

## Article

# Procurement Strategies with Inventory Swapping and Spot Market Under Demand Uncertainty

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## Abstract

Demand uncertainty and supply imbalances pose persistent challenges for procurement decisions. In practice, firms often mitigate such mismatches through inventory swapping and spot market procurement. Motivated by this observation, this paper develops a procurement model that integrates long-term wholesale procurement, ex-post inventory swapping, and spot-market sourcing within a unified framework. While prior studies have largely examined spot-market procurement and inventory swapping separately, little attention has been paid to how these two mechanisms jointly mitigate demand uncertainty. The analysis yields three main findings. First, inventory swapping remains economically valuable even when spot-market procurement is available, indicating that the two mechanisms are not pure substitutes. Second, the economic value of inventory swapping increases with higher spot-market prices and greater demand volatility, as reallocating existing inventory becomes more cost-effective than emergency spot-market procurement. Third, the impact of the partner buyer's order quantity depends critically on the swap transfer price, with lower transfer prices strengthening the value of inventory swapping. These findings contribute to the procurement and supply flexibility literature by clarifying how multiple procurement channels interact under demand uncertainty. The study also provides managerial implications for designing procurement systems that integrate inventory swapping with spot-market procurement.

**Keywords:** procurement strategies; inventory swapping; spot market; supply chain risk management

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## 1. Introduction

In recent years, firms operating in volatile supply chains, such as those in the energy, high-tech component, and commodity sectors, have increasingly relied on multiple procurement channels to cope with demand uncertainty, supply imbalances, and rapidly fluctuating market conditions. A representative example can be found in Japan's liquefied natural gas (LNG) market [1]. Firms typically secure LNG cargoes through long-term contracts under normal market conditions. However, when supply becomes constrained and demand increases, they procure additional cargoes from alternative procurement channels. One complementary approach is cargo swapping, under which cargoes are received from other firms when needed and equivalent cargoes are returned at a later date. Kansai Electric,

for example, issued an LNG cargo swap tender to receive a prompt cargo in exchange for delivering another cargo at a later date, thereby securing near-term supply without purchasing an additional cargo outright. Another important procurement channel is spot-market procurement, through which firms can obtain additional cargoes to satisfy urgent requirements. Although this channel provides immediate sourcing flexibility, spot-market prices are often high and highly volatile. More importantly, the ability to use the spot market as an immediate and economically viable emergency sourcing channel may differ across firms because of differences in trading scale, contractual flexibility, terminal access, credit capacity, and established trading relationships. An IEA–KEEI report documents that, among more than 200 general gas utilities, only nine imported LNG through their own terminals, whereas the remaining utilities purchased gas from LNG importers or domestic producers [2]. The report indicates that firms without their own terminals must rely on third-party terminal access or wholesale suppliers, as minimum cargo sizes, limited terminal capacity, and regional pipeline constraints may make direct spot procurement economically unviable. Consequently, firms with more limited spot-market access may depend more heavily on bilateral swapping arrangements to manage short-term supply–demand imbalances. These observations suggest that inventory swapping and spot-market procurement should be viewed as complementary rather than mutually exclusive procurement mechanisms.

The availability of multiple procurement channels, including long-term supply contracts, inventory-swapping arrangements, and spot-market procurement, gives rise to a broader procurement problem. Each procurement channel has distinct advantages and limitations. Long-term supply contracts provide relatively stable procurement at lower cost, but they lack flexibility in responding to demand uncertainty. Inventory swapping provides an additional source of supply flexibility; however, its effectiveness depends on the extent to which one buyer’s surplus inventory matches another buyer’s shortage. Spot-market procurement effectively alleviates supply shortages but is often associated with high and volatile prices, exposing buyers to substantial procurement cost risk. These trade-offs suggest that procurement decisions should account for interactions among multiple procurement channels rather than evaluating each channel independently.

While these procurement mechanisms are widely observed in practice, they have rarely been examined within a unified analytical framework. In particular, inventory swapping and spot-market procurement represent fundamentally different forms of procurement flexibility. Spot-market procurement provides external recourse through market-based replenishment, whereas inventory swapping relies on cooperative inventory sharing among buyers. Ignoring their interaction may lead to misleading evaluations of procurement strategies by overstating the value of spot-market sourcing or understating the role of cooperative inventory reallocation. As a result, the existing literature provides only a partial understanding of how multiple procurement channels jointly influence procurement decisions under demand uncertainty.

To address this research gap, we develop an integrated analytical framework that jointly incorporates long-term wholesale procurement, ex-post buyer-to-buyer inventory swapping, and spot-market sourcing under demand uncertainty. Within this framework, we investigate how inventory swapping and spot-market sourcing interact in shaping optimal procurement decisions. Specifically, we examine whether swap-based cooperation substitutes for or complements spot-market sourcing, how the value of swapping changes with spot prices and demand uncertainty, and how spot-market access affects the buyer’s optimal procurement decisions. This study makes three main contributions to the literature. First, to the best of our knowledge, this study is the first to develop an integrated analytical framework that jointly models long-term wholesale procurement, buyer-to-buyer inventory swapping, and spot-market sourcing within a unified newsvendor setting. Second, we

demonstrate the economic role of inventory swapping when the spot market is available, showing that swapping functions not only as an alternative source of flexibility but also as a complementary mechanism that mitigates reliance on costly spot-market purchases. Third, we show that the impact of the counterpart buyer's order quantity on the focal buyer's expected profit depends critically on the swap transfer price.

The remainder of this paper is organized as follows. Section 2 reviews the literature on spot-market procurement and inventory swapping and positions the present study within the existing literature. Section 3 presents the analytical models and the underlying assumptions used throughout the analysis. Section 4 presents the numerical analysis and discusses managerial insights derived from the results, illustrating the effects of key parameters across various scenarios. Finally, Section 5 concludes the paper by summarizing the main findings, discussing their implications, and suggesting directions for future research.

## 2. Literature Review

Relevant studies can be broadly divided into two streams: spot-market procurement and inventory swapping. The first stream examines how firms combine long-term or contractual sourcing with spot-market recourse under uncertainty. The second investigates how lateral cooperation through inventory reallocation can mitigate mismatch risks among firms. The present study lies at the intersection of these two streams.

### 2.1. Spot Market Procurement

The spot-market procurement literature emphasizes the flexibility of external emergency sourcing, but also highlights the associated price risk. Inderfurth and Kelle [3] examine capacity reservation decisions when firms can rely on the spot market after uncertainty is resolved. Fu et al. [4] analyze portfolio procurement with option contracts and random spot prices, showing that spot-market sourcing can serve as a valuable but risky recourse mechanism. Merzifonluoglu [5,6] further develops supply portfolio models that incorporate demand volatility, supply unreliability, risk aversion, and spot-market uncertainty. Hou et al. [7] develop a mathematical model to determine the retailer's optimal procurement strategy, considering an imperfect spot market and fluctuating spot prices, which demonstrates the importance of a comprehensive portfolio procurement strategy that incorporates forward contracts alongside option contracts and the spot market. Fleuren et al. [8] develop a procurement portfolio model in which a supplier faces uncertain customer demand and can rely on an uncertain spot market for expediting. They examine the joint effects of raw material procurement, customer selection, and capacity reservation on supply chain performance. Their study highlights the strategic value of spot-market recourse under uncertainty but does not consider cooperative inventory reallocation among buyers. Talaba and Koo [9] address portfolio procurement strategies in which forward and option contracts are integrated with spot-market access. They demonstrate that incorporating the spot market significantly enhances procurement flexibility and profitability, particularly in environments characterized by high demand uncertainty and price volatility. Collectively, this literature highlights the role of spot-market sourcing as a flexible but potentially costly recourse mechanism whose value depends on price uncertainty and advance procurement decisions. Existing studies primarily focus on portfolio procurement decisions with spot-market access and do not consider buyer-to-buyer inventory swapping as an additional post-demand recourse mechanism. To the best of our knowledge, no existing study has jointly examined procurement portfolio decisions that integrate both spot-market sourcing and inventory swapping within a unified analytical framework.

## 2.2. Inventory Swapping

Inventory swapping is closely related to lateral inventory transshipment, which involves reallocating inventory from surplus locations to shortage locations after demand uncertainty is realized. Prior studies show that lateral transshipment can reduce inventory mismatches, improve service levels, and generate inventory pooling benefits in multi-location inventory systems [10,11]. These studies provide an important operational foundation for understanding how ex-post inventory reallocation can mitigate demand–supply mismatches. Paterson et al. [12] provide a comprehensive review of inventory models with lateral transshipments. Traditional transshipment studies mainly focus on operational stock redistribution among locations, warehouses, or retailers, where the key decisions concern when and how much inventory to transfer. In contrast, inventory swapping in this study refers to buyer-to-buyer inventory reallocation under a predetermined swap transfer price. Therefore, swapping involves not only physical inventory reallocation but also bilateral profit incentives, transfer-price effects, and participation considerations between independent buyers.

A related line of research examines inventory swapping and lateral cooperation in settings characterized by high logistics costs, volatile demand, or geographically dispersed energy and resource allocation. Several studies investigate inventory swapping in energy, petroleum, and geographically dispersed logistics systems [13–18]. Collectively, they demonstrate that swapping reduces mismatch costs, transportation costs, and operational inefficiencies. In contrast, our study focuses on flexible procurement strategies in which buyers can use both inventory swapping and spot-market sourcing after demand realization.

Only a limited number of studies connect swapping to the supply contract literature. Park [19] examines how competing firms can benefit from structured inventory-swapping arrangements that guarantee positive transfer quantities and improve profits. Ryu and Koo [20] integrate swap agreements with long-term wholesale contracts and demonstrate that optimal order quantities differ from those under traditional wholesale settings. However, these studies do not explicitly incorporate spot-market recourse into the procurement framework. As a result, they cannot fully capture how internal inventory reallocation and external market procurement jointly shape post-demand adjustment behavior. More importantly, because spot-market sourcing and inventory swapping represent fundamentally different forms of procurement flexibility, external market-based recourse versus cooperative inventory sharing, analyzing them separately may lead to biased assessments of flexibility value and suboptimal characterization of procurement behavior under uncertainty.

The present paper addresses this gap by jointly modeling long-term wholesale procurement, inventory swapping, and spot-market sourcing within a single analytical framework. Unlike the spot-market literature, which typically treats buyers as isolated decision makers, our model allows buyer-to-buyer cooperation after demand realization. In contrast to the swap literature, which often abstracts from external emergency sourcing, our framework explicitly incorporates spot-market sourcing as an additional recourse channel. By explicitly analyzing the interaction between inventory swapping and spot-market sourcing under demand uncertainty, this paper provides a more comprehensive understanding of collaborative procurement decisions. Table 1 summarizes the positioning of the present study relative to representative works in the literature.

**Table 1.** Comparison between this study and existing studies.

| Literature               | Contract Type    | Spot Market | Post-Demand Procurement Mechanism | Decision Variables   |
|--------------------------|------------------|-------------|-----------------------------------|----------------------|
| Inderfurth and Kelle [3] | Option           | ✓           | Spot-market sourcing              | Capacity reservation |
| Hou et al. [7]           | Forward + Option | ✓           | Spot-market sourcing              | Option quantity      |

Table 1. Cont.

| Literature         | Contract Type    | Spot Market | Post-Demand Procurement Mechanism         | Decision Variables                    |
|--------------------|------------------|-------------|-------------------------------------------|---------------------------------------|
| Fleuren et al. [8] | Wholesale        | ✓           | Spot-market sourcing                      | Order quantity and customer selection |
| Talaba and Koo [9] | Forward + Option | ✓           | Spot-market sourcing                      | Order quantity + option quantity      |
| Park [19]          | Swap             | -           | Inventory swapping                        | Swapping quantity                     |
| Ryu and Koo [20]   | Wholesale + swap | -           | Inventory swapping                        | Order quantity                        |
| This work          | Wholesale + swap | ✓           | Inventory swapping + Spot-market sourcing | Order quantity                        |

### 3. Model Description

This paper studies the procurement strategy of a buyer operating in a multi-channel sourcing environment characterized by uncertain demand and long supply lead times. The supply chain consists of suppliers and multiple buyers procuring the same product. We focus on a representative buyer (hereafter, Buyer 1) and investigate how long-term wholesale procurement, inventory swapping, and spot-market procurement can be integrated to determine an optimal procurement strategy under demand uncertainty. Buyer 1 procures a baseline quantity through a long-term wholesale contract with a supplier. After demand is realized, shortages may be mitigated through peer-to-peer inventory swapping with another buyer (Buyer 2) procuring the same product. If any residual shortages remain after inventory swapping, Buyer 1 can procure additional units from the spot market. Buyer 2 is modeled as the counterpart in the inventory-swapping mechanism, whose procurement behavior is characterized solely by its wholesale order quantity and realized demand rather than by an additional procurement optimization problem. This modeling choice enables us to isolate the effects of inventory swapping and spot-market procurement on Buyer 1's ordering decision. Such an asymmetric setting is also consistent with many practical situations in which buyers differ in their effective access to spot-market procurement because of differences in market access, firm size, geographical restrictions, or transaction costs. Spot-market access is treated as an exogenous buyer characteristic because it depends on pre-existing infrastructure, trading scale, credit arrangements, and established market relationships that are beyond the scope of the single-period procurement decision. Accordingly, spot-market procurement is explicitly incorporated only for Buyer 1, allowing the analysis to focus on Buyer 1's procurement decisions while maintaining analytical tractability.

Buyer 1's demand is modeled as a random variable  $X$ , with probability density function (pdf)  $f_X(x)$  and cumulative distribution function (cdf)  $F_X(x)$ . It is assumed that Buyer 1 has information about the counterparty buyer's stochastic demand  $Y$ , characterized by pdf  $f_Y(y)$  and cdf  $F_Y(y)$ , as well as the counterparty's planned order quantity  $Q_2$ . The unit wholesale price from the supplier is denoted by  $w$ , and the retail price is  $p$ . A pre-specified swap price  $r$  governs transactions between buyers in the peer-to-peer reallocation channel. In addition, a spot market is available, where the unit price  $s$  is represented by an exogenous expected unit price  $\mu_s$ . All price parameters are assumed to be exogenously given. To ensure the economic relevance of the long-term wholesale contract, we assume that the price parameters satisfy  $p > \mu_s > w$ . The sequence of events of the proposed model is described as follows:

1. Ordering Stage: Buyer 1 determines its optimal wholesale order quantity  $Q_1$  based on demand forecasts.
2. Demand Realization Stage: Buyer 1's demand is realized. The buyer first uses its own inventory (through advance wholesale procurement) to satisfy the demand.
3. Swapping Stage: Inventory imbalances are identified. If one buyer has surplus inventory while the other experiences a shortage, the two parties execute a bilateral inventory swap at transfer price  $r$ .

4. Spot Procurement Stage: After the swapping stage, Buyer 1 procures additional units from the spot market to cover the residual shortage  $(X - Q_1 - q_1)^+$ , where  $(a)^+ = \max(a, 0)$ .

This sequence establishes a hierarchical procurement structure in which inventory pooling through swapping is prioritized, followed by spot procurement available to Buyer 1. Table 2 summarizes the key notations used in the paper.

**Table 2.** Notations and descriptions.

| Notations | Descriptions                                                         |
|-----------|----------------------------------------------------------------------|
| $w$       | Per-unit wholesale price                                             |
| $s$       | Per-unit spot market purchase price ( $\mu_s = E(s)$ )               |
| $p$       | Per-unit retail selling price to end customers                       |
| $r$       | Per-unit inventory swap price between buyers                         |
| $X$       | Market demand faced by Buyer 1, with pdf $f_X(x)$ and cdf $F_X(x)$   |
| $Y$       | Market demand faced by Buyer 2, with pdf $f_Y(y)$ and cdf $F_Y(y)$   |
| $Q$       | Order quantity procured from a supplier under the wholesale contract |
| $q$       | Swap quantity between collaborating buyers                           |
| $\pi$     | Profit function                                                      |
| $i$       | Index for buyers, $i = 1, 2$                                         |

### 3.1. Wholesale Contract with Spot Market (SPOT)

Under a traditional wholesale contract without spot-market access (WH), the buyer follows a standard newsvendor-type ordering framework: the buyer places an order before demand realization and sells the available inventory to the market. Any unmet demand is lost. This setting serves as the natural starting point for procurement analysis. When the procurement quantity is  $Q$  and the demand is  $X$ , the sales amount is  $\min(X, Q)$  and the stock-out amount is  $\max(X - Q, 0)$ . Then, the buyer's profit is  $\pi_1^{WH} = p \min(X, Q) - wQ - g \max(X - Q, 0)$  where  $g$  is per-unit stock-out penalty cost. The corresponding optimal order quantity follows directly from the standard newsvendor solution:  $Q^* = F_x^{-1}(\frac{p+g-w}{p+g})$ .

In this study, we extend the traditional wholesale benchmark by allowing Buyer 1 to procure additional units from the spot market after demand realization. This benchmark structure is adopted to evaluate the incremental value of inventory swapping introduced in the following subsection. Specifically, Buyer 1 places an order quantity  $Q_1$  with the upstream supplier at the wholesale price  $w$ . After demand realization, Buyer 1 sells products at the retail price  $p$ . If realized demand exceeds the initial order quantity, the buyer purchases additional units from the spot market at unit price  $s$ . The realized profit of Buyer 1 under the wholesale contract with spot-market access is given by:

$$\pi_1^{SPOT} = p \min(X, Q) + (p - s)(X - Q)^+ - wQ \quad (1)$$

The first term represents revenue from advance procurement, the second term captures the contribution of spot-market procurement, and the final term denotes the wholesale purchasing cost. The expected profit function of Buyer 1 is given by:

$$E[\pi_1^{SPOT}] = p \left( Q - \int_0^Q F(x) dx \right) + (p - \mu_s) \int_Q^\infty (1 - F(x)) dx - wQ \quad (2)$$

Note that  $E[\min(X, Q)] = Q - \int_0^Q F(x) dx$ ,  $E[s] = \mu_s$ ,  $E[(X - Q)^+] = \int_Q^\infty (1 - F(x)) dx$ , and spot price  $s$  and demand  $x$  are independent of each other. Now, the first-order derivative with respect to  $Q$  is given by:

$$\frac{dE[\pi_1^{SPOT}]}{dQ} = (\mu_s - w) - \mu_s F(Q) \quad (3)$$

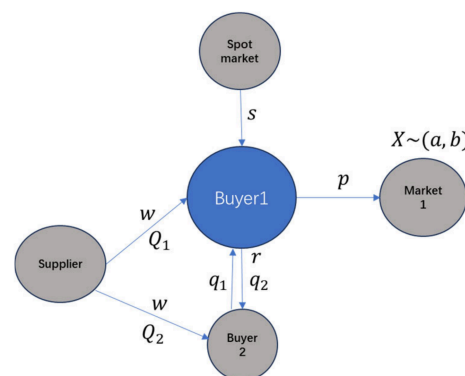
Since the second-order derivative with respect to  $Q$  is negative,  $\frac{d^2E[\pi_1^{SPOT}]}{dQ^2} = -\mu_s f(Q) < 0$ , the expected profit function is concave in the order quantity. Then, the optimal order quantity can be obtained by solving the first-order condition:

$$Q_{SPOT}^* = F_x^{-1}\left(\frac{\mu_s - w}{\mu_s}\right) \quad (4)$$

This benchmark solution provides a baseline against which the benefits of inventory swapping can be evaluated.

### 3.2. Inventory Swapping with the Spot Market (SPSW)

We next examine a procurement setting in which the two buyers are allowed to transfer their remaining inventories via lateral inventory swapping after demand realization, while Buyer 1 has access to the spot market as the final procurement channel. If one buyer experiences a shortage while the other has surplus inventory, the surplus units can be transferred through swapping at a predetermined transfer price  $r$ . Figure 1 illustrates the supply chain structure considered in this study.



**Figure 1.** The supply chain structure under the spot market and swap contract.

Let  $q_1$  denote the quantity transferred from Buyer 2 to Buyer 1 and  $q_2$  denote the quantity transferred from Buyer 1 to Buyer 2. Based on the realized demands  $x$  and  $y$ , the swap quantities can be expressed as:

$$q_1 = \min\left((x - Q_1)^+, (Q_2 - y)^+\right); q_2 = \min\left((Q_1 - x)^+, (y - Q_2)^+\right) \quad (5)$$

Buyer 1 pays  $rq_1$  when receiving products from Buyer 2, whereas Buyer 2 pays  $rq_2$  when the inventory transfer occurs in the opposite direction. After the swapping stage, Buyer 1 may procure additional units from the spot market at price  $\mu_s$  to satisfy the residual shortage  $(X - Q_1 - q_1)^+$ . To evaluate the economic value of the swapping mechanism, we next derive the expected swap quantities. By applying the tail integration method and exploiting the independence between  $X$  and  $Y$ , the expected swap supply can be expressed as follows:

$$E(q_1) = \int_0^\infty P(x > Q_1 + t, y < Q_2 - t) dt = \int_0^\infty P(x > Q_1 + t) \cdot P(y < Q_2 - t) dt \quad (6)$$

This can be transformed in terms of the distribution function as follows:

$$E(q_1) = \int_0^\infty [1 - F_X(Q_1 + t)] F_Y(Q_2 - t) dt \quad (7)$$

Similarly, we obtain:

$$E(q_2) = \int_0^\infty F_X(Q_1 - t)[1 - F_Y(Q_2 + t)]dt \quad (8)$$

After the swapping stage, Buyer 1 may still face a residual shortage if the incoming swapped quantity is insufficient to fully cover its initial demand. The spot procurement quantity can be denoted by  $(X - Q_1 - q_1)^+$ . Now, for the supply chain environment with procurement through long-term wholesale, inventory swapping, and the spot market, Buyer 1's profit can be expressed as follows:

$$\pi_1^{SPSW} = p \min(X, Q_1) - wQ_1 + (p - s)(X - Q_1 - q_1)^+ + (p - r)q_1 + rq_2 \quad (9)$$

The first term represents the revenue from the initial order, the second term accounts for the purchase cost under wholesale procurement, the third term signifies the revenue from the spot procurement, and the last two terms represent the revenue from inbound and outbound swapping inventory. Buyer 1's expected profit is expressed by:

$$E[\pi_1^{SPSW}] = p \left( Q_1 - \int_0^{Q_1} F_X(x)dx \right) - wQ_1 + (p - \mu_s)E(X - Q_1 - q_1)^+ + (p - r)E(q_1) + rE(q_2) \quad (10)$$

A key structural property of our swapping mechanism allows for an analytical simplification. Because the inbound swap quantity  $q_1$  is strictly bounded by the initial shortage  $(X - Q_1)^+$ , the spot procurement quantity can be restated as follows:  $(X - Q_1 - q_1)^+ = (X - Q_1)^+ - q_1$ . With  $E[(X - Q_1)^+] = \mu_X - Q_1 + \int_0^{Q_1} F_X(x)dx$ , Buyer 1's expected profit can be simplified as follows:

$$E[\pi_1^{SPSW}] = (\mu_s - w)Q_1 + (p - \mu_s)\mu_X - \mu_s \int_0^{Q_1} F_X(x)dx + (\mu_s - r)E(q_1) + rE(q_2) \quad (11)$$

For Buyer 2, the realized profit and the expected profit can be expressed as follows:

$$\pi_2^{SPSW} = p \min(Y, Q_2) - wQ_2 + (p - r)q_2 + rq_1 - g \max(Y - Q_2 - q_2, 0) \quad (12)$$

$$E[\pi_2^{SPSW}] = p \left( Q_2 - \int_0^{Q_2} F_Y(y)dy \right) - wQ_2 + (p - r)E(q_2) + rE(q_1) - g(\mu_Y - Q_2 - E(q_2) + \int_0^{Q_2+q_2} F_Y(y)dy) \quad (13)$$

**Proposition 1.** Buyer 1's expected profit under the SPSW mechanism is concave in  $Q_1$  when  $0 \leq r \leq \mu_s$ .

**Proof.** We prove the concavity of Buyer 1's expected profit by deriving its second derivative with respect to  $Q_1$  and showing that it is non-positive over the relevant domain. From expression (12), the first and second derivatives of Buyer 1's expected profit are given by:

$$\frac{dE[\pi_1^{SPSW}]}{dQ_1} = (\mu_s - w) - \mu_s F_X(Q_1) + (\mu_s - r) \frac{\partial E[q_1]}{\partial Q_1} + r \frac{\partial E[q_2]}{\partial Q_1} \quad (14)$$

$$\frac{d^2 E[\pi_1^{SPSW}]}{dQ_1^2} = -\mu_s f_X(Q_1) + (\mu_s - r) \frac{d^2 E[q_1]}{dQ_1^2} + r \frac{d^2 E[q_2]}{dQ_1^2} \quad (15)$$

We next derive the first and second derivatives of  $E[q_1]$  and  $E[q_2]$  with respect to  $Q_1$ . From Equation (7), the first derivative is given as follows:

$$\frac{dE[q_1]}{dQ_1} = - \int_0^\infty f_X(Q_1 + t)F_Y(Q_2 - t)dt \quad (16)$$

Letting  $v = Q_1 + t$ , we can rewrite the expression as follows:

$$\frac{dE[q_1]}{dQ_1} = - \int_{Q_1}^{\infty} f_X(v)F_Y(Q_1 + Q_2 - v) dv \quad (17)$$

Applying Leibniz's rule, the second derivative of  $E[q_1]$  can be expressed as follows:

$$\frac{d^2E[q_1]}{dQ_1^2} = f_X(Q_1)F_Y(Q_2) - \int_{Q_1}^{\infty} f_X(v)f_Y(Q_1 + Q_2 - v) dv \quad (18)$$

Differentiating  $E[q_2]$  with respect to  $Q_1$  yields:

$$\frac{dE[q_2]}{dQ_1} = \int_0^{Q_1} f_X(Q_1 - t)[1 - F_Y(Q_2 + t)]dt; \quad (19)$$

Following a procedure similar to that used for  $E[q_1]$ , the derivatives of  $E[q_2]$  can be obtained by using the substitution  $v = Q_1 - t$ , as follows:

$$\frac{dE[q_2]}{dQ_1} = \int_{-\infty}^{Q_1} f_X(v)[1 - F_Y(Q_1 + Q_2 - v)]dv; \quad (20)$$

$$\frac{d^2E[q_2]}{dQ_1^2} = f_X(Q_1)(1 - F_Y(Q_2)) - \int_{-\infty}^{Q_1} f_X(v)f_Y(Q_1 + Q_2 - v)dv \quad (21)$$

Define the two convolution-type terms  $C_1(Q_1, Q_2) = \int_{Q_1}^{\infty} f_X(v)f_Y(Q_1 + Q_2 - v)dv$  and  $C_2(Q_1, Q_2) = \int_{-\infty}^{Q_1} f_X(v)f_Y(Q_1 + Q_2 - v)dv$ . Then, the second derivative is given by:

$$\frac{d^2E[\pi_1^{SPSW}]}{dQ_1^2} = -f_X(Q_1)[(\mu_s - r)(1 - F_Y(Q_2)) + rF_Y(Q_2)] - (\mu_s - r)C_1(Q_1, Q_2) - rC_2(Q_1, Q_2) \quad (22)$$

Since  $C_1(Q_1, Q_2) \geq 0$  and  $C_2(Q_1, Q_2) \geq 0$ , the last two terms are nonpositive under  $0 \leq r \leq \mu_s$ . Moreover, since  $0 \leq F_Y(Q_2) \leq 1$  and  $0 \leq r \leq \mu_s$ , we have  $(\mu_s - r)(1 - F_Y(Q_2)) + rF_Y(Q_2) \geq 0$ . Therefore,  $\frac{d^2E[\pi_1^{SPSW}]}{dQ_1^2} \leq 0$ . This proves that Buyer 1's expected profit is concave in  $Q_1$ .  $\square$

**Proposition 2.** Let  $G(Q_1) = \frac{dE[\pi_1^{SPSW}]}{dQ_1}$  and  $Q_1'$  be solution satisfying  $G(Q_1) = 0$ . The optimal order quantity of Buyer 1,  $Q_1^*$ , is determined by the following equation.

$$Q_1^* = \begin{cases} 0, & G(0) \leq 0 \\ Q_1', & G(0) > 0 \end{cases} \quad (23)$$

**Proof.** See Appendix A.  $\square$

Proposition 2 provides a simple, operational decision rule for managers: the ordering decision can be reduced to checking whether the first unit is profitable in expectation. If it is not profitable to initiate ordering, the firm should optimally refrain from purchasing altogether, thereby avoiding unnecessary exposure to inventory holding costs and demand uncertainty. Conversely, if the initial marginal profit is positive, the firm should increase its wholesale order until the marginal profit becomes zero. This threshold structure substantially simplifies procurement decisions by replacing a potentially complex optimization problem with a straightforward marginal profitability test. From a practical perspective, managers can readily assess the attractiveness of committing to long-term wholesale procurement without solving the entire optimization model, making the decision process more transparent and easier to implement.

**Corollary 1.** Under the concavity condition  $0 \leq r \leq \mu_s$ , the optimal order quantity  $Q_1^*$  satisfies the following comparative-static properties.

$$\frac{dQ_1^*}{dr} \geq 0; \frac{dQ_1^*}{d\mu_s} \geq 0; \frac{dQ_1^*}{dQ_2} \leq 0 \quad (24)$$

**Proof.** See Appendix A.  $\square$

Corollary 1 illustrates how procurement decisions should be adjusted in response to changes in the procurement environment. A higher swap price increases the value of excess inventory that can be reallocated to peer buyers, encouraging managers to commit to larger wholesale orders. Similarly, a higher expected spot-market price makes emergency procurement more expensive, strengthening the incentive to secure inventory in advance through long-term contracts. In contrast, when Buyer 2 places a larger wholesale order, the availability of inventory through peer-to-peer reallocation increases, reducing the need for Buyer 1 to maintain a high initial inventory level. These results suggest that procurement managers should coordinate wholesale purchasing decisions with both market conditions and the inventory positions of potential swapping partners rather than relying solely on their own demand forecasts.

**Corollary 2.** For any given  $Q_1$  and  $Q_2$ , the inventory-swapping agreement satisfies the participation constraints of both buyers when  $0 \leq r \leq \mu_s$ .

**Proof.** See Appendix A.  $\square$

Corollary 2 establishes a sufficient condition on the transfer price under which inventory swapping is individually rational for both buyers. For Buyer 1, inventory received from Buyer 2 substitutes for units that would otherwise be purchased from the spot market. Thus, when  $r \leq \mu_s$ , inbound swapping costs no more than emergency spot procurement. Buyer 1 may also obtain additional revenue by transferring its surplus inventory to Buyer 2. For Buyer 2, supplying surplus inventory generates swap revenue, while receiving inventory from Buyer 1 creates additional sales opportunities and reduces the expected stock-out penalty. Consequently, neither buyer is worse off by participating in the inventory-swapping agreement when  $0 \leq r \leq \mu_s$ .

### 3.3. Optimal Order Quantity Under Uniform Distribution Conditions

To obtain analytical insights into the effects of inventory swapping and spot-market recourse, we consider a special case in which the demands faced by the two buyers follow independent uniform distributions. Specifically,

$$X \sim U[a, b], Y \sim U[l, u] \quad (25)$$

This assumption is widely adopted in supply chain models because it allows analytical evaluation of expectations involving piecewise functions while preserving the key economic insights of demand uncertainty. The cumulative distribution functions are given by:

$$F_X(x) = \frac{x-a}{b-a}, F_Y(y) = \frac{y-l}{u-l} \quad (26)$$

In Equation (8), we first need to evaluate  $1 - F_X(Q_1 + t)$  and  $F_Y(Q_2 - t)$  to calculate  $E(q_1)$ . The following expressions can be directly obtained under the uniform distribution assumption.

$$1 - F_X(Q_1 + t) = \frac{b - Q_1 - t}{b - a}, F_Y(Q_2 - t) = \frac{Q_2 - t - l}{u - l} \quad (27)$$

The feasible range of the swap quantity is  $T_1 = \min(b - Q_1, Q_2 - l)$ . Accordingly, the expected swap quantity  $E(q_1)$  can be written as follows:

$$E(q_1) = \int_0^{T_1} \left( \frac{b - Q_1 - t}{b - a} \right) \left( \frac{Q_2 - t - l}{u - l} \right) dt \quad (28)$$

By evaluating the integral in Equation (28), we obtain the following expression:

$$E(q_1) = \frac{1}{(b - a)(u - l)} \left[ (Q_2 - l)(b - Q_1)T_1 - \frac{(b - Q_1) + (Q_2 - l)}{2} T_1^2 + \frac{T_1^3}{3} \right] \quad (29)$$

By symmetry, the expected swap transfer quantity from Buyer 1 to Buyer 2 can be derived as follows, where  $T_2 = \min(Q_1 - a, u - Q_2)$ :

$$E(q_2) = \frac{1}{(b - a)(u - l)} \left[ (Q_1 - a)(u - Q_2)T_2 - \frac{(Q_1 - a) + (u - Q_2)}{2} T_2^2 + \frac{T_2^3}{3} \right] \quad (30)$$

The integral of the cumulative distribution function is given by  $\int_0^{Q_1} F_X(x) dx = \frac{(Q_1 - a)^2}{2(b - a)}$ , and the expected demand is  $\mu_X = \frac{(a + b)}{2}$ . Substituting these into the structurally simplified expected profit function derived in the previous section, the expected profit of Buyer 1 under uniform demand can be explicitly expressed as:

$$E[\pi_1^{SPSW}] = (\mu_s - w)Q_1 + \frac{(p - \mu_s)(a + b)}{2} - \mu_s \frac{(Q_1 - a)^2}{2(b - a)} + (\mu_s - r)E(q_1) + rE(q_2) \quad (31)$$

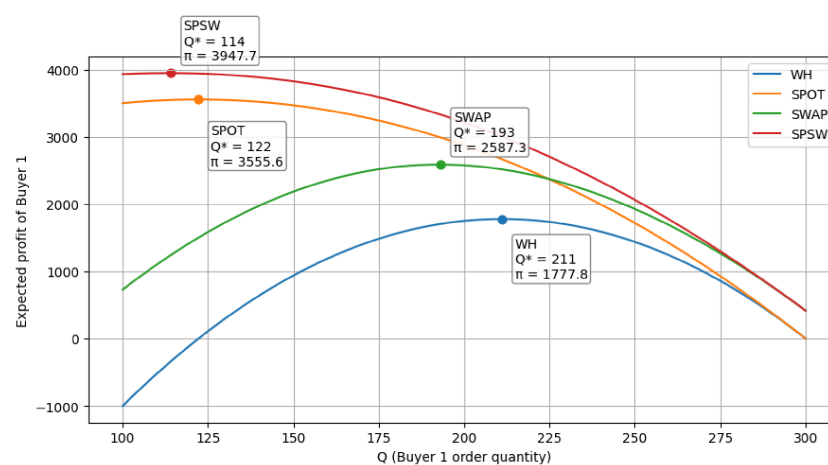
Under the uniform distribution assumption, the expected swap quantities can be expressed as explicit integral expressions, thereby making the model's economic structure more transparent. However, even in this case, the expected profit of Buyer 1 remains piecewise and nonlinear because the swap terms involve minimum operators and the spot-market recourse applies only after the swap stage. Accordingly, we rely on numerical optimization to determine the optimal order quantity and conduct comparative statics in the subsequent section [21].

#### 4. Numerical Analysis

This section evaluates the economic performance of the proposed procurement framework by comparing it with benchmark procurement mechanisms. Sensitivity analyses are then conducted to examine how key model parameters affect procurement decisions and expected profit. The numerical experiments are designed to evaluate the economic value of inventory swapping under spot-market access and to investigate how market conditions influence the interaction between these two recourse mechanisms. Buyer 1's demand  $X$  and Buyer 2's demand  $Y$  independently follow  $U(100, 300)$ . The main cost parameters are as follows: retail price  $p = 60$ , wholesale price  $w = 40$ , swap price  $r = 30$ , spot market price  $\mu_s = 45$ , and penalty cost for unmet demand  $g = 30$ , which are adapted from Ryu and Koo [20]. In the baseline analysis, Buyer 2's order quantity is fixed at 220. This value represents a moderate-to-high inventory position, under which Buyer 2 may face either a shortage or a surplus depending on realized demand, and thus serves as a meaningful benchmark for evaluating the value of inventory swapping and spot-market recourse. Based on this benchmark, we compare the optimal order quantities and expected profits across four procurement mechanisms: WH, SWAP, SPOT, and SPSW, which repre-

sent wholesale-only, wholesale + inventory swapping, wholesale + spot market access, and wholesale + spot market access + inventory swapping, respectively.

Figure 2 illustrates Buyer 1's expected profit as a function of the wholesale order quantity under the four procurement mechanisms. Several important observations can be made. The expected profit initially increases with the order quantity and subsequently decreases. Ordering too little leads to insufficient market coverage, whereas ordering too much increases the risk of overstock. Among the four mechanisms, the integrated SPSW strategy consistently achieves the highest expected profit at its optimum, demonstrating the complementary benefits of inventory swapping and spot-market procurement. It is also worth noting that although the SPSW mechanism yields the highest expected profit, its optimal wholesale order quantity is smaller than those under the other procurement mechanisms. This finding suggests that greater procurement flexibility through inventory swapping and spot-market procurement reduces the need for a large initial wholesale order.



**Figure 2.** Expected profit of buyer 1 under four procurement mechanisms.

Table 3 reports the optimal expected profits for each procurement mechanism together with the corresponding profit improvement ratios (expressed as percentage improvements relative to the wholesale-only benchmark, WH). The quantified results confirm that both spot procurement and inventory swapping individually improve Buyer 1's expected profit, while their combination achieves the largest percentage profit improvement. These results demonstrate the complementary economic value of combining alternative sourcing and inventory-sharing mechanisms.

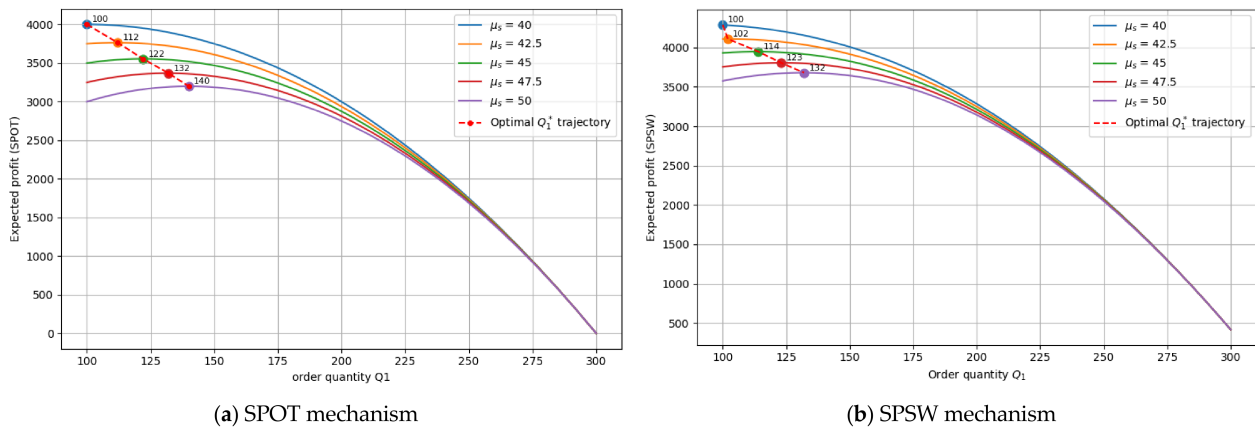
**Table 3.** Expected profits and profit improvement ratios under different procurement mechanisms.

| Mechanism               | Optimal Order Quantity | Expected Profit | % Improvement |
|-------------------------|------------------------|-----------------|---------------|
| WH (benchmark)          | 211                    | 1777.8          | -             |
| SWAP (WH + Swap)        | 193                    | 2587.3          | 45.5%         |
| SPOT (WH + Spot)        | 122                    | 3555.6          | 100.0%        |
| SPSW (WH + Spot + Swap) | 114                    | 3947.7          | 122.1%        |

% Improvement: Percentage increase in expected profit relative to the wholesale-only (WH) benchmark.

Figure 3 illustrates how the spot price affects Buyer 1's expected profit and optimal order quantity under the SPOT and SPSW mechanisms. In both panels, the profit curves shift downward as the spot price increases, indicating that a higher spot procurement cost reduces Buyer 1's profitability. At the same time, the optimal order quantity  $Q_1^*$  increases, suggesting that Buyer 1 tends to place a larger initial order when the spot market becomes more expensive. More specifically, in the SPOT case shown in Figure 3a, the optimal order

quantity increases from approximately 100 to 140 as the spot price rises. A similar pattern is observed in Figure 3b under the SPSW mechanism, where the optimal  $Q_1^*$  also increases with the spot price and the overall expected profit remains higher than that under the SPOT framework. This result suggests that inventory swapping partially mitigates the adverse effect of expensive spot procurement by providing an additional adjustment channel.

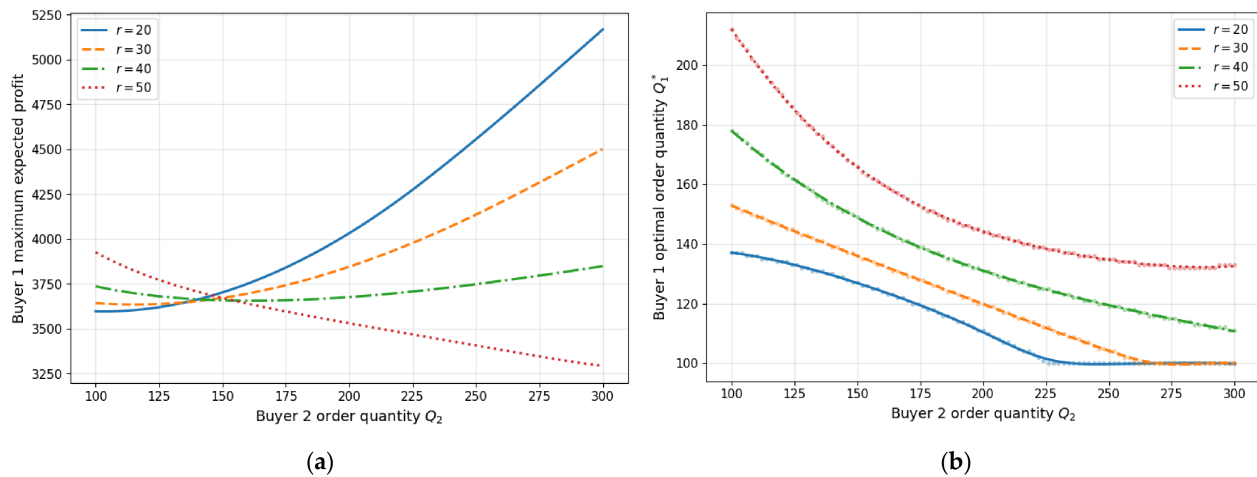


**Figure 3.** Impact of spot price on Buyer 1's expected profit and optimal order quantity under (a) SPOT and (b) SPSW mechanisms.

Figure 4 illustrates how the partner buyer's order quantity,  $Q_2$ , affects the focal buyer's optimal procurement decision under different swap transfer prices. The experiment is motivated by the idea that the value of inventory swapping depends not only on the availability of transferable inventory, but also on the transfer-price environment under which such inventory can be exchanged. First,  $Q_1^*$  and  $Q_2$  exhibit a pronounced substitution relationship. As Buyer 2 increases its order quantity, Buyer 1 consistently reduces its own initial order quantity across all values of  $r$ . This indicates that a higher inventory position held by Buyer 2 increases the expected availability of surplus units for ex-post reallocation, thereby reducing Buyer 1's reliance on advance procurement. Moreover, this substitution effect is stronger when the swap transfer price is lower. At lower values of  $r$ , Buyer 1's optimal order quantity declines more rapidly. On the other hand, at higher transfer prices, Buyer 1 maintains a relatively larger initial order, reflecting the weaker attractiveness of the swap channel. Second, the impact of  $Q_2$  on Buyer 1's maximum expected profit is strongly moderated by the transfer price. When the swap transfer price is relatively low, Buyer 1's profit increases substantially as  $Q_2$  becomes larger, following only a slight initial decline. This suggests that greater inventory held by Buyer 2 creates valuable reallocation opportunities that more than compensate for the focal buyer's reduced initial procurement. As the transfer price rises, the positive effect of  $Q_2$  on profit becomes less pronounced. The profit curve gradually flattens and, at a relatively high transfer price, shows only a mild U-shaped pattern. This implies that although a larger  $Q_2$  still improves the physical availability of transferable inventory, the higher transfer payment absorbs a considerable share of the economic benefit generated by swapping.

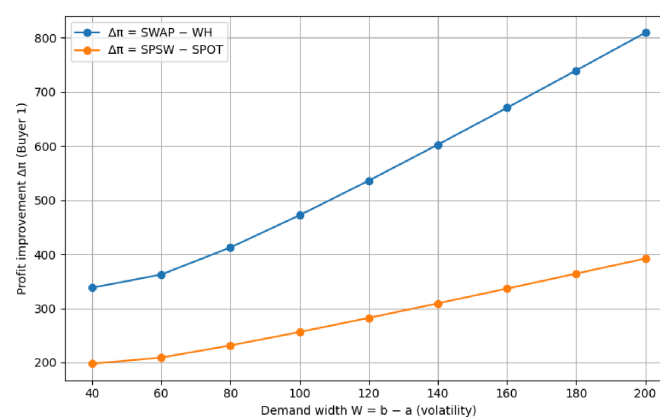
The results suggest that the value of the SPSW mechanism depends jointly on partner inventory availability and transfer pricing. A lower swap transfer price strengthens the economic value of inventory reallocation, allowing Buyer 1 to rely more on ex-post adjustment, reduce its initial ordering commitment, and achieve higher profitability. In contrast, a higher transfer price weakens this effect and limits the extent to which Buyer 2's larger inventory position improves Buyer 1's performance. This result further explains the strategic distortion caused by a high swap transfer price. A larger  $Q_2$  improves the physical availability of transferable inventory, but the economic value of this additional

availability depends on the transfer price. When  $r$  is low, Buyer 1 can use Buyer 2's surplus inventory as a cost-effective adjustment channel. When  $r$  is high, the transfer payment absorbs a larger share of the reallocation benefit. Therefore, a larger partner inventory pool does not necessarily lead to proportional profit improvement for Buyer 1, and its positive effect may be weakened or even offset.



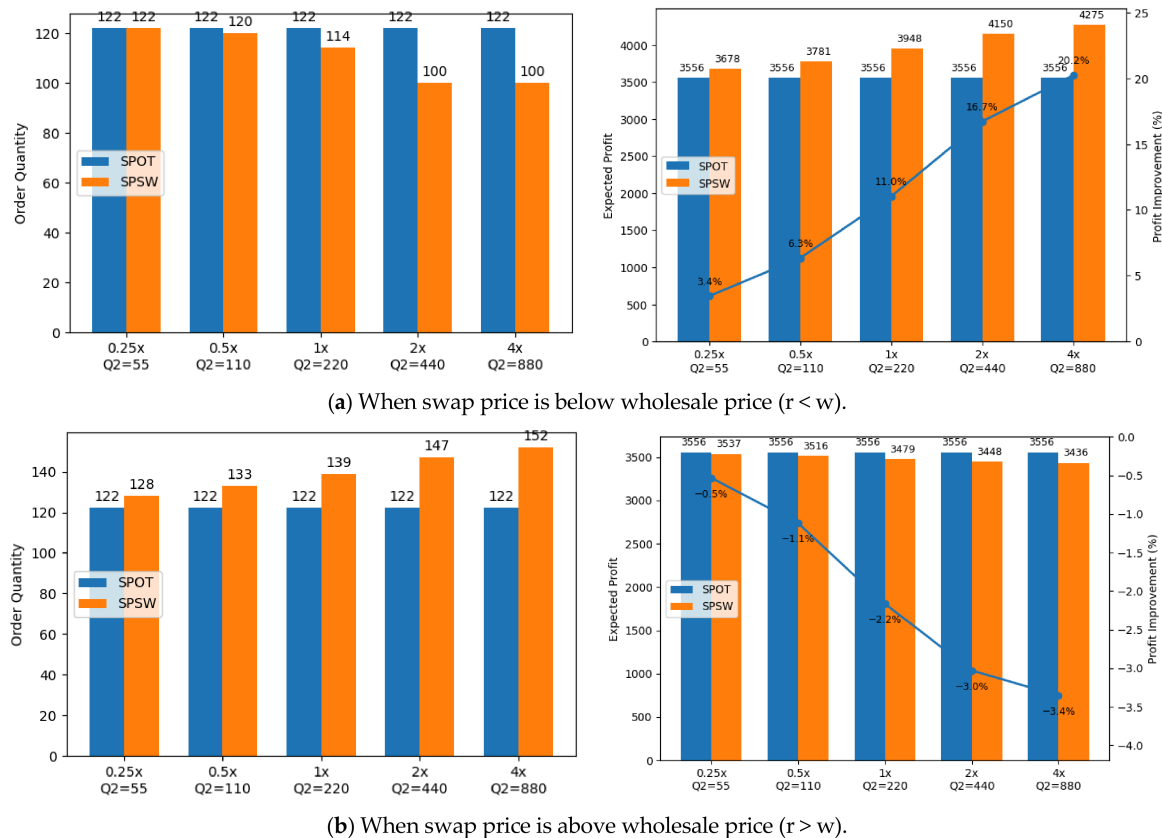
**Figure 4.** Impact of Buyer 2's order quantity on (a) Buyer 1's optimal expected profit and (b) optimal order quantity under the SPSW mechanism.

Figure 5 shows how demand variability affects the profit improvement generated by the swap mechanism under scenarios with and without spot-market access. In the baseline setting, demand variability is given by  $W = b - a = 200$ . To examine the robustness of this effect, a sensitivity analysis is conducted by varying  $W$  from 40 to 200. The vertical axis measures the incremental profit generated by introducing inventory swapping, namely  $SWAP - WH$  in the no-spot case and  $SPSW - SPOT$  in the spot-market case. The results indicate that the value of inventory swapping increases with demand uncertainty. When demand fluctuations are small, the benefit of inventory swapping is relatively limited because the initial procurement decision is less likely to deviate significantly from actual market needs. As volatility increases, the mismatch risk between ordered quantity and realized demand becomes more severe, and the flexibility provided by the swap mechanism becomes increasingly valuable. In addition, the profit improvement from the swapping is consistently greater in the absence of the spot market than in its presence. This suggests that when the spot market is unavailable, inventory swapping serves as a more critical adjustment channel and therefore creates greater incremental value.



**Figure 5.** Effect of demand volatility on the profit improvement of Swap: With and Without Spot Market.

To examine how the effectiveness of the swap mechanism depends on the size of the collaborating buyer, we conduct a sensitivity analysis by varying Buyer 2's market scale while keeping Buyer 1's market scale unchanged. Figure 6 compares the SPOT benchmark and the SPSW mechanism under two swap transfer-price settings: a baseline case with  $r < w$  and a high transfer-price case with  $r > w$ .



**Figure 6.** Effects of the SPSW mechanism on Buyer 1's order quantity and expected profit across different market scales of Buyer 2.

Under the SPOT benchmark, Buyer 1's optimal order quantity and expected profit remain unchanged across different market scales of Buyer 2, because Buyer 2's market characteristics do not affect Buyer 1's decision when inventory swapping is unavailable. In contrast, under the SPSW mechanism with  $r = 30$ , Buyer 1's expected profit increases as Buyer 2's market scale becomes larger. The profit improvement rises from 3.4% at the 0.25 scale to over 20% at the 4 scale. However, this positive scale effect depends strongly on the swap transfer price. When the transfer price is high, as shown in the case with  $r = 50$ , used as a stress-test scenario, the SPSW mechanism no longer generates a clear profit advantage over the SPOT benchmark and may even perform worse. Although a larger Buyer 2 can provide more potential inventory for swapping, the high transfer payment offsets much of the economic benefit from reallocation. Therefore, Figure 6 shows that partner size alone does not guarantee the success of inventory swapping. From a managerial perspective, firms should consider both the scale of the potential swapping partner and the transfer-price arrangement. A large partner is beneficial only when the swap price remains sufficiently attractive relative to other procurement channels, such as wholesale ordering and spot-market purchasing.

To examine the robustness of the proposed model to alternative demand distributions, we also consider truncated normal and Gamma distributions in addition to the baseline uniform distribution. The parameters of the alternative distributions are calibrated to

match the baseline distribution's mean and standard deviation, thereby isolating the effect of distributional shape. Table 4 shows that the optimal order quantities and expected profits remain broadly consistent across the three demand distributions, although slight numerical differences are observed. In particular, the integrated procurement strategy consistently yields profit improvements exceeding 100% relative to the wholesale-only benchmark across all three distributions, demonstrating that the economic value of combining inventory swapping with spot-market procurement is not sensitive to the assumed demand distribution. These results demonstrate that the main conclusions are robust to the underlying distributional assumptions and are not highly sensitive to the choice of demand distribution. Accordingly, the managerial insights derived from the proposed model remain valid not only under the baseline uniform distribution but also under alternative demand distributions with different distributional shapes.

**Table 4.** Robustness of the proposed model under alternative demand distributions.

| Distribution                                     | $Q_1^{WH}$ | $\pi_1^{WH}$ | $Q_1^{SPSW}$ | $\pi_1^{SPSW}$ | % Improvement |
|--------------------------------------------------|------------|--------------|--------------|----------------|---------------|
| Uniform $U(100, 300)$                            | 211.1      | 1777.8       | 113.5        | 3947.7         | 122.1%        |
| Truncated normal<br>$TN(199.9, 57.8; 0; \infty)$ | 208.0      | 1945.4       | 114.4        | 3896.3         | 100.3%        |
| Gamma<br>$\Gamma(\alpha = 12, \theta = 16.7)$    | 202.5      | 1943.3       | 118.2        | 3953.6         | 103.4%        |

% Improvement: Percentage increase in expected profit relative to the wholesale-only (WH) benchmark.

## 5. Conclusions

This paper investigates a procurement problem under demand uncertainty by developing a two-buyer model that jointly incorporates long-term wholesale ordering, inventory swapping, and spot-market recourse. The model examines buyers' optimal ordering decisions and the resulting procurement performance when multiple procurement channels exist. Unlike prior studies that typically examine inventory swapping and spot-market sourcing separately, this study integrates both mechanisms within a unified analytical framework.

### 5.1. Results and Discussion

The results show that inventory swapping is not merely a substitute for spot-market sourcing but a complementary coordination mechanism. By reallocating surplus inventory between buyers after demand realization, inventory swapping reduces supply–demand mismatches and remains economically valuable even when spot-market sourcing is available. The sensitivity analyses further show that the economic value of swapping is contingent on the market and contractual environment. When the expected spot-market procurement cost increases, Buyer 1 optimally increases its wholesale order quantity, reflecting the reduced attractiveness of external emergency procurement. When demand variability increases, the value of swapping also increases because inventory mismatches between the two buyers become more likely. In addition, the analyses of Buyer 2's order quantity show that partner inventory availability is an important driver of swapping value. However, this benefit is moderated by the swap transfer price: a larger partner inventory pool does not automatically translate into higher profit if the transfer price is too high. These findings suggest that when external emergency replenishment becomes costly or market conditions become more volatile, internal inventory coordination can effectively substitute for expensive spot purchases and enhance procurement flexibility.

From a theoretical perspective, this study contributes to the literature in three ways. First, it develops a unified framework that links ex-ante wholesale ordering with two ex-post recourse channels: buyer-to-buyer inventory swapping and spot-market procurement. Second, it clarifies that procurement flexibility depends not only on the presence of an

external market but also on the internal reallocation capability of the system. Third, it demonstrates that inventory swapping retains positive economic value even when an external emergency procurement channel is available, thereby extending the literature on multi-channel procurement flexibility. From a managerial perspective, the findings suggest that rather than relying exclusively on costly spot-market purchases, firms may benefit from establishing inventory-sharing arrangements with compatible partners as an additional post-demand procurement mechanism.

## 5.2. Limitations and Future Work

Despite these contributions, this study has several limitations that open avenues for future research. First, the current model is developed within a single-period framework. Future studies may extend the analysis to a multi-period setting in which inventory carry-over, repeated swapping, and dynamic spot procurement decisions are considered. Second, the two buyers' demand is assumed to be independent. Incorporating correlated demand would provide further insights into how common market shocks affect the value of inventory swapping. Third, the swap transfer price is treated as an exogenous parameter. Future research may endogenize the design-optimal swap price through risk-attitude or mechanism-design approaches. Fourth, the model assumes asymmetric spot-market access, with only Buyer 1 able to procure from the spot market. Although this assumption is motivated by procurement environments in which buyers differ in infrastructure access, trading scale, or market-access costs, it limits the generalizability of the results. Accordingly, the findings primarily apply to environments characterized by asymmetric spot-market access. Future research could extend the model by allowing both buyers to access the spot market at different effective spot prices. Finally, although the spot-market price is modeled as a random variable, the risk-neutral expected-profit framework implies that the optimal procurement decision depends only on its expected value. Future research could further enhance the present framework by incorporating higher-order moments of the spot-price distribution, risk-averse decision makers, or possible correlations between spot prices and demand.

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**Conflicts of Interest:** The authors declare no conflict of interest.

## Appendix A

**Proof of Proposition 2.** By Proposition 1,  $E[\pi_1^{SPSW}]$  is concave in  $Q_1$ . Hence, its first-order derivative  $G(Q_1) = \frac{dE[\pi_1^{SPSW}]}{dQ_1}$  is nonincreasing in  $Q_1$ . If  $G(0) \leq 0$ , then  $G(Q_1) \leq G(0) \leq 0$  for all  $Q_1 \geq 0$ . Therefore,  $E[\pi_1^{SPSW}]$  is nonincreasing in  $Q_1$  over the feasible region, and the optimal solution is the boundary solution  $Q_1^* = 0$ . If  $G(0) > 0$ , the optimal order quantity should be positive. As  $Q_1 \rightarrow \infty$ , additional units generate negligible additional revenue while still incurring the wholesale procurement cost  $w$ . Consequently,  $G(Q_1)$  becomes negative for sufficiently large  $Q_1$ . Furthermore, under the condition  $0 < r < \mu_s$

and  $f_X(Q_1) > 0$ , Equation (22) implies  $\frac{d^2 E[\pi_1^{SPSW}]}{dQ_1^2} < 0$ . Hence,  $E[\pi_1^{SPSW}]$  is strictly concave in  $Q_1$ , and  $G(Q_1)$  is strictly decreasing. Since  $G(0) > 0$  and  $G(Q_1) < 0$  for sufficiently large  $Q_1$ , there exists a unique  $Q_1^*$  satisfying the first-order condition  $G(Q_1) = 0$ . By strict concavity of  $E[\pi_1^{SPSW}]$ ,  $Q_1^*$  is the unique global maximizer.  $\square$

**Proof of Corollary 1.** Recall  $G(Q_1) = \frac{dE[\pi_1^{SPSW}]}{dQ_1} = (\mu_s - w) - \mu_s F_X(Q_1) - (\mu_s - r)C_1(Q_1, Q_2) + rC_2(Q_1, Q_2)$  where  $C_1(Q_1, Q_2) = \int_{Q_1}^{\infty} f_X(v)F_Y(Q_1 + Q_2 - v)dv$  and  $C_2(Q_1, Q_2) = \int_{-\infty}^{Q_1} f_X(v)[1 - F_Y(Q_1 + Q_2 - v)]dv$ .

Effect of  $r$ : By the Implicit Function Theorem (IFT),  $\frac{dQ_1^*}{dr} = -\frac{\frac{\partial G(Q_1^*, r)}{\partial r}}{\frac{\partial G(Q_1^*, r)}{\partial Q_1}}$ . Since  $\frac{\partial G(Q_1^*, r)}{\partial r} = C_1(Q_1^*, Q_2) + C_2(Q_1^*, Q_2) \geq 0$  and by concavity,  $\frac{\partial G(Q_1^*, r)}{\partial Q_1} < 0$ , it follows that  $\frac{dQ_1^*}{dr} \geq 0$ .

Effect of  $\mu_s$ : Applying the IFT yields  $\frac{dQ_1^*}{d\mu_s} = -\frac{\frac{\partial G(Q_1^*, \mu_s)}{\partial \mu_s}}{\frac{\partial G(Q_1^*, \mu_s)}{\partial Q_1}}$ . Note that  $\frac{\partial G(Q_1^*, \mu_s)}{\partial \mu_s} = 1 - F_X(Q_1^*) - C_1(Q_1^*, Q_2)$ . We have  $C_1(Q_1^*, Q_2) = \int_{Q_1^*}^{\infty} f_X(v)F_Y(Q_1^* + Q_2 - v)dv \leq \int_{Q_1^*}^{\infty} f_X(v)dv = 1 - F_X(Q_1^*)$ , which implies  $\frac{\partial G(Q_1^*, \mu_s)}{\partial \mu_s} \geq 0$ . Given that  $\frac{\partial G(Q_1^*, r)}{\partial Q_1} < 0$ , we obtain  $\frac{dQ_1^*}{d\mu_s} \geq 0$ .

Effect of  $Q_2$ : By the IFT,  $\frac{dQ_1^*}{dQ_2} = -\frac{\frac{\partial G(Q_1^*, Q_2)}{\partial Q_2}}{\frac{\partial G(Q_1^*, Q_2)}{\partial Q_1}}$ . Differentiating  $C_1$  and  $C_2$  with respect to  $Q_2$  yields  $\frac{\partial C_1(Q_1, Q_2)}{\partial Q_2} = \int_{Q_1}^{\infty} f_X(v)f_Y(Q_1 + Q_2 - v)dv \geq 0$  and  $\frac{\partial C_2(Q_1, Q_2)}{\partial Q_2} = -\int_0^{Q_1} f_X(v)f_Y(Q_1 + Q_2 - v)dv \leq 0$ . Since  $0 \leq r \leq \mu_s$ , it follows that  $\frac{\partial G(Q_1^*, Q_2)}{\partial Q_2} = -(\mu_s - r)\frac{\partial C_1(Q_1, Q_2)}{\partial Q_2} + r\frac{\partial C_2(Q_1, Q_2)}{\partial Q_2} \leq 0$ . Moreover, by concavity of  $G(\cdot)$ ,  $\frac{\partial G(Q_1^*, Q_2)}{\partial Q_1} < 0$ . Therefore,  $\frac{\partial Q_1^*}{\partial Q_2} \leq 0$ .  $\square$

**Proof of Corollary 2.** Relative to the corresponding no-swap benchmarks, the expected profit gains of Buyers 1 and 2 are  $\Delta\pi_1 = (\mu_s - r)E[q_1] + r[q_2]$ , and  $\Delta\pi_2 = rE[q_1] + (p + g - r)E[q_2]$ . Since  $p > \mu_s$ ,  $g \geq 0$ , and  $0 \leq r \leq \mu_s$ , all coefficients of  $E[q_1]$  and  $E[q_2]$  are nonnegative. Therefore,  $\Delta\pi_1 \geq 0$  and  $\Delta\pi_2 \geq 0$ . Hence, both buyers are willing to participate in the swapping agreement.  $\square$

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