

Article

Risk Gradient Based Evolution of Spatial Evolution Trends of Rock-Mass Instability in Deep Mining

Guanguan Li, Fang Yan *, Feifan He, Jiarui Wang and Jialei Chen

School of Resources and Safety Engineering, Central South University, Changsha 410083, China

* Correspondence: yanfang3543@csu.edu.cn

Abstract

Rock-mass instability in deep mining is a progressive process controlled by stress redistribution, fracture evolution, and excavation disturbance. Conventional risk assessment mainly identifies high-risk zones according to risk magnitude, but provides limited information on risk-transition boundaries and propagation directions. This study proposes a risk-gradient field method for identifying the propagation direction and migration path of rock-mass instability. An interpretable instability probability function was first established using the sure independence screening and sparsifying operator (SISSO), and the predicted probabilities were mapped into a three-dimensional probabilistic risk field. On this basis, the spatial risk-gradient magnitude and direction were calculated to identify high-gradient boundaries, potential risk fronts, and dominant propagation directions. The results show that the probabilistic risk field and risk-gradient field provide complementary information: the former indicates where instability risk accumulates, whereas the latter reveals where risk changes sharply. High-gradient zones were mainly concentrated near the margins of high-risk regions, suggesting that they represent transition zones rather than high-risk cores. Validation using subsequent microseismic activity showed that 72.53% of later events and 63.26% of released energy occurred within the high-gradient buffer zone, with an angular deviation of 37.9° between the gradient-centroid migration direction and the microseismic energy-centroid migration direction. These results indicate that the proposed method can extend rock-mass instability assessment from probability-based zoning to direction-oriented risk-evolution diagnosis, providing a useful basis for identifying potential instability migration zones in deep mining.

Keywords: rock-mass instability; microseismic monitoring; SISSO; risk field; risk gradient**MSC:** 91G70Academic Editors: Heung Soo Kim
and Salman Khalid

Received: 16 June 2026

Revised: 7 July 2026

Accepted: 9 July 2026

Published: 12 July 2026

Copyright: © 2026 by the authors.
Licensee MDPI, Basel, Switzerland.
This article is an open access article
distributed under the terms and con-
ditions of the [Creative Commons At-
tribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

1. Introduction

Deep metal mining is progressively extending into environments characterized by high in situ stress, strong excavation disturbance, and complex geological structures, making surrounding-rock stability an increasingly critical issue. Under the combined effects of in situ stress, fault structures, lithological heterogeneity, and excavation-induced unloading, cracks within deep surrounding rock are prone to initiation, propagation, and coalescence, which may further trigger dynamic hazards such as spalling, roof collapse, rockburst, or local instability [1–4]. Rock-mass instability is generally not an isolated in-

stantaneous failure process, but a progressive process controlled by local damage accumulation, stress redistribution, and fracture evolution. Therefore, under deep mining conditions, risk assessment of rock-mass instability should not only identify high-risk zones, but also reveal the spatial evolution characteristics and potential development trends of instability risk.

Microseismic (MS) monitoring can capture crack activity and energy-release processes within rock masses, and has become an important tool for characterizing damage evolution in deep surrounding rock [5–8]. The spatiotemporal distribution, energy release, event frequency, and source parameters of MS events can reflect the intensity and spatial clustering characteristics of mining-induced rock fracturing [9]. Accordingly, MS monitoring has been widely applied to ground-pressure monitoring, rock-mass stability assessment, and disaster early warning in mines. For example, Liu et al. [10] revealed the evolution process of rockbursts based on MS monitoring and true triaxial compression tests. Although MS events are indirect indicators of rock-mass damage, previous studies have shown that MS multiparameter analysis can provide useful information for identifying damage evolution and hazard precursors in underground engineering. For example, variations in b-value, spatial fractal dimension, energy index, and cumulative apparent volume have been used to characterize the precursory process of large-scale rock-mass collapse, while MS multiparameter analysis, high-magnitude event distribution, MS tomography, and field investigation have been jointly applied to rockburst risk identification [11,12]. Ma et al. [13] combined a one-dimensional convolutional neural network (1D CNN) with a fully connected neural network (FCNN) to predict rockburst locations using the spatial positions of MS events. Zheng et al. [14] used MS monitoring data and a hybrid convolutional long short-term memory network (HCLSTM) to assess and warn against rockburst occurrence. However, raw MS events are generally discrete, stochastic, and strongly heterogeneous in space and time. Relying solely on event distributions or individual MS indicators is therefore insufficient for robustly characterizing rock-mass instability risk [15,16]. It is therefore necessary to further transform MS monitoring information into integrated risk indicators that can represent the probability of rock-mass instability, thereby improving the objectivity and continuity of risk identification.

Statistical analysis, index-based comprehensive evaluation, and machine-learning methods have been widely used for MS risk identification and rock-mass instability prediction [17,18]. These methods have achieved useful results in characterizing MS activity intensity, identifying anomalous stages, and classifying risk levels. Nevertheless, several limitations remain. On the one hand, some models rely heavily on empirical thresholds or manually assigned weights, making their evaluation results susceptible to subjectivity. On the other hand, although complex machine-learning models have strong nonlinear fitting capability, their model structures are often black-box in nature, making it difficult to explicitly interpret the relationship between MS indicators and rock-mass instability probability. In addition, conventional risk-zoning results mainly focus on describing the magnitude of risk, but provide limited insight into risk boundaries, risk migration, and potential propagation directions.

To improve the interpretability of probabilistic modeling for rock-mass instability, the sure independence screening and sparsifying operator (SISSO) provides an effective approach for feature selection and explicit model construction from multiple MS indicators. SISSO can screen key descriptors with strong representational capability from a high-dimensional candidate feature space and establish a concise mathematical expression [19]. Compared with conventional black-box models, the SISSO model can not only describe the nonlinear relationship between MS features and rock-mass instability probability, but also enhance model interpretability and transferability through its explicit functional form [20,21]. Therefore, the SISSO-based probability function can serve as a key bridge between

MS monitoring information and probabilistic assessment of rock-mass instability. Risk assessment in engineering systems generally aims not only to estimate the likelihood of hazardous events, but also to support safety management, consequence control, and decision-making under uncertain conditions [22–24]. Therefore, after obtaining the probability of rock-mass instability, constructing a continuous spatial risk representation becomes essential for analyzing risk evolution [25]. Because MS events and assessment points are spatially discrete, the probability result at a single point can only reflect local instability potential and cannot fully characterize the continuous spatial variation in risk within the mining-affected region. Therefore, the instability probabilities calculated by the SISSO probability function can be mapped onto three-dimensional assessment points, and a probabilistic risk field can then be constructed through spatial interpolation and smoothing. This risk field transforms discrete MS responses into a continuous spatial probability distribution, thereby revealing the locations, extents, and temporal evolution characteristics of high-risk zones at different stages. Dong et al. [26] developed a rock-instability risk-field model based on MS monitoring data, in which risk was represented as a continuous function in three-dimensional space, and its applicability to regional risk assessment was verified using field disturbance data. Yan et al. [27] further integrated instability probability and consequence severity into a three-dimensional risk-field function, and the results showed good spatial consistency with energy-density clusters identified by Density-Based Spatial Clustering of Applications with Noise DBSCAN. Xue et al. [28] combined the Mann–Kendall trend test with a spatially smoothed seismicity model to generate contour maps of spatial hazard indices for identifying local rockburst-prone zones. Compared with conventional event-density maps or single-indicator anomaly identification, the probabilistic risk field not only preserves the spatiotemporal information of MS activity, but also expresses surrounding-rock instability risk in an explicit probabilistic form, making the risk-identification results more interpretable and comparable. However, the probabilistic risk field mainly answers the question of where risk accumulates, emphasizing risk magnitude and spatial distribution. It remains insufficient for determining whether risk boundaries are clear, whether risk tends to expand outward, and in which direction the risk may develop. Therefore, it is necessary to further analyze the spatial variation characteristics of the probabilistic risk field.

Risk-gradient analysis provides a further quantitative approach for revealing the boundary characteristics and development directions of rock-mass instability risk. The risk gradient represents the spatial rate of change in the probabilistic risk field in three-dimensional space; its magnitude reflects the intensity of probability variation between adjacent spatial locations, whereas its direction points toward the fastest increase in risk. Gradient information has been widely used for boundary detection in continuous spatial fields [29]. For example, Li et al. [30] constructed edge-detection filters using full gradient tensors in gravity and magnetic potential-field interpretation, which improved the identification of faults, lithological boundaries, and linear structures. This indicates that, for continuous spatial fields, gradients or spatial rates of change can effectively highlight locations where field variables vary sharply. By analogy, in a probabilistic risk field, regions with large risk-gradient magnitudes do not necessarily correspond to the highest-risk cores, but are more likely to indicate transition boundaries between high- and low-risk zones or potential risk fronts. Therefore, risk gradients can extract information that is difficult to express directly using conventional probability distributions, including high-gradient boundaries, risk fronts, and risk-propagation directions. In addition, the fracture and rockburst evolution processes of surrounding rock in deep mining exhibit clear spatial evolution characteristics. Xue et al. [31] delineated high-risk rockburst zones with high prediction difficulty based on three-dimensional MS monitoring and numerical simulation. Wang et al. [32] established horizontal and vertical distribution models of a working

face using multiple MS indicators, demonstrating that mining-induced fracture propagation and stress redistribution can be reflected by the evolution of MS indicators. Khan et al. [33] identified high-stress concentration zones, newly formed faults, and risk areas using MS energy-density clouds and three-dimensional fracture visualization. These studies indicate that rock-mass instability risk is expressed not only as local high-value accumulation, but also as fracture propagation, stress migration, and risk-boundary advancement. Furthermore, Lu et al. [34] found that seismic events in the deep mining of steeply inclined coal seams gradually extended toward the middle part of the inclined axis, accompanied by the migration of dominant rockburst-control zones. Gou et al. [35] showed that MS clustering is highly consistent with fault activation and that stress-anomaly zones can be visualized using seismic CT. Liu et al. [36] traced fracture propagation paths based on the spatial locations and topological relationships of MS events. These studies provide a basis for introducing risk gradients into probabilistic risk fields: the gradient magnitude can be used to identify abrupt risk-transition boundaries, while the gradient direction can indicate possible risk-expansion directions and migration paths. In other words, the probabilistic risk field reveals the current spatial distribution of instability risk, whereas the risk-gradient field further characterizes the evolution tendency of risk from local accumulation toward spatial expansion. Combining the two can advance rock-mass instability assessment from static risk zoning to direction-informed risk-evolution diagnosis, providing a more valuable basis for risk-front identification, migration-path judgment, and targeted support control in deep mining.

Based on the above considerations, this study develops a coupled framework of probabilistic risk field and risk gradient for analyzing the evolution of rock-mass instability risk using MS monitoring data from a deep metal mine. First, risk-related indicators are constructed from the spatiotemporal and energy characteristics of MSc events, and an explicit probability function for rock-mass instability is established using SISSO. Second, the calculated instability probabilities are mapped onto three-dimensional assessment points to construct a continuous probabilistic risk field. On this basis, the magnitude and direction of the risk gradient are further calculated to identify high-gradient boundaries, potential risk fronts, and risk-propagation directions. Finally, subsequent MS activity is used to validate the spatial indication capability of the risk-gradient field. The proposed framework aims to extend rock-mass instability assessment from probability-based risk identification to risk-boundary extraction and migration-direction judgment, thereby providing a new analytical approach for dynamic risk assessment under deep mining conditions.

2. Methodology

2.1. SISSO-Based Probability Function for Rock-Mass Instability

Before constructing the spatial risk field and extracting its gradient information, it is necessary to obtain the instability probability at discrete assessment points. In this study, the sure independence screening and sparsifying operator (SISSO) framework was used to establish an explicit probability function for rock-mass instability. For a three-dimensional assessment point $x_j = (x_j, y_j, z_j)$ and a time window t_k , the MS information before t_k was first aggregated to form a time-dependent feature vector:

$$u_{j,k} = [u_1(\mathbf{x}_j, t_k), u_2(\mathbf{x}_j, t_k), \dots, u_p(\mathbf{x}_j, t_k)]^T \quad (1)$$

where $u_{j,k}$ denotes the original input feature vector at assessment point x_j during time window t_k , and p is the number of original variables. To ensure that the probability model only uses information available before the prediction time, all features were calculated using historical data within the look-back window prior to t_k .

Because the raw MS variables differ substantially in dimension, magnitude, and statistical distribution, feature preprocessing was performed before SISSO modeling. For

each variable u_m , robust normalization was applied to reduce the influence of extreme MS events [37]:

$$\tilde{u}_m = \frac{u_m - \text{median}(u_m)}{\text{MAD}(u_m) + \varepsilon} \quad (2)$$

where $\text{MAD}(\cdot)$ is the median absolute deviation and ε is a small positive constant introduced to avoid division by zero. After normalization, quantile truncation was adopted to suppress abnormal outliers, and the processed variables were further scaled into a comparable numerical range. The target variable was defined according to the instability state of the assessment point in the corresponding prediction window:

$$y_{j,k} = \begin{cases} 1, & \text{unstable or high-risk state} \\ 0, & \text{stable or low-risk state} \end{cases} \quad (3)$$

where $y_{j,k}=1$ represents the occurrence of an instability-related state within the target window and $y_{j,k}=0$ represents a relatively stable state. The instability-related state can be determined using engineering records, abnormal MS activity, or dynamic risk classification criteria. In this way, the modeling problem is transformed into a supervised probabilistic classification problem.

On the basis of the normalized input variables, SISSO first constructs a candidate feature dictionary. The initial feature set is denoted as:

$$\Phi_0 = \{\tilde{u}_1, \tilde{u}_2, \dots, \tilde{u}_p\} \quad (4)$$

Then, a series of unary and binary mathematical operations are applied to generate nonlinear candidate descriptors:

$$\Phi = \mathcal{H}(\Phi_0) \quad (5)$$

where $\mathcal{H}(\cdot)$ represents the feature-generation operator. The allowed operations include physically meaningful transformations.

After generating the high-dimensional candidate dictionary Φ , the sure independence screening stage was used to rapidly identify candidate descriptors that are strongly related to the target variable [38]. Each descriptor $\phi_i \in \Phi$ was evaluated according to its statistical relevance to $y_{j,k}$. The descriptors with the highest relevance scores were retained to form a reduced candidate subset:

$$\Phi_{SIS} = \{\phi_1, \phi_2, \dots, \phi_K\} \quad (6)$$

where K is the number of retained descriptors after screening. This step substantially reduces the dimensionality of the candidate feature space while preserving variables with strong explanatory potential. To further reduce redundancy, descriptors with excessive correlation were removed so that the final candidate subset contained relatively independent and physically interpretable variables.

In the sparsifying stage, a logistic probability model was established using the selected descriptors. For each assessment point and time window, the logit score of rock-mass instability is expressed as:

$$Z(\mathbf{x}_j, t_k) = \beta_0 + \sum_{m=1}^M \beta_m \phi_m(\mathbf{x}_j, t_k) \quad (7)$$

where $Z(x_j, t_k)$ is the linear instability score, β_0 is the intercept, β_m is the regression coefficient of the m -th descriptor, ϕ_m is the descriptor selected by SISSO, and M is the final number of sparse descriptors. The instability probability is then obtained using the sigmoid function [39]:

$$P(\mathbf{x}_j, t_k) = \frac{1}{1 + \exp[-Z(\mathbf{x}_j, t_k)]} \quad (8)$$

where $P(x_j, t_k) \in [0, 1]$ denotes the predicted probability of rock mass instability at assessment point x_j during time window t_k .

For clarity, the final SISSO-based probability function used in this study is further given in a compact form. The instability probability was calculated using the following logistic-link equation:

$$Z(\mathbf{x}_j, t_k) = -0.2667 + \beta^T \Phi \quad (9)$$

where $Z(\mathbf{x}_j, t_k)$ is the linear instability score, Φ is the vector of the final SISSO-selected descriptors, and β is the corresponding coefficient vector. In this study, Φ and β are defined as:

$$\Phi = [\ln(\Delta t), \sqrt{\Delta t}, \Delta t, \sqrt{R_w}, R_{e,10}, \sqrt{R_{w,30}}, \ln(R_w), R_{w,10}, \ln(R_{e,10}), R_{e,roll}, \ln(R_e), R_e, \ln(R_{w,10}), r_{10}, \ln(r_{10})], \quad (10)$$

$$\beta = [-0.25, 0.35, -0.03, -1.588, -8.297, 3.00, 1.673, -0.15, 0.752, -6.81, -0.54, 43.737, -0.938, 0.127, -2.91] \quad (11)$$

Here, Δt denotes the time interval-related indicator; R_w and R_e denote window-level and energy-related MS risk indicators, respectively; $R_{w,10}$, $R_{w,30}$, and $R_{e,10}$ represent the corresponding indicators calculated within specific historical windows; $R_{e,roll}$ denotes the rolling energy-related indicator; and r_{10} denotes a local ratio-type MS indicator. A positive coefficient indicates that the corresponding descriptor increases the instability score, whereas a negative coefficient indicates that the descriptor reduces the score.

To avoid overfitting and information leakage, the training and validation procedures were performed in a time-aware manner. Samples in later time windows were not used to construct features for earlier windows, and all moving averages, cumulative variables, and temporal difference variables were calculated only from historical information. The sparse coefficients were estimated using regularized logistic regression, and the final descriptors were selected according to both predictive performance and physical interpretability.

2.2. Construction of the Probabilistic Risk Field

After obtaining the instability probability at discrete assessment points using the SISSO-based probability function, the next step is to extend these point-scale probability values into a continuous spatial field. The purpose of this step is to transform the discrete prediction results into a three-dimensional probabilistic risk field, so that the spatial distribution, local concentration, and subsequent gradient variation in instability risk can be analyzed within a unified field framework.

Let the study domain be denoted as $\Omega \subset \mathbb{R}^3$. At a given time window t_k , the SISSO-based probability function provides the instability probability at N assessment points:

$$\{P(\mathbf{x}_1, t_k), P(\mathbf{x}_2, t_k), \dots, P(\mathbf{x}_N, t_k)\} \quad (12)$$

where $x_j = (x_j, y_j, z_j)$ is the coordinate of the j -th assessment point, and $P(x_j, t_k)$ represents the predicted probability of rock-mass instability at this point during time window t_k . These probabilities are discrete in space and cannot directly describe the continuous spatial morphology of the unstable zone. Therefore, spatial field reconstruction is required.

A regular three-dimensional grid was first established in the study domain. The coordinate of an arbitrary grid node is denoted as:

$$\mathbf{x} = (x, y, z) \quad (13)$$

For each grid node $\mathbf{x}$, the probabilistic risk value was estimated by spatially interpolating the SISSO-predicted probabilities at surrounding assessment points Li and Heap [40]. The continuous risk field can be expressed as:

$$R(\mathbf{x}, t_k) = \sum_{j=1}^N w_j(\mathbf{x}) P(\mathbf{x}_j, t_k) \quad (14)$$

where $R(x, t_k)$ is the reconstructed risk value at grid node x and time window t_k , and $w_j(\mathbf{x})$ is the spatial contribution weight of assessment point x_j to grid node x . The weights satisfy:

$$\sum_{j=1}^N w_j(\mathbf{x}) = 1, \quad w_j(\mathbf{x}) \geq 0 \quad (15)$$

In this way, the discrete probability outputs are transformed into a continuous scalar field:

$$R = R(x, y, z, t) \quad (16)$$

where $R \in [0,1]$ represents the probabilistic risk level of rock-mass instability. A higher value of R indicates a greater probability of instability, while a lower value indicates a relatively stable state.

In this study, the spatial weights can be determined using a distance-based interpolation scheme. For example, when inverse-distance weighting is adopted, the weight of the j -th assessment point can be written as:

$$w_j(\mathbf{x}) = \frac{(d_j + \varepsilon)^{-q}}{\sum_{l=1}^N (d_l + \varepsilon)^{-q}} \quad (17)$$

where $d_j = \|\mathbf{x} - \mathbf{x}_j\|$ is the Euclidean distance between the grid node and the j -th assessment point, q is the distance attenuation exponent, and ε is a small positive constant used to avoid singularity when the grid node is close to an assessment point.

Because the risk-gradient calculation in Section 3.3 involves spatial derivatives of the risk field, the smoothness and stability of $R(\mathbf{x}, t_k)$ are particularly important. Direct interpolation of discrete probabilities may generate local oscillations or artificial abrupt changes, especially in regions with sparse assessment points or near the boundary of the study domain. To reduce this effect, a spatial smoothing operation was applied to the reconstructed risk field:

$$\bar{R}(\mathbf{x}, t_k) = \int_{\Omega} K_h(\mathbf{x} - \xi) R(\xi, t_k) d\xi \quad (18)$$

where $\bar{R}(\mathbf{x}, t_k)$ is the smoothed probabilistic risk field, $K_h(\cdot)$ is a smoothing kernel, and h is the smoothing bandwidth.

To further improve the reliability of the reconstructed field, boundary treatment was also considered. For each time window t_k , the above procedure produces a three-dimensional probabilistic risk field. By repeating the calculation over consecutive time windows, a time sequence of risk fields can be obtained:

$$\{R(\mathbf{x}, t_1), R(\mathbf{x}, t_2), \dots, R(\mathbf{x}, t_K)\} \quad (19)$$

This sequence provides the basis for analyzing not only the spatial distribution of instability probability, but also the temporal evolution of risk. In the present study, the probabilistic risk field is not treated merely as a visualization result. Instead, it is regarded as a continuous scalar field that contains spatial variation information. The spatial derivative of this field can therefore be used to extract the risk-gradient field, which further reveals the direction and intensity of risk development.

2.3. Risk-Gradient Calculation and Directional Analysis

After the probabilistic risk field is constructed, the risk value $R(x, t)$ can be regarded as a continuous scalar field in 3D space and time. Although the risk field itself can describe the spatial distribution of instability probability, it does not directly provide directional information on how the risk evolves or where the high-risk zone may propagate. To further extract the internal variation pattern of the risk field, a risk-gradient-based analysis method is introduced in this study. The basic idea is to use spatial and temporal derivatives of the continuous risk field to characterize the direction, intensity, and migration tendency of rock-mass instability risk.

For a grid node $\mathbf{x} = (x, y, z)$ at time window t_k , the spatial risk-gradient vector is defined as:

$$\nabla R(\mathbf{x}, t_k) = \left[\frac{\partial R(\mathbf{x}, t_k)}{\partial x}, \frac{\partial R(\mathbf{x}, t_k)}{\partial y}, \frac{\partial R(\mathbf{x}, t_k)}{\partial z} \right]^T \quad (20)$$

where $\nabla R(x, t_k)$ denotes the spatial rate of change in the probabilistic risk field. The three components represent the risk variation rates along the x , y , and z directions, respectively. In practical computation, the partial derivatives can be calculated using the finite-difference method on the regular three-dimensional grid [41]. For example, the derivative in the x -direction can be approximated as:

$$\frac{\partial R(\mathbf{x}, t_k)}{\partial x} \approx \frac{R(x + \Delta x, y, z, t_k) - R(x - \Delta x, y, z, t_k)}{2\Delta x} \quad (21)$$

where Δx is the grid spacing in the x -direction. The derivatives in the y - and z -directions can be calculated in the same way. The magnitude of the spatial risk gradient is defined as:

$$G_s(\mathbf{x}, t_k) = \|\nabla R(\mathbf{x}, t_k)\| = \sqrt{\left(\frac{\partial R}{\partial x}\right)^2 + \left(\frac{\partial R}{\partial y}\right)^2 + \left(\frac{\partial R}{\partial z}\right)^2} \quad (22)$$

where $G_s(x, t_k)$ represents the intensity of spatial risk variation. A larger value of G_s indicates that the risk changes sharply within a short spatial distance. Therefore, high-gradient zones usually correspond to transition regions between high-risk and low-risk areas [30,42]. These regions are important because they may indicate the boundary of risk accumulation, the front of risk propagation, or the sensitive zone where instability risk changes rapidly.

The unit vector of the spatial risk-gradient direction is defined as:

$$\mathbf{n}_s(\mathbf{x}, t_k) = \frac{\nabla R(\mathbf{x}, t_k)}{G_s(\mathbf{x}, t_k) + \varepsilon} \quad (23)$$

where ε is a small positive constant used to avoid division by zero. The vector n_s indicates the direction in which the risk increases most rapidly. In the context of rock-mass instability, this direction can be interpreted as the preferential direction of risk enhancement.

To further describe the directional characteristics of risk propagation, the spatial risk gradient is decomposed into horizontal and vertical components. The horizontal risk-gradient magnitude is defined as:

$$G_h(\mathbf{x}, t_k) = \sqrt{\left(\frac{\partial R}{\partial x}\right)^2 + \left(\frac{\partial R}{\partial y}\right)^2} \quad (24)$$

and the vertical risk-gradient magnitude is defined as:

$$G_v(\mathbf{x}, t_k) = \left| \frac{\partial R}{\partial z} \right| \quad (25)$$

Based on these two quantities, the vertical-gradient contribution ratio is defined as:

$$\eta_v(\mathbf{x}, t_k) = \frac{G_v(\mathbf{x}, t_k)}{G_h(\mathbf{x}, t_k) + G_v(\mathbf{x}, t_k) + \varepsilon} \quad (26)$$

where $\eta_v \in [0, 1]$. When η_v is close to 0, the spatial variation in risk is dominated by horizontal propagation, indicating that the risk mainly develops within the same mining level or along the roadway plane. When η_v is close to 1, the vertical component becomes dominant, suggesting that the risk may have an obvious interlayer transfer or vertical migration tendency.

$$G_t(\mathbf{x}, t_k) = \frac{\partial R(\mathbf{x}, t_k)}{\partial t} \quad (27)$$

In discrete time windows, it can be approximated as:

$$G_t(\mathbf{x}, t_k) \approx \frac{R(\mathbf{x}, t_k) - R(\mathbf{x}, t_{k-1})}{\Delta t} \quad (28)$$

where Δt is the time interval between two adjacent risk fields. A positive value of G_t indicates that the local risk is increasing, whereas a negative value indicates that the risk is decreasing. Therefore, the temporal gradient can be used to distinguish active risk-enhancement zones from relatively stable or decaying zones.

To identify areas where risk changes both strongly and directionally, the spatial risk gradient and temporal risk gradient are combined. A risk-enhancement zone can be defined as:

$$\Omega_{\text{enh}}(t_k) = \{\mathbf{x}: R(\mathbf{x}, t_k) > Q_R, \quad G_s(\mathbf{x}, t_k) > Q_G, \quad G_t(\mathbf{x}, t_k) > 0\} \quad (29)$$

where Q_R and Q_G are threshold values for the risk field and the spatial gradient magnitude, respectively. These thresholds can be determined using quantiles of the risk field and gradient field. On this basis, the risk front is further extracted. The risk front is defined as the region where the risk field exhibits both relatively high risk values and strong spatial variation:

$$F(t_k) = \{\mathbf{x}: R(\mathbf{x}, t_k) > Q_R, \quad G_s(\mathbf{x}, t_k) > Q_G\} \quad (30)$$

The physical meaning of $F(t_k)$ is different from that of the high-risk core. The high-risk core represents the area where instability probability has already accumulated to a high level, whereas the risk front represents the boundary or transition zone where risk changes sharply and may continue to propagate. Therefore, the risk front provides a more suitable basis for identifying potential expansion boundaries of rock-mass instability. In this study, Q_R was set as the 80th percentile of the risk-field values, and Q_G was set as the 90th percentile of the spatial risk–gradient magnitudes for each representative period. Thus, the high-risk zone corresponds to the upper 20% of risk values, whereas the high-gradient zone corresponds to the upper 10% of gradient magnitudes.

To characterize the overall migration tendency of the high-risk region or risk front, a risk-weighted centroid is calculated [43]:

$$\mathbf{C}(t_k) = \frac{\sum_{\mathbf{x} \in \Omega} R(\mathbf{x}, t_k) \mathbf{x}}{\sum_{\mathbf{x} \in \Omega} R(\mathbf{x}, t_k) + \varepsilon} \quad (31)$$

where $\mathbf{C}(t_k)$ denotes the centroid of the risk field at time t_k . If only the extracted risk front is considered, the summation domain can be replaced by $F(t_k)$. The migration vector between two adjacent time windows is then defined as:

$$\mathbf{D}(t_k) = \mathbf{C}(t_k) - \mathbf{C}(t_{k-1}) \quad (32)$$

The vector $\mathbf{D}(t_k)$ describes the macroscopic movement direction of the risk concentration zone. By comparing this vector with the local or regional average risk–gradient direction, the consistency between local risk increase and overall risk migration can be evaluated.

The directional consistency can be quantified by the included angle between the risk–gradient direction and the migration vector:

$$\cos \theta = \frac{\bar{\mathbf{n}}_s(t_k) \cdot \mathbf{D}(t_k)}{\|\bar{\mathbf{n}}_s(t_k)\| \|\mathbf{D}(t_k)\| + \varepsilon} \quad (33)$$

where $\bar{\mathbf{n}}_s(t_k)$ is the average risk–gradient direction within the high-gradient zone or risk front. A smaller angle θ indicates stronger consistency between the risk–gradient direction and the subsequent migration direction of the high-risk region. This provides a quantitative basis for evaluating whether the gradient direction can be used as an indicator of risk propagation tendency.

For validation, subsequent MS activity is used as an independent reference. After the risk–gradient field is calculated at time t_k , MS events occurring within the subsequent period $[t_k, t_k + \Delta T]$ are collected. The spatial distribution and migration direction of these subsequent events are then compared with the previously calculated gradient direction and risk front. The event capture rate of the risk front is defined as:

$$P_c = \frac{N_{\text{front}}}{N_{\text{total}}} \quad (34)$$

where N_{front} is the number of subsequent MS events falling within or near the risk front, and N_{total} is the total number of subsequent MS events in the validation period. For strong-event validation, the capture rate can be written as:

$$P_s = \frac{N_{strong,front}}{N_{strong,total}} \quad (35)$$

where $N_{strong,front}$ is the number of subsequent strong MS events captured by the risk front, and $N_{strong,total}$ is the total number of subsequent strong events. Strong events can be defined according to energy, moment magnitude, stress drop. In this way, the analysis is extended from static identification of high-risk zones to directional interpretation of risk evolution.

Overall, the proposed methodology establishes a complete analytical chain from MS monitoring data to risk-gradient-based directional diagnosis. As illustrated in Figure 1, historical MS information is first aggregated within time windows and transformed into risk-related indicators at assessment points. The SISSO-based probability function then converts these indicators into instability probabilities, which are further reconstructed into a continuous three-dimensional probabilistic risk field through spatial interpolation and smoothing. Based on this field, the risk-gradient magnitude and direction are calculated to identify high-gradient boundaries, potential risk fronts, and migration vectors. Finally, subsequent MS activity is used as an independent reference to evaluate the spatial indication capability of the extracted gradient features. This workflow enables the assessment of rock-mass instability to move beyond probability-based risk zoning toward gradient-based interpretation of risk propagation and migration.

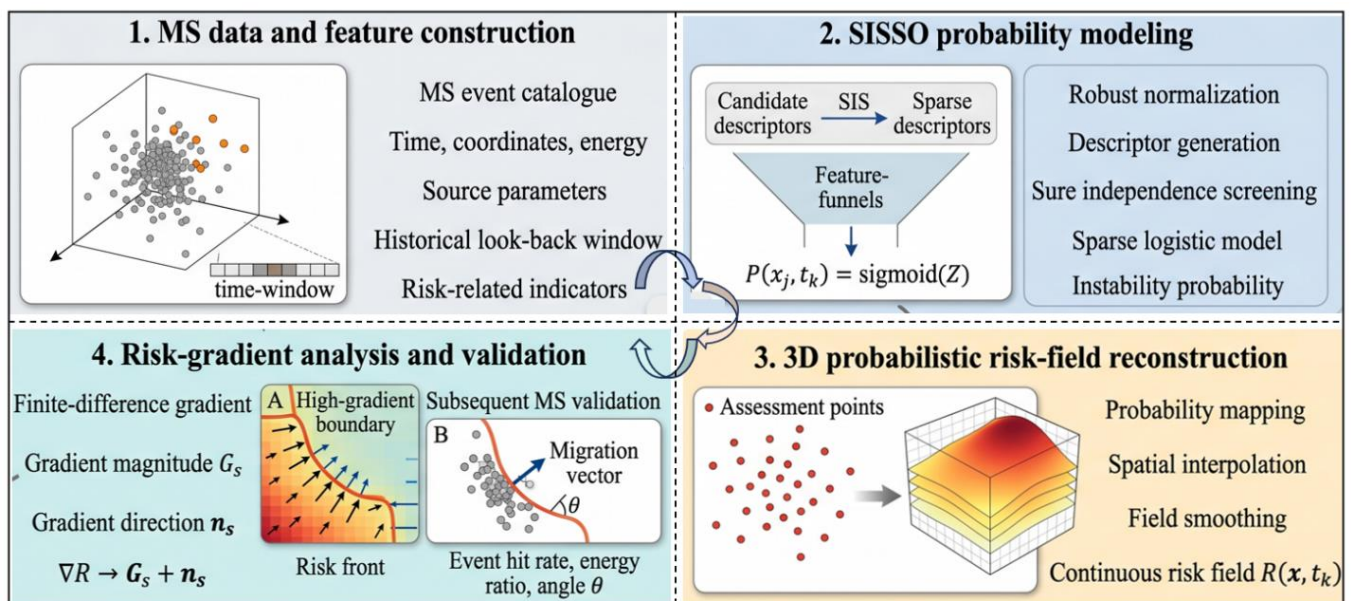


Figure 1. Workflow of the proposed risk-gradient field method for rock-mass instability analysis.

3. Case Study

3.1. Engineering Background

The study was conducted at the Dongtangzi lead–zinc deposit in Feng County, Baoji City, Shaanxi Province, China. The deposit is located in the central Qinling Orogenic Belt, a tectonically active transition zone between the North China Plate and the Yangtze Plate. The ore-bearing strata mainly occur near the contact between the Middle Devonian Gudaoling Formation and the Upper Devonian Xinghongpu Formation (Figure 2).

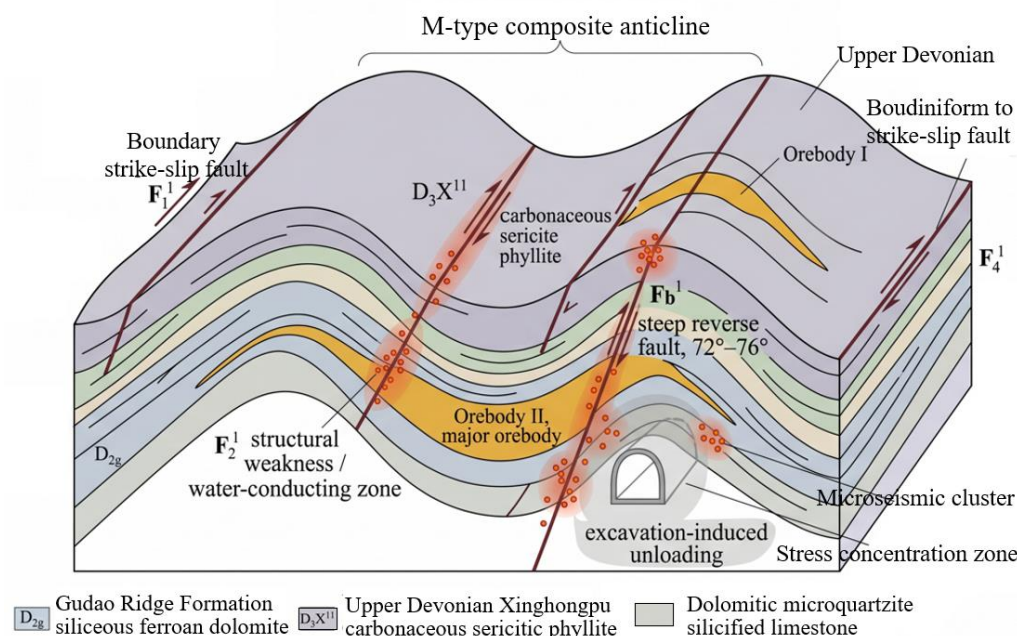


Figure 2. Conceptual geological and engineering setting of the Dongtangzi Pb–Zn deposit.

The main lithologies include siliceous ferroan dolomite, ferroan dolomitic microquartzite, silicified limestone, and carbonaceous sericite phyllite. The orebodies are mainly stratiform to lenticular and are controlled by the Qiandongshan–Dongtangzi composite anticline. The orebody is buried at a depth of approximately 500 m, resulting in a relatively high in situ stress environment during underground mining.

The mining area is affected by folds, strike-slip faults, reverse faults, and lithological interfaces. These structures disturb the continuity of the rock mass and locally control stress redistribution. In particular, fold hinges, fault intersections, and weak lithological contact zones tend to become areas of stress concentration and fracture development. Under mining-induced unloading, these zones are prone to brittle failure, crack propagation, and MS activity.

The mine contains multiple production levels, and excavation has gradually advanced toward deeper parts of the orebody. As mining progresses, the surrounding rock experiences continuous stress adjustment, which may cause local energy accumulation and instability.

3.2. MS Monitoring Data

MS monitoring data from the Dongtangzi lead–zinc mine were used in this study. The complete monitoring catalog covers the period from April 2022 to May 2024 and includes 127,703 valid records after data preprocessing. Each record contains event time, event type, spatial coordinates, seismic energy, seismic moment, source radius, stress drop, peak acceleration, magnitude-related parameters, dominant frequency, ringing count, and monitoring region.

To support forward validation of the proposed risk–gradient method, the dataset was divided into two parts. Events from April 2022 to March 2023 were used for probability modeling, risk-field construction, and risk–gradient calculation. Events from April 2023 to May 2024 were retained as independent validation data to examine whether the extracted risk–gradient direction and risk front were consistent with subsequent MS activity.

During the modeling period, 97,853 event records were included, consisting of 90,535 MS events and 7318 blasting events. The MS events were used as the main data source for risk analysis, whereas blasting events were identified separately to reduce the influence of artificial

excavation signals on instability-risk characterization. During the validation period, 29,850 records were retained, including 20,879 MS events and 8971 blasting events.

The monitoring events cover the main mining-affected region of the deposit and provide sufficient spatial and temporal information for constructing the probabilistic risk field. In this study, historical MS information was first transformed into event-level and window-level risk indicators, then used to calculate instability probability at assessment points, and finally extended into a continuous risk field for gradient-based directional analysis.

3.3. Assessment Points and Spatial Grid

A total of 120 assessment points were arranged in the main mining-affected area. These points were organized as a regular three-dimensional array with six points in the X-direction, five points in the Y-direction, and four layers in the Z-direction. The assessment points were uniformly distributed over the monitored mining area and were used not only as evaluation sites, but also as spatial calculation points for receiving and aggregating MS information. This arrangement provides three-dimensional coverage of the whole mining-affected region and allows the regional variation in the probability field to be reconstructed for subsequent gradient estimation. Each assessment point was assigned a unique site number and coordinate:

$$\mathbf{x}_j = (X_{c,j}, Y_{c,j}, Z_{c,j}) \quad (106)$$

The assessment points cover the main spatial range of MS activity and include four representative elevation layers. At each assessment point, historical MS events within the look-back window were aggregated to calculate event-level and window-level risk indicators. These indicators were then used in the SISSO-based probability function to obtain the instability probability $P(\mathbf{x}_j, t_k)$.

The grid nodes served as the basic units for risk-field reconstruction and gradient calculation (Figure 3). Before computing the gradient, spatial smoothing was applied to reduce local interpolation noise, and boundary grid nodes with low interpolation reliability were treated with caution. Thus, the assessment points provide discrete probability inputs, while the spatial grid supports continuous risk-field representation and directional risk-gradient analysis.

3.4. Selection of Representative Periods for Risk-Gradient Analysis

Risk-gradient analysis was performed on the time sequence of probabilistic risk fields rather than on isolated MS events. For the modeling period from April 2022 to March 2023, the MS catalog and assessment-point probabilities were aggregated using the same time interval. For each time window, the event count, cumulative seismic energy, mean instability probability, top 10% instability probability, and high-risk site ratio were calculated to characterize the temporal evolution of MS activity and risk response (Figure 4).

Representative periods were selected based on the joint variation in these indicators. As shown in the temporal evolution curves, four typical stages, denoted as P1–P4, were identified. P1 corresponds to an early risk-enhancement stage, during which the MS event count increased rapidly and the instability probability showed a clear rise. P2 is characterized by a pronounced event-count peak and relatively high energy release, indicating intensified MS activity. P3 reflects local risk enhancement, as indicated by the increase in the top 10% instability probability. P4 represents a typical high-risk stage, during which both the mean probability and the high-risk site ratio remained at relatively high levels.

These representative periods were used for detailed risk-field and risk-gradient analysis in the following sections. The selection was intended to cover different risk-evolution

states, including early enhancement, strong MS activity, local risk intensification, and typical high-risk development. Therefore, the subsequent gradient analysis was not restricted to a single abnormal event, but focused on representative stages extracted from the continuous temporal evolution of the monitoring data.

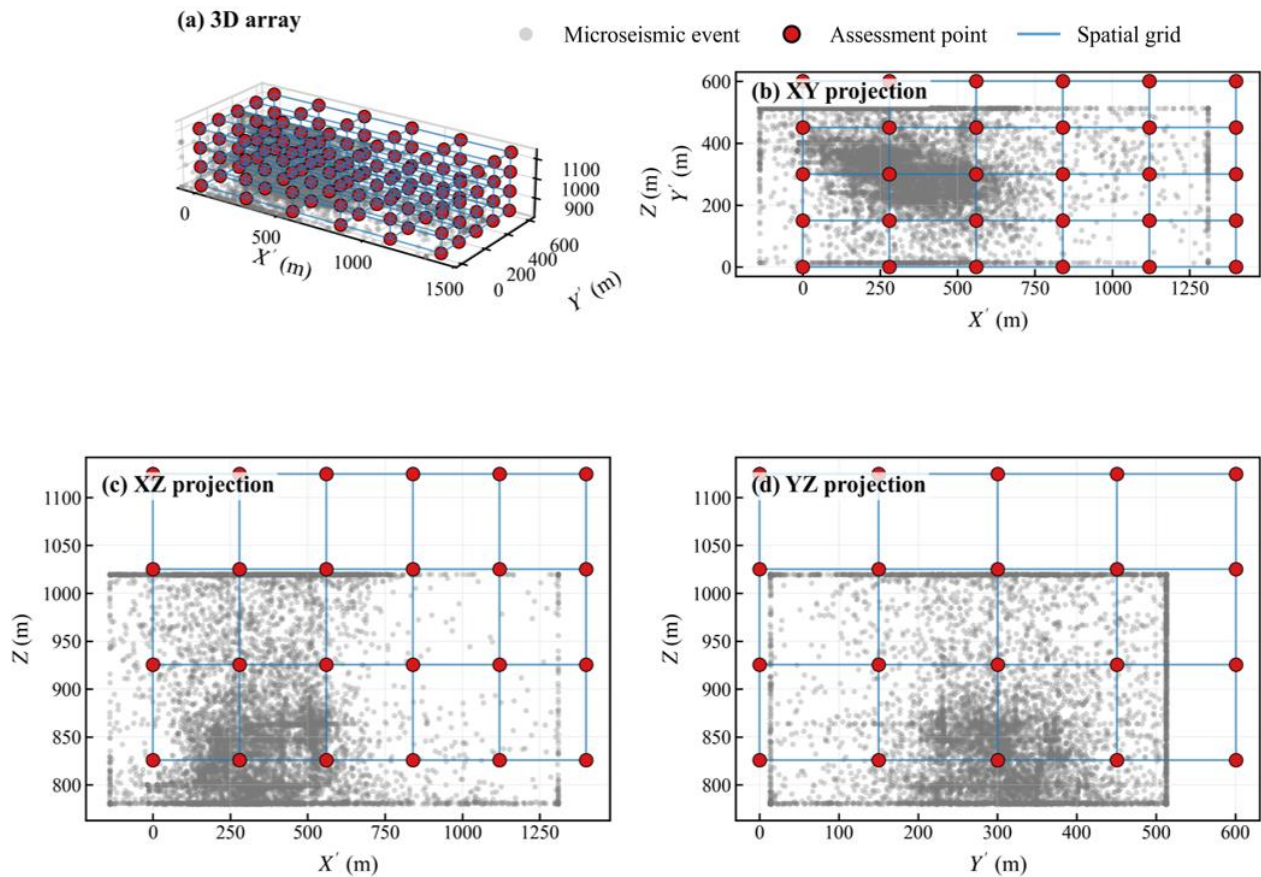


Figure 3. Assessment points and spatial grid used for risk-field reconstruction.

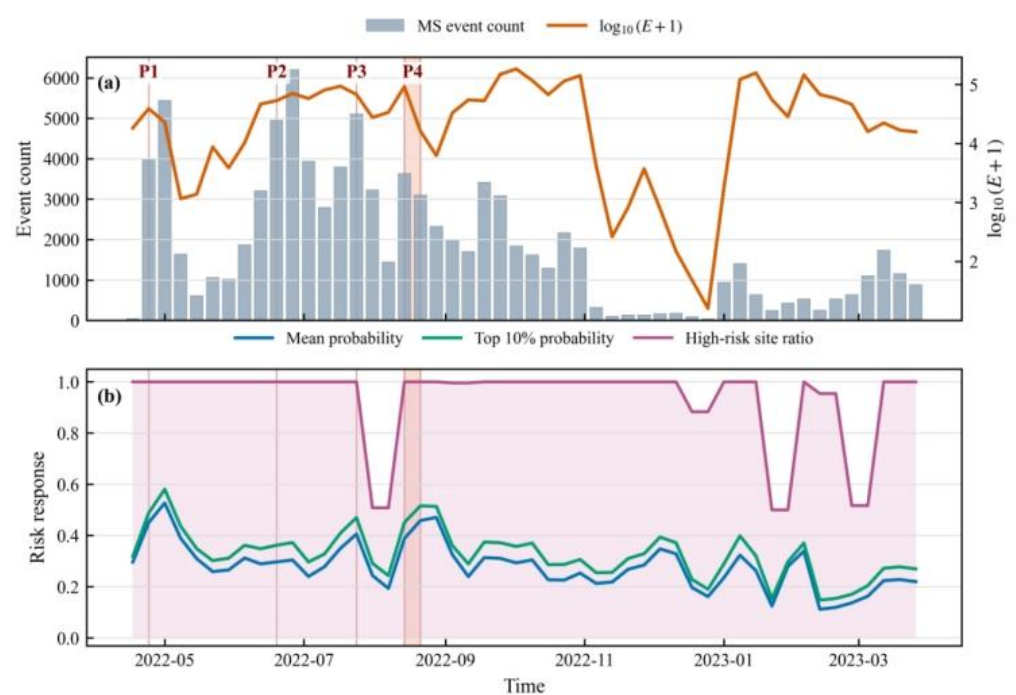


Figure 4. Selection of representative periods for risk–gradient analysis based on microseismic activity and risk–response indicators. (a) Temporal variation in MS event count and cumulative energy release; (b) temporal variation in mean instability probability, top 10% instability probability, and high-risk site ratio.

4. Results and Discussion

4.1. Spatial Distribution of the Risk–Gradient Field

Figure 5 shows the spatial distributions of the projected risk field and risk–gradient magnitude during the four representative periods. The upper row presents the maximum projected instability probability, while the lower row presents the corresponding risk–gradient magnitude.

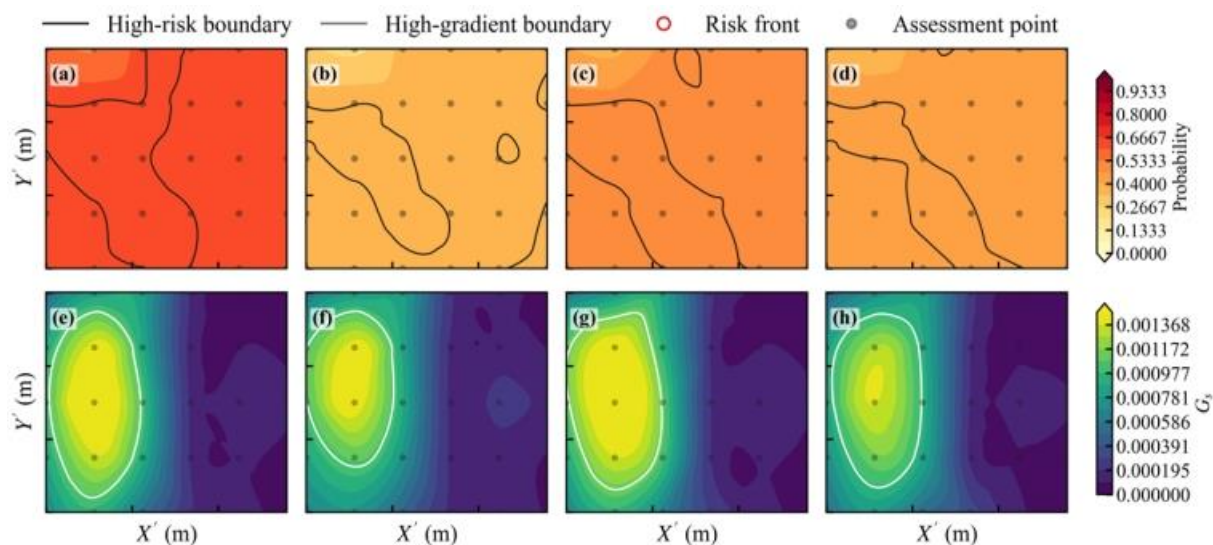


Figure 5. Spatial distribution of risk field and risk–gradient magnitude during representative periods. (a–d) Projected instability probability fields for P1–P4; (e–h) projected risk–gradient magnitude fields for P1–P4.

The risk field shows clear spatial heterogeneity in all four periods. High-risk zones are locally distributed rather than uniformly developed, indicating that instability probability is concentrated in specific areas. Compared with the risk field, the risk–gradient field exhibits more distinct spatial localization. The high-gradient zones mainly appear in the left part of the domain and form elliptical or banded regions, suggesting that risk changes most sharply in these areas.

The comparison between the two rows indicates that high-risk zones and high-gradient zones are not completely coincident. The risk field reflects where instability probability has accumulated, whereas the risk–gradient field identifies where the probability changes rapidly. Therefore, high-gradient zones can provide additional boundary information and may represent potential risk fronts or transition zones for instability development.

Among the four periods, P1 and P3 show relatively stronger gradient responses, while P2 and P4 show similar but slightly weaker high-gradient patterns. These results indicate that risk–gradient analysis can reveal spatial risk–transition characteristics beyond the probability field itself and provides a basis for subsequent analysis of risk propagation direction.

4.2. Directional Characteristics of Risk Propagation

Figure 6 presents the directional characteristics of the risk–gradient field during the four representative periods. The background color shows the projected risk–gradient

magnitude, while the black arrows indicate local horizontal gradient directions. The red dashed arrows further represent the dominant propagation direction extracted from the high-gradient region.

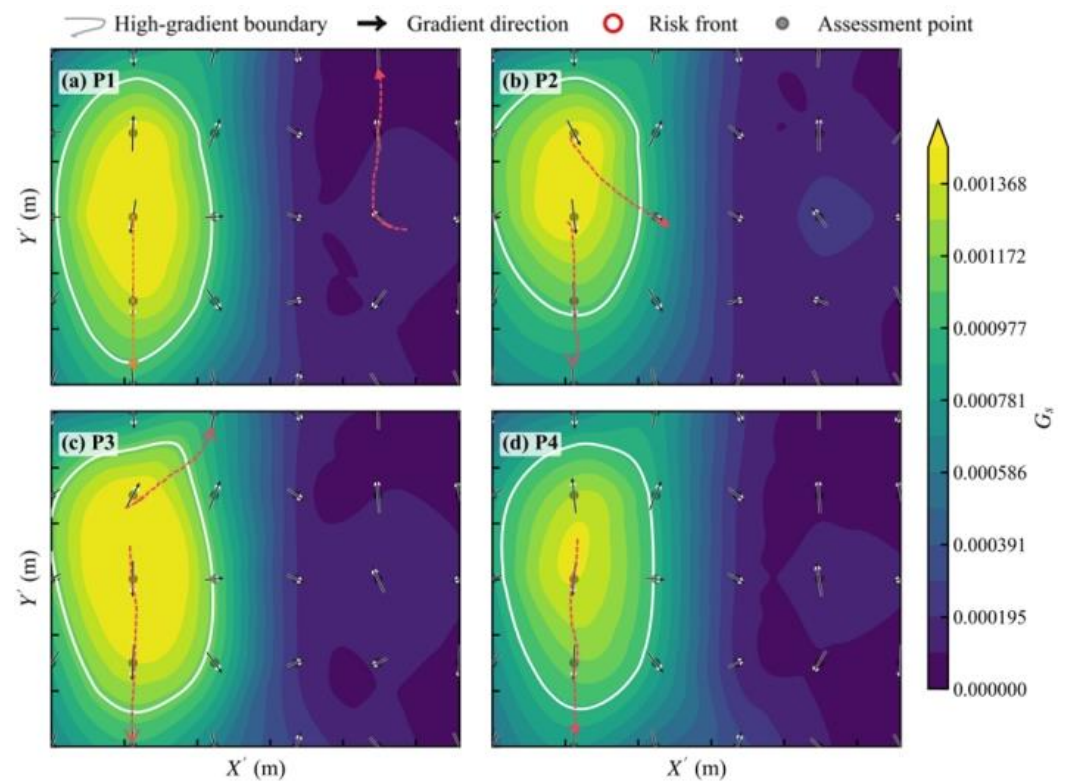


Figure 6. Directional characteristics of the risk–gradient field during representative periods. (a–d) Projected risk–gradient magnitude and horizontal gradient directions for P1–P4.

The results show that the risk–gradient directions are not randomly distributed. In all four periods, the strongest gradient responses are mainly concentrated in the left part of the domain, and the local gradient vectors generally point from the high-gradient core toward the surrounding lower-gradient region. This indicates that the risk field has a clear directional transition near the high-gradient boundary.

During P1, the dominant direction is mainly downward along the high-gradient zone, suggesting that the risk transition was concentrated around a vertically elongated boundary. In P2 and P3, the dominant direction shows an oblique component, indicating that the risk propagation tendency was not limited to a single vertical direction but included lateral migration. In P4, the dominant direction becomes more localized and is mainly distributed near the central high-gradient region, implying a relatively concentrated risk-transfer pattern.

Overall, the directional analysis confirms that the risk–gradient field can provide propagation information beyond the risk magnitude itself. The high-gradient boundary identifies where the risk changes sharply, while the gradient direction indicates the preferential direction of risk increase or transfer.

4.3. Identification of Risk Fronts and High-Gradient Boundaries

Figure 7 shows the identification results of high-risk zones, high-gradient boundaries, and potential risk-front regions during the representative periods. The background represents the projected instability probability field, the black solid lines denote high-risk boundaries, and the blue dashed lines represent high-gradient boundaries.

The high-risk zones and high-gradient boundaries show clear spatial differences. The high-risk boundaries mainly enclose areas with elevated instability probability, whereas the high-gradient boundaries are concentrated along the zones where the risk field changes sharply. In all four periods, the high-gradient boundaries are mainly distributed in the left part of the study domain, indicating that this region represents a persistent transition zone of instability risk.

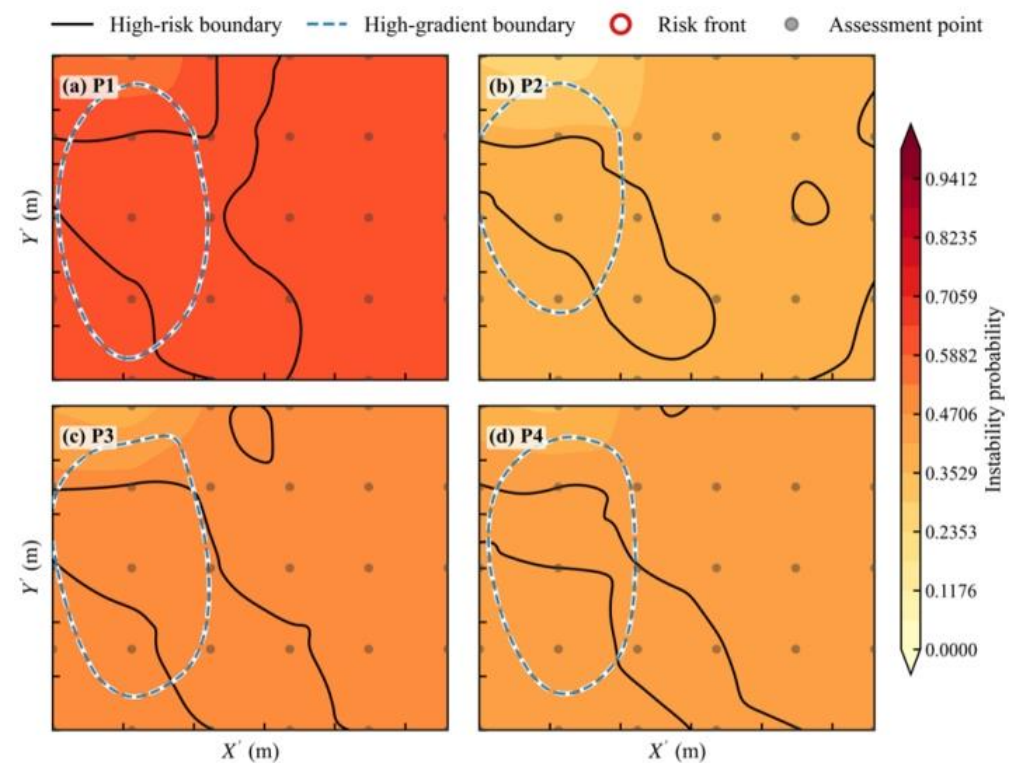


Figure 7. Identification of risk fronts and high-gradient boundaries during representative periods. The background color represents the maximum projected instability probability.

Table 1 summarizes the statistical characteristics of the high-risk zones and high-gradient boundaries. Each period contains 120 assessment points. Under the selected thresholds, 24 high-risk points and 12 high-gradient points were extracted for each period. P1 shows the highest mean instability probability, with a mean R of 0.605 and a maximum R of 0.634, indicating the strongest overall risk level. P3 has the highest mean and maximum gradient magnitude, with a mean G_s of 0.000280 and a maximum G_s of 0.001614, suggesting the strongest spatial risk variation.

Table 1. Statistical characteristics of high-risk zones and high-gradient boundaries during representative periods.

Period	Total Points	High-Risk Points	High-Gradient Points	Mean (R)	Mean (G_s)	Max (R)	Max (G_s)
P1	120	24	12	0.605	0.000264	0.634	0.001590
P2	120	24	12	0.337	0.000269	0.366	0.001442
P3	120	24	12	0.459	0.000280	0.488	0.001614
P4	120	24	12	0.419	0.000244	0.443	0.001374

According to the strict definition in Equation (28), the direct intersection between high-risk points and high-gradient points was empty in the discrete three-dimensional assessment grid. Therefore, no strictly defined point-type risk front was obtained, and the

threshold conditions were not relaxed to force a non-empty intersection. Instead, the risk front was interpreted as a transition belt formed by high-gradient points near the margin of the high-risk region.

This interpretation is consistent with the gradient-based definition, because the maximum gradient is expected to occur where the risk field changes most sharply, usually near the boundary rather than at the center of a high-risk region. The width of the transition belt was not determined by an additional empirical threshold, but was controlled by the reconstructed grid resolution and the smoothing scale used before gradient calculation. Therefore, the extracted high-gradient boundary should be regarded as a regional risk-transition zone supporting the analysis of potential risk expansion and instability migration, although its exact position may still be influenced by assessment-point density and interpolation.

4.4. Migration Path and Validation Using Subsequent MS Activity

Figure 8 compares the risk–gradient migration pattern with the subsequent MS activity. The projected risk–gradient magnitude in P4 is used as the background field, while the high-gradient boundary, risk centroid path, gradient centroid path, and monthly energy-weighted MS centroids are superimposed for comparison.

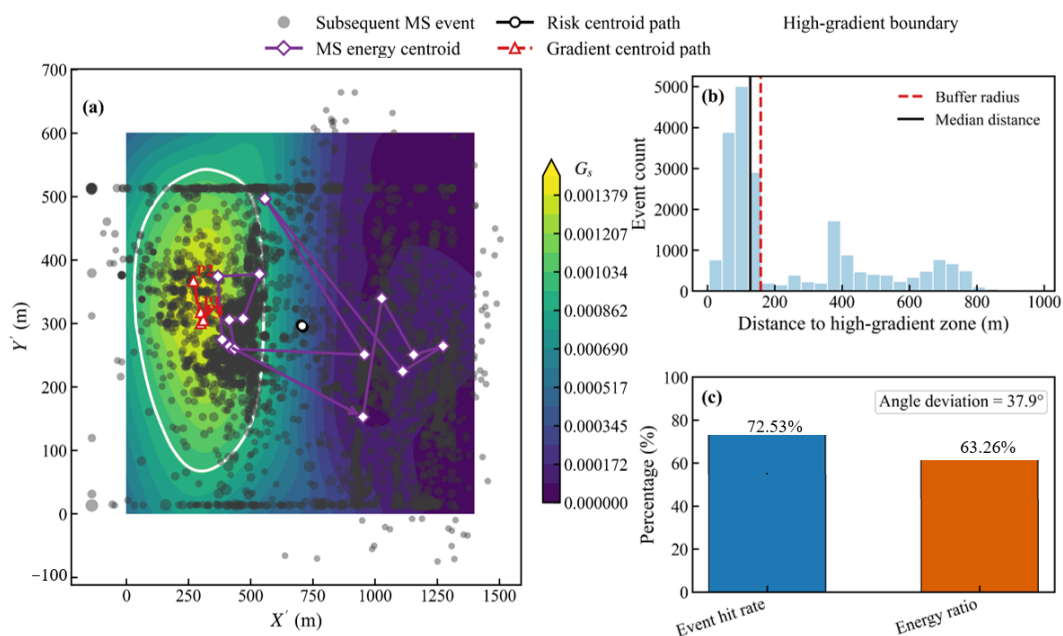


Figure 8. Migration path and validation using subsequent MS activity. (a) Spatial comparison between risk–gradient migration and subsequent MS activity; (b) distance distribution from subsequent MS events to the high-gradient zone; (c) event hit rate and energy ratio within the high-gradient buffer zone.

The subsequent MS events show evident spatial clustering near the high-gradient zone. As shown in Figure 8a, a large number of events are distributed around the left-side high-gradient boundary, indicating that the extracted high-gradient zone has a clear spatial correspondence with later MS activity. The risk and gradient centroid paths are also located close to this active region, suggesting that the risk–gradient field can capture the potential migration area of instability-related activity.

The distance-based validation further supports this observation. In Figure 8b, most subsequent events are concentrated within a short distance from the high-gradient zone, and the median distance is smaller than the buffer radius. Quantitatively, 72.53% of subsequent MS events fall within the high-gradient buffer zone, and these events account for

63.26% of the total released energy (Figure 8c). The migration direction of the gradient centroid path and the MS energy centroid path has an angular deviation of 37.9° , indicating a moderate directional consistency.

To further examine the spatial organization of subsequent MS activity, DBSCAN clustering was performed using the three-dimensional coordinates of MS events in the validation period. The neighborhood radius was determined from the k-distance curve, giving $\text{eps} = 55.91$ m with $\text{MinPts} = 20$ (Figure 9). A total of 20,879 subsequent MS events were included in the analysis.

The clustering results show that the subsequent MS events were highly clustered rather than randomly distributed. Two clusters were identified, and clustered events accounted for 96.97% of all subsequent MS events, while noise events accounted for only 3.03%. The dominant cluster C0 contained 96.87% of the events and contributed 96.70% of the total released energy, indicating that later fracture activity was mainly concentrated in one dominant spatial cluster.

The spatial relationship between the DBSCAN clusters and the high-gradient points further indicates that the extracted high-gradient boundary is mainly located near the margin of the dominant MS cluster rather than within its central region. The energy-weighted centroid of C0 was 258.47 m from the nearest high-gradient point.

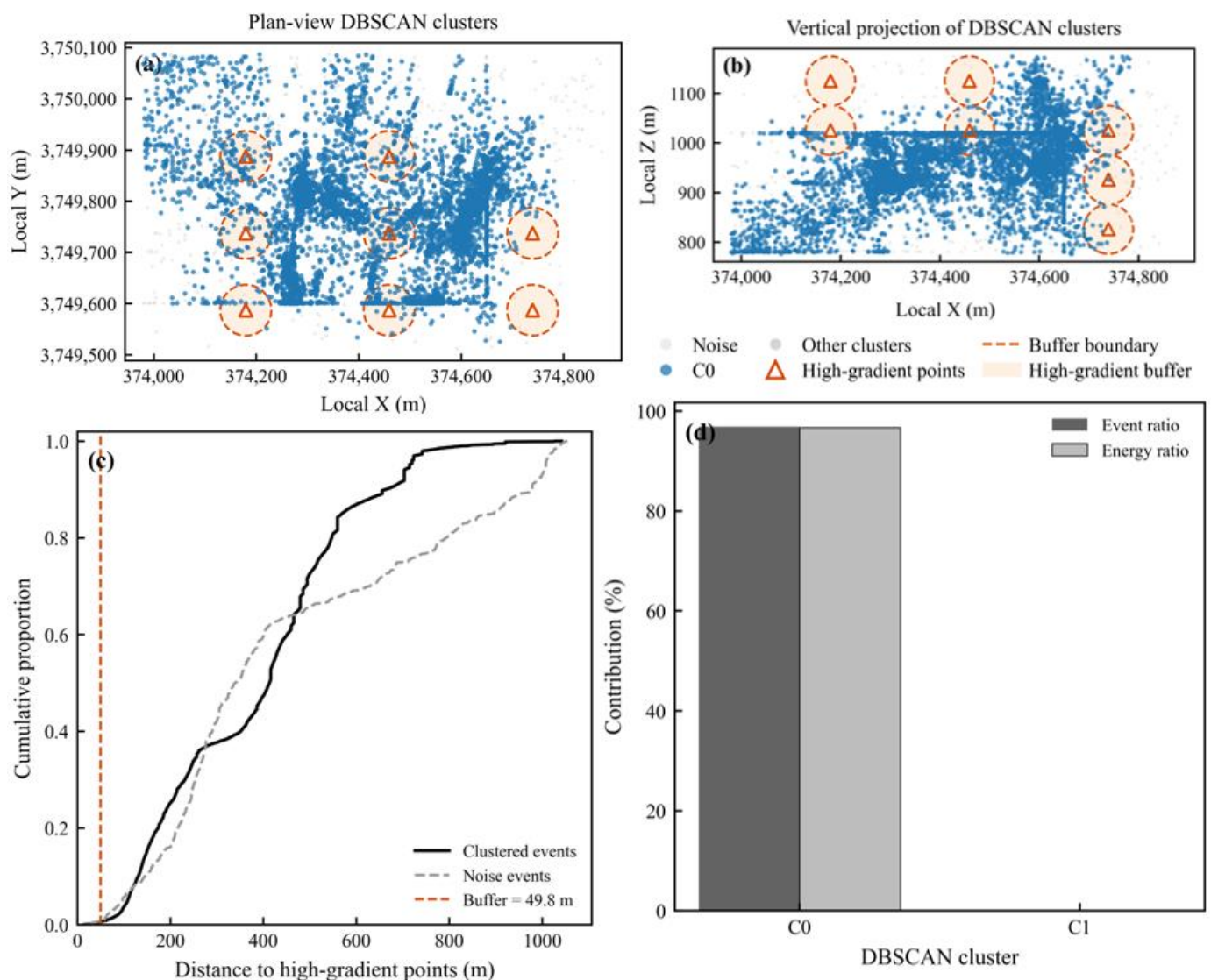


Figure 9. DBSCAN clustering analysis of subsequent microseismic activity and its spatial relationship with the high-gradient boundary. (a) Plan-view projection of DBSCAN clusters and high-gradient points; (b) vertical projection of DBSCAN clusters and high-gradient points; (c) cumulative distance distribution from clustered and noise events to the high-gradient points; (d) event and energy contributions of the identified DBSCAN clusters.

Overall, the DBSCAN results indicate that subsequent MS activity developed as a spatially clustered process, while the risk–gradient field highlighted the adjacent transition boundary of this clustered activity. Therefore, the high-gradient boundary can provide supplementary information for identifying potential risk-transition zones and migration margins. Nevertheless, this relationship should be interpreted as a spatial indication rather than deterministic prediction, because subsequent microseismicity may also be influenced by excavation disturbance, structural discontinuities, and heterogeneous rock-mass conditions.

4.5. Sensitivity Analysis and Quantitative Comparison

The proposed risk–gradient field involves several user-defined parameters, including the interpolation method, smoothing bandwidth, grid resolution, and quantile thresholds used to define high-risk and high-gradient regions. To examine the influence of these parameters on the extracted high-gradient zones, a sensitivity analysis was performed using P4 as the representative period. The baseline setting adopted inverse-distance weighting interpolation, the original spatial grid, a Gaussian smoothing parameter of 0.8, and quantile thresholds of $Q_R = 0.80$ and $Q_G = 0.90$. Several perturbed cases were then tested by changing one parameter group at a time, including linear and nearest-neighbor interpolation, lower and higher smoothing levels, coarser and finer grids, and looser or stricter threshold combinations.

The sensitivity analysis was evaluated using the spatial overlap ratio between the perturbed and baseline high-gradient zones, the displacement of the high-gradient centroid, and the event hit rate and energy ratio of subsequent MS activity within the high-gradient influence zone. The overlap ratio was used to measure the spatial consistency of the extracted high-gradient zones, whereas the centroid displacement reflected the overall spatial shift of the high-gradient region under parameter perturbation.

As shown in Figure 10, the interpolation method had a limited influence on the extracted high-gradient zones. The linear and nearest-neighbor interpolation cases showed an overlap ratio of 1.00 with the baseline result, and their centroid displacements were nearly zero. This indicates that the main high-gradient locations were not sensitive to the interpolation scheme under the present assessment-point configuration. The threshold perturbation also produced relatively stable results. When the thresholds were relaxed to $Q_R = 0.75$ and $Q_G = 0.85$, the overlap ratio remained 0.67, and the centroid displacement was only 11.35 m. When the thresholds were tightened to $Q_R = 0.85$ and $Q_G = 0.95$, the overlap ratio decreased to 0.50, but the centroid displacement remained limited at 27.94 m. These results indicate that the threshold setting mainly affects the number and local extent of extracted high-gradient points, but does not substantially change the dominant high-gradient region.

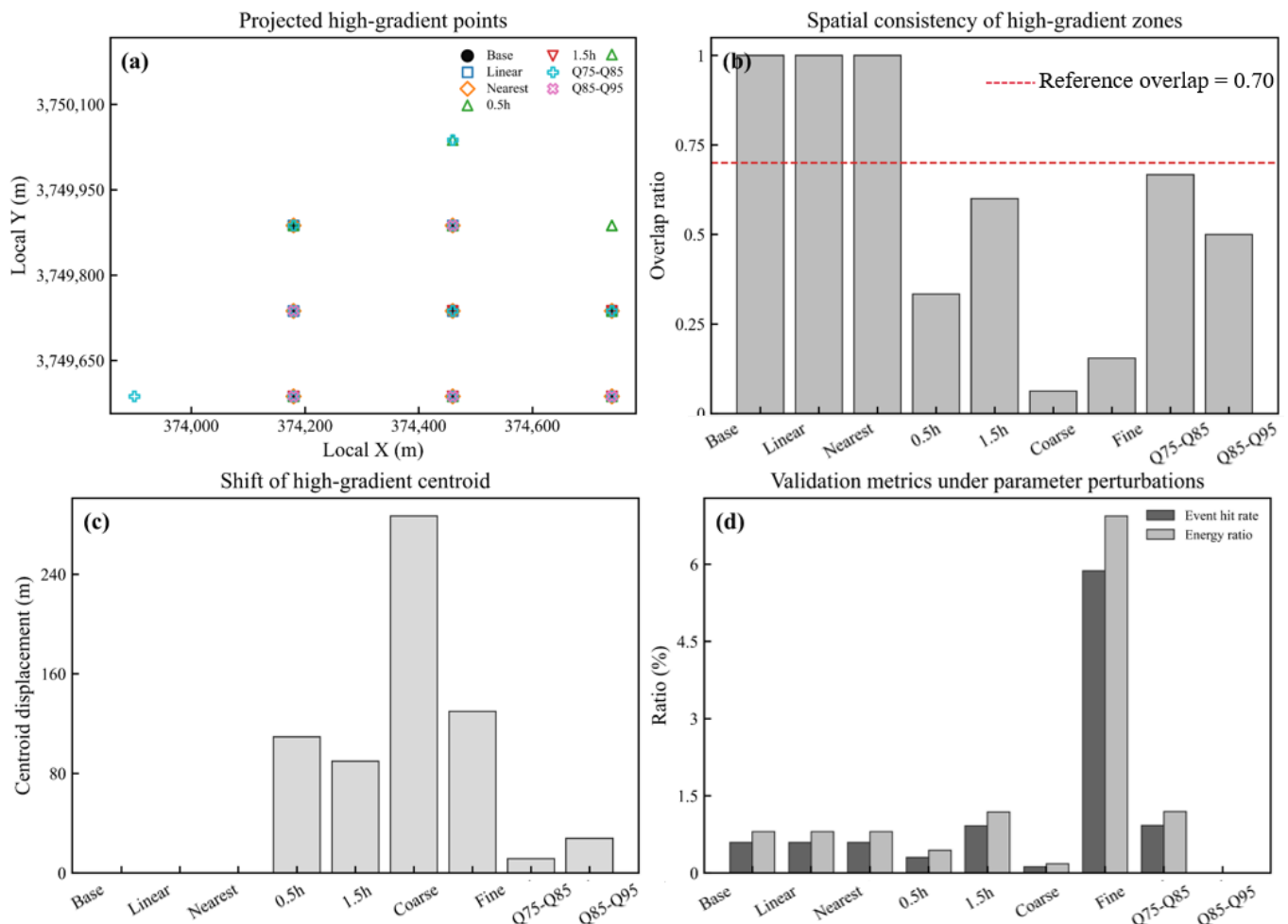


Figure 10. Sensitivity analysis of high-gradient boundary identification under different parameter settings. (a) Projected high-gradient points under representative parameter settings; (b) overlap ratio between perturbed and baseline high-gradient zones; (c) displacement of the high-gradient centroid; (d) event hit rate and energy ratio of subsequent MS activity within the high-gradient influence zone.

In contrast, the smoothing bandwidth and grid resolution had a stronger influence on the local morphology of the high-gradient field. Reducing the smoothing parameter to 0.5 h led to an overlap ratio of 0.33 and a centroid displacement of 109.67 m, while increasing the smoothing parameter to 1.5 h improved the overlap ratio to 0.60 and reduced the centroid displacement to 89.61 m. The grid-resolution tests showed the largest variation. The coarse-grid case had an overlap ratio of 0.06 and a centroid displacement of 286.85 m, whereas the fine-grid case had an overlap ratio of 0.15 and a centroid displacement of 129.50 m. This suggests that gradient-based boundary extraction is relatively sensitive to spatial discretization, because the gradient magnitude is calculated from spatial differences of the reconstructed risk field.

Overall, the sensitivity analysis indicates that the proposed risk-gradient framework is relatively robust to changes in interpolation method and moderate threshold perturbation, but is more sensitive to smoothing bandwidth and grid resolution. This is consistent with the derivative-based nature of the risk-gradient calculation. The main risk-transition area remains identifiable under moderate parameter changes, although the exact number, shape, and local position of high-gradient points may vary. Therefore, the risk-gradient field should be interpreted as a regional diagnostic tool for identifying risk-transition

boundaries and migration margins, rather than as a point-wise deterministic prediction of subsequent MS events.

To further compare the proposed method with conventional risk-assessment indicators, the probability–risk field, MS event-density field, MS energy-density field, and risk–gradient field were evaluated using the same subsequent MS dataset. For each method, the top 10% assessment points were extracted, corresponding to the same spatial proportion of 10%.

As shown in Table 2, the risk–gradient field achieved the highest event capture rate of 59.3%, the highest energy capture ratio of 38.1%, and the shortest median distance of 71.848 m. The probability–risk field also showed relatively high capture performance, with an event capture rate of 28.7% and an energy capture ratio of 35.3%, but its median distance was larger than that of the risk–gradient field. In contrast, the MS event-density and energy-density fields showed lower event and energy capture rates.

Table 2. Quantitative comparison between the proposed risk–gradient field and conventional risk-assessment indicators.

Method	Extracted Zone	Spatial Proportion (%)	Event Capture Rate (%)	Energy Capture Ratio (%)	Median Distance (m)
Probability–risk field	Top 10% R	10	28.7	35.3	95.635
MS event-density field	Top 10% event density	10	20.4	18.0	155.227
MS energy-density field	Top 10% energy density	10	20.6	18.6	148.977
Risk–gradient field	Top 10% Gs	10	59.3	38.1	71.848

These results indicate that the risk–gradient field provides additional quantitative information beyond conventional probability and density-based indicators. It is more effective in capturing subsequent MS activity near risk-transition boundaries, supporting its use for identifying potential risk-propagation margins and migration-related zones.

4.6. Prospects and Limitations

This study developed a risk–gradient analysis framework for rock-mass instability in deep mining by combining MS monitoring, SISSO-based probabilistic modeling, three-dimensional probabilistic risk-field construction, and directional gradient analysis. The results indicate that the proposed framework can not only describe the spatial distribution of instability probability, but also extract high-gradient boundaries, identify potential risk fronts, and characterize the dominant direction of risk propagation. Compared with conventional risk zoning based only on probability magnitude, the proposed method provides a more informative representation of risk evolution and offers a potential basis for targeted monitoring and engineering control. Nevertheless, several aspects still require further improvement and extension.

- (1) The proposed framework extends risk assessment from probability mapping to directional risk-evolution analysis, but the interpretation of risk propagation still needs further validation. In this study, the risk–gradient field was used to identify high-gradient boundaries and dominant propagation directions, and subsequent MS activity was introduced to verify its spatial indication capability. The results suggest that high-gradient zones have a certain correspondence with later MS clustering, indicating that the gradient field can provide useful information beyond the probability field itself. However, the validation remains mainly based on spatial consistency and direction comparison. Future studies may incorporate longer monitoring sequences, multiple mining stages, and independent case studies to further evaluate the stability and general applicability of risk-gradient-based propagation analysis.

- (2) The current risk–gradient field provides a continuous spatial description of risk variation, but its accuracy is still affected by data density and field-reconstruction parameters. The assessment points used in this study allow the discrete SISSO-based probabilities to be transformed into a continuous three-dimensional risk field, thereby supporting gradient calculation and directional analysis. However, the calculated gradient magnitude and direction are sensitive to grid resolution, interpolation strategy, smoothing parameters, and boundary effects. In regions with sparse monitoring coverage or strong geological heterogeneity, the reconstructed gradient field may contain local uncertainty. Future work should therefore introduce denser assessment grids, adaptive interpolation methods, uncertainty quantification, and geological constraints to improve the robustness of risk-front and high-gradient boundary identification.
- (3) The proposed method has shown potential for identifying risk fronts and migration paths, but the risk representation can be further expanded toward multi-factor and real-time applications. In the present study, instability probability derived from MS indicators was used as the main field quantity, and risk–gradient analysis was performed on this probability field. From an engineering safety perspective, however, risk is also related to consequence severity, exposure level, excavation sequence, geological structures, and support conditions. Future studies may integrate these factors into a more comprehensive risk field and update the field dynamically with real-time MS data. This would help transform the current post-analysis framework into a more practical tool for short-term risk tracking, early warning, and targeted support optimization in deep mining.

5. Conclusions

This study proposed a risk–gradient field method for identifying the propagation direction and migration path of rock-mass instability in deep mining. By integrating SISSO-based probabilistic modeling, three-dimensional risk-field reconstruction, and gradient-based directional analysis, the proposed framework extends conventional risk assessment from static probability zoning to direction-oriented risk-evolution diagnosis. The main conclusions are as follows.

- (1) An interpretable SISSO-based probability function was used to transform MS indicators into rock-mass instability probabilities, and these probabilities were further reconstructed into a continuous three-dimensional probabilistic risk field. This provides a quantitative basis for representing the spatial distribution of instability risk from discrete MS responses.
- (2) The risk–gradient field reveals information that cannot be obtained from the probability field alone. The probabilistic risk field identifies where instability risk accumulates, whereas the risk–gradient field highlights where risk changes sharply. The results show that high-gradient zones are mainly located near the margins of high-risk regions, indicating that they can be interpreted as potential risk-transition boundaries or risk-front zones.
- (3) The direction of the risk gradient provides a basis for identifying the preferential propagation direction of instability risk. During the representative periods, the high-gradient regions showed clear directional characteristics, demonstrating that the proposed method can characterize not only the intensity of risk variation, but also the possible migration tendency of instability risk.
- (4) Validation using subsequent MS activity confirms the spatial indication capability of the risk–gradient field. A total of 72.53% of subsequent MS events and 63.26% of released energy were located within the high-gradient buffer zone, and the angular deviation between the risk–gradient migration direction and the MS energy-centroid

migration direction was 37.9°. These results indicate that the high-gradient boundary can serve as an effective indicator for identifying potential instability migration zones in deep mining.

Author Contributions: Conceptualization, G.L., F.Y. and F.H.; Methodology, G.L. and F.Y.; Software, G.L., F.Y. and J.C.; Validation, G.L., F.Y. and F.H.; Formal analysis, G.L. and F.H.; Investigation, F.Y. and J.W.; Resources, F.Y.; Data curation, F.Y. and J.W.; Writing—original draft, G.L.; Writing—review and editing, F.Y.; Visualization, G.L. and J.C.; Supervision, F.Y.; Project administration, F.Y.; Funding acquisition, F.Y. All authors have read and agreed to the published version of the manuscript.

Funding: The study is supported by the National Natural Science Foundation of China (52474280, 52104108).

Data Availability Statement: The data presented in this study are available on request from the corresponding author due to confidentiality agreements and operational safety restrictions associated with the operating mine and its microseismic monitoring records.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Zhao, Z.; Tan, Y.; Chen, S.; Ma, Q.; Gao, X. Theoretical analyses of stress field in surrounding rocks of weakly consolidated tunnel in a high-humidity deep environment. *Int. J. Rock Mech. Min. Sci.* **2019**, *122*, 104064. <https://doi.org/10.1016/j.ijrmms.2019.104064>.
2. Wu, S.; Shi, C.; Zhang, J.; Zhang, C.; Zhang, Y.; Li, W. Analytical solution for the stress field in surrounding rock of a near-fault tunnel considering tectonic stress boundary conditions. *Rock Mech. Rock Eng.* **2024**, *57*, 7013–7031. <https://doi.org/10.1007/S00603-024-03893-8>.
3. Ming, C.; He, M.; Wang, J.; Liu, J.; Qi, J.; Coli, M. Experimental and numerical study of stress compensation and optimization for non-pillar retaining roadway. *Rock Mech. Rock Eng.* **2025**, *59*, 5263–5290. <https://doi.org/10.1007/S00603-025-05039-W>.
4. Liu, H.; Fall, M. Mechanical response and pore pressure evolution of cemented paste backfill under deep mine-like multiaxial stress and temperature conditions. *Int. J. Min. Sci. Technol.* **2026**, *36*, 457–474. <https://doi.org/10.1016/J.IJMST.2025.09.013>.
5. Li, B.; Xu, N.; Xiao, P.; Xia, Y.; Zhou, X.; Gu, G.; Yang, X. Microseismic monitoring and forecasting of dynamic disasters in underground hydropower projects in southwest China: A review. *J. Rock Mech. Geotech. Eng.* **2023**, *15*, 2158–2177. <https://doi.org/10.1016/J.JRMGE.2022.10.017>.
6. Liu, F.; Wang, Y.; Kou, M.; Liang, C. Applications of microseismic monitoring technique in coal mines: A state-of-the-art review. *Appl. Sci.* **2024**, *14*, 1509. <https://doi.org/10.3390/APP14041509>.
7. Shu, H.; Dawod, A.Y. Microseismic monitoring signal waveform recognition and classification: Review of contemporary techniques. *Appl. Sci.* **2023**, *13*, 12739. <https://doi.org/10.3390/APP132312739>.
8. Cheng, T.; He, M.; Li, H.; Liu, D.; Qiao, Y. Effects of structural plane on rockburst in tunnel: A review based on published cases. *Eng. Geol.* **2025**, *356*, 108245. <https://doi.org/10.1016/J.ENGGEOL.2025.108245>.
9. Yan, F.; Li, G.; Dong, L.; Du, S.; He, F. Temporal analysis and early warning of rock instability in hard rock mines based on CNN-LSTM and CUSUM. *J. Cent. South Univ.* **2026**, *in press*. <https://doi.org/10.1007/s11771-026-6325-z>.
10. Liu, Z.; Li, Y.; Wang, H.; Huang, R.; Wang, L. Characteristics and formation mechanism of axial chain rockbursts in deep-buried tunnels traversed by stiff rock vein through gneiss strata. *Eng. Geol.* **2026**, *366*, 108664. <https://doi.org/10.1016/J.ENGGEOL.2026.108664>.
11. Liu, J.; Yang, S.; Li, L.; Pan, X.; Yin, X.; Hu, J.; Peng, F.-H. Precursory characteristics of large-scale rockmass collapse based on multiple microseismic parameters. *J. Cent. South Univ.* **2024**, *31*, 878–889. <https://doi.org/10.1007/s11771-024-5610-y>.
12. Dong, L.; Yan, X.; Wang, J.; Tang, Z. Case study of microseismic tomography and multi-parameter characteristics under mining disturbances. *J. Cent. South Univ.* **2023**, *30*, 2252–2265. <https://doi.org/10.1007/s11771-023-5358-9>.
13. Ma, K.; Shen, Q.; Zhang, Z.; Wang, T. Development and validation of intelligent models for predicting rockburst location based on microseismic monitoring and machine learning. *Rock Mech. Rock Eng.* **2025**, *58*, 5093–5113. <https://doi.org/10.1007/S00603-025-04388-W>.

14. Zheng, L.; Lin, S.; Guo, H.; Cao, X.; Zheng, H. Real-time rockburst assessment based on a novel hybrid convolutional long short-term memory network based on microseismic monitoring data. *Eng. Fail. Anal.* **2025**, *169*, 109191. <https://doi.org/10.1016/J.ENGFAILANAL.2024.109191>.
15. Xia, K.; Chen, C.; Deng, Y.; Xiao, G.; Zheng, Y.; Liu, X.; Fu, H.; Song, X.; Chen, L. In situ monitoring and analysis of the mining-induced deep ground movement in a metal mine. *Int. J. Rock Mech. Min. Sci.* **2018**, *109*, 32–51. <https://doi.org/10.1016/j.ijrmms.2018.06.014>.
16. Yu, J.; He, H.; Liu, Z.; He, X.; Zhou, F.; Song, Z.; Yang, D. Analysis of spatiotemporal characteristics of microseismic monitoring data in deep mining based on ST-DBSCAN clustering algorithm. *Processes* **2025**, *13*, 2359. <https://doi.org/10.3390/PR13082359>.
17. Basnet, P.M.S.; Jin, A.; Mahtab, S. Applying machine learning approach in predicting short-term rockburst risks using MS information: A comparison of parametric and non-parametric models. *Nat. Hazards* **2024**, *121*, 731–758. <https://doi.org/10.1007/S11069-024-06794-9>.
18. Mahmoodzadeh, A.; Ghazouani, N.; Mohammed, A.H.; Ibrahim, H.H.; Alghamdi, A.; Albaijan, I.; El Ouni, M.H. Predicting rockbursts in deep tunnels based on ejection velocity and kinetic energy measurements using advanced machine learning. *Autom. Constr.* **2024**, *166*, 105671. <https://doi.org/10.1016/J.AUTCON.2024.105671>.
19. Ouyang, R.; Curtarolo, S.; Ahmetcik, E.; Scheffler, M.; Ghiringhelli, L.M. SISSO: A compressed-sensing method for identifying the best low-dimensional descriptor in an immensity of offered candidates. *Phys. Rev. Mater.* **2018**, *2*, 083802. <https://doi.org/10.1103/physrevmaterials.2.083802>.
20. Maihom, T.; Sittiwong, J.; Probst, M.; Boekfa, B.; Wattanakit, C.; Limtrakul, J. Predicting transition state and activation energies in n-hexane cracking over zeolites: Combined DFT calculations and estimations with the SISSO method. *J. Catal.* **2024**, *437*, 115656. <https://doi.org/10.1016/J.JCAT.2024.115656>.
21. Zhang, L.; Wang, Y.; Xiong, K. Axial ligand engineering of FeN₃O single-atom catalysts for enhanced trifunctional HER, OER, and ORR performance: A combined DFT and SISSO study. *Int. J. Hydrogen Energy* **2026**, *207*, 153518. <https://doi.org/10.1016/J.IJHYDENE.2026.153518>.
22. Li, C.Y.; Zhang, Z.R.; Yang, H.Y.; Larranaga, M.; Wu, M.Z.; McIntosh, J.; Wang, Q. The role of in-house safety professionals in influencing organizational safety performance across global industries. *J. Saf. Sustain.* **2025**, *2*, 95–103. <https://doi.org/10.1016/j.jsasus.2025.04.002>.
23. Stemn, E.; Martey, I.M.; Fosu, S. Descriptive and bowtie analysis of commercial fire outbreaks in Ghana: Threats, consequences and control measures. *J. Saf. Sustain.* **2025**, *2*, 293–306. <https://doi.org/10.1016/j.jsasus.2025.06.002>.
24. Lu, Y.T.; Zhang, B.K.; Qiu, Z.K.; Huang, S.Z.; Jiang, Y.; Song, C.H. TSC model for distribution networks considering extreme ice-coating conditions. *J. Saf. Sustain.* **2025**, *2*, 285–292. <https://doi.org/10.1016/j.jsasus.2025.08.001>.
25. Einstein, H.H. Risk and risk analysis in rock engineering. *Tunn. Undergr. Space Technol.* **1996**, *11*, 141–155. [https://doi.org/10.1016/0886-7798\(96\)00014-4](https://doi.org/10.1016/0886-7798(96)00014-4).
26. Dong, L.; Zhu, H.; Yan, F.; Bi, S. Risk field of rock instability using microseismic monitoring data in deep mining. *Sensors* **2023**, *23*, 1300. <https://doi.org/10.3390/S23031300>.
27. Yan, F.; Du, S.; Dong, L.; Li, X.; Li, G.; He, F. Evaluating rock instability risk areas in deep mining using microseismic monitoring-based risk fields. *J. Mt. Sci.* **2026**, *23*, 1528–1546. <https://doi.org/10.1007/s11629-025-0017-2>.
28. Xue, Y.; Song, D.; Chen, J.; Li, Z.; He, X.; Wang, H.; Zhou, C.; Sobolev, A. Integrated rockburst hazard estimation methodology based on spatially smoothed seismicity model and Mann-Kendall trend test. *Int. J. Rock Mech. Min. Sci.* **2023**, *163*, 105329. <https://doi.org/10.1016/j.ijrmms.2023.105329>.
29. Yan, F.; Cao, R.; Jin, C.; Li, Z.; Xu, K. Research and development of field theory-based three-dimensional risk assessment. *Part I Optim. Risk Reduction. Saf. Sci.* **2019**, *120*, 312–322. <https://doi.org/10.1016/j.ssci.2019.07.018>.
30. Li, G.; Liu, S.; Shi, K.; Hu, X. Normalized edge detectors using full gradient tensors of potential field. *Pure Appl. Geophys.* **2023**, *180*, 2327–2349. <https://doi.org/10.1007/s00024-023-03274-2>.
31. Xue, R.; Liang, Z.; Xu, N. Rockburst prediction and analysis of activity characteristics within surrounding rock based on microseismic monitoring and numerical simulation. *Int. J. Rock Mech. Min. Sci.* **2021**, *142*, 104750. <https://doi.org/10.1016/j.ijrmms.2021.104750>.
32. Wang, K.; Ma, K.; Tang, C.; Liu, H.; Wang, X.; Li, Q. Study on deep mining-induced strata behavior based on the evolutionary laws of multiple indices from microseismic monitoring. *Rock Mech. Rock Eng.* **2023**, *56*, 6481–6501. <https://doi.org/10.1007/s00603-023-03411-2>.

33. Khan, M.; He, X.; Song, D.; Tian, X.; Li, Z.; Xue, Y.; Aslam, K.S. Extracting and predicting rock mechanical behavior based on microseismic spatio-temporal response in an ultra-thick coal seam mine. *Rock Mech. Rock Eng.* **2023**, *56*, 3725–3754. <https://doi.org/10.1007/s00603-023-03247-w>.
34. Lu, A.; Song, D.; Li, Z.; He, X.; Zhou, C.; Qiu, L. Numerical simulation study on microseismic characteristics and rockburst hazard prediction in deep mining of steeply inclined coal seams. *Rock Mech. Rock Eng.* **2025**, *58*, 2465–2486. <https://doi.org/10.1007/s00603-024-04288-5>.
35. Gou, R.; Song, D.; He, X.; Zhang, J.; Guo, M.; He, S.; Li, Z.; Yang, G.; Qiu, L. Microseismic response characteristics and stress anomaly zoning in deep outburst-prone coal seams with multi-concealed structures: A case study. *J. Appl. Geophys.* **2025**, *242*, 105932. <https://doi.org/10.1016/j.jappgeo.2025.105932>.
36. Liu, H.L.; Huang, H.T.; Zhang, P.H.; Li, L.C.; Hou, J.X.; Zhang, L.Y.; Zhang, Z.L. Propagation path tracing of hydraulically created fractures based on microseismic monitoring. *Front. Earth Sci.* **2022**, *10*, 952694. <https://doi.org/10.3389/feart.2022.952694>.
37. Rousseeuw, P.J.; Croux, C. Alternatives to the median absolute deviation. *J. Am. Stat. Assoc.* **1993**, *88*, 1273–1283. <https://doi.org/10.1080/01621459.1993.10476408>.
38. Fan, J.; Lv, J. Sure independence screening for ultrahigh dimensional feature space. *J. R. Stat. Soc. Ser. B* **2008**, *70*, 849–911. <https://doi.org/10.1111/j.1467-9868.2008.00674.x>.
39. Hosmer, D.W.; Lemeshow, S.; Sturdivant, R.X. *Applied Logistic Regression*, 3rd ed.; Wiley: Hoboken, NJ, USA, 2013. <https://doi.org/10.1002/9781118548387>.
40. Li, J.; Heap, A.D. Spatial interpolation methods applied in the environmental sciences: A review. *Environ. Model. Softw.* **2014**, *53*, 173–189. <https://doi.org/10.1016/j.envsoft.2013.12.008>.
41. LeVeque, R.J. *Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems*; Society for Industrial and Applied Mathematics: Philadelphia, PA, USA, 2007. <https://doi.org/10.1137/1.9780898717839>.
42. Canny, J. A computational approach to edge detection. *IEEE Trans. Pattern Anal. Mach. Intell.* **1986**, *PAMI-8*, 679–698. <https://doi.org/10.1109/TPAMI.1986.4767851>.
43. Dong, G.; Ma, J.; Wei, R.; Haycox, J. Electric vehicle charging point placement optimisation by exploiting spatial statistics and maximal coverage location models. *Transp. Res. Part D Transp. Environ.* **2019**, *67*, 77–88. <https://doi.org/10.1016/j.trd.2018.11.005>.

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.