

Article

Leader-Following Mean-Square Consensus of Stochastic Multi-Agent Systems via Distributed Impulsive Control

Dianqiang Li ^{1,*}, Yijia Song ¹ and Kangkang Sun ^{2,*}¹ School of Mathematical and Physical Sciences, Suqian University, Suqian 223800, China² School of Astronautics, Harbin Institute of Technology, Harbin 150080, China

* Correspondence: 23183@squ.edu.cn (D.L.); kksun@hit.edu.cn (K.S.)

Abstract

This paper investigates the leader-following mean-square consensus of stochastic linear multi-agent systems (MASs) via distributed impulsive control. A distributed impulsive control law relying on local neighbor information and discrete sampled data is designed. By using Lyapunov theory, stochastic analysis, algebraic graph theory and the linear matrix inequality (LMI) technique, sufficient conditions for mean-square consensus are derived. We provide methods for coupling gains and impulsive control gains and deduce the consensus result of deterministic linear MASs as a corollary. Numerical simulations demonstrate the effectiveness of the proposed control.

Keywords: stochastic linear MASs; leader-following consensus; distributed impulsive control; mean-square asymptotic stability; linear matrix inequality

MSC: 93E15, 93A16, 93C30, 93D50

1. Introduction

In recent years, distributed cooperative control of MASs has drawn broad research focus in control and systems, and the consensus theory of deterministic homogeneous MASs has been well developed [1–6]. As a typical research topic in cooperative control, leader-following consensus aims to make all the followers asymptotically track the leader's state via the designed control protocols, which provides theoretical support for formation control, tracking control and a wide range of other engineering applications [7–12].

Among the various control strategies, impulsive control stands out as an efficient discontinuous control method, which only implements control actions at discrete sampling instants rather than continuous time [13–18]. Accordingly, the research on multi-agent systems with impulsive control has advanced rapidly in recent years. We will present a categorized discussion based on the order of agent systems [19–24]. For first-order multi-agent systems, Cao et al. developed a variable structure-based distributed coordinated tracking approach that achieves consensus and swarm tracking without velocity/acceleration measurements under both fixed and switching topologies [19]. Gao et al. investigated the finite-time and fixed-time bipartite consensus over signed graphs by using discontinuous impulsive control strategies [20]. For second-order multi-agent systems, Liu et al. developed a distributed impulsive group consensus strategy for directed topologies that realized subgroup coordination through an impulsive gain design [21]. Wang et al. developed a distributed impulsive consensus protocol using only delayed position information that achieved dynamic average consensus under heterogeneous communication delays



Academic Editors: Quanxin Zhu and Paolo Mercorelli

Received: 2 May 2026

Revised: 24 June 2026

Accepted: 6 July 2026

Published: 10 July 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

via model transformation techniques [22]. For high-order multi-agent systems, Li et al. developed a low-gain distributed impulsive strategy that achieved semi-global consensus through low-gain feedback control and Lyapunov-based parametric discrete-time Riccati equation design [23]. Zhou et al. developed a novel distributed impulsive control approach for leader-following consensus that extends to the output feedback case [24]. Despite these advances, nearly all the aforementioned impulsive control results are established for deterministic MASs, ignoring noise in real environments.

However, in practical engineering environments, MASs are inevitably disturbed by stochastic factors, such as environmental noise, measurement errors and communication packet loss [25–29]. These stochastic factors may affect the consensus performance of systems. Therefore, it is of great theoretical significance and practical value to study the leader-following mean-square consensus of stochastic MASs. At present, some progress has been made in the leader-following mean-square consensus control of stochastic MASs [30–33]. By stochastic analysis, algebraic graph theory and a common Lyapunov function, Ma et al. designed consensus protocols with time-varying gains that ensured strong mean-square consensus under fixed topologies [30]. Li et al. presented necessary and sufficient conditions for bounded distributed mean-square tracking of second-order agents under directed network topologies [31]. By using the stochastic stability theorem and the generalized algebraic Riccati equation, Zong et al. studied stochastic consensus of continuous-time linear homogeneous multi-agent systems with time delays [32]. Li et al. proposed admissible distributed observation and cooperative output feedback control strategies that ensured mean-square bounded output tracking [33]. Apart from continuous-time schemes, discrete strategies, including sampled-data control, event-triggered transmission and impulsive control, have been exploited to reduce communication cost. Sun et al. investigated sampled-data consensus subject to successive packet losses and random sampling [34]; Li et al. addressed periodic event-triggered consensus under switching topologies for stochastic MASs [35]; Liu et al. developed event-triggered impulsive control against external disturbances for nonlinear stochastic systems [36]. In addition, Li et al. studied leader-following full-state consensus for feedforward nonlinear stochastic multi-agent systems under fixed directed topologies with stealthy actuator deception attacks, constructed a novel attack model driven by stochastic inverse dynamics, and proposed a distributed output-feedback control algorithm using only neighbor output information to realize bounded consensus in probability [37]. Zhang and Che designed a fully distributed fuzzy secure control protocol resisting asynchronous DoS attacks and derived sum-of-squares-based mean-square bounded consensus criteria [38]. Xiao G et al. constructed a layered control framework with a settling-time monitoring subsystem and designed adaptive fault-tolerant predefined-time protocols to realize optimal-time consensus [39]. The above works focus on continuous-time control or simple impulsive control. To fill the research gaps, this paper investigates the leader-following mean-square consensus problem of stochastic linear MASs via distributed impulsive control. The main contributions of this paper are summarized as follows.

(i) Compared with the deterministic framework in [24], we extend the analysis to stochastic multi-agent systems and derive sufficient conditions for leader-following mean-square consensus under distributed impulsive control. Moreover, we present explicit design procedures for both coupling gain and impulsive control gain, which can provide a certain reference for the design of impulsive control laws for stochastic systems. Compared with Ref. [12], which adopts event-triggered impulsive control to reduce redundant communications via preset event thresholds yet requires continuous real-time monitoring of agent states to judge triggering conditions, our scheme only executes information interaction

and control updates at discrete impulse time instants without continuous state detection throughout the whole process, thus achieving lower communication overhead.

(ii) By combining Lyapunov stability theory, stochastic analysis, algebraic graph theory and the LMI technique, sufficient conditions for leader-following mean-square consensus are derived, and the stochastic perturbation term is explicitly embedded into the stability constraints. The proposed distributed impulsive control is executed solely at discrete sampling instants, which can mitigate the communication overhead and energy consumption of the system to a certain extent and better conforms to practical engineering scenarios with constrained resources.

(iii) The derived consensus conditions possess good structural generality: when the stochastic disturbance matrix is set to zero, the obtained theoretical results can naturally degenerate into the consensus criteria for deterministic multi-agent systems, which establishes a unified theoretical framework bridging deterministic impulsive consensus analysis and stochastic impulsive consensus research.

The rest of this paper is organized as follows. Section 2 formulates the leader-following stochastic linear MAS model, introduces the basic concepts of algebraic graph theory and gives necessary assumptions and lemmas. Section 3 presents the main results, including the sufficient conditions for mean-square consensus and the design of distributed impulsive control gains. Section 4 gives a numerical simulation to demonstrate the effectiveness of the proposed control laws. Section 5 concludes the paper.

Notation: The symbols $\mathbb{R}$ and $\mathbb{R}_+$ denote real and non-negative numbers, respectively; $\mathbb{R}^n$ denotes the set of n -dimensional real column vectors; $\mathbb{R}^{m \times n}$ denotes the set of $m \times n$ dimensional real matrices; $\mathbf{0}_{m \times n}$ represents the $m \times n$ dimensional zero matrix; I_m denotes the $m \times m$ dimensional identity matrix; $\text{diag}\{A_1, \dots, A_N\}$ represents the block diagonal matrix, with entries being $A_1, \dots, A_N$. For a given vector or matrix X , X^T denotes its transpose, $\text{Tr}(X)$ denotes its trace, and $\|X\|$ represents its 2-norm. For a given real matrix $A \in \mathbb{R}^{n \times n}$, $\sigma(A)$ represents the spectrum of A , and $\lambda_i(A)$ ($i = 1, \dots, n$) represents the i th eigenvalue of A arranged in order of ascending real part. For a given real symmetric matrix $B \in \mathbb{R}^{n \times n}$, $\lambda_{\min}(B)$ is the minimum eigenvalue of B , and $\lambda_{\max}(B)$ is the maximum eigenvalue of B . $A > 0$ (or $A \geq 0$) denotes that A is positive definite (or positive semi-definite), and $A < 0$ (or $A \leq 0$) denotes that A is negative definite (or negative semi-definite). For two real symmetric matrices A and B , $A > B$ (or $A \geq B$) denotes that $A - B$ is positive definite (or $A - B$ is positive semi-definite), and $A < B$ (or $A \leq B$) denotes that $A - B$ is negative definite (or $A - B$ is negative semi-definite). For two matrices C and D , $C \otimes D$ denotes their Kronecker product. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq t_0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t \geq t_0}$ satisfying the usual conditions. It is right continuous and increasing while $\mathcal{F}_0$ contains all the $\mathbb{P}$ -null sets; $w(t) = (w_1(t), \dots, w_m(t))^T$ denotes an m -dimensional standard Brownian motion defined in $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq t_0}, \mathbb{P})$. For a given random variable X , the mathematical expectation of X is denoted by $\mathbb{E}[X]$. The symbol $L^p([a, b]; \mathbb{R}^d)$ denotes the family of $\mathbb{R}^d$ -valued $\mathcal{F}_t$ -adapted processes $\{f(t)\}_{a \leq t \leq b}$ such that $\int_a^b |f(t)|^p dt < \infty$ a.s.; $\mathcal{L}^p(\mathbb{R}_+; \mathbb{R}^d)$ denotes the family of processes $\{f(t)\}_{t \geq 0}$ such that, for every $T > 0$, $\{f(t)\}_{0 \leq t \leq T} \in L^p([0, T]; \mathbb{R}^d)$; $\mathcal{C}^{2,1}(\mathbb{R}^d \times \mathbb{R}_+; \mathbb{R})$ denotes the family of all real-valued functions $V(x, t)$ defined in $\mathbb{R}^d \times \mathbb{R}_+$, which are continuously twice differentiable in $x \in \mathbb{R}^d$ and once differentiable in $t \in \mathbb{R}_+$. $\delta(t - t_k)$ is the Dirac delta function, and $\{t_k\}_{k=1}^\infty$ is the impulsive time sequence satisfying $0 = t_0 < t_1 < \dots < t_k$ and $\lim_{k \rightarrow \infty} t_k = \infty$.

2. Problem Formulation

Consider a leader-following stochastic multi-agent system consisting of a leader and N followers. The dynamics of the i -th follower ($i = 1, 2, \dots, N$) is described as

$$dx_i(t) = (Ax_i(t) + Bu_i(t))dt + Dx_i(t)dw(t), \quad (1)$$

where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^m$ are the state and control input of the i -th follower, respectively; $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, and $D \in \mathbb{R}^{n \times n}$ are constant matrices; $w(t)$ is a standard one-dimensional Brownian motion.

The dynamics of the leader (labeled as agent 0) is given by

$$dx_0(t) = Ax_0(t)dt + Dx_0(t)dw(t), \quad (2)$$

where $x_0(t) \in \mathbb{R}^n$ is the state of the leader.

Remark 1. The stochastic diffusion term $Dx_i(t)dw(t)$ adopted in system (1) corresponds to state-dependent multiplicative noise, which is essentially different from additive measurement noise only imposed on system output. Such multiplicative noise is utilized to characterize random communication gain fluctuations and time-varying random parameter uncertainties widely existing in practical multi-agent networks, where the disturbance intensity varies proportionally with the agent's real-time state. This modeling framework based on Itô stochastic differential equations has been extensively applied in the research of stochastic multi-agent consensus, unmanned aerial vehicle swarms and wireless sensor networks; see [26,32,33] and references therein for detailed physical interpretations and relevant theoretical studies.

We use $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$ to represent a weighted graph formed by the leader and N followers and use $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ to represent the subgraph formed by N followers, where the set of nodes $\bar{\mathcal{V}} = \{0, 1, 2, \dots, N\}$ and $\mathcal{V} = \bar{\mathcal{V}} \setminus \{0\}$ and the set of edges $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Denote the neighbors of the i th follower by $\mathcal{N}_i$. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$, $\bar{\mathcal{A}} = \begin{bmatrix} 0 & \mathbf{0}_{1 \times N} \\ a_0 & \mathcal{A} \end{bmatrix} \in \mathbb{R}^{(N+1) \times (N+1)}$, and, if $j \in \mathcal{N}_i$, then $a_{ij} = 1$. Otherwise, $a_{ij} = 0$; $a = [a_{10}, a_{20}, \dots, a_{N0}]^T$, and, if $0 \in \mathcal{N}_i$, then $a_{i0} = 1$. Otherwise, $a_{i0} = 0$. The Laplacian matrix of $\bar{\mathcal{G}}$ is given by $\bar{\mathcal{L}} = \begin{bmatrix} 0 & \mathbf{0}_{1 \times N} \\ -a & H \end{bmatrix} \in \mathbb{R}^{(N+1) \times (N+1)}$, where $H = \mathcal{L} + F$, $\mathcal{L}$ is the Laplacian matrix of $\mathcal{G}$ and $F = \text{diag}(a_{10}, a_{20}, \dots, a_{N0})$.

2.1. Distributed Impulsive Control Law

We adopt the distributed impulsive control law for each follower, which only uses the sampled state information at discrete impulsive instants t_k :

$$u_i(t) = cK \sum_{k=1}^{\infty} \sum_{j=0}^N a_{ij}(x_i(t) - x_j(t))\delta(t - t_k), \quad (3)$$

where $c > 0$ is the coupling gain; $K \in \mathbb{R}^{m \times n}$ is the impulsive control gain to be designed; $\delta(\cdot)$ is the Dirac delta function, which characterizes the impulsive input at $t = t_k$. In addition, the impulsive time sequence $\{t_k\}_{k=1}^{\infty}$ satisfies $0 = t_0 < t_1 < \dots < t_k < \dots$, $\lim_{k \rightarrow \infty} t_k = \infty$. It is assumed that the impulsive interval is bounded; i.e., there exist positive scalars $0 < \tau_0 \leq \tau_1 < \infty$ such that $\tau_0 \leq t_k - t_{k-1} \leq \tau_1$. For simplicity, we use the notation $t_k \in S(\tau_0, \tau_1)$ to denote the class of impulsive time sequences t_k satisfying the above assumption.

2.2. Definitions

In this section, we present the definitions used in this paper.

Definition 1 ([40], Itô's formula). Let $x(t)$ be a d -dimensional Itô process on $t \geq 0$ with the stochastic differential

$$dx(t) = f(t)dt + g(t)dw(t),$$

where $f \in \mathcal{L}^1(\mathbb{R}_+; \mathbb{R}^d)$ and $g \in \mathcal{L}^2(\mathbb{R}_+; \mathbb{R}^{d \times m})$. Let $V \in C^{2,1}(\mathbb{R}^d \times \mathbb{R}_+; \mathbb{R})$. Then $V(x(t), t)$ is also an Itô process with the stochastic differential given by

$$dV(x(t), t) = [V_t(x(t), t) + V_x(x(t), t)f(t) + \frac{1}{2}\text{Tr}(g^T(t)V_{xx}(x(t), t)g(t))]dt + V_x(x(t), t)g(t)dw(t).$$

Definition 2. The leader-following stochastic MASs (1) and (2) under the distributed control law (3) are said to achieve leader-following mean-square consensus if, for any given initial values $x_0(0)$, $x_i(0)$, $i = 1, \dots, N$, the following equality holds:

$$\lim_{t \rightarrow \infty} \mathbb{E}[\|x_i(t) - x_0(t)\|^2] = 0, \quad i = 1, 2, \dots, N.$$

2.3. Assumption

In this section, we give the following assumption on the communication graph to achieve leader-following mean-square consensus.

Assumption 1. The digraph $\bar{\mathcal{G}}$ contains a spanning tree, and the graph $\mathcal{G}$ is undirected.

Remark 2. By this assumption, we know that the leader's information can be transmitted to its adjacent followers through the communication network, which is a necessary condition for leader-following consensus. What is more, we obtain that $\mathcal{L} + F$ is a positive definite matrix. Hence, there exists a unitary matrix Φ such that $\Phi^T(\mathcal{L} + F)\Phi = \text{diag}(\lambda_1(\mathcal{L} + F), \dots, \lambda_N(\mathcal{L} + F)) = \text{diag}(\lambda_1(H), \dots, \lambda_N(H)) \triangleq \Lambda$.

Remark 3. In subsequent comparative simulations, we further design a weakly connected topology that does not contain a directed spanning tree as a contrasting case to reveal the necessity of this spanning tree assumption for consensus achievement and illustrate how graph connectivity affects system convergence performance.

3. Main Results

Theorem 1. Suppose that Assumption 1 is satisfied, and the impulsive time sequence $\{t_k\}_{k \in \mathbb{N}}$ is such that $t_k \in S(\tau_0, \tau_1)$ for each integer k . If there exist a positive definite matrix P , a positive real constant α , and a real number β with $0 < \beta < e^{\alpha\tau_1} - 1$ such that

$$\begin{bmatrix} PA + A^T P + \alpha P & DP \\ PD^T & -P \end{bmatrix} < 0, \quad (4)$$

$$\begin{bmatrix} -\beta P & PB \\ B^T P & -(I_m + B^T PB) \end{bmatrix} < 0, \quad (5)$$

then, under the distributed impulsive control law (3), the leader-following mean-square consensus is achieved, where the coupling gain is chosen as $c \in [0, \frac{1+\sqrt{2}}{\lambda_N(H)}]$, and the impulsive control gain is designed as $K = -(I_m + B^T PB)^{-1} B^T P$.

Proof. Denote $e_i(t) = x_i(t) - x_0(t)$, $i = 1, 2, \dots, N$.

By (1), (2), (3) and the definition of $e_i(t)$, we get

$$\begin{aligned} de_i(t) &= dx_i(t) - dx_0(t) \\ &= (Ax_i(t) + Bu_i(t))dt + Dx_i(t)dw(t) - Ax_0(t)dt - Dx_0(t)dw(t) \\ &= A(x_i(t) - x_0(t))dt + D(x_i(t) - x_0(t))dw(t) + Bu_i(t)dt \\ &= Ae_i(t)dt + De_i(t)dw(t) + Bu_i(t)dt. \end{aligned} \quad (6)$$

By (3), we obtain

$$u_i(t) = \begin{cases} 0, & t \in (t_{k-1}, t_k), \\ cK \sum_{j \in \mathcal{N}_i} a_{ij}(x_j(t_k) - x_i(t_k)), & t = t_k \end{cases} \quad (7)$$

When $t \neq t_k$, that is, $t \in (t_{k-1}, t_k)$, by (6) and (7), we have

$$de_i(t) = Ae_i(t)dt + De_i(t)dw(t). \quad (8)$$

At the impulse instant t_k , we integrate both sides of Equation (6) over the infinitesimal interval $[t_k^-, t_k^+]$:

$$\int_{t_k^-}^{t_k^+} de_i(t) = \int_{t_k^-}^{t_k^+} Ae_i(t)dt + \int_{t_k^-}^{t_k^+} De_i(t)dw + \int_{t_k^-}^{t_k^+} Bu_i(t)dt. \quad (9)$$

Over an infinitesimal time interval, the ordinary Riemann integral $\int_{t_k^-}^{t_k^+} Ae_i(t)dt$ and the Itô stochastic integral $\int_{t_k^-}^{t_k^+} De_i(t)dw(t)$ both converge to zero.

The left-hand side yields the state jump of the tracking error:

$$\int_{t_k^-}^{t_k^+} de_i(t) = e_i(t_k^+) - e_i(t_k^-) \triangleq \Delta e_i(t_k). \quad (10)$$

By (7), (9) and (10), we obtain

$$\Delta e_i(t_k) = cBK \sum_{j \in \mathcal{N}_i} a_{ij}(x_i(t_k^-) - x_j(t_k^-)).$$

Using the relation $e_i(t) - e_j(t) = x_i(t) - x_j(t)$, the above expression can be rearranged as

$$\Delta e_i(t_k) = e_i(t_k) - e_i(t_k^-) = cBK \sum_{j \in \mathcal{N}_i} a_{ij}(e_i(t_k^-) - e_j(t_k^-)). \quad (11)$$

By (6), (8) and (11), we obtain

$$\begin{cases} de_i(t) = Ae_i(t)dt + De_i(t)dw(t), & t \neq t_k, \\ \Delta e_i(t_k) = cBK \sum_{j \in \mathcal{N}_i} a_{ij}(e_i(t_k^-) - e_j(t_k^-)), & t = t_k. \end{cases} \quad (12)$$

Define $e(t) = (e_1(t), e_2(t), \dots, e_N(t)) \in \mathbb{R}^{Nn}$ and $\Delta e(t) = (\Delta e_1(t), \Delta e_2(t), \dots, \Delta e_N(t)) \in \mathbb{R}^{Nn}$. By (12) and the definition of $e(t)$ and $\Delta e(t)$, we have

$$\begin{cases} de(t) = (I_N \otimes A)e(t)dt + (I_N \otimes D)e(t)dw(t), & t \neq t_k, \\ \Delta e(t) = (I_{Nn} + cH \otimes BK)e(t^-), & t = t_k. \end{cases} \quad (13)$$

Denote $\bar{e}(t) = (\Phi^T \otimes I_n)e(t)$. By the definition of $\bar{e}(t)$ and the above equation, we obtain

$$\begin{cases} d\bar{e}(t) = (I_N \otimes A)\bar{e}(t)dt + (I_N \otimes D)\bar{e}(t)dw(t), & t \neq t_k, \\ \Delta \bar{e}(t) = (I_{Nn} + c\Phi^T H \Phi \otimes BK)\bar{e}(t^-), & t = t_k. \end{cases} \quad (14)$$

Construct the following stochastic Lyapunov function:

$$V(t, \bar{e}(t)) = \bar{e}^T(I_N \otimes P)\bar{e}(t).$$

It satisfies the norm bound:

$$\lambda_{\min}(P)\mathbb{E}\left[\|\bar{e}(t)\|^2\right] \leq \mathbb{E}[V(t, \bar{e}(t))] \leq \lambda_{\max}(P)\mathbb{E}\left[\|\bar{e}(t)\|^2\right]. \quad (15)$$

When $t \neq t_k$, by Itô's formula and (14), we get

$$\begin{aligned} dV(t) &= 2\bar{e}^T(t)(I_N \otimes P)(I_N \otimes A)\bar{e}(t)dt \\ &\quad + 2\bar{e}^T(t)(I_N \otimes P)(I_N \otimes D)\bar{e}(t)dw(t) \\ &\quad + \bar{e}^T(t)\left[I_N \otimes (D^T P D)\right]\bar{e}(t)dt, \\ &= \bar{e}^T(t)\left[I_N \otimes (PA + A^T P)\right]\bar{e}(t)dt \\ &\quad + \bar{e}^T(t)\left[I_N \otimes (D^T P D)\right]\bar{e}(t)dt, \\ &\quad + 2\bar{e}^T(t)(I_N \otimes P)(I_N \otimes D)\bar{e}(t)dw(t) \\ &= \bar{e}^T(t)\left[\left(I_N \otimes (PA + A^T P)\right) + I_N \otimes (D^T P D)\right]\bar{e}(t)dt \\ &\quad + 2\bar{e}^T(t)(I_N \otimes P)(I_N \otimes D)\bar{e}(t)dw(t). \end{aligned}$$

Integrating both sides of the above inequality from 0 to t and taking the mathematical expectation, we obtain

$$\begin{aligned} \mathbb{E}[V(t)] &\leq \mathbb{E}[V(0)] + \mathbb{E}\left[\int_0^t \bar{e}^T(s)\left[\left(I_N \otimes (PA + A^T P)\right) + I_N \otimes (D^T P D)\right]\bar{e}(s)ds\right] \\ &= \mathbb{E}[V(0)] + \mathbb{E}\left[\int_0^t \bar{e}^T(s)\left[I_N \otimes (PA + A^T P + D^T P D)\right]\bar{e}(s)ds\right]. \end{aligned}$$

By (4) and Schur lemma, we obtain

$$PA + A^T P + D^T P D < -\alpha P,$$

Substituting the above matrix inequality into the integral, we obtain

$$\mathbb{E}[V(t)] \leq \mathbb{E}[V(0)] - \alpha \int_0^t \mathbb{E}[V(s)]ds.$$

Differentiating both sides with respect to t gives the differential estimate

$$\frac{d}{dt}\mathbb{E}[V(t)] \leq -\alpha\mathbb{E}[V(t)].$$

Recall the bounded impulsive interval condition $t_k \in S(\tau_0, \tau_1)$, i.e.,

$$\tau_0 \leq t_k - t_{k-1} \leq \tau_1, \quad \forall k \in \mathbb{Z}_+.$$

Solve the differential inequality $\frac{d}{dt}\mathbb{E}[V(t)] \leq -\alpha\mathbb{E}[V(t)]$ over $[t_{k-1}, t]$:

$$\mathbb{E}[V(t)] \leq \mathbb{E}[V(t_{k-1})]e^{-\alpha(t-t_{k-1})}.$$

Set $t = t_k^-$ (the instant right before impulse), and use the upper bound $t_k - t_{k-1} \leq \tau_1$:

$$\mathbb{E}[V(t_k^-)] \leq \mathbb{E}[V(t_{k-1})]e^{-\alpha\tau_1}. \quad (16)$$

Define the jump of V at t_k as:

$$\Delta V(t_k) = V(t_k, \bar{e}(t_k)) - V(t_k^-, \bar{e}(t_k^-)).$$

By substituting $\bar{e}(t_k) = (I_N + cH \otimes BK)\bar{e}(t_k^-)$ into the above equality, we get

$$\begin{aligned}
\Delta V(t_k) &= \bar{e}^T(t_k)(I_N \otimes P)\bar{e}(t_k) - \bar{e}^T(t_k^-)(I_N \otimes P)\bar{e}(t_k^-) \\
&= [(I_{Nn} + c\Phi^T H\Phi \otimes BK)\bar{e}(t_k^-)]^T (I_N \otimes P) [(I_{Nn} + c\Phi^T H\Phi \otimes BK)\bar{e}(t_k^-)] \\
&\quad - \bar{e}^T(t_k^-)(I_N \otimes P)\bar{e}(t_k^-) \\
&= \bar{e}^T(t_k^-) \left[(I_{Nn} + c\Phi^T H\Phi \otimes BK)^T (I_N \otimes P) (I_{Nn} + c\Phi^T H\Phi \otimes BK) - (I_N \otimes P) \right] \bar{e}(t_k^-) \\
&\triangleq \bar{e}^T(t_k^-) M \bar{e}(t_k^-),
\end{aligned} \tag{17}$$

where $M = (I_{Nn} + c\Phi^T H\Phi \otimes BK)^T (I_N \otimes P) (I_{Nn} + c\Phi^T H\Phi \otimes BK) - I_N \otimes P$.

By (5) and Schur lemma, we obtain

$$-\beta P + PB(I_m + B^T PB)^{-1} B^T P < 0 \tag{18}$$

By Assumption 1, we know that there exists a unitary matrix Φ such that $\Phi^T(\mathcal{L} + F)\Phi = \Phi^T H\Phi = \text{diag}(\lambda_1(H), \dots, \lambda_N(H)) \triangleq \Lambda$.

By Assumption 1, the definition of M , the impulsive control gain $K = -(I_m + B^T PB)^{-1} B^T P$, the designed range of coupling gain c and (18), we have

$$\begin{aligned}
M &= (I_{Nn} + c\Phi^T H\Phi \otimes BK)^T (I_N \otimes P) (I_{Nn} + c\Phi^T H\Phi \otimes BK) - I_N \otimes P \\
&= (I_N \otimes I_n + c\Lambda \otimes BK)^T (I_N \otimes P) (I_N \otimes I_n + c\Lambda \otimes BK) - I_N \otimes P \\
&= I_N \otimes P + c\Lambda \otimes [(BK)^T P + PBK] + c^2 \Lambda^2 \otimes [(BK)^T PBK] - I_N \otimes P \\
&= c\Lambda \otimes [(BK)^T P + PBK] + c^2 \Lambda^2 \otimes [(BK)^T PBK] \\
&= -2c\Lambda \otimes [PB(I_m + B^T PB)^{-1} B^T P] + c^2 \Lambda^2 \otimes [K^T B^T PBK] \\
&= -2c\Lambda \otimes [PB(I_m + B^T PB)^{-1} B^T P] \\
&\quad + c^2 \Lambda^2 \otimes \left[\left(-PB(I_m + B^T PB)^{-1} \right) B^T PB \left(-(I_m + B^T PB)^{-1} B^T P \right) \right] \\
&= -2c\Lambda \otimes [PB(I_m + B^T PB)^{-1} B^T P] \\
&\quad + c^2 \Lambda^2 \otimes \left[\left(PB(I_m + B^T PB)^{-1} \right) B^T PB \left((I_m + B^T PB)^{-1} B^T P \right) \right] \\
&\leq -2c\Lambda \otimes [PB(I_m + B^T PB)^{-1} B^T P] + c^2 \Lambda^2 \otimes [PB(I_m + B^T PB)^{-1} B^T P] \\
&= (-2c\Lambda + c^2 \Lambda^2) \otimes [PB(I_m + B^T PB)^{-1} B^T P] \\
&\leq I_N \otimes [PB(I_m + B^T PB)^{-1} B^T P] \\
&\leq \beta(I_N \otimes P)
\end{aligned} \tag{19}$$

By the above equality and (17), we obtain

$$\begin{aligned}
\Delta V(t_k) &= \bar{e}^T(t_k^-) M \bar{e}(t_k^-) \\
&< \beta \bar{e}^T(t_k^-)(I_N \otimes P)\bar{e}(t_k^-) \\
&= \beta V(t_k^-).
\end{aligned} \tag{20}$$

Take expectation on both sides:

$$\mathbb{E}[V(t_k)] - \mathbb{E}[V(t_k^-)] \leq \beta \mathbb{E}[V(t_k^-)],$$

which simplifies to

$$\mathbb{E}[V(t_k)] \leq (1 + \beta) \mathbb{E}[V(t_k^-)]. \tag{21}$$

By (16) and (21), we have

$$\mathbb{E}[V(t_k)] \leq (1 + \beta) e^{-\alpha \tau_1} \mathbb{E}[V(t_{k-1})].$$

Let $\rho = (1 + \beta)e^{-\alpha\tau_1}$. Since $0 < \beta < e^{\alpha\tau_1} - 1$, we have $0 < \rho < 1$. Iterate the recursive inequality from $k = 1$ to arbitrary integer k :

$$\mathbb{E}[V(t_k)] \leq \rho^k \mathbb{E}[V(0)].$$

From the lower bound of impulsive interval $t_k - t_{k-1} \geq \tau_0$, we have $t_k \geq k\tau_0 \Rightarrow k \leq \frac{t_k}{\tau_0}$. Substitute it into the above formula:

$$\mathbb{E}[V(t_k)] \leq \mathbb{E}[V(0)]e^{\frac{\ln \rho}{\tau_0} t_k}.$$

For arbitrary $t > 0$, there exists integer k such that $t \in (t_k, t_{k+1})$. Using the continuous decay property again:

$$\begin{aligned} \mathbb{E}[V(t)] &\leq \mathbb{E}[V(t_k)]e^{-\alpha(t-t_k)} \\ &\leq \mathbb{E}[V(0)]e^{\frac{\ln \rho}{\tau_0} t_k - \alpha(t-t_k)} \\ &\leq \mathbb{E}[V(0)]e^{-\gamma t}, \end{aligned}$$

where $\gamma = \min\left\{\alpha, -\frac{\ln \rho}{\tau_0}\right\} > 0$. Taking limit as $t \rightarrow \infty$, we immediately obtain

$$\lim_{t \rightarrow \infty} \mathbb{E}[V(t)] = 0.$$

By (15) and the above equality, we have

$$\lim_{t \rightarrow \infty} \mathbb{E}[\|\bar{e}(t)\|^2] = 0.$$

Since $\bar{e}(t) = (\Phi^\top \otimes I_n)e(t)$ and Φ is unitary (norm-preserving),

$$\lim_{t \rightarrow \infty} \mathbb{E}[\|e(t)\|^2] = 0,$$

which completes the proof. $\square$

Remark 4. Such intrinsic conservatism stems from the inequality scaling transformations exploited to resolve the coupling between stochastic diffusion dynamics and impulsive jump evolutions. Hence, leader-following mean-square consensus is still guaranteed even if the formulated LMIs admit no feasible solutions. The scalar tuning parameter α characterizes the upper bound of the exponential decay rate of tracking error trajectories within inter-impulse intervals. A larger α imposes more restrictive linear matrix inequality constraints and compresses the feasible domain of control gains, whereas a moderately reduced α improves the solvability of matrix inequalities at the cost of slower convergence rate. Furthermore, the feasible parameter region is highly sensitive to two factors: graph algebraic connectivity and multiplicative noise intensity. Specifically, low algebraic connectivity impairs information interchange among agents and narrows the admissible gain interval; stronger state-dependent multiplicative noise amplifies stochastic divergence propensity, raises the conservatism level of the stability criteria, and further shrinks the feasible set of coupling strengths and impulsive feedback gains.

Remark 5. In contrast to the deterministic framework established in [24], this work incorporates state-dependent stochastic perturbations, transforming the original ordinary differential system into an Itô-type stochastic differential system. Conventional deterministic Lyapunov-based stability analysis fails to accommodate the intricate coupling arising between stochastic diffusion dynamics and impulsive jump behaviors, which substantially complicates the rigorous stability derivation. To address this challenge, we resort to stochastic Lyapunov functional theory and stochastic calculus,

together with linear matrix inequality techniques, to explicitly encapsulate stochastic disturbances within the derived stability criteria for establishing leader-following mean-square consensus. As opposed to the continuous-time control strategy reported in [32], the proposed distributed impulsive protocol only triggers state information exchange and control actuation at discrete impulse instants, while no inter-agent data interaction is required over inter-impulse intervals. As concluded in the existing impulsive control literature [41], such inherent discrete execution property effectively reduces data transmission frequency and actuator operation times, alleviating network communication overhead and energy expenditure for resource-limited multi-agent nodes.

If the stochastic perturbation matrix $D = 0$, the system reduces to a deterministic linear multi-agent system.

Corollary 1. Suppose that Assumption 1 is satisfied, and the impulsive time sequence $\{t_k\}_{k \in \mathbb{N}}$ is such that $t_k \in S(\tau_0, \tau_1)$ for each integer k . If there exist a positive definite matrix P , a positive real constant α , and a real number β with $0 < \beta < e^{\alpha\tau_1} - 1$ such that

$$\begin{cases} PA + A^T P + \alpha P < 0 \\ \begin{bmatrix} -\beta P & PB \\ B^T P & -(I_m + B^T P B) \end{bmatrix} < 0 \end{cases}$$

hold, then, under the distributed impulsive control law (3), the leader-following mean-square consensus is achieved, where the coupling gain is chosen as $c \in \left[0, \frac{1+\sqrt{2}}{\lambda_N(H)}\right]$ and the impulsive control gain is designed as $K = -(I_m + B^T P B)^{-1} B^T P$.

Proof. Set $D = 0$. Similar to the proof of Theorem 1, we know that the leader-following consensus of the linear MASs is achieved. $\square$

4. Numerical Simulation

In this section, we provide numerical simulations to verify the correctness and effectiveness of the distributed impulsive control protocol derived in Theorem 1. We consider a leader-following stochastic multi-agent system consisting of one leader (agent 0) and three followers (agents 1, 2, and 3). The dynamics of each follower and the leader are described by (1) and (2), respectively, with system matrices

$$A = \begin{bmatrix} 0 & 1 \\ -4 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix},$$

where $x_i(t) \in \mathbb{R}^2$ is the agent state, and the distributed impulsive control law is defined in (3). The communication topology $\bar{\mathcal{G}}$ for the base example is illustrated in Figure 1 with adjacency matrix

$$\bar{\mathcal{A}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Compute the Laplacian matrix $H = L + F$, whose maximum eigenvalue satisfies $\lambda_3 = \frac{3+\sqrt{5}}{2}$. As the coupling gain satisfies $c \in \left[0, \frac{1+\sqrt{2}}{\lambda_3(H)}\right]$, we select $c = 0.7$. Initial states of all agents are set as

$$x_0(0) = \begin{bmatrix} 1.0 \\ -0.5 \end{bmatrix}, \quad x_1(0) = \begin{bmatrix} -1.0 \\ 0.5 \end{bmatrix}, \quad x_2(0) = \begin{bmatrix} 0.8 \\ -0.3 \end{bmatrix}, \quad x_3(0) = \begin{bmatrix} -0.6 \\ 0.4 \end{bmatrix}.$$

Consistent with the assumption in Theorem 1, the impulsive sequence satisfies bounded interval $t_k \in S(\tau_0, \tau_1)$ with $\tau_0 = 0.05$ s, $\tau_1 = 0.1$ s. We preset tuning parameter $\alpha = 0.2$

and select $\beta = 0.01$, satisfying $0 < \beta < e^{\alpha\tau_1} - 1$. By jointly solving the coupled matrix inequalities (4) and (5) in Theorem 1, the feasible positive definite matrix is obtained as $P = \begin{bmatrix} 1.52 & 0.08 \\ 0.08 & 1.37 \end{bmatrix}$. According to the designed form of the impulsive gain matrix, we deduce that $K = -(I_2 + B^T P B)^{-1} B^T P = \begin{bmatrix} -0.601 & -0.030 \\ -0.011 & -0.579 \end{bmatrix}$. By Figure 2, we know that, under the distributed control law (3), the leader-following mean-square consensus of the stochastic linear MASs is achieved.

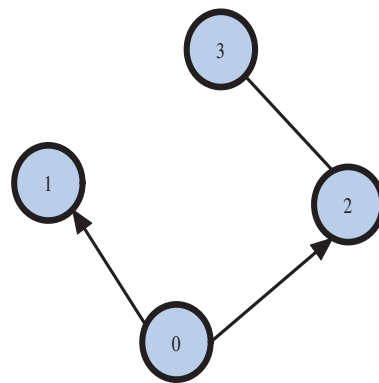


Figure 1. The communication topology graph.

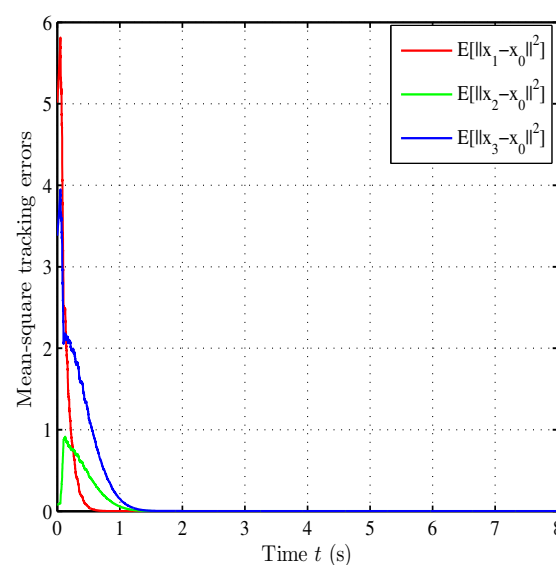


Figure 2. Mean-square tracking errors.

In the following, we will design multiple comparative simulation cases to investigate the effects of the tuning parameter α , multiplicative noise intensity, and graph algebraic connectivity on consensus performance.

1. Influence of tuning parameter α (Figure 3)

The scalar tuning parameter α introduced in LMI (4) imposes an upper bound on the exponential decay rate of the constructed stochastic Lyapunov functional. Three representative values $\alpha = 0.1, 0.2, 0.5$ are selected for comparative numerical experiments, with the corresponding mean-square tracking error responses illustrated in Figure 3. A larger α enforces more stringent stability constraints within the linear matrix inequalities, which narrows the feasible domain of the coupling gain c and accelerates the exponential convergence of tracking error trajectories. By contrast, reducing α to 0.1 relaxes the LMI feasibility conditions and facilitates the acquisition of a positive definite solution P yet

inevitably deteriorates the system's convergence speed, which is fully consistent with the theoretical results established in Theorem 1.

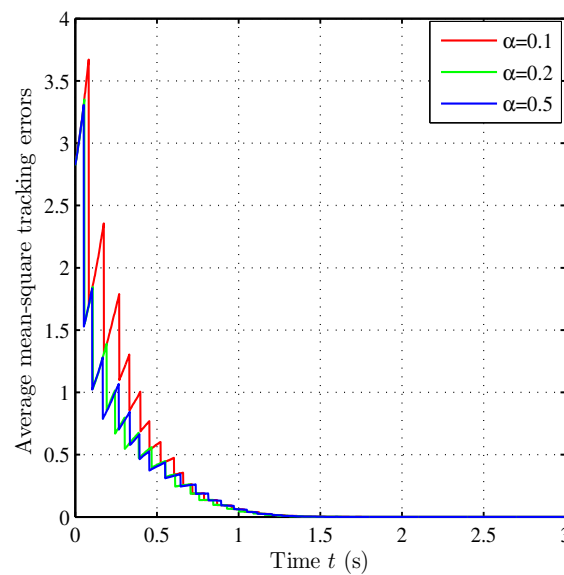


Figure 3. Influence of tuning parameter α on tracking error convergence.

2. Effect of stochastic disturbance intensity D (Figure 4)

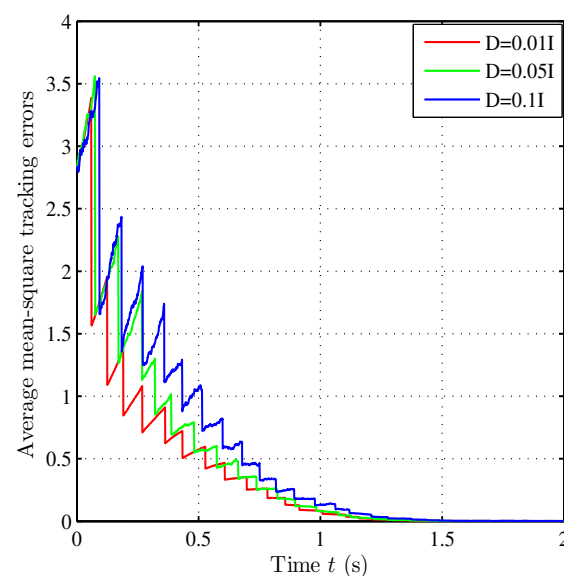


Figure 4. Effect of disturbance intensity D on tracking error convergence.

The matrix D quantifies the intensity of state-dependent multiplicative noise embedded in LMI (4). Three distinct noise levels $D = 0.01I$, $0.05I$, and $0.1I$ are adopted for comparative analysis, with the corresponding mean-square tracking error responses depicted in Figure 4. As the stochastic disturbance intensity escalates, the system exhibits a strengthened divergent tendency, which renders the consensus conditions more conservative and induces more pronounced steady-state fluctuations in the tracking error trajectories. In this case, a weak noise intensity ensures the simultaneous feasibility of the two coupled matrix inequalities, which further corroborates the favorable disturbance rejection capability of the devised impulsive control protocol.

3. Sensitivity to communication topology (Figures 5–8)

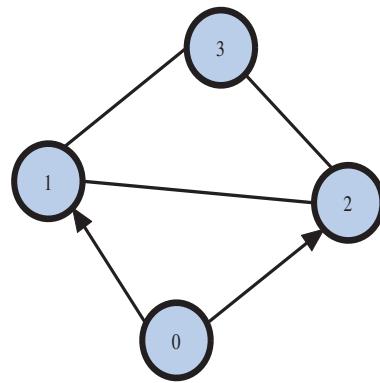


Figure 5. Strongly connected topology graph.

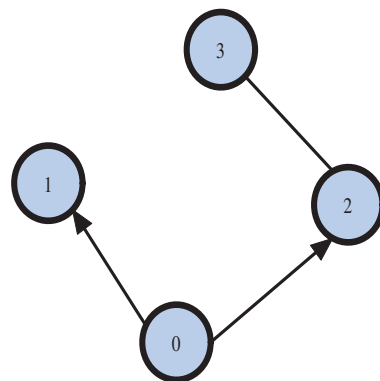


Figure 6. Spanning tree topology graph.

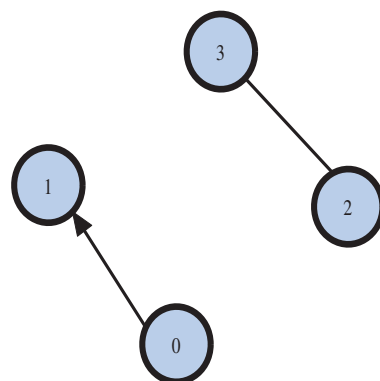


Figure 7. Weakly connected topology graph.

Three typical communication topologies are constructed for comparative simulations, including strongly connected topology, spanning tree topology and weakly connected topology, as illustrated in Figures 5, 6 and 7, respectively. The upper feasible bound of coupling gain c derived in Theorem 1 is inversely related to the maximum eigenvalue $\lambda_3(H)$ of the Laplacian matrix formed by follower agents. A topology with higher algebraic connectivity corresponds to a larger algebraic connectivity value, a wider feasible interval for coupling gain, and accordingly achieves a faster convergence rate of tracking errors. In particular, the weakly connected topology in Figure 7 fails to transmit the leader's information to all agents and violates Assumption 1. Such a contrasting case intuitively demonstrates that insufficient graph connectivity will deteriorate the consensus performance. By comparison, weakly connected topology yields slower system convergence and larger steady-state tracking deviation. The comparative convergence curves under three topologies are plotted in Figure 8, which fully verifies the topology-dependent constraint on coupling gain presented in Theorem 1 and further confirms that graph connectivity

acts as a critical factor governing the leader-following mean-square consensus dynamics of stochastic multi-agent systems.

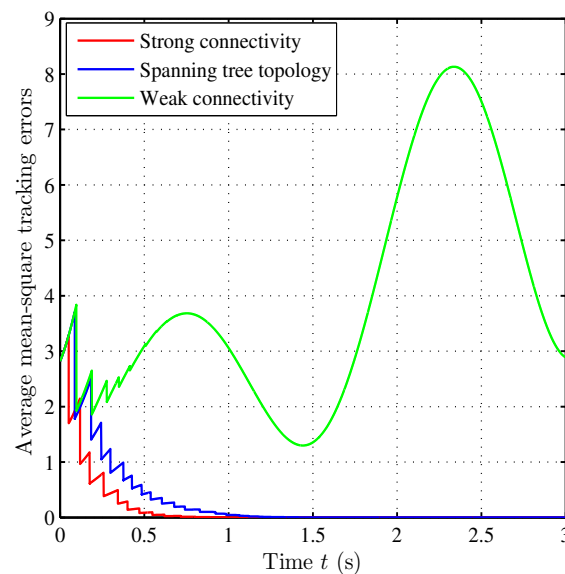


Figure 8. Effects of graph connectivity on mean-square tracking errors.

5. Conclusions

This paper has studied the leader-following mean-square consensus of stochastic linear multi-agent systems by using distributed impulsive control. A distributed impulsive control protocol for mean-square consensus is established. The coupling gain and impulsive control gain are designed in explicit forms, and the consensus of deterministic linear multi-agent systems is derived as a corollary. The numerical results demonstrate that all the followers can asymptotically track the leader's state in the mean-square sense. Future work will extend the results to systems with communication delays, switching topologies, and nonlinear dynamics, as well as the scenario with unmeasurable system states, by constructing state observers to develop output-feedback impulsive control strategies.

Author Contributions: Conceptualization, D.L. and K.S.; methodology, D.L.; software, K.S.; validation, D.L.; writing—original draft, D.L.; writing—review and editing, D.L., K.S. and Y.S.; visualization, Y.S.; supervision, D.L. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by Suqian Sci & Tech Program (Grant No. K202440) and Startup Foundation for Newly Recruited Employees of Suqian University (Grant No. 2024XRC005).

Data Availability Statement: The original contributions presented in this study are included in the article. For further inquiries, please contact the corresponding authors.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Peng, J.H.; Zhang, K. Consensus of linear multi-agent system with unknown input time-varying delay by linear time-varying protocol. *Complex Syst. Stab. Control* **2026**, *2*, 5.
2. Chen J, Wang Y, Wang F. H_∞ control for multi-agent systems under two-way amplify-and-forward relay networks: the event-trigger case. *Complex Syst. Stab. Control* **2026**, *2*, 6.
3. Yan, Q.; Han, X.; Zhou, L.; Zheng, M. Adaptive synchronization of a category of neural networks with proportional delays. *Complex Syst. Stab. Control* **2025**, *1*, 6.
4. Sun, K.; Fang, Y.; Qiu, J.; Wei, Q.; Chen, H.; Wang, T.; Fontanelli, D. A Survey of Attack Detection and Security Control for Advanced Cyber-Physical Systems and Their Applications. *IEEE Trans. Syst. Man. Cybern. Syst.* **2026**, 1–18. <https://doi.org/10.1109/TSMC.2026.3690568>.

5. Xiao, F.; Wang, L. Consensus problems for high-dimensional multi-agent systems. *IET Control Theory Appl.* **2007**, *1*, 830–837.
6. Olfati-Saber, R.; Murray, R.M. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Trans. Autom. Control* **2004**, *49*, 1520–1533.
7. Fax, J.A.; Murray, R.M. Information flow and cooperative control of vehicle formations. *IEEE Trans. Autom. Control* **2004**, *49*, 1465–1476.
8. Ren, W.; Beard, R.W. Consensus seeking in multiagent systems under dynamically changing interaction topologies. *IEEE Trans. Autom. Control* **2005**, *50*, 655–661.
9. Ni, W.; Cheng, D. Leader-following consensus of multi-agent systems under fixed and switching topologies. *Syst. Control Lett.* **2010**, *59*, 209–217.
10. Zhao, W.; Yu, W.; Zhang, H. Observer-based formation tracking control for leader–follower multi-agent systems. *IET Control Theory Appl.* **2019**, *13*, 239–247.
11. Guo, T.; Han, J.; Zhou, C.; Zhou, J. Multi-leader-follower group consensus of stochastic time-delay multi-agent systems subject to Markov switching topology. *Math. Biosci. Eng.* **2022**, *19*, 7504–7520.
12. Zhang, Z.; Peng, S.; Liu, D.; Wang, Y.; Chen, T. Leader-following mean-square consensus of stochastic multiagent systems with ROUs and RONs via distributed event-triggered impulsive control. *IEEE Trans. Cybern.* **2020**, *52*, 1836–1849.
13. Naghshtabrizi, P.; Hespanha, J.P.; Teel, A.R. Exponential stability of impulsive systems with application to uncertain sampled-data systems. *Syst. Control Lett.* **2008**, *57*, 378–385.
14. Naghshtabrizi, P.; Hespanha, J.P.; Teel, A.R. Stability of delay impulsive systems with application to networked control systems. *Trans. Inst. Meas. Control* **2010**, *32*, 511–528.
15. Lu, Y.; Ruan, D.; Zhu, Q. Symmetric analysis of stability criteria for nonlinear systems with multi-delayed periodic impulses: Intensity periodicity and averaged delay. *Symmetry* **2025**, *17*, 1481.
16. Xu, L.; Bao, B.; Hu, H. Stability of impulsive delayed switched systems with conformable fractional-order derivatives. *Int. J. Syst. Sci.* **2025**, *56*, 1271–1288.
17. Wu, J.; Li, X.; Xie, X. Finite-time stability for time-varying nonlinear impulsive systems. *Math. Methods Appl. Sci.* **2026**, *49*, 1250–1265.
18. Xu, H.; Zhu, Q. Stability of discrete-time impulsive stochastic systems with hybrid non-deterministic delays. *IEEE Trans. Autom. Control* **2026**, 1–8. <https://doi.org/10.1109/TAC.2026.3666702>.
19. Cao, Y.; Ren, W. Distributed coordinated tracking with reduced interaction via a variable structure approach. *IEEE Trans. Autom. Control* **2011**, *57*, 33–48.
20. Gao, S.; Wen, G.; Zhai, X.; Zheng, P. Finite-fixed-time bipartite consensus for first-order multi-agent systems via impulsive control. *Appl. Math. Comput.* **2023**, *442*, 127740.
21. Liu, J.; Zhou, J. Distributed impulsive group consensus in second-order multi-agent systems under directed topology. *Int. J. Control* **2015**, *88*, 910–919.
22. Wang, Y.W.; Yi, J.W. Consensus in second-order multiagent systems via impulsive control using position-only information with heterogeneous delays. *IET Control Theory Appl.* **2015**, *9*, 336–345.
23. Li, Z.; Fang, J.; Tang, Y.; Huang, T. Consensus of linear discrete-time multi-agent systems: A low-gain distributed impulsive strategy. *IEEE Trans. Syst. Man. Cybern. Syst.* **2017**, *49*, 1041–1052.
24. Zhou, D.; Chen, W.H.; Lu, X. Leader-following consensus of linear multi-agent systems via distributed impulsive control. *IEEE Trans. Autom. Control* **2026**, *71*, 552–558.
25. Liu, Y.; Candell, R.; Moayeri, N. Effects of wireless packet loss in industrial process control systems. *ISA Trans.* **2017**, *68*, 412–424.
26. Ma, L.; Wang, Z.; Han, Q.L.; Liu, Y. Consensus control of stochastic multi-agent systems: A survey. *Sci. China Inf. Sci.* **2017**, *60*, 120201.
27. Zhu, Q. Stabilization of stochastic nonlinear delay systems with exogenous disturbances and the event-triggered feedback control. *IEEE Trans. Autom. Control* **2019**, *64*, 3764–3771.
28. Zhu, Q.; Wang, H. Output feedback stabilization of stochastic feedforward systems with unknown control coefficients and unknown output function. *Automatica* **2018**, *87*, 166–175.
29. Hao, L.; Wang, C.; Shi, Y. Quadratic tracking control of linear stochastic systems with unknown dynamics using average off-policy q-learning method. *Mathematics* **2024**, *12*, 1533.
30. Ma, C.; Li, T.; Zhang, J. Consensus control for leader-following multi-agent systems with measurement noises. *J. Syst. Sci. Complex.* **2010**, *23*, 35–49.
31. Li, W.; Li, T.; Xie, L.; Zhang, J.F. Necessary and sufficient conditions for bounded distributed mean square tracking of multi-agent systems with noises. *Int. J. Robust. Nonlinear Control* **2016**, *26*, 631–645.
32. Zong, X.; Li, T.; Yin, G.; Wang, L.Y.; Zhang, J.F. Stochastic consentability of linear systems with time delays and multiplicative noises. *IEEE Trans. Autom. Control* **2017**, *63*, 1059–1074.

33. Li, D.; Li, T. Cooperative output feedback tracking control of stochastic linear heterogeneous multi-agent systems. *IEEE Trans. Autom. Control* **2023**, *68*, 47–62.
34. Sun, H.Y.; Han, H.G.; Sun, J.; Qiao, J.F. Leader-following sampled-data consensus of multi-agent systems with successive packet losses and stochastic sampling. *IEEE Trans. Cybern.* **2024**, *54*, 7381–7391.
35. Li, B.; Zhao, L.; Wen, S. Periodic event-triggered consensus of stochastic multi-agent systems under switching topology. *Artif. Intell. Sci. Eng.* **2025**, *1*, 147–156.
36. Liu, L.; Pan, C.; Fang, J. Event-triggered impulsive control of nonlinear stochastic systems with exogenous disturbances. *Int. J. Robust. Nonlinear Control* **2025**, *35*, 1654–1665.
37. Li, K.; Hua, C.; You, X.; Xu, Z. Distributed consensus of feedforward nonlinear stochastic multi-agent systems subject to actuator attacks. *IEEE Trans. Cybern.* **2025**, *55*, 4842–4851.
38. Zhang, Z.; Che, W.W. SOS-based secure consensus control for stochastic nonlinear MASs against multiple DoS attacks. *Appl. Math. Comput.* **2025**, *507*, 129595.
39. Xiao, G.; Ren, L.; Guan, Y. Adaptive Optimal-time consensus control for stochastic multi-agent systems with layered control approach. *Commun. Nonlinear Sci. Numer. Simul.* **2026**, *157*, 109701.
40. Mao, X. *Stochastic Differential Equations and Applications*; Horwood: Chichester, UK, 2006.
41. Yang, T. *Impulsive Systems and Control: Theory and Applications*; Nova Science Publishers, Inc.: Hauppauge, NY, USA, 2001.

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.