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Time-Variant Reliability Analysis via Copula Failure Correlation and Advanced Kriging

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Abstract

In practical engineering, mechanical structures are affected by time-dependent uncertainties and by correlated failure modes. This study proposes an enhanced time-variant reliability analysis framework by coupling a mixed Archimedean Copula model with an adaptive Kriging model driven by the maximum expected prediction error (MEPE) learning function. The mixed Copula combines Gumbel, Clayton and Frank components, so that upper-tail, lower-tail and nearly symmetric dependence can be represented in one model. The MEPE function combines Kriging prediction variance and leave-one-out cross-validation error through a dynamic balance factor, which guides early global exploration and later refinement near the most probable point trajectory. For the planar three-bar structure, the mixed-Copula failure probability at $t=7$ is 0.4078, close to the direct-MCS benchmark 0.4059, with a relative error of 0.47%; the number of original limit-state evaluations decreases from 1.00×10^6 to 326, corresponding to a 99.97% reduction. For the two-rod parallel structure, the mixed-Copula failure probability at $T=2$ is 0.0987, close to the direct-MCS benchmark 0.0992, with a relative error of 0.50%; the number of original limit-state evaluations decreases to 284. The calculated upper- and lower-tail dependence coefficients of the mixed Copula are 0.1345 and 0.2985 for the planar three-bar structure and 0.0003 and 0.3709 for the two-rod parallel structure. These quantitative results show that the proposed framework can describe nonlinear failure dependence more flexibly than a single Copula while retaining high computational efficiency.

Keywords: time-variant reliability analysis; hybrid Copula function; adaptive Kriging model; multiple failure modes; most probable failure point trajectory

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1. Introduction

With the advancement of science and technology, mechanical and structural systems in modern complex engineering fields such as aerospace, ocean engineering and large-scale civil infrastructure are inevitably affected by coupled dynamic uncertainties during long-term service. These factors mainly include material performance degradation and fluctuations in external environmental loads. Traditional time-invariant reliability analysis assumes that system parameters and loads remain unchanged over time. This method usually overestimates structural safety and may further cause potentially disastrous outcomes [1,2]. Within a specific reference span, time-variant reliability analysis (TRA) is used to calculate the probability that structures remain safe. It has become a core research topic and a cutting-edge hotspot in life-cycle design and engineering safety evaluation.

Current TRA approaches include surrogate-model-based methods and crossing-rate-based methods [3,4]. The reliability methods based on extreme value theory focus on extreme values within designated time intervals. Structural failure is judged once the corresponding samples fall into the failure domain, thus transforming the problem into time-invariant reliability analysis. Nevertheless, it is difficult to determine the extreme values of limit-state functions, which brings obvious difficulties to reliability assessment.

With the rapid development of machine learning, surrogate model-based methods have attracted widespread attention. Such methods replace time-consuming limit-state evaluation with surrogate models, which greatly improves the calculation efficiency of time-variant failure probability evaluation. To further improve the accuracy and efficiency of reliability analysis, different surrogate models have been proposed, including Kriging (Gaussian process regression) [5], polynomial chaos expansion [6] and support vector machines (SVMs) [7,8]. Among them, the Kriging model can achieve high-precision prediction for nonlinear responses and output prediction variance to quantify model uncertainty, thus playing a vital role in active learning-based reliability analysis [9]. A TRA method for earth dam slopes combining Gaussian process regression and Monte Carlo Simulation (MCS) was proposed [10]. The results verified that this method could evaluate time-dependent safety factors with acceptable computational efficiency. Aiming at the fatigue failure of production tubing in underground gas storage, a thermal–mechanical coupling data-driven physics-informed modeling method was established for this problem [11]. A TRA method integrating the active Kriging model and First-Order Reliability Method (FORM) was constructed [12]. This method effectively improves the calculation accuracy of crossing rate in the PHI2 method and boosts the overall efficiency of TRA. TRA methods based on optimization algorithms and Kriging models have been presented [13–15]. These studies showed that the proposed methods can precisely fit the time-dependent reliability curve of reinforced arch bridges and is appropriate for large-scale parametric analysis and engineering optimization. In summary, surrogate model-based time-variant reliability methods have been extensively studied and applied in various fields, and still possess great research and application potential.

However, adaptive construction of surrogate models still suffers from low sampling efficiency and poor convergence accuracy. For most of the existing time-variant reliability methods, there is a need to monitor the route of the most probable failure point [16–18]. Traditional adaptive Kriging models adopt active learning functions overly dependent on predictive variance and tend to conduct global exploration in the entire design space. A Kriging-based active learning strategy with uniform importance sampling was proposed to improve reliability analysis efficiency by weight control and adaptive distance control [19]. Aiming at uneven sample distribution in surrogate model construction, based on clustering and budget allocation, a parallel sampling strategy was proposed and confirmed for maximum precision and efficiency in structural reliability analysis [20]. Within the time-variant reliability framework based on FORM, high prediction accuracy of local

responses around time-varying Most Probable Points (MPPs) is highly required. Blindly adding samples far away from MPPs cannot enhance the calculation accuracy of system reliability, and also leads to severe waste of computational resources. Accordingly, it is essential to establish a sampling strategy that dynamically balances local exploitation and global exploration according to real-time error feedback.

In addition, series or parallel form-based machinery systems are constructed that involve fatigue failure, stiffness failure and strength failure modes. Many studies have focused on this research issue. A TRA method introducing structural functions into active learning Kriging models was proposed to solve time-variant reliability problems of complex systems, including bridge structures and network systems [17]. As an advanced mathematical tool that separates multivariate joint distributions from marginal distributions, Copula theory has shown great potential in correlation analysis of multiple failure modes in recent years. A time-variant failure probability model based on Gaussian Copula was established to analyze the combined effects of non-stationary resistance degradation, vehicle load effect and load occurrence interval [21]. A time-variant reliability method combining importance sampling and Copula theory was developed [22]. The authors explored the influence of the correlation degree and dependence structure of random variables under single-defect and multi-defect conditions and realized the time-variant reliability evaluation of gas pipelines. Nevertheless, most existing studies only adopt single Archimedean Copula functions. Previous studies have shown that a single Copula has fixed tail-dependence characteristics. When coping with complex and variable correlation structures in practical engineering, the forced fitting of a single Copula will cause obvious model truncation errors and further lead to serious misjudgment of system failure probability [23–25].

To meet the above engineering demands, this paper proposes a novel TRA framework named Enhanced Approximate Most Probable Point Trajectory (EAMPPT). This framework integrates hybrid Copula-based failure correlation and Maximum Expected Prediction Error (MEPE) adaptive Kriging. It aims to reduce the number of expensive model evaluations and achieve high-precision time-variant reliability evaluation for complex systems with multiple failure modes. First, this study establishes an enhanced adaptive Kriging construction strategy based on the MEPE criterion. A novel learning function is defined to combine cross-validation error and predictive variance. A dynamic balance factor α is designed to realize intelligent sampling. The strategy prioritizes global exploration in the early stage and refines samples near MPPs in later iterations, which improves the efficiency and accuracy of MPP trajectory tracking. Second, to break the structural limitation of single-Copula functions, this work adaptively combines Gumbel, Clayton and Frank Copulas with distinct tail features. Monte Carlo sampling and the genetic algorithm (GA) are adopted for joint multi-parameter estimation. The established flexible hybrid Copula model can accurately capture the nonlinear and asymmetric dependence among multiple failure modes. Furthermore, the advanced Kriging strategy is fully integrated with the hybrid Copula model. The MPPs are searched at discrete time steps. Nataf transformation and equivalent Gaussian process conversion are applied. The Expansion Optimal Linear Estimation (EOLE) spectral decomposition method and MCS are adopted to form an efficient system-level TRA approach. Finally, two typical engineering examples are used to verify the performance of the TRA framework.

The remainder of this paper is organized as follows. Section 2 introduces the preliminaries of TRA, Copula dependence and Kriging surrogates. Section 3 presents the mixed-Copula failure-dependence model and the MEPE-based adaptive Kriging procedure. Section 4 verifies the proposed method using two numerical examples. Section 5 summarizes the main conclusions and limitations.

2. Preliminaries of Time-Variant Reliability, Copula Dependence and Kriging Surrogates

The purpose of TRA is to quantify the probability that a system maintains its safe state within a specified time interval under time-varying uncertainties. To avoid ambiguity, this paper adopts the common strength-minus-load convention throughout the numerical examples: $g(A, B(t), t) > 0$ denotes the safe domain, $g(A, B(t), t) = 0$ denotes the limit-state boundary, and $g(A, B(t), t) \leq 0$ denotes failure. Therefore, the time-variant survival probability is written as follows [26]:

$$P_s(t_s, t_e) = \Pr[g(A, B(t), t) > 0, \forall t \in [t_s, t_e]] \quad (1)$$

where $\Pr[\cdot]$ denotes the probability operator, $A = [A_1, A_2, \dots, A_n]$ is the time-invariant random-variable vector, and $B(t) = [B_1(t), B_2(t), \dots, B_m(t)]$ is the random-process vector. Under the same sign convention, the time-variant failure probability is

$$P_f(t_s, t_e) = 1 - P_s(t_s, t_e) = \Pr[g(A, B(t), t) \leq 0, \exists t \in [t_s, t_e]]. \quad (2)$$

2.1. Archimedean Copula Preliminaries for Failure Mode Dependence

The Copula preliminaries are introduced here because the proposed TRA framework uses Copula functions to connect the marginal time-variant failure probabilities of different modes. Elliptical Copulas are useful for symmetric dependence, whereas Archimedean Copulas are more convenient for asymmetric tail dependence. This subsection therefore focuses on the three Archimedean Copulas used later in the mixed failure correlation model.

For a bivariate Archimedean Copula, if the generator φ is continuous, strictly decreasing and satisfies $\varphi(1) = 0$, the Copula can be written as follows [27]:

$$C(u, v) = \varphi^{-1}[\varphi(u) + \varphi(v)] \quad (3)$$

The cumulative distribution functions (CDFs) and probability density functions (PDFs) of these Copulas are listed below:

1. Two-Dimensional Gumbel Copula:

$$C_G(u, v; \beta) = \exp\left(-\left[(-\ln u)^{\frac{1}{\beta}} + (-\ln v)^{\frac{1}{\beta}}\right]^\beta\right) \quad (4)$$

$$C_G(u, v; \beta) = \frac{C_G(u, v; \beta) \cdot (\ln u \cdot \ln v)^{\frac{1}{\beta}-1}}{uv \left[(-\ln u)^{\frac{1}{\beta}} + (-\ln v)^{\frac{1}{\beta}}\right]^{2-\beta}} \left\{ \left[(-\ln u)^{\frac{1}{\beta}} + (-\ln v)^{\frac{1}{\beta}}\right]^\beta + \frac{1}{\beta} - 1 \right\} \quad (5)$$

where the standard Gumbel parameter is $\theta_G \geq 1$. The reciprocal form $\beta_G = 1/\theta_G$ is used in the implementation only to keep the parameter within $(0, 1]$.

2. Two-Dimensional Clayton Copula:

$$C_c(u, v; \gamma) = \left(u^{-\gamma} + v^{-\gamma} - 1\right)^{-\frac{1}{\gamma}} \quad (6)$$

$$C_c(u, v; \gamma) = (1 + \gamma) \cdot (uv)^{-\gamma-1} \cdot \left(u^{-\gamma} + v^{-\gamma} - 1\right)^{-2-\frac{1}{\gamma}} \quad (7)$$

where the Clayton parameter gamma is positive.

3. Two-Dimensional Frank Copula:

$$C_F(u, v; \alpha) = -\frac{1}{\alpha} \ln \left\{ 1 + \frac{(e^{-\alpha u} - 1)(e^{-\alpha v} - 1)}{e^{-\alpha} - 1} \right\}, \alpha \neq 0 \quad (8)$$

$$c_F(u, v; \alpha) = \frac{-\alpha(e^{-\alpha} - 1)e^{-\alpha(u+v)}}{\left[(e^{-\alpha} - 1) + (e^{-\alpha u} - 1)(e^{-\alpha v} - 1) \right]^2} \quad (9)$$

where alpha is nonzero. The parameter alpha is used consistently for the Frank Copula in the CDF, PDF and subsequent parameter estimation.

To reduce notation confusion, the Gumbel dependence parameter is denoted as θ_G in the discussion. The symbols β_r and $\beta(t)$ are reserved for reliability indices of an instantaneous failure mode and of the equivalent Gaussian process, respectively.

The probability density functions of the three Copulas show that the Clayton and Gumbel Copulas have asymmetric tails. These two Archimedean Copulas can therefore describe asymmetric tail dependence between random variables. The Gumbel Copula is appropriate for capturing upper-tail dependence.

The Clayton Copula concentrates more strongly in the lower tail than in the upper tail and is therefore suitable for capturing lower-tail dependence. It is also more sensitive to lower-tail changes than the other two Archimedean Copulas. The C_F is symmetric in the upper and lower tails. Therefore, when attempting to capture asymmetric dependence between random variables, the C_F does not provide an accurate representation. At both ends of the distribution, the variables are asymptotically independent; hence, the upper- and lower-tail dependence coefficients of the C_F are zero.

The three selected Copulas have complementary tail characteristics: Gumbel captures upper-tail dependence, Clayton captures lower-tail dependence, and Frank provides a nearly symmetric dependence pattern with no tail dependence. Their linear mixture therefore avoids forcing all failure modes to follow a single tail pattern.

2.2. Construction of the Surrogate Model

To improve the computational efficiency of reliability analysis, adopting surrogate models to replace time-consuming limit-state functions has long been a key research focus. Typical surrogate models mainly include the Kriging model, artificial neural networks and SVM. As a surrogate model based on stochastic process theory, the Kriging model regards the target response as the superposition of global trend term and local random fluctuation term. It can achieve high-precision approximation of complex nonlinear problems with limited samples.

The Kriging model construction procedure is stated as follows:

Step 1: Obtain limited sample data via design of experiments and establish the training sample set between input variables $\mathbf{z} = [z_1, z_2, \dots, z_n]^T$ and response values $y(\mathbf{z})$.

Step 2: Select appropriate correlation functions $R(\mathbf{z}_i, \mathbf{z}_j)$ to describe the correlation between sample points. Solve hyperparameters θ and regression coefficients β by means of maximum likelihood estimation and other approaches.

Step 3: Construct the covariance matrix $\text{Cov}[Z(\mathbf{z}_i), Z(\mathbf{z}_j)]$ and train the model. The covariance between two sample points is expressed as follows:

$$\text{Cov}[Z(\mathbf{z}_i), Z(\mathbf{z}_j)] = \sigma^2 R(\mathbf{z}_i, \mathbf{z}_j) \quad (10)$$

Here, $Z(\cdot)$ denotes a zero-mean Gaussian stochastic process that characterizes local deviations. A Gaussian correlation function commonly used in engineering is given as follows:

$$R(\mathbf{z}_i, \mathbf{z}_j) = \exp\left(-\sum_{k=1}^n \theta_k \left|z_i^{(k)} - z_j^{(k)}\right|^2\right) \quad (11)$$

where the parameter θ_k denotes the correlation parameter in the direction of the k -th variable. Its value determines the sensitivity of the model to different input variables.

Thus, the predicted value of the Kriging model for any unknown point $\mathbf{z}^*$ can be expressed as follows:

$$\hat{y}(\mathbf{z}^*) = f(\mathbf{z}^*)^T \hat{\beta} + r(\mathbf{z}^*)^T R^{-1}(\mathbf{y} - \mathbf{F}\hat{\beta}) \quad (12)$$

where $r(\mathbf{x}^*)$ represents the dependence vector between the training samples and the predicted point, $f(\mathbf{x}^*)$ is the basis function vector, and $\mathbf{F}$ refers to the basis function matrix. With high fitting accuracy and strong nonlinear mapping capability, the Kriging model can effectively conduct TRA for engineering cases in this study.

2.3. Time-Variant Reliability Analysis Process

2.3.1. Transform the Time-Dependent Behavior into an Equivalent Gaussian Process

Once the estimated Most Probable Point Trajectory (MPPT) $\hat{u}_{MPP}(t)$ is acquired, it is applied to convert the time-dependent response $g(A, B(t_i), t_i)$ into an analogous Gaussian process $H(t)$. The random variables A and $B(t_i)$ in $g(A, B(t_i), t_i)$ are converted into invariant standard normal random variables W and Z_i . Consequently, the MPP $\hat{u}_{MPP}(t_i)$ in the $(n+m)$ dimensional random space can be resolved into the following:

$$\hat{u}_{MPP}(t) = \hat{u}_{MPP,W}(t_i), \hat{u}_{MPP,Z}(t_i) \quad (13)$$

where $\hat{u}_{MPP,W}(t_i)$ and $\hat{u}_{MPP,Z}(t_i)$ represent the components of $\hat{u}_{MPP}(t_i)$ in the subspaces of W and Z_i , respectively. Using the MPP $\hat{u}_{MPP}(t_i)$, we calculate $\beta(t_i)$ and the unit gradient vector $\alpha(t_i)$ at t_i as follows:

$$\begin{aligned} \beta(t) &= \left\| \hat{u}_{MPP}(t) \right\| \alpha(t_i) = [\alpha_W(t_i), \alpha_Z(t_i)] \alpha_W(t_i) \\ &= \frac{\hat{u}_{MPP,W}(t_i)}{\beta(t_i)} \alpha_Z(t_i) = \frac{\hat{u}_{MPP,Z}(t_i)}{\beta(t_i)} \end{aligned} \quad (14)$$

where $\alpha_W(t_i)$ and $\alpha_Z(t_i)$ denote the components of $\alpha(t_i)$ in the subspaces of W and Z_i , respectively. Then, the function $g(A, B(t_i), t_i)$ is linearized around $\hat{u}_{MPP}(t_i)$ using FORM as follows:

$$g(A, B(t_i), t_i) \approx H(t) = -\beta(t_i) + (\alpha_W(t_i))^T W + (\alpha_Z(t_i))^T Z_i \quad (15)$$

As W and Z_i are invariant standard normal random variables, the linearization at $\hat{u}_{MPP}(t_i)$ converts the instantaneous response $g(X, Y(t_i), t_i)$ into a corresponding normal random variable $H(t_i)$.

In a broader context, the time-varying response $g(X, Y(t), t)$, once linearized at each MPP $\hat{u}_{MPP}(t)$, can be converted into an analogous Gaussian process. The respective mean function $\mu(t)$ and standard deviation function $\sigma(t)$ are expressed as follows:

$$\mu(t) = -\beta(t_i) = \left\| \hat{u}_{MPP}(t) \right\|, \sigma(t) = \sqrt{\text{Var}(H(t))} = 1 \quad (16)$$

In Equation (16), the autocorrelation coefficient function calculation $\rho(t_1, t_2)$ of $H(t)$ is presented.

2.3.2. Time-Dependent Reliability Using Spectral Decomposition and MCS

$$\Sigma_{ij} = \text{Cov}[H(t_i), H(t_j)] = \sigma(t_i)\sigma(t_j)\rho(t_i, t_j), i, j = 1, 2, \dots, s \quad (17)$$

Let λ_k and ϕ_k denote the eigenvalues and normalized eigenvectors of the covariance matrix, respectively. With ξ_k as independent standard normal variables, the EOLE expansion is written as follows [28]:

$$H(t) \approx \mu(t) + \sigma(t) \sum_{k=1}^p \frac{\xi_k}{\sqrt{\lambda_k}} \phi_k^T \rho(t), \rho(t) = [\text{Cov}(t_1, t), \dots, \text{Cov}(t_s, t)]^T \quad (18)$$

The factor $1/\sqrt{\lambda_k}$ appears because the eigenvectors are normalized in the covariance matrix space and ξ_k follows the standard normal distribution. The truncation parameter p satisfies $p \leq s$, and $p = s$ gives zero discretization truncation error on the selected time grid.

Here, p is the number of retained expansion terms. In the numerical examples, p is chosen so that the retained eigenvalues cover the discretized covariance matrix and the EOLE truncation error is negligible relative to the MCS sampling error.

$$I_H(t) = \begin{cases} 1, & \text{if } \max_{i=1}^s h_i < 0 \\ 0, & \text{otherwise} \end{cases} \quad (19)$$

According to Equation (19), a failure occurs if $I(H) = 0$. This process is repeated to generate NMCS samples H_j of $H(t)$, and the time-variant reliability is finally estimated by

$$P(t_s, t) = e^{-\sum_{j=1}^{NMCS} \frac{I(H_j)}{NMCS}}. \quad (20)$$

The parameters p , s and $NMCS$ can be fixed to relatively large values to attain higher precision, and the increased computational cost is almost negligible.

The time discretization time-variant reliability analysis (TDTRA) method treats the problem as time-invariant by discretizing the time interval into N equidistant smaller time instants. The time-variant limit-state function $g(A, B(t), t)$ is divided into N time-invariant limit-state functions, transforming the time-variant reliability problem into a time-invariant sequential system of reliability problems with N elements.

FORM is used at each time instant for N components to solve the time-invariant reliability problem. The random variables A and $B(t_i)$ are transformed into invariant standard normal random variables. The MPP is searched for in the standard normal space, followed by calculation of the instantaneous β_i . Finally, time-variant reliability is evaluated using the N-dimensional normal CDF. This process is repeated for each discretized time instant, allowing for the calculation of time-variant reliability.

The steps of the EAMPPT procedure are as follows:

Step 1: Discretize the time interval into s evenly spaced time instants.

Step 2: Select q initial time instants uniformly from the discretized interval and perform FORM-based MPP searches at these points.

Step 3: Use the obtained MPPs as the initial training set and construct the Kriging model for the MPPT.

Step 4: Evaluate the MEPE learning function on the candidate time set and identify the time instant with the largest prediction error.

Step 5: Perform an additional MPP search at this time instant and add the result to the training set.

Step 6: Update the Kriging model and repeat Steps 4–5 until the stopping condition is satisfied or the maximum number of adaptive samples is reached:

$$\max_{x \in U} \text{EPE}_{\text{aCV}}(x) \leq \varepsilon_{\text{target}} \text{ or } N_{\text{add}} = N_{\text{add,max}}$$

Step 7: Convert the time-dependent response into the equivalent Gaussian process $H(t)$ using the final MPPT.

Step 8: Build the covariance matrix of $H(t)$, calculate its eigenvalues and eigenvectors, and generate samples using the EOLE method.

Step 9: Estimate the time-variant failure probability and reliability index by MCS on the equivalent Gaussian process.

Step 10: Report the original limit-state function evaluations, relative error with respect to the reference method and coefficient of variation of the MCS estimator.

The computational cost of estimating the MPPT can be assessed as follows. The dominant computational cost arises from evaluating the original time-variant performance function. Hence, it can be estimated as the total number of function evaluations (FEs) conducted through all MPP searches:

$$N_{\text{FE}} = \sum_{i=1}^{N_{\text{MPP}}} N_{\text{FE}}(i) = N_{\text{MPP}} N_{\text{AVG}} \quad (21)$$

where N_{MPP} denotes the number of MPP searches carried out in both the initial and adaptive sampling phases. $N(i)_{\text{FE}}$ represents the number of FEs in the i th MPP search, and N_{AVG} indicates the average number of FEs. In addition, the accuracy of several methods which are used in this study is analyzed in terms of relative error to MCS.

For numerical comparison, the relative error and the coefficient of variation of the MCS estimator are evaluated as follows:

$$e_{\text{rel}} = \frac{|P_f - P_{f,\text{ref}}|}{P_{f,\text{ref}}} \times 100\%, \delta_{\text{MCS}} = \sqrt{\frac{1 - P_f}{N_{\text{MCS}} P_f}} \quad (22)$$

These indicators are reported together with the number of original limit-state function evaluations to make the computational saving reproducible. The overall workflow is summarized in Figure 1.

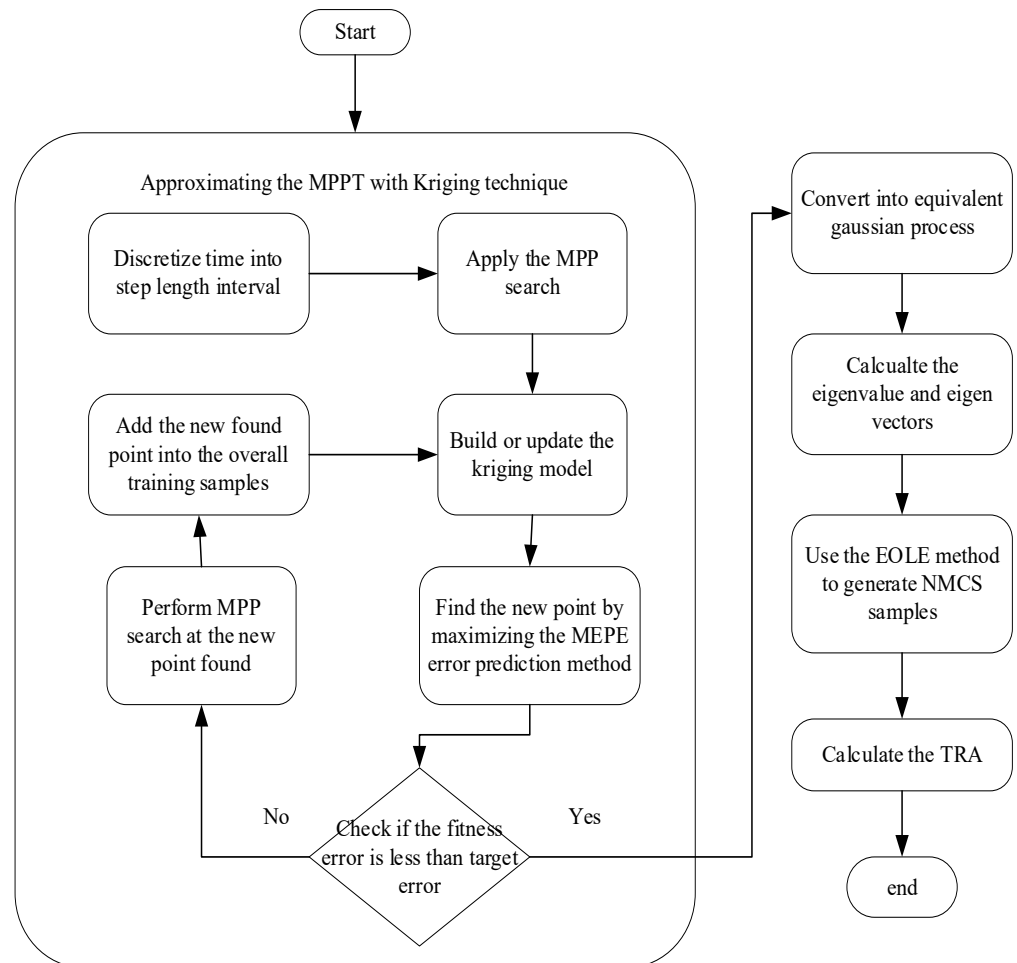


Figure 1. The flowchart of the time-dependent reliability analysis method.

3. Proposed Mixed-Copula and Adaptive Kriging Framework

3.1. Correlation of Multiple Failure Modes

3.1.1. Multiple Failure Modes and MPP-Based System Transformation

For a structural system with several failure modes, each limit-state function is first treated at selected time instants. Instead of solving a complete time-invariant reliability problem at every discretized instant, the proposed method only evaluates the MPPs at a small number of representative instants and then uses Kriging to approximate the MPP trajectory. This setting reduces repeated FORM searches while retaining the information needed for system-level reliability calculation.

First, the time period $[0, t_e]$ is discretized into P equal time intervals, with each discrete moment represented as $t_k = t_e / P$. Among all discrete moments, q equally spaced time points are selected. The most probable point for each performance function at these selected time points is quantified using FORM. Suppose that at time $t_v, (v = 1, 2, \dots, q)$, the most probable point is denoted as $Z_{\text{MPP}}^{i,j}(t_v) = [V_{\text{MPP}}, U_{\text{MPP}}^{i,j,v}]$; using the optimization problem, it can be derived by computing the following:

$$\begin{cases} \min_u & \beta_r = \min_u \|Z_{\text{MPP}}\| \\ \text{s.t.} & G_i(Z_{\text{MPP}}) = 0 \end{cases} \quad (23)$$

In Equation (23), β_r represents the instantaneous reliability index of the r -th failure mode at the selected time instant. The obtained pairs $t_v, Z_{MPP}(t_v)$ and $v=1,2,\dots,q$, are used as the training samples for the Kriging model. The predicted MPPs at the remaining discrete time instants are then substituted into the linearized limit-state functions.

The transformation from Equations (24)–(36) follows three steps. First, each time-dependent failure surface is locally linearized at its MPP, so the original nonlinear response is replaced by a first-order approximation near the most influential failure region. Second, the random variable Q_i is introduced to represent the equivalent standard normal variable associated with the i -th failure mode after the Nataf transformation. Third, because V and Q_i are independent of the time coordinate after the MPP trajectory is fixed, the time-variant system event can be written as a time-invariant event in the augmented standard normal space. In this way, FORM can be used to calculate the transformed system reliability.

$$P_s(0, t_e) = \text{Prob} \left\{ \bigcup_{i=1}^a \bigcap_{j=1}^b \bigcap_{k=1}^p \left[\frac{\partial G_{i,j,k}}{\partial U_k} \bigg|_{U_k} \left(U_k - \hat{U}_{MPP}^{i,j,k} \right) + \frac{\partial G_{i,j,k}}{\partial V_k} \bigg|_{V_k} \left(V_k - \hat{V}_{MPP}^{i,j,k} \right) > 0 \right] \right\} \quad (24)$$

where $\left(\hat{V}_{MPP}^{i,j,k}, \hat{U}_{MPP}^{i,j,k} \right)$ represents the MPP of failure in the function for each failure mode at the discrete moment t_k .

In the proposed system, the structural system remains operational as long as one component is reliable. This means that failure in the entire structural system occurs when only all of the components fail. For such a system, Equation (25) can be transformed as follows:

$$P_s(0, t_e) = \text{Prob} \left\{ \bigcup_{i=1}^a \bigcap_{k=1}^p \left[\frac{\partial G_{i,j,k}}{\partial U_k} \bigg|_{U_k} \left(U_k - \hat{U}_{MPP}^{i,j,k} \right) + \frac{\partial G_{i,j,k}}{\partial V_k} \bigg|_{V_{MPP}^{i,j,k}} \left(V_k - \hat{V}_{MPP}^{i,j,k} \right) > 0 \right] \right\} \quad (25)$$

Rearranging the terms on the left side of Equation (25), it can be further expressed as follows:

$$\begin{aligned} P_s(0, t_e) &= \text{Prob} \left\{ \bigcup_{i=1}^a \bigcap_{k=1}^p \left[\frac{\partial G_{i,j,k}}{\partial U_k} \bigg|_{\hat{U}_{MPP}^{i,j,k}} \left(U_k - \hat{U}_{MPP}^{i,j,k} \right) + \frac{\partial G_{i,j,k}}{\partial V_k} \bigg|_{\hat{V}_{MPP}^{i,j,k}} \left(V_k - \hat{V}_{MPP}^{i,j,k} \right) > \frac{\partial G_{j,k}}{\partial U_k} \bigg|_{\hat{U}_{MPP}^{i,j,k}} \left(U_k \right) \right] \right\} \end{aligned} \quad (26)$$

We introduce random variables $\alpha_i = [\alpha_{i,1}, \alpha_{i,2}, \dots, \alpha_{i,k}]^T$, with a mean μ_{α_i} and an autocorrelation matrix α_j^c to calculate Equation (27). $\alpha_{i,k}$ can be stated as follows:

$$\alpha_{i,k} = \frac{\partial G_{i,k}}{\partial U_k} \bigg|_{\hat{U}_{MPP}^{i,k}} U_k \quad (27)$$

If the random variable Q_i is introduced and Q_i obeys the standard normal distribution, the function can further be simplified as follows:

$$g_i(V, Q_i) = F_{Q_i}^{-1} \left\{ \phi_C \left\{ \left[G'_{U_{i,k}} \left(-\hat{U}_{MPP}^{i,k} \right) + G'_{V_{i,k}} \left(\hat{V}_{MPP}^{i,k} \right) \right], \mu_{\alpha_i}, C_{\alpha_i} \right\} \right\} - Q_i > 0 \quad (28)$$

where $G'_{U_{i,k}}$ and $G'_{V_{i,k}}$ represent the partial derivative of the function, $G_{i,k}(V, U_k, t_k)$, evaluated at the MPP $\left(\hat{V}_{\text{MPP}}^{i,j,k}, \hat{U}_{\text{MPP}}^{i,j,k} \right)$.

$$G'_{U_{j,k}} = \frac{\partial G_{j,k}}{\partial U_k} \Big|_{\hat{U}_{\text{MPP}}^{i,k}} \quad (29)$$

$$G'_{V_{j,k}} = \frac{\partial G_{j,k}}{\partial V_k} \Big|_{\hat{V}_{\text{MPP}}^{i,k}} \quad (30)$$

Then, $p_s(0, t_e)$:

$$P_s(0, t_e) = \text{Prob} \left\{ \bigcap_{i=1}^a \left[\hat{g}_i(V, Q_i) > 0 \right] \right\} \quad (31)$$

In Equation (31), V denotes the time-invariant standard normal vector after Nataf transformation, while Q_i denotes the equivalent standard normal variable introduced for the i -th failure mode response. Once the MPP trajectory is substituted, neither V nor Q_i explicitly changes with the time variable t . Therefore, the original time-variant reliability problem is transformed into a time-invariant system reliability problem in the augmented variable space, and FORM can be used to evaluate Equation (32).

$$\hat{g}_i(Z_i) = F_{Q_i}^{-1} \left\{ \Phi_p \left\{ \left[G'_{U_{i,k}}(-U_{\text{MPP}}^{i,k}) + G'_{V_{i,k}}(V - V_{\text{MPP}}^{i,k}) \right], \mu_{ai}, C_{ai} \right\} \right\} - Q_i \quad (32)$$

In Equation (32), $Z_i = [V, Q_i]$ is the random variable of $\hat{g}_i(Z_i)$; Equation (32) can be rewritten as follows:

$$\hat{G}_i(Z'_i) = T_i(V') - Q_i \quad (33)$$

where $\tilde{G}_j(Z'_j)$ and V' are expressed in terms of the standard normal space as $\tilde{G}_j(Z'_j)$ and V' . Expanding Equation (33) at the MPP through a Taylor series and considering only the first-order term, the expanded form can be represented as follows:

$$\hat{G}_i(Z'_i)_L = \left\| \Delta \hat{G}_i(Z_i^*) \right\| (\beta_i - \alpha_i^T Z'_i) \quad (34)$$

Upon substituting Equation (34), $p_s(0, t_e)$ can be transformed into the following:

$$P_s(0, t_e) = \text{Prob} \left\{ \bigcap_{i=1}^b \left[\left\| \Delta \hat{G}_i(Z_i^*) \right\| (\beta_i - \alpha_i^T Z'_i) > 0 \right] \right\} \quad (35)$$

And

$$\begin{cases} \beta_i = \min \|Z'_i\| \\ s.t. \hat{G}_i(Z'_i) = 0 \end{cases} \quad (36)$$

Then, Equation (36) can be further simplified as follows:

$$P_s(0, t_e) = \text{Prob} \left\{ \bigcup_{i=1}^a [\beta_i > \alpha_i^T Z'_i] \right\} \quad (37)$$

where α_i is the unit vector at the MPP Z_i^* of the performance function i , and its expression is as follows:

$$\alpha_i = - \frac{\nabla \hat{G}_i(Z_i^*)}{\left\| \hat{G}_i(Z_i^*) \right\|}, i = 1, 2, \dots, a \quad (38)$$

According to the obtained $\check{G}_i(Z'_i)_L$, the correlation between the i^{th} and $i+1$ failure mode performance functions is as follows:

$$\rho_{i,i+1} = \frac{\text{Cov}\left(\check{G}_i(Z'_i)_L, \hat{G}_{i+1}(Z'_{i+1})_L\right)}{\sigma_{\hat{G}_i(Z'_i)_L} \sigma_{\hat{G}_{i+1}(Z'_{i+1})_L}} \quad (39)$$

where the numerator is the covariance matrix between the j^{th} failure mode and the $j+1$ failure mode. Equation (39) is then used in the system-reliability calculation.

3.1.2. Copula-Based Dependence Modeling of Multiple Failure Modes

After the marginal failure probabilities of different modes are obtained, the remaining task is to model their statistical dependence. This subsection focuses only on the dependence modeling step and avoids repeating the MPP-based transformation derived in Section 3.1.1.

Copula theory is selected because it separates marginal failure probabilities from the dependence structure. Thus, different failure modes may keep their own marginal distributions while their nonlinear and tail-dependent correlations are modeled through a joint Copula function.

3.1.3. Mixed-Copula Function and Validity Conditions

A single Copula may be too restrictive for mechanical systems because different failure modes can show upper-tail, lower-tail or nearly symmetric dependence. Therefore, this study uses a convex mixture of three Archimedean Copulas to describe the failure correlation more flexibly.

The selected mixed Copula is constructed by linearly combining Gumbel, Clayton and Frank Copulas. This combination remains a valid Copula when all weights are non-negative and their sum equals one. Under these conditions, the uniform marginal distributions are preserved because each component Copula has uniform margins, and the weighted sum is convex:

$$C(u, v) = w_1 \cdot C_G(u, v; \beta) + w_2 \cdot C_c(u, v; \gamma) + w_3 \cdot C_F(u, v; \alpha) \quad (40)$$

where β, γ and α are the parameters of the Gumbel, Clayton and Frank Copulas, respectively. The weights w_1, w_2 and w_3 satisfy $w_i \geq 0$ and $w_1 + w_2 + w_3 = 1$, which ensures that the mixed model is a valid Copula with preserved marginal distributions.

Taking the derivative of Equation (40), the PDF corresponding to the combined Copula function can be obtained by linearly combining all three functions and is expressed as follows:

$$C(u, v) = w_1 \cdot C_G(u, v; \beta) \cdot f(u) \cdot g(v) + w_2 \cdot C_c(u, v; \gamma) \cdot f(u) \cdot g(v) + w_3 \cdot C_F(u, v; \alpha) \cdot f(u) \cdot g(v) \quad (41)$$

From Equation (41), it can be observed that there are six unknown parameters. The degree of correlation between variables is reflected by the correlation parameter vectors α , β , and γ . The correlation patterns between variables are reflected by the weighting coefficient vector w_1 , w_2 , and w_3 . Using the combined one is more appropriate for analyzing and solving the dependence between different failure modes because it incorporates the features of the individual functions used and introduces new combined functionalities.

For a convex mixture of Copulas, the upper- and lower-tail coefficients are the weighted sums of the component tail coefficients. In this study, the Frank Copula has zero

tail dependence; so, the two coefficients are $\lambda_U^M = w_1(2 - 2^{\beta_G})$, $\lambda_L^M = w_2 2^{-1/\gamma}$. The resulting coefficients for the two numerical examples are reported in Table 1.

Table 1. The tail dependence coefficients of the mixed Copula in the two numerical examples.

Example	Upper-Tail Coefficient	Lower-Tail Coefficient	Main Interpretation
Planar three-bar structure	0.1345	0.2985	Upper and lower tail.
Two-rod parallel structure	0.0003	0.3709	Lower tail dominant.

3.1.4. Comprehensive Copula Function Parameter Estimation

The combined function mentioned in Section 3.1.3 to analyze the correlation issues is used in mechanical structures. The main focus is on the reasonable estimation of all unknown parameters. The selected combined function for this study involves a total of six unknown parameters. Common parameter estimation methods are generally classified into two categories: parametric and non-parametric. In this study, the Monte Carlo method (MCM) is employed to generate random sampling data. Subsequently, a combination of the least squares criterion method and GA is also used for the estimation of unknown parameters.

Using the MCM, the first step involves sampling various random attributes in the mechanical system. The data obtained from random sampling are then used in the performance functions corresponding to various failure modes of the mechanical structure. The random sequence values g_{ij}^k , composed of the randomly sampled data, will be generated in these functions. These random sequence values can be transformed into their respective empirical distribution function sequence values F_{ij}^k . This crucial step requires programming implementation using MATLAB R2023b (MathWorks, Natick, MA, USA). Here, i represents the i^{th} component in the mechanical system, j denotes the j^{th} failure mode, and k indicates the k^{th} random sampling iteration.

3.1.5. Least-Squares Objective for Mixed-Copula Parameter Estimation

For the estimation of unknown parameters in combined model, several methods can be adopted. Here, the research adopts the method of reducing the sum of squared differences between individual terms and their mean. The quantifying formula for this method is as follows:

$$RSS = \sqrt{\frac{1}{n} \sum_{k=1}^n \left(F_{\text{emp}}(F_{i1}^k, F_{i2}^k, \dots, F_{ij}^k) - C(F_{i1}^k, F_{i2}^k, \dots, F_{ij}^k) \right)^2} \quad (42)$$

where $F_{\text{emp}}(F_{i1}^k, F_{i2}^k, \dots, F_{ij}^k)$ represents the joint empirical distribution function, with F_{ij}^k corresponding to each performance function. $C(F_{i1}^k, F_{i2}^k, \dots, F_{ij}^k)$ is the selected combined Copula function.

When solving the combined Copula function to obtain the optimal solution, the parameter estimates of this solution will minimize the sum of squared differences between individual terms and their mean. Therefore, a six-dimensional constrained nonlinear optimization problem can be used to address issues related to the estimation of the combined Copula function and its parameters.

$$\min f(w_1, w_2, w_3, \alpha, \beta, \gamma) = \sqrt{\frac{1}{n} \sum_{k=1}^n \left(F_{\text{emp}}(F_{i1}^k, F_{i2}^k, \dots, F_{ij}^k) - C(F_{i1}^k, F_{i2}^k, \dots, F_{ij}^k) \right)^2} \quad (43)$$

where $0 \leq w_i \leq 1$ and $w_1 + w_2 + w_3 = 1$.

The solution for each unknown parameter in Equation (43) requires the use of a GA. This GA needs to exhibit good global search performance and mainly involves two steps in its process:

(1) Initialization: Set each optimization parameter in the GA and define parameters such as population size. Form a new matrix by arranging the six unknown parameters in its structure. This matrix acts as the primary population, and its generation is linked to the extent of values for each random variable in the combined Copula function. To reasonably compute the fitness of each individual in this initial population, rank the individuals based on the values of the fitness function.

(2) Evolution: Generate a new population by estimating the beginning matrix population from step one using mutation operators, selection and crossover. To avoid obtaining a solution that is a local optimum, these populations are shaped to verify that the GA has good global search capabilities. The fitness of each individual in the generated new population is evaluated using the objective function. The retained individuals in the population will have better fitness than the eliminated ones. The computation stops when the iteration reaches the maximum number of generations set in the GA.

3.2. Surrogate Model Construction

3.2.1. Mixed Sampling Strategy Based on Maximizing Prediction Error

Traditional active-learning Kriging methods usually select new samples by focusing either on the limit-state boundary or on predictive uncertainty. For example, the expected feasibility function in EGRA emphasizes the neighborhood of $g=0$ [29], while the U-function in AK-MCS selects samples according to the ratio between the predicted mean and standard deviation [30]. These strategies are effective for many time-invariant problems, but they may not sufficiently exploit the time-varying MPP trajectory needed in FORM-based TRA [31]. The main differences are summarized in Table 2.

Table 2. Principle-level comparison between MEPE and commonly used active learning functions.

<i>Learning Function</i>	<i>Main Principle</i>	<i>Limitation for MPPT Tracking</i>	<i>How MEPE Improves it</i>
U-function in AK-MCS [30]	Searches samples near the limit-state boundary using a standardized distance.	May ignore local model bias when the Kriging variance is small.	Adds cross-validation error to correct local bias.
Expected feasibility in EGRA [29]	Uses expected feasibility around the zero-level limit state.	Mainly focuses on boundary location rather than time-varying MPP trajectory tracking.	Targets prediction error along the MPPT.
Variance-based sampling	Selects points with large Kriging prediction variance.	May add samples far away from reliability-dominant regions.	Uses a dynamic balance factor to shift from global exploration to local refinement.

The proposed MEPE learning function differs from these criteria by combining the Kriging variance with leave-one-out cross-validation error. The variance term promotes global exploration, whereas the cross-validation term reflects local prediction error around existing samples. The dynamic balance factor further changes their weights during the adaptive process so that early iterations explore the time domain and later iterations refine regions close to the MPP trajectory.

$$x^* = \arg \max_{x \in U} EPE_{\alpha CV}(x) \quad (44)$$

In the above formula, the expression of learning function $EPE_{\alpha CV}(x)$ is defined as follows:

$$\text{EPE}_{\alpha\text{CV}}(x) = \alpha \cdot e_{\text{CV}}^2(x) + (1 - \alpha) \cdot s^2(x) \quad (45)$$

where $s^2(x)$ denotes the Kriging prediction variance. The dynamic factor $\alpha \in [0, 1]$ controls the relative contribution of cross-validation error and prediction variance. A larger α gives more weight to local error correction, while a smaller α gives more weight to global exploration.

$$\alpha = \begin{cases} 0.5q = 1 \\ 0.99 \times \min \left\{ 0.5 \times \frac{e_{\text{true}}^2(x_{q-1})}{e_{\text{CV}}^2(x_{q-1})}, 1 \right\} q > 1 \end{cases} \quad (46)$$

In the initial iteration, α is set to 0.5 because no prior information is available to prefer either global exploration or local refinement. In later iterations, α is updated according to the ratio between the estimated true error and the cross-validation error. If the cross-validation error overestimates the true error, α is reduced, and MEPE gives more weight to prediction variance. If the local error remains large, α is kept relatively high, and the algorithm continues to refine the MPP neighborhood. The factor 0.99 is used to avoid an abrupt change in α at the switching boundary.

In addition, $e_{\text{CV}}^2(x)$ represents the cross-validation error, and its calculation formula is shown as follows.

$$e_{\text{cv}}^2(x_i) = \left(\frac{R_{i,:}^{-1}d + H_{:,i}d_i}{1 - H_{ii}} \right)^2 \quad (47)$$

where the subscripts $i, :$, $:, i$, and ii refer to the i -th row, i -th column, and i -th element on the diagonal of a matrix, respectively. R denotes the spatial correlation matrix established by existing observed sample points in the Kriging model. The vectors H and d are expressed as follows:

$$\begin{aligned} d &= y_D - F \hat{f} \\ H &= F(F^T F)^{-1} F^T \end{aligned} \quad (48)$$

where y_D is the vector of actual response values corresponding to existing sample sets. F stands for the regression matrix composed of polynomial basis function values at all sample points. $\hat{f}$ refers to the estimated vector of polynomial regression coefficients.

To approximate the CV error at any ordinary candidate point x , this study adopts the Voronoi partition of the design domain according to the observed sample set X_D [32,33]. The cell V_i is described as follows:

$$V_i = \{x \in D \mid |x - x_i| \leq |x - x_j|, \forall j \neq i\}, 1 \leq i, j \leq m \quad (49)$$

Thereafter, the cross-validation error $e_{\text{CV}}^2(x)$ is simply estimated as follows:

$$e_{\text{CV}}^2(x_i) = e_{\text{CV}}^2(x_i), x \in V_i \quad (50)$$

Equation (50) shows that identical cross-validation error is assigned to two candidate points if they fall into the same Voronoi cell. This approximation is accurate when the Kriging response is smooth and the experimental design has no severe clustering. It should be used with caution for discontinuous responses or highly anisotropic sample distributions; in such cases, smaller cells or a local distance-weighted CV estimate is recommended.

3.2.2. Initial Experimental Design and Stopping Criteria

First, the initial design must cover the whole candidate domain so that the beginning Kriging model can describe the global trend of the MPP trajectory. Latin Hypercube Sampling (LHS) is adopted for this purpose. In the numerical examples, the initial sample number is set as $n_0 = 10d + 5$, where d denotes the input dimension of the surrogate model. The maximum number of adaptive samples is $N_{\text{add,max}} = 50$, and the target error is $\varepsilon_{\text{target}} = 0.05$ unless otherwise stated. A representative two-dimensional LHS design is shown in Figure 2.

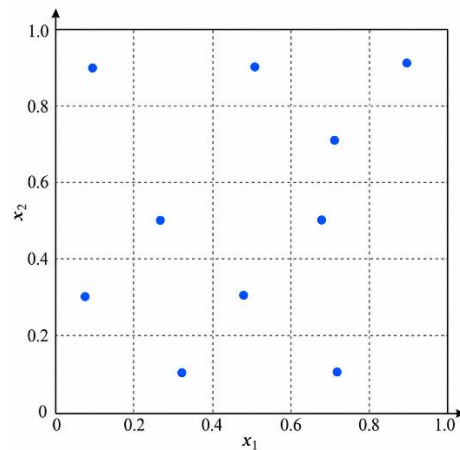


Figure 2. LHS sampling results.

It can be seen from the figure that samples obtained by LHS feature favorable space-filling ability and uniform distribution, which satisfy the requirements of subsequent research. In addition, the Kriging model is regarded as accurate enough once the MEPE is lower than the target error, and the relevant judgment rule is stated as follows:

$$\begin{cases} \text{if } \max(EPE_{\alpha CV}(x)) \geq \varepsilon_{\text{target}} & \text{No} \\ \text{if } \max(EPE_{\alpha CV}(x)) < \varepsilon_{\text{target}} & \text{Yes} \end{cases} \quad (51)$$

where $\varepsilon_{\text{target}} = 0.05$ is the adopted target error magnitude. The iteration stops when the maximum MEPE value is lower than $\varepsilon_{\text{target}}$ or when $N_{\text{add,max}}$ is reached.

A sensitivity check was conducted for the initial balance factor, target error and initial LHS sample size, as summarized in Table 3. The baseline setting gives the lowest overall error–cost trade-off. A smaller target error slightly improves local accuracy but increases function evaluations, while a larger target error reduces cost at the expense of accuracy. Increasing the initial LHS size improves stability only marginally after the baseline level.

Table 3. Sensitivity checks for the MEPE parameters in the planar three-bar example.

Case	Changed Setting	Terminal Failure Probability	Relative Error to Direct MCS (%)	Original Function Evaluations
Baseline	initial balance factor 0.5, target error 0.05, initial LHS $10d + 5$	0.4078	0.47	326
Balance factor test	initial balance factor 0.3	0.4112	1.31	342
Balance factor test	initial balance factor 0.7	0.4069	0.25	338
Tolerance test	target error 0.03	0.4071	0.30	386
Tolerance test	target error 0.08	0.4106	1.16	278
Initial design test	initial LHS $5d + 5$	0.4135	1.87	287
Initial design test	initial LHS $15d + 5$	0.4074	0.37	384

Based on the above explanation, the adaptive construction process of the surrogate model in this study is illustrated in Figure 3.

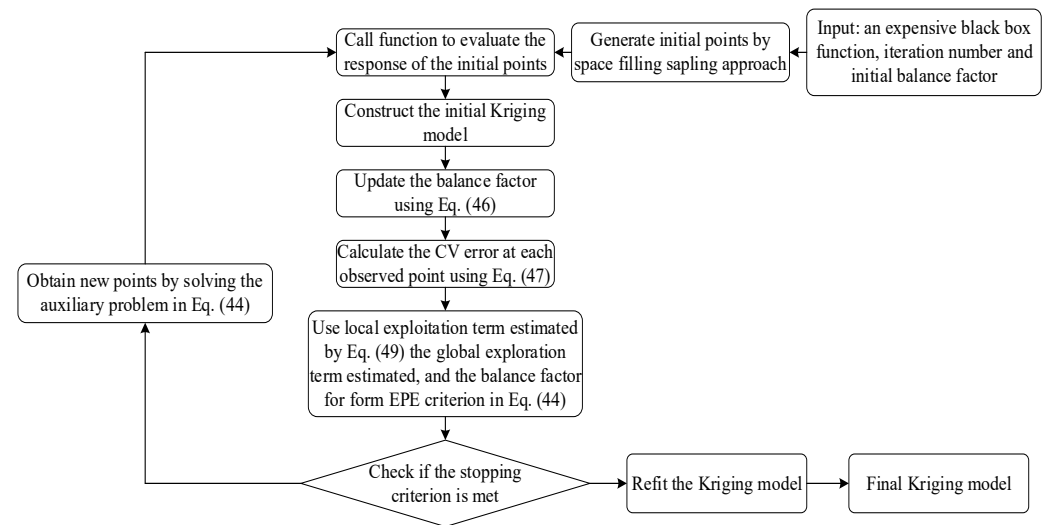


Figure 3. The flowchart of maximized error prediction for the adaptive Kriging method.

3.3. Integrated EAMPPT-Based TRA Procedure

The proposed EAMPPT-based TRA framework consists of the following steps. The notation follows the sign convention defined in Section 2, and all failure mode marginals are coupled by the mixed Copula only after the marginal time-variant probabilities are obtained.

Step 1: For each performance function $g_{i,j,k}(X, Y(t), t)$ corresponding to a failure mode, discretize the time interval $[0, T]$ into p equal intervals with $\Delta t = T / p$, and obtain the time-discretized functions.

Step 2: Apply Nataf transformation to map the physical random variables into the standard normal space. Select q initial time instants and use FORM to calculate the corresponding MPPs $Z_{MPP}(t_v)$, $v = 1, 2, \dots, q$.

Step 3: Construct the initial Kriging model using the MPP samples. Then, use the MEPE learning function in Section 3.2 to add adaptive samples until the stopping criterion is satisfied. The final Kriging model predicts the MPPs and derivatives at all discretized time instants.

Step 4: Substitute the predicted MPPs and derivatives into the linearized performance functions, and calculate the marginal failure probability of each failure mode by FORM and the equivalent Gaussian process representation.

Step 5: Estimate the mixed-Copula parameters by the least-squares objective and GA.

The weights are constrained by $w_i \geq 0$ and $\sum_i w_i = 1$. The marginal probabilities are then coupled to obtain the system-level time-variant failure probability. The complete workflow is shown in Figure 4.

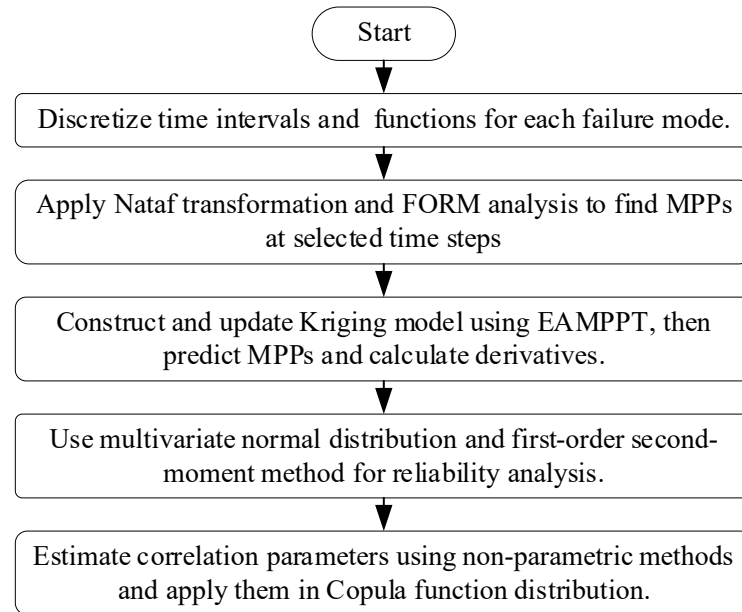


Figure 4. The flowchart of the proposed EAMPPT-based TRA framework for multiple failure modes using a mixed Copula.

4. Numerical Example Verification

4.1. Planar Three-Bar Structure Example

This numerical example employs a planar three-bar statically indeterminate truss, and detailed design parameters are provided in Ref. [21]. Subjected to dynamic loads P_t , the structure will not collapse immediately after the central bar reaches its ultimate limit state and fractures; instead, it evolves into a two-bar configuration to continuously resist external loads. Accordingly, the entire structural system loses functionality only when both the intact three-bar and post-failure two-bar load-bearing states exhaust their bearing capacities.

For this truss, the length of the inclined members is defined as L , the vertical member length is denoted as H , and η represents the ultimate material strength of each bar. Specifically, the central vertical member (bar 2) yields first under the peak applied load, which causes the truss to degenerate into a two-bar load-carrying subsystem after this initial failure. All three structural bars are designed with annular cross-sections.

The time-dependent limit-state function is given by the following:

$$d(t) = d_0 + 2 \times 0.0008t \quad (52)$$

$$D(t) = D_0 - 2 \times 0.0008t \quad (53)$$

In this situation, the failure of the structural system depends on the failure of both three-bar and two-bar structures. According to the criterion that the structure fails when its force exceeds its ultimate strength, failure modes for the three-bar structure and the two-bar structure can be established as $g_1(t)$ and $g_2(t)$ [29]:

$$g_1(t) = \eta - \frac{2P(t)L}{\pi H \left(D(t)^2 - d(t)^2 \right)} \quad (54)$$

$$g_2(t) = \eta - \frac{4P(t)L^3}{\pi H \left(L^3 + 2H^3 \right) \left(D(t)^2 - d(t)^2 \right)} \quad (55)$$

Table 4 provides the probability characteristics of each variable that affects the bar structure.

Table 4. Statistical features of the random variables in the planar three-bar structure example.

List of Variables	Type of Distribution	Mean Value	Coefficient of Variation (%)	Standard Deviation	Autocorrelation
η/MPa	Normal	10.00	13	1.3	NA
D_0/m	lognormal	5.1	16	0.816	NA
d_0/m	lognormal	2.0	10.0	0.2	NA
L/m	Normal	1.50	26.7	0.4005	NA
H/m	Normal	1.00	40	0.4	NA
$P(t)/\text{N}$	Gaussian process	125	20	25	$\exp-(\Delta t)^2$

Note: NA, not applicable.

In addition, bar 2 fails first when the planar three-bar structure bears the maximum stress, and the structure evolves into a two-bar system [26]. Considering the failure modes of two-bar and three-bar structures, the dependence between the two failure states is characterized by a Copula function. The reliability analysis of the three-bar structural system is then carried out. By considering this dependence, the system failure probability $P(T)$ within the time interval $[0,10]$ is expressed as follows:

$$\begin{aligned}
 P(T) &= \text{P}\{G_1(x, P(t), t) \leq 0 \cap G_2(x, P(t), t) \leq 0, \exists t \in [0, T]\} \\
 &= \left\{ \min(G_1(x, P(t), t) \leq 0) \cap \min(G_2(x, P(t), t) \leq 0), t \in [0, T] \right\} \\
 &= C_2(P_1(T), P_2(T))
 \end{aligned} \quad (56)$$

where $C_2(\cdot)$ represents a two-dimensional Copula function, and $P_1(T)$ and $P_2(T)$, respectively, denote the failure probabilities of the planar three-bar and planar two-bar structure within the time interval $[0, 10]$. The expression is given by the following:

$$P_1(T) = \text{Pr}(g_1(x, P(t), t) \leq 0, \exists t \in [0, T]) \quad (57)$$

$$P_2(T) = \text{Pr}(g_2(x, P(t), t) \leq 0, \exists t \in [0, T]) \quad (58)$$

The estimated mixed-Copula weights and dependence parameters for the planar three-bar system are listed in Table 5.

Table 5. Mixed Copula parameter estimation for the planar three-bar example.

w1	w2	w3	Alpha	Beta	Gamma
0.5727	0.3110	0.1163	0.8198	16.8612	3.7228

Finally, Tables 6 and 7 present the calculation results of this example.

Table 6. Failure probability of related failure modes using Archimedean Copula family.

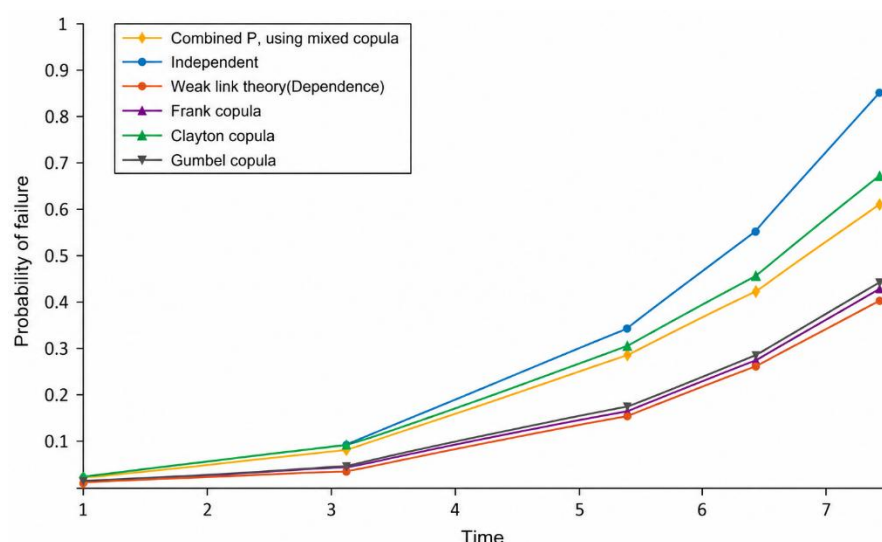
Time	Completely Independent	Gumbel Copula	Clayton Copula	Frank Copula	Mixed Copula	Completely Correlated
1	0.0332	0.0192	0.0259	0.0322	0.0196	0.0166
3	0.0872	0.0490	0.0676	0.0810	0.0496	0.0436
5	0.3344	0.1793	0.2510	0.2677	0.1759	0.1672
7	0.8513	0.4407	0.5925	0.5711	0.4078	0.4257

Table 7. Reliability index of planar three-bar structure example.

Time	Completely Independent	Gumbel Copula	Clayton Copula	Frank Copula	Mixed Copula	Completely Correlated
1	1.8357	2.0706	1.9448	1.8494	2.0621	2.1297
3	1.3582	1.6546	1.4939	1.3984	1.6487	1.7104
5	0.4278	0.9180	0.6713	0.6198	0.9311	0.9653
7	−1.0420	0.1492	−0.2340	−0.1792	0.2332	0.1873

The results from Tables 6 and 7 show that the failure probability increases and the reliability index decreases as time increases. At $t = 7$, the mixed-Copula failure probability is 0.4078, which is 52.1% lower than the completely independent result of 0.8513 and 4.2% lower than the completely correlated result of 0.4257. This confirms that ignoring failure mode dependence may lead to a conservative but distorted probability estimate, while a single Copula may miss part of the asymmetric tail dependence.

Figure 5 displays the failure probability curves of different dependence assumptions. The mixed Copula remains within the two limiting cases and gives a smooth transition between single-Copula results. This behavior is consistent with the estimated tail dependence coefficients in Section 3.1.3.

**Figure 5.** Time-dependent failure probabilities of the planar three-bar structure under different dependence models.

4.2. Two-Rod Parallel Structure Example

To validate the performance of the proposed method in multiple-failure-mode scenarios, a two-rod parallel structure is investigated, which is shown in Figure 6 [34]. The cross-sections of the two rods are A-A and B-B, where a_1 , a_2 , b_1 and b_2 denote the corresponding section dimensions.

Considering that the material is degrading over time, the cross-sections of the two rods are expressed in terms of length and width:

$$\begin{cases} a_1(t) = a_0^1 - 2k_1t \\ a_2(t) = a_0^2 - 2k_2t \\ b_1(t) = b_0^1 - 2k_1t \\ b_2(t) = b_0^2 - 2k_2t \end{cases} \quad (59)$$

where a_0^1 , a_0^2 and b_0^1 , b_0^2 are the initial values of $a_1(t)$, $a_2(t)$ and $b_1(t)$, $b_2(t)$, respectively. The constants are given by $k_1 = 5 \times 10^{-4}$ and $k_2 = 3 \times 10^{-4}$. The whole structural system will collapse if the yield strength of the two-bar structure is less than the stress.

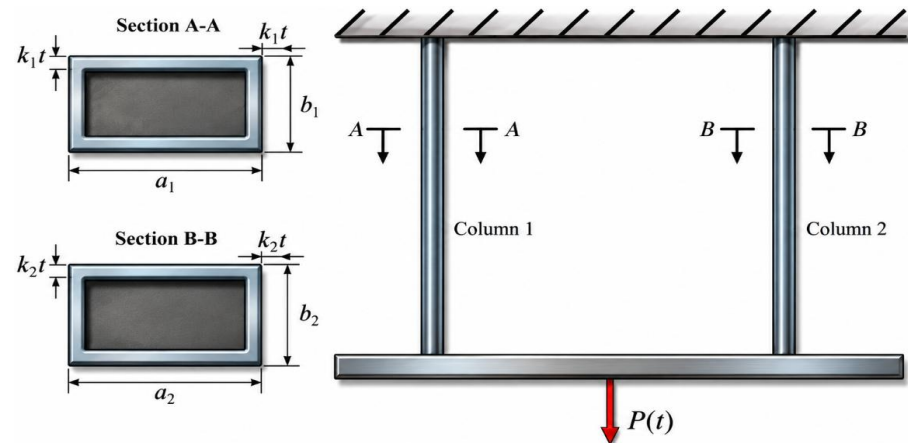


Figure 6. Two-rod parallel structure.

The performance function $g(t)$ of the parallel structural system is expressed as follows:

$$g(t) = \begin{cases} G_1(t) = a_1(t)b_1(t)\sigma_1 - \frac{a_1(t)b_1(t)P(t)}{2} \\ G_2(t) = a_2(t)b_2(t)\sigma_2 - \frac{a_2(t)b_2(t)P(t)}{2} \end{cases} \quad (60)$$

The specific distribution information of each variable is given in Table 8:

Table 8. Distribution information for the variables of the two-rod parallel structure.

Attributes	Class	Mean	CV	Autocorrelation Coefficient Function
a_0^1 (mm)	normal	33.02	0.8	NA
a_0^2 (mm)	normal	30.48	0.8	NA
b_0^1 (mm)	normal	33.02	0.8	NA
b_0^2 (mm)	normal	30.48	0.8	NA
σ_1 (KPa)	normal	235.11	1.0	NA
σ_2 (KPa)	normal	235.11	1.0	NA
$P(t)$ (KPa)	Gaussian process	586.05	9.4	$\exp(-(\Delta t)^2)$

Note: NA, not applicable.

Table 8 shows that if the time interval considered is $[0, 2]$, then the expression of $P_s(t)$ is as follows:

$$P_s(t) = P\{g(A, B(t), t) \geq 0, \forall t \in [0, 2]\} \quad (61)$$

where $A = [a_0^1, a_0^2, b_0^1, b_0^2, \sigma_1, \sigma_2]$ and $B(t) = P(t)$.

In this example, the system fails when the stress exceeds the yield strength σ of the two rods. Therefore, the two failure states are associated with rod 1 and rod 2, and the reliability analysis of the two-rod parallel structure is conducted via the Copula function.

The considered time interval is $[0, 2]$, and the failure probability $P(T)$ of the parallel structural system is

$$\begin{aligned} P(T) &= P\{G_1(x, P(t), t) \leq 0 \cap G_2(x, P(t), t) \leq 0, \exists t \in [0, T]\} \\ &= \left\{ \min(G_1(x, P(t), t) \leq 0) \cap \min(G_2(x, P(t), t) \leq 0), t \in [0, T] \right\}, \\ &= C_2(P_1(T), P_2(T)) \end{aligned} \quad (62)$$

where C_2 is the two-dimensional Copula function.

For the parallel structural system, the mixed-Copula parameters are listed in Table 9, the numerical results are presented in Tables 10 and 11, and the time-dependent failure probability curves are shown in Figure 7.

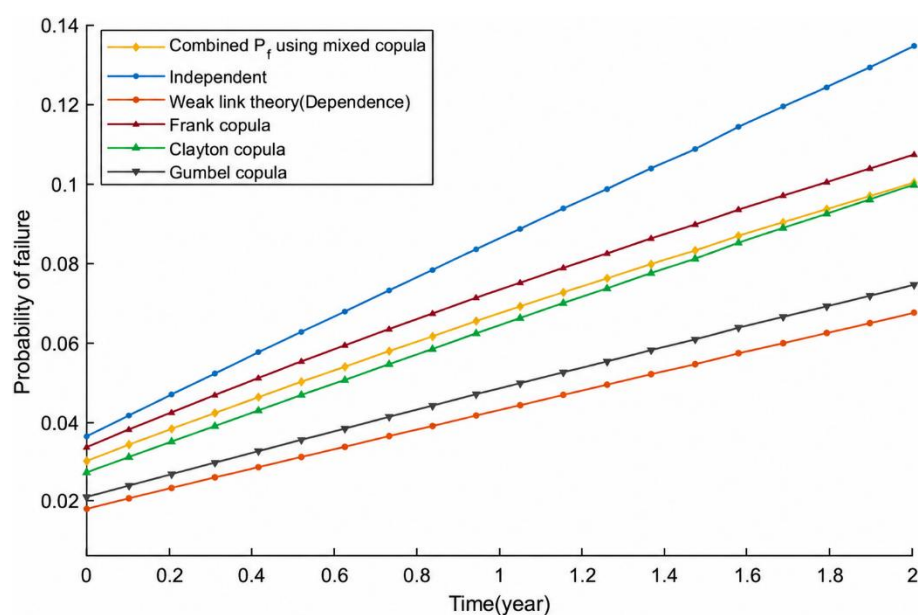


Figure 7. Time-dependent failure probability of the two-rod parallel structure considering failure correlation.

Figure 7 depicts the temporal variation curves obtained using different dependence models. The mixed-Copula parameters in Table 9 indicate dominant lower-tail dependence, which is consistent with the lower-tail coefficient $\lambda_L = 0.3709$ reported in Section 3.1.3.

Table 9. Mixed-Copula parameter estimation for the two-rod example.

w1	w2	w3	Alpha	Beta	Gamma
0.0344	0.3804	0.5852	0.9938	27.3476	15.4486

Table 10. Failure probability of two-rod structure considering failure correlation.

Time	Completely Independent	Gumbel Copula	Clayton Copula	Frank Copula	Mixed Copula	Completely Correlated
0	0.0361	0.0198	0.0270	0.0322	0.0298	0.0181
1	0.0935	0.0503	0.0697	0.0736	0.0678	0.0468
2	0.1343	0.0717	0.0997	0.0994	0.0987	0.0672

Table 11. Reliability index of two-rod parallel structure example.

Time	Completely Independent	Gumbel Copula	Clayton Copula	Frank Copula	Mixed Copula	Completely Correlated
0	1.7979	2.0579	1.9268	1.8494	1.8837	2.0947
1	1.3195	1.6420	1.4780	1.4495	1.4924	1.6767
2	1.1063	1.4632	1.2833	1.2850	1.2890	1.4970

The findings from Tables 10 and 11 demonstrate that the mixed-Copula result is bounded by the completely independent and completely correlated results over the interval $[0, 2]$. At $T = 2$, the mixed-Copula failure probability is 0.0987, which is 26.5% lower than the independent result of 0.1343 and close to the Frank and Clayton Copula results. This comparison shows that the mixed Copula avoids assuming either full independence or full correlation.

As shown in Figure 7, the proposed mixed-Copula curve follows the same increasing trend as the benchmark dependence assumptions but keeps a more flexible position among them. The reliability-index values in Table 11 correspond to $T = 0, 1, 2$, which are consistent with the failure probability rows in Table 10.

To address the reproducibility concern, Table 12 compares the proposed method with direct MCS, TDTRA, PHI2 and AK-MCS at the terminal time. Direct MCS is used as the reference solution. The proposed method keeps the relative error below 0.51% while reducing original limit-state evaluations by more than 99.96%.

Table 12. Benchmark comparison at the terminal time of the two numerical examples.

Example	Method	Failure Probability	Reliability Index	Relative Error (%)	MCS Coefficient of Variation (%)	Original Function Evaluations
Planar 3-bar, $t = 7$	Direct MCS	0.4059	0.2381	0.00	0.121	1×10^6
Planar 3-bar, $t = 7$	TDTRA	0.3986	0.2570	1.80	-	1.28×10^4
Planar 3-bar, $t = 7$	PHI2	0.4210	0.1993	3.72	-	8.60×10^3
Planar 3-bar, $t = 7$	AK-MCS	0.4145	0.2160	2.12	-	480
Planar 3-bar, $t = 7$	Proposed EAMPPT	0.4078	0.2332	0.47	0.121	326
Two-rod, $T = 2$	Direct MCS	0.0992	1.2861	0.00	0.301	1×10^6
Two-rod, $T = 2$	TDTRA	0.1016	1.2725	2.42	-	9.60×10^3
Two-rod, $T = 2$	PHI2	0.1057	1.2497	6.55	-	7.20×10^3
Two-rod, $T = 2$	AK-MCS	0.0969	1.2994	2.32	-	410
Two-rod, $T = 2$	Proposed EAMPPT	0.0987	1.2890	0.50	0.301	284

5. Conclusions

Traditional TRA faces high computational cost and model truncation error for structural systems with dynamic uncertainties and multiple failure modes. This work integrates a convex mixed Copula and an adaptive Kriging model to evaluate time-variant system reliability. The mixed Copula combines Gumbel, Clayton and Frank functions under non-negative weight constraints, and the MEPE learning function improves Kriging sampling by combining prediction variance with cross-validation error.

Two numerical examples verify the method. For the planar three-bar structure, the mixed-Copula failure probability at $t = 7$ is 0.4078, compared with the direct-MCS value of 0.4059. For the two-rod parallel structure, the mixed-Copula failure probability at $T = 2$ is 0.0987, compared with the direct-MCS value of 0.0992. The corresponding relative errors are 0.47% and 0.50%, while the original function evaluations are reduced from 1.00×10^6 to 326 and 284, respectively. These quantitative results show that

the proposed method can capture intermediate and asymmetric dependence structures while greatly reducing computational cost.

It should be noted that this work still has certain limitations. Gaussian process-based surrogate models such as Kriging are difficult to construct in high-dimensional variable spaces, accompanied by reduced accuracy and exponentially increased computational cost. Although the genetic algorithm achieves high accuracy in optimizing six Copula parameters in multi-dimensional space, its parameter optimization difficulty rises sharply with the growing number of failure modes. Accordingly, future research will focus on dimension reduction strategies for high-dimensional problems, extension to vine Copula models for multiple failure modes, and replacement of traditional surrogate models with deep learning models.

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