

Article

Image Reconstruction by Frequency Extrapolation and Deep Learning in Three-Layer Medium

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Abstract

This paper proposes a novel multi-frequency extended Deep Learning (DL) model for electromagnetic image reconstruction under Transverse Magnetic (TM) wave incidence in layered media, inspired by conventional microwave imaging techniques that combine nonlinear inversion algorithms with neural networks to improve reconstruction performance. The proposed framework adopts a two-stage neural network architecture. In the first stage, a Deep Residual Convolutional Neural Network (DRCNN) is employed to extrapolate multi-frequency scattered fields from single-frequency input data, thereby enriching the frequency-dependent scattering information available for reconstruction. Subsequently, the extrapolated multi-frequency scattered fields are fed into a Deep Convolutional Encoder–Decoder (DCED) network to reconstruct an accurate dielectric constant distribution within the imaging domain. To validate the effectiveness of the proposed approach, two representative comparison methods are considered: (1) a hybrid framework combining the Back-Propagation Scheme (BPS) with a Convolutional Neural Network (CNN), and (2) a framework integrating the Dominant Current Scheme (DCS) with a CNN. In both approaches, conventional inversion algorithms are first utilized to generate coarse initial reconstructions, which are subsequently refined by the neural network. Numerical simulations and experimental results show that the proposed multi-frequency extension model achieves lower reconstruction error and higher structural similarity than the reference methods. These results confirm the effectiveness and potential of the proposed framework for advanced electromagnetic imaging applications.

Keywords: frequency extrapolation; deep learning; three-layer medium; microwave imaging; multi-frequency; Deep Residual Convolutional Neural Network; Deep Convolutional Encoder–Decoder

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1. Introduction

Electromagnetic imaging technology has found widespread application across various fields, primarily including medicine, subsurface exploration, and national defense. In the medical field, electromagnetic imaging has rapidly advanced in non-invasive diagnostics. For example, microwave imaging is being applied in breast cancer detection, as

well as in brain imaging and lung monitoring for real-time observation. Electromagnetic imaging is used in archaeological surveys, mineral exploration, and groundwater monitoring to detect subsurface structures. Geophysical exploration, for instance, employs electromagnetic waves to detect mineral distributions or voids within geological formations, aiding in resource development and environmental monitoring. In defense applications, electromagnetic imaging can be utilized for through-wall imaging to support search and rescue or surveillance missions carried out by law enforcement agencies. Moreover, millimeter-wave scanners used in airport security are another practical application of electromagnetic imaging. Other areas, such as industrial non-destructive testing and smart sensing, are also showing great potential as electromagnetic imaging technologies continue to improve. However, despite its broad applications, electromagnetic imaging still faces numerous challenges and technical difficulties. These include mathematical ill-posedness and nonlinearity, high computational cost and convergence issues, as well as limitations in measurement conditions and noise interference. In recent years, two primary approaches have been developed to address Electromagnetic Inverse Scattering (EMIS) problems: one based on traditional algorithms [1–10], and the other based on DL methods [11–21].

In the field of electromagnetic inverse scattering, various traditional algorithms have been proposed to improve imaging accuracy and computational efficiency, including the Born iterative method [1,6,7], subspace-based optimization methods [3], and Tikhonov regularization techniques [9]. Although these approaches have demonstrated effectiveness in solving inverse scattering problems, they generally require solving large-scale linear or nonlinear systems, particularly in scenarios involving strong scattering effects or inhomogeneous media. Consequently, these methods often suffer from high computational complexity and computational cost, especially for iterative-based optimization procedures that require repeated forward and inverse calculations. Owing to these limitations, DL-based approaches have gradually attracted increasing attention in recent years due to their capability to learn nonlinear mappings directly from data while reducing reconstruction time during the inference stage. DL methods can be categorized into three main types: DL-based inversion methods [11–15], multi-stage DL methods [16–18], and the multiple-frequency learning methods [19–21]. DL-based inversion methods aim to replace traditional inverse scattering algorithms with DL models by directly using the measured scattered field data or the preliminarily reconstructed images obtained from conventional methods as input, enabling efficient reconstruction of the dielectric properties of the target object. In 2021, Zhou proposed an improved deep learning framework for electromagnetic inverse scattering by introducing the Modified Contrast Scheme (MCS). The proposed method can alleviate the strong nonlinearity of inverse scattering problems and improve the suitability of the data for deep learning reconstruction. Numerical results demonstrated that the proposed approach achieved more accurate reconstruction performance for high-contrast scatterers compared with the conventional DCS [11]. In 2022, Liu proposed an unrolling deep learning framework based on the subspace-based optimization method (SOM) for solving full-wave inverse scattering problems. The proposed network embedded the Lippmann–Schwinger physical model into the network architecture and iteratively updated the induced current and permittivity distribution by mimicking the conventional SOM procedure. Numerical and experimental results demonstrated that this method achieved superior reconstruction performance [12]. In 2023, Wang proposed an Early Fusion Framework (EFF) for solving electromagnetic inverse scattering problems by integrating scattered-field features and BPS prior information within a unified deep learning architecture. The proposed framework combined a scattered field encoder and a back-propagation encoder to simultaneously capture external multiple-scattering characteristics and internal structural information of the target object. Numerical and experimental

results demonstrated that the proposed EFF achieved superior reconstruction accuracy, stability and convergence speed [13]. In 2024, Hu proposed a priori knowledge-based Physics-Informed Neural Networks (PINNs) for electromagnetic inverse scattering problems. The proposed two-step framework first utilized the distorted finite-difference-frequency-domain-based iterative method (DFIM) to generate initial permittivity distributions and total fields, which were subsequently incorporated into the PINN as prior information to improve network training. Numerical and experimental results demonstrated that the proposed method achieved high reconstruction accuracy, computational efficiency, and robustness against noise [14]. In 2025, Maricar proposed an Attention-Unet (ATTN-Unet) architecture with attention gates for microwave imaging inverse-scattering problems. The proposed network utilized backpropagated scattered-field data as input and reconstructed the complex permittivity distribution of the target object, while attention gates were introduced to enhance important spatial features during the reconstruction process. Experimental results demonstrated that the ATTN-Unet achieved superior image quality and lower computational time compared with conventional Contrast Source Inversion (CSI) and other CNN-based architectures [15].

Multi-stage DL methods involve cascading multiple DL modules with specific functions to sequentially process EMIS tasks, aiming to enhance reconstruction accuracy and overall system performance. These methods fully leverage the strengths of different models tailored to specific tasks, making the image reconstruction process more flexible and interpretable. These multi-stage architectures illustrate that, through staged learning and modular processing, DL can achieve higher accuracy and better generalization in EMIS, offering an efficient and reliable strategy for handling complex scattered field data. In 2022, Luo proposed a Cascaded Complex U-Net (CCU-net) framework for solving phaseless-data inverse scattering problems in the complex domain. The proposed architecture consisted of a Phase Retrieval Network (PRNet) for recovering the phase and amplitude of scattered fields and an Image Reconstruction Network (IRNet) for reconstructing the dielectric profile from the recovered fields. Numerical and experimental results demonstrated that the proposed CCU-net achieved strong robustness, generalization ability, and accurate reconstruction performance for high-contrast scatterers in real-time applications [16]. In 2023, Zhang proposed an enhanced two-step deep learning framework for electromagnetic inverse scattering problems by combining frequency extrapolation and scatterer reconstruction. The proposed method first employed a DRCNN to predict multifrequency scattered fields from single-frequency measurements, and subsequently utilized a DCED network to reconstruct the dielectric distribution of the target object. Numerical results demonstrated that the proposed approach achieved accurate and efficient reconstruction performance for inhomogeneous and high-contrast scatterers [17]. In 2024, Yao proposed a two-step deep learning electromagnetic inversion solver for reconstructing high-contrast and heterogeneous scatterers. The proposed framework first utilized a Deep Convolutional Asymmetric Encoder–Decoder Structure (DCAEDS) to generate coarse permittivity reconstructions from single-frequency scattered-field measurements, followed by a DRCNN to refine the reconstructed dielectric distribution using both the initial reconstruction and the original scattered-field data. Numerical results demonstrated that the proposed method achieved high reconstruction accuracy and computational efficiency while overcoming the limitations of conventional inverse scattering methods, such as ill-posedness and high computational cost [18].

Multiple-frequency learning methods utilize scattered-field information at different frequencies to enrich the physical features available for electromagnetic inverse scattering reconstruction. In general, low-frequency data provide stable global structural information, while high-frequency data contain finer spatial details that are useful for improving boundary resolution and image sharpness. By learning the complementary relation-

ship among different frequency components, DL models can enhance reconstruction accuracy and robustness compared with single-frequency approaches. In 2021, Li proposed a multifrequency U-Net convolutional neural network framework for solving electromagnetic inverse scattering problems. The proposed method utilized multifrequency back-propagation results as training data and demonstrated that incorporating multifrequency information significantly improved reconstruction stability and accuracy compared with single-frequency approaches. Numerical and experimental results further showed that the proposed multifrequency U-Net CNN achieved robust reconstruction performance for high-contrast and complex scatterers [19]. In 2024, Chiu proposed a convolutional neural network framework incorporating multifrequency information and structural similarity loss functions for electromagnetic imaging of anisotropic objects. The proposed method first utilized the Backpropagation Scheme (BPS) to estimate the multifrequency permittivity distribution and subsequently employed a CNN optimized with a combined structural similarity index (SSIM) and root mean square error (RMSE) loss function to improve reconstruction quality. Numerical results demonstrated that the proposed multifrequency framework achieved more stable and accurate reconstructions than single-frequency approaches under both TM and Transverse Electric (TE) wave conditions, even in the presence of noise interference [20]. In the same year, Martin investigated three different deep learning strategies for incorporating multifrequency data into microwave imaging, including a multichannel U-Net-like network, a cascaded multifrequency framework, and a Long Short-Term Memory (LSTM)-based recurrent network. The proposed cascaded and LSTM-based approaches utilized lower-frequency reconstruction results as prior information for higher-frequency imaging in order to improve reconstruction performance. Numerical and experimental results demonstrated that the LSTM-based framework significantly outperformed the other methods in terms of reconstruction accuracy and imaging capability [21].

Overall, DL-based inversion methods demonstrate great potential in overcoming the limitations of traditional approaches in handling nonlinearity and high-dimensional data, opening new opportunities for solving EMIS problems. Electromagnetic imaging is essential in real-world scenarios, such as airport security for detecting concealed explosives or metals in luggage and non-destructive testing of artifacts to reveal hidden fractures or layers that are challenging to inspect visually. These applications demand high-precision imaging techniques for reliable and rapid assessments. To meet this need, we introduce a novel two-stage multi-frequency deep learning framework for electromagnetic image reconstruction under TM wave incidence in layered media. The main contributions of this work are as follows:

1. This paper designs a two-stage DL model that combines DRCNN and DCED to effectively integrate single-frequency scattered fields with extrapolated multi-frequency information for dielectric target reconstruction. Compared with Zhang [17], the present study extends the two-stage DL framework to the more complex and practically relevant problem of electromagnetic imaging in a three-layer medium. Unlike Zhang's work, which focused only on homogeneous media in free space, the proposed approach addresses heterogeneous backgrounds and three-layer structures, while requiring fewer training samples, making the imaging task significantly more challenging.
2. Through the DRCNN model, the proposed method successfully extrapolates single-frequency scattered field data to the multi-frequency domain, overcoming the practical limitation of incomplete frequency-band measurements. This approach effectively resolves the challenge of acquiring multi-frequency data, making it particularly suitable for extreme environments.

3. Although a two-stage training strategy is employed, both stages utilize data from the same synthetic dataset. This eliminates the need for extra dataset construction, thereby avoiding additional cost in data preparation.
4. The proposed framework does not require complex preprocessing of scattered field data, which enhances system practicality and improves real-time applicability, facilitating future deployment in real-world scenarios.
5. Compared with conventional approaches that directly measure multi-frequency data, the proposed method reduces hardware burden and testing time while still achieving reliable results under limited data availability.
6. Numerical experiments confirm that the proposed approach outperforms other methods in terms of image quality, boundary reconstruction, and noise robustness, demonstrating strong practical potential.
7. Compared with Chiu [22], the present work extends the two-stage frequency extrapolation framework from a homogeneous free-space background to a three-layer slab medium. This extension requires a reformulated forward scattering model based on the Green's function of the layered medium, in which the reflected and transmitted wave components at the dielectric interfaces are explicitly incorporated. The incident fields are also derived separately for upper-side and lower-side incidences. In addition, the slab-medium configuration imposes more restricted illumination and observation conditions than the free-space case; therefore, the incident-wave angles are carefully controlled to generate physically consistent scattered-field data.

Figure 1 presents the overall architecture of the proposed two-stage reconstruction framework. In the first stage, the measured single-frequency scattered-field data are processed by the DRCNN to generate the corresponding scattered-field responses at additional frequencies. These measured and extrapolated data are subsequently combined as the input to the DCED in the second stage, where the relative permittivity distribution of the target is reconstructed.

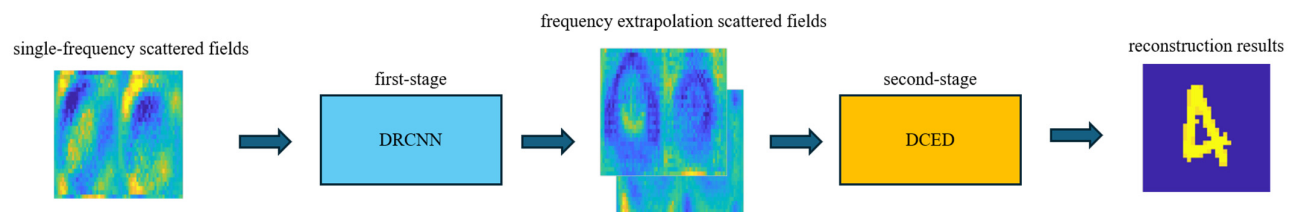


Figure 1. Overall workflow of the proposed two-stage reconstruction framework.

Section 2 introduces the theoretical formulation of the electromagnetic inverse scattering problem, including the forward model and the sources of nonlinearity and ill-posedness. Section 3 presents the proposed multi-frequency extended DL model, which incorporates multi-frequency information to improve reconstruction stability and accuracy. Section 4 provides numerical results through a series of simulation studies and comparative evaluations. Finally, Section 5 concludes the paper by summarizing the main contributions and outlining potential limitations and directions for future work.

2. Theory

Consider a dielectric object embedded in the second layer of a three-layer medium in a two-dimensional space. Each layer is characterized by its own permittivity ε_i and electrical conductivity σ_i , where $(i = 1, 2, 3)$ denote the three regions from top to bottom, respectively. The permittivity values are set as follows: Region 1 and Region 3 have a relative permittivity of 1, while Region 2 has a relative permittivity of 2.56.

In a three-layer spatial structure, there are two interfaces located between Region 1 and Region 2 (at $y = a$) and between Region 2 and Region 3 (at $y = -a$), as shown in Figures 2 and 3. According to Maxwell's equations, the electromagnetic fields satisfy the following relations: [23]

$$\nabla \times \vec{H} = j\omega\epsilon_2\epsilon\vec{E} \quad (1)$$

$$\nabla \times \vec{E} = -j\omega\mu_0\vec{H} \quad (2)$$

The total electric field in the z-direction E_z^{tot} can be expressed as:

$$E_z^{tot} = E_z^i + E_z^s \quad (3)$$

The total electric field can be expressed in terms of the two-dimensional Green's function as follows:

$$E_z^{tot}(\vec{r}) = E_z^i(x, y) + \int_s G(\vec{r}, \vec{r}')(-k_2^2)(\epsilon_r(\vec{r}') - 1)E_z^{tot}(\vec{r}')ds' \quad (4)$$

where E_z^i denotes the incident field in the z-direction, E_z^s represents the scattered field received in the z-direction. G is Green's function, which satisfies the equation $(\nabla^2 + k_i^2)G(x, y; x', y') = \delta(x - x')\delta(y - y')$.

Since the medium consists of three layers, Green's function can be decomposed into three parts [24,25].

$$G(\vec{r}, \vec{r}') = G(x, y; x', y') = \begin{cases} G_1(x, y; x', y'), & y \geq a \\ G_2(x, y; x', y'), & a > y > -a \\ G_3(x, y; x', y'), & y \leq -a \end{cases} \quad (5)$$

$$G_1(x, y; x', y') = \frac{1}{2\pi} \int_{-\infty}^{\infty} j e^{-j\gamma_1(y-a)} \frac{(\gamma_2 + \gamma_3)e^{j\gamma_2(y'+a)} + (\gamma_2 - \gamma_3)e^{-j\gamma_2(y'+a)}}{(\gamma_1 + \gamma_2)(\gamma_2 + \gamma_3)e^{j\gamma_2(2a)} + (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)e^{-j\gamma_2(2a)}} e^{-j\alpha(x-x')} d\alpha \quad (6)$$

$$G_2(x, y; x', y') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{j}{2\gamma_2} \left[\frac{(\gamma_2 + \gamma_1)(\gamma_2 + \gamma_3)e^{-j\gamma_2[|y-y'|-2a]} + (\gamma_2 - \gamma_1)(\gamma_2 - \gamma_3)e^{j\gamma_2[|y-y'|-2a]}}{(\gamma_2 + \gamma_1)(\gamma_2 + \gamma_3)e^{j\gamma_2(2a)} + (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)e^{-j\gamma_2(2a)}} + \frac{(\gamma_2 - \gamma_1)(\gamma_2 + \gamma_3)e^{j\gamma_2[y+y']}}{(\gamma_2 + \gamma_1)(\gamma_2 + \gamma_3)e^{j\gamma_2(2a)} + (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)e^{-j\gamma_2(2a)}} + \frac{(\gamma_2 - \gamma_3)(\gamma_2 + \gamma_1)e^{-j\gamma_2[y+y']}}{(\gamma_2 + \gamma_1)(\gamma_2 + \gamma_3)e^{j\gamma_2(2a)} + (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)e^{-j\gamma_2(2a)}} \right] e^{-j\alpha(x-x')} d\alpha \quad (7)$$

$$G_3(x, y; x', y') = \frac{1}{2\pi} \int_{-\infty}^{\infty} j e^{j\gamma_3(y+a)} \frac{(\gamma_2 + \gamma_1)e^{-j\gamma_2(y'-a)} + (\gamma_2 - \gamma_1)e^{j\gamma_2(y'-a)}}{(\gamma_2 + \gamma_1)(\gamma_2 + \gamma_3)e^{j\gamma_2(2a)} + (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)e^{-j\gamma_2(2a)}} e^{-j\alpha(x-x')} d\alpha \quad (8)$$

$$\gamma_i^2 = k_i^2 - \alpha^2, \quad i = 1, 2, 3, \quad \text{Im}(\gamma_i) \leq 0, \quad -a < y' < a$$

Here, $G(x, y; x', y')$ is the Green's function of the three-layer medium, which can be derived using the Fourier Transform. k_i denotes the wavenumber in the i -th region, and α is the Fourier variable. When dealing with the propagation of plane waves, we assume that the incident wave has a time-harmonic form of $e^{j\omega t}$.

$$E_z^i(\vec{r}) = E_z^i(x, y)\hat{z} = E_z^i e^{-j\vec{k}_z \cdot \vec{r}} \hat{z} \quad (9)$$

$e^{-j\vec{k}_z \cdot \vec{r}}$ is the phase term of the plane wave, representing the propagation direction and phase variation in the wave.

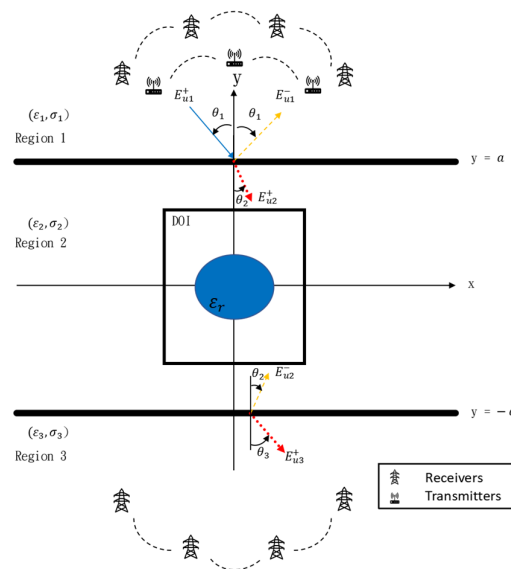


Figure 2. Schematic diagram of a two-dimensional medium object in three-layer space (Region 1 incident).

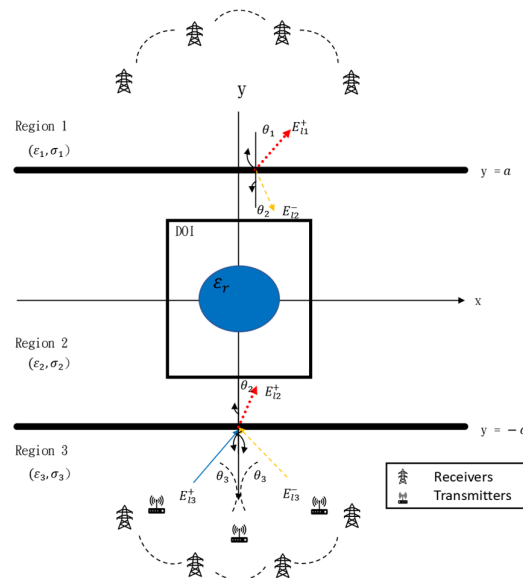


Figure 3. Schematic diagram of a two-dimensional medium object in three-layer space (Region 3 incident).

In the three-layer spatial structure, assuming that Region 1 and Region 3 are both air, electromagnetic waves can be incident and received from these two regions, respectively. When an electromagnetic wave propagates from one region to another, changes in permittivity or conductivity at the interface cause the incident wave to generate two fundamental wave phenomena: reflected waves and transmitted waves. Accordingly, the incident-field E_z^i is considered separately for upper incidence from Region 1 and lower incidence from Region 3, as illustrated in Figures 2 and 3. The incident field for the upper incidence $E_{uz}^i(x, y)$ can be expressed as:

$$E_{uz}^i(x, y)\hat{z} = \begin{cases} E_{u1}^+ e^{jk_1 \cos \theta_1 y} e^{-jk_1 \sin \theta_1 x} \hat{z} + E_{u1}^- e^{-jk_1 \cos \theta_1 y} e^{-jk_1 \sin \theta_1 x} \hat{z}, & y \geq a \\ E_{u2}^+ e^{jk_2 \cos \theta_2 y} e^{-jk_2 \sin \theta_2 x} \hat{z} + E_{u2}^- e^{-jk_2 \cos \theta_2 y} e^{-jk_2 \sin \theta_2 x} \hat{z}, & a > y > -a \\ E_{u3}^+ e^{jk_3 \cos \theta_3 y} e^{-jk_3 \sin \theta_3 x} \hat{z}, & y \leq -a \end{cases} \quad (10)$$

E_{u1}^+ , E_{u2}^+ , and E_{u3}^+ are the transmission coefficients in Region 1, Region 2, and Region 3, respectively, while E_{u1}^- and E_{u2}^- are the reflection coefficients in Region 1 and Region 2. By setting $E_{u1}^+ = 1$, we can obtain:

$$E_{u1}^- = \frac{e^{j2k_1 \cos \theta_1 a} [(z_1 + z_2)(z_3 - z_2)e^{-j2k_2 \cos \theta_2 a} - (z_1 - z_2)(z_3 + z_2)e^{j2k_2 \cos \theta_2 a}]}{(z_1 + z_2)(z_3 + z_2)e^{j2k_2 \cos \theta_2 a} - (z_1 - z_2)(z_3 - z_2)e^{-j2k_2 \cos \theta_2 a}} \quad (11)$$

$$E_{u2}^+ = \frac{1}{2} e^{jk_2(\sin \theta_2 x - \cos \theta_2 a)} \left[\frac{z_1 + z_2}{z_1} e^{-jk_1(\sin \theta_1 x - \cos \theta_1 a)} + \frac{z_1 - z_2}{z_1} E_1^- e^{-jk_1(\sin \theta_1 x + \cos \theta_1 a)} \right] \quad (12)$$

$$E_{u2}^- = \frac{1}{2} e^{jk_2(\sin \theta_2 x + \cos \theta_2 a)} \left[\frac{z_1 - z_2}{z_1} e^{-jk_1(\sin \theta_1 x - \cos \theta_1 a)} + \frac{z_1 + z_2}{z_1} E_1^- e^{-jk_1(\sin \theta_1 x + \cos \theta_1 a)} \right] \quad (13)$$

$$E_{u3}^+ = \frac{2z_3}{z_2 + z_3} E_2^+ e^{-jk_2(\sin \theta_2 x + \cos \theta_2 a)} e^{jk_3(\sin \theta_3 x + \cos \theta_3 a)} \quad (14)$$

where $z_i = \frac{\eta_i}{\cos \theta_i}$, $i = 1, 2, 3$, η is the characteristic impedance of the medium, θ_1 is the incident angle, and θ_2 and θ_3 are the refraction angles.

The incident field for the lower incidence $E_{lz}^i(x, y)$ can be expressed as:

$$E_{lz}^i(x, y) \hat{z} = \begin{cases} E_{l3}^+ e^{-jk_3 \cos \theta_3 y} e^{-jk_3 \sin \theta_3 x} \hat{z} + E_{l3}^- e^{jk_3 \cos \theta_3 y} e^{-jk_3 \sin \theta_3 x} \hat{z}, & y \leq -a \\ E_{l2}^+ e^{-jk_2 \cos \theta_2 y} e^{-jk_2 \sin \theta_2 x} \hat{z} + E_{l2}^- e^{jk_2 \cos \theta_2 y} e^{-jk_2 \sin \theta_2 x} \hat{z}, & -a < y < a \\ E_{l1}^+ e^{-jk_1 \cos \theta_1 y} e^{-jk_1 \sin \theta_1 x} \hat{z}, & y \geq a \end{cases} \quad (15)$$

E_{l1}^+ , E_{l2}^+ , and E_{l3}^+ are the transmission coefficients in Region 1, Region 2, and Region 3, respectively, while E_{l2}^- and E_{l3}^- are the reflection coefficients in Region 2 and Region 3. By setting $E_{l3}^+ = 1$, we can obtain:

$$E_{l3}^- = \frac{e^{j2k_3 \cos \theta_3 a} [(z_3 + z_2)(z_1 - z_2)e^{-j2k_2 \cos \theta_2 a} - (z_3 - z_2)(z_1 + z_2)e^{j2k_2 \cos \theta_2 a}]}{(z_3 + z_2)(z_1 + z_2)e^{j2k_2 \cos \theta_2 a} - (z_3 - z_2)(z_1 - z_2)e^{-j2k_2 \cos \theta_2 a}} \quad (16)$$

$$E_{l2}^+ = \frac{1}{2} e^{jk_2(\sin \theta_2 x - \cos \theta_2 a)} \left[\frac{z_3 + z_2}{z_3} e^{-jk_3(\sin \theta_3 x - \cos \theta_3 a)} + \frac{z_3 - z_2}{z_3} E_3^- e^{-jk_3(\sin \theta_3 x + \cos \theta_3 a)} \right] \quad (17)$$

$$E_{l2}^- = \frac{1}{2} e^{jk_2(\sin \theta_2 x + \cos \theta_2 a)} \left[\frac{z_3 - z_2}{z_3} e^{-jk_3(\sin \theta_3 x - \cos \theta_3 a)} + \frac{z_3 + z_2}{z_3} E_3^- e^{-jk_3(\sin \theta_3 x + \cos \theta_3 a)} \right] \quad (18)$$

$$E_{l1}^+ = \frac{2z_1}{z_1 + z_2} E_2^+ e^{-jk_2(\sin \theta_2 x + \cos \theta_2 a)} e^{jk_1(\sin \theta_1 x + \cos \theta_1 a)} \quad (19)$$

Here, θ_3 is the incident angle, θ_1 and θ_2 are the refraction angles. For numerical computation, this study employs the Method of Moments (MoM) to solve the integral equations. The closed surface of the object is divided into several small subregions, assuming that the electric field and permittivity are constant within each subregion. The integral equations are then converted into matrix form:

$$(E_z^i) = [-[G_2][\tau_z] + [I]](E_z^{tot}) \quad (20)$$

$$(E_z^s) = [G_1][\tau_z] (E_z^{tot}) \quad (y > a) \quad (21)$$

$$(E_z^s) = [G_3][\tau_z] (E_z^{tot}) \quad (y < -a) \quad (22)$$

$$[G_1]_{mn} = -[k_z^2 \cdot G_1(x_m, y_m; x_n, y_n)] \pi a_n^2 \quad (23)$$

$$[G_2]_{mn} = \begin{cases} -\frac{j\pi k_2 a_n}{2} J_1(k_2 a_n) H_0^{(2)}(k_2 \rho_{mn}) - k_z^2 [G_s(x_m, y_m; x_n, y_n)] \pi a_n^2, & M \neq N \\ -\frac{j}{2} [\pi(k_2 a_n) H_1^{(2)}(k_2 a_n) - 2j] - k_z^2 [G_s(x_m, y_m; x_n, y_n)] \pi a_n^2, & M = N \end{cases} \quad (24)$$

$$[G_3]_{mn} = -[k_z^2 \cdot G_3(x_m, y_m; x_n, y_n)] \pi a_n^2 \quad (25)$$

To simplify the integration or numerical analysis, each sufficiently small region with area ΔS is approximated by an equivalent circular region of the same area. We assume $\Delta S = \pi a_n^2$, where a_n denotes the radius of the equivalent circle. This approximation is valid when the discretization is sufficiently fine to facilitate subsequent analysis and derivations. Here, (x_m, y_m) denotes the coordinates of the m -th observation point, (x_n, y_n) denotes the coordinates of the n -th source point, and $\rho_{mn} = \sqrt{(x_m - x_n)^2 + (y_m - y_n)^2}$. $H_1^{(2)}$ is the first-order Hankel function of the second kind, and J_1 is the first-order Bessel function.

3. Neural Network

The method proposed in this paper is a multi-frequency extended DL model, whose core architecture adopts a two-stage neural network design. In the first stage, DRCNN is introduced to integrate multi-frequency information and expand feature representations. In the second stage, a DCED is employed to achieve accurate reconstruction of the permittivity distribution. This staged design facilitates the effective extraction and utilization of scattering features across different frequencies, thereby enhancing the final reconstruction quality and stability.

The DRCNN, whose architecture is illustrated in Figure 4, primarily focuses on the task of frequency extrapolation of electromagnetic scattered fields. The convolutional layers are followed by batch normalization and a rectified linear unit (ReLU) activation function to improve nonlinear feature learning and training stability. In traditional electromagnetic inversion methods, obtaining scattering data of the target at multiple frequencies is usually required to improve imaging quality and reconstruction stability. However, in practical applications, because of instrumental limitations, lengthy measurement procedures, or hardware constraints, acquiring complete multi-frequency scattering data is often highly challenging. Therefore, developing techniques capable of estimating scattered fields at other frequencies from a single frequency holds significant value for enhancing the efficiency and flexibility of practical systems.

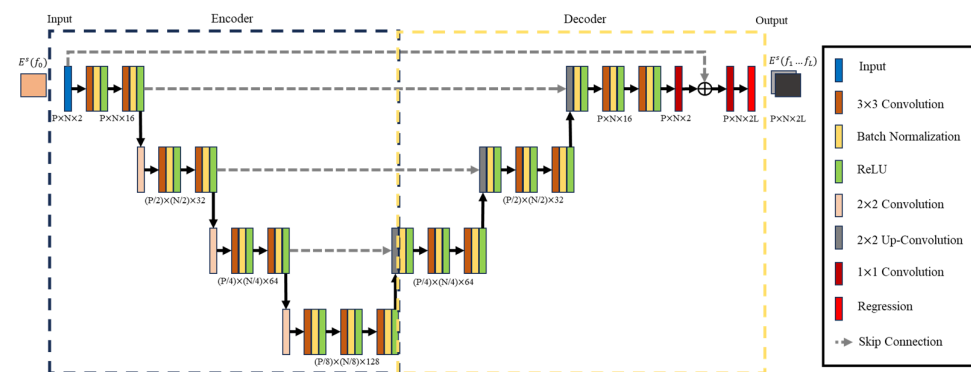


Figure 4. DRCNN Architecture.

The DRCNN model is designed to address this need, with its core objective being to learn the scattering information at a single frequency f_0 and predict the corresponding scattered field data at other frequencies $f_1, f_2, \dots, f_L$, thereby achieving a single-to-multiple frequency mapping. The input size is $P \times N \times 2L$, where P denotes the number of transmitters, N the number of receivers, and 2 represents the real and imaginary parts of the scattered field. Through this frequency extension mechanism, the network can supplement the missing frequency information in measurements or simulations, further improving the accuracy and stability of subsequent reconstruction tasks, such as restoring the permittivity distribution.

The key advantage of this method lies in its extrapolation capability along the frequency dimension. In practical applications, the DRCNN can learn the nonlinear relationships between different frequencies, including patterns arising from phase delay, amplitude attenuation, and variations in multiple scattering effects with frequency. Owing to the residual learning architecture, which effectively mitigates the common issue of gradient vanishing in deep networks while facilitating the preservation of high-frequency features and the smooth integration of low-frequency features, the model is particularly well-suited for constructing complex mapping functions across frequencies.

Moreover, frequency extrapolation contributes to improved reconstruction quality in electromagnetic imaging. According to wave theory, higher-frequency electromagnetic waves have shorter wavelengths, providing higher-resolution spatial information, whereas lower frequencies can penetrate deeper into the medium, covering a broader scattering range. Therefore, by predicting high-frequency scattered fields from low-frequency information, it is possible to combine the advantages of multiple frequency bands, achieving more accurate image reconstruction with both deep penetration and high surface resolution.

From the perspective of electromagnetic inverse scattering, scattered fields at different frequencies contain complementary information about the target. Lower-frequency fields generally provide greater penetration and more stable information regarding the global geometry and average dielectric contrast of the object, whereas higher-frequency fields contain richer spatial-frequency components and exhibit greater sensitivity to material interfaces, object boundaries, and local permittivity variations. Reconstruction based on a single frequency is therefore limited by incomplete spatial-frequency information and may produce blurred boundaries or ambiguity in the estimated dielectric distribution. The extrapolated multi-frequency scattered fields allow the proposed framework to combine the global information provided by lower frequencies with the localized structural information provided by higher frequencies, thereby introducing additional constraints into the inverse problem and reducing reconstruction ambiguity. This capability is particularly important in a three-layer medium, where the measured fields contain both target-induced scattering and reflection–transmission components generated at the dielectric interfaces. The multi-frequency representation enables the reconstruction network to better characterize these frequency-dependent responses, distinguish target-related features from background interface effects, and improve the stability, boundary representation, and structural accuracy of the reconstructed permittivity distribution.

In summary, the DRCNN plays a crucial role in electromagnetic imaging by performing frequency extrapolation and data completion. It improves the quality and resolution of image reconstruction while reducing the cost and practical barriers associated with acquiring multi-frequency data, providing a feasible and efficient technological pathway for future intelligent, low-cost electromagnetic imaging systems.

The main task of the second stage is the reconstruction of the permittivity distribution in the target region, which involves learning a mapping from the electromagnetic scattered fields back to the material properties. To achieve this, we adopt a network model based on a DCED architecture, with the overall structure shown in Figure 5. The network takes as input both the single-frequency scattered field from the first stage and the predicted multi-frequency scattering data. After a series of deep convolution and up-convolution operations, it outputs an image that accurately represents the permittivity distribution of the region under investigation.

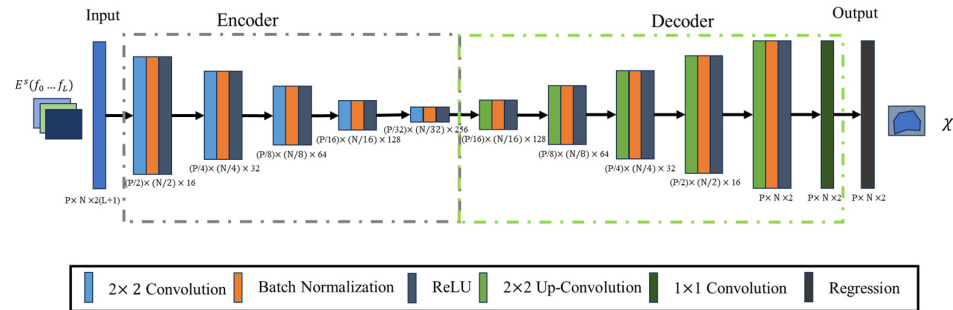


Figure 5. DCED Architecture.

The DCED architecture essentially constitutes a DL framework that combines feature extraction with image reconstruction. In the encoder, the multi-frequency scattering data first pass through multiple convolutional layers to progressively extract spatial and frequency features, effectively compressing them into a low-dimensional feature vector with rich semantic representation. Subsequently, in the decoder, the network uses a series of up-convolution operations to map the feature vector back into the spatial image domain, thereby reconstructing the permittivity distribution of the target region. The DCED architecture offers several key advantages:

1. **Multi-frequency information fusion:** The DCED can fully leverage the multi-frequency scattering data predicted in the first stage, learning the spatial interactions across different frequencies. This helps mitigate common issues in single-frequency reconstructions, such as blurring or insufficient information.
2. **Deep nonlinear modeling capability:** Through multiple layers of nonlinear transformations, the network can model the complex nonlinear relationship between the scattered field and the permittivity distribution, overcoming the inversion difficulties faced by traditional algorithms in high-contrast or strong-scattering scenarios.
3. **Local and global structural awareness:** The encoder can extract both local and global structural features of the image, aiding in maintaining clear target boundaries and reasonable internal distributions during reconstruction, thereby enhancing reconstruction accuracy and visual quality.
4. **Parameter sharing and regularization:** Convolutional operations inherently share parameters, keeping the total number of model parameters relatively controlled, which benefits learning from small datasets and prevents overfitting. Moreover, combined with batch normalization and carefully designed activation functions, this accelerates training convergence and improves model stability.

In this paper, the input multi-frequency data consist of a total of $L+1$ frequency components. The input data are stacked along the channel dimension and processed by the DCED model to produce a permittivity image. This image provides quantitative estimates of the relative permittivity values at each pixel and represents the geometric shape and spatial distribution of the target objects, providing a reliable basis for subsequent classification, recognition, or material analysis. Through the design of this stage, the study achieves a high-precision mapping from scattered field data to images of the target's physical properties, demonstrating the potential of DL in electromagnetic inversion by combining “physics-driven data” with “feature learning optimization.” Compared to traditional inversion workflows based on numerical solutions (such as the Born approximation or least-squares inversion), the DCED approach offers superior learning capability and inference speed, making it well-suited for real-time, high-resolution, and low-cost electromagnetic imaging applications.

In this paper, the relative permittivity is assumed to be frequency independent within the considered operating band. This assumption is adopted because the frequency range investigated in this work is relatively narrow, namely 20–40 MHz, over which the dielectric-property distribution of the target is approximated as invariant with respect to frequency. Under this formulation, the proposed DRCNN is designed to learn the nonlinear mapping between scattered-field responses at different frequencies for a fixed material-property distribution, rather than to explicitly model the dispersive characteristics of the medium. The nonlinear relationship $\mathcal{F}$ between the single-frequency measured scattered field (f_0) and the multi-frequency predicted scattered fields ($f_1, \dots, f_L$) can be expressed as:

$$E^s(f_1, \dots, f_L) = \mathcal{F}(E^s(f_0)) \quad (26)$$

The loss function is defined as:

$$Q(f_1, \dots, f_L) = \sum_{i=1}^N \|E_i^s(f_1, \dots, f_L) - \mathcal{F}(E_i^s(f_0))\| \quad (27)$$

The measured scattered fields E_i^s induced by N different incident fields E^i can be approximated through an iterative process using the optimized scattered fields $E_i^s(\chi)'$. Here, β denotes a constant normalization factor, and R represents the regularization term.

$$\min: f(\chi) = \sum_{i=1}^N \|E_i^s - E_i^s(\chi)'\| + \beta R(\chi) \quad (28)$$

4. Numerical Results

4.1. Experimental Conditions

This section presents a simulation platform for investigating the electromagnetic inverse scattering behavior of objects embedded in a three-layer medium and reconstructing their relative permittivity distributions. Unlike a single free-space background, the three-layer structure provides more realistic and challenging simulation conditions, which help evaluate the performance of inverse scattering techniques in inhomogeneous media. To obtain more comprehensive scattered-field data, this paper employs TM waves incident from multiple directions and angles, with the expectation that multi-angle observations can enhance the reconstruction quality and accuracy.

In the simulation study, the target scatterer is discretized into a series of small grid cells with side lengths less than $\frac{0.2\lambda_0}{\sqrt{\epsilon_r}}$, where λ_0 is the wavelength in free space and ϵ_r denotes the relative permittivity of the scatterer. The entire simulation is set within a layered medium background, reflecting the complex structures commonly encountered in real-world environments. The incident wave frequency is set to 30 MHz, with 32 transmitting positions and 32 receiving positions arranged. The training process employs the Adam optimizer, with a learning rate between 10^{-3} and 10^{-4} . The Modified National Institute of Standards and Technology (MNIST) dataset is used to construct digit-shaped scatterers with relative permittivity values ranging from 2.56 to 9. The larger dataset evaluated in Section 4.6 contains 10,000 MNIST-based samples. For each digit class, 1000 samples are randomly selected for each of the 10 digits, resulting in a total of 10,000 samples. The dataset is divided into 80% training, 10% validation, and 10% testing sets. The validation set is used to monitor model convergence, whereas the testing set is reserved only for final performance evaluation and is not used during network training.

No additional data augmentation is applied during the training process. This setting is used to ensure a controlled and consistent comparison among BPS-CNN, DCS-CNN, and the proposed two-stage method under the same dataset configuration.

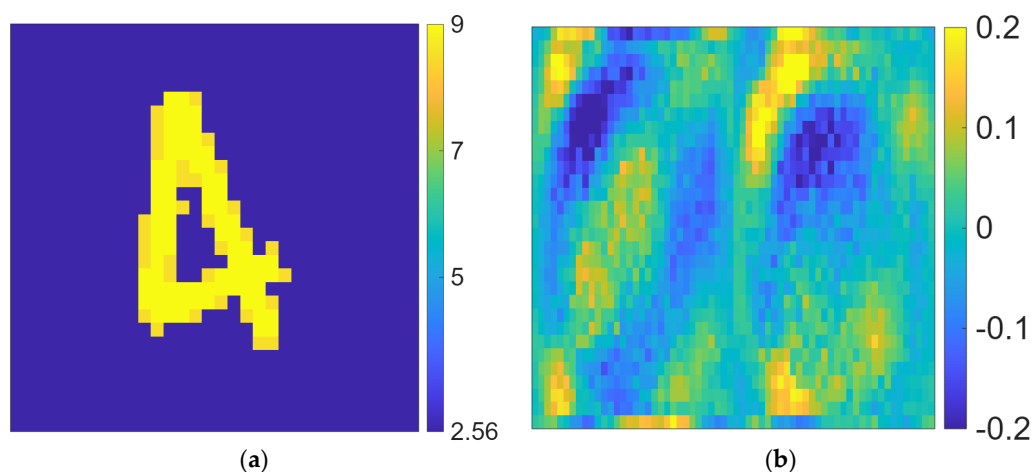
4.2. Performance of DRCNN for Frequency Extrapolation

The overall electromagnetic image reconstruction process is divided into two stages, and a different evaluation metric is adopted for each stage according to its specific objective. The first stage is frequency extrapolation, whose main objective is to predict the electromagnetic scattered fields at other frequencies from single-frequency scattering data. In the first stage, the mean squared error (MSE) is employed as the evaluation metric. MSE focuses on the pointwise squared error between the model predictions and the actual scattered field values, and its numerical value effectively reflects the accuracy of the network in predicting scattering intensities. Since the scattered field values themselves have physical dimensions and a fixed numerical range, using an absolute error-based metric such as MSE is a reasonable choice.

In this process, the frequency $f_0 = 30$ MHz is used as the input, and the DRCNN is employed to predict the scattered field data at $f_1 = 20$ MHz and $f_2 = 40$ MHz. Through network training, the DRCNN effectively learns the physical transformation patterns between single-frequency input and multi-frequency output, demonstrating very high prediction accuracy in numerical simulations, while maintaining stable performance even for high-contrast and strongly scattering targets.

The proposed method reconstructs the relative permittivity distribution using multifrequency scattering data and is compared with the BPS-CNN and DCS-CNN methods [25]. To quantitatively evaluate the reconstruction performance, we used Normalized Mean Square Error (NMSE) and SSIM as evaluation metrics to assess the reconstruction accuracy and image quality of the proposed method.

Figure 6 presents the frequency extrapolation results obtained using the DRCNN. The reconstructed object is shown in Figure 6a. The real and imaginary parts of the input scattered field at f_0 (30 MHz) are shown in Figure 6b. The predicted and actual scattered fields at f_1 (20 MHz) are shown in Figure 6c and 6d, respectively, while the predicted and actual scattered fields at f_2 (40 MHz) are shown in Figure 6e and 6f, respectively. For each scattered-field map, the left panel represents the real part, and the right panel represents the imaginary part. Based on MSE evaluation, the prediction results at frequencies f_1 and f_2 are 1.1×10^{-3} and 9.5×10^{-4} , respectively. These low MSE values indicate that the prediction model achieves high accuracy, effectively capturing the scattering field characteristics at different frequencies and demonstrating robust performance in the reconstruction process.



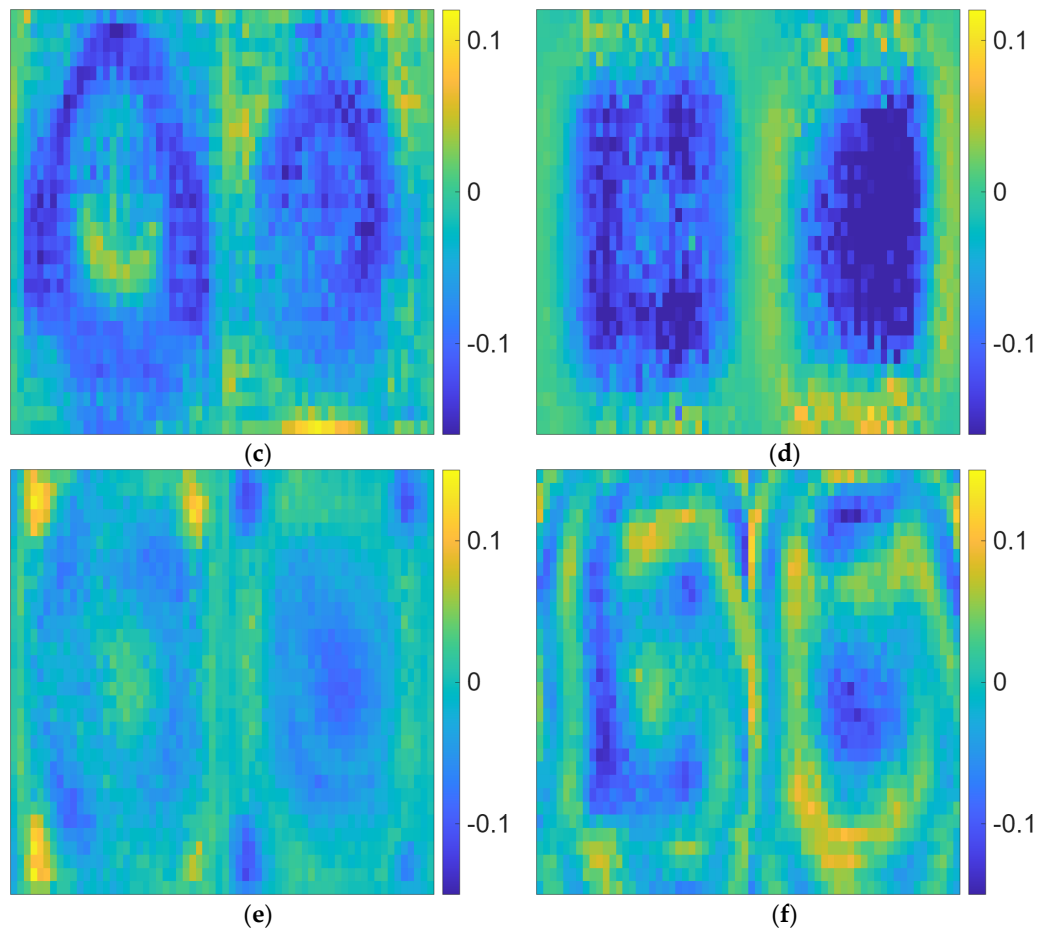


Figure 6. Frequency extrapolation results obtained using the DRCNN: (a) ground-truth relative permittivity distribution; (b) real and imaginary parts of the input scattered field at $f_0 = 30$ MHz; (c) predicted scattered field at $f_1 = 20$ MHz; (d) corresponding ground-truth scattered field at $f_1 = 20$ MHz; (e) predicted scattered field at $f_2 = 40$ MHz; and (f) corresponding ground-truth scattered field at $f_2 = 40$ MHz. For panels (b–f), the left and right images represent the real and imaginary components, respectively.

4.3. Noise Enhancement

To comprehensively evaluate the performance of the proposed multi-frequency extended DL model under potential measurement errors in practical applications, reconstruction experiments are specifically designed with varying noise levels to assess its stability and robustness. Specifically, Gaussian white noise was added to the scattered-field data during the testing phase, with five different noise intensities set at 5%, 10%, 20%, 30%, and 40%, simulating scenarios ranging from low to high measurement errors. All reconstruction procedures are conducted using the same network architecture and training parameters to ensure experimental fairness. The noisy scattered-field data are defined as:

$$\hat{E}^s = E^s + n \quad (29)$$

where $\hat{E}^s$ denotes the noisy scattered-field vector and n represents the added Gaussian noise. Since the scattered-field data are complex-valued, both the real and imaginary components are perturbed by the added Gaussian noise. For a prescribed noise level ψ , the corresponding signal-to-noise ratio (SNR), expressed in decibels (dB), is defined as

$$SNR_{dB} = 10 \log_{10} \left(\frac{1}{\psi} \right) \quad (30)$$

In this paper, the noise was added based on the measured average power of the scattered-field vector. Therefore, the noise power is given by

$$P_n = \psi P_s \quad (31)$$

$$P_s = \frac{1}{N} \sum_{m=1}^N |E_s(m)|^2 \quad (32)$$

where P_n denotes the noise power and P_s denotes the measured average power of E^s .

In Figure 7, Figure 7 (a) presents the ground truth. Figure 7 (b)–(f) illustrate the reconstruction results obtained by applying the proposed method under increasing noise levels of 5%, 10%, 20%, 30% and 40%, respectively. It can be observed that at low noise level (e.g., 5%), the reconstructed permittivity distribution is almost identical to the ideal target, with high alignment at the boundaries and a clearly restored permittivity gradient within the target. As the noise intensity increases, some blurring and boundary distortions appear in the reconstructed images; however, the overall target contour remains accurately depicted. In particular, the major regions of permittivity variation still maintain good contrast, demonstrating the excellent noise resistance of the proposed model.

Furthermore, to quantitatively evaluate the reconstruction performance under different noise conditions, Table 1 summarizes the results in terms of NMSE and SSIM. As the noise level increases from 5% to 40%, the NMSE exhibits a gradual upward trend, indicating the accumulation of reconstruction errors due to stronger noise interference. Meanwhile, the SSIM values show a corresponding decreasing tendency, reflecting the degradation in structural similarity between the reconstructed images and the ground truth.

Nevertheless, it is noteworthy that even under high noise levels (e.g., 30% and 40%), the proposed method still maintains relatively low NMSE and high SSIM values compared to typical expectations in noisy inverse problems. This demonstrates that the reconstructed results preserve both quantitative accuracy and structural integrity to a considerable extent. The consistency between the quantitative metrics in Table 1 and the visual results in Figure 7 further validates the robustness and stability of the proposed multi-frequency extended DL model under severe noise contamination.

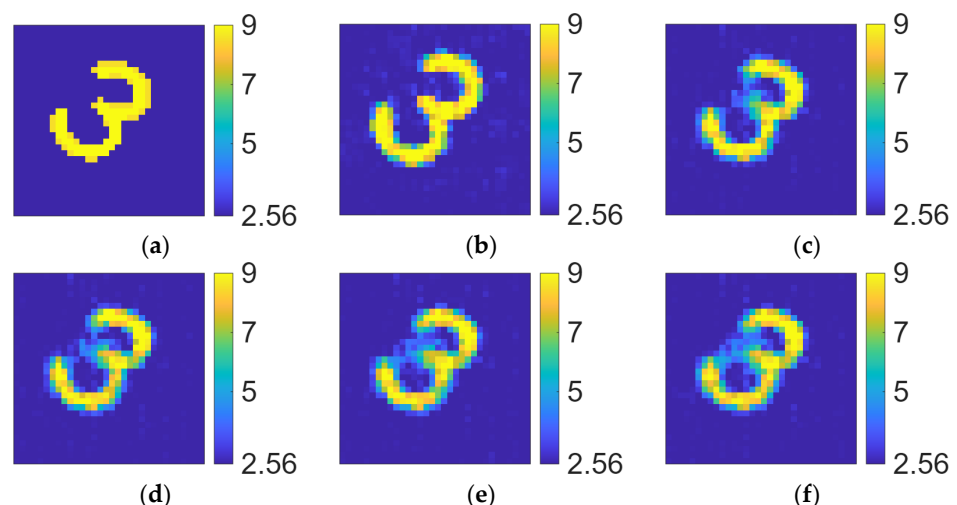


Figure 7. Comparisons of reconstructed relative permittivities of digit-shaped scatterers in MNIST under different noise levels: (a) ground truth; (b) 5% noise; (c) 10% noise; (d) 20% noise; (e) 30% noise; and (f) 40% noise.

Compared with conventional single-frequency DL models, the proposed multi-frequency extended framework demonstrates greater reconstruction stability and accuracy under the same noise levels. This improvement is attributed to the multi-frequency feature

extension strategy introduced in the first stage, which effectively integrates complementary information carried by different frequencies, thereby enhancing the model's feature recognition capability under noise interference. Overall, the quantitative evaluation indicates that the proposed multi-frequency extended DL model remains stable under the tested noise conditions, as reflected by the observed trends in NMSE and SSIM.

Table 1. Performance evaluation of reconstructed relative permittivity under different noise conditions using NMSE and SSIM.

Performance	5% Noise	10% Noise	20% Noise	30% Noise	40% Noise
NMSE	2.37%	2.47%	4.57%	3.1%	4.31%
SSIM	93.06%	92.62%	87.21%	85.67%	77.97%

4.4. MNIST Dataset Reconstruction

To validate the effectiveness of the proposed method, initial reconstruction images are first generated from the scattered-field data using the DCS and BPS methods. These initial images are then used as inputs to a CNN to obtain more accurate reconstruction results, which are subsequently compared with the proposed method. To ensure fairness in evaluation, the same dataset is employed and divided into 80% for training and 20% for testing. Moreover, the training process adopts the same parameter settings as those used in the multi-frequency extended two-stage method, aiming to enhance the accuracy and consistency of the reconstructed images.

Figure 8 and Table 2 present a comparative evaluation of the reconstruction performance of three different methods—BPS-CNN, DCS-CNN, and the proposed two-stage method—using a given ground truth dataset. Figure 8 displays the reconstructed images for two distinct shapes (a “9” and a “2”) across the three methods, alongside the original ground truth images for reference. The color scales in the images indicate the intensity values, ranging from 2.56 to 9, providing a visual assessment of reconstruction quality. Table 2 uses NMSE and SSIM to evaluate the reconstructed results of the three different methods.

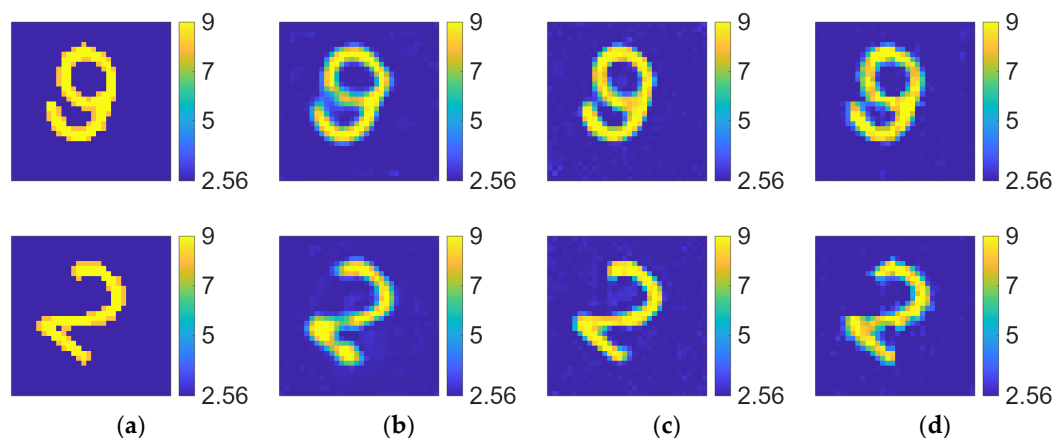


Figure 8. Comparisons of reconstructed relative permittivities of digit-shaped scatterers: (a) ground truth; (b) BPS-CNN result; (c) DCS-CNN result; and (d) result obtained using the proposed two-stage method.

Table 2. Reconstructed relative dielectric constant results using different methods.

Performance	BPS-CNN	DCS-CNN	Two-Stage
NMSE	21.44%	10.94%	2.73%
SSIM	64.13%	65.29%	71.82%

As shown in Figure 8, the reconstructed results of the BPS-CNN method are noticeably blurred, with significant loss of detail, especially along the shape boundaries. Analyses of the DCS-CNN method produces slightly clearer reconstructions compared to the BPS-CNN, but artifacts and distortions are still present. In contrast, the two-stage method yields the most accurate results, visually the most faithful, and highly similar to the ground truth image, with minimal blurring and well-preserved structural details. This visual superiority of the two-stage method is corroborated by the quantitative data in Table 2. These results collectively demonstrate that the proposed two-stage method outperforms both BPS-CNN and DCS-CNN, delivering the highest reconstruction accuracy and structural fidelity across the evaluated dataset.

4.5. Generalization to Unseen Patterns

To systematically evaluate the generalization capability of the proposed two-stage image reconstruction method, this paper introduces an additional square-shaped dataset as supplementary training and testing data. In this simulation environment, 10 scatterers with different permittivity distributions are placed at 50 different positions within a 32×32 -pixel area. This setup is designed to examine the model's performance on completely unseen shapes and structural features, as well as to assess its adaptability to variations in geometric distribution, stroke styles, and structural complexity.

In terms of data, all samples are strictly maintained with the same pixel size as the MNIST handwritten digit dataset to ensure consistency in image specifications. They are then placed into the same simulation space as the MNIST scatterer experiments, using identical boundary conditions and background medium settings. To better simulate realistic variations in material properties, the relative permittivity of the scatterers is set within the range of (2.56, 10). Regarding the training strategy, the same parameters are adopted as those used with the MNIST handwritten digit dataset for training, in order to test the model's transferability and generalization capability to unseen scatterer shapes.

Figure 9 presents the reconstruction results of square scatterers using different methods, where the three columns correspond to the ground truth, the BPS-CNN method, and the proposed two-stage method, respectively. In this paper, the proposed multi-frequency extended two-stage method achieved an MSE of 3.02% and an SSIM of 56.63%. As shown, BPS-CNN is able to reconstruct the general position and shape of the scatterers; however, noticeable blurring occurs at the boundaries, and the internal permittivity distribution tends to be averaged, deviating from the ground truth. In contrast, the proposed two-stage method can more accurately recover the geometric structure and boundary details of the scatterers, while producing a permittivity distribution closer to the ground truth. Although slight granular noise remains, the method still demonstrates superior imaging accuracy and robustness.

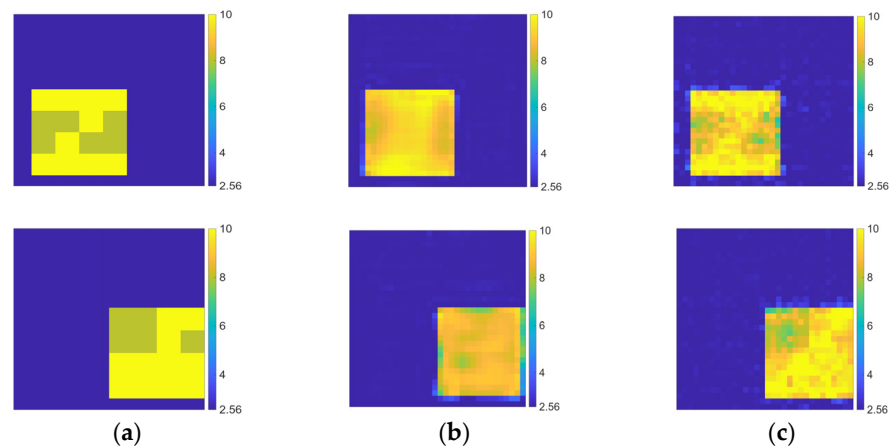
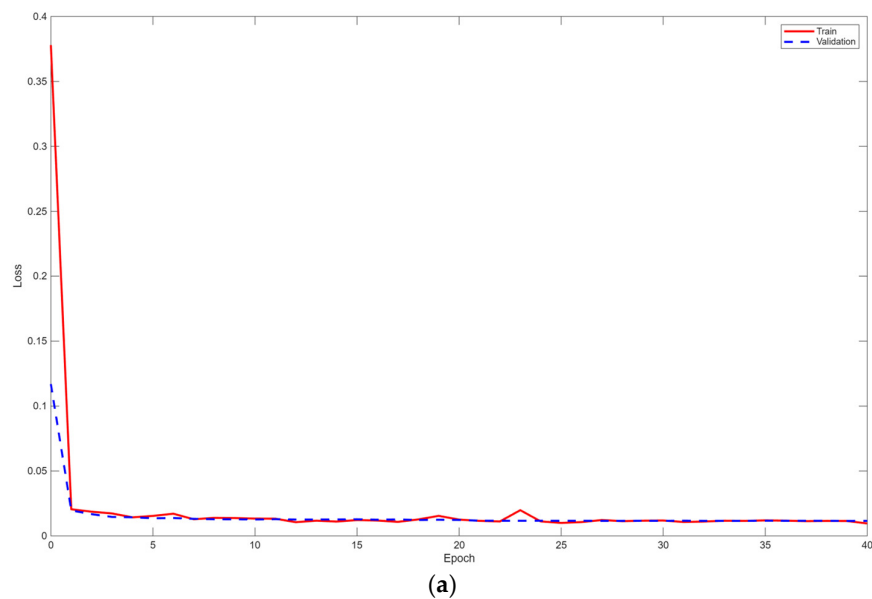


Figure 9. Comparisons of reconstructed relative permittivities of square-shaped scatterers. (a) ground truth; (b) reconstruction obtained using BPS-CNN; and (c) reconstruction obtained using the proposed two-stage method.

4.6. Training Stability and Validation

To further validate the training stability and robustness of the proposed two-stage method, the training and validation loss curves of the DRCNN and DCED networks are presented in Figure 10, and an additional experiment using the full MNIST-based dataset is shown in Figure 11. As shown in Figure 10, both the training and validation losses decrease steadily with increasing epochs and gradually converge to stable values. The validation loss follows a trend similar to the training loss, indicating that the proposed networks were trained stably without severe overfitting. Furthermore, to evaluate the influence of dataset size on reconstruction performance, the full MNIST-based dataset, including 1000 styles for each of the 10 digit classes, is used for additional validation. The same training and testing procedures are applied to ensure a fair evaluation. As shown in Figure 11, the proposed two-stage method maintains stable reconstruction performance under the larger dataset setting.



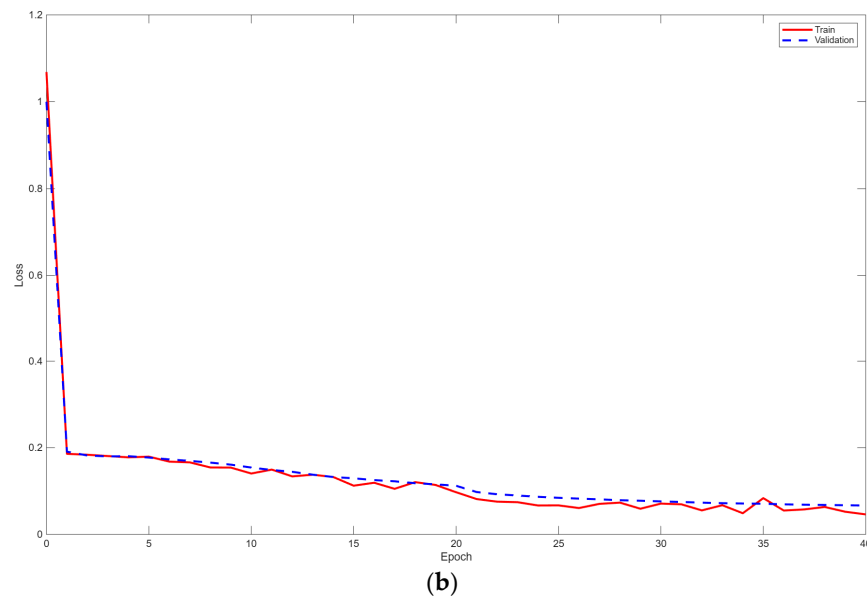


Figure 10. Training and validation loss curves of the proposed two-stage method: (a) DRCNN frequency-extrapolation stage and (b) DCED image-reconstruction stage.

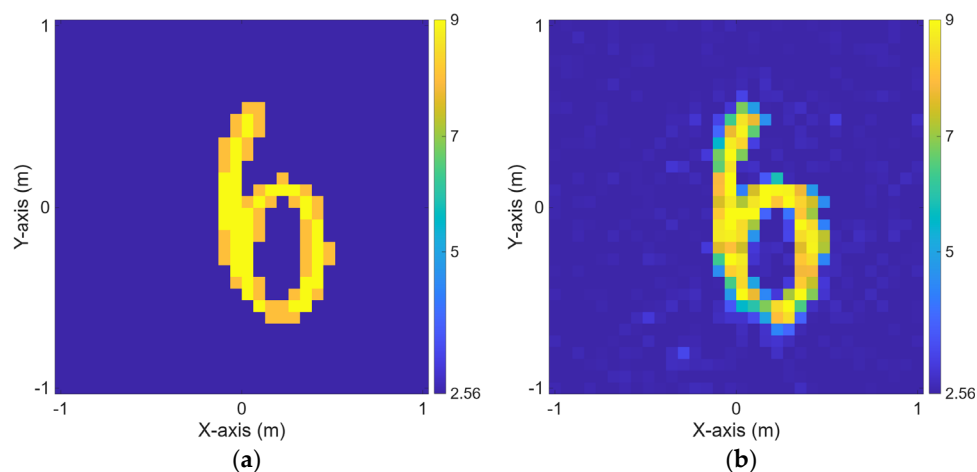


Figure 11. Reconstruction results obtained using the larger MNIST-based dataset to evaluate reconstruction stability under an increased dataset size: (a) ground truth and (b) reconstruction obtained using the proposed two-stage method.

All numerical simulations and network training procedures were performed on a workstation equipped with a 14th-generation Intel Core i7 CPU, 128 GB RAM, and an NVIDIA RTX 50-series GPU. The training time of BPS-CNN and DCS-CNN is approximately 3 min for each model. For the proposed two-stage method, the total training time is approximately 6 min, including approximately 3 min for the DRCNN stage and 3 min for the DCED stage. Once the models are trained, the average inference time for reconstructing one testing sample is less than 1 s for all compared methods. Therefore, the additional computational cost of the proposed method is mainly associated with the offline training stage, whereas the online reconstruction process remains highly efficient. This confirms that the proposed two-stage method is suitable for real-time or near-real-time electromagnetic imaging applications.

5. Conclusions

This paper proposes a multi-frequency extended two-stage DL framework for solving the problem of electromagnetic image reconstruction under TM wave incidence in layered media. The framework integrates two stages: frequency extrapolation using the DRCNN and image reconstruction using the DCED to effectively overcome the limitations of scarce multi-frequency data and complex preprocessing, while reducing the hardware burden and measurement time. Numerical experiments demonstrate that the proposed method outperforms BPS-CNN and DCS-CNN in terms of image quality, boundary detail, and high-contrast scatterer reconstruction. DRCNN accurately extrapolates multi-frequency scattered fields, while DCED produces lower NMSE values and higher SSIM values than the comparison methods. Even under 40% Gaussian white noise, the method preserves clear target contours and accurate permittivity distributions, indicating strong noise robustness. Moreover, even when trained with scatterers of different shapes, the framework is still able to achieve reliable reconstruction results, demonstrating its generalization capability across various shapes and structures. In summary, the proposed method demonstrates significant advantages in imaging accuracy, robustness, and generalization, providing an efficient and feasible solution for electromagnetic image reconstruction, with potential for further application to more complex environments and experimental data. Although the proposed framework shows promising reconstruction performance, the present study remains limited by its reliance on primarily synthetic datasets and by the assumption that the relative permittivity is frequency independent within the considered operating band. The present validation mainly considers MNIST-based digit-shaped scatterers and square-shaped scatterers. More complex geometries with richer fine-scale structural features would require larger and more diverse simulation datasets, as well as additional evaluation under different material distributions, spatial resolutions, and frequency configurations. Future work will focus on extending the proposed framework to more complex structural datasets, more realistic imaging scenarios, and highly dispersive media with frequency-dependent relative permittivity, thereby enabling a more comprehensive assessment of its robustness, generalization capability, and applicability under practical material conditions. In addition, the effects of modeling errors, calibration uncertainties, correlated noise, and antenna-position deviations will be systematically investigated to further evaluate the reliability of the proposed method under practical measurement conditions.

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