

Article

Research on Early Warning Threshold Setting for Inventory Pledge Financing Under Supervisory Credit Restructuring

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Abstract

From the perspective of supervisory credit reconstruction, this study proposes the setting of early warning points. It employs a combined approach of evolutionary game theory and the stochastic cusp model to calculate the replenishment point and liquidation point, and simulates the impact of disruptive factors on these two points. The study reveals that: (1) The optimal equilibrium in the game between the two parties is reached when the 3PL enterprise chooses to positive supervise, the financial institution opts not to provide incentives, and instead relies on penalty deterrence, market discipline, and reputation mechanisms to ensure the 3PL enterprise implements active supervision activities; (2) During the evolutionary process of supervision, whether a sudden change in supervisory behavior occurs is determined by the bifurcation point of the stochastic cusp catastrophe model, which in turn dictates the settings of the replenishment and liquidation points; (3) The 3PL enterprise's negative supervision penalty and the 3PL enterprise's reputational loss are positively correlated with the replenishment point and negatively correlated with the liquidation point. This study provides theoretical and methodological support for supervisory credit reconstruction and quantitative implementation, which bolsters financial institutions' confidence in inventory pledge financing and promotes the development of inclusive finance.

Keywords: supervisory credit; replenishment point; liquidation point; mutation model; evolutionary game

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1. Introduction

Inventory pledge financing is a model in which a borrowing enterprise applies for financing from financial institutions such as banks by pledging its own finished goods, semi-finished goods, raw materials, and other assets. The pledged collaterals are then transferred to a qualified 3PL enterprise for custody [1]. As a typical business model in supply chain finance, it demonstrates a particularly pronounced selection preference among core enterprises compared to accounts receivable financing and prepayment financing models, serving as a crucial channel for financing small and medium-sized enterprises [2]. Currently, with the introduction of the “Ten Prohibitions” policy by the State-Owned Assets Supervision and Administration Commission of the State Council and the emphasis on the importance of financial services supporting the real economy in the report of the

20th National Congress of the Communist Party of China, inventory pledge financing has once again become a focal point in the sector due to its authentic transaction scenarios, actual control over goods ownership, and clear commercial substance. However, due to the frequent “explosions” of inventory pledge financing, supervisory misconduct such as unauthorized movement or substitution of goods, duplicate pledging, illegal release, missing records, and collusive loan fraud has become a significant risk in the industry [3]. For example, the 2012 Shanghai steel trade crisis was triggered by the lack of a price mark-to-market mechanism. In 2014, the Qingdao Port incident involved fraudulent loans obtained through duplicate pledge financing. The 2022 Foshan aluminum ingot case involved internal collusion leading to a sudden default, and the 2024 Yunnan Huijia case resulted in direct losses for state-owned enterprises due to forged cargo ownership documents. A series of risk incidents have heightened distrust between financial institutions and supervised enterprises, as the institutions face difficulties in controlling the value and status of collateral [4]. The lack of supervisory credit from supervisory distrust has led financial institutions to adopt a generally cautious approach to access in inventory pledge financing, resulting in the slow development of this business. The underlying causes can be identified from past risk incidents. These include risks arising from conflicts of interest, such as bribery, abuse of power, falsification of cargo ownership, and fraudulent attestations, as well as risks stemming from human manipulation, such as information asymmetry, failure to conduct physical inventory checks, and fictitious inventory. Consequently, research has been initiated focusing on resolving conflicts of interest and reducing human manipulation.

Regarding conflicts of interest, most scholars point out that supervisory incentives can enhance the motivation of supervisory authority, thereby reducing such conflicts. For instance, Wang et al. [5] analyzed how banks can design optimal contract mechanisms to incentivize 3PL enterprises to improve their supervision quality under three scenarios: single moral hazard, dual information asymmetry, and the Nash bargaining model. He et al. [6] introduced a repeated game and reputation model under conditions of incomplete information. By leveraging the implicit incentives generated from long-term cooperation, this model induces logistics enterprises to fully cooperate with banks, thereby preventing collusion between borrowing enterprises and logistics enterprises. Furthermore, a considerable number of scholars have employed methods such as three-stage game theory, dynamic game theory, evolutionary game theory, and multi-case studies. By comprehensively considering factors like the risk-bearing proportion of logistics enterprises, fairness preferences, capability information, and reputation mechanisms, they design optimal contract mechanisms to foster the formation of economic communities between 3PL enterprises and financial institutions such as banks. This approach aims to reduce the risks associated with conflicts of interest [7–10]. Regarding human manipulation, research primarily focuses on employing technological means such as the Internet of Things (IoT) and blockchain to reduce human intervention. Chu & Li [11] emphasize using technology to prevent and control supervisory risks, being the first to propose a QR code technology-based supervision solution. Subsequently, Dong et al. [12] utilized the IoT to propose image recognition technology for monitoring changes in goods. Zhou et al. [13] analyzed how core technologies like blockchain’s distributed ledger facilitate the sharing of pledged goods information, achieve tripartite consensus, ensure information security, and automatically execute contracts, thereby supervising inventory pledged goods and ensuring the realization of the pledge right. Yang et al. [14] further analyzed the path and mechanism of its penetrating asset supervision based on blockchain. While supervisory incentives can reduce some conflicts of interest to a certain extent, such incentives primarily operate at the corporate level. In contrast, the supervisory risks in inventory pledge financing arise from individual actions rather than corporate behavior. For example, in the Yunnan

Huijia goods rights case, Liu and Yu, the responsible persons of the Guangzhou Yihai warehouse, were bribed. Therefore, whether supervisory incentives for enterprises can directly influence the individuals ultimately responsible for cargo supervision is a question worthy of examination. Furthermore, even if incentives do reach individuals, whether the benefits of such incentives can outweigh the potential gains from being bribed remains a difficult problem to resolve. Consequently, relying solely on supervisory incentives is insufficient to effectively mitigate the actual risks inherent in inventory pledge financing. The application of technologies such as the IoT and blockchain to reduce human manipulation offers insights, to some extent, for controlling supervisory risks. Trust can reduce uncertainty and effectively mitigate risks. Technologies such as blockchain can enhance the authenticity and transparency of information verification for inventories, logistics, and capital flows within supply chains. By integrating data for verification, monitoring, and tracking, they establish digital trust and generate a range of optional credit enhancement schemes backed by guarantees. This progression—from technology to trust, to digital trust, and finally to credit enhancement—provides guidance for establishing supervisory credit.

Supervisory credit serves as a complementary form of credit to the entity credit reflected in traditional financial statements. It is a credit assessment metric generated by collecting data related to business processes with the aid of third-party tools. In inventory pledge financing, the supervisory credit is a credit assessment indicator. It is formed by the supervising entity, which acts as an external credit enhancement source, collecting data related to the value and status of pledged collateral. This process utilizes digital technology and is predicated on digital trust. However, current research on supervisory incentives and technological support remains relatively isolated, lacking integration, which results in limited effectiveness in enhancing supervisory credit. Therefore, can we leverage the application of current technologies such as the IoT, blockchain, and artificial intelligence in inventory pledge financing to construct an evaluation metric? This would involve using IoT to collect data on the status of pledged inventory, including its location, quantity, volume, temperature, and humidity. Concurrently, a blockchain consortium chain could be employed to gather data related to the value of the pledged goods, such as market prices, the operating conditions of the financing enterprises, and relevant industry sentiment. The resulting metric would then be linked to the behavior of the supervising entity and the fulfillment capability of the financing enterprise, thereby ensuring that the pledged collateral maintains sufficient value. Simultaneously, it enables the dynamic monitoring of indicators through technologies such as the IoT and blockchain, facilitating early warning and automated execution. This process establishes digital trust, forms digital credit, and reconstructs supervisory credit. Consequently, it addresses the deficiency of supervisory credit in inventory pledge financing caused by conflicts of interest and human manipulation. This constitutes an important, albeit understudied, issue that urgently requires research and resolution. Inspired by theories of stock position management and inventory management, when inventory falls below a certain threshold, it must be replenished promptly to prevent stockouts and supply chain disruptions. Similarly, when the value of a stock position declines to a certain threshold, it should be liquidated in a timely manner to mitigate loss risk. Therefore, using the replenishment point and the liquidation point—two data points closely related to the value of the pledged collateral and the status of the position—as assessment indicators for inventory pledge financing, setting corresponding supervisory thresholds to form warning points, and establishing an intelligent supervisory system for inventory pledge financing represent an innovative measure for reconstructing supervisory credit. This system enables financial institutions to maintain control over the value and status of the pledged collateral.

Accordingly, this study first reviews the literature related to the quantification of early warning points, clarifies the limitations of existing research, and presents the innovations of this study. Second, it describes dynamic game theory, stochastic mutation theory, and cargo value issues in inventory pledge financing, thereby establishing the underlying assumptions of the study. Third, based on the problem description, specific models are developed step by step and subjected to relevant model analysis. Subsequently, MATLAB R2025a is employed for simulation analysis to clarify the impact of various factors on early warning points, as well as how adjusting these factors influence replenishment points and liquidation points. Finally, the model is further verified against actual case events, leading to the conclusions of this research and the corresponding policy recommendations.

2. Literature Review

As outlined in the introduction, the composition of early warning values is related to research on early warning values for inventory pledge financing, which primarily encompasses two aspects: the quantification of supply chain finance risks and the valuation of pledged collateral. Therefore, this study will review and summarize the relevant literature from these two perspectives.

2.1. Quantification of Supply Chain Finance Risks

Supply chain finance risks are typically categorized into eight major types: credit risk, operational risk, transaction risk, technological risk, supply chain risk, environmental risk, personnel risk, and physical asset risk. Currently, there are four primary methods for quantifying these risks. (1) Empirical research methods. Zhang et al. [15] measured corporate debt default risk by constructing a distance-to-default indicator, concluding that supply chain finance can significantly reduce the default risk of core enterprises. Ling et al. [16] quantified operational risk using the standard deviation of cash flow from operating activities, examining the mitigating effect of supply chain finance on investment and financing maturity mismatch. Zhang & Zhao [17] employed a two-way fixed effects model to analyze the impact of supply chain financing on the resilience of agricultural enterprises, using the standard deviation of annualized stock returns to measure corporate risk-taking capacity. (2) Network topology method. Liu et al. [18] quantified systemic risk losses, contagion paths, and node vulnerability in supply chain finance based on network topology. Wen et al. [19] constructed a supply chain finance risk assessment model based on complex network theory, quantifying the scale of risk after default contagion by calculating default probabilities. Ye et al. [20] integrated complex adaptive system theory with epidemic dynamics to build an SEIJR risk propagation model for analyzing the transmission mechanism of supply chain disruption risks. (3) Game theory method. Sun et al. [3] employed an evolutionary game model to analyze the impact of introducing blockchain technology on accounts receivable factoring business. They quantified credit risk and operational risk using parameters such as the pledge rate and default costs, and solved for the evolutionary stable strategy through Jacobian matrix stability analysis. Xie et al. [21] constructed a tripartite supply chain game model to analyze the contagion and diffusion effects of default risk under two financing modes: bank credit and trade credit. Wang et al. [22] introduced evolutionary game theory to analyze the strategic interaction between financial institutions and small and medium-sized enterprises (SMEs) in a blockchain environment. By incorporating blockchain technology maturity and financial institutions' risk appetite into the game model, they found that blockchain technology can significantly reduce the probability of collusion in accounts receivable financing. (4) Nonlinear modeling methods. Chang et al. [23] employed ten machine learning algorithms—including logistic regression, decision trees, random forests, and gradient boosting—to compare and analyze

their performance in predicting bank creditworthiness. They found that the gradient boosting algorithm demonstrated the best performance. Through SHAP interpretability analysis, the study identified that features related to repayment status were the most significant factors influencing credit risk. Zhao [24] employed a BP neural network to construct a comprehensive evaluation model for green supply chain finance risk. Through network training, the model achieved the automatic classification of risk levels, demonstrating that the BP neural network can effectively identify risk variations in green finance operations across different regions.

While the research methodologies for quantifying supply chain finance risks are already quite diverse, and have provided some reference for reconstructing early warning values for inventory pledge financing—such as game theory approaches—the scenarios examined in current studies are predominantly based on systemic risk perspectives or rely on data from listed companies, resulting in insufficient adaptability for risk quantification in small, medium, and micro enterprises. Apart from game theory methods, most other approaches are not closely linked to actual operational practices, making their applicability difficult to assess. As an important model serving the real economy, inventory pledge financing requires its risk quantification to be highly operational.

2.2. Collateral Valuation

Current methods related to valuation and pricing in pledge financing mainly focus on pledge services, pledge ratios, and pledge interest rates. For instance, Liu Lei [25] employed a 0–1 mixed integer programming model to construct a dynamic pricing framework for inventory pledge financing. The objective function was set to maximize the profit of 3PL enterprises, and a genetic algorithm was applied to solve for the optimal pricing strategy. The dynamic pledge model was found to offer higher service pricing than the static model, which can generate greater profits for logistics enterprises. Zhu et al. [26] constructed a Stackelberg game model under deferred payment conditions to analyze how 3PL enterprises set service fees for inventory pledge financing. Kuang et al. [27] incorporated the cost of carbon emission governance into the loan interest rate pricing framework for inventory pledge financing. They constructed a decision-making model that included an option-adjusted spread and a carbon-adjusted spread, and employed the value-at-risk (VaR) method to determine the optimal pledge rate. Shi et al. [28] developed an inventory pledge financing model with downside risk aversion. They analyzed the bank's decisions on the pledge rate and financing interest rate under two scenarios where the price of the pledged collateral followed either a normal distribution or a log-normal distribution. Liu & An [29] introduced liquidity measurement methods from the securities market into the field of inventory pledge financing. They applied the liquidity-adjusted VaR method based on liquidation time to set the pledge rate, demonstrating that the scale of the inventory position is a key factor affecting both liquidity and the pledge rate. Ouyang & Li [30] investigated the policy effect of the People's Bank of China including bonds issued by AA-rated and above small and SMEs as eligible collateral for the medium-term lending facility. The study found that this policy significantly expanded SMEs' access to loans.

The valuation or pricing of pledged collateral itself is primarily applied in scenarios such as stock pledges and digital asset pledges, utilizing option pricing methodologies. For instance, Wong & Wong [31] employed an exponential phase-type Lévy model to study the valuation of stock pledge loans. They formulated the problem as an optimal stopping problem for a perpetual American option and derived a constant integro-differential equation along with a closed-form pricing formula. Separately, Prage & Zhang [32] investigated the valuation of European-style stock pledge loans under a regime-switching model involving both a geometric Brownian motion model and a mean-reversion model. Unlike traditional hedging approaches,

this study treated loan value as the expected value of future stock prices minus the repayment amount. Brigo [33] established an arbitrage-free valuation framework for bilateral counterparty credit risk, noting that the valuation adjustment consists of the sum of two option payoff terms, each dependent on the net risk exposure, defined as the difference between the exposure at default and the pre-default collateral account. Bormetti et al. [34] investigated the valuation of interest rate derivatives under credit risk and collateral modeling, discussing the impact of the stochastic basis on pricing the credit valuation adjustment for interest rate swaps and basis swaps. Zhang et al. [35] introduced an insurance mechanism into the valuation of data asset pledges, constructing a unified framework based on VaR. They proposed the Pmax model to determine the feasibility conditions under which insurance enhances pledge value.

While current research on collateral valuation primarily focuses on financial assets such as stock pledges, the related methodologies provide some reference for quantifying the value of goods in inventory pledge financing. For instance, the loan value can be viewed as the expected future stock price minus the repayment amount, or as the difference between the exposure at default and the pre-default collateral account. Therefore, by integrating game theory and the concept of “difference”, a quantitative model for early-warning indicators can be constructed. However, representative approaches currently used for early-warning quantification include dynamic signaling games, moral hazard games, evolutionary games, stochastic catastrophe models, and transfer learning models, among others, as shown in Table 1. The specific form of the game to be adopted should be further specified.

Although classical dynamic signaling games and moral hazard games can constrain the default behavior of 3PL enterprises in inventory pledge financing from ex-ante and ex-post perspectives, the assumption of complete rationality among participants and the reality that financial institutions impose access thresholds for 3PL enterprises reduce the adaptability of these models. In contrast, evolutionary game theory, which emphasizes bounded rationality and dynamic equilibrium processes, is better suited to depict the game process between the two groups of 3PL enterprises and financial institutions in the inventory pledge financing market. Simultaneously, the supervision of 3PL enterprises exhibits nonlinear characteristics. When selecting enterprises to supervise, financial institutions typically apply certain standards, such as the character and capability of the supervisor, leading to strict supervision at the initial stage. Simultaneously, the supervision of 3PL enterprises exhibits nonlinear characteristics. Supervisory status changes abruptly only when events such as collusion occur. The random mutation model, by incorporating the impact of stochastic disturbances on the game, can capture the mutation behavior of game participants and is therefore suitable for analyzing supervisory mutation behavior in 3PL enterprises. To this end, this study proposes a method combining evolutionary game theory and catastrophe models to quantify two early-warning indicators in supervisory credit restructuring: the replenishment point and the liquidation point. The corresponding advantages and disadvantages are also given in Table 1.

Based on the above analysis, the innovations of this study are as follows. (1) Innovation in research perspective. This study adopts the perspective of supervisory credit reconstruction. By integrating the application of the IoT and blockchain in inventory pledge financing, it proposes the setting of early warning indicators, such as the replenishment point and the liquidation point. These indicators are designed to quantify the supervisory effectiveness of 3PL enterprises, thereby providing a new approach for assessing supervisory credit in inventory pledge financing. (2) Innovation in research methodology. By analyzing issues in inventory pledge financing, this study employs a combined approach of “evolutionary game theory + catastrophe model” to derive a quantitative method for determining warning points. Through simulation analysis, the impact of

parameters is clarified, providing a methodological attempt for integrating an evolutionary game theory, catastrophe model.

Table 1. Quantification models for early warning values and their characteristics.

Method Category	Research Content	Method Advantages	Method Disadvantages
Game	The dynamic signaling game primarily focuses on ex-ante analysis, achieving different equilibria by controlling signaling costs [36].	It aids 3PL companies in their selection process by signaling quality and capability.	This approach assumes fully rational participants and allows for the existence of multiple equilibria.
	The moral hazard game primarily focuses on ex-post situations, aiming to control moral hazard through incentives [37].	(1) It is simple and easy to implement; (2) it has a certain inhibitory effect on 3PL defaults	It overlooks the selectivity of financial institutions toward 3PL enterprises in inventory pledge financing. Any change in the supervisory status of a qualified 3PL enterprise must be triggered by external stimuli.
	Evolutionary game theory identifies stable strategies through dynamic adjustment [38].	It emphasizes the bounded rationality of participants, where equilibrium is a dynamic process.	Multiple stable strategies can exist.
Random Mutation Model	(1) Introducing stochastic disturbance factors into the evolutionary game model transforms it into a cusp catastrophe model [39]; (2) a resilience metric is introduced to describe the system's capacity to absorb disturbances [40].	(1) It incorporates the impact of stochastic disturbances on the game; (2) it can capture the mutation behavior of game participants.	While it can reflect the degree of influence that changes in stochastic disturbances have on the system, it cannot conversely quantify the stochastic disturbances themselves.
	The content of this study is based on evolutionary game theory + mutation model [41].	(1) It incorporates the impact of stochastic disturbance factors on the game; (2) it can capture the mutation behavior of game participants; (3) it enables the quantification of stochastic disturbance factors.	The relative complexity requires careful handling of methodological integration.
Transfer Learning Model	The approach is based on the Knockoff and Trans Lasso Logistic models [42].	Addressing the high-dimensional, small-sample modeling problem by transferring data from other similar business operations	This approach requires the availability of similar business data and cannot resolve modeling challenges involving multi-source heterogeneous data

3. Problem Description and Assumptions

3.1. Problem Description

In inventory pledge financing, the participating entities include the financing enterprise, the financial institution, and the 3PL enterprise. The relationship between the financing enterprise and the financial institution is one of inventory pledge and financing services. The relationship between the financing enterprise and the 3PL enterprise involves logistics services and payment of fees. The relationship between the financial institution and the 3PL enterprise is one of entrusting supervision and providing supervision services. Once the financial institution grants credit to the financing enterprise based on the value of the pledged inventory, the financial institution and the 3PL enterprise become the two entities directly involved with the collateral. The financial institution holds rights over the pledged inventory, including possession, priority of compensation, control, and the

right of disposal in accordance with the law. Ensuring the value of the pledged inventory thus becomes a crucial risk control measure for the financial institution after extending credit. As the entity responsible for supervising the pledged inventory, the 3PL enterprises are also the source of supervisory credit. Therefore, research focusing on the supervisory credit between financial institutions and 3PL enterprises becomes critical.

In inventory pledge financing, 3PL enterprises serve as supervisory entities whose behavior is influenced by both positive and negative incentives from financial institutions, creating a game-theoretic relationship between the parties. Driven by profit maximization objectives, both parties continuously learn and accumulate experience through repeated games, dynamically adjusting their strategic choices [5]. The strategy space for 3PL enterprises consists of {positive supervision, negative supervision}, while the strategy space for financial institutions comprises {positive incentives, negative incentives}. Through evolutionary game theory, an evolutionary description under specific conditions can be derived. Nevertheless, in complex business relationships, the evolution of the game is subject to numerous interfering factors and exhibits uncertainty, potentially leading to abrupt changes in the supervisory attitude of the supervising enterprise. For instance, in inventory pledge financing, the supervisory conduct of 3PL enterprises during the decision-making process is subject to influence from financing enterprises. When the market price of collateralized goods experiences significant fluctuations, financing enterprises may adopt speculative operational practices. Similarly, when confronted with debt repayment pressures, these enterprises might engage in collusive activities to temporarily “plug financial gaps”. Through illicit practices such as unauthorized relocation or substitution of goods within warehouses, repeated pledging of collateral, unauthorized release of goods, and record omissions, they collude with the supervising entity to extract rent, thereby enabling the supervising entity to achieve returns exceeding the original equilibrium. Such actions disrupt the pre-existing game equilibrium between the supervising entity and financial institutions, precipitating an abrupt shift in the supervising enterprise’s behavior from positive supervision to negative supervision. Consequently, numerous divergence points emerge based on various influencing factors.

To this end, referring to the research on stochastic catastrophe models by Wei et al. [40], white noise is introduced into the evolutionary game to reflect random interference, and stochastic catastrophe theory is incorporated to transform the game model into a stochastic cusp catastrophe model. The analysis yields the bifurcation point set corresponding to the sudden evolution of 3PL enterprises from a relatively stable state of positive supervision (or negative supervision) to a state of negative supervision (positive supervision). The collateral values at the equilibrium point and at the bifurcation point are calculated, and the value differences under these two states are analyzed to establish the replenishment point and liquidation point indicators corresponding to the warning points, thereby reconstructing the supervision credit. Meanwhile, by analyzing the extent to which parameters—such as the operating conditions of financing enterprises and rent-seeking behaviors triggered by market price fluctuations of collateral—affect warning points, this study quantitatively describes the patterns of these warning points. Furthermore, it provides guidance for warning strategies of financial institutions, thereby establishing the research framework for the entire study, as specifically illustrated in Figure 1.

3.2. Model Assumptions

Assumption 1: Participants. Both 3PL enterprises and financial institutions are bounded rational economic agents. Financial institutions have certain selection criteria for 3PL enterprises. Therefore, only when there is interference from external fac-

tors or the temptation of excess returns is it possible for 3PL enterprises to change their supervisory attitude.

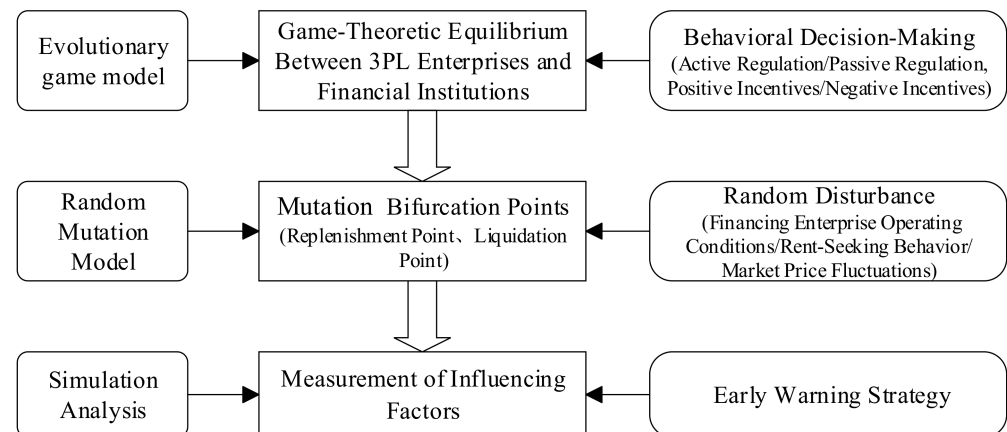


Figure 1. Research framework diagram.

Assumption 2: Game theory strategies. The game theory strategies for 3PL enterprises are “positive supervision” and “negative supervision”, with the probability of selecting “positive supervision” being x . $0 \leq x \leq 1$, The probability of selecting “negative supervision” is denoted as $1 - x$; financial institutions employ game strategies of “positive incentive” and “negative incentive”, with the probability of choosing “positive incentive” being y , $0 \leq y \leq 1$, and consequently, the probability of selecting “negative incentive” is $1 - y$.

Assumption 3: Related revenues. In actual business operations, when 3PL enterprises conduct inventory pledge financing supervision, they typically generate fixed revenue R_1 during positive supervision. During negative supervision, revenue is influenced by the supervision success rate k , $0 \leq k \leq 1$, and R_1 is generally related to the cargo value V ; the higher the cargo value, the greater the supervision responsibility, and R_1 also increases. The revenue obtained by financial institutions from inventory pledge financing operations is R_2 , and R_2 is generally related to the credit limit L and credit rate of return ρ . Additionally, when financial institutions implement positive incentives, 3PL enterprises receive positive incentive revenue r ($r > 0$); similarly, when implementing negative incentives, 3PL enterprises incur penalties e ($e > 0$) for negative incentives. However, when 3PL enterprises adopt negative supervision due to collusion and rent-seeking, they obtain excess benefits typically greater than e , and these excess benefits have a linear relationship with the operating condition G of the financing enterprise. The operating condition of the financing enterprise affects rent-seeking willingness and the available benefit space. When a financing enterprise approaches bankruptcy, its rent-seeking motivation intensifies, resulting in larger excess returns that typically manifest as substantial one-time bribes. For financing enterprises with relatively stable operations, excess returns tend to be consistent over the long-term, though smaller in magnitude per instance. In the case of high-quality leading enterprises, there is a propensity for establishing long-term cooperative relationships with supervising enterprises, which generates greater expected cumulative excess returns.

Assumption 4: Related costs. The surveillance cost for 3PL enterprises to implement positive surveillance is denoted as C_1 , while the cost for implementing negative surveillance is denoted as C_2 , with the condition that $C_1 > C_2 > 0$; furthermore, to deter 3PL enterprises from selecting the negative surveillance strategy, $e + C_2 > C_1 - r$ must be satisfied, otherwise 3PL enterprises will opt for negative surveillance. For financial institutions, considering only the costs arising from surveillance activities while excluding capital costs, monitoring costs, and transaction costs, the negative surveillance by 3PL enterprises results

in a loss M to financial institutions; simultaneously, when both parties adopt negative measures, the negative surveillance behavior of 3PL enterprises propagates throughout the industry, causing them a reputation loss S ($S > 0$).

The corresponding notations for the game-theoretic interaction between the above-mentioned 3PL enterprises and financial institutions are detailed in Table 2.

Table 2. Symbols and explanations.

Symbols	Explanations	Economic and Practical Significance
x	The probability of 3PL enterprises selecting “positive supervision”, $0 \leq x \leq 1$	In inventory pledge financing, financial institutions and 3PL enterprise providers are in a principal–agent relationship. This indicator reflects the extent to which incentive compatibility constraints are satisfied.
y	The probability of financial institutions selecting “positive incentive”, $0 \leq y \leq 1$	This reflects whether the bank is willing to pay agency costs in exchange for monitoring efficiency.
R_1	The benefits of positive supervision in inventory pledge financing for 3PL enterprises	Driving forces for the development of inventory pledge financing supervision services.
k	Success rate of 3PL companies in implementing inventory pledge financing under negative supervision	This measures the probability that a 3PL enterprise can evade punishment for failing to fulfill its supervisory duties. A higher value of this indicator corresponds to a greater expected return from negative supervision.
R_2	The income derived by financial institutions from inventory pledge financing activities.	The driving force of inventory pledge financing credit.
V	Collateral value	The establishment of a credit line for inventory pledge financing is based on the loan-to-value ratio, which is the credit line divided by the value of the pledged collateral.
P	Collateral market price	The basis for establishing the value of pledged collateral.
Q	Collateral amount	The basis for establishing the value of pledged collateral.
L	Credit limit for inventory pledge financing by financial institutions	The final representation of inventory pledge financing.
φ	Pledge ratio for inventory pledge financing	Basis for establishing credit line in inventory pledge financing.
θ	Return on inventory pledge supervision for 3PL enterprises	Supervisory return ratio.
ρ	Return on inventory pledge financing operations by financial institutions	Inventory pledge financing return ratio.
r	3PL enterprises obtain positive incentive benefits	The expenditure incurred by financial institutions to incentivize 3PL enterprises to enhance supervisory efficiency.
e	Penalties for negative incentives in 3PL enterprises	The penalty imposed on financial institutions due to losses arising from inadequate supervision by 3PL enterprises, which is reflected as income.
G	Operating performance of financed enterprises (business income)	The payment capacity affecting rent-seeking incentives and excess benefits.
h	Linear coefficient of excess returns and business operations of financing enterprises	The extent to which excess benefits are influenced by the actual payment capacity of financing enterprises.
M	Negative supervision of 3PL enterprises causes losses to financial institutions.	The portion of credit extended by financial institutions that remains unrecovered.
S	The reputational losses suffered by 3PL enterprises due to negative supervision	Losses incurred by 3PL enterprises due to future business obstructions resulting from the spread of negative supervision actions as adverse signals within the industry.

Table 2. Cont.

Symbols	Explanations	Economic and Practical Significance
C_1	Supervisory costs incurred by 3PL enterprises through positive supervision	Supervisory costs encompass both online visual supervision and offline inspections, such as personnel patrols and inventory checks.
C_2	Supervisory costs arising from negative supervision in 3PL enterprises	Same connotation as C_1 .

4. Model Establishment

4.1. Evolutionary Game Analysis

4.1.1. Construction of the Evolutionary Game Model

Based on the assumptions, the positive supervision revenue of the 3PL enterprise is linearly related to the cargo value. The revenue obtained by financial institutions from inventory pledge financing services is associated with the credit line and credit rate of return, where the credit line is linearly related to both the loan-to-value ratio and the cargo value. The cargo value, in turn, depends on the market price and the quantity of the pledged goods. Therefore, the relationship between R_1 , R_2 and the cargo value V can be expressed as:

$$\begin{aligned} R_1(V) &= \theta V = \theta PQ \\ R_2(V) &= \rho L = \rho \varphi V = \rho \varphi PQ \end{aligned} \quad (1)$$

Based on the assumptions, the game between 3PL enterprises and financial institutions is represented in Table 3.

Table 3. Evolutionary game representation.

(3PL, Financial Institutions)	Financial Institutions Positive Incentives	Financial Institutions Negative Incentives
3PL Positive Supervision	$(R_1 + r - C_1, R_2 - r)$	$(R_1 - C_1, R_2)$
3PL Negative Supervision	$(kR_1 + hG - C_2, R_2 - M)$	$(kR_1 - e + hG - C_2 - S, R_2 - e - M)$

Accordingly, the expected returns for the 3PL enterprises under positive supervision E_1 , negative supervision E_2 , and the overall expected return $\bar{E}_1$ are as follows:

$$E_1 = y(R_1 + r - C_1) + (1 - y)(R_1 - C_1) \quad (2)$$

$$E_2 = y(kR_1 + hG - C_2) + (1 - y)(kR_1 - e + hG - C_2 - S) \quad (3)$$

$$\bar{E}_1 = xE_1 + (1 - x)E_2 \quad (4)$$

According to the replication dynamics equation, we have:

$$\begin{aligned} F(x) = \frac{dx}{dt} &= x(E_1 - \bar{E}_1) \\ &= x(1 - x)[y(R_1 + r - C_1) + (1 - y)(R_1 - C_1) - y(kR_1 + hG - C_2) \\ &\quad - (1 - y)(kR_1 - e + hG - C_2 - S)] \\ &= x(1 - x)[(1 - k)R_1 + C_2 - C_1 - hG + S + e + y(r - e - S)] \end{aligned} \quad (5)$$

Let $F(x) = 0$, then $x_1 = 0$, $x_2 = 1$, $y^* = \frac{(1-k)R_1 + C_2 - C_1 - hG + e + S}{e + S - r}$ can be obtained. Similarly, the expected returns for financial institutions adopting positive incentives E_3 , negative incentives E_4 , and the overall expected return $\bar{E}_2$ are respectively:

$$E_3 = x(R_2 - r) + (1 - x)R_2 - M \quad (6)$$

$$E_4 = xR_2 + (1 - x)(R_2 - e - M) \quad (7)$$

$$\bar{E}_2 = yE_3 + (1 - y)E_4 \quad (8)$$

According to the replication dynamics equation, we have:

$$\begin{aligned} F(y) = \frac{dy}{dt} &= y(E_3 - \bar{E}_2) = y(1 - y)[x(R_2 - r) + (1 - x)R_2 - M - xR_2 - (1 - x)(R_2 + e - M)] \\ &= y(1 - y)[e - x(e + r)] \end{aligned} \quad (9)$$

Let $F(x) = 0$, as $-x^2r - (1 - x)^2e < 0$ then $y_1 = 0, y_2 = 1, x^* = \frac{e}{e+r}$ can be obtained. Consequently, when $0 < x^* < 1, 0 < y^* < 1$, the game yields five equilibrium points, specifically $(0, 0), (0, 1), (1, 0), (1, 1), (x^*, y^*)$.

4.1.2. Stability Analysis of Evolutionary Game Equilibrium Points

According to Friedman's method for analyzing equilibrium point stability using Jacobian matrices, the determinant of the Jacobian matrix is calculated. If the determinant $\text{Det}(J) > 0$ and trace $\text{Tr}(J) < 0$ satisfy the stability conditions, it indicates the existence of an evolutionarily stable strategy at that equilibrium point.

$$J = \begin{bmatrix} \frac{\partial F(x)}{\partial x} & \frac{\partial F(x)}{\partial y} \\ \frac{\partial F(y)}{\partial x} & \frac{\partial F(y)}{\partial y} \end{bmatrix} = \begin{bmatrix} (1 - 2x)[A + (r - e - S)y] & x(1 - x)[r - e - S] \\ y(1 - y)(-e - r) & (1 - 2y)[e - x(e + r)] \end{bmatrix} \quad (10)$$

where $A = (1 - k)R_1 + C_2 - C_1 - hG + e + S$.

The determinant $\text{det}(J)$ and trace $\text{Tr}(J)$ of the Jacobian matrix at each of the five equilibrium points were computed to conduct stability analysis. The conditions for the existence of a stable equilibrium point are presented in Table 4.

Table 4. Stable analysis.

Equilibrium Point	Det(J)	Tr(J)	Stability Condition
(0, 0)	Ae	$A + e$	Unstable
(0, 1)	$-(A - e - S + r)e$	$A + r - S - 2e$	saddle point
(1, 0)	Ar	$-A - r$	$A > 0$ ESS
(1, 1)	$-(A - e - S + r)r$	$-(A - e - S) - 2r$	saddle point
(x^*, y^*)	$x^* y^* (1 - x^*)(1 - y^*)(r - e - S)(r + e)$	0	saddle point

As shown in Table 4, among the five local equilibrium points, only $(0, 0)$ and $(1, 0)$ have conditions for stability, with the magnitude of A becoming the key determinant for judging stability. A represents the constraining force of the penalty e imposed by financial institutions and the 3PL reputational losses S on the difference between the net benefit $R_1 - C_1$ from active supervision and the baseline benefit $kR_1 - C_2 + hG$ from passive supervision. When the reputational losses and penalties are sufficiently significant, the 3PL enterprises will proactively lean toward positive supervision. Such positive supervision serves as a confidence guarantee for financial institutions in conducting inventory pledge financing. This aligns with the earlier analysis, which indicates that financial institutions typically establish certain criteria when selecting supervising entities, and the initial phase of supervision tends to be strict. Therefore, when $A > 0$ equals $R_1 > \frac{C_1 - C_2 + hG - e - S}{1 - k}$, the equilibrium point $(1, 0)$ is a stable point. In this scenario, financial institutions do not provide incentives. Instead, the deterrence of penalties, coupled with market discipline and reputational mechanisms, drives the 3PL enterprises to implement positive supervision activities. This aligns with real-world practices where market discipline is exerted on 3PL enterprises through reputational impacts, such as case disclosures. For instance, in May 2025, a cold storage facility in Tianjin Dongjiang was publicly disclosed and handed over to the economic crime investigation authorities. It was alleged to have re-pledged the client's

imported frozen goods for cash, plunging the importers into a predicament where the ownership of the goods was unclear and retrieval was impossible.

Based on the equilibrium condition $R_1 > \frac{C_1 - C_2 + hG - e - S}{1 - k}$ and Equation (1), the collateral value at the equilibrium point is:

$$V = PQ > \frac{C_1 - C_2 + hG - e - S}{(1 - k)\theta} \quad (11)$$

4.2. Stochastic Cusp Catastrophe Model and Replenishment Point

4.2.1. Construction of the Stochastic Cusp Catastrophe Model

Inventory pledge financing involves multiple participants in its operational framework. The supervisory behavior of 3PL enterprises is inevitably influenced by external uncertainties. Among these, collusive rent-seeking activities by financing enterprises toward 3PL enterprises represent the most significant disruptive factor. Drawing on the methodology employed in existing studies for measuring uncertainty, this study introduces white noise to reflect stochastic disturbances into the evolutionary dynamics of 3PL enterprises under deterministic conditions.

To reduce the two-dimensional replicator dynamic equations to a standard stochastic cusp catastrophe model, this study introduces the symmetric dynamic path condition $x = y$. When financial institutions and 3PL firms have similar learning capabilities, information acquisition costs, and strategy adjustment speeds, the evolutionary paths of their strategy adoption proportions exhibit approximately symmetric features, i.e., $x(t) = y(t)$; hence it is assumed that $x = y$. To verify the robustness of this assumption, a perturbation test is presented in the Appendix A, demonstrating that the core conclusions are not sensitive to this approximation.

Substituting $x = y$ into Equation (5) and setting $\alpha = S + e - r$, $\beta = -(1 - k)R_1 + C_1 - C_2 + hG + r - 2e - 2S$, $\gamma = (1 - k)R_1 + C_2 - C_1 - hG + S + e$, Equation (5) can be simplified as:

$$F(x) = \frac{dx}{dt} = \alpha x^3 + \beta x^2 + \gamma x \quad (12)$$

Equation (12) is extended into a stochastic differential equation of $It\hat{o}$ form, namely:

$$dx = [\alpha x^3(t) + \beta x^2(t) + \gamma x(t)]dt + \varepsilon(x)dw(t) \quad (13)$$

In Equation (13), $w(t)$ is a one-dimensional standard Brown motion. At a given time t , $w(t)$ follows a normal distribution $N(0, t)$, and $w(t)$ effectively captures the random disturbances acting on the system. Meanwhile, $dw(t)$ represents Gaussian white noise, and $\varepsilon(x)$ quantifies the extent of the influence of perturbative factors on the system.

Given that supervisory mutations in 3PL enterprises are driven by interests, and collusive rent-seeking by financing enterprises constitutes the most significant nonlinear factor of interest-driven behavior, the stochastic mutation theory is applied to treat $x(t)$ in Equation (13) as a stochastic process. The probability density function of $x(t)$ at time t is expressed as:

$$f(\mu, t, x_0) = \frac{d}{d\mu} \text{Prob}\{x(t) < \mu | x(0) = x_0\} \quad (14)$$

Let $V(x, g)$ denote the system's potential function and g represent the control variable. Then, the following holds:

$$V(x, g) = \int_{x_0}^x [\alpha \mu^3 + \beta \mu^2(t) + \gamma \mu] d\mu \quad (15)$$

Therefore, Equation (13) can be expressed as:

$$dx(t) = \frac{\partial V(x, g)}{\partial x} dt + \varepsilon(x) dw(t) \quad (16)$$

As $t \rightarrow \infty$ approaches infinity, Equation (14) converges to a limit $f(\mu, t, x_0) \rightarrow f^*$, implying that the limiting probability density function can be expressed as:

$$f^*(x) = N_c \exp[-V_{sto}(x)] \quad (17)$$

In Equation (17), $c \in [-1, 1]$ is a constant, and N_c is also a constant. The expression for $V_{sto}(x)$ is given as follows:

$$V_{sto}(x) = -2 \int_c^x \frac{-\frac{\partial V(w, g)}{\partial w} - \frac{1}{2} [\varepsilon^2(w)]'}{\varepsilon^2(w)} dw \quad (18)$$

Assume that $\varepsilon(w)$ is constant, indicating a uniform degree of influence from perturbing factors. The transition of the stochastic process $x(t)$ corresponds to a topological change in the limiting probability density function $f^*(x)$ with respect to its modes and antimodes, and coincides with the minima and maxima of the potential function given in Equation (15), which is derived from Equation (12). Based on Equations (17) and (18), the differentiable mode points of $f^*(x)$ arise from:

$$f^{*'}(x) = N_c \exp[-V_{sto}(x)] V'_{sto}(x) = 0$$

Namely,

$$V'_{sto}(x) = 0$$

Available as,

$$\alpha x^3 + \beta x^2 + \gamma x = 0 \quad (19)$$

When $\alpha \neq 0$, the quadratic term elimination method is applied by introducing $g = x + \frac{\beta}{3\alpha}$, $u = \frac{\gamma}{\alpha} - \frac{\beta^2}{3\alpha^2}$, $v = \frac{2\beta^3}{27\alpha^3} - \frac{\beta\gamma}{3\alpha^2}$, and Equation (19) can be transformed into the standard cusp catastrophe model's manifold equation, namely:

$$g^3 + ug + v = 0 \quad (20)$$

In Equation (20), g is a state variable representing the positive and negative supervision states of the 3PL enterprise, while u and v are control variables.

4.2.2. Stochastic Cusp Catastrophe Bifurcation Point Set

According to catastrophe theory, the critical manifold satisfies:

$$\begin{cases} f^{*'}(g) = g^3 + ug + v = 0 \\ f^{*''}(g) = 3g^2 + u = 0 \end{cases}$$

Simplifying yields the bifurcation set:

$$4u^3 + 27v^2 = 0 \quad (21)$$

Substituting u and v into the equation and simplifying yields:

$$\gamma^2(4\alpha\gamma - \beta^2) = 0 \quad (22)$$

In light of the definitions of α , β , γ in Equation (12), Equation (22) can be decomposed as follows:

$$(1-k)R_1 + C_2 - C_1 - hG + r = 0 \text{ or } (1-k)R_1 + C_2 - C_1 - hG + e + S = 0$$

According to the stability conditions for evolutionary game ESS discussed earlier, $A = (1-k)R_1 + C_2 - C_1 - hG + e + S > 0$. Therefore, the condition for Equation (22) to hold is:

$$(1-k)R_1 + C_2 - C_1 - hG + r = 0 \quad (23)$$

Therefore, as the parameter varies continuously, when condition $4u^3 + 27v^2 = 0$, i.e., $(1-k)R_1 + C_2 - C_1 - hG + r = 0$ is satisfied, the supervisory behavior of the 3PL enterprise undergoes a sudden change.

The catastrophe model and bifurcation point set are plotted in MATLAB, as shown in Figure 2. When $u < 0$, the catastrophe model exhibits an accessible bifurcation point set. The supervisory behavior of 3PL enterprises undergoes a catastrophe when $(1-k)R_1 + C_2 - C_1 - hG + r = 0$ is satisfied, which occurs when the collateral market price, financing enterprise's operating condition, supervisory costs, and reputation loss satisfy Equation (23). As shown in Figure 2, when the evolutionary path transitions from $N \rightarrow M$, it passes through a bifurcation point set and undergoes a catastrophic change to M' , resulting in the supervisory behavior of 3PL companies shifting abruptly from positive supervision to negative supervision. Conversely, if the evolutionary path follows $M' \rightarrow P' \rightarrow Q'$, the supervisory behavior of 3PL companies transitions from negative supervision to positive supervision. Thus, as external factors such as the market prices of pledged collateral and reputational losses, the dynamic equilibrium between 3PL enterprise and financial institutions can be disrupted once these factors reach a certain threshold. Regardless of whether a 3PL enterprise's supervisory stance is positive or negative, an abrupt shift can occur. If the underlying conditions are not altered, this supervisory stance may become uncontrollable. For example, the collective default of Shanghai's steel trade mutual guarantee loans erupted because changes in factors such as surveillance costs and penalty severity transformed previously manageable risks into uncontrollable ones [43].

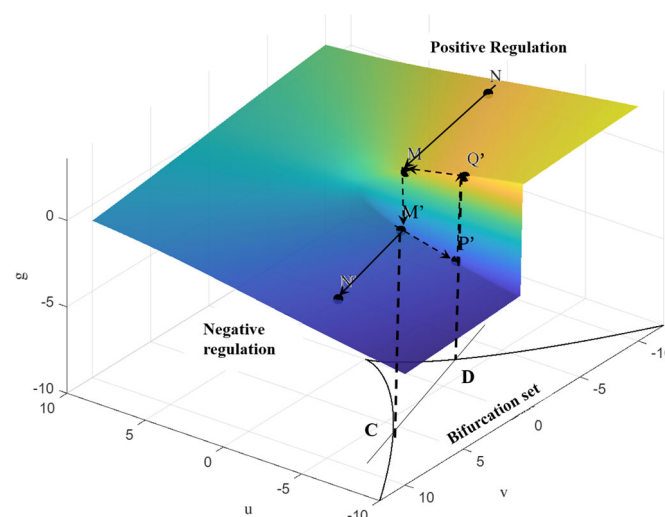


Figure 2. Schematic diagram of the catastrophe in the supervisory behavior of 3PL enterprises.

4.2.3. Replenishment Point

This study elucidates the pathway and process of behavioral shifts in the supervision provided by 3PL enterprises within inventory pledge financing. To mitigate supervisory

risks, financial institutions should implement controls over the value of pledged inventory before such behavioral shifts occur. Establishing replenishment points and liquidation points serves as early-warning indicators for supervision.

In the stochastic cusp catastrophe model, the control variable u determines whether a catastrophe occurs. By fixing a value of u_i (where $u_i < 0$), a cross-section of the catastrophe model is obtained (as shown in Figure 3). It can be observed that when $v = -v_i$, at point N_{t_0} , the system enters an early warning state. As v continues to change to v_i , at point M_{t_1} , the system reaches the catastrophe state. Therefore, to control the collateral value at which the system reaches the catastrophe state, the collateral value at point M_{t_1} and the value at the game equilibrium point are calculated. The difference between these two values forms the replenishment point, ensuring that the collateral value does not fall below the threshold that triggers the catastrophe, thereby achieving early warning.

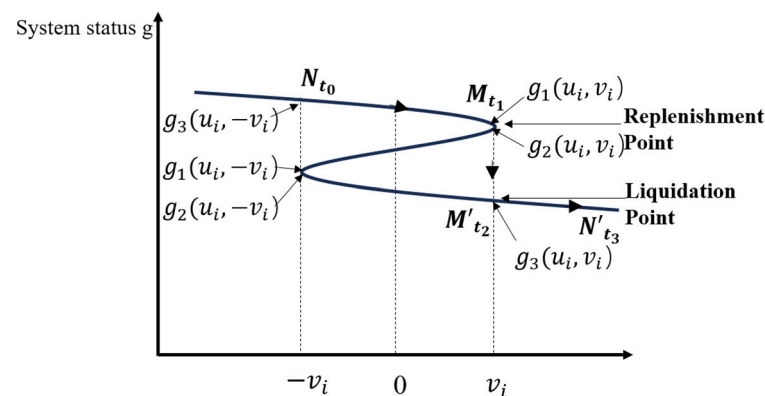


Figure 3. Catastrophe cross-section of supervision behaviors in 3PL enterprises.

M_{t_1} represents the point traversing the catastrophe bifurcation set, satisfying Equation (23). The corresponding value expression can thus be derived as follows:

$$V = PQ = \frac{C_1 + hG - C_2 - r}{(1 - k)\theta} \quad (24)$$

Equations (24)—(11) yield: $\frac{e+S-r}{(1-k)\theta}$. Based on the definitions of the parameters, where $0 \leq k \leq 1, 0 < \theta < 1$, the validity of A and Equation (23), it follows that $e + s - r > 0$. This indicates that when the supervisory behavior of the 3PL enterprise undergoes a catastrophe, the value of the pledged collateral indeed changes. Therefore, replenishing the collateral value at point M_{t_1} can effectively mitigate risks.

$$V_b = PQ_b = \frac{e + S - r}{(1 - k)\theta} \quad (25)$$

The replenishment point quantity Q_b can be expressed as:

$$Q_b = \frac{e + S - r}{(1 - k)\theta P} \quad (26)$$

4.2.4. Liquidation Points

As shown in Figure 3, when the system undergoes a catastrophe from M_{t_1} to M'_{t_2} , the supervision by the 3PL enterprises has entirely shifted to a negative mode. Based on the calculation process of the replenishment point, it can be inferred that the actual value of the goods has already decreased substantially at this stage, making it impossible to mitigate risks through replenishment. Consequently, financial institutions can only resort to liquidation to monetize the collateral and minimize losses. Here, M'_{t_2} represents the

liquidation point. Therefore, the pledged collateral value at which the system reaches the mutation M_{t_1} , minus the actual value loss ΔV incurred due to the supervisory mutation, is the pledged collateral value at the liquidation point.

From the calculation of the replenishment point, it follows that the difference between the value of the pledged collateral at equilibrium—Equation (11)—and the value of the pledged collateral when the system undergoes a catastrophe—Equation (24)—is exactly the value lost in fact due to the supervisory catastrophe. Hence, we have $\Delta V = V_b$. Suppose that after the supervisory action completes a shift from positive to negative, the pledged collateral value declines further from V in Equation (24), and the decline is proportional to the replenishment point V_b ; then we have:

$$V_p = V - \lambda V_b \quad (27)$$

where λ is the post-mutation loss coefficient for pledged collateral value, with $0 < \lambda \leq 1$, reflecting the extent of pledged collateral value loss caused by negative supervision. If the most conservative risk control strategy is adopted, that is, if negative supervision leads to a complete loss of pledged collateral value to the degree corresponding to the replenishment point, set $\lambda = 1$, we have:

$$V_p = V - V_b = \frac{C_1 + hG - C_2 - r}{(1 - k)\theta} - \frac{e + S - r}{(1 - k)\theta}$$

This means that

$$V_p = \frac{C_1 + hG - C_2 - e - S}{(1 - k)\theta} \quad (28)$$

From Equations (26) and (28), both the replenishment point and the liquidation point are affected by the penalties for negative supervision e , the reputational loss S , the regulatory return θ , and the success rate of negative supervision. At the same time, the replenishment point is also influenced by the incentive benefits r and the collateral market price P , whereas the liquidation point is also influenced by the supervisory costs C_1 and C_2 and the operating condition of the financed enterprise G . To further discern the specific functional relationships between parameter changes and the replenishment and liquidation points, it is necessary to employ simulation analysis for elucidation.

5. Simulation Analysis

The early warning indicators established through the replenishment point and liquidation point provide a quantitative framework for supervisory credit reconstruction, yet these indicators are influenced by various parameters. Analyzing the extent of each parameter's impact on the warning indicators facilitates the implementation of relevant strategies through parameter control, thereby promoting the positive development of inventory pledge financing business. To this end, this study sets relevant parameter values (see Table 5), conducts simulations using MATLAB, and focuses on examining the effects of the collateral market price P , incentive benefit r , penalties for negative supervision e , reputational loss S , and supervisory costs C_1 and C_2 on the early warning indicators.

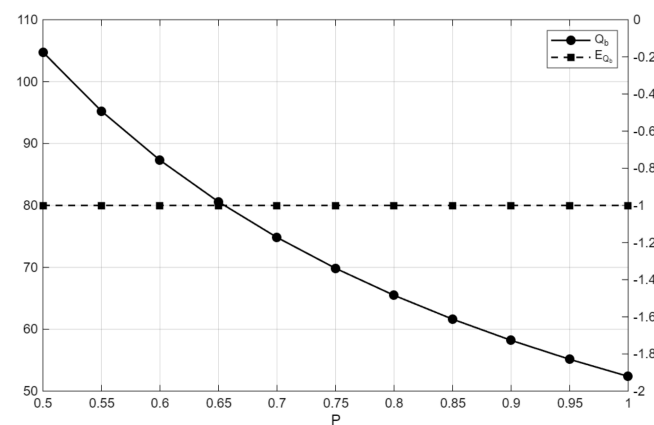
5.1. Collateral Market Price

Parameter settings follow Table 5; a simulation analysis was conducted by varying the collateral market price P from 0.5 to 1 in increments of 0.05. The resulting trends of the early-warning indicators—replenishment point Q_b —is illustrated in Figure 4.

Table 5. Parameter setting table.

Parameter Symbol	Parameter Value	Explanation
k	0.3	Negative supervision success rate $0 \leq k \leq 1$
θ	0.03	Based on research into companies such as CMST Development Co., Ltd., the industry-standard logistics supervision fee typically ranges from 1% to 3% of the collateral's value
h	0.04	Excess return coefficient, higher than the supervisory rate of return
C_1	15	Positive supervision cost $C_1 > C_2$
C_2	5	Negative supervision cost
P	0.5	Collateral market price
Q	200	Initial collateral quantity
G	200	Financing enterprise operating condition
e	14	Penalties for negative supervision
S	3	Reputational loss
r	15.9	Incentive benefit

Since the above parameters are constrained by Equation (23), $(1-k)R_1 + C_2 - C_1 - hG + r = 0$ and $A = (1-k)R_1 + C_2 - C_1 - hG + e + S > 0$.

Figure 4. Trends and sensitivity of the replenishment point Q_b with collateral market price P .

It can be observed that as the collateral market price P of the collateral increases, the replenishment point Q_b exhibits a declining trend. In other words, a higher collateral market price of the collateral corresponds to a smaller required replenishment quantity, whereas a lower collateral market price necessitates a larger replenishment volume. This finding aligns with the earlier discussion in the study regarding financing enterprises' speculative operational behavior induced by fluctuations in the collateral market price. Typically, when the collateral market price rises, financing enterprises are more likely to collude with 3PL enterprises for rent-seeking and speculative activities. By replenishing, the value of the collateral can be stabilized. This also aligns with practical implications. Collateral embodies value, meaning that when a financing enterprise lacks repayment capacity, financial institutions can liquidate the collateral to obtain compensation, thereby reducing credit risk. The purpose of the replenishment point is to ensure that the value of the collateral does not fall below the credit line established under a given loan-to-value ratio. To maintain a constant collateral value, the product of the collateral market price and its quantity must remain unchanged—implying an inverse relationship between the two. Consequently, when selecting collateral, financial institutions should prioritize assets with high liquidity, broad acceptability, strong stability, and reliable value retention. This enables them to achieve the required collateral value with minimal restocking, thereby mitigating risk. Meanwhile, the sensitivity analysis shows that the sensitivity E_{Q_b} of the

replenishment points Q_b to the collateral market price P exhibits unit negative elasticity. Specifically, a 1% increase in the collateral market price leads to a 1% decrease in the re-quired collateral replenishment quantity, while a 1% decrease in the price necessitates a 1% increase in replenishment to cover the risk. This adjustment of the replenishment quantity serves to hedge against the risk of collateral market price volatility, thereby ensuring the stability of financing recovery for financial institutions. Therefore, from the perspective of financial services supporting the real economy and the development of industrial chain finance, financial institutions should establish the capability to mark-to-market the prices of pledged assets, either independently or in collaboration with relevant specialized institutions, thereby enhancing their own risk management capacity.

5.2. Incentive Benefits

Parameter settings follow Table 5; a simulation analysis was conducted by varying the incentive benefits r from 14 to 16.8 in increments of 0.5. The resulting trends of the early-warning indicators—replenishment point Q_b —is illustrated in Figure 5.

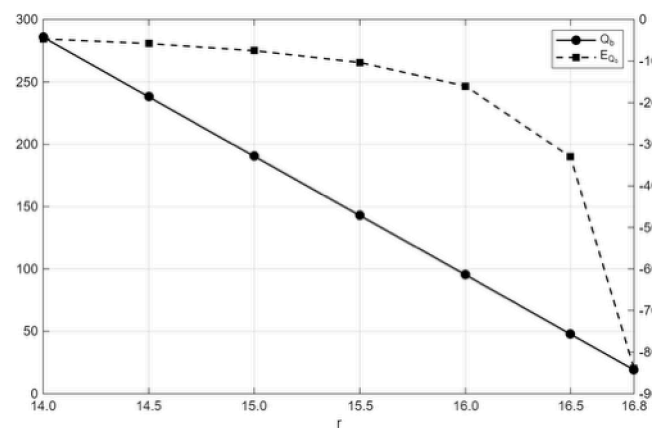


Figure 5. Trends and sensitivity of the replenishment point Q_b with incentive benefits r .

As shown in Figure 5, the incentive benefits r received by the 3PL enterprise is negatively correlated with the inventory pledge financing replenishment point Q_b . The higher the incentive benefits r received by the 3PL enterprise, the lower the replenishment point, and the smaller the quantity that the financing enterprise needs to replenish. This is also consistent with reality: the higher the incentive benefits, the stronger the 3PL enterprise's incentive to positive supervise, the weaker the incentive for collusion and rent-seeking, and given a stable collateral market price, the smaller the change in the pledged collateral's value, which in turn reduces the quantity of pledged collateral that must be replenished. Meanwhile, the sensitivity E_{Q_b} of the replenishment point Q_b to the incentive benefits r obtained by the 3PL enterprise exhibits negative elasticity. When r is low, the replenishment point Q_b is insensitive to its variation; when r exceeds 16, it becomes highly sensitive. At this point, the incentive benefits received by the 3PL enterprise are sufficiently large, and the 3PL enterprise does not engage in rent-seeking behavior but instead actively fulfills its supervising duties to ensure the value of pledged collateral, so that the quantity of additional pledged collateral required will be low. Therefore, in inventory pledge financing, financial institutions should cooperate deeply with 3PL enterprises and encourage them to supervise actively through supervisory incentives, thereby forming a virtuous cooperative cycle.

5.3. Penalties for Negative Supervision

Parameter settings follow Table 5; a simulation analysis was conducted by varying the penalties for negative supervision e from 14 to 22 in increments of 0.5. The resulting trends of the early-warning indicators—replenishment point Q_b and liquidation point V_p —are illustrated in Figure 6.

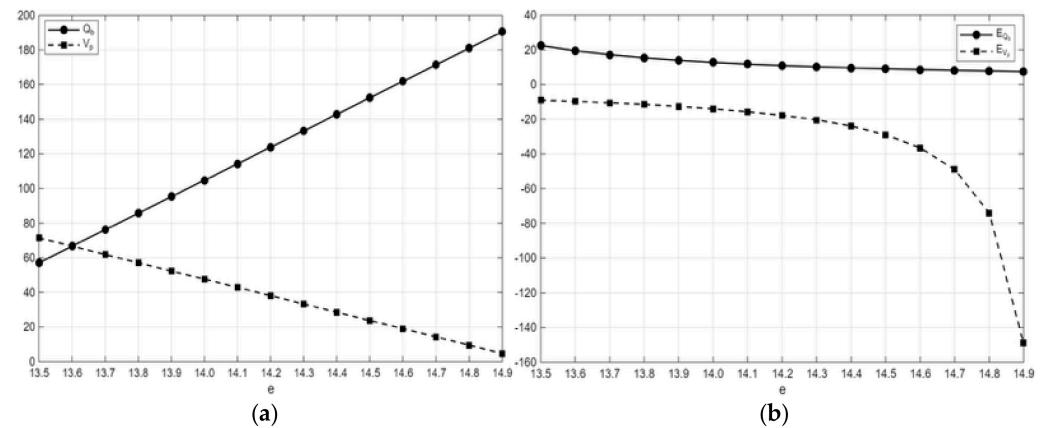


Figure 6. (a) Trends of replenishment point Q_b and liquidation point V_p with penalties for negative supervision e ; (b) sensitivity of the replenishment point Q_b and liquidation point V_p to penalties for negative supervision e .

The replenishment points Q_b increases as the penalty e for the 3PL enterprise's negative supervisory behavior rises. This indicates that a higher penalty intensity leads to greater supervisory slack and stronger incentives for collusion and rent-seeking by the 3PL enterprise. Due to lax oversight, the quality and quantity of pledged collateral cannot be reliably assured, and its market value is subject to significant fluctuations. To maintain the collateral's value and mitigate risks for financial institutions, the financing enterprise must frequently supplement a substantial amount of collateral. Conversely, the liquidation point V_p decreases as the penalty e for the 3PL enterprise's negative supervision increases, which aligns with practical risk control perspectives. A higher penalty for negative supervision indicates a greater probability of risk materializing in the collateral. Under such circumstances, setting a high liquidation point becomes unrealistic. Only a lower trigger point for liquidation can effectively prevent greater losses. Furthermore, the sensitivity of the replenishment point Q_b and the liquidation point V_p to supervisory penalties was analyzed, as shown in Figure 6b. The elasticity of the replenishment points Q_b with respect to supervisory penalties E_{Q_b} was positive and exhibited a diminishing trend. This implies that as the penalties for negative supervision increase by a certain amount, the replenishment quantity also increases by a certain amount, but the sensitivity is low. Hence, a sufficiently large penalty substantially reduces the 3PL enterprise's incentive to remain negative supervision, increases its supervisory effort, and reduces the magnitude of fluctuation in the pledged collateral value, thereby also reducing the corresponding replenishment quantity of the pledged collateral. The liquidation point V_p has a negative elasticity with respect to penalties for negative supervision; when e is low, the liquidation point V_p is insensitive to its changes, whereas it exhibits high sensitivity when $e > 14.5$. This means that as penalties for negative supervision increase, the liquidation point V_p declines more sharply.

Therefore, supervisory penalties exert a certain deterrent effect on 3PL enterprises, leading to a reduction in negative supervisory behavior and consequently lowering the thresholds for replenishment and liquidation points. However, financial institutions should establish an optimal penalty range to avoid excessive penalties, which could dampen the

willingness of 3PL enterprises to participate in inventory pledge financing and hinder its development.

5.4. Reputational Loss

Parameter settings follow Table 5; a simulation analysis was conducted by varying the reputational loss S from 2 to 3.8 in increments of 0.1. The resulting trends of the early-warning indicators—replenishment point Q_b and liquidation point V_p —are illustrated in Figure 7.

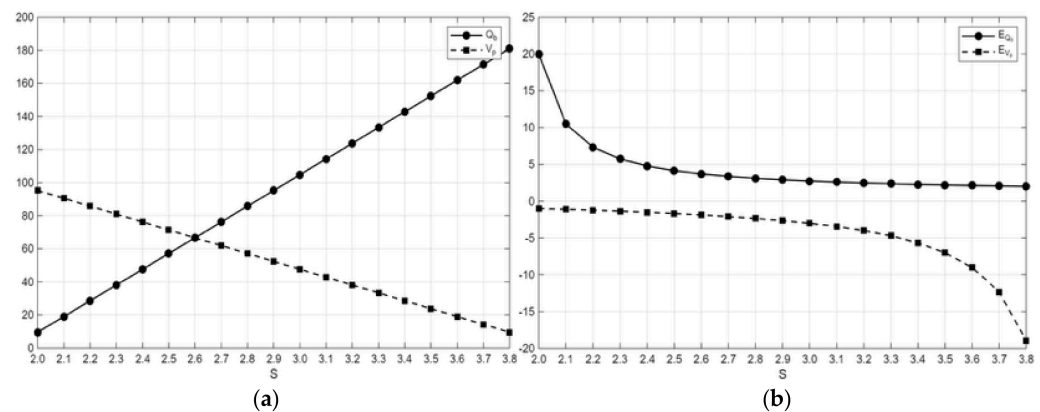


Figure 7. (a) Trends of replenishment point Q_b and liquidation point V_p with reputational loss S ; (b) sensitivity of the replenishment point Q_b and liquidation point V_p to reputational loss S .

Reputation, as a character trait of 3PL enterprises, typically exhibits a cumulative effect. Reputation loss represents a diminution of intangible assets resulting from 3PL enterprises' lax supervision. As shown in Figure 7a, the replenishment point Q_b increases as reputation loss rises. This indicates that the reduction in collateral value caused by negative supervision must be offset by increasing the quantity of pledged collaterals. Moreover, the more relaxed the supervision, the greater the value loss, and consequently, the higher the required replenishment point. The liquidation point V_p decreases as reputational losses increase. When reputational losses are minor, meaning that the negative supervisory reputation of the 3PL enterprise spreads only to a limited extent within the industry, the negative public sentiment impact on financial institutions from liquidating collateral is also limited. In such cases, the 3PL enterprise retains the motivation to restore its reputation. Consequently, it can choose a higher cargo value point for liquidation based on market trends. However, when negative supervisory reputation accumulates to a certain level, the increased reputational damage can cause inventory pledge financing business to become a focal point of public sentiment. This development impedes the liquidation of pledged collateral. Consequently, a lower liquidation threshold setting is more likely to trigger a forced sale to achieve liquidation promptly. In a financing incident in Inner Mongolia involving the misrepresentation of duck meat as beef and mutton, Inner Mongolia Green Land Agriculture and Animal Husbandry Co., Ltd. had, since 2019, used its inventory of "beef and mutton" as collateral to apply for "warehouse financing" loans from the Xilinhaote Rural Cooperative Bank. Because the supervisory company failed to exercise close monitoring over the inventory, Green Land, facing cash flow difficulties, sold part of the collateral privately. This went undetected until the loans became overdue in 2023, when it was discovered that 808.74 metric tons of the pledged goods were water-injected duck meat that had been packaged and labeled as beef and mutton, while genuine beef and mutton products accounted for roughly only 65 metric tons. The supervisory company was branded neglectful in its duties and complicit in theft, its reputation within the industry ruined, and the disposal of the pledged goods became difficult. The sensitivity of the

replenishment points Q_b and the liquidation point V_p to reputational loss is shown in Figure 7b. The elasticity of the replenishment points Q_b with respect to reputational loss E_{V_p} is positive, with diminishing marginal sensitivity. A higher reputational loss leads to a higher replenishment point. As reputational loss increases, the rate of increase in the replenishment point gradually diminishes and stabilizes. The relationship between the liquidation points V_p and the elasticity of reputational loss E_{V_p} is initially negative. When reputation losses are high, a high degree of sensitivity emerges, which corroborates the cumulative effect of reputation losses: once the losses reach a certain threshold, a low liquidation point should be set to realize returns as quickly as possible.

Therefore, on the one hand, financial institutions should consider the industry reputation of 3PL supervision enterprises when selecting them to ensure the effective implementation of inventory pledge financing. On the other hand, during the cooperation process, the magnitude of the reputation loss for the 3PL supervising enterprise can be controlled by means of the desired liquidation point, thereby enabling the liquidation behavior to be triggered or suppressed.

5.5. Supervisory Costs

Parameter settings follow Table 5; a simulation analysis was conducted by varying the 3PL enterprise's positive supervision costs C_1 from 14.5 to 15.4 in increments of 0.1 and the 3PL enterprise's negative supervision costs C_2 from 4 to 5.4 in increments of 0.1. The resulting trends of the early-warning indicators—liquidation point V_p —are illustrated in Figure 8.

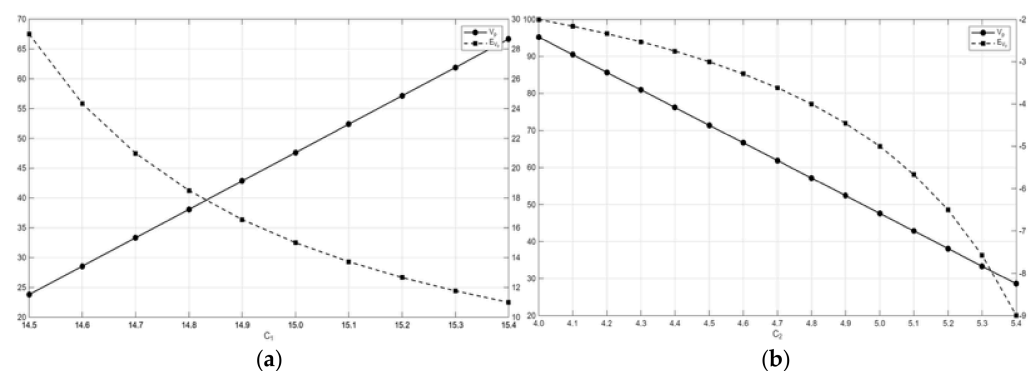


Figure 8. (a) Trends and sensitivity of liquidation point V_p with positive supervision costs C_1 ; (b) trends and sensitivity of liquidation point V_p with negative supervision costs C_2 .

As can be seen from Figure 8a, the liquidation points V_p increases with the increase in the 3PL enterprise's positive supervision costs C_1 . An increase in the 3PL enterprise's positive supervision costs C_1 implies that 3PL enterprises are more inclined to choose low-cost negative supervision. Negative supervision prevents financial institutions from timely and accurately ascertaining the actual level of the pledged collateral's value, leading to an increase in pledged collateral risk. For this reason, financial institutions, to guard against risk, need to set the liquidation points warning at a higher pledged collateral value level. Similarly, the elasticity of the liquidation points V_p with respect to the 3PL enterprise's positive supervision costs C_1 is positive and exhibits a monotonically decreasing trend, which also indicates that the higher the 3PL enterprise's positive supervision costs, the higher the liquidation point V_p . When the positive supervision cost is low, adjusting the cost can significantly alter the liquidation point, whereas when the positive supervision cost is high, the adjustment of the liquidation point is not significant. For the negative supervision costs C_2 , it can be seen from Figure 8b that the liquidation points V_p decreases as the 3PL enterprise's negative supervision costs C_2 increases. This means that when negative

supervision becomes less beneficial, the 3PL enterprise is more inclined to supervise actively. Positive supervision helps detect early signs of default by the financing enterprise, enabling the financial institution to act sooner and reduce losses. The financial institution has greater confidence in the value of the pledged collateral under high-quality supervising and can therefore set the liquidation warning at a lower pledged collateral value. At the same time, the elasticity of the liquidation points V_p with respect to the 3PL enterprise's negative supervision costs C_2 is negative, and its absolute value is increasing. This indicates that the larger the cost of negative supervision, the larger the magnitude of the decrease in the liquidation point.

Therefore, based on selecting the 3PL supervisory enterprise considering reputational loss, the financial institution should also, together with penalties for negative supervision, raise the 3PL enterprise's cost of negative supervision, thus inducing the 3PL enterprise to commit to positive supervising. At the same time, the incentive benefits for positive supervising can be raised in parallel, so that inventory pledge financing enters a virtuous cycle.

6. Conclusions and Suggestions

The lack of supervisory credit is the primary cause of defaults in inventory pledge financing. Financial institutions require supervisory credit for lending authorization, financing enterprises depend on it for credit enhancement, and 3PL enterprises need it to maintain credit stability. Starting from the necessity of reconstructing supervisory credit, this study focuses on early warning indicators for replenishment points and liquidation points in inventory pledge financing. First, an evolutionary game model involving financial institutions and 3PL supervisory enterprises is established to analyze the equilibrium points of the game in the absence of stochastic disturbances. Second, by introducing stochastic disturbance terms, a stochastic cusp catastrophe model is constructed using probability density functions and potential functions. This allows for analysis of the catastrophe bifurcation set and the depiction of the mutation path of 3PL enterprises' supervisory behavior. Third, the expression for the replenishment point is derived by calculating the difference in collateral value at the catastrophe point and the collateral value at the game equilibrium. Next, the study extends the mutation model by adding a replenishment point, from which we then derive the liquidation point. Finally, through simulation analysis and real-world case studies, the study examines the impacts of the collateral market price, the incentive benefits obtained by the 3PL enterprise, the penalties for negative supervision imposed on the 3PL enterprise, and the reputational losses incurred by the 3PL enterprise on the replenishment point and the liquidation point. The paper derives the following conclusions and insights.

(1) When external factors such as fluctuations in the collateral market price and reputational losses for 3PL enterprises are disregarded, the optimal equilibrium in the game between the two parties is reached when the 3PL enterprise chooses to positively supervise, the financial institution opts not to provide incentives, and instead relies on penalty deterrence, market discipline, and reputation mechanisms to ensure that the 3PL enterprise implements active supervision activities. However, in practice, this equilibrium is easily disrupted. Since August 2023, a series of defaults by steel trading firms in Sichuan, Jiangsu, Zhejiang, and Guizhou have occurred, many involving supervisory risks such as repeated pledging. Therefore, reconstructing supervisory credibility and strengthening the application of technologies such as blockchain and the IoT in the monitoring process, enabling full visibility and traceability throughout the inventory-based financing cycle, are critical considerations for maintaining the strategic equilibrium between the two parties.

(2) During the supervisory evolution process, whether a supervisory shift occurs abruptly is determined by the bifurcation point in a stochastic cusp catastrophe model, which in turn influences the setting of replenishment and liquidation thresholds. By calculating the difference between the bifurcation point and the equilibrium value of inventory, replenishment thresholds can be established to effectively prevent insufficient collateral value when abrupt changes occur, thereby reducing risks for financial institutions. Likewise, once a supervisory shift has materially taken place and the collateral value has significantly declined, the liquidation point is determined by the actual loss in pledged collateral value. This approach allows financial institutions to implement flexible liquidation measures and mitigate realization risks. Therefore, in inventory-secured financing operations, early-warning indicators for replenishment and liquidation thresholds should be established. On the one hand, it enables the quantification of 3PL enterprise supervisory behavior; on the other hand, it provides actionable entry points for the implementation of emerging technologies such as blockchain, the IoT, and AI. Through the technology-driven dynamic monitoring of key indicators, early warnings can be triggered and automatically executed, thereby reconstructing supervisory credibility.

(3) The 3PL enterprise's negative supervision penalty and the 3PL enterprise's reputational loss are positively correlated with the replenishment point and negatively correlated with the liquidation point. The collateral market price and the incentive benefits are negatively correlated with the replenishment point in inventory pledge financing, whereas the supervisory cost of the 3PL enterprise is both positively and negatively correlated with the liquidation point depending on its source. Therefore, when selecting collateral for inventory pledge financing, financial institutions should, to the extent possible, choose pledged collateral that are highly liquid, widely accepted, stable in quality, and strong in value retention. When selecting a 3PL enterprise for supervising, they should consider the enterprise's reputation in the industry, engage in deep cooperation, and encourage positive supervision through supervisory incentives, thereby fostering a virtuous cycle of collaboration. At the same time, financial institutions should combine penalties for negative supervision to raise the cost of negative supervision for 3PL enterprises, so that 3PL enterprises commit to supervise positively; they should also complement this by increasing the incentive benefits for positive supervision, thereby promoting the healthy development of inventory pledge financing.

Furthermore, although this study has yielded some meaningful conclusions, it can be further extended and refined in two aspects. On the one hand, potential interactions among parameters may influence the simulation outcomes. Subsequent research could conduct simulation analyses based on variations in two or three factors to assess the robustness of the results. On the other hand, future work could enhance the research through methods such as interviews or questionnaire surveys.

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Abbreviations

The following abbreviations are used in this manuscript:

3PL	Third-Party Logistics
IoT	Internet of Things
SMEs	Small and Medium-Sized Enterprises
VaR	Value-at-Risk

Appendix A

1. Robustness test based on $x = y$:

To examine the robustness of the dynamic path symmetry condition $x = y$, this study introduces a perturbation term δ , sets $x = y + \delta$, $|\delta| \ll 1$, and performs a linear stability analysis of the two-dimensional replicator dynamic system at the equilibrium point $(1, 0)$. According to Equation (10), at equilibrium point $(1, 0)$, since $J = \begin{bmatrix} -A & 0 \\ 0 & -r \end{bmatrix}$, $A = (1 - k)R_1 + C_2 - C_1 - hG + e + S > 0$, $r > 0$ and $\text{Det}(J) = Ar > 0$, $\text{Tr}(J) = -A - r < 0$, making equilibrium point $(1, 0)$ stable. We introduce the $x = y + \delta$ test for stability. When $y = 0$, $x = \delta$, Equation (9) yields $F(y) = \frac{dy}{dt} = y(1 - y)[-e + x(e - r)] = 0$; regardless of the value x takes, $y = 0$ is always a stable point of $F(y)$, and y converges toward 0. When $x = 1$, $y = 1 - \delta$, Equation (5) yields $F(x) = \frac{dx}{dt} = x(1 - x)[(1 - k)R_1 + C_2 - C_1 - hG + S + e + y(r - e - S)] = 0$; regardless of the value y takes, $x = 1$ is always a stable point of $F(x)$, and x converges toward.

2. The derivation process for $g^3 + ug + v = 0$ is as follows:

Let $a(x) = \frac{\partial V(x, g)}{\partial x}$, then Equation (16) can be expressed as $dx(t) = a(x)dt + \varepsilon(x)dw(t)$. As $t \rightarrow \infty$, $f(\mu, t, x_0) \rightarrow f^*$, it follows that $\frac{d}{dx}[a(x)f^*(x)] - \frac{1}{2}\frac{d^2}{dx^2}[\varepsilon^2(x)f^*(x)] = 0$. Substituting $a(x)$ yields $\frac{d}{dx}\left(\frac{\partial V(x, g)}{\partial x}f^*\right) - \frac{1}{2}\frac{d^2}{dx^2}[\varepsilon^2(x)f^*] = 0$. Integrate with respect to x once to obtain $-\frac{\partial V(x, g)}{\partial x}f^*(x) - \frac{1}{2}\frac{d}{dx}(\varepsilon^2(x)f^*(x)) = 0$, express in logarithmic derivative form $\frac{1}{f^*(x)}\frac{df^*(x)}{dx} = -2 * \frac{\frac{\partial V(x, g)}{\partial x} - \frac{1}{2}[\varepsilon^2(x)]'}{\varepsilon^2(x)}$, regarding the integration of x from the constant c to x , $\ln f^*(x) = -2 \int_c^x \frac{\frac{\partial V(w, g)}{\partial w} - \frac{1}{2}[\varepsilon^2(w)]'}{\varepsilon^2(w)} dw + \ln N_c$, where N_c is the normalization constant. Combined with the exponential form of the limiting probability density in Equation (17), it follows that $V_{sto}(x) = -2 \int_c^x \left\{ \frac{\frac{\partial V(w, g)}{\partial w} - \frac{1}{2}[\varepsilon^2(w)]'}{\varepsilon^2(w)} \right\} dw$.

Differentiating Equation (18), it follows that $V'_{sto}(x) = -2 * \frac{\frac{\partial V(x, g)}{\partial x} - \frac{1}{2}[\varepsilon^2(x)]'}{\varepsilon^2(x)} = 0$, it follows that $-\frac{\partial V(x, g)}{\partial x} - \frac{1}{2}[\varepsilon^2(x)]' = 0$. Combining Equation (15), its derivative is $\frac{\partial V(x, g)}{\partial x} = \alpha x^3 + \beta x^2 + \gamma x$, Substituting into the previous equation yields Equation (19), namely $\alpha x^3 + \beta x^2 + \gamma x = 0$. When $\alpha \neq 0$, by performing a variable substitution on Equation (19) to eliminate the quadratic term, it is transformed into the standard form of a cusp catastrophe, $x^3 + \frac{\beta}{\alpha}x^2 + \frac{\gamma}{\alpha}x = 0$, when $x = g - \frac{\beta}{3\alpha}$, it follows that $\left(g - \frac{\beta}{3\alpha}\right)^3 + \frac{\beta}{\alpha}\left(g - \frac{\beta}{3\alpha}\right)^2 + \frac{\gamma}{\alpha}\left(g - \frac{\beta}{3\alpha}\right) = 0$. Expanding yields $g^3 + \left(\frac{\gamma}{\alpha} - \frac{\beta^2}{3\alpha^2}\right)g + \left(\frac{27\beta^3}{27\alpha^3} - \frac{\beta\gamma}{3\alpha^2}\right) = 0$, substitute $u = \frac{\gamma}{\alpha} - \frac{\beta^2}{3\alpha^2}$, $v = \frac{2\beta^3}{27\alpha^3} - \frac{\beta\gamma}{3\alpha^2}$, then Equation (20) is obtained as $g^3 + ug + v = 0$.

Appendix B

Case reference

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