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On the Performance of User Selection for Zero-Forcing Beamforming in MU-MIMO Downlink Channels with One-Bit ADCs

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Abstract

This paper analyzes the ergodic sum rate of zero-forcing beamforming with semi-orthogonal user selection (ZFBF-SUS) in multiuser multiple-input multiple-output (MU-MIMO) downlink channels with one-bit analog-to-digital converters (ADCs) at the receivers. Although ZFBF-SUS has been widely studied under the conventional assumption of infinite-resolution ADCs, its ergodic performance with one-bit ADC receivers has not been fully characterized. To address this issue, we derive analytical approximations for the ergodic sum rate by considering the achievable rate expression induced by one-bit quantization. For mathematical tractability, the achievable rate of each selected user is approximated as a function of its channel gain under the condition that the selected users are nearly orthogonal and the number of users is sufficiently large. Moreover, the channel gain of each scheduled user is characterized using maximum order statistics of Gamma random variables, while the size of the SUS candidate set is approximated by exploiting the semi-orthogonality condition. We further derive an asymptotic sum-rate expression with respect to the number of users by applying extreme value theory. Simulation results verify the validity of the proposed analytical approximations and show that the derived expressions accurately capture the ergodic sum-rate behavior of ZFBF-SUS with one-bit ADC receivers, especially when the number of users is large and the SUS threshold is properly chosen.

Keywords: multi-user multi-antenna systems; one-bit ADCs; zero-forcing beamforming; semi-orthogonal user selection

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1. Introduction

Multiple-input multiple-output (MIMO) techniques have been widely studied as an effective method for improving the spectral efficiency of wireless communication systems. In a point-to-point MIMO link, spatial multiplexing gains can be achieved by increasing the numbers of transmit and receive antennas. However, equipping user terminals with a large number of receive antennas is often impractical due to hardware complexity, power consumption, device size, and mobility constraints. To overcome this limitation, multiuser MIMO (MU-MIMO) has been considered as a practical approach that exploits multiple users in the system. By serving multiple users simultaneously, MU-MIMO can obtain spatial multiplexing gains even when each user is equipped with a single antenna or a

small number of antennas. It has been shown that the capacity of MU-MIMO downlink channels can be achieved by dirty paper coding (DPC) [1,2]. However, DPC is difficult to implement in practical systems due to its high computational complexity [3]. As low-complexity alternatives, linear transmit strategies such as zero-forcing beamforming (ZFBF) and block diagonalization (BD) have been extensively studied [4,5]. These linear schemes can provide substantial throughput gains in MIMO broadcast channels with relatively low computational complexity [6,7].

Among these linear schemes, ZFBF is particularly attractive for MU-MIMO downlink systems because it suppresses inter-user interference by constructing beamforming vectors that are orthogonal to the channel directions of the other scheduled users. However, the performance of ZFBF strongly depends on the channel directions of the selected users. When users with highly correlated channel vectors are scheduled together, the zero-forcing operation may suffer from a significant power loss. Therefore, when ZFBF is employed as the spatial division multiple access (SDMA) scheme, user scheduling plays a crucial role in exploiting the spatial multiplexing and multiuser diversity gains available in MU-MIMO downlink channels [8–10]. Semi-orthogonal user selection (SUS) is a well-known scheduling method for ZFBF-based MU-MIMO systems [8,9]. The SUS algorithm sequentially selects users whose channel vectors are nearly orthogonal to those of the previously selected users, thereby reducing the power loss caused by zero-forcing. Under perfect channel state information at the transmitter (CSIT), ZFBF combined with SUS has been shown to achieve the optimal asymptotic sum-capacity growth rate in MISO broadcast channels as the number of users increases [8]. Owing to its low complexity and favorable asymptotic performance, ZFBF-SUS has been widely considered as a practical transmission strategy for MU-MIMO downlink systems [10].

1.1. Related Work

Although ZFBF-SUS has been extensively studied for MU-MIMO downlink systems, most existing analyses assume receivers with infinite-resolution analog-to-digital converters (ADCs). This assumption becomes less suitable for practical wideband systems, where high-resolution ADCs may account for a substantial portion of the receiver power consumption and hardware cost. Accordingly, low-resolution ADCs, particularly one-bit ADCs, have been actively studied as energy-efficient receiver components [11]. Several prior works have analyzed the information-theoretic limits and achievable rates of quantized communication systems [12–16]. The capacity of the one-bit-quantized single-input single-output (SISO) Gaussian channel was studied in [12], while achievable rates for MIMO systems with one-bit ADCs were investigated in [13]. In particular, [13] derived the exact capacity of MISO channels and provided upper bounds on the capacities of SIMO and MIMO channels under perfect CSIT. Receiver architectures employing few-bit ADCs and their energy–rate tradeoffs were further investigated in [14], while the impact of limited-feedback-based partial CSIT in systems with finite-resolution ADCs was studied in [15].

A number of studies have also investigated precoding and downlink transmission in systems with quantized receiver front ends. In [17], a hybrid precoding scheme was developed for a MIMO system with one-bit ADCs at the receiver. In [18], the achievable downlink performance of a multiuser massive MIMO transceiver with low-resolution DACs at the transmitter and finite-resolution ADCs at the users was analyzed. The effects of transmit- and receive-side quantization were characterized through large-system analysis. This framework was extended in [19] to spatially correlated channels, where the regularization parameter and multiuser loading ratio of quantized massive MIMO systems were investigated. These studies mainly focus on the effects of DAC and ADC quantization and on multiuser loading, rather than channel-dependent user scheduling.

More general quantized transceiver designs have also been studied. Joint symbol-level precoding and receive combining for MIMO-OFDM systems equipped with one-bit DACs and ADCs were developed in [20]. The influence of finite-bit DACs and ADCs on indoor THz communication systems was analyzed in [21]. Although [21] considers a broader finite-resolution setting rather than focusing exclusively on one-bit ADCs, it demonstrates the importance of incorporating converter resolution into the analysis and design of high-bandwidth MIMO transceivers. Transmit-side one-bit quantization has also been investigated in multiuser downlink systems. In [22], quantized linear precoding, including ZF precoding, was analyzed for a multiuser massive MIMO downlink with one-bit DACs at the BS. The resulting quantization distortion and its effects on the signal-to-quantization-plus-interference-and-noise ratio were characterized. In [23], uplink–downlink duality was employed to jointly design downlink beamformers and power allocation under one-bit converter constraints. These studies mainly address transmit-side quantization and quantization-aware precoder design. In contrast, the present work considers one-bit ADCs at the scheduled user terminals and focuses on the scheduling-induced statistics of ZFBF-SUS. More recently, ZFBF transmission with receiver-side one-bit quantization was directly considered in [24]. In that work, a multiuser MIMO cellular downlink with limited feedback, intercell interference, and one-bit ADC receivers was analyzed, and an approximation of the achievable spectral efficiency was used to determine the optimal CSI feedback rate. However, the served users were not selected through a channel-dependent semi-orthogonal scheduling procedure. Consequently, the analysis does not account for the stage-dependent candidate sets or the selected-user order statistics generated by a specific user scheduling.

Another line of research has focused on receiver architectures and communication strategies for MIMO systems with one-bit ADCs. The fundamental limits of multiterminal MIMO communication with one-bit ADCs at the receivers were investigated in [25]. Receiver structures and communication strategies employing analog processing and one-bit ADCs were further developed in [26]. In [27], nonlinear analog operations prior to one-bit sampling and quantization were introduced to improve the capacity of MIMO systems with one-bit ADCs. In [28], a receive spatial modulation architecture was proposed for a single-user massive MIMO downlink, where one-bit ADC-based threshold detectors were employed to recover the spatial symbols. These studies provide important results on receiver design, capacity, and signal detection but do not address semi-orthogonal user scheduling in a ZFBF-based multiuser downlink.

1.2. Contributions

Despite the aforementioned progress, existing studies have not analytically characterized the ergodic sum-rate performance of downlink ZFBF-SUS when the scheduled users employ one-bit ADCs. In particular, even closely related studies on downlink systems [18,19,24] do not jointly account for the stage-dependent cardinalities of the SUS candidate sets and the order statistics of the corresponding scheduled-user channel gains. To the best of the authors' knowledge, the ergodic sum rate of downlink ZFBF-SUS with one-bit ADC receivers has not been analytically characterized by incorporating these scheduling-induced statistics. To fill this research gap, this paper derives finite-user and asymptotic approximations for the ergodic sum rate of a MU-MIMO downlink system employing ZFBF-SUS with one-bit ADC receivers. The main contributions of this paper are summarized as follows:

- This paper derives analytical approximations for the ergodic sum rate of a MU-MIMO downlink employing ZFBF-SUS with one-bit ADC receivers. To the best of the authors' knowledge, no prior work has theoretically characterized the achievable ergodic per-

formance of this combination. Since analytical performance characterization plays an important role in understanding the fundamental behavior of wireless communication systems without relying solely on Monte Carlo simulations, the proposed analysis provides valuable theoretical insights into the achievable performance of such systems.

- The derived asymptotic analysis enables accurate prediction of the achievable ergodic sum rate for systems with a large number of users. Unlike existing analyses of SUS-based systems, whose accuracy often deteriorates as the number of users increases due to the difficulty of modeling the scheduling process, the proposed approximation becomes increasingly accurate in the large-user regime. This makes the proposed analysis particularly useful for massive-user scenarios with one-bit ADCs.
- Because the derived analytical results closely approximate the actual ergodic sum rate, they can be employed to efficiently optimize the semi-orthogonality threshold, α , used in SUS scheduling. Instead of relying on exhaustive Monte Carlo simulations, the proposed analysis provides an effective means of selecting the threshold that maximizes the system performance while significantly reducing computational complexity.
- The proposed analytical framework also provides practical design guidelines for system designers and network operators by offering quantitative insight into the achievable performance limits of ZFBF-SUS systems employing one-bit ADC receivers. Such insights facilitate practical system design and parameter optimization without requiring extensive numerical simulations.

1.3. Organization

The remainder of this paper is organized as follows. Section 2 describes the system and channel models, beamforming strategy, and scheduling method. Section 3 presents the derivation of the ergodic sum rate. Section 4 provides simulation results, and Section 5 concludes the paper.

1.4. Notations

Matrices and column vectors are represented by upper-case and lower-case boldface letters, respectively. The superscript $(\cdot)^H$ denotes the Hermitian transpose, and $\|\mathbf{a}\|$ denotes the Euclidean norm of vector $\mathbf{a}$. The operators $\Pr(\cdot)$, $\mathbb{E}[\cdot]$, $\text{Re}(\cdot)$, and $\text{Im}(\cdot)$ denote probability, expectation, real part, and imaginary part, respectively. Finally, $\stackrel{d}{=}$ denotes equality in distribution.

2. System Model

We consider a downlink MIMO broadcast channel in which a base station with M transmit antennas serves K single-antenna users. The received signal vector is expressed as

$$\mathbf{y} = \mathbf{H}^H \mathbf{x} + \mathbf{w}, \quad (1)$$

where $\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_K] \in \mathbb{C}^{M \times K}$ denotes the channel matrix, $\mathbf{x} = [x_1, \dots, x_M]^T \in \mathbb{C}^{M \times 1}$ denotes the transmit signal vector, and $\mathbf{w} = [w_1, \dots, w_K]^T \in \mathbb{C}^{K \times 1}$ denotes the Gaussian noise vector. The elements of $\mathbf{w}$ are independent circularly symmetric complex Gaussian random variables with zero mean and unit variance. The channel coefficients are assumed to be independent and identically distributed circularly symmetric complex Gaussian random variables with zero mean and unit variance.

For $k = 1, \dots, K$, user k employs one-bit resolution ADCs to separately quantize the real and imaginary components of its received signal y_k . Let r_k denote the quantized signal at user k . Then, the quantization operation is described by

$$r_k = f_Q(\text{Re}\{y_k\}) + \mathbf{j}f_Q(\text{Im}\{y_k\}), \quad (2)$$

where $\mathbf{j}$ denotes the imaginary unit and the quantization function f_Q is defined as

$$v = f_Q(u) = \begin{cases} 1, & u \geq 0 \\ -1 & u < 0 \end{cases}. \quad (3)$$

2.1. User Selection

In this paper, we consider the general scenario where the number of users K is larger than the number of transmit antennas M , and ZFBF is adopted as the transmission strategy. To ensure proper interference suppression, the base station does not simultaneously serve all K users. Instead, it first selects a subset of users whose cardinality does not exceed M , and then performs spatial multiplexing for the selected users using ZFBF. For user selection, we employ the well-known SUS algorithm. SUS is one of the most widely used user selection schemes, and it is particularly suitable for ZFBF since it tends to select users with nearly orthogonal channel vectors. The basic procedure of the SUS algorithm is summarized as follows [9]:

Step (1) Initialization ($i = 1$):

$$\mathcal{T}_1 = \{1, \dots, K\}, \quad \mathcal{S} = \{\}. \quad (4)$$

Step (2) For each $k \in \mathcal{T}_i$, we calculate

$$\mathbf{g}_k = \mathbf{h}_k - \sum_{j=1}^{i-1} \frac{\mathbf{g}_{(j)}^H \mathbf{h}_k}{\|\mathbf{g}_{(j)}\|^2} \mathbf{g}_{(j)} \quad (5)$$

$$= \left(\mathbf{I} - \sum_{j=1}^{i-1} \frac{\mathbf{g}_{(j)} \mathbf{g}_{(j)}^H}{\|\mathbf{g}_{(j)}\|^2} \right) \mathbf{h}_k. \quad (6)$$

When $i = 1$, it is clear that

$$\mathbf{g}_k = \mathbf{h}_k.$$

Step (3) i -th user selection:

$$\pi(i) = \arg \max_{k \in \mathcal{T}_i} \|\mathbf{g}_k\| \quad (7)$$

$$\mathcal{S} \leftarrow \mathcal{S} \cup \{\pi(i)\} \quad (8)$$

$$\mathbf{h}_{(i)} = \mathbf{h}_{\pi(i)} \quad (9)$$

$$\mathbf{g}_{(i)} = \mathbf{g}_{\pi(i)}. \quad (10)$$

Step (4) If $|\mathcal{S}| < M$, we calculate

$$\mathcal{T}_{i+1} = \left\{ k \in \mathcal{T}_i, k \neq \pi(i) \mid \frac{|\mathbf{g}_{(i)}^H \mathbf{h}_k|}{\|\mathbf{h}_k\| \|\mathbf{g}_{(i)}\|} < \alpha \right\} \quad (11)$$

$$i \leftarrow i + 1. \quad (12)$$

If $|\mathcal{T}_{i+1}| > 0$ and $|\mathcal{S}| < M$, return to Step 2 and repeat the procedure. Otherwise, i.e., if $|\mathcal{T}_{i+1}| = 0$ or $|\mathcal{S}| = M$, terminate the algorithm. The pseudo-code of the SUS algorithm can be summarized as follows Algorithm 1:

Algorithm 1 Semi-Orthogonal User Selection for ZFBF

```

1: Initialize  $i = 1$ ,  $\mathcal{T}_1 = \{1, \dots, K\}$ , and  $\mathcal{S} = \emptyset$ .
2: while  $\mathcal{T}_i \neq \emptyset$  and  $|\mathcal{S}| < M$  do
3:    $\mathbf{g}_k = \mathbf{h}_k - \sum_{j=1}^{i-1} \frac{\mathbf{g}_{(j)}^H \mathbf{h}_k}{\|\mathbf{g}_{(j)}\|^2} \mathbf{g}_{(j)}$ ,  $k \in \mathcal{T}_i$ .
4:    $\pi(i) = \arg \max_{k \in \mathcal{T}_i} \|\mathbf{g}_k\|^2$ ,  $\mathbf{g}_{(i)} = \mathbf{g}_{\pi(i)}$ .
5:    $\mathcal{S} \leftarrow \mathcal{S} \cup \{\pi(i)\}$ .
6:   if  $|\mathcal{S}| = M$ , break.
7:   Set  $\mathcal{T}_{i+1} = \left\{ k \in \mathcal{T}_i \setminus \{\pi(i)\} : \frac{|\mathbf{g}_{(i)}^H \mathbf{h}_k|}{\|\mathbf{g}_{(i)}\| \|\mathbf{h}_k\|} < \alpha \right\}$ .
8:    $i \leftarrow i + 1$ .
9: end while

```

2.2. Transmit Signal Construction: Zero-Forcing Beamforming

In the SUS algorithm described in Section 2.1, the set of selected users is denoted by $\mathcal{S} = \{\pi(1), \dots, \pi(N_s)\}$, where $N_s \leq M$ represents the cardinality of the set $\mathcal{S}$. In this work, the users in $\mathcal{S}$ are simultaneously served in each frame via ZFBF. Accordingly, N_s independent data streams are generated, one for each scheduled user. Based on this, the transmit signal $\mathbf{x}$ is defined as follows: $\mathbf{x} = \mathbf{V}\mathbf{s}$, where $\mathbf{V} \in \mathbb{C}^{M \times N_s}$ is the beamforming matrix, $\mathbf{s} = [s_1, \dots, s_{N_s}]^T$ is the information symbol vector, whose entries are mutually independent, with s_j containing the data for j -th scheduled user $\pi(j)$. The power allocated to each signal is denoted by $P_i = \mathbb{E}[|s_i|^2]$. Defining $\mathbf{H}_\mathcal{S}$ as $\mathbf{H}_\mathcal{S} \triangleq [\mathbf{h}_{\pi(1)}, \mathbf{h}_{\pi(2)}, \dots, \mathbf{h}_{\pi(N_s)}]$, ZFBF matrix is constructed as

$$\mathbf{V} = \mathbf{H}_\mathcal{S} (\mathbf{H}_\mathcal{S}^H \mathbf{H}_\mathcal{S})^{-1}. \quad (13)$$

Figure 1 illustrates the block diagram of the system considered in this study.

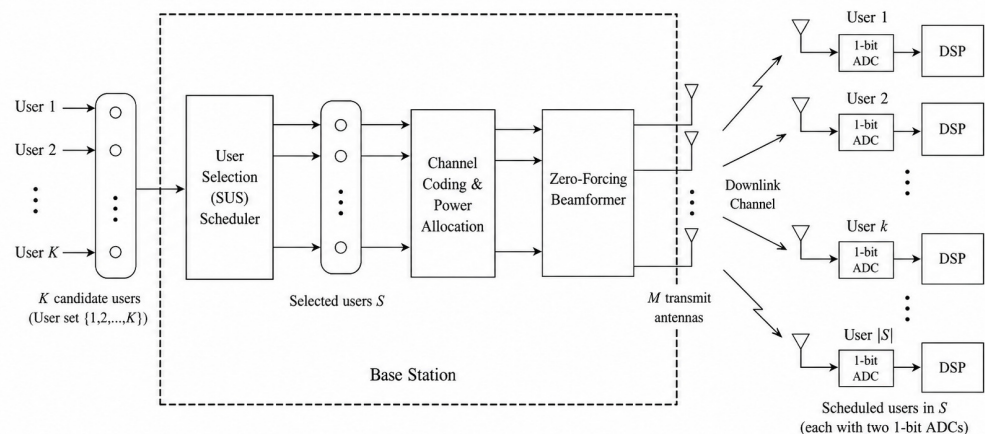


Figure 1. Block diagram of the considered MU-MIMO downlink system with SUS-based user selection, zero-forcing beamforming, and one-bit ADC receivers.

3. Performance Analysis

3.1. Sum Achievable Rate

From the definition of ZFBF matrix $\mathbf{V}$, the received signal of i -th scheduled user is given by

$$y_i = s_i + w_i, \quad (14)$$

and thus, instantaneous achievable rate of i -th scheduled user is given by [13]

$$R_i = 2 \left(1 - H_b \left(Q \left(\sqrt{P_i} \right) \right) \right), \quad (15)$$

where $H_b(\cdot)$ and $Q(\cdot)$ denote binary entropy function and Gaussian Q-function, respectively.

Accordingly, for given values of $P_1, \dots, P_{N_s}$, the sum achievable rate is

$$R(\mathcal{S}) = \sum_{i \in \mathcal{S}} \left[2 - 2H_b \left(Q \left(\sqrt{P_i} \right) \right) \right]. \quad (16)$$

3.2. Ergodic Sum-Rate Approximation

In this subsection, the main objective is to obtain an approximate expression for the achievable ergodic sum rate of ZFBF-SUS with one-bit ADCs. In particular, to enable mathematical analysis, this subsection assumes equal power allocation, i.e., $P_i \|\mathbf{v}_i\|^2 = P/N_s$ for all $i \in \mathcal{S}$. Under the equal power allocation assumption, the rate of each scheduled user in (16) can be expressed as:

$$R_i = 2 - 2H_b \left(Q \left(\sqrt{P/(N_s \|\mathbf{v}_i\|^2)} \right) \right), \quad (17)$$

where

$$\|\mathbf{v}_i\|^2 = [\mathbf{V}^H \mathbf{V}]_{i,i} = \left[(\mathbf{H}_S^H \mathbf{H}_S)^{-1} \right]_{i,i}. \quad (18)$$

Therefore, the sum rate is computed as follows:

$$R(\mathcal{S}) = \sum_{i \in \mathcal{S}} \left[2 - 2H_b \left(Q \left(\sqrt{P/(N_s \|\mathbf{v}_i\|^2)} \right) \right) \right]. \quad (19)$$

In the SUS algorithm, the parameter α controls the angle between the channel vectors of the selected users. As α decreases, the selected channel vectors become closer to being mutually orthogonal, thereby reducing the loss incurred during the zero-forcing process. Therefore, the following relationship generally holds: $\|\mathbf{h}_{\pi(i)}\|^2 \geq \frac{1}{\|\mathbf{v}_i\|^2}$. Combining this with Equation (35) in [9], we obtain the following inequality:

$$\|\mathbf{h}_{\pi(i)}\|^2 \geq \frac{1}{\|\mathbf{v}_i\|^2} \geq \frac{\|\mathbf{g}_{\pi(i)}\|^2}{1 + \frac{(M-1)^4 \alpha^2}{1 - (M-1) \alpha^2}}. \quad (20)$$

As α approaches zero in this expression, the denominator on the right-hand side converges to 1, and the numerator $\|\mathbf{g}_{\pi(i)}\|^2$ converges to $\|\mathbf{h}_{\pi(i)}\|^2$ because, as α approaches zero, the vectors of the selected users become perfectly orthogonal. Therefore, assuming that the cardinality of $\mathcal{T}_i$ is fixed, the individual user rate in (17) gradually increases as α decreases. However, if α becomes excessively small, the cardinality of the candidate user set $\mathcal{T}_i$ constructed at each step of the SUS algorithm decreases. This may reduce the final number of selected users N_s , which can eventually lead to a loss in the sum rate. Therefore, from a practical operation perspective, properly selecting the value of α can significantly affect the system performance.

However, the main objective of this study is to analyze the potential of ZFBF-SUS when combined with one-bit ADC receivers and the multiuser gain with respect to the number of users. Therefore, in the performance analysis, we assume that the number of users K is

sufficiently large. Under this assumption, even when α is chosen to be relatively small, M users can still be selected. Accordingly, by assuming sufficiently small value of α , the loss term $1/\|\mathbf{v}_i\|^2$ incurred during the ZFBF process can be approximated by $\|\mathbf{h}_{\pi(i)}\|^2$. In other words, summarizing the discussion so far, this section adopts the following assumptions for efficient mathematical performance analysis:

Two main assumptions in this section used for performance analysis:

1. Equal power allocation;
2. $\frac{1}{\|\mathbf{v}_i\|^2} \approx \|\mathbf{h}_{\pi(i)}\|^2$, provided that α is sufficiently small and K is sufficiently large.

With these assumptions, the long-term ergodic sum rate can be approximated as

$$\begin{aligned} \mathbb{E}[R(S)] &\approx \mathbb{E} \left[\sum_{i \in S} \left[2 - 2H_b \left(Q \left(\sqrt{P \|\mathbf{h}_{\pi(i)}\|^2 / N_s} \right) \right) \right] \right] \\ &\stackrel{(a)}{\leq} \sum_{i \in S} \left[2 - 2H_b \left(Q \left(\sqrt{P \mathbb{E} [\|\mathbf{h}_{\pi(i)}\|^2] / N_s} \right) \right) \right], \end{aligned} \quad (21)$$

where (a) follows from Jensen's inequality.

In general, approximations based on Jensen's inequality may become inaccurate depending on the probability distribution of the random variable of interest. However, in the case of one-bit ADCs considered in this study, the function $H_b(Q(\sqrt{\cdot}))$ that determines the data rate is a convex function and is known to be well approximated by a linear function in the low-SNR and medium-SNR regimes [13]. Therefore, Jensen's inequality used for (a) is expected to provide an accurate approximation in the low-to-moderate SNR regime, which are the main operating ranges of practical one-bit ADC systems. To further analyze the ergodic sum rate, the probability distribution of $\|\mathbf{h}_{\pi(i)}\|^2$ is required.

In the SUS algorithm, the i -th selected user $\pi(i)$ denotes the user with the largest $\|\mathbf{g}_k\|$ among the users in the set $\mathcal{T}_i$. In line with the second main assumption above, under the assumption that α is sufficiently small, we approximate $\mathbf{h} \approx \mathbf{g}$. Consequently, $\|\mathbf{h}_{\pi(i)}\|^2$ can be approximated as follows:

$$\|\mathbf{h}_{\pi(i)}\|^2 \approx \max_{k \in \mathcal{T}_i} \|\mathbf{h}_k\|^2. \quad (22)$$

Moreover, since $\|\mathbf{h}_k\|^2$ follows a Gamma distribution with parameters $(M, 1)$, the following result is obtained from maximum order statistics:

$$\mathbb{E} [\|\mathbf{h}_{\pi(i)}\|^2] \approx \mathbb{E} \left[\max_{k \in \mathcal{T}_i} \|\mathbf{h}_k\|^2 \right] \quad (23)$$

$$\stackrel{(a)}{\approx} \int_0^\infty \left[1 - \left(\frac{\gamma(M, x)}{\Gamma(M)} \right)^{\mathbb{E}[\|\mathcal{T}_i\|]} \right] dx, \quad (24)$$

where in (a), the cardinality of the set $\mathcal{T}_i$ is replaced with its expected value as it varies at each frame. Furthermore, the exact value of $\mathbb{E}[\|\mathcal{T}_i\|]$ is difficult to calculate, so we consider the following approximation:

$$\mathbb{E}[\|\mathcal{T}_i\|] = K \cdot \Pr[k \in \mathcal{T}_i] \quad (25)$$

$$\approx K \cdot \prod_{j=1}^{i-1} \Pr \left[\frac{|\mathbf{g}_{(j)}^H \mathbf{h}_k|}{\|\mathbf{h}_k\| \|\mathbf{g}_{(j)}\|} < \alpha \right] \quad (26)$$

$$\stackrel{(a)}{=} K \cdot \prod_{j=1}^{i-1} [\mathcal{B}_{1,M-1}(\alpha^2)] = K [\mathcal{B}_{1,M-1}(\alpha^2)]^{i-1} \triangleq K_i, \quad (27)$$

where $\mathcal{B}_{a,b}(\cdot)$ denotes the CDF of beta random variable with parameters a, b , and (a) follows because the square of the inner product between two independent and isotropically distributed unit vectors follows a beta distribution with parameters 1 and $M - 1$, when M is the dimension of two vectors.

By substituting these expressions into (21), the ergodic sum rate can be approximated as follows:

$$\mathbb{E}[R(S)] \approx \sum_{i \in \mathcal{S}} \left[2 - 2H_b \left(Q \left(\sqrt{P \int_0^\infty \left[1 - \left(\frac{\gamma(M, x)}{\Gamma(M)} \right)^{\lceil K_i \rceil} \right] dx / N_s} \right) \right) \right]. \quad (28)$$

Although the integral in this expression cannot be evaluated explicitly, (28) can be computed through numerical integration. This enables the analysis of the downlink ergodic performance of ZFBF-SUS when combined with receiver terminals employing one-bit ADCs.

3.3. Asymptotic Performance with Respect to K

Although the value in (28) can be computed through numerical integration as discussed in the previous subsection, numerical integration is not always guaranteed to be feasible depending on the system parameters. Therefore, in this subsection, we provide a performance analysis result that can be computed without numerical integration. However, since mathematical analysis is practically intractable in general cases, we theoretically analyze the asymptotic performance under the assumption that K is sufficiently large. To analyze the asymptotic performance of the ergodic sum rate, it is necessary to analyze the asymptotic behavior of $\mathbb{E}[\max_{k \in \mathcal{T}_i} \|\mathbf{h}_k\|^2]$. To this end, we use the maximum order statistics of Gamma random variables.

Since the number of transmit antennas M is an integer and $\|\mathbf{h}_k\|^2$ is a Gamma random variable with parameters $(M, 1)$, the PDF and CDF of $\|\mathbf{h}_k\|^2$ are respectively given by:

$$f(x) = \frac{x^{M-1}e^{-x}}{\Gamma(M)}, \quad x \geq 0 \quad (29)$$

$$F(x) = \frac{\gamma(M, x)}{\Gamma(M)} = 1 - e^{-x} \sum_{n=0}^{M-1} \frac{x^n}{n!}, \quad x \geq 0, \quad (30)$$

where $\gamma(\cdot, \cdot)$ denotes the lower incomplete gamma function. Therefore, the following holds:

$$\frac{1 - F(x)}{f(x)} = \left(e^{-x} \sum_{n=0}^{M-1} \frac{x^n}{n!} \right) \frac{\Gamma(M)}{x^{M-1}e^{-x}} \quad (31)$$

$$= \Gamma(M) \sum_{n=0}^{M-1} \frac{x^{n+1-M}}{n!}. \quad (32)$$

This implies that

$$\lim_{x \rightarrow \infty} \frac{d}{dx} \left[\frac{1 - F(x)}{f(x)} \right] = \Gamma(M) \lim_{x \rightarrow \infty} \sum_{n=0}^{M-1} \frac{(n+1-M)x^{n-M}}{n!} = 0. \quad (33)$$

Thus, by applying Theorem 10.5.2 in [29], we know that $\|\mathbf{h}_k\|^2$ belongs to a Gumbel-type distribution, and it satisfies

$$\Pr \left[\frac{M_n - b_n}{a_n} \leq x \right] = F^n(a_n x + b_n) \rightarrow e^{-e^{-x}} \text{ as } n \rightarrow \infty, \quad (34)$$

with $a_n = [nf(b_n)]^{-1}$ and $b_n = F^{-1}(1 - \frac{1}{n})$, where $M_n = \max(\|\mathbf{h}_1\|^2, \dots, \|\mathbf{h}_n\|^2)$. For the calculation of $a_n = [nf(b_n)]^{-1}$ and $b_n = F^{-1}(1 - \frac{1}{n})$, please refer to Appendix A. This further implies that $\frac{M_n - b_n}{a_n}$ converges in distribution to a random variable, say G , that follows the Gumbel distribution. That is,

$$\frac{M_n - b_n}{a_n} \xrightarrow{d} G \implies \mathbb{E}[M_n] = a_n \mathbb{E}[G] + b_n. \quad (35)$$

Based on these results, assuming sufficiently large K , we finally obtain

$$\mathbb{E} \left[\max_{k \in \mathcal{T}_i} \|\mathbf{h}_k\|^2 \right] \approx a_{|\mathcal{T}_i|} \mathbb{E}[G] + b_{|\mathcal{T}_i|} \quad (36)$$

$$\stackrel{(a)}{=} \log |\mathcal{T}_i| + (M - 1) \log \log |\mathcal{T}_i| - \log \Gamma(M) + \gamma_E \quad (37)$$

$$\approx \log K_i + (M - 1) \log \log K_i - \log \Gamma(M) + \gamma_E, \quad (38)$$

where $\gamma_E \approx 0.5772156649$ denotes the expected value of a Gumbel random variable which equals Euler's constant, and (a) follows from (A14). Substituting this into (21), we obtain the following asymptotic approximation of the sum rate:

$$\begin{aligned} & \mathbb{E}[R(S)] \\ & \approx \sum_{i \in \mathcal{S}} \left[2 - 2H_b \left(Q \left(\sqrt{P[\log K_i + (M - 1) \log \log K_i - \log \Gamma(M) + \gamma_E] / N_s} \right) \right) \right]. \end{aligned} \quad (39)$$

Remark 1. In analyzing the asymptotic behavior of Gamma random variables, one may consider Lemma 6 in [30]. However, the values of a_n and b_n derived therein do not provide an accurate approximation for the sum rate in our scenario. Therefore, this study adopts a different approach.

4. Simulation Results

In this section, the accuracy of the proposed approximations is verified by comparing the Monte Carlo simulation results for the exact sum rate in (19) with the analytical approximation derived in (39). Accordingly, Figures 2 and 3 directly compare the ergodic sum rate in (19) with the approximate expression in (39) as a function of the semi-orthogonality threshold α . The ergodic sum rate in (19) is evaluated through Monte Carlo simulations, where the optimal power allocation derived in [31] is employed to achieve the maximum attainable performance. As can be observed from the figures, at relatively low SNRs, discrepancies arise between the analytical approximation and simulation results when α is either very small or very large. These discrepancies decrease as the SNR increases. In particular, as shown in Figure 3, the analytical approximation exhibits high accuracy when $P = 5$ and $P = 10$ dB. Although some discrepancies remain for certain values of α when $P = 5$ and 10 dB, the analytical and simulation results are nearly identical in the vicinity of the optimal α .

As stated in the contribution summary in the Introduction, the performance analysis developed in this study closely approximates the actual ergodic sum rate. Therefore, the derived analytical result can also be used to determine an appropriate value of the semi-orthogonality threshold α employed in SUS. In the SUS algorithm, a smaller value of α generally reduces the performance loss incurred during ZFBF by enforcing stronger semi-orthogonality among the selected users. However, it also reduces the pool of eligible users and may consequently decrease the number of users that can be simultaneously served through ZFBF. This tradeoff may cause a performance degradation at relatively high SNRs. This explains why, in Figures 2 and 3, the sum rate initially increases with α when α is relatively small. On the other hand, once α becomes sufficiently large, full

scheduling is guaranteed. Beyond this point, the loss caused by imperfect orthogonality in ZFBF and the gain resulting from an enlarged candidate-user pool balance each other. Consequently, the sum rate no longer increases significantly and eventually saturates, as illustrated in Figures 2 and 3. Except when the number of users is small or the SNR is extremely low, e.g., below 0 dB, the actual sum rate obtained through simulations is close to the analytical approximation derived in the previous section. Thus, the optimal value of α can be identified directly from the analytical performance expression without conducting computationally intensive Monte Carlo simulations. This constitutes another important advantage of the proposed analytical approximation.

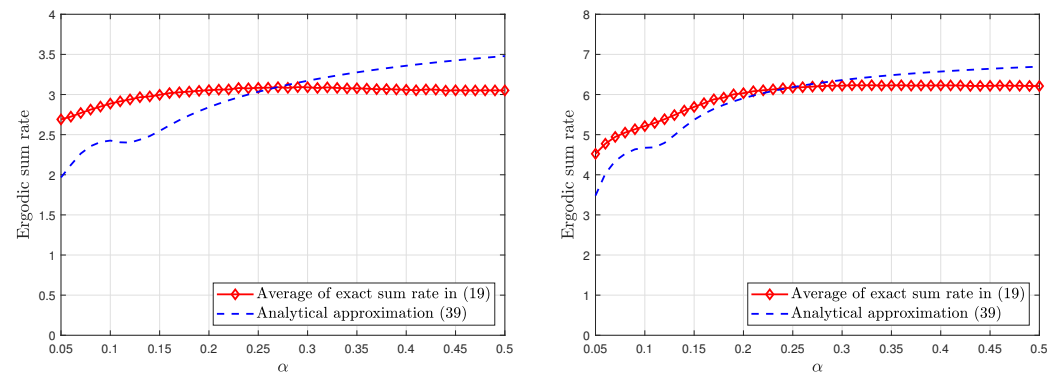


Figure 2. Ergodic sum rate vs. α with $M = 4$, $K = 50,000$. $P = -5$ dB for the left figure and $P = 0$ dB for the right figure.

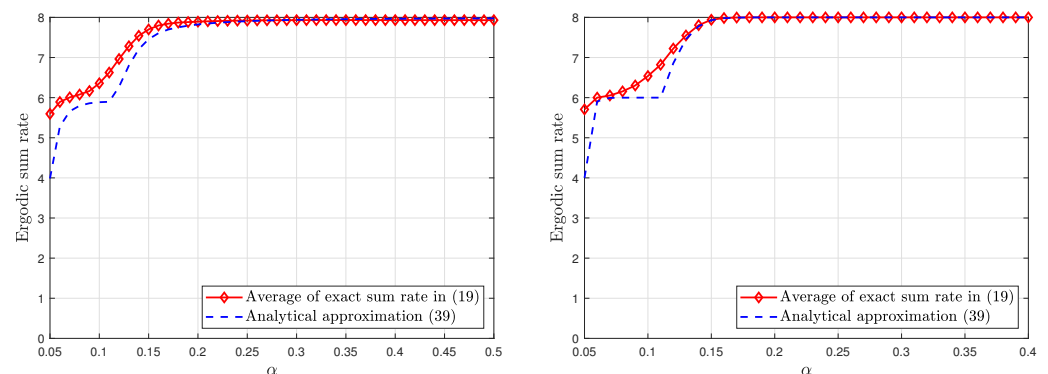


Figure 3. Ergodic sum rate vs. α with $M = 4$, $K = 50,000$. $P = 5$ dB for the left figure and $P = 10$ dB for the right figure.

Furthermore, after the onset of saturation, which occurs near $\alpha = 0.2$ in Figures 2 and 3, the sum rate either slightly decreases or remains nearly constant. Therefore, selecting α near the saturation point makes it possible to maintain near-optimal performance while satisfying the assumptions adopted in the performance analysis, including the assumptions that K is sufficiently large and that the optimal value of α can be sufficiently small. The left-hand graph of Figure 4 plots, as a function of the number of users, the minimum value of α at which saturation begins, where the saturation point is defined as the smallest α that achieves a sum rate within 1% of the maximum sum rate. As expected, in the moderate- and high-SNR regimes, a smaller value of α is sufficient to achieve near-optimal performance as the total number of users increases. In the left-hand graph of Figure 4, the theoretically predicted optimal values of α , obtained using the analytical approximation developed in this study, are marked by diamonds. At a relatively low SNR of 5 dB, a slight difference is observed between the actual optimal value and its analytical estimate. Nevertheless, as shown in Figure 3, the sum rate as a function of α becomes nearly saturated after reaching

its optimum. Therefore, the difference between the two values of α causes only a negligible difference in the achievable sum rate, as confirmed by the right-hand graph of Figure 4, where the sum-rate loss remains within the predefined tolerance of 1%.

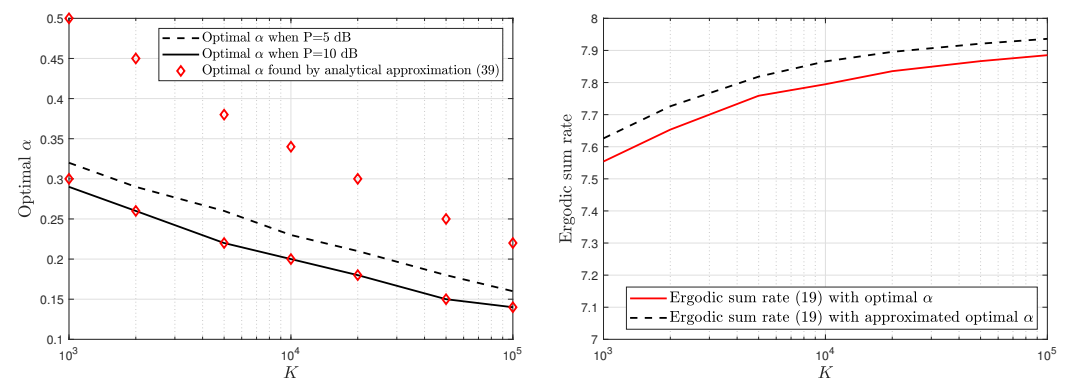


Figure 4. Optimal values of α and their approximation (diamond markers) are shown in the left figure for $M = 4$ and $P = 5$ and 10 dB. The right figure shows the ergodic sum rate versus K using the optimal α obtained from the left figure for $M = 4$ and $P = 5$ dB.

Next, Figure 5 experimentally validates the accuracy of the approximation for $\mathbb{E}[|\mathcal{T}_i|]$ employed in the mathematical derivation of the ergodic sum rate. The definition of K_i is given in (27). The red solid curves represent $\mathbb{E}[|\mathcal{T}_i|]$, whereas the black dashed curves represent K_i . As can be observed from the figure, K_i closely approximates the average size of the SUS candidate-user pool $\mathbb{E}[|\mathcal{T}_i|]$ when K is sufficiently large. The subsequent figures verify the accuracy of the derived ergodic sum-rate approximation as a function of P and K , respectively. The value of α used in these figures is selected near the onset of saturation so that the approximations introduced in the preceding sections remain sufficiently accurate.

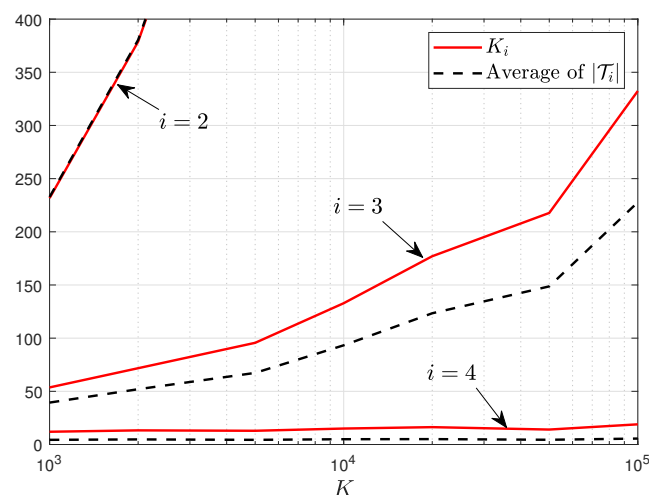


Figure 5. Average of $|\mathcal{T}_i|$ vs. K when $M = 4$ and $P = 10$ [dB].

Figures 6 and 7 compare the analytical approximations of the ergodic sum rate derived in Sections 3.2 and 3.3, given in (28) and (39), respectively, with the ergodic average of the actual sum rate. In Figure 6, we set $K = 20,000$, assuming that a sufficiently large number of users exists in the system. Therefore, the approximations used in the analysis are generally accurate, and the analytical results provide good approximations regardless of the value of P (SNR). Figure 7 plots the ergodic sum rate as a function of the number of users, thereby examining the impact of the number of users on the accuracy of the performance analysis. It can be observed that when the number of users is relatively small

and the number of transmit antennas is large, the error of the analytical approximation in (39) becomes relatively large. This is because the estimated cardinality of $\mathcal{T}_i$ used in deriving the approximation may become less accurate as the number of antennas increases. Therefore, when $M = 2$, almost no approximation error is observed.

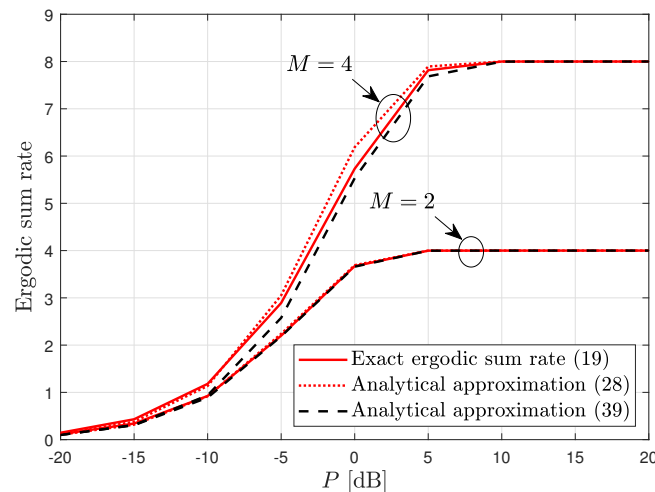


Figure 6. Ergodic sum rate vs. P with $M = 2, 4$, $\alpha = 0.2$, $K = 20,000$.

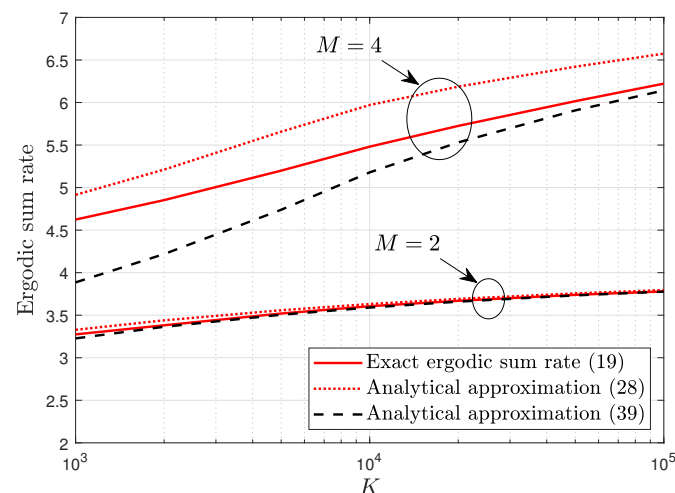


Figure 7. Ergodic sum rate vs. K with $M = 2, 4$, $P = 0$ [dB], $\alpha = 0.2$.

5. Conclusions

This paper analyzed the ergodic sum-rate performance of ZFBF-SUS in MU-MIMO downlink channels with one-bit ADC receivers. To this end, we first formulated the achievable rate of each scheduled user under one-bit quantization and ZFBF transmission. For analytical tractability, we adopted equal power allocation and approximated the channel gain of the scheduled user when the selected users are nearly orthogonal and the number of users is sufficiently large. Based on these approximations, we derived an analytical expression for the ergodic sum rate by using maximum order statistics of Gamma random variables and by approximating the cardinality of the SUS candidate user set. In addition, an asymptotic sum-rate expression with respect to the number of users was derived by applying extreme value theory. Simulation results verified that the derived analytical expressions closely approximate the actual ergodic sum rate, particularly when the number of users is large and the SUS threshold is properly chosen. These results provide useful ana-

lytical insight into the performance of ZFBF-SUS with one-bit ADC receivers in MU-MIMO downlink systems.

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Appendix A. Calculation of a_n and b_n in (34)

Let $\bar{F}(x) = 1 - F(x)$. Then, for the Gamma distribution,

$$\bar{F}(x) = \frac{\Gamma(M, x)}{\Gamma(M)}, \quad (\text{A1})$$

where $\Gamma(M, x)$ is the upper incomplete gamma function. Using the well-known asymptotic expansion, we have

$$\Gamma(M, x) = x^{M-1}e^{-x} \left(1 + \frac{M-1}{x} + O(x^{-2}) \right), \quad (\text{A2})$$

and thus obtain

$$\bar{F}(x) = \frac{x^{M-1}e^{-x}}{\Gamma(M)} (1 + o(1)). \quad (\text{A3})$$

As b_n is given by $F^{-1}(1 - \frac{1}{n})$ from Theorem 10.5.2. in [29], it should satisfy

$$F(b_n) = 1 - \frac{1}{n} \iff n(\bar{F}(b_n)) = 1. \quad (\text{A4})$$

Combining (A3) and (A4), and Substituting the tail approximation gives

$$n \frac{b_n^{M-1}e^{-b_n}}{\Gamma(M)} \approx 1. \quad (\text{A5})$$

Taking logarithms yields

$$\log n + (M-1) \log b_n - b_n - \log \Gamma(M) \approx 0. \quad (\text{A6})$$

Hence

$$b_n = \log n + (M-1) \log b_n - \log \Gamma(M). \quad (\text{A7})$$

Now observe that

$$b_n \sim \log n. \quad (\text{A8})$$

Therefore,

$$\log b_n = \log \log n + o(1). \quad (\text{A9})$$

Substituting this into (A7) gives

$$b_n = \log n + (M - 1) \log \log n - \log \Gamma(M) + o(1). \quad (\text{A10})$$

Second, we calculate $a_n = [nf(b_n)]^{-1}$ as

$$a_n = \frac{1}{nf(b_n)} \stackrel{(a)}{=} \frac{\bar{F}(b_n)}{f(b_n)}, \quad (\text{A11})$$

where (a) follows from (A4).

Combining (29), (A1), and (A2), we have

$$a_n = \frac{\bar{F}(b_n)}{f(b_n)} = 1 + \frac{M - 1}{b_n} + O(b_n^{-2}). \quad (\text{A12})$$

From (A10), as $b_n = \log n + (M - 1) \log \log n - \log \Gamma(M) + o(1)$ and it goes to infinity as n increases, a_n can be written as

$$a_n = 1 + O\left(\frac{1}{\log n}\right). \quad (\text{A13})$$

Thus, assuming sufficiently large n , a_n and b_n can be approximated as

$$a_n \approx 1, \quad b_n \approx \log n + (M - 1) \log \log n - \log \Gamma(M). \quad (\text{A14})$$

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