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A NUMERICAL SIMULATION OF MELTING OF ICE HEATED FROM ABOVE

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Abstract

Melting of ice in a cubical enclosure partially heated from above was studied. Half of the upper surface was maintained at room temperature and the other half at 70°C. The ice cube was maintained at its melting point at the bottom. The other side surfaces were insulated. The process was first modeled by ignoring the effect of natural convection in the liquid phase. The resulting equations of conservation of energy were solved in each phase. The motion of melting front was governed by an energy balance at the interface. This conduction model was verified by applying it to a 1-D phase change problem for which an analytical solution is available. Preliminary experiments conducted resulted in a progress of the phase front faster than that predicted by the conduction model and the interface was smoother due to strong effects of natural convection in the liquid phase, except for the initial start of melting. The model was then extended to include convective heat transfer in such a way that the liquid phase was assumed to be a mixed body subjected to natural convection from the top surface and the liquid-solid interface. The flux at the interface was obtained by finding a heat transfer coefficient for natural convection with a cold plate facing upward. The predictions of this convection model agreed well with the experimental results.

1. INTRODUCTION

Melting/solidification problems belong to a class of heat transfer where there exists a phase change and its location is not known a priori. Phase change problems are encountered extensively in nature and in a variety of technologically important processes. Such processes include melting of ice, freezing/thawing of moist soil, crystal growth, latent heat-of-fusion, thermal energy storage, purification and casting of metals, welding and plastics manufacturing.

Phase change problems have been the subject of intensive research over recent years. A number of studies dealing with analytical or numerical aspects of particular melting or freezing heat transfer problems have appeared in the literature [1-9]. It has been observed that most studies are for the case of cooling or heating from vertical walls and many models lack quantitative comparison with experiments. Moreover, models for phase change problems are in general based on conduction type of heat transfer, both in solid and liquid. However, the actual physical processes show that convection type of heat transfer may be present in the liquid and often plays an important role [10-13]. The objective of this study was to simulate the melting of ice in a rectangular enclosure partially heated from above taking into account natural convection in the liquid phase by assuming the liquid to be a mixed body subjected to natural convection from the heated surface above and the phase front below. This is a short cut method for inclusion of natural convection in the phase change analysis in a simplified manner which reduces the computational time substantially.

2. MATHEMATICAL FORMULATION

In the following subsections, the governing equations for melting of ice in a cubical enclosure ($0.20\times0.20\times0.20$ m^3) are presented. Ice at an initial temperature of $T_o = -30°C$ is subjected to a temperature of $T_{hot} = 70°C$ at half of the upper boundary ($0.10 \le x \le 0.20$ m, $z = 0$) and 20°C at the other half ($0 \le x < 0.10$ m, $z = 0$). The temperature of the solid at the bottom ($z = 0.20$ m) is maintained at $T_m = 0°C$. Other side surfaces are insulated.

2.1. Mathematical Formulation With Conduction

Conservation of energy in the liquid and solid phases can be written as [14]

$$\frac{\partial T}{\partial t} = \alpha_l \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) \qquad \text{for} \quad 0 < z < s(x,t), \quad t > 0 \tag{1}$$

and

$$\frac{\partial T}{\partial t} = \alpha_s \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) \qquad \text{for} \quad s(x,t) > z > 0.20 \text{ m}, \quad t > 0 \tag{2}$$

where T is temperature, t is time, α_l is liquid thermal diffusivity, α_s is solid thermal diffusivity and s is the interface location in z-direction.

The energy balance at the interface [15] is

$$\rho_l L \frac{\partial s}{\partial t} = \left[1 + \left(\frac{\partial s}{\partial x} \right)^2 \right] \left(k_s \frac{\partial T_s}{\partial z} - k_l \frac{\partial T_l}{\partial z} \right) \tag{3}$$

where ρ is density, L is latent heat of melting, k is thermal conductivity and subscripts l and s refer to liquid and solid, respectively.

The initial and boundary conditions are as follows

$$T(x,z,t) = T_0 = -30°C \qquad \text{at} \quad t = 0 \tag{4}$$

$$T(x,z,t) = T_{hot} = 70°C \qquad \text{at} \quad t \ge 0, \ z = 0, \ 0.10 \le x \le 0.20 \text{ m} \tag{5}$$

$$T(x,z,t) = T_{room} = 20°C \qquad \text{at} \quad t \ge 0, \ z = 0, \ 0 \le x < 0.10 \text{ m} \tag{6}$$

$$T(x,z,t) = T_m = 0°C \qquad \text{at} \quad t \ge 0, \ z = 0.20 \text{ m} \tag{7}$$

$$\frac{\partial T}{\partial x} = 0 \qquad \text{at} \quad t \ge 0, \ x = 0 \tag{8}$$

$$\frac{\partial T}{\partial x} = 0 \qquad \text{at} \quad t \geq 0, \ x = 0.20 \text{ m} \tag{9}$$

2.2. Mathematical Formulation With Convection

Preliminary experiments showed that ice was melting faster than predicted by the conduction model. The melt front was little inclined but smooth (again, not as predicted by the conduction model). Upon the establishment of the strong natural convection effects in the liquid phase in the preliminary experiments, it was decided to use a simplified approach neglecting the small inclination of the melt front to model the averaged convection effects. Liquid water was assumed to be a mixed body subjected to natural convection from the top hot surface and the cold ice surface. Once liquid starts forming at the top, a heat balance can be written on the liquid as

$$m_l C_{p_l}\left(T_l^{t+\Delta t} - T_l^t\right) = \left[h_{hot}(T_{hot} - T^*)\tfrac{A}{2} + h_{room}(T_{room} - T^*)\tfrac{A}{2} + h_{melt}(T_m - T^*)A\right]\Delta t \tag{10}$$

where T_l is the liquid bulk temperature, $T^* = 0.5(T_l^{t+\Delta t}+T_l^t)$, m is mass, C_p is specific heat capacity, h is heat transfer coefficient, A is cross-sectional area of the ice cube and subscripts *hot, room* and *melt* refer to top heated surface, top unheated surface and melt front (liquid-solid interface), respectively. Therefore, $T_l^{t+\Delta t}$ can be determined from the above equation. The heat transfer coefficients are calculated from the following correlation involving Rayleigh number, Ra, (for hot surface facing down or cold surface facing up) [14]

$$Nu = 0.27 Ra^{1/4} \qquad (\text{for } 3\text{x}10^5 < Ra < 10^{10}) \tag{11}$$

where Nu is Nusselt number.

The energy balance at the interface will then be

$$\rho_l L \frac{\partial s}{\partial t} = k_s \frac{\partial T_s}{\partial z} - h_{melt}(T_m - T^*) \tag{12}$$

3. NUMERICAL SOLUTION TECHNIQUE

Finite differencing with explicit technique was utilized. A fixed grid structure (11×11) was used. At grid points near the melt front, string-intersected approximations to derivatives were used [16-19]. Central differencing was performed to approximate the spatial derivatives at all interior grid points. The discretization details for the case of conduction type of heat transfer, both in solid and liquid, are as follows.

The conservation of energy equation in the liquid can be discretized as

$$T_{i,k}^{n+1} = T_{i,k}^n + \frac{\Delta t}{(\Delta x)^2}\alpha_l(T_{i+1,k}^n - 2T_{i,k}^n + T_{i-1,k}^n) + \frac{\Delta t}{(\Delta z)^2}\alpha_l(T_{i,k+1}^n - 2T_{i,k}^n + T_{i,k-1}^n) \tag{13}$$

and in the solid as

$$T_{i,k}^{n+1} = T_{i,k}^n + \frac{\Delta t}{(\Delta x)^2}\alpha_s(T_{i+1,k}^n - 2T_{i,k}^n + T_{i-1,k}^n) + \frac{\Delta t}{(\Delta z)^2}\alpha_s(T_{i,k+1}^n - 2T_{i,k}^n + T_{i,k-1}^n) \tag{14}$$

except at a grid point close to the solid-liquid boundary which requires special attention. This can be accommodated using string-intersected-boundary approximation. The second derivatives at the grid points near the liquid-solid interface are written using the string-intersected formulae below [16-19]

$$\frac{\partial^2 T}{\partial z^2}=\frac{2}{\Delta z}\left(\frac{T_m}{(\delta z/\Delta z)(\Delta z+\delta z)}-\frac{T_{i,k}}{\delta z}+\frac{T_{i,k-1}}{\Delta z+\delta z}\right) \quad \text{in the liquid} \tag{15}$$

and

$$\frac{\partial^2 T}{\partial z^2}=\frac{2}{\Delta z}\left(\frac{T_{i,k+1}}{\Delta z+\delta' z}-\frac{T_{i,k}}{\delta' z}+\frac{T_m}{(\delta' z/\Delta z)(\Delta z+\delta' z)}\right) \quad \text{in the solid} \tag{16}$$

when the melting is in the positive z-direction, and

$$\frac{\partial^2 T}{\partial x^2}=\frac{2}{\Delta x}\left(\frac{T_{i+1,k}}{\Delta x+\delta x}-\frac{T_{i,k}}{\delta x}+\frac{T_m}{(\delta x/\Delta x)(\Delta x+\delta x)}\right) \quad \text{in the liquid} \tag{17}$$

and

$$\frac{\partial^2 T}{\partial x^2}=\frac{2}{\Delta x}\left(\frac{T_m}{(\delta' x/\Delta x)(\Delta x+\delta' x)}-\frac{T_{i,k}}{\delta' x}+\frac{T_{i-1,k}}{\Delta x+\delta' x}\right) \quad \text{in the solid} \tag{18}$$

when the liquid is in the positive x-direction and the solid in the negative x-direction relative to the grid point, as is the case in this study. In the above equations, δx and δz are the distances between the interface and the closest node to the interface in the liquid in x and z directions, respectively, and $\delta' x$ and $\delta' z$ are similar distances in the solid.

The index (k_i^*) of the closest grid point to the interface in the liquid in z-direction can be determined from

$$k_i^* = \frac{s_{i,t}}{\Delta z}+1 \tag{19}$$

when the right hand side is to be taken as rounded to the next lower integer. The closest grid point to the interface in the solid in z-direction is then k_i^*+1.

Then, one can write

$$\delta z = s_{i,t}-\left(k_i^*-1\right)\Delta z \tag{20}$$

and

$$\delta' z = k_i^* \Delta z - s_{i,t} \qquad \text{or} \qquad \delta' z = \Delta z - \delta z \tag{21}$$

The method of the congruence of triangles was used to determine δx and $\delta' x$.

The energy balance at the interface can be discretized as

$$s_i^{n+1}=s_i^n+\left[1+\left(\frac{s_{i+1}^n-s_{i-1}^n}{2\,\Delta x}\right)^2\right]\left[\frac{k_s}{\rho_l L}\frac{\Delta t}{\delta' z}\left(T_{k_i^*+1}-T_m\right)-\frac{k_l}{\rho_l L}\frac{\Delta t}{\delta z}\left(T_m-T_{k_i^*}\right)\right] \tag{22}$$

where $T_{k_i^*}$ is the temperature at the grid point k_i^* and $T_{k_i^*+1}$ is the temperature at the grid point k_i^*+1.

The singularities that would arise when any of δz, $\delta' z$, δx, and $\delta' x$ approaches zero are prevented by assuming that the temperature at the grid point near the interface is at its boundary value of T_m = 0°C whenever δz, $\delta' z$, δx, or $\delta' x$ is smaller than a set criterion; for example 10^{-5}.

In that case, the interface condition is discretized as

$$s_i^{n+1} = s_i^n + \left[1 + \left(\frac{s_{i+1}^n - s_{i-1}^n}{2\,\Delta x} \right)^2 \right] \left[\frac{k_s}{\rho_l L} \frac{\Delta t}{\delta' z} \left(T_{k_i^*+1} - T_m \right) - \frac{k_l}{\rho_l L} \frac{\Delta t}{\Delta z} \left(T_m - T_{k_i^*-1} \right) \right] \tag{23}$$

when $\delta z \le 10^{-5}$ m, and

$$s_i^{n+1} = s_i^n + \left[1 + \left(\frac{s_{i+1}^n - s_{i-1}^n}{2\,\Delta x} \right)^2 \right] \left[\frac{k_s}{\rho_l L} \frac{\Delta t}{\Delta z} \left(T_{k_i^*+2} - T_m \right) - \frac{k_l}{\rho_l L} \frac{\Delta t}{\delta z} \left(T_m - T_{k_i^*} \right) \right] \tag{24}$$

when $\delta' z \le 10^{-5}$ m. In these equations, $T_{k_i^*-1}$ is the temperature at grid point $k_i^* - 1$ and $T_{k_i^*+2}$ is the temperature at the grid point $k_i^* + 2$.

The discretization for the case of numerical modeling involving convection is similarly done except that heat transfer in the liquid phase is determined by utilizing equations (10-12).

The physical properties used in the solutions are: liquid density, ρ_l = 993 kg/m^3; liquid thermal conductivity, k_l = 0.624 W/m °C; solid thermal conductivity, k_s = 2.367 W/m °C; liquid thermal diffusivity, $\alpha_l = 1.505\times10^{-7}$ m^2/s; solid thermal diffusivity, $\alpha_s = 1.3164\times0^{-6}$ m^2/s; liquid specific heat capacity, C_{P_l} = 4174 J/kg °C and latent heat of melting, L = 333,790 J/kg.

4. EXPERIMENTAL

Water in a copper container (0.2×0.2×0.2 m^3) was frozen to -30°C with several thermocouples imbedded within. The ice was placed on an ice/water bath. Half of the top surface of the ice was contacted with a copper box with hot water at 70°C circulating from a reservoir. The other half was covered by a thin copper plate. The other side surfaces were insulated. The thermocouple readings and the location of the ice front were recorded during the experiment. There were 10 thermocouples from top to bottom in the ice box arranged using a solid wire. The melt thickness was measured by inserting a cylindrical ruler in the melting box through a few openings at the top.

5. RESULTS AND DISCUSSIONS

Advance of the melting front obtained from the conduction model is shown in Figure 1. The time required for the ice in the box to melt completely was determined to be about 97 hours (when natural convection in the liquid was ignored). Temperature profiles in two dimensions of the phase change system at two specified times predicted by the conduction model are shown in Figures 2 and 3. The lower curves in both figures are for the case when the solid thickness is at its initial value. The top curves of the figures, on the other hand, are for the case when no solid is left in the system.

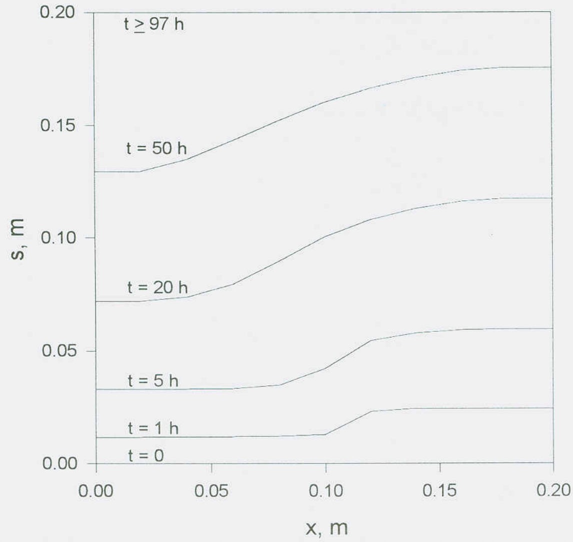

Figure 1. Melt thickness profile obtained from the conduction model ($\Delta x = \Delta z = 0.02$ m, $\Delta t = 1$ s).

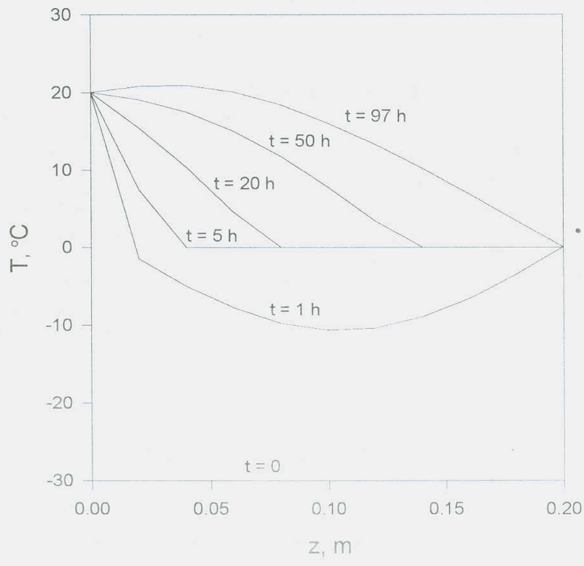

Figure 2 Temperature profiles at x = 0.04 m obtained from the conduction model ($\Delta x = \Delta z = 0.02$ m, $\Delta t = 1$ s).

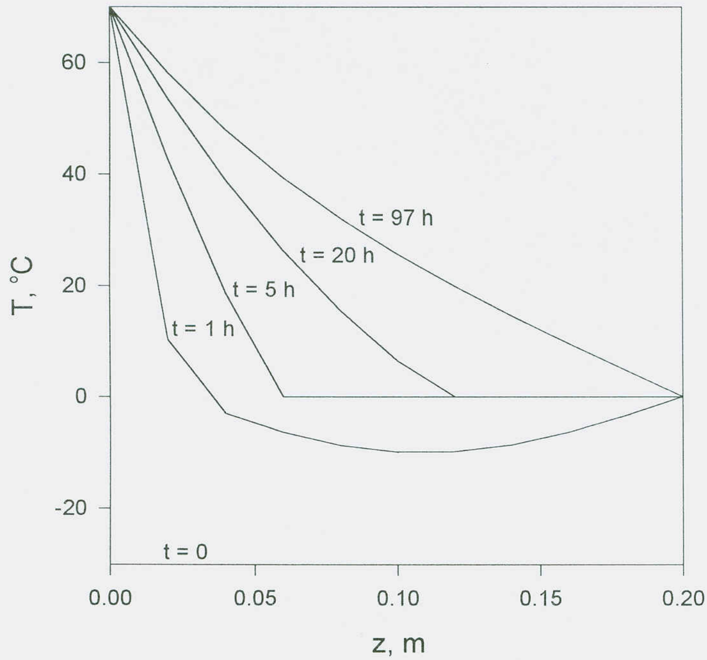

Figure 3. Temperature profiles at x = 0.16 m obtained from the conduction model ($\Delta x = \Delta z = 0.02$ m, $\Delta t = 1$ s).

The grid and time step sensitivities of the model were also checked, as shown in Figure 4. The results of the model were almost identical for spatial to time step size ratios ($\Delta x/\Delta t$ or $\Delta z/\Delta t$) of 0.02, 0.04 and 0.2 for the grid structure used (11×11). The stability and/or accuracy of the model predictions suffered significantly when higher or lower spatial to step size ratios were used. The results were not significantly different when a grid structure of 21×21 was devised instead of 11×11.

The 2-D conduction model was also tested by applying it to 1-D melting of a solid which is at the melting temperature and subjected to a higher temperature at a boundary, for which an analytical solution is available if the solid phase stays at the melting temperature throughout and convection type of heat transfer in the liquid phase is ignored [15].

Figures 5 and 6 compare the results of the numerical model with those obtained by the analytical solution. Good agreement with the analytical solution verifies the numerical model for the case of conduction type of heat transfer.

Figure 7 compares the advance of melting front obtained from the numerical solution involving convection heat transfer with that obtained experimentally for a water/ice system. Since ice lifts after a certain time, limited experimental values are reported. The results of the simplified numerical model considering natural convection in the liquid showed good agreement with the preliminary experimental work.

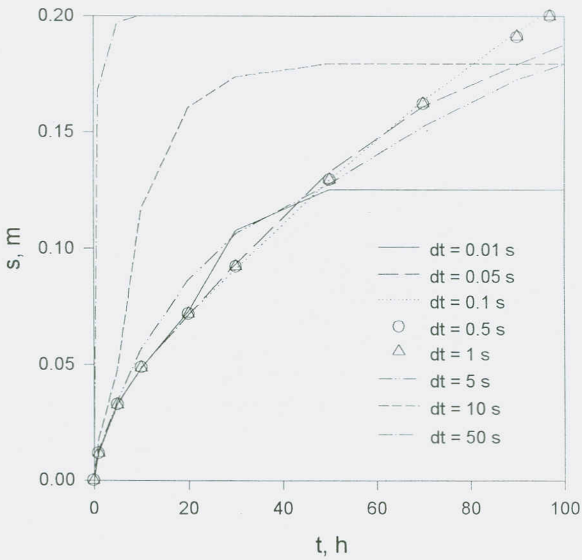

Figure 4. Melt thickness vs time plot at x=0 obtained from the conduction model for several time step size values (while the spatial step sizes were fixed at 0.02 m).

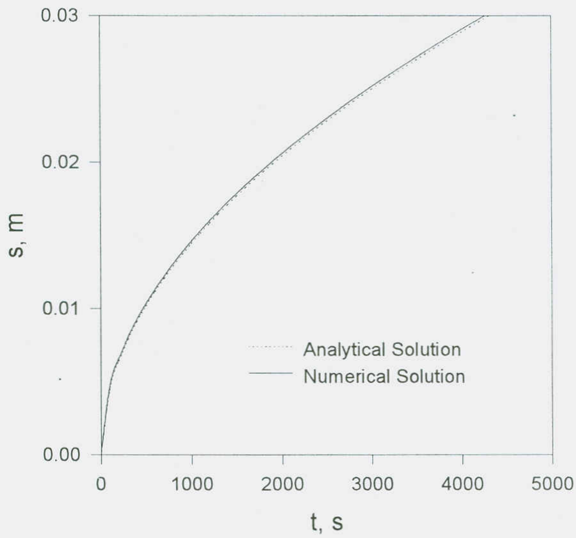

Figure 5. Melt thickness vs time plot obtained from the conduction model and the analytical solution for melting of ice subjected to 70°C at one boundary while the solid phase was maintained at the melting temperature.

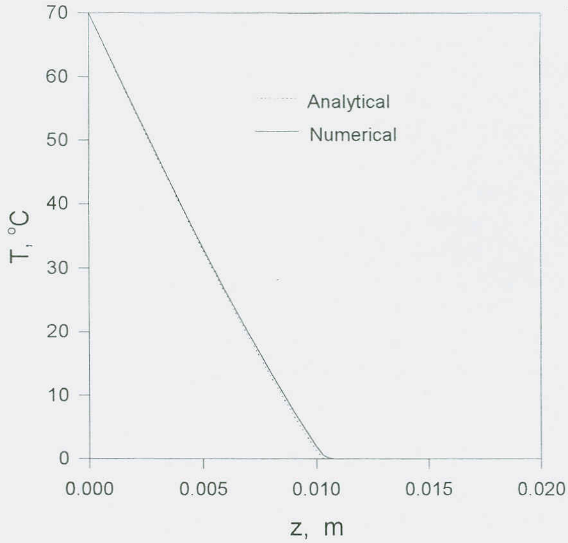

Figure 6. Temperature profiles at time = 500 s obtained from the conduction model and the analytical solution for melting of ice subjected to 70°C at one boundary while the solid phase was maintained at the melting temperature.

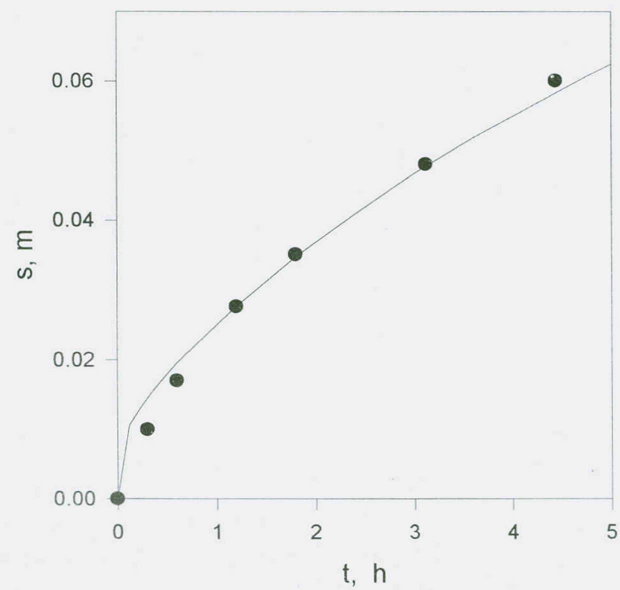

Figure 7. Melt thickness vs time plot obtained experimentally and from the numerical model including convection.

6. CONCLUSIONS

Melting of ice in a rectangular enclosure heated from above was modeled, first, by ignoring the effect of natural convection in the liquid phase. This numerical study for the case of conduction type of heat transfer was verified by application to a 1-D system for which an analytical solution is available. The experimental investigation showed that the melting process was affected significantly by natural convection in the liquid phase. Upon this observation, the model was extended to include convective heat transfer in the liquid phase by assuming the liquid to be a mixed body subjected to natural convection from the heating surface above and the ice surface below. The model, modified to take into account natural convection in the liquid phase in a simplified manner, agreed well with the experimental work. Future work is planned to develop a more rigorous and general model considering the temperature dependency of water density and using all components of the equations of motion and continuity, and also verify the model by sophisticated experimentation.

7. ACKNOWLEDGMENT

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