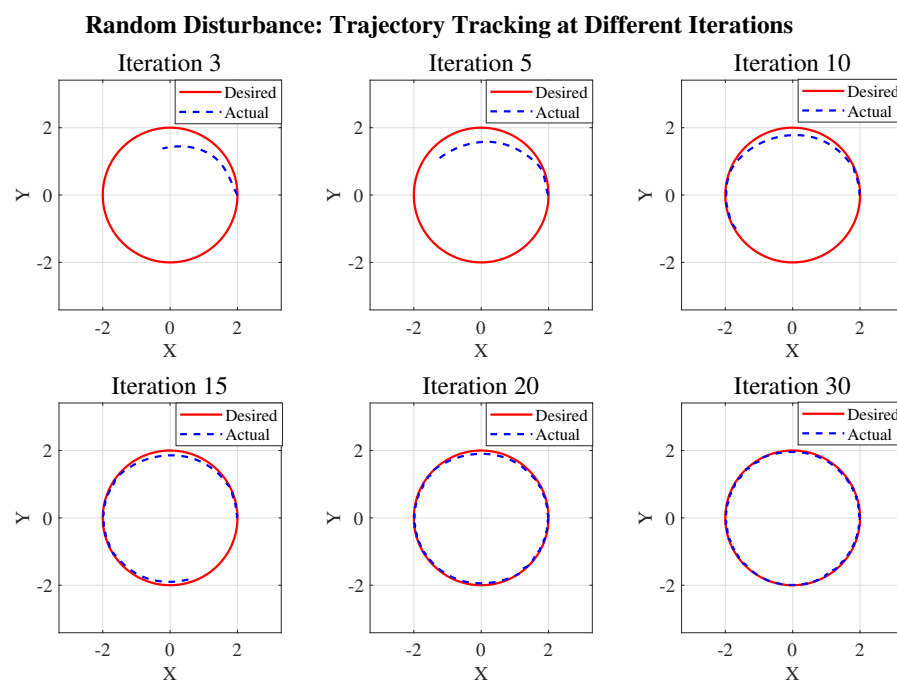


Figure 5. Trajectory tracking effects at multiple iterations under random disturbance.

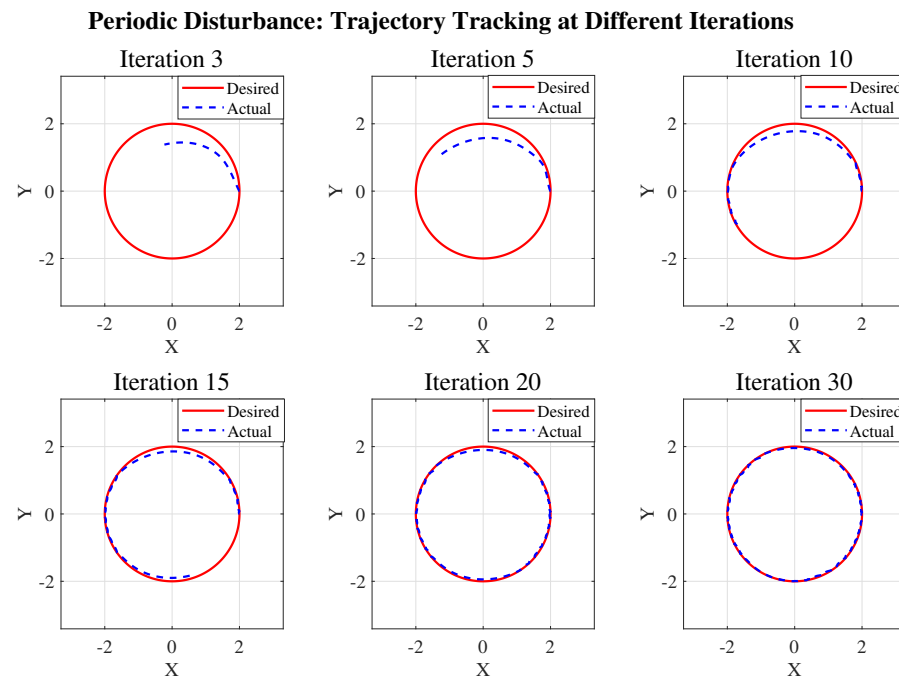


Figure 6. Trajectory tracking effects at multiple iterations under periodic disturbance.

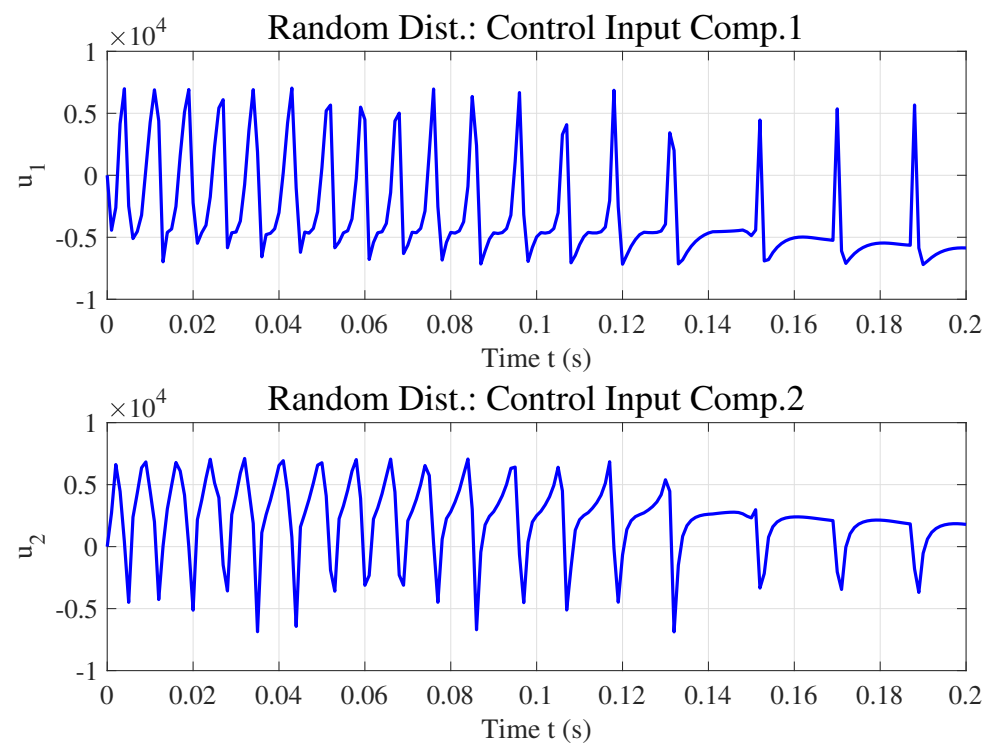


Figure 7. Control input response under random disturbance.

In summary, the proposed adaptive iterative learning algorithm exhibits good convergence performance and tracking accuracy under both typical environments of random disturbance and sinusoidal periodic disturbance, effectively suppressing the influence of external disturbances. It provides a feasible solution for trajectory tracking tasks of underwater robots in complex marine environments.

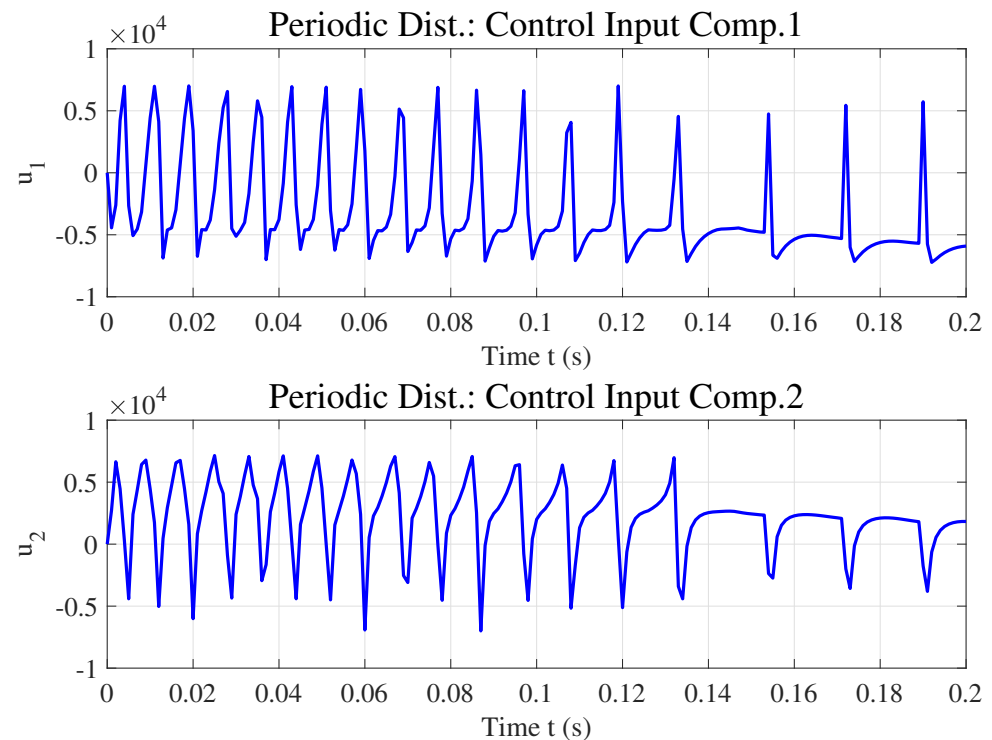


Figure 8. Control input response under periodic disturbance.

6. Conclusions

For multi-input multi-output nonlinear systems with uncertain control gain matrices, when the trial length of the system varies with the number of iterations, the system can still ensure that the tracking error tends to zero by designing an adaptive iterative learning control algorithm. Compared with existing research on adaptive iterative learning control with non-uniform trial lengths, this paper assumes that the control gain matrix of the system is uncertain and proves the convergence of the method by designing a Lyapunov composite energy function. The underwater robotic system has successfully validated the effectiveness of the proposed control strategy through simulation-based verification experiments.

In future research, the adaptive iterative learning control method proposed in this paper can be further expanded step by step from the perspectives of theoretical system improvement, key mechanism analysis, and engineering application verification. First, the basic assumptions adopted in this paper can be further relaxed. The iterative behavior of the system can be investigated when Assumption 1 is not satisfied, such as the cases of unbounded initial errors and incomplete resetting of initial iterative conditions. By taking into account the actual operating conditions of the underwater manipulator, systematic parameter-tuning optimization rules can be established for different scenarios. Meanwhile, fault-tolerant control strategies for unexpected iterative interruptions can be designed to improve the applicability and reliability of the algorithm under complex non-ideal conditions. Second, the physical meaning and engineering selection criteria of the iterative error threshold λ_k can be deeply analyzed, and its influences on control accuracy, iterative stability and practical execution efficiency can be explored so as to provide a clearer theoretical basis for parameter tuning. On this basis, to address the shortcoming that existing theoretical analysis only focuses on iterative convergence, the convergence speed in the iterative domain can be strictly quantitatively characterized in the future. By introducing convergence rate indexes and establishing convergence bound estimations, the theoretical analysis system of convergence performance can be further improved. Furthermore, subse-

quent research will focus on extreme scenarios, such as sudden equipment failure, sensor faults, and strong time-varying flow disturbances, to further verify the robustness of the algorithm under complex practical disturbances. Although the efficiency of the proposed adaptive iterative learning control strategy has been thoroughly validated via high-fidelity numerical simulations of a 2-DOF underwater manipulator, executing physical experiments in real-world sea environments remains an indispensable next step. Our future endeavors will focus on deploying the developed algorithm onto an embedded onboard platform. A staged roadmap has been established, moving from hardware-in-the-loop (HIL) testing to open-water experimental trials, aiming to systematically evaluate the control law under unmodeled hydrodynamics and severe communicative constraints.

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