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Nonlocal Low-Rank Residual Modeling for Hyperspectral Image Mixed Noise Removal

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Abstract

Hyperspectral image (HSI) denoising remains a critical challenge due to noise corruption during acquisition. While nonlocal low-rank (LR) tensor methods leverage spatial-spectral correlations, they usually fail under heavy or complex noise, as directly estimating LR tensors from noisy observations usually leads to residual noise accumulation. To address this limitation, we propose a novel nonlocal low-rank residual (NLRR) approach, which reformulates LR tensor recovery as a progressive residual minimization problem. Unlike conventional methods that exclusively approximate LR tensors directly from degraded observations, the proposed NLRR approach iteratively refines the latent LR tensor structure by minimizing the rank residual, thereby decoupling noise suppression from tensor approximation. This residual-driven framework uniquely integrates two complementary priors: (1) a nonlocal LR residual prior that exploits spatial self-similarity, and (2) a global spectral LR prior that suppresses spectral redundancy. To generalize the proposed NLRR approach to real-world scenarios with mixed noise, we develop the NLRR-robust principal component analysis (NLRR-RPCA) framework, which incorporates the LR residual along with global spectral LR and sparse tensor priors for mixed noise removal. Additionally, to ensure both numerical stability and computational tractability, we develop an adaptive rank-adjusted alternating minimization algorithm, which dynamically adjusts the ranks of the estimated tensors to better handle different noise scenarios. Extensive experiments on both simulated and real HSI datasets demonstrate that our proposed NLRR approach outperforms numerous popular or state-of-the-art methods in both quantitative evaluation and visual perception.

Keywords: hyperspectral image denoising; nonlocal low-rank residual; nonlocal self-similarity; robust principal component analysis; adaptive rank-adjusted alternating minimization



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1. Introduction

Hyperspectral image (HSI) provides rich spectral information, enabling fine-grained representation and analysis of real-world scenes. However, the unique imaging mechanism of HSIs makes them susceptible to various types of noise, including Gaussian noise, salt-and-pepper noise, stripes, and deadline noise, which significantly deteriorate their quality and limit their practical applications [1–3]. Consequently, developing robust and effective HSI denoising algorithms is essential for ensuring the reliability of subsequent analysis

tasks, such as object detection [4,5], anomaly detection [6,7], medical diagnostics [8,9], and pollution monitoring [10,11].

HSI denoising has been extensively studied using various techniques, including band-wise image denoising [12–15], low-rank (LR) matrix approximation [16,17], tensor decomposition [18–22], nonlocal LR tensor modeling [23–25], and deep learning methods [26–30]. Among these approaches, nonlocal LR tensor modeling has recently shown promising results [23–25,31]. This method leverages the nonlocal self-similarity (NSS) prior [32] to cluster similar full-band patches into three-dimensional nonlocal full-band groups (tensors), followed by enforcing an LR constraint on each nonlocal full-band group to enhance denoising performance. However, most existing nonlocal LR tensor modeling approaches suffer from two critical limitations:

- **Pitfalls of Direct LR Approximation:** Conventional LR tensor approximation methods directly estimate the latent LR tensor structure from degraded HSIs, which often leads to suboptimal estimation, especially in the presence of heavy noise. As a result, residual noise or excessive detail loss occurs.
- **Inability to Handle Mixed Noise Robustly:** While LR tensor modeling is effective for Gaussian noise removal, its direct application to mixed noise scenarios is challenging due to the presence of sparse outliers (e.g., salt-and-pepper noise, stripes, and deadline noise).

To address these limitations, we propose a novel nonlocal low-rank residual (NLRR) approach, which significantly enhances the robustness of HSI denoising by introducing an innovative residual minimization strategy. The main contributions of this paper are summarized as follows:

- (1) Unlike conventional methods that directly approximate LR tensors, we progressively refine the underlying LR tensor structure by minimizing the rank residual. This novel approach reduces LR estimation errors and enhances robustness to heavy noise. This residual-driven framework integrates two complementary priors: a nonlocal LR residual prior leveraging spatial self-similarity and a global spectral LR prior that reduces spectral redundancy. Figure 1 provides an overview of the proposed NLRR approach for HSI denoising.
- (2) To demonstrate the applicability of our proposed NLRR approach, we generalize NLRR to the NLRR-robust principal component analysis (NLRR-RPCA) framework, which integrates LR residual in conjunction with global spectral LR and sparse tensor priors to effectively remove mixed noise, making it more applicable to real-world noisy HSI data.
- (3) To ensure efficient and stable optimization, we design an alternating minimization algorithm that dynamically adjusts the rank during the iterative refinement process, improving both performance and computational efficiency.
- (4) We conduct extensive experiments on both simulated and real HSI data, demonstrating that our proposed NLRR algorithm outperforms many popular or state-of-the-art methods in terms of both quantitative and visual performance metrics.

The remainder of this paper is organized as follows. Section 2 provides a brief overview of previous HSI denoising studies. Section 3 introduces the necessary notations and some preliminaries. Section 4 presents the proposed NLRR model with its application to HSI denoising and develops effective algorithms to address HSI denoising challenges. Experimental results are presented in Section 5, while Section 6 contains concluding remarks.

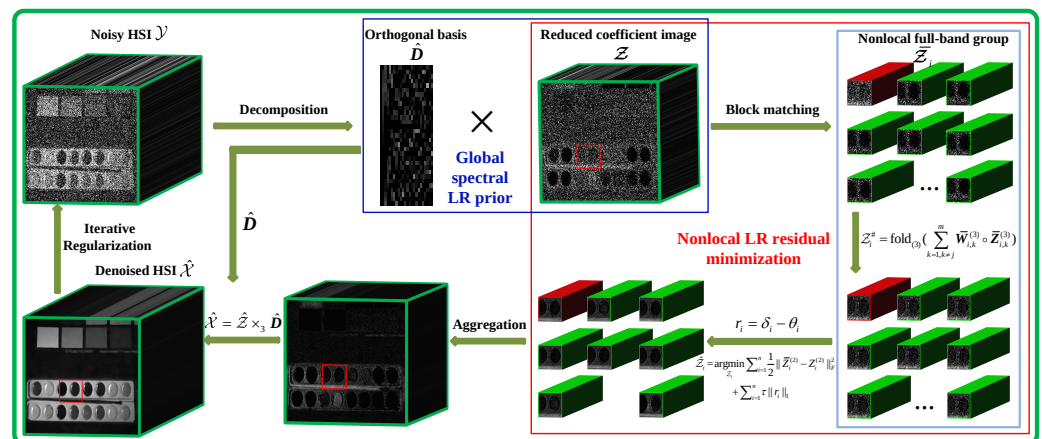


Figure 1. Flowchart depicting the proposed NLRR model for HSI denoising. The proposed NLRR algorithm incorporates two complementary priors: the global spectral prior and nonlocal LR residual prior, harnessing the global LR and spatial NSS properties of HSI.

2. Related Work

Numerous studies have sought to address mixed noise (e.g., both Gaussian noise and sparse noise) by modeling the noise with sophisticated non-independence and identical distribution (non-i.i.d.) statistical structures. It is important to note that the mixed noise scene closely resembles the realistic noise scene in hyperspectral imaging [25,32,33]. Leveraging the assumption that HSI has LR properties due to the high correlation between bands, Zhang et al. [16] were the first to apply the LR matrix recovery (LRMR) method to remove mixed noise. Chen et al. [17] developed a non-convex LR matrix approximation model for HSI denoising. Another intriguing method that utilizes the LR property of HSI is LR matrix factorization (LRMF) for mixed noise removal, which applies to several flexible noise distribution hypotheses, such as a mixture of Exponential Power (MoEP) distribution [34] and a non-i.i.d. mix of Gaussians (NMoGs) noise distribution [35]. Given that an HSI can be viewed as a third-order tensor, various tensor decomposition/tensor rank minimization methods, such as Tucker [36], CANDECOMP / PARAFAC (CP) [37], and tensor singular value decomposition (t-SVD) [38], have been employed to leverage the global LR property of HSI for mixed noise removal [18–22]. For example, Renard et al. [18] proposed a LR tensor approximation (LRTA) model that utilizes Tucker decomposition for HSI denoising. Zheng et al. [20] proposed a low-fibered-rank regularization model for removing mixed noise. Liu et al. [21] developed a fibered rank constrained tensor restoration (FRCTR) framework for removing mixed noise. However, the above-mentioned methods often fail to consider the important spatial NSS property of HSI [32], which plays a crucial role in preserving structural details during the denoising process. In response to this, several recent studies have incorporated NSS priors to address mixed noise removal [25,39–42]. For example, He et al. [40] developed a tensor subspace LR learning method incorporating a nonlocal prior for eliminating mixed noise from HSI. Sun et al. [41] proposed a tensor cascaded-rank minimization model for HSI denoising. Su et al. [25] proposed a weighted multidirectional low-rank tensor recovery (WMLTR) method for effectively removing mixed noise from HSI. However, these nonlocal-based methods struggle to distinguish between real image structures and structured noise, leading to residual noise or excessive detail loss. Recently, Chang et al. [43] leveraged deep learning techniques to extend the gray-scale image denoising architecture, denoising convolutional neural networks (DnCNNs) [44], into an HSI-DeNet mixed denoising method that preserves spectral–spatial correlation. Since then, various deep learning-based methods [28–31,45–49] have been proposed for removing mixed noise, such as 3D quasi-recurrent neural network (QRNN3D) [28], trainable spectral–

spatial sparse coding (T3SC) [48], and deep unfolding transformer (DUT) [31]. However, mixed noise exhibits a high degree of variability, and deep learning models trained on specific noise patterns often struggle to generalize to unseen noise conditions.

3. Notations and Preliminaries

3.1. Notations

In this paper, we adhere to the tensor notation defined in [50], where a tensor, matrix, vector, and scalar are denoted by a calligraphic letter (e.g., $\mathcal{X}$), a boldface capital letter (e.g., $\mathbf{X}$), a bold lowercase letter (e.g., $\mathbf{x}$), and a lowercase letter (e.g., x), respectively. We represent an N -dimensional tensor as $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \dots \times I_N}$, with each element $(i_1, i_2, \dots, i_N)$ of $\mathcal{X}$ defined as $x_{i_1 \times i_2 \times \dots \times i_N}$. Each element (i, j) of the matrix $\mathbf{X}$ is denoted as $x_{i,j}$, and the i -th entry of the vector $\mathbf{x}$ is denoted as x_i . $\mathbf{X}^T$ and $\mathcal{X}^T$ are the transpose of the matrix $\mathbf{X}$ and the tensor $\mathcal{X}$, respectively. The Frobenius norm of tensor $\mathcal{X}$ is defined as $\|\mathcal{X}\|_F = \sqrt{\sum_{i_1=1}^{I_1} \sum_{i_2=1}^{I_2} \dots \sum_{i_N=1}^{I_N} x_{i_1 \times i_2 \times \dots \times i_N}^2}$, which equals the square root of the sum of the squares of its elements. We represent the model- n unfolding operation of the tensor $\mathcal{X}$ as $\mathbf{X}^{(n)} \in \mathbb{R}^{I_n \times (I_1 \dots I_{n-1} I_{n+1} \dots I_N)}$. Conversely, the unfolding matrix $\mathbf{X}$ along the n -mode can be folded back into a tensor $\mathcal{X} = \text{fold}_{(n)}(\mathbf{X})$. Additionally, the mode- n multiplication of the tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \dots \times I_N}$ and matrix $\mathbf{Y} \in \mathbb{R}^{I_n \times K}$ is represented by $\mathcal{Z} = \mathcal{X} \times_n \mathbf{Y}$, where $\mathcal{Z} \in \mathbb{R}^{I_1 \times I_2 \times \dots \times K}$, and $\mathcal{X} \times_n \mathbf{Y} = \text{fold}_{(n)}(\mathbf{Y}\mathbf{X}^{(n)})$.

3.2. Nonlocal LR Modeling

The nonlocal LR model assembles similar nonlocal patches into groups and harnesses the LR property of each group, exhibiting significant potential across various image restoration applications [14,24,25,51]. Specifically, an image $\mathbf{x} \in \mathbb{R}^N$ is initially divided into n overlapped patches $\{\mathbf{x}_i\}_{i=1}^n \in \mathbb{R}^{b \times 1}$. Subsequently, with each $\mathbf{x}_i$ as the exemplar patch, the m closest patches to $\mathbf{x}_i$ are gathered to form a group, denoted as $\mathbf{R}_i \mathbf{x} = \mathbf{X}_i = \{\mathbf{x}_{i,1}, \dots, \mathbf{x}_{i,m}\}$. Each $\mathbf{R}_i : \mathbf{x} \mapsto \mathbf{R}_i \mathbf{x} \in \mathbb{R}^{b \times m}$ represents a K-Nearest Neighbor (KNN) operator [52], which collects the m closest patches (determined by Euclidean distance) to the i -th patch $\mathbf{x}_i$. Since all patches in each group $\mathbf{X}_i$ possess similar structures, the resulting data matrix exhibits the LR property. Hence, the LR matrix $\mathbf{L}_i$ can be determined from each group $\mathbf{X}_i$ by addressing the following minimization problem,

$$\hat{\mathbf{L}}_i = \arg \min_{\mathbf{L}_i} \frac{1}{2} \|\mathbf{X}_i - \mathbf{L}_i\|_F^2 + \lambda P(\mathbf{L}_i) \quad \forall i, \quad (1)$$

where $P(\mathbf{L}_i)$ is the regularizer encapsulating the LR property of $\mathbf{L}_i$, with λ being a positive constant. Popular choices for the regularizer $P(\mathbf{L}_i)$ include nuclear norm [53], truncated nuclear norm [54], and weighted nuclear norm [14].

Moreover, unlike gray-scale images, many images contain multi-channel information. As such, nonlocal LR tensor modeling has been widely employed in high-dimensional image restoration tasks, with HSI serving as a prime example [23–25]. To be specific, consider an HSI $\mathcal{X} \in \mathbb{R}^{H \times W \times B}$ that is divided into n overlapping full-band patches, denoted as $\mathcal{X}_{i,fbp} \in \mathbb{R}^{\sqrt{d} \times \sqrt{d} \times B}$, $i = 1, \dots, n$. Each $\mathcal{X}_{i,fbp}$ serves as an exemplar full-band patch, and then the KNN algorithm [52] is used to identify the m most similar full-band patches to $\mathcal{X}_{i,fbp}$. These selected m full-band patches are sorted in descending order based on their Euclidean distance from the exemplar full-band patch $\mathcal{X}_{i,fbp}$, forming the columns of the data tensor $\mathcal{R}_i \mathcal{X} \in \mathbb{R}^{d \times m \times B}$, i.e., $\mathcal{R}_i \mathcal{X} = \mathbf{X}_i = \{\mathbf{X}_{i,1}^{(3)}, \dots, \mathbf{X}_{i,m}^{(3)}\}$, where $\mathcal{R}_i$ represents the operator that extracts the data tensor $\mathcal{X}_i$ from $\mathcal{X}$, and $\mathbf{X}_{i,m}^{(3)} \in \mathbb{R}^{B \times d}$ denotes the m -th similar full-band patch of the i -th data tensor $\mathcal{X}_i$. As a result, each data tensor, $\mathcal{X}_i$, recognized as a nonlocal full-band group, consists of structurally similar patches and

exhibits the LR property. Typically, the LR tensor method integrates the NSS prior to effectively address the associated optimization problem,

$$\hat{\mathcal{L}}_i = \arg \min_{\mathcal{L}_i} \frac{1}{2} \|\mathcal{X}_i - \mathcal{L}_i\|_F^2 + \lambda \mathcal{P}(\mathcal{L}_i) \quad \forall i, \quad (2)$$

where $\mathcal{P}(\mathcal{L}_i)$ signifies the regularizer employed to represent the LR property of the tensor $\mathcal{L}_i$. The estimation of tensor rank is generally performed by assuming the rank along each mode, denoted as $\mathcal{P}(\mathcal{L}_i) = \sum_{k=1}^3 \text{rank}(\text{Unfold}_{(k)}(\mathcal{L}_i))$ [20], where k is the k -mode of the tensor $\mathcal{L}_i$. However, these approximations, which often assign an equal rank to each subspace, lack necessary physical justification, which in turn limits their applicability in real-world scenarios [23–25].

3.3. Global Spectral LR Prior

It is widely acknowledged that different bands of HSI exhibit strong correlations, resulting in the LR property of HSI in the spectral direction. This implies that the intrinsic dimension of the spectral subspace is smaller than the spectral dimension of the original HSI subspace [55]. Inspired by the subspace LR representation [56], we decompose a clean HSI $\mathcal{X} \in \mathbb{R}^{H \times W \times B}$ into $\mathcal{X} = \mathcal{Z} \times_3 \mathbf{D}$. In this context, $\mathbf{D} \in \mathbb{R}^{B \times M}$ ($M \ll B$) is an orthogonal basis matrix used to encapsulate the common subspace across all spectral bands. $\mathcal{Z}$ signifies the reduced coefficient image (RCI) with dimensions $H \times W \times M$. By employing the HySime algorithm [57] or SVD, the orthogonal basis matrix $\mathbf{D}$ can be initially learned from the degraded HSI $\mathcal{Y} \in \mathbb{R}^{H \times W \times B}$. This enables the recovery of the latent clean HSI $\mathcal{X}$ by leveraging prior knowledge of the RCI $\mathcal{Z}$. Therefore, the global spectral LR prior-based regularization model can be represented as

$$\begin{aligned} \{\hat{\mathcal{Z}}, \hat{\mathbf{D}}\} &= \arg \min_{\mathcal{Z}, \mathbf{D}} \frac{1}{2} \|\mathcal{Y} - \mathcal{Z} \times_3 \mathbf{D}\|_F^2 + \lambda \mathbf{R}(\mathcal{Z}), \\ \text{s.t.} \quad &\mathbf{D}^T \mathbf{D} = \mathbf{I}, \end{aligned} \quad (3)$$

where $\mathbf{R}(\mathcal{Z})$ is a regularizer that is based on the prior information of the RCI $\mathcal{Z}$, and $\mathbf{I}$ is an identity matrix. Upon determining the solutions for $\hat{\mathbf{D}}$ and $\hat{\mathcal{Z}}$ by resolving Equation (3), the latent clean HSI $\hat{\mathcal{X}}$ can be reassembled through $\hat{\mathcal{Z}} \times_3 \hat{\mathbf{D}}$.

4. HSI Denoising via NLRR Modeling

4.1. Modeling of NLRR

As previously mentioned, although existing nonlocal tensor modeling methods have shown promising capabilities in HSI denoising [23–25,58–63], they depend exclusively on approximating LR tensors directly from degraded nonlocal full-band groups, making them susceptible to challenges, especially in heavy or complex noise environments. As a result, they may generate suboptimal LR tensor approximations, leading to unsatisfactory denoising results. Furthermore, while expanding the spectral bands of an HSI can improve denoising performance, it also significantly increases computation when handling nonlocal full-band groups using tensor decomposition or tensor rank minimization methods [36–38]. To address these issues, we propose a novel NLRR approach for highly effective HSI denoising in this paper. The specifics of this method will be elaborated in the subsequent discourse.

Nonlocal LR Residual Minimization

In light of the above considerations, and against the findings about weak LR properties in both spectral and local modes of the nonlocal full-band group (a nonlocal full-band

group is a three-order tensor, which is typically represented by the local spatial domain in mode-1, nonlocal domain in mode-2 (nonlocal mode) and spectral domain in mode-3 [23], we propose a single-mode LR tensor recovery approach to explore the LR property of each nonlocal full-band group at the nonlocal mode via the LR residual minimization. Specifically, given a degraded HSI $\mathcal{Y} \in \mathbb{R}^{H \times W \times B}$ and utilizing the group construction operation in Section 3.2, we construct n degraded nonlocal full-band groups $\{\mathcal{Y}_i\}_{i=1}^n \in \mathbb{R}^{d \times m \times B}$, where $\mathcal{Y}_i = \{Y_{i,1}^{(3)}, \dots, Y_{i,m}^{(3)}\}$, and $Y_{i,m}^{(3)} \in \mathbb{R}^{B \times d}$ is the m -th similar nonlocal full-band patch of the i -th nonlocal full-band group $\mathcal{Y}_i$. For each nonlocal full-band group $\mathcal{Y}_i$, it is first unfolded into a matrix $Y_i^{(2)} \in \mathbb{R}^{m \times b}$ at the nonlocal mode, where $b = d \times B$. Considering that each unfolded matrix $Y_i^{(2)}$ resides within a low-dimensional subspace [24,25], LR matrix approximation methods, such as nuclear norm minimization (NNM) [53], truncated nuclear norm minimization (TNNM) [54], and WNNM [14], have proven effective in estimating the LR matrix. However, these methods depend exclusively on approximating LR matrices directly from observations of the degraded nonlocal full-band group. This dependence may render them vulnerable to certain challenges, such as heavy or complex noise. Thus, the singular values of the estimated LR matrix using conventional LR matrix approximation methods might deviate from their true values (refer to Figure 2 for an example), leading to suboptimal results in real applications.

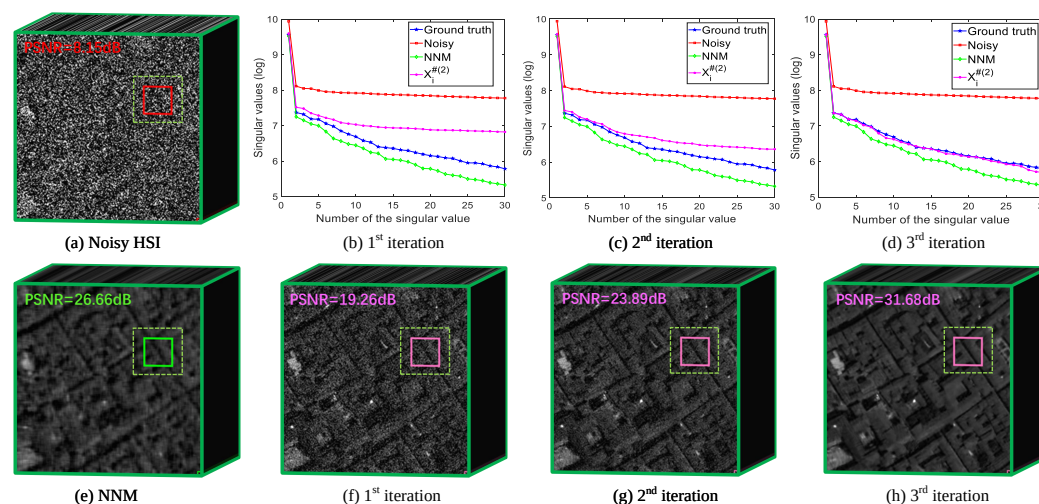


Figure 2. Demonstration of the proposed LR residual minimization model for HSI denoising. We utilize the Pavia City Center (PAC) dataset, which is corrupted by zero-mean white Gaussian additive noise with a standard deviation $\sigma_n = 100$. In subfigure (a), we construct the nonlocal full-band group using an exemplar full-band patch (highlighted by the small red square). Subfigures (b–d) depict the singular values of the constructed nonlocal full-band group unfolded at the nonlocal mode. The values are taken from the ground truth (blue line), the noisy HSI (red line), the HSI recovered by NNM [53] (green line), and the proposed reference matrix $X_i^{(2)}$ (pink line) at the 1st, 2nd, and 3rd iterations of our algorithm. Subfigures (b–d) displays the denoising results obtained by NNM. Subfigures (f–h) display the reconstructed HSIs at the 1st, 2nd, and 3rd iterations using our proposed LR residual minimization model. As visible, the singular values of the reference matrix $X_i^{(2)}$ progressively approximate the ground truth, and the reconstructed HSI is getting close to the original HSI.

To alleviate this issue, we propose an LR residual minimization model for estimating the LR matrix. In LR matrix approximation, due to degradation, we hope that the singular values of the recovered LR matrix $X_i^{(2)} \in \mathbb{R}^{m \times b}$ are as close as possible to the singular values of the original matrix $X_i^{(2)} \in \mathbb{R}^{m \times b}$. Therefore, we define the *rank residual* as

$$r_i^* \stackrel{\text{def}}{=} \delta_i - \theta_i^*, \quad (4)$$

where δ_i and θ_i^* denote the singular values of $X_i^{(2)}$ and $X_i^{*(2)}$, respectively. It is evident that the rank estimation of the matrix $X_i^{(2)}$ depends greatly on the level of the rank residual r_i^* . However, in HSI denoising, the original HSI $\mathcal{X}$ is unknown, making the original matrix $X_i^{*(2)}$ unavailable. As a result, we aspire to attain a reliable estimate of $X_i^{*(2)}$, denoted as $X_i^{\#(2)} \in \mathbb{R}^{m \times b}$.

- **Estimation of the unknown Reference Matrix $X_i^{\#(2)}$:** Inspired by the observation that an HSI usually consists of repetitive structures (nonlocal redundancies) [23–25,61], we utilize nonlocal image statistics to estimate $X_i^{\#(2)}$, which is the unfolding form of the reference tensor $\mathcal{X}_i^{\#} \in \mathbb{R}^{d \times m \times B}$ at the nonlocal mode. To be concrete, for a given degraded nonlocal full-band group $\mathcal{Y}_i = \{Y_{i,1}^{(3)}, \dots, Y_{i,m}^{(3)}\}$, each reference patch $X_{i,j}^{\#(3)}$ in $\mathcal{X}_i^{\#}$ can be computed as a weighted average of patches $\{Y_{i,k}^{(3)}\}$ in $\mathcal{Y}_i$. This can be expressed as

$$X_{i,j}^{\#(3)} = \sum_{k=1, k \neq j}^m W_{i,k}^{(3)} \circ Y_{i,k}^{(3)}, \quad (5)$$

where $\circ$ is the Hadamard product [64], and $X_{i,j}^{\#(3)}$ and $Y_{i,k}^{(3)}$ are the j -th and k -th patch of $\mathcal{X}_i^{\#}$ and $\mathcal{Y}_i$, respectively. Following the principles of nonlocal means filtering [12], we define the weighted matrix $W_{i,k}^{(3)}$ to be inversely proportional to the distance between the exemplar patch $Y_{i,fbp}^{(3)}$ and the similar patch $Y_{i,k}^{(3)}$. This can be formulated as

$$W_{i,k}^{(3)} = \frac{1}{W} \exp(-\|Y_{i,fbp}^{(3)} - Y_{i,k}^{(3)}\|_F^2/h), \quad (6)$$

where $Y_{i,fbp}^{(3)}$ represents the unfolding form of the exemplar full-band $\mathcal{Y}_{i,fbp} \in \mathbb{R}^{\sqrt{d} \times \sqrt{d} \times B}$ at the spectral mode. W is a normalization factor, and h is a predefined constant [12]. By applying Equation (5), we can derive the reference tensor $\mathcal{X}_i^{\#} = \{X_{i,1}^{\#(3)}, \dots, X_{i,m}^{\#(3)}\}$, and subsequently obtain $X_i^{\#(2)}$ by unfolding the reference tensor $\mathcal{X}_i^{\#}$ at the nonlocal mode. Notably, each reference tensor $\mathcal{X}_i^{\#}$, along with its corresponding degraded nonlocal full-band group $\mathcal{Y}_i$ (when the iteration number $t > 1$, the corresponding degraded nonlocal full-band group $\mathcal{Y}_i$ is transformed into the recovered LR tensor $\hat{\mathcal{X}}_i^t$), undergoes a gradual and mutually inclusive update process with each iteration.

By introducing the reference matrix $X_i^{\#(2)}$ and defining the rank residual, where $r_i \stackrel{\text{def}}{=} \delta_i - \theta_i$ (θ_i denotes the singular values of $X_i^{\#(2)}$), we can formulate the proposed nonlocal LR residual minimization model as follows:

$$\hat{X}_i^{(2)} = \arg \min_{X_i^{(2)}} \frac{1}{2} \|Y_i^{(2)} - X_i^{(2)}\|_F^2 + \lambda \|r_i\|_p \quad \forall i, \quad (7)$$

where $\|\cdot\|_p$ represents the p -norm, which is usually set to either 1 or 2.

- **Determination of the Value of p :** One significant concern related to the proposed nonlocal LR residual minimization model in Equation (7) is the determination of the value for p . To investigate this, we conduct several experiments to explore the statistical properties of the rank residual r_i . We use the original HSI to construct the reference matrices $\{X_i^{\#(2)}\}_{i=1}^n$. In these experiments, we utilize the Washington DC Mall (WDC) dataset to simulate the generation of the noisy HSI by adding zero-mean white Gaussian additive noise with standard deviation $\sigma_n = 30$ and mixed noise (scenario 4 in Section 5.1), respectively. Figure 3 presents the fitting results of the empirical distribution of $\{r_i\}_{i=1}^n$ under these two different noise scenarios. We can observe that the empirical distribution (represented by the pink dashed line) can be

well approximated by the Laplacian distribution (depicted by the blue line), which is usually modeled by the ℓ_1 -norm. Hence, we recalibrate Equation (7) using the ℓ_1 -norm as follows:

$$\hat{\mathbf{X}}_i^{(2)} = \arg \min_{\mathbf{X}_i^{(2)}} \frac{1}{2} \|\mathbf{Y}_i^{(2)} - \mathbf{X}_i^{(2)}\|_F^2 + \lambda \|\mathbf{r}_i\|_1 \quad \forall i. \quad (8)$$

Upon solving Equation (8), the recovered LR matrix $\hat{\mathbf{X}}_i^{(2)}$ is attained, from which we can derive the LR tensor $\hat{\mathcal{X}}_i$ by folding the matrix $\hat{\mathbf{X}}_i^{(2)}$ along the nonlocal mode. It is important to note that our proposed NLRR algorithm gradually refines both the recovered LR tensor $\hat{\mathcal{X}}_i$ and the reference tensor $\mathcal{X}_i^\#$ in a joint manner with each iteration. As demonstrated in the preceding analysis, it becomes evident that our proposed nonlocal LR residual minimization model progressively approximates the underlying LR tensor by minimizing the rank residual, marking a significant departure from most existing nonlocal LR tensor approximation methods [23–25,58–61]. The working principle of the proposed nonlocal LR residual minimization model for HSI denoising is illustrated in Figure 2. It is apparent from the subfigures that the singular values of the constructed nonlocal full-band group unfolded at the nonlocal mode using the conventional LR matrix approximation method, NNM [53] (green line), significantly deviate from its true values (blue line), thus leading to unsatisfactory denoising results in Figure 2e. In contrast, the singular values of our proposed reference matrix $\mathbf{X}_i^{\#(2)}$ (pink line) progressively approximate the singular values of the ground truth (blue line). This improvement can also be observed in the recovered HSI shown in Figure 2f–h. Furthermore, in contrast to many conventional LR tensor modeling methods [23,25,58–61], our approach takes into account the intrinsic property of the constructed nonlocal full-band group, allowing us to design a streamlined single-mode LR tensor method that drastically cuts down on computation time (for more details, please refer to Section 5.4).

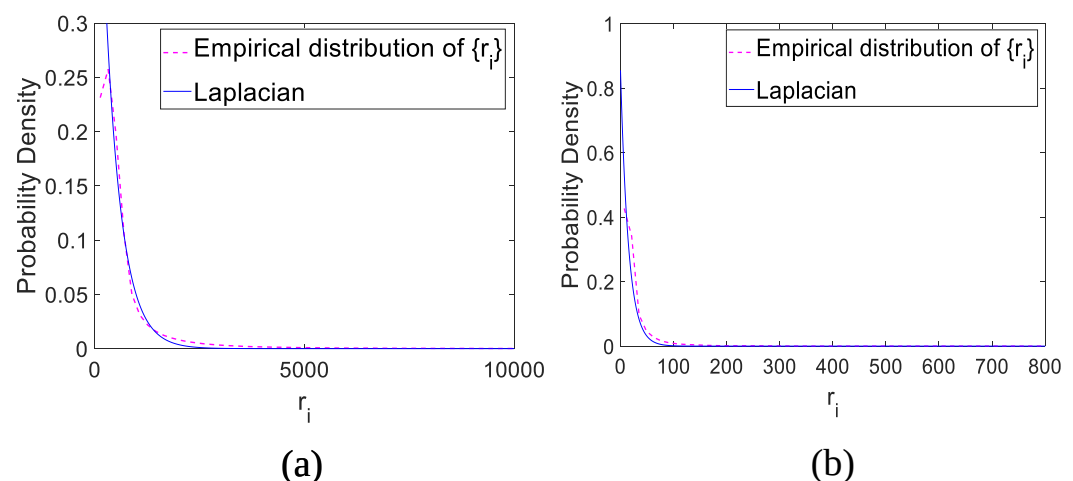


Figure 3. The distributions of the rank residual $\{r_i\}_{i=1}^n$ (pink dashed line) for the WDC dataset. (a) Zero-mean white Gaussian additive noise ($\sigma_n = 30$); (b) mixed noise with scenario 4.

4.2. NLRR Model for Mixed Noise Removal

Many degraded HSIs suffer from not only Gaussian noise but also other types of noise, making noise modeling significantly more complex than when dealing with Gaussian noise alone. In this subsection, we explore using the proposed NLRR model for mixed noise removal in HSI. To be specific, assuming that a noisy HSI $\mathcal{Y} \in \mathbb{R}^{H \times W \times B}$ is corrupted by various types of noise, including additive Gaussian noise $\mathcal{N}$ and additive sparse noise $\mathcal{S}$

(which includes salt and pepper noise, stripes, and deadline noise), the degradation model of HSI mixed noise can be represented as

$$\mathcal{Y} = \mathcal{X} + \mathcal{N} + \mathcal{S}. \quad (9)$$

The critical challenge in recovering the latent clean HSI $\mathcal{X}$ from the noisy HSI $\mathcal{Y}$ lies in accurately characterizing the HSI prior by devising reliable regularization terms. As such, the generalized HSI mixed noise removal model can be mathematically formulated as [16,20]

$$\{\hat{\mathcal{X}}, \hat{\mathcal{S}}\} = \arg \min_{\mathcal{X}, \mathcal{S}} \frac{1}{2} \|\mathcal{Y} - \mathcal{X} - \mathcal{S}\|_F^2 + \alpha \mathcal{R}(\mathcal{X}) + \beta \|\mathcal{S}\|_1, \quad (10)$$

where $\mathcal{R}(\mathcal{X})$ denotes the regularization term that is dependent on the underlying prior knowledge of the clean HSI, while $\|\mathcal{S}\|_1$ serves as a sparse regularization term for accounting for sparse noise. α and β are regularization parameters. In this subsection, we utilize the proposed NLRR model within the framework of RPCA [65] to address mixed noise removal in HSIs. This approach integrates the global spectral LR prior as per Equation (3), the nonlocal LR residual minimization model in Equation (8), and sparse tensor prior in Equation (10). As a result, the mixed noise removal process using the proposed NLRR modeling can be represented as

$$\begin{aligned} \{\hat{\mathcal{Z}}, \hat{\mathcal{D}}, \hat{\mathcal{S}}, \hat{\mathcal{L}}_i\} = \arg \min_{\mathcal{Z}, \mathcal{D}, \mathcal{S}, \mathcal{L}_i} & \frac{1}{2} \|\mathcal{Y} - \mathcal{Z} \times_3 \mathcal{D} - \mathcal{S}\|_F^2 \\ & + \alpha \sum_{i=1}^n \left(\frac{1}{\sigma_n^2} \|\mathcal{R}_i \mathcal{Z} - \mathcal{L}_i\|_F^2 + \tau \mathcal{P}_{NLRR}(\mathcal{L}_i) \right) \\ & + \beta \|\mathcal{S}\|_1, \quad s.t. \quad \mathcal{D}^T \mathcal{D} = \mathbf{I}, \end{aligned} \quad (11)$$

where $\mathcal{R}_i$ denotes the operator extracting the data tensor $\mathcal{Z}_i$ from $\mathcal{Z}$ (refer to Section 3.2 for more details). $\tau \mathcal{P}_{NLRR}(\mathcal{L}_i)$ stands for the proposed nonlocal LR residual regularizer described in Equation (8), which captures the LR property of $\mathcal{L}_i$. α acts as a balance factor, whereas β is a regularization parameter. Evidently, the mixed noise removal problem in HSI, using the proposed NLRR mode in Equation (11), is transformed into solving four sub-problems, $\mathcal{Z}$, $\mathcal{D}$, $\mathcal{S}$, and $\mathcal{L}_i$. Fortunately, each of these has a closed-form solution using alternating minimization as elaborated below.

4.2.1. Nonlocal LR Tensor $\mathcal{L}_i$ Sub-Problem

With the remaining variables in Equation (11) kept fixed, $\mathcal{L}_i$ sub-problem can be represented by the following equation:

$$\hat{\mathcal{L}}_i = \arg \min_{\mathcal{L}_i} \frac{1}{\sigma_n^2} \|\mathcal{R}_i \mathcal{Z} - \mathcal{L}_i\|_F^2 + \tau \mathcal{P}_{NLRR}(\mathcal{L}_i). \quad (12)$$

We utilize the nonlocal LR residual minimization model, as detailed in Equation (8), to solve for the LR tensor $\mathcal{L}_i$ sub-problem. Then, invoking the strategy in Equation (8), we can reformulate Equation (12) as follows:

$$\hat{\mathcal{L}}_i^{(2)} = \arg \min_{\mathcal{L}_i^{(2)}} \frac{1}{\sigma_n^2} \|\mathcal{S}_i^{(2)} - \mathcal{L}_i^{(2)}\|_F^2 + \tau \|\mathcal{r}_i\|_1, \quad (13)$$

where $\mathcal{S}_i = \mathcal{R}_i \mathcal{Z}$ and $\mathcal{S}_i^{(2)}$ represents the unfolding of $\mathcal{S}_i$ along the nonlocal mode.

To derive an efficient solution for Equation (13), we introduce the following conclusions.

Lemma 1. The minimization problem defined by

$$\hat{x} = \arg \min_x \left(\frac{1}{2} \|a - x\|_2^2 + \tau \|x - b\|_1 \right), \quad (14)$$

has a closed-form solution given by

$$\hat{x} = \text{soft}(a - b, \tau) + b, \quad (15)$$

where $\text{soft}(a, \tau) = \text{sgn}(a) \odot \max(\text{abs}(a) - \tau, 0)$; $\odot$ represents the element-wise product, and a, b, x are vectors of identical dimension.

Proof. See [66]. $\square$

Theorem 1. (von Neumann's Inequality) For any two matrices $A, B \in \mathbb{R}^{m \times n}$, the inequality $\text{Tr}(A^T B) \leq \text{Tr}(\delta(A)^T \delta(B))$ holds true, where Tr computes the trace of a given matrix; $\delta(A)$ and $\delta(B)$ denote the singular value matrices of A and B arranged in the same order, respectively.

Proof. See [67]. $\square$

Theorem 2. Consider $\bar{Z}_i^{(2)} = U_i \Lambda_i V_i^T$ is the SVD of $\bar{Z}_i^{(2)} \in \mathbb{R}^{m \times c}$, where $\Lambda_i = \text{diag}(\sigma_{i,1}, \dots, \sigma_{i,j})$. Similarly, the reference matrix $Z_i^{\#(2)}$ (for details on deriving the reference matrix $Z_i^{\#(2)}$ from $\bar{Z}_i^{(2)}$, please refer to Section 4.1) $= R_i \Theta_i Q_i^T$ represents the SVD of $Z_i^{\#(2)} \in \mathbb{R}^{m \times c}$ with $\Theta_i = \text{diag}(\theta_{i,1}, \dots, \theta_{i,j})$, where $j = \min(m, c)$. The optimal solution to the problem in Equation (13) is defined as $Z_i^{(2)} = U_i \Sigma_i V_i^T$, where $\Sigma_i = \text{diag}(\delta_{i,1}, \dots, \delta_{i,j})$. The k^{th} diagonal element, referred to as $\delta_{i,k}$, is determined as follows:

$$\min_{\delta_{i,k} \geq 0} \left(\frac{1}{2} (\sigma_{i,k} - \delta_{i,k})^2 + \tau |\delta_{i,k} - \theta_{i,k}| \right) \quad \forall k. \quad (16)$$

Proof. Assuming the SVD of $\bar{Z}_i^{(2)}$, $Z_i^{\#(2)}$ and $Z_i^{(2)}$ are $\bar{Z}_i^{(2)} = P_i \Lambda_i G_i^T$, $Z_i^{\#(2)} = R_i \Theta_i Q_i^T$, and $Z_i^{(2)} = U_i \Sigma_i V_i^T$, respectively, where Λ_i , Θ_i , and Σ_i represent ordered singular value matrices in the same order. By referring back to Equation (13) and considering Theorem 1, we can deduce that

$$\begin{aligned} \|\bar{Z}_i^{(2)} - Z_i^{(2)}\|_F^2 &= \|P_i \Lambda_i G_i^T - U_i \Sigma_i V_i^T\|_F^2 \\ &= \text{Tr}(\Lambda_i \Lambda_i^T) + \text{Tr}(\Sigma_i \Sigma_i^T) - 2\text{Tr}(Z_i^{(2)T} \bar{Z}_i^{(2)}) \\ &\geq \text{Tr}(\Lambda_i \Lambda_i^T) + \text{Tr}(\Sigma_i \Sigma_i^T) - 2\text{Tr}(\Sigma_i^T \Lambda_i) \\ &= \|\Lambda_i - \Sigma_i\|_F^2, \end{aligned} \quad (17)$$

where the equality holds only when $P_i = U_i$ and $G_i = V_i$. As such, the minimum of Equation (17) is achieved when $P_i = U_i$ and $G_i = V_i$, and the optimal solution for Σ_i can be found by solving

$$\begin{aligned} \min_{\Sigma_i \geq 0} \frac{1}{2} \|\Lambda_i - \Sigma_i\|_F^2 + \tau \sum_{k=1}^j |r_{i,k}| \\ = \min_{\delta_{i,k} \geq 0} \sum_{k=1}^j \left(\frac{1}{2} (\sigma_{i,k} - \delta_{i,k})^2 + \tau |\delta_{i,k} - \psi_{i,k}| \right), \end{aligned} \quad (18)$$

where $\sigma_{i,k}$, $\delta_{i,k}$ and $\psi_{i,k}$ represent the k -th singular value of $\bar{Z}_i^{(2)}$, $Z_i^{(2)}$, and $Z_i^{\#(2)}$, respectively. $\square$

Therefore, based on Theorem 2, we can convert Equation (13) into the problem of solving Equation (18). Then, relying on Lemma 1, we can derive a closed-form solution of Equation (18) as follows:

$$\hat{\delta}_{i,k} = \text{Soft}(\sigma_{i,k} - \theta_{i,k}, \frac{\tau\sigma_n^2}{2}) + \theta_{i,k}, \quad (19)$$

where $\delta_{i,k}$, $\sigma_{i,k}$, and $\theta_{i,k}$ are denoted the k -th singular value of $\mathbf{L}_i^{(2)}$, $\mathbf{S}_i^{(2)}$, and $\mathbf{S}_i^{\#(2)}$, respectively. For more information regarding the acquisition of $\mathbf{S}_i^{\#(2)}$ from $\mathbf{S}_i^{(2)}$, please refer to Section 4.1. After using Equation (19) to obtain the solution for $\hat{\Sigma}_i = \text{diag}(\hat{\delta}_{i,1}, \dots, \hat{\delta}_{i,j})$, the recovered LR matrix $\hat{\mathbf{L}}_i^{(2)}$ can be calculated as $\hat{\mathbf{L}}_i^{(2)} = \mathbf{U}_i \hat{\Sigma}_i \mathbf{V}_i^T$, where $\mathbf{U}_i$ and $\mathbf{V}_i$ denote left and right singular vector matrices of $\mathbf{S}_i^{(2)}$, respectively. As such, the recovered LR tensor $\hat{\mathcal{L}}_i$ can be determined using $\hat{\mathcal{L}}_i = \text{fold}_{(2)}(\hat{\mathbf{L}}_i)$.

4.2.2. LR Subspace $\mathbf{D}$ Sub-Problem

For fixed other variables, the $\mathbf{D}$ sub-problem is reduced to

$$\hat{\mathbf{D}} = \arg \min_{\mathbf{D}} \frac{1}{2} \|\mathcal{Y} - \mathcal{Z} \times_3 \mathbf{D} - \mathcal{S}\|_F^2 \quad \text{s.t.} \quad \mathbf{D}^T \mathbf{D} = \mathbf{I}. \quad (20)$$

According to [58], a closed-form solution for the $\mathbf{D}$ sub-problem can be derived as follows:

$$\hat{\mathbf{D}} = \mathbf{U} \mathbf{V}^T. \quad (21)$$

In this case, $\mathbf{U}$ and $\mathbf{V}$ are the left and right singular vector matrices of $\text{Unfold}_{(3)}(\mathcal{Y} - \mathcal{S})(\text{Unfold}_{(3)}(\mathcal{Z}))^T$, respectively.

4.2.3. RCI $\mathcal{Z}$ Reconstruction Sub-Problem

With the other variables fixed, the RCI $\mathcal{Z}$ reconstruction sub-problem in Equation (11) is simplified as follows:

$$\begin{aligned} \hat{\mathcal{Z}} = \arg \min_{\mathcal{Z}} & \frac{1}{2} \|\mathcal{Y} - \mathcal{Z} \times_3 \mathbf{D} - \mathcal{S}\|_F^2 \\ & + \alpha \sum_{i=1}^n \left(\frac{1}{\sigma_n^2} \|\mathcal{R}_i \mathcal{Z} - \mathcal{L}_i\|_F^2 \right). \end{aligned} \quad (22)$$

As seen from Equation (22), this is a quadratic optimization problem, which has a closed-form solution as follows:

$$\begin{aligned} \hat{\mathcal{Z}} = & \left(\mathcal{I} + 2\alpha \sum_{i=1}^n \frac{1}{\sigma_n^2} \mathcal{R}_i^T \mathcal{R}_i \right)^{-1} \\ & \times \left((\mathcal{Y} - \mathcal{S}) \times_3 \mathbf{D}^T + 2\alpha \sum_{i=1}^n \frac{1}{\sigma_n^2} \mathcal{R}_i^T \mathcal{L}_i \right). \end{aligned} \quad (23)$$

4.2.4. Sparse Noise $\mathcal{S}$ Sub-Problem

For fixed other variables, the $\mathcal{S}$ sub-problem in Equation (11) is reduced to

$$\hat{\mathcal{S}} = \arg \min_{\mathcal{S}} \frac{1}{2} \|\mathcal{Y} - \mathcal{Z} \times_3 \mathbf{D} - \mathcal{S}\|_F^2 + \beta \|\mathcal{S}\|_1. \quad (24)$$

This can be directly solved by employing the soft-thresholding operator as presented in [15,59],

$$\hat{\mathcal{S}} = \text{soft}(\mathcal{Y} - \mathcal{Z} \times_3 \mathbf{D}, \beta). \quad (25)$$

For further details regarding the definition of $\text{soft}(\cdot)$, please refer to Lemma 1.

At this point, we have successfully derived effective solutions for the aforementioned four sub-problems. Upon reaching convergence, the final denoised HSI can be obtained as $\hat{\mathcal{X}} = \hat{\mathcal{Z}} \times_3 \hat{\mathbf{D}}$. A summary of the entire process of the proposed NLRR model for mixed noise removal in HSI is provided in Algorithm 1.

Algorithm 1 The proposed NLRR algorithm for mixed noise removal

Require: Degraded HSI $\mathcal{Y}$.

1: **Initialize:**

- Set initial estimates $\hat{\mathcal{X}}^0 = \mathcal{Y}$ and $\mathcal{Y}^0 = \mathcal{Y}$.
- Estimate the rank M using HySime [57].
- Set parameters α and β .

2: **for** $t = 1$ to Max-Iter **do**

3: Learn the subspace basis $\hat{\mathbf{D}}$ from $\mathcal{Y}$ and estimate the RCI $\mathcal{Z} = \mathcal{Y} \times \hat{\mathbf{D}}^T$.

4: **for** each full-band patch $\mathcal{Z}_{i,fbp}$ in $\mathcal{Z}$ **do**

5: Form a nonlocal full-band group $\mathcal{S}_i = \mathcal{R}_i \mathcal{Z}$ from similar full-band patches.

6: Unfolding $\mathcal{S}_i$ into matrix $\mathcal{S}_i^{(2)}$ along the nonlocal mode.

7: Compute $[\mathbf{U}_i, \mathbf{\Delta}_i, \mathbf{V}_i] = \text{SVD}(\mathcal{S}_i^{(2)})$.

8: Estimate the reference matrix $\mathcal{S}_i^{\#(2)}$ using the strategy in Equation (5).

9: Compute $[\mathbf{R}_i, \mathbf{\Theta}_i, \mathbf{Q}_i] = \text{SVD}(\mathcal{S}_i^{\#(2)})$.

10: Compute the LR matrix $\hat{\mathcal{L}}_i^{(2)}$ using Equation (19).

11: Derive the latent clean nonlocal full-band group $\hat{\mathcal{L}}_i$ from $\text{fold}_{(2)}(\hat{\mathcal{L}}_i^{(2)})$.

12: **end for**

13: **while** not converged **do**

14: Update $\hat{\mathcal{Z}}$ using Equation (23).

15: Update $\hat{\mathbf{D}}$ using Equation (21).

16: Update $\hat{\mathcal{S}}$ using Equation (25).

17: **end while**

18: Algorithm Refinement (Section 4.3).

19: **end for**

20: **Output:** The final denoised HSI $\hat{\mathcal{X}} = \hat{\mathcal{Z}} \times_3 \hat{\mathbf{D}}$.

4.3. Algorithm Refinement

In this subsection, we introduce several advantageous strategies that enhance the denoising performance of the proposed NLRR algorithm. These strategies encompass iterative regularization, updating the low-dimensional subspace, and adaptive regularization parameters, all of which synergistically contribute to improving denoising results.

4.3.1. Iterative Regularization

Iterative regularization [68] is a proficient technique for image denoising that has gained widespread usage due to its effectiveness in enhancing the performance of denoising algorithms [14,23,24]. In this study, we leverage the iterative regularization technique to update the next input noisy HSI and to re-estimate the noise variance σ_n^2 . This aids in refining the orthogonal basis $\mathbf{D}$ during the LR subspace learning phase. The fundamental premise of iterative regularization is the incorporation of filtered noise into the denoised HSI, which, in turn, leads to the update of the next input noisy HSI as described below,

$$\mathcal{Y}^{t+1} = \rho \hat{\mathcal{X}}^t + (1 - \rho) \mathcal{Y}, \quad (26)$$

where t is the t -th iteration and ρ is a relaxation parameter that governs the trade-off between the denoised HSI $\hat{\mathcal{X}}$ and the original noisy HSI $\mathcal{Y}$.

As the noise level undergoes change throughout the iterations, it becomes essential to recompute the noise standard deviation σ_n , especially when making use of the proposed nonlocal LR residual minimization regularizer (Equation (8)). Given that the subspace dictionary $\mathbf{D}$ is orthogonal, it is reasonable to assert that the RCI $\mathcal{Z}$ retains the same noise level as the noisy HSI $\mathcal{Y}$ [69]. In light of this, we draw upon the iterative regularization approach to update the estimation of the noise standard deviation σ_n . Specifically, in the t -th iteration, the standard deviation of noise σ_n is calculated as follows [68]:

$$\sigma_n^t = \mu \sqrt{(\sigma_n^2 - \|\mathcal{Y} - \mathcal{Y}^{t+1}\|_F^2)}. \quad (27)$$

Here, μ serves as a scaling factor used to regulate the re-estimation of the noise variance. This iterative regularization strategy is particularly effective in refining the subspace dictionary $\mathbf{D}$ in scenarios when the input HSI is heavily contaminated with noise.

4.3.2. Update Low-Dimensional Subspace

Since the noise level varies with each iteration, it becomes necessary to adjust the rank M of the subspace $\mathbf{D}$ correspondingly. This rank acts as a balancing factor between preserving image information, which is favored by higher rank values, and reducing noise, which benefits from lower rank values. Generally speaking, when dealing with an image severely degraded by noise, setting a lower value for the rank M can effectively suppress the noise, resulting in more satisfactory denoising results. To this end, we initially use HySime [57] to obtain a small value for M as a starting point. As the number of iterations increases, the intensity of the noise gradually diminishes. Additionally, it is worth noting that the rank M does not exceed the number of bands B present in the original HSI. Considering these points, we heuristically update the rank M of subspace $\mathbf{D}$ as follows:

$$M^{t+1} = \min(M^t + q \times t, B), \quad (28)$$

where q is a constant used to control the trade-off between the number of iterations and the rank M .

4.3.3. Update Regularization Parameter

The regularization parameter τ plays a crucial role in the proposed NLRR denoising algorithms. It is common to use empirical methods to set its value, but solely relying on empirical methods may result in the algorithm not being robust enough to different noise scenarios. To overcome this limitation, we are inspired by previous research [51] to design an adaptive regularization parameter τ , namely,

$$\tau = \frac{2\sqrt{2} \sigma_n^{2(t)}}{(\varphi_i + \epsilon)}, \quad (29)$$

where φ_i denotes the estimated standard deviation of $\mathbf{r}_i$, and ϵ is a small positive constant introduced to prevent a division by zero scenario.

Up to this point, we have provided a detailed introduction on the utilization of the proposed NLRR algorithm for the removal of Gaussian noise as well as mixed noise in HSI. In the next section, we will present a wealth of experimental results to highlight the high effectiveness of the proposed NLRR algorithms.

5. Experimental Results

In this section, an exhaustive set of experiments is undertaken to validate the effectiveness of the proposed NLRR algorithm for HSI denoising.

- Implementation Details** The NLRR algorithm is applied to both simulated and real-world HSIs. The basic parameters are set as follows. In all our experiments, we establish the patch size of $\sqrt{d} \times \sqrt{d}$ to be 5×5 . The spectral dimension M is first initialized using HySime [57] and then updated according to Equation (28). We empirically assign the values of 2, 0.9, and 20 to the parameters ϱ , ρ , and h , respectively. The nonlocal full-band patches m is set to 200. The parameters α , β , and μ are empirically set to 0.01, 0.2, and 0.07, respectively. All experiments are carried out on a PC powered by an Intel (R) Core (TM) i9-10920X CPU running at 3.50 GHz, equipped with 64GB of memory, under the Matlab 2019b environment.
- Image Quality Evaluation Metrics** HSIs contain both spatial and spectral information, each playing a pivotal role in the quality of reconstructed images. To comprehensively evaluate the performance of HSI denoising algorithms, we use four quantitative image quality metrics, including the peak signal-to-noise ratio (PSNR), structural similarity index (SSIM) [70], erreur relative globale adimensionnelle de synthese (ERGAS) [71], and spectral angle mapper (SAM) [72]. PSNR and SSIM stand as prevalent spatial quality metrics for image restoration. PSNR quantifies the similarity between target and reference images by measuring the mean squared error (MSE), while SSIM focuses on assessing structural consistency between the two images. ERGAS, a distinguished spectral quality metric, evaluates denoising accuracy via a weighted sum of MSE across all bands, while SAM quantifies spectral variation between the target and reference HSI. It is noteworthy that lower ERGAS and SAM values indicate better denoising performance, whereas higher PSNR and SSIM values are preferred.

5.1. Mixed Noise Removal

In this subsection, we evaluate the denoising of our proposed NLRR algorithm, based on Equation (11), for mixed noise removal. We take into consideration the following four noise scenarios:

- Scenario 1: Gaussian noise, adhering to the principles of independence and identical distribution (i.i.d.), is added to each band, with variances randomly selected from a range between 0.1 and 0.2.
- Scenario 2: The Gaussian noise added is identical to that in scenario 1. Moreover, an additional 20% of bands are randomly chosen to incorporate salt and pepper noise, affecting 20% of each selected band.
- Scenario 3: This case adheres to the same noise conditions as in scenario 2, and an additional deadline of width ranging between 1 and 3 is imposed on 20% of the bands.
- Scenario 4: Maintaining the same addition of noise as in scenario 3, but with an additional corruption of ten bands, specifically bands 6 to 15, by stripes affecting 40% of each of these bands.

Two widely used HSI datasets, namely the PAC and WDC, are used to evaluate the denoising performance of different algorithms. To demonstrate the superiority of the proposed NLRR algorithm, we compare it with eleven popular or state-of-the-art methods, including LRMR [16], LRTV [73], NMoG [35], L1HyMixDe [69], 3DLogTNN [20], FRCTR [21], NG-Meet [24], WMLTR [25], HSI-SDeCNN [27], QRNN3D [28], and MAC-Net [29]. The methods of LRMR, LRTV, NMoG, L1HyMixDe, 3DLogTNN, and FRCTR utilize the global spectral LR property, while NG-Meet and WMLTR are grounded in NSS properties. On the other hand, HSI-SDeCNN, QRNN3D, and MAC-Net being deep learning

approaches, leverage an end-to-end scheme to achieve superior denoising performance. Among these, WMLTR has recently been recognized as a state-of-the-art HSI denoising method for addressing mixed noise. It is noteworthy that HSI-SDeCNN, and NG-Meet are inherently designed with the assumption of dealing with Gaussian noise. Therefore, we apply the tensor PCA (TRPCA) strategy [74] to eliminate sparse noise before using them. In all cases, we have meticulously fine-tuned the results of each competing method to attain the optimal denoising performance.

For all competing methods, the quantitative results for these two HSI datasets under four noise scenarios are presented in Table 1. It is evident that the proposed NLRR consistently outperforms the other competing methods across all cases, as measured by four different quantitative metrics. The only exception is scenario 4 of the PAC dataset where the L1HyDeMix method scores better than the proposed NLRR algorithm based on the ERGAS metric. In particular, the proposed NLRR algorithm exhibits a substantial edge over other competing methods when addressing more complex noise scenarios. Moreover, we display the average PSNR values per band for all competing methods across these two HSI datasets under noise scenario 2 and noise scenario 4 in Figure 4. In a comprehensive analysis, our proposed NLRR approach (indicated by the red line) outperforms the other competing methods in the majority of the bands. We further present the visual comparison results of HSI WDC with bands 99, 160, and 180 under noise scenario 2 in Figure 5. It is evident that LRMR, and NMoG methods struggle to remove the noise effectively. The LRTV, 3DLogTNN, WMLTR, and HSI-SDeCNN methods are apt to over-smooth the image, while L1HyMixDe, FRCTR, NG-Meet, QRNN3D, and MAC-Net methods often result in unwanted visual artifacts. In contrast, the proposed NLRR approach preserves finer details and causes fewer visual artifacts compared to other competing methods. Furthermore, the error images of all competing methods further highlight the superior visual quality achieved by the proposed NLRR approach.

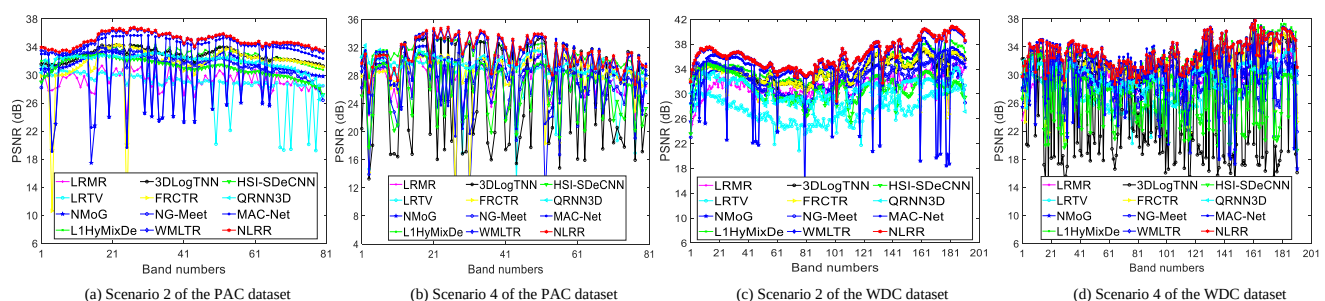


Figure 4. PSNR values for each band in the PAC and WDC datasets under scenario 2 and scenario 4, respectively.

5.2. Real Data Denoising

Previously, we presented extensive experimental results that underscore the excellent performance of the proposed NLRR algorithm in simulated HSI denoising. In this subsection, to further exemplify the superiority of our NLRR approach, we apply it to real HSI data denoising. We consider two well-known real noisy HSI datasets: the AVIRIS Indian Pines dataset (<https://engineering.purdue.edu/~biehl/MultiSpec/>, accessed on 1 June 2026) and the HYDICE Urban dataset (<https://www.erd.usace.army.mil/Media/Fact-Sheets/Fact-Sheet-Article-View/Article/610433/hypercube/Scenes>, accessed on 1 June 2026), with respective dimensions of $145 \times 145 \times 220$ and $307 \times 307 \times 210$. The original AVIRIS Indian Pines data comprises 220 bands, but we remove 20 water absorption bands (104–108, 150–163, and 220) that do not contribute valuable information, reducing the dataset dimensions to $145 \times 145 \times 200$.

Table 1. Average denoising results for PSNR (dB), SSIM, ERGAS and SAM metrics, utilizing LRMR [16], LRTV [73], NMoG [35], L1HyMixDe [69], 3DLogTNN [20], FRCTR [21], NG-Meet [24], WMLTR [25], HSI-SDeCNN [27], QRNN3D [28], and MAC-Net [29] and the proposed NLRR methods, across the PAC and WDC datasets with four complex noise scenarios.

PAC Dataset													
Noise scenarios	Index	Global Spectral LR					Nonlocal LR			Deep Learning			NLRR
		LRMR	LRTV	NMoG	L1HyMixDe	3DLogTNN	FRCTR	NG-Meet	WMLTR	HSI-SDeCNN	QRNN3D	MAC-Net	
Scenario 1	PSNR	29.71	28.87	32.15	33.24	33.16	32.72	34.79	36.15	33.55	31.95	35.09	36.23
	SSIM	0.8510	0.7961	0.9098	0.9331	0.9368	0.9092	0.9514	0.9632	0.9305	0.9122	0.9526	0.9641
	ERGAS	111.26	140.98	85.63	73.00	74.17	80.80	63.13	51.79	70.70	84.72	59.20	51.34
	SAM	0.1395	0.1529	0.0885	0.0708	0.0589	0.0850	0.0600	0.0451	0.0760	0.0805	0.0560	0.0447
Scenario 2	PSNR	29.19	28.55	29.82	32.77	32.80	31.62	31.66	34.74	31.06	31.14	31.85	34.99
	SSIM	0.8378	0.7893	0.8741	0.9289	0.9325	0.8982	0.9118	0.9531	0.8978	0.9161	0.9304	0.9542
	ERGAS	118.43	140.33	143.40	77.12	76.98	192.81	88.30	62.01	93.64	92.26	127.87	60.05
	SAM	0.1410	0.1552	0.1515	0.0760	0.0606	0.2001	0.0809	0.0555	0.0723	0.0806	0.1347	0.0550
Scenario 3	PSNR	28.33	28.23	28.80	32.04	29.64	29.28	29.12	33.33	28.84	30.93	30.75	33.60
	SSIM	0.8277	0.7751	0.8518	0.9203	0.8940	0.8571	0.8613	0.9421	0.8575	0.9132	0.9196	0.9433
	ERGAS	146.21	198.95	200.14	84.67	215.29	194.54	150.22	74.60	152.48	94.49	137.32	72.25
	SAM	0.1701	0.2229	0.2182	0.0896	0.2043	0.2279	0.1504	0.0663	0.1519	0.0814	0.1424	0.0663
Scenario 4	PSNR	27.20	27.89	27.24	31.26	25.08	27.94	26.83	31.00	25.87	29.71	28.77	31.35
	SSIM	0.8052	0.7679	0.8379	0.9008	0.7932	0.8344	0.8031	0.9270	0.7775	0.9050	0.9071	0.9288
	ERGAS	158.09	185.40	247.26	92.64	314.82	210.82	190.26	99.61	201.49	109.53	155.68	95.67
	SAM	0.1748	0.1976	0.2447	0.0818	0.3423	0.2274	0.1839	0.0780	0.1886	0.0898	0.1381	0.0772
Average	PSNR	28.61	28.39	29.50	32.33	30.17	30.39	30.60	33.80	29.83	30.93	31.61	34.04
	SSIM	0.8304	0.7821	0.8684	0.9208	0.8891	0.8747	0.8819	0.9463	0.8658	0.9116	0.9274	0.9476
	ERGAS	133.50	166.42	169.11	81.86	170.32	169.74	122.98	72.00	129.58	95.25	120.01	69.83
	SAM	0.1563	0.1822	0.1757	0.0795	0.1665	0.1851	0.1188	0.0610	0.1222	0.0831	0.1178	0.0608

Table 1. Cont.

WDC Dataset													
Noise scenarios	Index	Global Spectral LR					Nonlocal LR			Deep Learning			
		LRMR	LRTV	NMoG	L1HyMixDe	3DLogTNN	FRCTR	NG-Meet	WMLTR	HSI-SDeCNN	QRNN3D	MAC-Net	NLRR
Scenario 1	PSNR	31.48	27.52	33.67	34.18	33.95	34.14	35.58	36.50	33.09	30.59	35.19	36.56
	SSIM	0.8923	0.7174	0.9283	0.9379	0.9441	0.9461	0.9613	0.9654	0.9240	0.9014	0.9555	0.9660
	ERGAS	100.39	158.52	78.13	72.79	74.72	73.15	63.15	56.16	83.84	115.06	65.39	55.79
	SAM	0.1282	0.1266	0.0924	0.0798	0.0753	0.0742	0.0650	0.0607	0.0977	0.1164	0.0711	0.0603
Scenario 2	PSNR	31.17	27.42	32.14	34.08	33.81	33.53	32.24	36.07	30.98	30.56	33.66	36.30
	SSIM	0.8888	0.7153	0.9014	0.9379	0.9423	0.9375	0.9202	0.9626	0.8803	0.8990	0.9362	0.9642
	ERGAS	106.21	161.20	155.05	75.29	76.38	84.45	95.36	59.32	111.32	115.92	143.72	57.39
	SAM	0.1337	0.1315	0.1817	0.0833	0.0773	0.0908	0.1085	0.0632	0.1178	0.1184	0.1572	0.0611
Scenario 3	PSNR	30.70	27.34	31.10	33.82	30.20	32.42	30.37	34.26	28.78	30.43	32.28	34.35
	SSIM	0.8840	0.7104	0.8945	0.9342	0.9095	0.9281	0.8903	0.9525	0.8454	0.8970	0.9293	0.9528
	ERGAS	112.49	163.23	143.57	76.72	247.94	107.59	130.49	73.46	176.65	118.09	135.06	72.66
	SAM	0.1412	0.1311	0.1629	0.0851	0.2489	0.1122	0.1459	0.0740	0.1864	0.1207	0.1512	0.0728
Scenario 4	PSNR	29.78	27.23	29.84	32.55	25.48	31.36	28.76	32.92	26.37	29.92	30.86	33.11
	SSIM	0.8733	0.7071	0.8819	0.9195	0.8285	0.9089	0.8545	0.9454	0.7857	0.8943	0.9186	0.9457
	ERGAS	129.09	164.60	166.92	99.14	358.98	117.90	161.38	86.01	224.91	123.89	161.32	83.87
	SAM	0.1585	0.1376	0.1775	0.1108	0.3794	0.1305	0.1758	0.0841	0.2305	0.1229	0.1680	0.0830
Average	PSNR	30.78	27.38	31.69	33.66	30.86	32.86	31.74	34.94	29.80	30.38	33.00	35.08
	SSIM	0.8846	0.7125	0.9015	0.9324	0.9061	0.9302	0.9066	0.9565	0.8589	0.8979	0.9349	0.9572
	ERGAS	112.05	161.89	135.92	80.99	189.51	95.77	112.59	68.74	149.18	118.24	126.37	67.43
	SAM	0.1404	0.1317	0.1536	0.0897	0.1952	0.1019	0.1238	0.0700	0.1581	0.1196	0.1369	0.0693

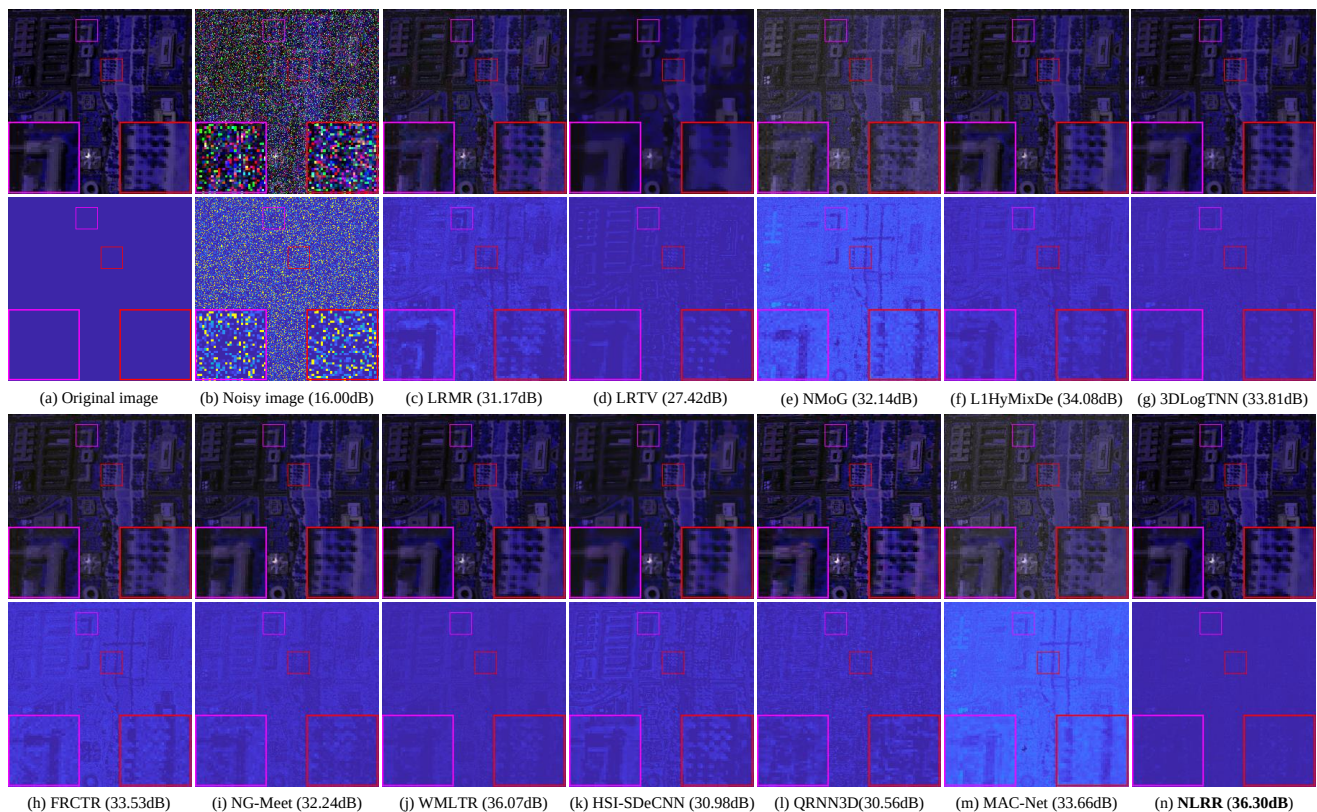


Figure 5. Visual comparison results of HSI WDC by different methods (the error images in the second and fourth rows correspond to the denoised images in the first and third rows, respectively). (a) Original image (bands: 99, 160, 180); (b) Noisy image (16.00 dB); (c) LRMR [16] (31.17 dB); (d) LRTV [73] (27.42 dB); (e) NMoG [35] (32.14 dB); (f) L1HyMixDe [69] (34.08 dB); (g) 3DLogTNN [20] (33.81 dB); (h) FRCTR [21] (33.53 dB); (i) NG-Meet [24] (32.24 dB); (j) WMLTR [25] (36.07 dB); (k) HSI-SDeCNN [27] (30.98 dB); (l) QRNN3D [28] (30.56 dB); (m) MAC-Net [29] (33.66 dB); (n) NLRR (36.30 dB).

In real-world scenarios, HSI noise is rarely independent across bands. To validate the ability of our proposed NLRR approach to handle band-dependent noise coupling, we compare it against deep learning-based methods [27–29], which are designed to learn spectral dependencies through pre-training. Unlike some deep learning methods [29,48], which require prior training on synthetic noise distributions, our proposed NLRR approach explicitly enforces spectral coherence through LR modeling. This allows our NLRR approach to adaptively suppress correlated noise across bands without relying on predefined noise distributions. The parameters for all competing methods are configured identically to those used in the simulated experiments.

The visual comparison results of these two real HSI datasets are depicted in Figures 6 and 7. Observing the denoising results of the AVIRIS Indian Pines dataset in Figure 6, it becomes evident that the LRMR and FRCTR methods fail to effectively remove the noise. The NMoG, L1HyMixDe, 3DLogTNN, NG-Meet, and MAC-Net methods still give rise to some undesired visual artifacts. The LRTV, WMLTR, and QRNN3D methods, on the other hand, tend to over-smooth the image. Conversely, our proposed NLRR method delivers superior visual results in comparison to the other competing methods. Analyzing the denoising results of the HYDICE Urban dataset in Figure 7, it is noticeable that the FRCTR method still retains noise. The NMoG, 3DLogTNN, NG-Meet, and HSI-SDeCNN methods markedly generate undesired striped artifacts, whereas the LRTV, WMLTR, QRNN3D, and MAC-Net methods result in substantial over-smoothing. When compared to the LRTV, NMoG, 3DLogTNN, FRCTR, NG-Meet, WMLTR, HSI-SDeCNN,

QRNN3D, and MAC-Net methods, better visual results are achieved by the L1HyMixDe, and our proposed NLRR methods. Furthermore, we employ a well-known no-reference image quality metric, namely BRISQUE [75], to evaluate the denoising performance of all competing methods in real scenarios. It is worth mentioning that a smaller BRISQUE value indicates superior denoising performance. As depicted in Figures 6 and 7, our proposed NLRR approach achieves the lowest BRISQUE values, indicating the superior denoising performance on real HSI data compared to all other competing methods.



Figure 6. Denoising results of the real AVIRIS Indian Pines dataset by different methods. (a) Noisy image (bands: 4, 35, 200); (b) LRMR [16] (27.81); (c) LRTV [73] (16.84); (d) NMoG [35] (14.59); (e) L1HyMixDe [69] (16.56); (f) 3DLogTNN [20] (35.05); (g) FRCTR [21] (35.67); (h) NG-Meet [24] (24.99); (i) WMLTR [25] (16.02); (j) HSI-SDeCNN [27] (31.50); (k) QRNN3D [28] (28.40); (l) MAC-Net [29] (21.01); (m) NLRR (11.03).

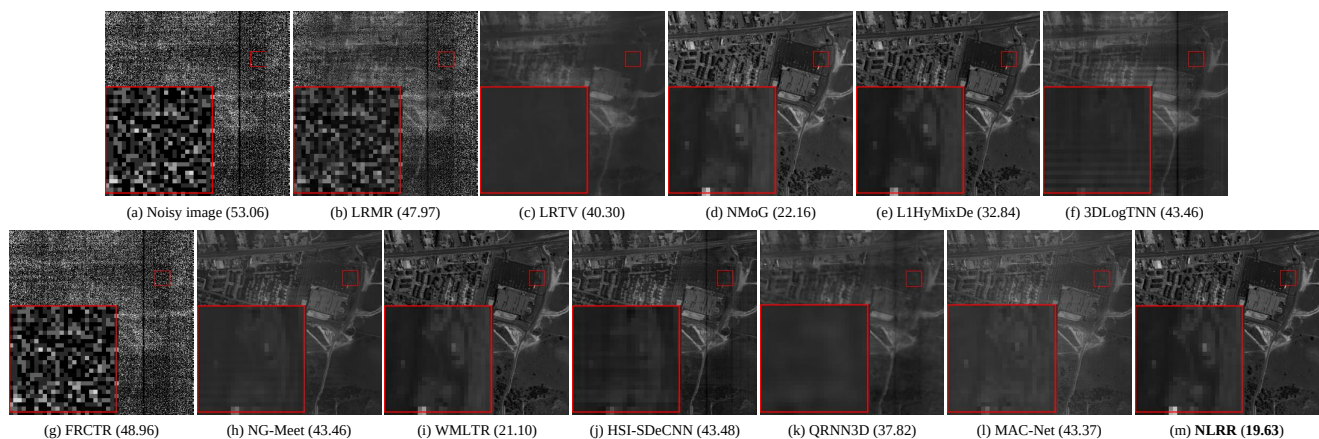


Figure 7. Denoising results of the real HYDICE Urban dataset by different methods. (a) Noisy image (band 208); (b) LRMR [16] (47.97); (c) LRTV [73] (40.30); (d) NMoG [35] (22.16); (e) L1HyMixDe [69] (32.84); (f) 3DLogTNN [20] (43.46); (g) FRCTR [21] (48.96); (h) NG-Meet [24] (43.46); (i) WMLTR [25] (21.10); (j) HSI-SDeCNN [27] (43.48); (k) QRNN3D [28] (37.82); (l) MAC-Net [29] (43.37); (m) NLRR (19.63).

5.3. Algorithm Complexity

The computational complexity of our proposed NLRR algorithm primarily stems from solving LR subspace and nonlocal LR tensor problems. The complexity of the LR subspace problem is estimated to be $O(HWB^2)$, whereas the complexity of the nonlocal LR tensor problem is approximated as $O(nT_b + nm^2c)$, with T_b representing the operator for searching similar full-band patches. Consequently, the overall computational complexity of the proposed NLRR algorithm is approximately $O(t(HWB^2 + nT_b + nm^2c))$.

5.4. Computational Efficiency

It is essential to recognize that HSI denoising is a significant real-world issue often plagued by a variety of noises. Therefore, computational efficiency serves as another vital consideration when appraising an HSI denoising algorithm. We provide the average computational time for the competing methods, assessed during mixed noise removal tasks under four different noise scenarios within the PAC and WDC datasets as shown in Table 2. The results underscore that our proposed NLRR method outperforms others by delivering the fastest execution time. Please note that QRNN3D and MAC-Net methods were implemented on a Linux platform using GPU hardware. Therefore, their computational times are not included in this analysis.

Table 2. Average running times (in seconds) of different methods for HSI denoising.

Methods	LRMR	LRTV	NMoG	L1HyMixDe	3DLogTNN	FRCTR	NG-Meet	WMLTR	SDeCNN	NLRR
Time	85.82	328.30	110.54	176.17	280.90	459.62	150.79	54.29	148.78	46.10

6. Conclusions

Conventional nonlocal LR tensor modeling has garnered widespread attention in HSI denoising. However, such methods often fail to deliver superior denoising performance in the presence of heavy or complex noise, which in turn limits the broader application of HSI. In this paper, we have proposed a novel NLRR approach for highly effective HSI denoising, which progressively refines the underlying LR tensor structure by minimizing the rank residual. This residual-driven approach integrated two complementary priors: a nonlocal LR residual prior leveraging spatial self-similarity and a global spectral LR prior that reduces spectral redundancy. In order to illustrate the effectiveness of the proposed NLRR approach in handling various types of noise, we have expanded it into the NLRR-RPCA, incorporating the LR residual in conjunction with global spectral LR and sparse tensor priors for mixed noise removal. Furthermore, we have devised an effective alternating minimization algorithm with adaptive rank adjustment to tackle the proposed NLRR-based HSI denoising problems. Through extensive experimental results on both simulated and real data, it has been demonstrated that the proposed NLRR algorithm outperforms many popular or state-of-the-art HSI denoising methods regarding quantitative and visual performance metrics.

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