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Dynamic Event-Triggered Prescribed-Time Consensus of Second-Order Multi-Agent Systems Under Disconnected Time-Varying Topologies

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Abstract

This article concentrates on the practical prescribed-time leader-following consensus problem of second-order multi-agent systems (MASs) under disconnected time-varying topologies. Firstly, a new practical prescribed-time stability criterion is presented, where the time-varying scaling function is related to connected moments and the derivative inequality is increasing in the disconnected intervals. Secondly, inspired by the backstepping design framework, the practical prescribed-time control protocol is designed, including the distributed velocity estimator, virtual velocity and dynamic event-triggered controller. Thirdly, some consensus conditions for achieving leader-following consensus are established and the Zeno behavior is excluded. Finally, simulation results on unmanned aerial vehicle (UAV) formation tracking are provided to demonstrate the effectiveness of the proposed method.

Keywords: multi-agent systems; practical prescribed-time stability; disconnected time-varying topology; dynamic event-triggered mechanism

MSC: 93A16; 93C15; 93D05; 37N35

1. Introduction

MASs have attracted a surge of interest owing to their superior application prospects in many engineering fields, such as intelligent transportation, unmanned aerial vehicles (UAVs), smart grids and so on [1–3]. Consensus is one of the fundamental objectives in the coordination of MASs, aiming to ensure that all agents achieve consistent goals through local interactions. Up to now, consensus can be categorized into leaderless consensus and leader-following consensus. Recently, a significant number of studies have been conducted with regard to the leader-following consensus of MASs [4–6].

In [5,6], the systems usually achieve consensus asymptotically. However, in practice, it is usually expected that all agents can achieve consensus with a rapid convergence rate and high control accuracy. For example, a group of UAVs tracks and destroys the opponent's target within the specified time. In recent years, a significant number of works have focused on the finite/fixed-time consensus for MASs in challenging scenarios and diverse environments [7–10]. The difference between finite-time and fixed-time consensus lies in the dependence on initial conditions. The fixed-time consensus can effectively address the estimation of settling time without the information of initial values. However, it usually leads to overestimation because the settling time is related to the control parameters. In



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addition, if there is a significant difference in initial states, the fixed-time controller typically requires a larger initial control input, which results in consuming more energy to complete the collaborative task and may damage the physical motor or cause input saturation issues. To alleviate this problem, the prescribed-time control approach has been exploited by introducing a time-varying function [11], where the settling time is a directly tunable parameter that determines the convergence of system states. In practical scenarios such as UAV swarms, multi-robot coordination, and smart grids, the temporal coordination of tasks is of critical importance. For instance, when executing time-critical military missions like precision strikes, it is essential to ensure that the UAV swarm forms a specific formation or reaches the target area synchronously within a prescribed time. Therefore, significant efforts have been made to develop the prescribed-time control protocol to achieve consensus for MASs [12–15]. For example, in [12], the distributed predefined-time formation control problem was studied for specific heterogeneous MASs, including completely different unmanned autonomous helicopters, unmanned ground vehicles, and autonomous underwater vehicles in the presence of actuator faults; in [14], by adopting an adaptive sliding mode control, the velocity-free prescribed-time consensus issue for MASs with input and output quantization was addressed; in [15], a fault-tolerant formation control strategy was proposed to guarantee the prescribed-time consensus for heterogeneous switched nonlinear MASs with actuator faults. It is worth noting that, despite those extensive studies on prescribed-time control in the literature [12–15], the controller updates in the aforementioned control methods are continuous. Therefore, it is of both theoretical and practical significance to investigate how to achieve prescribed-time consensus for MASs while drastically reducing the frequency of controller updates, which is precisely the main focus of this paper.

An event-triggered mechanism (ETM) is an effective communication and control updating strategy that reduces unnecessary transmissions by updating signals only when certain state-dependent conditions are satisfied. There is little relevant work considering the prescribed-time control protocol with an ETM [16,17]. To further effectively utilize communication and computation resources, the dynamic event-triggered mechanism (DETM) has been presented [18]. As a special case of an ETM, a DETM typically introduces internal dynamic variables, thereby providing additional design freedom and improving triggering performance. This mechanism can significantly decrease communication burden, actuator updates, and energy consumption while still preserving desired closed-loop stability properties. Based on this advantage, the combination of a DETM and prescribed-time control has yielded fruitful results [19–22]. To mention just a few, in [21], a modified DETM was established to guarantee the realization of prescribed-time consensus for pure-feedback MASs under input dead-zone nonlinearities and output constraints.

Unfortunately, the prescribed-time consensus issues in [19–22] have been solved assuming a connected and fixed topology, which implies that agents can communicate with each other at all times. However, in practice, the physical constraints cause the limitation of agents' sensing range. It implies that the communication topology of MASs is typically dynamic and even disconnected at certain moments; e.g., communication between unmanned systems may become disconnected when the communication network is interfered with and/or blocked; UAVs may have to disconnect from their neighbors to avoid obstacles during flight. In [23], the time-varying disconnected topologies were considered, where the agents were divided into three types and a local bipartite pinning algorithm was introduced to reestablish the communication channels between disconnected and connected subnetworks. However, this algorithm only guarantees the fixed-time consensus. Therefore, in the context of time-varying disconnected topologies, how to design a novel DETM-based prescribed-time consensus scheme, while ensuring both

communication efficiency and prescribed-time convergence, is a challenging problem that deserves further investigation.

Following the abovementioned analysis, this article is dedicated to addressing the practical prescribed-time consensus for second-order MASs under a time-varying disconnected topology by utilizing a DETM. A summary of the main contributions is as follows.

- (1) A new practical prescribed-time stability criterion is developed for disconnected time-varying topologies. The proposed criterion explicitly incorporates connected time instants into the time-varying scaling function and allows the derivative inequality to increase during disconnected intervals. Compared with the period case in [24] and the stability result without interval division in [25–28], the developed practical prescribed-time stability results reflect a more general and practically relevant communication scenario.
- (2) The time-varying disconnected topology is considered, where the time interval from the interruption of the communication channel between Type I and Type II followers and isolated followers to the completion of its reconstruction is introduced. Compared with the results in [23], the proposed topology model is more realistic and possesses wider applicability.
- (3) By combining the backstepping design idea, an integrated control framework is constructed, including a distributed velocity estimator, virtual velocity, and a dynamic event-triggered controller. This framework effectively resolves the issue that the leader's velocity information is unavailable to certain followers while also solving the practical prescribed-time consensus problem.

The remainder of this paper is organized as follows. Section 2 gives some preliminaries. Section 3 displays the second-order MASs, topology model, practical prescribed-time stability theorem and control objective. In Section 4, the control protocol is designed. In Section 5, the main results are given. In Section 6, the simulation results based on UAVs are shown. The conclusion is finally drawn in Section 7.

2. Preliminaries

2.1. Notations

$\mathbb{R}^n$ denotes n -dimensional real vector; I_n is n -dimensional identity matrix; A^T and $\lambda_{\min}(A)$ represent the transpose and minimum eigenvalue of matrix A , respectively; $\|x\|$ is the Euclidean norm of x ; $\|x\|_1$ is the 1-norm of x ; $\mathcal{I}$ denotes the set of Type I and Type II followers; $\hat{\mathcal{I}}$ denotes the set of isolated followers. $\mathbf{1}_n$ is the n -dimensional vector of all elements being 1.

2.2. Graph Theory

Suppose that the symbol $\mathcal{G}(\mathcal{V}, \mathcal{E})$ represents the undirected communication topology among N followers, where $\mathcal{V} = \{1, 2, \dots, N\}$ denotes the set of followers and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. N_i represents the set of neighbors of follower i . If there is a link from follower j to follower i , then $(j, i) \in \mathcal{E}$. Define the adjacency matrix as $\mathcal{A} = (a_{ij})_{N \times N}$, in which, if $(j, i) \in \mathcal{E}$, then $a_{ij} > 0$; otherwise, $a_{ij} = 0$. Let $L = (l_{ij})_{N \times N}$ be the Laplacian matrix, where its elements satisfy $l_{ij} = -a_{ij}$, $i \neq j$, and $l_{ii} = -\sum_{j \in N_i} l_{ij}$. Define $B = \text{diag}\{b_1, b_2, \dots, b_N\} \geq 0$ as the adjacency matrix between leader and followers. We assume that there is at least one edge connecting leader and followers. For simplicity, we introduce a topology matrix $H = (h_{ij})_{N \times N} = L + B$.

3. Problem Formulation

Consider the second-order MASs with N followers and one leader. The dynamics of i -th follower and leader are, respectively, described by

$$\begin{aligned}\dot{p}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= q_i(p_i(t), v_i(t)) + u_i(t), i = 1, \dots, N,\end{aligned}\quad (1)$$

and

$$\begin{aligned}\dot{p}_0(t) &= v_0(t), \\ \dot{v}_0(t) &= u_0(t),\end{aligned}\quad (2)$$

where the vectors $p_i(t) \in \mathbb{R}^n$ and $v_i(t) \in \mathbb{R}^n$ are the position and velocity of i -th follower, respectively; $p_0(t) \in \mathbb{R}^n$ and $v_0(t) \in \mathbb{R}^n$ are the position and velocity of leader, respectively; $q_i(p_i(t), v_i(t)) \in \mathbb{R}^n$, denoted simply as $q_i(t) \in \mathbb{R}^n$, represents the external disturbance of i -th follower, with $\|q_i(t)\| \leq Q$ for all i ; Q is a positive constant; $u_i(t) \in \mathbb{R}^n$ and $u_0(t) \in \mathbb{R}^n$ are the control inputs of i -th follower and leader, respectively.

Assumption 1. For any $u_0(t) \in \mathbb{R}^n$, there exists a constant $\mathcal{U}$ such that $\|u_0(t)\| \leq \mathcal{U}$.

Definition 1. For MASs (1) and (2), for any initial conditions and the designed control strategies, if the position and velocity states satisfy

$$\begin{aligned}\lim_{t \rightarrow T_2^-} \|p_i(t) - p_0(t)\| &\leq \tau_1, \\ \|p_i(t) - p_0(t)\| &\leq \tau_1, t \geq T_2, \\ \lim_{t \rightarrow T_2^-} \|v_i(t) - v_0(t)\| &\leq \tau_2, \\ \|v_i(t) - v_0(t)\| &\leq \tau_2, t \geq T_2,\end{aligned}$$

where T_2 is the prescribed time and $i = 1, \dots, N$, τ_1 and τ_2 are the positive constants, then MASs (1) and (2) can achieve the practical prescribed-time consensus.

3.1. Communication Topology

In practice, the information of an agent is usually exchanged within the sensing radius. However, the limitation of the sensing radius may cause disconnected communication at some moments. According to the communication links among agents, we classify the agents into the following three types:

- (1) Type I Followers: the agents directly connect to the leader (yellow followers in Figure 1).
- (2) Type II Followers: the agents connect to Type I followers (green followers in Figure 1).
- (3) Isolated Followers: the agents can neither directly communicate with the leader nor Type I or Type II followers during a certain time interval (black followers in Figure 1).

It should be noted that, even if isolated followers do not exchange information with leaders and Type I followers, it is still possible to exchange information with Type II followers. But, if the isolated follower does not exchange information with Type II followers, it indicates that the followers are completely isolated and unable to communicate with any neighbors. For isolated followers, the time intervals are divided into connected and disconnected intervals, denoted as $[t_s, t_s^a)$ and $[t_s^a, t_{s+1})$, $s = 0, 1, \dots$, respectively. In these intervals, the communication topologies are invariant. Define $S(0, t)$ and $S^a(0, t)$ as the total connected and disconnected intervals, respectively.

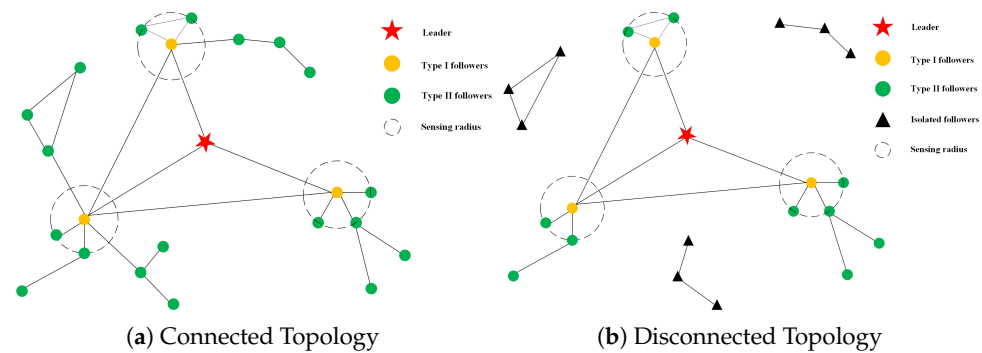


Figure 1. The schematic diagram of connected and disconnected topologies.

Assumption 2. Assume that there exists a positive constant τ such that $|S^a(0, T_m)| \leq \frac{T_m}{\tau}$ holds, where $|S^a(0, T_m)|$ denotes the total length of disconnected intervals within $[0, T_m)$, $m = 0, 1, 2$.

Remark 1. In [23], a local pinning algorithm was presented to rebuild the communication between Type I and Type II followers as well as isolated followers. The key of this algorithm is: (1) The identification of Type II followers and isolated followers by employing depth/breadth first search algorithm. (2) The selection of pinning followers. (3) The establishment of links. In this paper, we adopt the algorithm in [23] to establish the communication between isolated and nearest Type I followers. Furthermore, we observe that the time intervals between communication interruption and restoration are disconnected, but the results in [23] have not been taken into account at this point. This implies that the rebuilding of links in [23] was completed within a very short time, even within a few sampling periods. In reality, this may be idealized. In this paper, we further consider a more general topology model to improve the results in [23], where the connected and disconnected intervals are modeled.

Remark 2. In this paper, the topology may be disconnected at any time, and the disconnection may last for an arbitrarily long period. Instead of assuming global connectivity, we actively resolve the disconnection issue through the proposed pinning algorithm in [23]. In contrast, the jointly connected switching topology requires that the union of the graphs over any finite time interval is connected, while the Markovian switching topology assumes that the switching obeys a known Markov chain with a prescribed transition probability matrix. Both switching topologies impose probabilistic or cumulative connectivity assumptions. They do not handle true disconnections but rather assume that connectivity is restored frequently enough or with high probability.

Remark 3. Taking cooperative search-and-rescue by UAV swarms as an example, the leader UAV determines the task point and action trajectory, while the follower UAVs are responsible for target search, environmental perception, map construction, and communication relay. According to their connectivity with the leader and the main communication network, the follower UAVs can be classified into three categories. Type I followers can establish direct communication links with the leader. They are mainly responsible for receiving global mission references, maintaining the main communication, and disseminating task commands and state information to other UAVs. Type II followers cannot directly communicate with the leader but can access the main network indirectly through Type I followers. These UAVs are mainly employed to expand the search coverage area, collect local environmental sensing data, and exchange information with the main network via Type I followers. Isolated followers are UAVs that temporarily lose communication connections with the leader, Type I followers, or Type II followers due to limited communication range, dynamic obstacle-avoidance maneuvers, and so on.

When an isolated subnetwork appears, a UAV is selected as a local pinned follower. The purpose is to restore effective connections between isolated subnetworks and leader reference information at lower communication and control costs. Therefore, when restoring communication to isolated subnetworks, priority should be given to rebuilding communication connections with UAVs that have strong local connectivity and better link quality. Compared to Type II followers, Type I followers have a stronger global reference anchoring effect. If an isolated follower is connected to Type II followers, the leader information needs to go through more communication hops to be transmitted, which may introduce greater latency, packet loss risk, cumulative error and slower consensus rate. Therefore, the connection between the pinned isolated follower and the nearest Type I follower can more reliably restore information exchange between isolated subnetworks and the main network and facilitate overall collaborative and consistent control.

3.2. Practical Prescribed-Time Stability Lemma

The practical prescribed-time stability aims to achieve stability within an arbitrary time range $[0, T_m)$. Because T_m is finite, $[0, T_m)$ can be divided into the finite time intervals. Meanwhile, based on the considered communication topology model, s is finite within $[0, T_m)$. Thus, for isolated followers, we assume that there exist r_m intervals composed of connected and disconnected topology intervals within $[0, T_m)$. In addition, the frequency of disconnected communication topologies among agents is random; $\cup_{s=0}^{r_m-1} [t_s, t_{s+1})$ may not fully encompass $[0, T_m)$. Therefore, we assume that, if there exists the interval $[t_{r_m}, T_m)$, it is connected. To represent time intervals more clearly, let $[t_{r_m}, t_{r_m+1}) = [t_{r_m}, T_m)$ and $\cup_{s=0}^{r_m} [t_s, t_{s+1}) = [0, T_m)$. Define $\mathcal{R}_{m-1} = \{0, 1, \dots, r_m - 1\}$ and $\mathcal{R}_m = \{0, 1, \dots, r_m\}$. For Type I or Type II followers, we adopt the same notation about the time intervals for simplicity. The main difference is that the interval $[t_s, t_{s+1})$ denotes the connected interval for Type I or Type II followers. Therefore, the m -th time-varying function is designed as

$$\varepsilon_m(t) = \begin{cases} \frac{T_m}{T_m - t + t_s}, t \in [t_s, t_{s+1}), s \in \mathcal{R}_m, \\ 1, t \in [T_m, +\infty), \end{cases} \quad (3)$$

where $m = 0, 1, 2, 0 < T_0 < T_1 < T_2$ is the prescribed time. The derivative of $\varepsilon_m(t)$ can be obtained as follows:

$$\dot{\varepsilon}_m(t) = \begin{cases} \frac{1}{T_m} \varepsilon_m^2(t), t \in [t_s, t_{s+1}), s \in \mathcal{R}_m, \\ 0, t \in [T_m, +\infty). \end{cases}$$

Definition 2 ([29]). Considering the nonlinear system $\dot{x}(t) = g(x(t))$, $x(0) = x_0$, if $\lim_{t \rightarrow T_m^-} \|x(t)\| \leq \tau_3$ and $\|x(t)\| \leq \tau_3, t \geq T_m$, where τ_3 is the positive constant, then $x(t)$ is practical prescribed-time stable.

In [30], the following practical prescribed-time stability is presented.

Lemma 1 ([30]). Considering the nonlinear system $\dot{x}(t) = g(x(t))$, $x(0) = x_0$, if there exist a continuous differentiable function $V(x(t)) = V(t)$ and positive constants a_1, a_2 such that

$$\dot{V}(t) \leq -a_1 \frac{\dot{\varepsilon}_m(t)}{\varepsilon_m(t)} V(t) + a_2,$$

holds, then $x(t)$ is practical prescribed-time stable with settling time T_m .

Based on the disconnected time-varying topology, we can find that the systems (1) and (2) cannot achieve the prescribed-time consensus based on Lemma 1. The reason is that

the function $V(t)$ may be increasing within the disconnected intervals. To overcome this shortcoming, we propose the following lemma. Meanwhile, it should be pointed out that the interval $[t_{r_m}, t_{r_m+1})$ is replaced as $[t_{r_m}, t_{r_m}^a)$ because the interval $[t_{r_m}, t_{r_m+1})$ is connected.

Lemma 2. *Considering the nonlinear system $\dot{x}(t) = g(x(t))$, $x(0) = x_0$, if there exist a continuous differentiable function $V(x(t)) = V(t)$ and positive real constants $a_1, \dots, a_6$ such that*

$$\begin{cases} \dot{V}(t) \leq -a_1 \frac{\dot{\varepsilon}_m(t)}{\varepsilon_m(t)} V(t) + a_2, & t \in [t_s, t_s^a), s \in \mathcal{R}_m, \\ \dot{V}(t) \leq a_3 V(t) + a_4, & t \in [t_s^a, t_{s+1}), s \in \mathcal{R}_{m-1}, \\ \dot{V}(t) \leq -\frac{a_5}{T_m} V(t) + a_6, & t \in [T_m, +\infty), \end{cases} \quad (4)$$

hold, then $x(t)$ is practical prescribed-time stable within settling time T_m .

Proof. Because the function $\varepsilon_m(t)$ is piecewise continuous, we introduce the Filippov solution. For the system $\dot{x}(t) = g(x(t))$, suppose that $x(t)$ is a Filippov solution to the differential inclusion $\dot{x}(t) \in \mathcal{K}[g(x(t))]$, where $\mathcal{K}[\cdot]$ is the closure of the convex hull. $\square$

Then $\dot{V}(t) \in \dot{V}(t)$, where $\dot{V}(t) = \cap_{\mathbb{N} \in \partial V(t)} \mathbb{N}^T [\mathcal{K}[g](x(t))]^T, 1]^T$ is the generalized time derivative of $V(t)$ along the Filippov trajectories of $\dot{x}(t) = g(x(t))$, where $\partial V(t)$ is the Clarke generalized gradient of $V(t)$. At this point, we have

$$\begin{cases} \dot{V}(t) \leq -a_1 \frac{\dot{\varepsilon}_m(t)}{\varepsilon_m(t)} V(t) + a_2, & a.e. t \in [t_s, t_s^a), s \in \mathcal{R}_m, \\ \dot{V}(t) \leq a_3 V(t) + a_4, & a.e. t \in [t_s^a, t_{s+1}), s \in \mathcal{R}_{m-1}, \\ \dot{V}(t) \leq -\frac{a_5}{T_m} V(t) + a_6, & a.e. t \in [T_m, +\infty). \end{cases} \quad (5)$$

Assume that the function $f(t) > 0$ is an integrable function in the set of real numbers. When $a.e. t \in [t_s, t_s^a)$, multiplying function $f(t)$ by both sides of the first inequality of (5), we have

$$f(t) \dot{V}(t) \leq -a_1 f(t) \frac{\dot{\varepsilon}_m(t)}{\varepsilon_m(t)} V(t) + a_2 f(t). \quad (6)$$

Moving the term $a_1 f(t) \frac{\dot{\varepsilon}_m(t)}{\varepsilon_m(t)} V(t)$ to the left of the inequality (6), we have $f(t) \dot{V}(t) + a_1 f(t) \frac{\dot{\varepsilon}_m(t)}{\varepsilon_m(t)} V(t) \leq a_2 f(t)$.

Combining $(f(t)V(t))' = \dot{f}(t)V(t) + f(t)\dot{V}(t)$, we can further obtain

$$\dot{f}(t) = a_1 f(t) \frac{\dot{\varepsilon}_m(t)}{\varepsilon_m(t)}, \quad (7)$$

and

$$(f(t)V(t))' \leq a_2 f(t). \quad (8)$$

By integrating both sides of Equation (7) and Inequality (8) over $[t_s, t)$, we can get

$$\ln \left(\frac{f(t)}{f(t_s)} \right) = a_1 \ln \left(\frac{\varepsilon_m(t)}{\varepsilon_m(t_s)} \right), \quad (9)$$

and

$$f(t)V(t) \leq f(t_s)V(t_s) + \int_{t_s}^t a_2 f(s) ds. \quad (10)$$

Therefore, we have

$$f(t) = f(t_s) \left(\frac{\varepsilon_m(t)}{\varepsilon_m(t_s)} \right)^{a_1}. \quad (11)$$

Combining Inequality (10) and Equation (11), we have

$$f(t_s) \left(\frac{\varepsilon_m(t)}{\varepsilon_m(t_s)} \right)^{a_1} V(t) \leq f(t_s) V(t_s) + \int_{t_s}^t a_2 f(t_s) \left(\frac{\varepsilon_m(s)}{\varepsilon_m(t_s)} \right)^{a_1} ds.$$

Due to $f(t) > 0$, $\varepsilon_m(t_s) > 0$, multiply both sides of the above inequality by $\frac{\varepsilon_m(t_s)^{a_1}}{f(t_s)}$ and one arrives at

$$\frac{\varepsilon_m(t_s)^{a_1}}{f(t_s)} f(t_s) \left(\frac{\varepsilon_m(t)}{\varepsilon_m(t_s)} \right)^{a_1} V(t) \leq \frac{\varepsilon_m(t_s)^{a_1}}{f(t_s)} f(t_s) V(t_s) + \frac{\varepsilon_m(t_s)^{a_1}}{f(t_s)} \int_{t_s}^t a_2 f(t_s) \left(\frac{\varepsilon_m(s)}{\varepsilon_m(t_s)} \right)^{a_1} ds.$$

Now, we further derive

$$\varepsilon_m(t_s)^{a_1} V(t) \leq V(t_s) + \int_{t_s}^t a_2 \varepsilon_m(t_s)^{a_1} ds.$$

At this point, integrating both sides of the above inequality, we can derive

$$V(t) \leq \varepsilon_m^{-a_1}(t) V(t_s) + \frac{a_2 T_m}{a_1 - 1} (\varepsilon_m^{-1}(t) - \varepsilon_m^{-a_1}(t)). \quad (12)$$

If $a_1 > 1$, since $\varepsilon_m(t) > 1$, then $\varepsilon_m^{-1}(t) > \varepsilon_m^{-a_1}(t)$. Therefore,

$$V(t) \leq \varepsilon_m^{-1}(t) \left(V(t_s) + \frac{a_2 T_m}{a_1 - 1} \right).$$

If $a_1 < 1$, since $\varepsilon_m(t) > 1$, then $\varepsilon_m^{-1}(t) < \varepsilon_m^{-a_1}(t)$. Therefore,

$$V(t) \leq \varepsilon_m^{-a_1}(t) \left(V(t_s) - \frac{a_2 T_m}{a_1 - 1} \right).$$

If $a_1 = 1$, we have

$$V(t) \leq \varepsilon_m^{-a_1}(t) \left(V(t_s) + a_2 T_m \ln \frac{T_m}{T_m - t_s^a + t_s} \right).$$

Owing to $a_1 > 0$, let $\hat{a} = \max\{-1, -a_1\} < 0$ and we get

$$V(t) \leq (V(t_s) + \varkappa) \varepsilon_m^{\hat{a}}(t), \quad (13)$$

where $\varkappa = \max \left\{ \left| \frac{a_2 T_m}{a_1 - 1} \right|, a_2 T_m \ln \frac{T_m}{T_m - t_s^a + t_s} \right\}$.

In a similar way, when $t \in [t_s^a, t_{s+1}^a)$, $s \in \mathcal{R}_{m-1}$, we can get from the second inequality of Inequality (5) that

$$V(t) \leq V(t_s^a) e^{a_3(t-t_s^a)} + \frac{a_4}{a_3} (e^{a_3(t-t_s^a)} - 1). \quad (14)$$

Let $\gamma(\sigma) = \frac{a_4}{a_3} (e^{a_3\sigma} - 1)$ and $\sigma = t - t_s^a$. Then, we have

$$V(t) \leq V(t_s^a) e^{a_3(t-t_s^a)} + \gamma(t - t_s^a). \quad (15)$$

Combining Inequalities (13) and (15), one arrives at

$$V(t) \leq V(t_s) \varepsilon_m^{\hat{a}}(t_s^a) e^{a_3(t-t_s^a)} + \varkappa \varepsilon_m^{\hat{a}}(t_s^a) e^{a_3(t-t_s^a)} + \gamma(t-t_s^a).$$

Similarly, when $t \in [t_0, T_m)$, by employing a mathematical recursive method, we can obtain

$$\begin{aligned} V(t) &\leq V(t_s) \varepsilon_m^{\hat{a}}(t_s^a) e^{a_3(t-t_s^a)} + \varkappa \varepsilon_m^{\hat{a}}(t_s^a) e^{a_3(t-t_s^a)} + \gamma(t-t_s^a) \\ &\leq V(t_{s-1}^a) \varepsilon_m^{\hat{a}}(t_{s-1}^a) \varepsilon_m^{\hat{a}}(t_s^a) e^{a_3[(t-t_{s-1}^a)+(t_s-t_{s-1}^a)]} \\ &\quad + \varkappa \varepsilon_m^{\hat{a}}(t_{s-1}^a) \varepsilon_m^{\hat{a}}(t_s^a) e^{a_3[(t-t_{s-1}^a)+(t_s-t_{s-1}^a)]} \\ &\quad + \gamma(t_s-t_{s-1}^a) \varepsilon_m^{\hat{a}}(t_s^a) e^{a_3(t-t_s^a)} + \varkappa \varepsilon_m^{\hat{a}}(t_s^a) e^{a_3(t-t_s^a)} + \gamma(t-t_s^a) \\ &\leq \dots \\ &\leq V(0) \varepsilon_m^{\hat{a}}(t_s^a) \dots \varepsilon_m^{\hat{a}}(t_0^a) e^{a_3[(t-t_s^a)+\dots+(t_1-t_0^a)]} \\ &\quad + \varkappa \varepsilon_m^{\hat{a}}(t_{s-1}^a) \dots \varepsilon_m^{\hat{a}}(t_0^a) e^{a_3[(t-t_s^a)+\dots+(t_1-t_0^a)]} + \dots + \varkappa \varepsilon_m^{\hat{a}}(t_s^a) e^{a_3(t-t_s^a)} \\ &\quad + \gamma(t_s-t_{s-1}^a) \varepsilon_m^{\hat{a}}(t_s^a) \dots \varepsilon_m^{\hat{a}}(t_0^a) e^{a_3(t-t_s^a)} + \dots + \gamma(t-t_s^a). \end{aligned}$$

According to the definition of the disconnected intervals and $\hat{a} < 0$, we have

$$\left(\frac{T_m}{T_m - t_j^a + t_s} \right)^{\hat{a}} \leq 1, \text{ i.e., } \left(\varepsilon_m(t_j^a) \right)^{\hat{a}} \leq 1, j = 1, 2, \dots, r_m.$$

Further, one can obtain

$$\begin{aligned} V(t) &\leq V(0) e^{a_3[(t-t_s^a)+\dots+(t_1-t_0^a)]} + \varkappa e^{a_3[(t-t_s^a)+\dots+(t_1-t_0^a)]} + \dots + \varkappa e^{a_3(t-t_s^a)} \\ &\quad + \gamma(t_s-t_{s-1}^a) e^{a_3(t-t_s^a)} + \dots + \gamma(t-t_s^a). \end{aligned}$$

Furthermore, according to Assumption 2, we have $|S^a(0, t)| < \frac{T_m}{\tau}$, so

$$V(t) \leq V(0) e^{\frac{a_3 T_m}{\tau}} + (r_m + 1) \varkappa e^{\frac{a_3 T_m}{\tau}} + (r_m + 1) \gamma \left(\frac{T_m}{\tau} \right) e^{\frac{a_3 T_m}{\tau}}.$$

Let $\Theta_1 = V(0) e^{\frac{a_3 T_m}{\tau}} + (r_m + 1) \varkappa e^{\frac{a_3 T_m}{\tau}} + (r_m + 1) \gamma \left(\frac{T_m}{\tau} \right) e^{\frac{a_3 T_m}{\tau}}$ and we have $V(t) \leq \Theta_1$. When $t \in [T_m, +\infty)$, it is easy to obtain from Inequality (5) that

$$V(t) \leq V(T_m) e^{-\frac{a_5}{T_m}(t-T_m)} + \frac{a_6 T_m}{a_5} \left(1 - e^{-\frac{a_5}{T_m}(t-T_m)} \right). \quad (16)$$

Therefore, we derive that $V(t) \leq \Theta_2$, where $\Theta_2 = \frac{a_6 T_m}{a_5} + V(T_m)$. Further, we can get $\lim_{t \rightarrow T_m^-} \|V(t)\| \leq \Theta_3$, $\|V(t)\| \leq \Theta_3$, $t \geq T_m$, where $\Theta_3 = \max\{\Theta_1, \Theta_2\}$ is a positive constant. Because the function $V(t)$ is related to $x(t)$, $\|x(t)\|$ is bounded. Based on Definition 2, we have that $x(t)$ is the practical prescribed-time stable. The proof is completed.

Remark 4. Time-varying scaling function (3) in previous works [26–28] was generally designed only related to the prescribed-time T_m , such as $\frac{T_m}{T_m-t}$, $\frac{T_m}{T_m-\gamma t}$, $(\frac{T_m}{T_m-t})^\alpha$. These functions are only applicable in the case of continuous communication topology. For the disconnected case, the authors in [24] presented a new time-varying function $(\frac{T_m}{T_m-t+lT})^\alpha$ with regard to the intermittent control period lT . In [31], the authors further relaxed the period restriction to aperiodic case; i.e., $(\frac{T_m}{T_m-t+t_0})^\alpha$, $T_m \in [t_s, t_s^a)$ and $(\frac{T_m}{T_m-t+t_0-c_{\max}})^\alpha$, $T_m \in [t_s^a, t_{s+1})$, $c_{\max} \geq \sup\{t_{s+1} - t_s^a\}$. Evidently, although the function in [31] was related to the aperiodic constant, the function is piecewise and $c_{\max}$ is conservative. This may increase the complexity of controller design and decrease the flexibility of application. To further overcome this drawback, we proposed the time-

varying function (3), where the aperiodic constant t_s is introduced to change according to the disconnected interval. Meanwhile, this united function can cope with different intervals. Therefore, it is more advanced and general. By employing this function, we proposed Lemma 2, where the derivative inequality of $V(t)$ is different under distinct intervals.

Remark 5. For isolated agents, due to the disconnection of the communication topology, no control input is available, so the system errors tend to grow naturally. Consequently, $\dot{V}(t)$ may increase. To address this, we design $\dot{V}(t) \leq a_3 V(t) + a_4$ in the disconnected intervals. This design relaxes the traditional requirement that $V(t)$ must be monotonically decreasing. Even though $V(t)$ may increase during the disconnected intervals, as long as a_3 and a_4 are bounded and controllable and the total duration of disconnections is constrained within the prescribed time T_m , once the communication topology is reconnected, the increment of $V(t)$ can be sufficiently compensated by the negative terms of $V(t)$ during the connected intervals. In the proof of Lemma 2, a recursive method is employed to concatenate the connected and disconnected intervals over the entire time horizon, thereby ensuring system stability.

3.3. Control Objective

To achieve the second-order practical prescribed-time consensus, the control target is divided into three parts as follows:

- (1) Design a practical prescribed-time velocity observer to estimate the leader velocity within a prescribed time T_0 .
- (2) Design a practical prescribed-time controller to ensure that the actual velocity converges to the virtual velocity within a prescribed time T_1 .
- (3) Design the virtual velocity to drive the agents to achieve consensus within prescribed time T_2 .

The control process in this paper is shown in Figure 2.

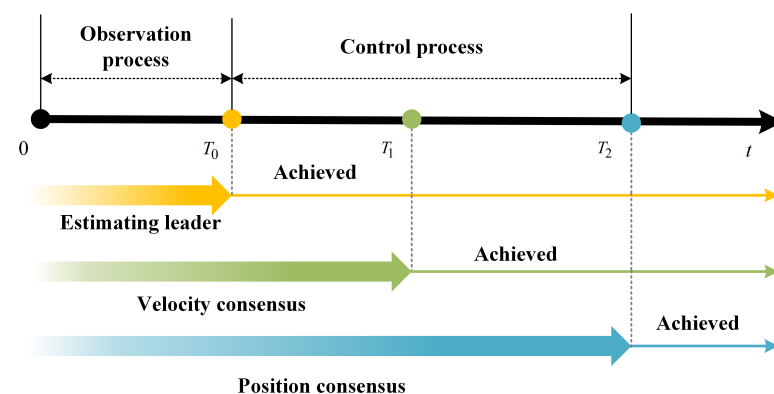


Figure 2. Schematic diagram of control process.

4. The Design of Control Protocol

In this section, according to the backstepping control design philosophy, the velocity observer, virtual velocity and controller with a DETM are designed to achieve the position and velocity practical prescribed-time consensus. The design of controllers typically utilizes information from neighbors. In the disconnected intervals, the isolated followers may be an isolated subnetwork or completely isolated followers. In the case of complete isolation, the controller may fail, while, in the case of subnetworks, the controller may still play a role in controlling the state. This means that designing controllers for isolated followers in completely isolated situations on disconnected intervals is more general. Therefore, the controller design in this subsection focuses on the completely isolated case to obtain more general consensus conditions.

First, we are ready to design a distributed observer for leader. It has the following structure:

$$\dot{\hat{\vartheta}}_i(t) = \begin{cases} -k_0(t)\Phi_i(t) - c_1\Phi_i(t), i \in \mathcal{I} \text{ or } t \in [t_s, t_s^a), s \in \mathcal{R}_0, i \in \hat{\mathcal{I}}, \\ 0, t \in [t_s^a, t_{s+1}), s \in \mathcal{R}_{0-1}, i \in \hat{\mathcal{I}}, \end{cases} \quad (17)$$

where $k_0(t) = \frac{n_0}{T_0}\varepsilon_0(t)$, $n_0 > 0$, $\Phi_i(t) = \sum_{j \in N_i} a_{ij}(\hat{\vartheta}_i(t) - \hat{\vartheta}_j(t) + b_i(\hat{\vartheta}_i(t) - v_0(t)))$ and $c_1 > 0$ is a constant.

Design the virtual velocity in the following:

$$v_i^*(t) = \begin{cases} \hat{\vartheta}_i(t) - k_2(t)\Psi_i(t) - c_2\Psi_i(t), i \in \mathcal{I} \text{ or } t \in [t_s, t_s^a), s \in \mathcal{R}_2, i \in \hat{\mathcal{I}}, \\ \hat{\vartheta}_i(t), t \in [t_s^a, t_{s+1}), s \in \mathcal{R}_{2-1}, i \in \hat{\mathcal{I}}, \end{cases} \quad (18)$$

where $k_2(t) = \frac{n_2}{T_2}\varepsilon_2(t)$, $n_2 > 0$, $\Psi_i(t) = \sum_{j \in N_i} a_{ij}(p_i(t) - p_j(t)) + b_i(p_i(t) - p_0(t))$ and $c_2 > 0$ is a constant.

Define $\dot{\Psi}_i(t) = Z_i(t) = \sum_{j \in N_i} a_{ij}(v_i(t) - v_j(t)) + b_i(v_i(t) - v_0(t))$, $e_i(t) = v_i^*(t) - v_i(t)$, $\tilde{p}_i(t) = p_i(t) - p_0(t)$; the practical prescribed-time controller with a DETM is constructed as

$$u_i(t) = \begin{cases} \hat{\vartheta}_i(t_k^i) - k_2(t_k^i)\Psi_i(t_k^i) - k_2(t_k^i)Z_i(t_k^i) - c_2Z_i(t_k^i) - k_1(t_k^i)e_i(t_k^i) \\ -c_3e_i(t_k^i), i \in \mathcal{I} \text{ or } t \in [t_s, t_s^a), s \in \mathcal{R}_1, i \in \hat{\mathcal{I}}, \\ -k_1(t_k^i)e_i(t_k^i) - c_3e_i(t_k^i), t \in [t_s^a, t_{s+1}), s \in \mathcal{R}_{1-1}, i \in \hat{\mathcal{I}}, \end{cases} \quad (19)$$

where the monotonically increasing time sequence t_k^i , $k = 0, 1, \dots$ represents the triggering moment of i -th agent, $t \in [t_k^i, t_{k+1}^i)$, $k_1(t_k^i) = \frac{n_1}{T_1}\varepsilon_1(t_k^i)$, $n_1 > 0$ and $c_3 > 0$ is a constant.

Let $t_0^i = 0$, and the measurement error is as follows:

$$E_i(t) = \begin{cases} \hat{\vartheta}_i(t) - k_2(t)\Psi_i(t) - k_2(t)Z_i(t) - c_2Z_i(t) - k_1(t)e_i(t) \\ -c_3e_i(t) - \hat{\vartheta}_i(t_k^i) + k_2(t_k^i)\Psi_i(t_k^i) + k_2(t_k^i)Z_i(t_k^i) + c_2Z_i(t_k^i) \\ +k_1(t_k^i)e_i(t_k^i) + c_3e_i(t_k^i), i \in \mathcal{I} \text{ or } t \in [t_s, t_s^a), s \in \mathcal{R}_1, i \in \hat{\mathcal{I}}, \\ -k_1(t)e_i(t) - c_3e_i(t) + k_1(t_k^i)e_i(t_k^i) \\ +c_3e_i(t_k^i), t \in [t_s^a, t_{s+1}), s \in \mathcal{R}_{1-1}, i \in \hat{\mathcal{I}}. \end{cases}$$

The following triggering condition is designed to determine the next triggered instant

$$t_{k+1}^i = \inf \left\{ t > t_k^i \mid \|E_i(t)\|^2 \geq d_1\|e_i(t)\|^2 + d_2 + d_3\tilde{\xi}_i(t) \right\}, \quad (20)$$

where $\tilde{\xi}_i(t)$ is the internal variable, $\tilde{\xi}_i(t_0^i) > 0$, and the dynamic equation is given as follows:

$$\dot{\tilde{\xi}}_i(t) = -d_4\tilde{\xi}_i(t) - d_5k_1(t)\tilde{\xi}_i(t) + d_6(d_1\|e_i(t)\|^2 + d_2 - \|E_i(t)\|^2), \quad (21)$$

where $d_1, \dots, d_6$ are positive constants.

Lemma 3. For the DETM (20) and (21), the inequality $\tilde{\xi}_i(t) \geq \tilde{\xi}_i(t_0^i)e^{-\alpha(t-t_0^i)} \geq 0$ holds, where $\alpha = d_4 + d_5 + d_6d_3 > 0$.

Proof. According to (20) and (21), we have $\dot{\tilde{\xi}}_i(t) \geq -d_4\tilde{\xi}_i(t) - d_5k_1(t)\tilde{\xi}_i(t) - d_6d_3\tilde{\xi}_i(t)$. Meanwhile, combining with the definition of $k_1(t)$ and $\frac{T_1}{T_1-t+t_s} \geq 1$, we can derive that $\dot{\tilde{\xi}}_i(t) \geq -(d_4 + d_5 + d_6d_3)\tilde{\xi}_i(t) = -\alpha\tilde{\xi}_i(t)$, where $\alpha = d_4 + d_5 + d_6d_3 > 0$. By integrating both sides of the above equation from t_k^i to t , we get $\tilde{\xi}_i(t) \geq \tilde{\xi}_i(t_k^i)e^{-\alpha(t-t_k^i)}$. Therefore, by employing a mathematical recursive method, we have $\tilde{\xi}_i(t) \geq \tilde{\xi}_i(t_k^i)e^{-\alpha(t-t_k^i)} \geq \tilde{\xi}_i(t_{k-1}^i)e^{-\alpha(t-t_{k-1}^i)} \geq \dots \geq \tilde{\xi}_i(t_0^i)e^{-\alpha(t-t_0^i)} \geq 0$. The proof is completed. $\square$

Remark 6. For leader-following consensus goal, the velocity consensus goal is divided into $\|v_i(t) - v_i^*(t)\| \leq \rho_1$ and $\|v_i^*(t) - v_0(t)\| \leq \rho_2$, where ρ_1 and ρ_2 are positive constants. This means that the design of virtual velocity needs to consider the dynamics of leader. However, following the methods in [30], the virtual velocity will be designed as $\tilde{v}_i^*(t) = v_0(t) - k_2(t)\Psi_i(t) - c_2\Psi_i(t)$. Obviously, each virtual velocity needs the information of leader. Although the communication topology between followers' virtual velocity is the same as actual velocity, it is impossible that all followers can obtain the state of leader. Therefore, we introduce the velocity observer, where the state of leader is distributively estimated by utilizing the information of neighbors.

5. Main Results

In this section, the theoretical results of the practical prescribed-time consensus of second-order MASs will be formulated.

Theorem 1. Under Assumptions 1 and 2, if there exist positive constants $\beta_1, \beta_2, \beta_3$, and $d_1, \dots, d_6$ such that the following inequalities

$$(1) \ c_1(H \otimes I_n) - \frac{\beta_1}{2}I_{Nn} \geq 0, \quad (2) \ 1 + \frac{\beta_2}{2} - c_3 + \frac{d_1}{2\beta_2} \leq 0,$$

$$(3) \ d_4 - \left(\frac{1}{2\beta_2} - d_6\right)d_3 > 0, \quad (4) \ c_2\lambda_1 - \frac{\beta_3}{2} \leq 0,$$

hold, then the systems (1) and (2) can achieve the practical prescribed-time consensus within T_2 .

Proof. The proof consists of five steps.

Step 1: The velocity observer and leader velocity can achieve the practical prescribed-time consensus within T_0 .

Define the observation error $\tilde{v}_i(t) = \hat{v}_i(t) - v_0(t)$, and then

$$\dot{\tilde{v}}_i(t) = \begin{cases} -k_0(t)\Phi_i(t) - c_1\Phi_i(t) - u_0(t), & i \in \mathcal{I} \text{ or } t \in [t_s, t_s^a), s \in \mathcal{R}_0, i \in \hat{\mathcal{I}}, \\ -u_0(t), & t \in [t_s^a, t_{s+1}), s \in \mathcal{R}_{0-1}, i \in \hat{\mathcal{I}}. \end{cases} \quad (22)$$

Let the right-hand side of Equation (22) be equal to $\Pi(\tilde{v}_i(t))$. Because the function $\varepsilon_0(t)$ is piecewise continuous, similar to Lemma 2, we suppose that $\tilde{v}_i(t)$ is a Filippov solution to the differential inclusion $\dot{\tilde{v}}_i(t) = \mathcal{K}[\Pi](\tilde{v}_i(t))$. Consider the Lyapunov function $V_1(t) = \frac{1}{2} \sum_{i=1}^N \tilde{v}_i^T(t) \tilde{v}_i(t)$; we further derive $\dot{V}_1(t) \in \dot{\mathcal{V}}_1(t)$, where $\dot{\mathcal{V}}_1(t) = \cap_{\hat{v} \in \partial V_1(t)} \hat{v}^T [\mathcal{K}[\Pi](\tilde{v}_i(t))^T, 1]^T$. Further, in the case of Type I or Type II followers, for a.e. $t \in [t_s, t_{s+1})$, $s \in \mathcal{R}_0$, calculating the time derivative of $V_1(t)$ along solutions of system (22), according to condition (1), we have

$$\begin{aligned} \dot{V}_1(t) &= - \sum_{i=1}^N \tilde{v}_i^T(t) k_0(t) \Phi_i(t) - \sum_{i=1}^N \tilde{v}_i^T(t) c_1 \Phi_i(t) - \sum_{i=1}^N \tilde{v}_i^T(t) u_0(t) \\ &\leq - \sum_{i=1}^N \tilde{v}_i^T(t) k_0(t) \Phi_i(t) - \sum_{i=1}^N \tilde{v}_i^T(t) c_1 \Phi_i(t) + \frac{\beta_1}{2} \sum_{i=1}^N \tilde{v}_i^T(t) \tilde{v}_i(t) + \frac{|N|}{2\beta_1} \sum_{i=1}^N u_0^T(t) u_0(t) \\ &\leq -k_0(t) \lambda_{\min}(H \otimes I_n) \tilde{v}^T(t) \tilde{v}(t) - \tilde{v}^T(t) \left(c_1(H \otimes I_n) - \frac{\beta_1}{2} I_{Nn} \right) \tilde{v}(t) + \frac{|N|}{2\beta_1} \mathcal{U}^2 \\ &\leq -2k_0(t) \lambda_1 V_1(t) + \frac{|N|}{2\beta_1} \mathcal{U}^2 \delta_1, \end{aligned} \quad (23)$$

where $\tilde{v}(t) = [\tilde{v}_1^T(t), \tilde{v}_2^T(t), \dots, \tilde{v}_N^T(t)]^T$, $|N|$ is the total number of neighbors of i -th agent, $\lambda_1 = \lambda_{\min}(H \otimes I_n)$, $\delta_1 = \frac{|N|}{2\beta_1} \mathcal{U}^2$.

Next, we consider that agent i is isolated. For a.e. $t \in [t_s, t_s^a)$, we can obtain the same result: $\dot{V}_1(t) \leq -2k_0(t)\lambda_1 V_1(t) + \delta_1$. When $t \in [t_s^a, t_{s+1})$, taking the derivative of $V_1(t)$, we have

$$\begin{aligned}\dot{V}_1(t) &= \sum_{i=1}^N \tilde{v}_i^T(t) \dot{\tilde{v}}_i(t) = - \sum_{i=1}^N \tilde{v}_i^T(t) u_0 \\ &\leq \frac{\beta_1}{2} \sum_{i=1}^N \tilde{v}_i^T(t) \tilde{v}_i(t) + \frac{|N|}{2\beta_1} \mathcal{U}^2 \leq \beta_1 V_1(t) + \delta_1.\end{aligned}\quad (24)$$

For a.e. $t \in [T_0, +\infty)$, according to the Equation (23) and $\varepsilon_0(t) = 1$, we get $\dot{V}_1(t) \leq -2\frac{\eta_0}{T_0}\lambda_1 V_1(t) + \delta_1$. Based on Lemma 2, we can obtain $\|\tilde{v}(t)\| \leq \sqrt{2\hat{\Theta}_3}$, where $\hat{\Theta}_1 = V(0)e^{\frac{\beta_1}{\tau}T_0} + (r_0 + 1)\mathcal{K}e^{\frac{\beta_1}{\tau}T_0} + (r_0 + 1)\gamma(T_0)e^{\frac{\beta_1}{\tau}T_0}$. Further, according to Definition 1, the velocity observer (17) and leader velocity can achieve the practical prescribed-time consensus within T_0 .

Step 2: The actual velocity and virtual velocity can achieve the practical prescribed-time consensus within T_1 .

The error equation can be formulated as

$$e_i(t) = \begin{cases} \hat{v}_i(t) - v_i(t) - k_2(t)\Psi_i(t) - c_2\Psi_i(t), & i \in \mathcal{I} \text{ or } t \in [t_s, t_s^a), s \in \mathcal{R}_2, i \in \hat{\mathcal{I}}, \\ \hat{v}_i(t) - v_i(t), & t \in [t_s^a, t_{s+1}), s \in \mathcal{R}_{2-1}, i \in \hat{\mathcal{I}}. \end{cases}\quad (25)$$

By employing the same method as **Step 1**, let the right-hand side of Equation (25) be equal to $\Pi_1(e_i(t))$. Assume that $e_i(t)$ is a Filippov solution to the differential inclusion $\dot{e}_i(t) = \mathcal{K}[\Pi_1](e_i(t))$. Consider the Lyapunov function $V_2(t) = \frac{1}{2} \sum_{i=1}^N e_i^T(t) e_i(t) + \sum_{i=1}^N \xi_i(t)$ and we have $\dot{V}_2(t) \in \dot{\mathcal{V}}_2(t)$, where $\dot{\mathcal{V}}_2(t) = \cap_{\check{\mathcal{K}} \in \partial V_2(t)} \check{\mathcal{K}}^T[\mathcal{K}[\Pi_1](e_i(t))^T, 1]^T$. In the case of Type I or Type II followers, for a.e. $t \in [t_s, t_{s+1}) \cap [t_k^i, t_{k+1}^i)$, calculating the time derivative of $V_2(t)$ and substituting the measurement error and Equation (1), we have

$$\begin{aligned}\dot{V}_2(t) &= \sum_{i=1}^N e_i^T(t) \dot{q}_i(t) - \sum_{i=1}^N e_i^T(t) E_i(t) - \sum_{i=1}^N e_i^T(t) k_1(t) e_i(t) - \sum_{i=1}^N e_i^T(t) c_3 e_i(t) \\ &\quad - \sum_{i=1}^N d_4 \xi_i(t) - \sum_{i=1}^N d_5 k_1(t) \xi_i(t) + \sum_{i=1}^N d_6 (d_1 \|e_i(t)\|^2 + d_2 - \|E_i(t)\|^2).\end{aligned}$$

Applying the triggering mechanism (20) and conditions (2) and (3), we can get

$$\begin{aligned}\dot{V}_2(t) &\leq \sum_{i=1}^N e_i^T(t) e_i(t) + \sum_{i=1}^N Q^2 + \sum_{i=1}^N \frac{\beta_2}{2} e_i^T(t) e_i(t) + \sum_{i=1}^N \frac{1}{2\beta_2} d_1 \|e_i(t)\|^2 \\ &\quad + \sum_{i=1}^N \frac{1}{2\beta_2} d_2 - \sum_{i=1}^N e_i^T(t) k_1(t) e_i(t) - \sum_{i=1}^N e_i^T(t) c_3 e_i(t) - \sum_{i=1}^N d_4 \xi_i(t) \\ &\quad - \sum_{i=1}^N d_5 k_1(t) \xi_i(t) + \sum_{i=1}^N \left(\frac{1}{2\beta_2} - d_6 \right) d_3 \xi_i(t) \\ &\leq \sum_{i=1}^N e_i^T(t) \left(1 + \frac{\beta_2}{2} - c_3 + \frac{d_1}{2\beta_2} \right) e_i(t) + |N|Q^2 - \sum_{i=1}^N e_i^T(t) k_1(t) e_i(t) \\ &\quad - \sum_{i=1}^N \left(d_4 - \left(\frac{1}{2\beta_2} - d_6 \right) d_3 \right) \xi_i(t) + \sum_{i=1}^N \frac{d_2}{2\beta_2} - \sum_{i=1}^N d_5 k_1(t) \xi_i(t) \\ &\leq -2k_1(t) \frac{1}{2} e^T(t) e(t) - d_5 k_1(t) \sum_{i=1}^N \xi_i(t) + \frac{|N|d_2}{2\beta_2} + |N|Q^2 \\ &\leq -\delta_2 k_1(t) V_2(t) + \delta_3,\end{aligned}\quad (26)$$

where $\delta_2 = \min\{2, d_5\}$, $\delta_3 = \frac{|N|d_2}{2\beta_2} + |N|Q^2$.

When agent i is isolated, for a.e. $t \in [t_s, t_{s+1}) \cap [t_k^i, t_{k+1}^i)$, it is easy to derive that $\dot{V}_2(t) \leq -\delta_2 k_1(t) V_2(t) + \delta_3$. The reason is that the communication between the actual and virtual velocities does not include information from other neighbors. By adopting Lemmas 1 and 2, we can obtain $\|e_i(t)\| \leq 2\sqrt{(V(0) + \hat{\kappa})}$, where $\hat{\kappa} = \max\left\{\left|\frac{a_2 T_1}{a_1 - 1}\right|, a_2 T_1 \ln \frac{T_1}{T_1 - t_1^a}\right\}$. Then, according to Definition 1, the actual and virtual velocities can achieve the practical prescribed-time consensus within T_1 .

Step 3: The followers' and leader's position can achieve the practical prescribed-time consensus within T_2 .

According to the position error $\tilde{p}_i(t)$, the derivative of $\tilde{p}_i(t)$ is calculated as

$$\dot{\tilde{p}}_i(t) = \begin{cases} \tilde{v}_i(t) - k_2(t)\Psi_i(t) - c_2\Psi_i(t), & i \in \mathcal{I} \text{ or } t \in [t_s, t_s^a), s \in \mathcal{R}_2, i \in \hat{\mathcal{I}}, \\ \tilde{v}_i(t), & t \in [t_s^a, t_{s+1}), s \in \mathcal{R}_{2-1}, i \in \hat{\mathcal{I}}. \end{cases} \quad (27)$$

Similarly, let the right-hand side of Equation (27) be equal to $\Pi_2(\tilde{p}_i(t))$. Assume that $\tilde{p}_i(t)$ is a Filippov solution to the differential inclusion $\dot{\tilde{p}}_i(t) = \mathcal{K}[\Pi_2](\tilde{p}_i(t))$. Consider the Lyapunov function $V_3(t) = \frac{1}{2} \sum_{i=1}^N \tilde{p}_i^T(t) \tilde{p}_i(t)$ and one arrives at $\dot{V}_3(t) \in \dot{V}_3(t)$, where $\dot{V}_3(t) = \cap_{\tilde{p} \in \partial V_3(t)} \tilde{p}^T [\mathcal{K}[\Pi_2](\tilde{p}_i(t))^T, 1]^T$. When agent i is a Type I or Type II follower, for a.e. $t \in [t_s, t_{s+1})$, calculating the time derivative of $V_3(t)$, according to Equation (18) and condition (4), we arrive at

$$\begin{aligned} \dot{V}_3(t) &= \sum_{i=1}^N \tilde{p}_i^T(t) \tilde{v}_i(t) - \sum_{i=1}^N \tilde{p}_i^T(t) k_2(t) \Psi_i(t) - \sum_{i=1}^N \tilde{p}_i^T(t) c_2 \Psi_i(t) \\ &\leq -k_2(t) \tilde{p}^T(t) (H \otimes I_n) \tilde{p}(t) - \tilde{p}^T(t) c_2 (H \otimes I_n) \tilde{p}(t) \\ &\quad + \frac{\beta_3}{2} \sum_{i=1}^N \tilde{p}_i^T(t) \tilde{p}_i(t) + \frac{1}{2\beta_3} \sum_{i=1}^N \tilde{v}_i^T(t) \tilde{v}_i(t) \\ &\leq -2k_2(t) \lambda_1 V_3(t) + \frac{1}{2\beta_3} \sum_{i=1}^N \|\tilde{v}_i(t)\|^2, \end{aligned} \quad (28)$$

where $\tilde{p}(t) = [\tilde{p}_1^T(t), \tilde{p}_2^T(t), \dots, \tilde{p}_N^T(t)]^T$, $c_2 \lambda_1 - \frac{\beta_3}{2} \leq 0$. According to **Step 1**, it is obvious that $\tilde{v}_i(t)$ is bounded; i.e., $\|\tilde{v}_i(t)\| \leq \delta_4$. Hence, we have

$$\dot{V}_3(t) \leq -2k_2(t) \lambda_1 V_3(t) + \delta_5, \quad (29)$$

where $\delta_5 = \frac{1}{2\beta_3} |N| \delta_4^2$.

When agent i is isolated, the same result $\dot{V}_3(t) \leq -2k_2(t) \lambda_1 V_3(t) + \delta_5$ is derived, $t \in [t_s, t_s^a)$. For a.e. $t \in [t_s^a, t_{s+1})$, we compute the derivative of $V_3(t)$. It holds that

$$\dot{V}_3(t) \leq \sum_{i=1}^N \tilde{p}_i^T(t) \tilde{v}_i(t) \leq \frac{\beta_3}{2} \sum_{i=1}^N \tilde{p}_i^T(t) \tilde{p}_i(t) + \frac{1}{2\beta_3} \sum_{i=1}^N \tilde{v}_i^T(t) \tilde{v}_i(t) \leq \beta_3 V_3(t) + \delta_5. \quad (30)$$

Combining Inequalities (29) and (30) and Lemma 2, we have $\|\tilde{p}(t)\| \leq \sqrt{2\hat{\Theta}_2}$, where $\hat{\Theta}_2 = V(0)e^{\beta_3 T_2} + (r_2 + 1)\kappa e^{\beta_3 T_2} + (r_2 + 1)\gamma(T_2)e^{\beta_3 T_2}$. Further, based on Definition 1, the position of followers and leader can achieve the practical prescribed-time consensus within T_2 .

Step 4: $\|v_i^*(t) - v_0(t)\|$ is bounded within T_2 .

Based on the equation of $v_i^*(t)$, one obtains $\|v_i^*(t) - v_0(t)\| \leq \|\tilde{v}_i(t)\| + \|(k_2(t) + c_2)(H \otimes I_n) \tilde{p}(t)\| \leq \delta_4 + \frac{\tau}{\tau-1} \|H_i \otimes I_n\| \delta_6 = \delta_7$, where $\|\tilde{p}(t)\| \leq \delta_6$, H_i is the i -th row element of H . Therefore, $\|v_i^*(t) - v_0(t)\|$ is bounded within T_2 .

Step 5: Excluding the Zeno behavior.

Calculating the Dini derivative of $E_i(t)$, we can derive $D^+ \|E_i(t)\|_1 \leq \|\dot{E}_i(t)\|_1 = \|\ddot{\tilde{v}}_i(t) - \ddot{k}_2(t)\Psi_i(t) - 2\dot{k}_2(t)Z_i(t) - k_2(t)\dot{Z}_i(t) - c_2\dot{Z}_i(t) - \dot{k}_1(t)e_i(t) - k_1(t)\dot{e}_i(t) - c_3\dot{e}_i(t)\|_1$. Since $\tilde{v}(t) = \hat{v}(t) - v_0(t)$, we have

$$\ddot{\tilde{v}}_i(t) = \begin{cases} -k_0(t)(H_i \otimes I_n)\dot{\tilde{v}}(t) - \dot{k}_0(t)(H_i \otimes I_n)\tilde{v}(t) \\ -c_1(H_i \otimes I_n)\tilde{v}(t), i \in \mathcal{I} \text{ or } t \in [t_s, t_s^a], s \in \mathcal{R}_0, i \in \hat{\mathcal{I}}, \\ 0, t \in [t_s^a, t_{s+1}), s \in \mathcal{R}_{0-1}, i \in \hat{\mathcal{I}}. \end{cases}$$

According to Equation (17), we get

$$\dot{\tilde{v}}(t) = \begin{cases} -k_0(t)(H_i \otimes I_n)\tilde{v}(t) - c_1(H_i \otimes I_n)\tilde{v}(t) - u_0(t), i \in \mathcal{I} \text{ or } t \in [t_s, t_s^a], s \in \mathcal{R}_0, i \in \hat{\mathcal{I}}, \\ -u_0(t), t \in [t_s^a, t_{s+1}), s \in \mathcal{R}_{0-1}, i \in \hat{\mathcal{I}}. \end{cases}$$

Therefore, for $t \in [t_s, t_s^a], s \in \mathcal{R}_0, i \in \hat{\mathcal{I}}$, based on **Step 1**, one has

$$\begin{aligned} \|\ddot{\tilde{v}}_i(t)\| &= \|(k_0(t) + c_1)(H_i \otimes I_n)\dot{\tilde{v}}(t) + \dot{k}_0(t)(H_i \otimes I_n)\tilde{v}(t)\| \\ &= \|(k_0(t) + c_1)(H_i \otimes I_n)((k_0(t) + c_1)(H_i \otimes I_n)\tilde{v}(t) + u_0(t)) + \dot{k}_0(t)(H_i \otimes I_n)\tilde{v}(t)\| \\ &= \|(k_0(t) + c_1)^2(H_i \otimes I_n)^2\tilde{v}(t) + (k_0(t) + c_1)(H_i \otimes I_n)u_0(t) + \dot{k}_0(t)(H_i \otimes I_n)\tilde{v}(t)\| \\ &\leq \left\| \left(\left(\frac{T_0}{t_s} + c_1 \right)^2 (H_i \otimes I_n)^2 + \frac{T_0}{t_s^2} \right) \right\| \|\tilde{v}(t)\| + \left\| \frac{T_0}{t_s} + c_1 \right\| (H_i \otimes I_n) \|u_0(t)\| \\ &\leq \left\| \left(\left(\frac{T_0}{t_s} + c_1 \right)^2 (H_i \otimes I_n)^2 + \frac{T_0}{t_s^2} \right) \right\| 2\sqrt{\hat{\Theta}_1} + \left\| \frac{T_0}{t_s} + c_1 \right\| (H_i \otimes I_n) \|\mathcal{U}\|, \end{aligned}$$

where $\hat{\Theta}_1 = V(0)e^{\frac{\beta_1}{\tau}T_0} + (r_0 + 1)\mathcal{K}e^{\frac{\beta_1}{\tau}T_0} + (r_0 + 1)\gamma(T_0)e^{\frac{\beta_1}{\tau}T_0}$.

Furthermore, the second-order derivative of $\varepsilon_m(t)$ is

$$\ddot{\varepsilon}_m(t) = \begin{cases} \frac{2}{T_m^2}\varepsilon_m^3(t), t \in [t_s, t_{s+1}), s \in \mathcal{R}_m, \\ 0, t \in [T_m, +\infty). \end{cases}$$

Therefore, the second-order derivative of $k_2(t)$ can be calculated as $\ddot{k}_2(t) = \frac{2n_2}{T_2^3}\varepsilon_2^3(t)$. When $t \rightarrow T_2^-$, $\|\ddot{k}_2(t)\| \leq \frac{2n_2}{T_2^3}$. This means that $\ddot{k}_2(t)$ is bounded.

Based on the proof of **Steps 1–4**, it is easy to obtain that $D^+ \|E_i(t)\|_1$ is bounded; i.e., $D^+ \|E_i(t)\|_1 \leq \delta_8$. By integrating both sides of the above equation, it holds $E_i(t) - E_i(t_k^i) \leq \delta_8(t - t_k^i)$. When $t = t_{k+1}^i$, we have $\sqrt{d_2} \leq E_i(t_{k+1}^i) - E_i(t_k^i) \leq \delta_8(t_{k+1}^i - t_k^i)$. Further, it goes $t_{k+1}^i - t_k^i \geq \frac{\sqrt{d_2}}{\delta_8} > 0$. The Zeno behavior is excluded. The proof is completed. $\square$

Remark 7. The convergence domains δ_1, δ_3 , and δ_5 appearing in Equations (24), (26) and (30) are adjustable through the design parameters. These bounds are determined by the total number of neighbors of i -th agent, the parameters of the scaling functions (e.g., β_1, β_2 , and β_3), $\mathcal{U}$, Q and so on. Specifically, the value of δ_1 can be reduced by increasing β_1 or decreasing $|N|$ and $\mathcal{U}$, the value of δ_3 can be reduced by increasing β_2 or decreasing $|N|$, d_2 and Q , the value of δ_5 can be reduced by increasing β_3 or decreasing $|N|$ and δ_4 , and vice versa. The appropriate parameters can be chosen based on the trade-off between accuracy and energy consumption.

Remark 8. When designing the parameters, the main considerations are their effects on closed-loop system performance and their practical selection criteria. Specifically, first, the prescribed times T_0, T_1 , and T_2 are user-specified based on mission requirements but are constrained by actuator capabilities. Smaller values demand faster convergence speeds and consequently require larger control magnitudes; therefore, they should be set according to the mission deadline while respecting actuator saturation limits. Second, the controller gains n_0, n_1, n_2, c_1, c_2 , and c_3 should be tuned to achieve prescribed-time stability. However, overly large gains may cause control input saturation,

aggravated actuator wear, and other undesirable effects. Third, the convergence bounds δ_1 , δ_3 , and δ_5 reflect the steady-state accuracy achievable after the prescribed time. The smaller the bounds, the higher the accuracy. Hence, these bounds need to be verified against engineering precision requirements. In conclusion, the appropriate parameters should be chosen by balancing the trade-off between accuracy and control effort.

6. Simulation Results

In this section, a simulation example associated with the position formation tracking of UAVs is provided to illustrate the effectiveness of the control protocol and the correctness of the theoretical analysis.

In this part, the height of UAVs is ignored for simplicity. The simplified model of UAV is shown in Figure 3, where $O_1X_1Y_1$ represents the Earth coordinate system and $O_2X_2Y_2$ represents the body coordinate system. Define $p_i(t) = [p_{xi}(t), p_{yi}(t)]^T$ and $v_i(t) = [v_{xi}(t), v_{yi}(t)]^T$ as the position and velocity information of the i -th UAV position subsystem, where the subscripts x and y denote the X - and Y -directions. Let $\Theta_i = [\mu_i, \theta_i, \phi_i]^T$ be the Euler angles of the i -th UAV attitude subsystem, where μ_i , θ_i and ϕ_i denote the yaw, pitch and roll angles, respectively.

Based on the works in [32], the UAV systems can be described by

$$\begin{aligned}\dot{p}_i(t) &= v_i(t), \\ m_i \dot{v}_i(t) &= F_i R(\Theta_i) \omega,\end{aligned}$$

where $m_i \in \mathbb{R}$ denotes the mass of i -th UAV, $F_i \in \mathbb{R}$ is the total lift, $\omega = [0, 0, 1]^T$, and $R(\Theta_i)$ denotes the planar projection of the rotation matrix, described by

$$\begin{pmatrix} C_i^\theta C_i^\phi & S_i^\mu S_i^\theta C_i^\phi - C_i^\mu S_i^\phi & S_i^\mu S_i^\theta C_i^\phi + C_i^\mu S_i^\phi C_i^\phi \\ C_i^\theta S_i^\phi & S_i^\mu S_i^\theta S_i^\phi + C_i^\mu S_i^\phi & S_i^\mu S_i^\theta S_i^\phi - S_i^\mu C_i^\phi \end{pmatrix},$$

where $C_i = \cos(\cdot)$, $S_i = \sin(\cdot)$. Define an auxiliary control signal as $u_i = F_i R(\Theta_i) \omega$ and consider the external disturbance $q_1(t) = 0.01 \sin(0.01t) \mathbf{1}_2$, $q_2(t) = 0.02 \sin(0.02t) \mathbf{1}_2$, $q_3(t) = 0.01 \cos(0.01t) \mathbf{1}_2$, $q_4(t) = 0.025 \tanh(0.025t) \mathbf{1}_2$, $q_5(t) = 0.01 \cos(0.01t) \mathbf{1}_2$, $q_6(t) = 0.02 \sin(0.02t) \mathbf{1}_2$, $q_7(t) = 0.01^2 \sin(0.01t) \mathbf{1}_2$, $q_8(t) = 0.025 \tanh(0.01t) \mathbf{1}_2$ and $q_9(t) = 0.05e^{-0.05t} \mathbf{1}_2$. It is easy to verify that the external disturbance is bounded; i.e., $Q = 0.05\sqrt{2}$. In this case, the UAV systems can be described by Equations (1) and (2).

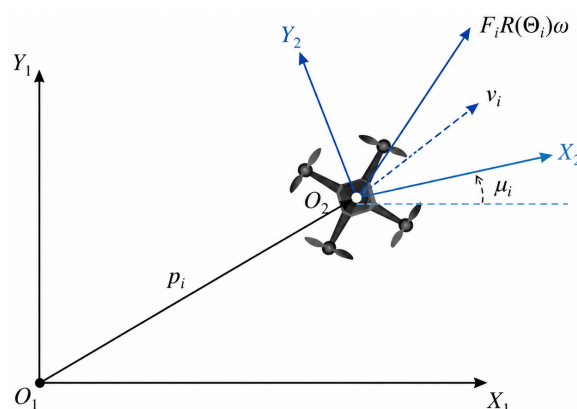


Figure 3. Diagram of UAV coordinate system.

Therefore, in this simulation part, the agent represents UAV, and nine following UAVs and one leader UAV are considered. The initial values are given as follows: $p_0(0) = [1, 2]^T$, $p_1(0) = [1.5, 1.5]^T$, $p_2(0) = [1.4, 1]^T$, $p_3(0) = [2, 2]^T$, $p_4(0) = [0.1, 1]^T$,

$p_5(0) = [1, 1]^T$, $p_6(0) = [0.2, 1.2]^T$, $p_7(0) = [1, 2]^T$, $p_8(0) = [1, 0.2]^T$, $p_9(0) = [2, 2]^T$, $v_0(0) = [1, 2]^T$, $v_1(0) = [3, 1.5]^T$, $v_2(0) = [2, 2.3]^T$, $v_3(0) = [1, 1.7]^T$, $v_4(0) = [0.5, 1]^T$, $v_5(0) = [2.3, 1.5]^T$, $v_6(0) = [0.7, 2]^T$, $v_7(0) = [2, 1]^T$, $v_8(0) = [1, 1.1]^T$, and $v_9(0) = [2, 2.5]^T$. Let $u_0(t) = \sin(t)\mathbf{1}_2$.

The communication links are displayed in Figure 4. When UAVs are the Type I and Type II followers, the links are shown in the first subfigure (Figure 4a). When UAVs are the isolated followers, the connection is characterized in the second subfigure (Figure 4b). In the case of disconnected communication, the pinning UAV is selected as UAV 7 by the pinning algorithm in [23]. In addition, let the disconnected intervals be $[0.8, 1.5)$, $[2, 2.6)$, $[2.9, 3.3)$, $[3.9, 4.2)$, $[6, 6.5)$, $[8, 8.5)$, $[10.8, 11.5)$ and $[12.5, 13.5)$. The connected and disconnected intervals are displayed in Figure 5, where the value 1 on Y-axis denotes the disconnected case and the value 0 on Y-axis denotes the connected case.

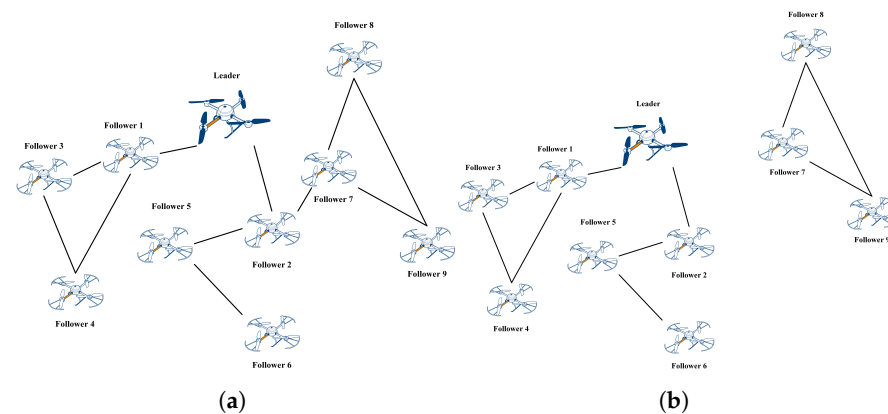


Figure 4. The communication topology of ten UAVs.

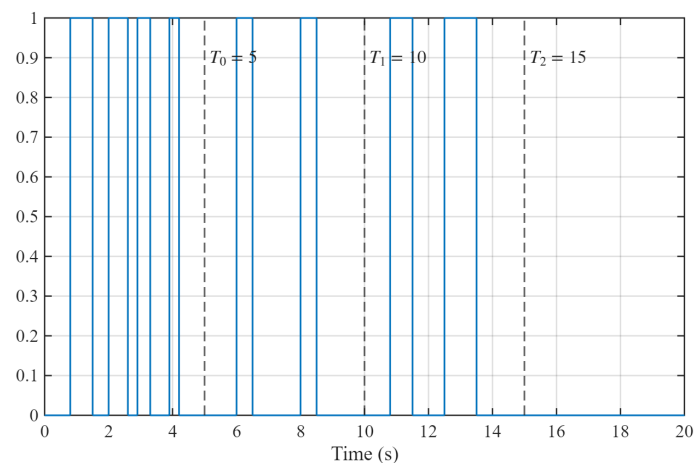


Figure 5. The communication intervals, where the value 0/1 denotes the connected/disconnected case.

Choose the prescribed-time $T_0 = 5$ s, $T_1 = 10$ s and $T_2 = 15$ s. For the distributed velocity observer, the initial conditions are as follows: $\hat{v}_1(0) = [1.5, 2.5]^T$, $\hat{v}_2(0) = [2.1, 1.1]^T$, $\hat{v}_3(0) = [0.8, 1.8]^T$, $\hat{v}_4(0) = [2, 3]^T$, $\hat{v}_5(0) = [1.2, 2.2]^T$, $\hat{v}_6(0) = [2.1, 3.1]^T$, $\hat{v}_7(0) = [1.5, 5.7]^T$, $\hat{v}_8(0) = [3.4, 6.4]^T$, and $\hat{v}_9(0) = [2.5, 3.5]^T$. The control gain n_0 and c_1 are selected as 7.5 and 15.5, respectively. It should be pointed out that the choice of control gain is related to the prescribed-time T_0 because the larger control gain leads to the faster convergence speed. Figures 6 and 7 display the error trajectories of velocity observers and leader velocity in two-dimensional plane (2D plane) and three-dimensional space (3D space), respectively. It can be seen that the velocity observers can observe the leader velocity within our prescribed-time $T_0 = 5$ s. In addition, the real convergence time of the velocity observation error

is around 4 s, so the real convergence time is within our predefined time. The evolution trajectories indicate the effectiveness of the designed distributed observers.

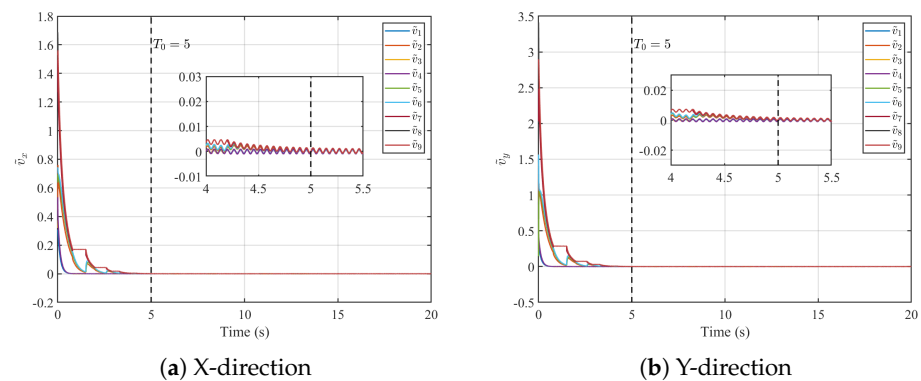


Figure 6. The error trajectories of velocity observer and leader velocity in X- and Y-directions.

For the virtual velocity, select the initial values as: $v_1^*(0) = [0.5, 1.5]^T$, $v_2^*(0) = [1, 2.5]^T$, $v_3^*(0) = [1, 0.5]^T$, $v_4^*(0) = [1, 0.8]^T$, $v_5^*(0) = [1, 1.4]^T$, $v_6^*(0) = [1, 0.5]^T$, $v_7^*(0) = [1, 1.3]^T$, $v_8^*(0) = [1, 2.7]^T$, and $v_9^*(0) = [1, 3.5]^T$. Meanwhile, we take the parameters $n_1 = 0.01$, $n_2 = 3$, $c_2 = 3.5$, and $c_3 = 0.02$. The triggering parameters are selected as $d_1 = 0.01$, $d_2 = 0.005$, $d_3 = 0.1$, $d_4 = 0.8$, $d_5 = 0.1$, and $d_6 = 0.01$, respectively. The trajectories of internal variable $\zeta_i(t)$ are shown in Figure 8. It is evident that it verifies the correctness of Lemma 3; i.e., $\zeta_i(t) \geq 0$.

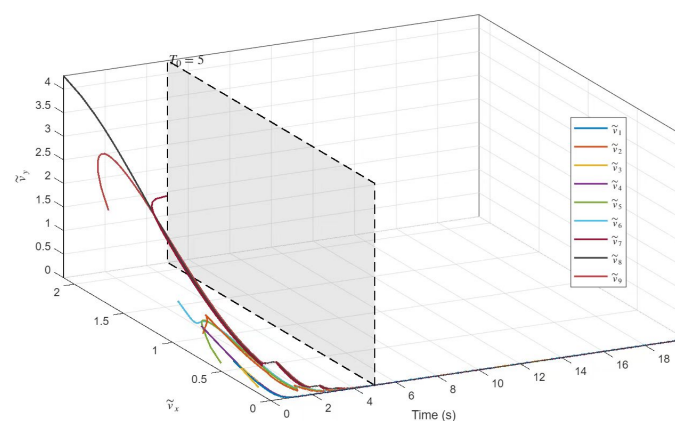


Figure 7. The error trajectories of velocity observer and leader velocity in a 3D space.

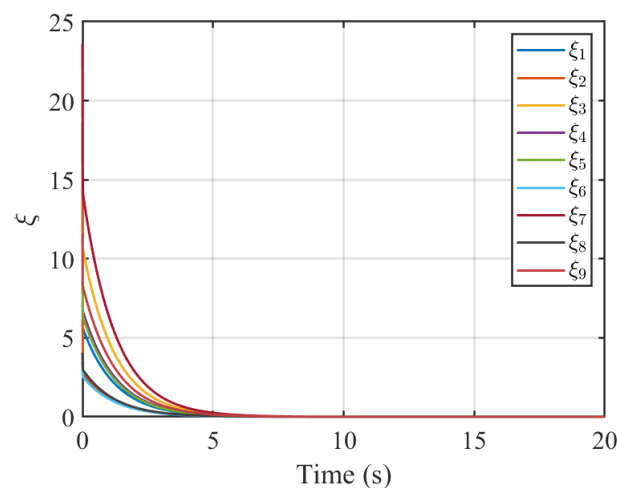


Figure 8. The trajectories of $\zeta_i(t)$.

Figures 9 and 10 give the velocity error trajectories under the prescribed-time controller with a DETM in the 2D plane and 3D space, respectively. From them, it is easy to see that the virtual velocity can attain the actual velocity with basic accuracy as time evolves. The controller trajectories are shown in Figure 11. It can be observed that the control signals exhibit large magnitudes at the initial stage to drive the system toward consensus and then gradually decay to a small neighborhood of zero as the virtual velocity approaches the actual velocity. Thereby, it demonstrates the effectiveness of the proposed controller. In addition, we record the triggering times within the DETM in Figure 12. As can be seen from this figure, both the triggering intervals and the number of triggering events vary across agents. In addition, the system exhibits no Zeno behavior.

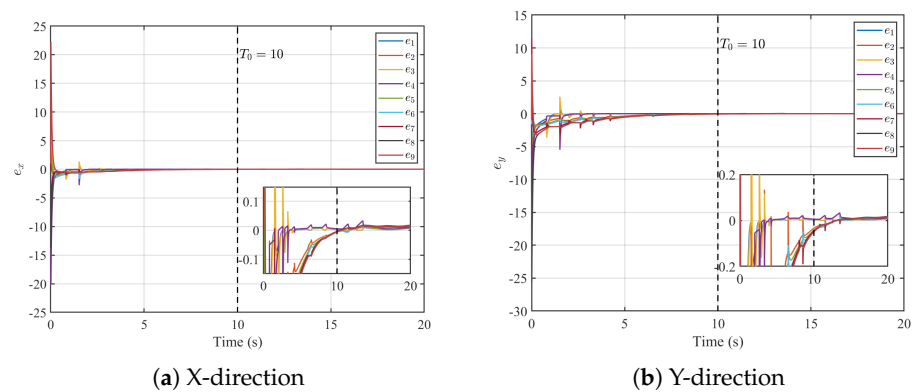


Figure 9. The error trajectories of virtual velocity and actual velocity in X- and Y-directions.

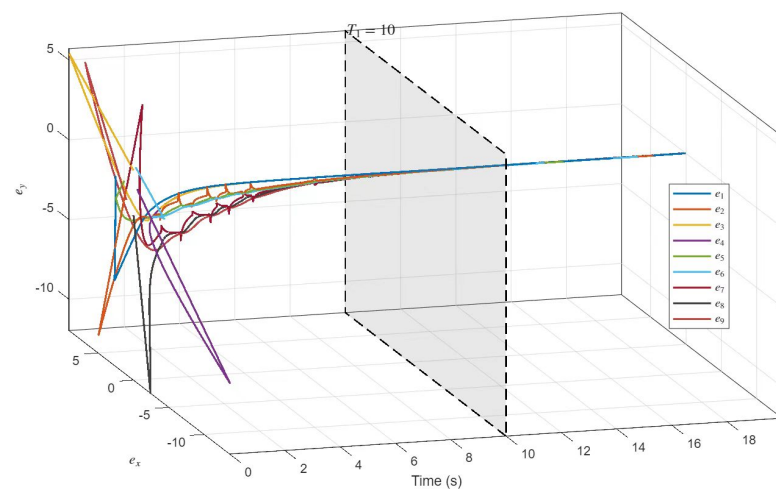


Figure 10. The error trajectories of virtual velocity and actual velocity in a 3D space.

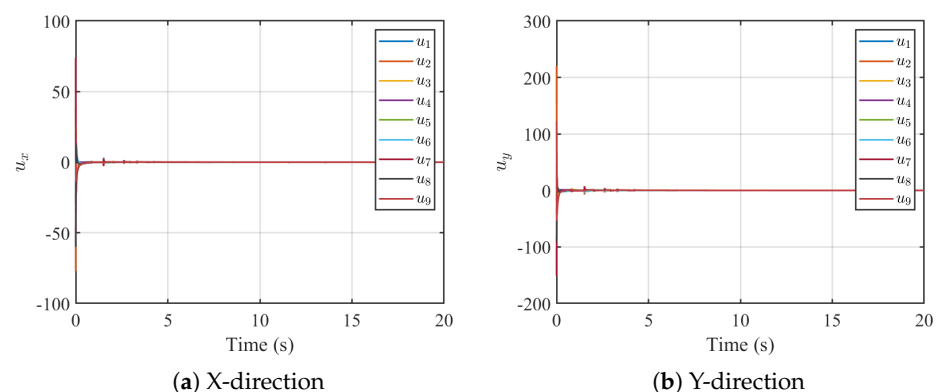


Figure 11. The trajectories of control input in X- and Y-directions.

For the position state, we consider the formation tracking issue. The formation pattern is a time-invariant formation. The distance between UAVs' positions is set as 1. Figures 13 and 14 show the trajectories of position formation error under the designed virtual velocity in 2D plane and 3D space, respectively. From them, it can be seen that the position formation error converges to an acceptable level within the predefined-time $T_2 = 15$ s, which implies that the nine following UAVs and leader UAV achieve the formation tracking and the formation error is bounded. This indicates that the virtual velocity, including velocity observer, is effective. By the above facts, the correctness of Theorem 1 is verified.

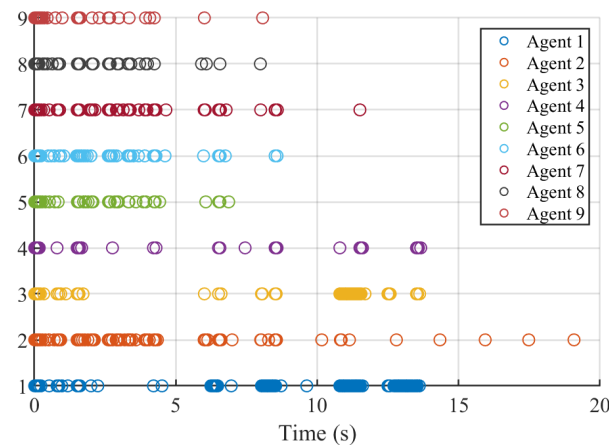


Figure 12. The triggering times.

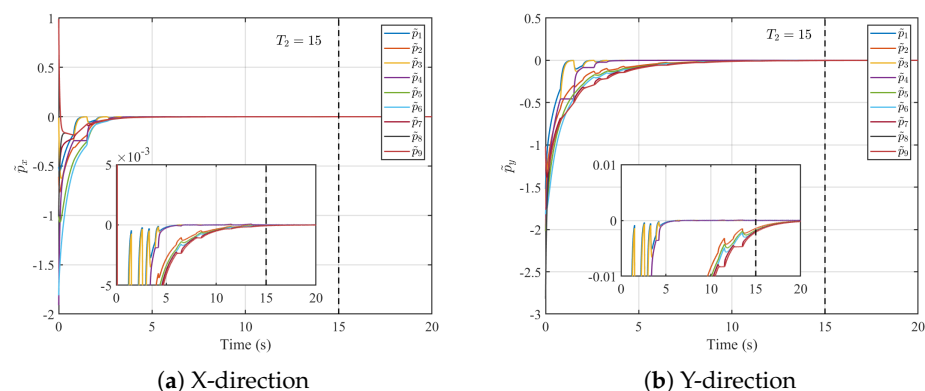


Figure 13. The error trajectories of followers' and leader's positions in X- and Y-directions.

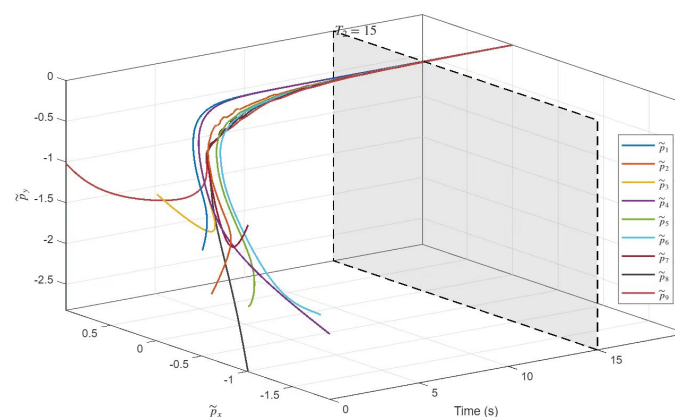


Figure 14. The error trajectories of followers' and leader's positions in a 3D space.

To further validate the superiority of the proposed DETM controller against the existing ETM in [31], which is designed to reduce the system resources, a comparative analysis is conducted based on the simulation mentioned above. For the sake of fairness, the relevant parameters of the controller in [31] are kept consistent with the original source and the initial state values of the aforementioned UAV systems are adopted. The triggering times under a DETM and an ETM are illustrated in Table 1. As shown, the designed DETM controllers can reduce the triggering times. Meanwhile, from Figure 12, we can observe that the Zeno phenomenon does not occur. The above facts indicate the superiority of the designed control scheme in Equation (19) for reducing control consumption and system resources.

Table 1. Triggering times under DETM and ETM.

UAV	Proposed DETM	ETM in [31]
1	117	207
2	114	151
3	62	155
4	39	66
5	49	107
6	67	99
7	68	124
8	45	65
9	41	75

To illustrate the influence of isolated pinned followers connected to different types of followers, we test three cases: (1) Connected to Type I follower, i.e., follower 2. (2) Connected to Type II follower, i.e., follower 5. (3) Connected to Type II follower, i.e., follower 6. Meanwhile, we define the global error norm function $EE(t) = [\|\tilde{v}(t)\|, \|e(t)\|, \|\tilde{p}(t)\|]$. The norm curves under different conditions can be seen in Figure 15. From Figure 15, we can find the following facts:

- (1) All three connection methods are able to converge $EE(t)$ to around 0, indicating that connecting follower 5, follower 6, and follower 2 can restore system consensus.
- (2) The convergence speed of connecting follower 2 is faster, indicating that, when the isolated pinned follower is connected to follower 2, the error attenuation is the fastest and the collaborative recovery efficiency is the highest.
- (3) Due to the number of communication hops between follower 5 and the leader being less than follower 6, the convergence speed of connecting follower 5 is faster than that of follower 6.
- (4) The curve convergence rate of follower 6 is slowest, indicating that follower 6, as a reconnection node, has a weak restraining effect on isolated subnetworks or a poor effective path to the leader information source.

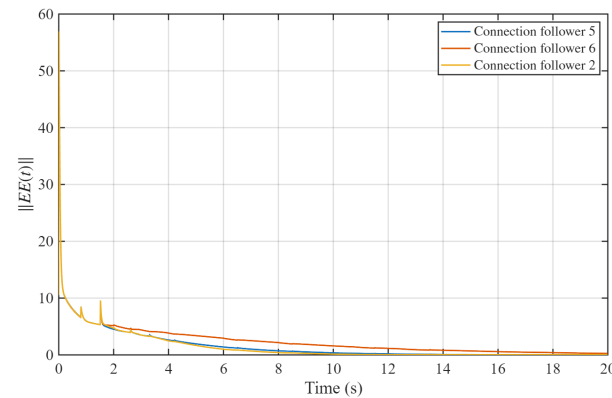


Figure 15. The curve of $\|EE(t)\|$.

7. Conclusions

This paper tackled the practical prescribed-time leader-following consensus control issue for second-order MASs under a time-varying disconnected topology. A novel practical prescribed-time stability result was presented by incorporating the communication-related time into the time-varying scaling function and considering a different derivative inequality. Further, drawing on the design concept of backstepping control, the virtual velocity and controller with a DETM were proposed to achieve the practical prescribed-time control goal, where the velocity estimator was designed to evaluate the state of the leader velocity and incorporate it into the virtual velocity. With the help of the developed stability theorem, the main results about the consensus conditions were given. Finally, the theoretical outcomes were proved to be effective via the simulation of the formation tracking of UAVs. In this paper, all the followers maintain a cooperative relationship, but, in reality, both collaborators and competitors often coexist. Therefore, further research will be conducted on the bipartite prescribed-time consensus of second-order MASs under disconnected time-varying topologies.

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References

1. Chen, H.; Cong, Y.; Wang, X.; Xu, X.; Shen, L. Coordinated path-following control of fixed-wing unmanned aerial vehicles. *IEEE Trans. Syst. Man. Cybern. Syst.* **2022**, *52*, 2540–2554.
2. Golchin, M.; Bayat, F.; Mobayen, S.; Fekih, A. Adaptive formation tracking control for heterogeneous unmanned air/ground vehicles: A barrier function approach for minimizing energy consumption. *Iran. J. Sci. Technol. Trans. Electr. Eng.* **2025**, *49*, 1521–1543.
3. Zhou, Z.; Liu, J.; Yu, J. A survey of underwater multi-robot systems. *IEEE/CAA J. Autom. Sin.* **2022**, *9*, 1–18.
4. Li, H.; Ning, J.; Tong, S. Distributed reinforcement learning optimal cluster consensus control for Takagi-Sugeno fuzzy multiagent systems. *IEEE Trans. Artif. Intell.* **2026**, *7*, 1792–1802.

5. Zhao, Y.; Sun, H.; Yang, D.; Dai, B.; Hou, L. Fully distributed leader-following bipartite consensus for multiagent systems under disturbances and actuator faults via dual-channel event-triggered control. *IEEE Internet Things J.* **2026**, *13*, 2557–2573.
6. Huang, Y.; Xu, C.; Song, Z.; Zhang, G. Dynamic event-triggered consensus of matrix-weighted linear multiagent systems. *IEEE Syst. J.* **2025**, *19*, 975–982.
7. Li, R.; Feng, Z.; Shi, Y. Distributed finite-time bipartite consensus control for constrained nonlinear MASs: A switched function approach based on prioritized strategy. *IEEE Trans. Autom. Control* **2026**, *71*, 520–527.
8. Han, J.; Jiang, H. Dynamic event-triggered, fixed-time control for heterogeneous multi-agent systems with hybrid DoS attacks. *Mathematics* **2025**, *13*, 4009.
9. Lin, G.; Chen, X.; Long, J.; Sun, J.; Lu, R. NN-based fixed-time human-in-the-loop formation control for nonlinear MASs with time-varying delay. *Neurocomputing* **2026**, *678*, 133101.
10. Yue, H.Y.; Zhao, Y.; Wang, J.Q.; Xue, D.P. Distributed event-triggered fixed-time time-varying formation control for multi-agent systems. *Mathematics* **2026**, *14*, 588.
11. Sánchez-Torres, J.; Gómez-Gutiérrez, D.; López, E.; Loukianov, A. A class of predefined-time stable dynamical systems. *IMA J. Math. Control Inf.* **2018**, *35*, 1–29.
12. Cheng, W.; Zhang, K.; Jiang, B. Practical predefined-time fault-tolerant optimal control for heterogeneous multiagent systems under directed graph. *IEEE Trans. Cybern.* **2025**, *56*, 2135–2147.
13. Yue, X.; Zhang, H.; Ma, J. Predefined-time safe cooperative control for multiagent systems with privacy preservation and unknown disturbances. *IEEE Trans. Cybern.* **2026**, *56*, 404–413.
14. Jiang, T.; Yan, Y.; Ge, S. Adaptive predefined-time bounded consensus tracking control of multiagent systems under input/output quantization. *IEEE Trans. Syst. Man. Cybern. Syst.* **2025**, *55*, 3372–3383.
15. Zhang, H.; Chen, C.; Xiang, Z. Prescribed-time fault-tolerant formation for heterogeneous switched multiagent systems. *IEEE Trans. Syst. Man. Cybern. Syst.* **2026**, *56*, 2793–2802.
16. Wang, J.; Hu, Z.; Zhang, Y.; Chen, C.L.P.; Liu, Z.; Chen, K. Dual-channel adaptive event-triggered predefined-time consensus control for multi-agent systems. *Nonlinear Dyn.* **2025**, *113*, 29681–29695.
17. Liu, H.; Li, L.; Tian, X.; Mai, Q. Distributed prescribed-time optimal formation control for high-order heterogeneous multiagent systems based on adaptive flexible performance functions and switching threshold event-triggered. *Commun. Nonlinear Sci. Numer. Simul.* **2026**, *160*, 109940.
18. Girard, A. Dynamic triggering mechanisms for event-triggered control. *IEEE Trans. Autom. Control* **2015**, *60*, 1992–1997.
19. Zhang, Z.; Wu, Y.; Wang, X.; Liu, J.; Sun, C. Practical prescribed-time consensus of uncertain multi-agent systems via intermittent dynamic event-triggered control. *IEEE Trans. Circuits Syst. I Regul. Pap.* **2025**, *72*, 7358–7370.
20. Zheng, X.; Ma, H.; Zhou, Q.; Li, H. Neural-based prescribed-time consensus control for multiagent systems via dynamic memory event-triggered mechanism. *Sci. China Technol. Sci.* **2025**, *68*, 1320402.
21. Wang, X.; Nie, X. Dynamic event-triggered-based prescribed-time consensus for pure-feedback multi-agent systems with input dead-zone nonlinearities and deferred output constraints. *Nonlinear Dyn.* **2026**, *114*, 365.
22. Wei, F.; Xu, N.; Zhao, X.; Li, L.; Al-Barakati, A.A. Dynamic memory event-triggered adaptive neural prescribed-time bipartite consensus control for high-order MASs with privacy preservation. *Commun. Nonlinear Sci. Numer. Simul.* **2025**, *145*, 108693.
23. Liu, X.; He, H.; Jiang, Z.; Fan, L.; Yu, W. Fixed-time event-triggered bipartite consensus of multiagent systems under time-varying disconnected topologies. *IEEE Trans. Syst. Man. Cybern. Syst.* **2025**, *55*, 5017–5026.
24. Xu, X.; Yu, Z.; Jiang, H.; Mei, X. Prescribed-time practical consensus of nonlinear multi-agent systems subject to DoS attacks via event-triggered mechanism. *IEEE Trans. Autom. Sci. Eng.* **2025**, *22*, 14525–14537.
25. Yin, S.; Yin, X.; Gao, Z. Dynamic event-triggered prescribed time bipartite consensus for matrix-weighted HMASs under asynchronous attacks. *Appl. Math. Comput.* **2026**, *520*, 129957.
26. Chen, C.; Han, Y.; Zhu, S.; Zeng, Z. Prescribed-time cooperative output regulation of heterogeneous multiagent systems. *IEEE Trans. Ind. Inform.* **2024**, *20*, 2432–2443.
27. Liu, S.; Zhang, L.; Hua, C.; Liu, S. A new framework of distributed prescribed-time consensus homogeneous domination control for feedforward multiagent systems. *IEEE Trans. Cybern.* **2026**, *56*, 1108–1116.
28. Yang, H.; Wang, Y.; Jiang, K.; Ong, Y.S.; Song, Y. Prescribed-time consensus tracking of multiagent systems under denial-of-service attacks with application to unmanned vehicles. *IEEE Trans. Syst. Man. Cybern. Syst.* **2026**, *56*, 3075–3085.
29. Cao, Y.; Cao, J.; Song, Y. Practical prescribed time tracking control over infinite time interval involving mismatched uncertainties and non-vanishing disturbances. *Automatica* **2022**, *136*, 110050.
30. Shi, J.; Liu, J.; Xue, L.; Wu, Y.; Liu, J.; Sun, C. Dynamic event/self-triggered prescribed-time consensus of second-order disturbed MASs under switching topologies. *Nonlinear Dyn.* **2025**, *113*, 26197–26211.

31. Zhang, S.; Yin, X.; Gao, Z. Prescribed-time consensus control of nonlinear time-delayed multi-agent systems under DoS attacks. *ISA Trans.* **2025**, *175*, 269–285.
32. Sun, F.; Su, J.; Zhu, W.; Guan, Z.; Kurths, J. Predefined-time formation control of UAV swarm under spatiotemporal constraints. *ISA Trans.* **2026**, *170*, 187–199.

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