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ERROR ANALYSIS OF DETERMINING AIRPLANE LOCATION BY GLOBAL POSITIONING SYSTEM

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Abstract -This paper studies the error analysis of determining airplane location by global positioning system (GPS) using statistical testing method. The Newton Rhapson method positions the airplane at the intersection point of four spheres. Absolute errors, relative errors and standard deviation have been calculated. The results show that the positioning error of the airplane varies with the coordinates of GPS satellite and the airplane.

1. INTRODUCTION

The artificial satellites are still successfully utilized to solve navigation problems. The artificial satellite can be described as an orientation object that orbits twenty thousand kilometers away from the earth. If this orientation object successfully orbits at the originally calculated altitude according to the gravitational forces, and if it has a radio transmitter (that allows for the measurement of navigational parameters from the earth), then it will be a fully functioning space lighthouse. It will serve the same purpose with the orientation objects on earth that are used to calculate the location of an airplane. Most essentially, it will be available on a 24-hour basis regardless of weather conditions. The methods used by navigation satellites to locate moving airplanes and other objects on earth are: distance measurement, difference of distance measurement and radial speed.[1,2]

The steps to be followed when locating the position of airplanes or other objects: (a) the ground stations calculate the orbit parameters of the satellite; (b) these parameters are transmitted back to the satellite, which corrects the clock; (c) the satellite transmits signals that tell its orbit coordinates at a certain time; (d) these signals are picked up and accepted by the airplane's receiver antenna; (e) signals are filtered; (f) depending on the radionavigation system being used, the elapsed time or Doppler frequency deviation is measured; (g) all this data is entered into the computer of the airplane; (h) the operator enters the Greenwich time, the speed of the airplane and the yaw angle of the plane into the computer; (i) using all the data given, computer calculates the current coordinates of the airplane.

The publications on GPS studies [1 - 5] do not investigate the exactness of airplane coordinates. There is no information pertaining to positioning errors or why these errors vary. This paper examines the dynamics of the variation of error analysis when locating airplanes and the reasons as to why this variation occurs.

2. HOW THE GPS WORKS

An airplane can be located using the distance measurement method by the help of at least three satellites. This time, the distance from the satellite to the airplane is taken as the navigation parameter. The position of the airplane is found as the intersection point of three spheres, each having a satellite at its center and a radius equal to the distance between the satellite and the airplane. Figure 1 demonstrates how the coordinates of the airplane are calculated using the measurement of distance between three satellites with different orbits and the airplane.

The coordinates of the airplane are found as the intersection point of three spheres with D_1, D_2, and D_3 radii. However, because there are two intersection points to three overlapping spheres, the result of this method is not very clear. In order to clarify this issue, a fourth navigation satellite or an additional navigational method that does not have to be precise should be utilized.

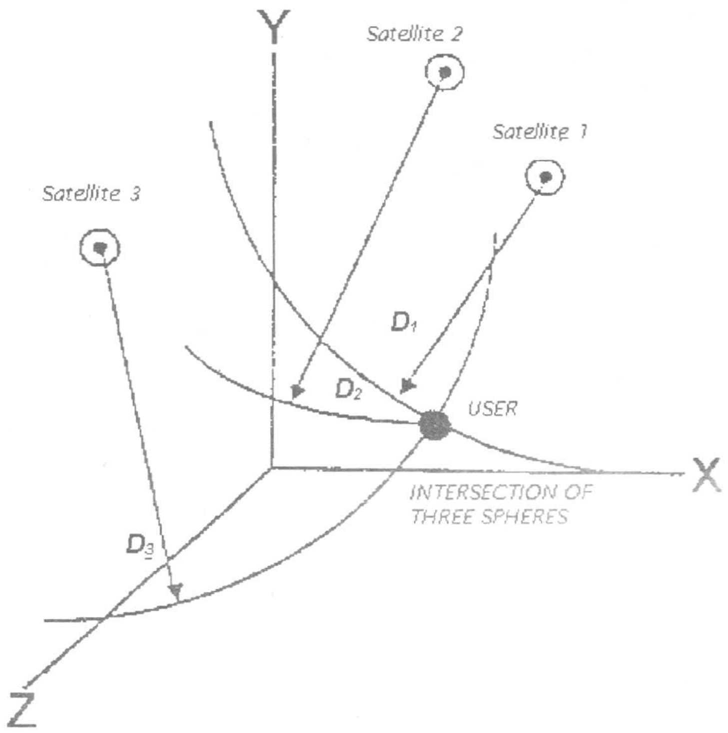

Figure 1. Locating the coordinates of an airplane using navigation satellites

Contemporary navigational systems are still being placed at mid-altitude orbits. NAVSTAR satellite radionavigational system (GPS) manufactured in 1978 in the USA, is an example of a system that is used to locate the position of various moving objects in the air.

NAVSTAR GPS consists of 24 navigational satellites. These satellites have been placed in 6 different orbits in groups of four. They go around the world twice within 24 hours. The observer can detect between 4 to 7 satellites at any given radio observation area. The coordinates of a given object are found by the distance measurement method (with the distances from the object to the four satellites). In order to find the Descartes coordinates, the following equation is solved for four variables:

$$(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2 = (D_i - \delta_i)^2, \quad i = 1, \dots, 4 \qquad (1)$$

In this equation x_i, y_i, z_i the Descartes coordinates of satellite i (i = 1,...,4); x_c, y_c, z_c is the Descartes coordinates of the moving object; D_i is the distance between the moving object and the satellite; δ_t is the distance error that results because of the difference between the observer's time scale and the system's main time scale.

The equation system that consists of four nonlinear equations can be solved easily by the computer using the iteration technique. At this point, using the Newton-Rhapson method is more effective. As a result, the Descartes coordinates x_c, y_c, z_c of the moving object is found.

3. THE FACTORS THAT AFFECT THE ACCURACY OF LOCATING THE AIRPLANE WHEN USING GPS

Figure 2 describes how the coordinates of the airplane are found with the help of GPS using the distance measurement technique.

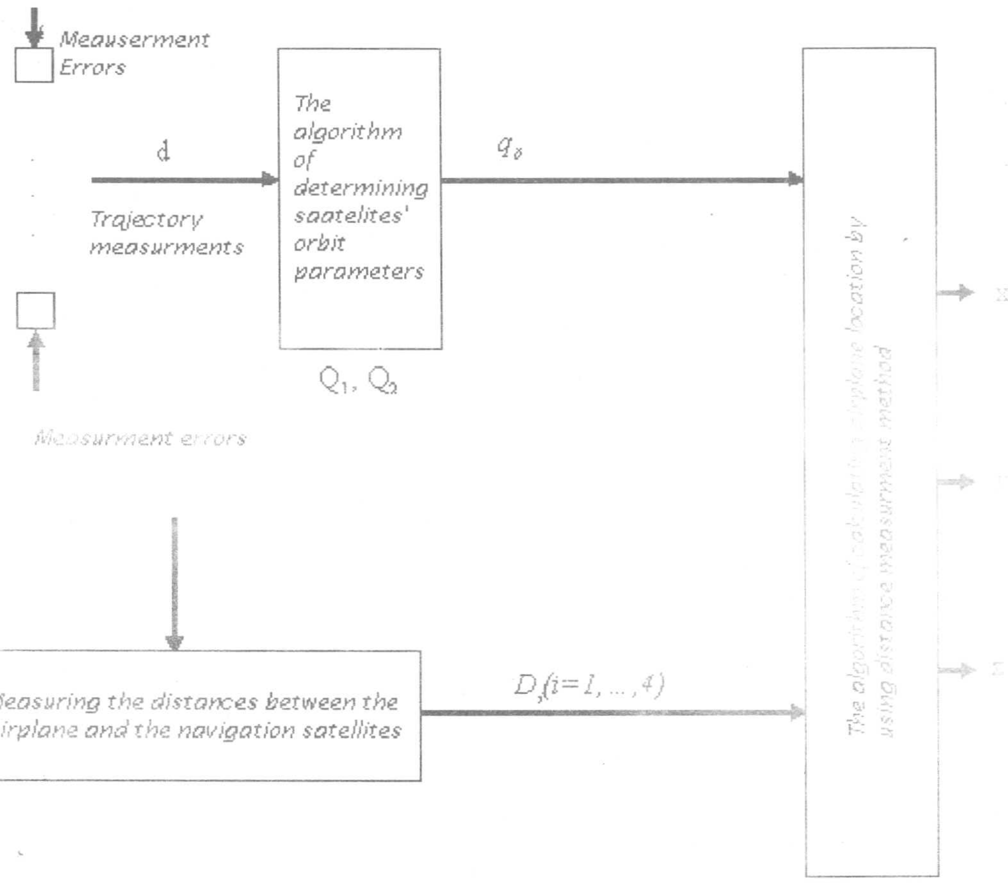

Figure 2 Locating the airplane's coordinates with the help of GPS using the distance measurement technique

The following procedures need to be completed to locate the airplane:

a) Find the orbit parameters of the navigation satellites

$q = Q_1 d,$ Q_1 is the algorithm to find the orbit parameters.

b) Proposing the orbit parameters

$q_{\delta} = Q_2\, d,$ Q_2 is the algorithm of orbit parameter proposition.

c) Locating the airplane

$$x = f_x(\{x_i\, ,\, y_i\, ,\, z_i\, ,\, D_i\, ;\, i = 1,\ldots,4\}),$$
$$y = f_y(\{x_i\, ,\, y_i\, ,\, z_i\, ,D_i\, ;\, i = 1,\ldots,4\}),$$
$$z = f_z(\{x_i\, ,\, y_i\, ,\, z_i\, ,\, D_i\, ;\, i = 1,\ldots,4\}),$$

Let us look at the factors that affect the accuracy of locating the airplane using the distance measurement technique with the GPS.

- The measurement errors by trajectory. These errors are related to the measuring device and orbit parameters. The reductions of these errors are possible by choosing appropriate measuring devices and placing them in the most optimal positions.
- Making errors when finding the orbit parameters. There are two ways these errors can be made: 1) the errors made because of trajectory measurement errors; 2) erroneous proposition of orbit parameters. Proposition error depends on the range of proposition and the selection of proposition technique. The reduction of errors is possible by the use of statistical analysis to test trajectory measurement results and by the selection of a better mathematical model.
- The erroneous measurement of the distance between the airplane and the navigation satellite. This error is related to the accuracy of the distance measurement stations and the first operation algorithms of measurements.
- The erroneous algorithm of locating the airplane with the distance measurement method. This error is related to the selection of orbits of navigation satellites and the number of satellites present in the observer's radio observation field and their location in space.

4. SIMULATION

In the article, a simulation has been made with Matlab program. This is a simulation of an airplane in a horizontal flight and three satellites moving within a 0-50 seconds time frame. Descartes coordinates have been taken as the reference coordinate system. First of all, the program calculates the distances between the airplane and the satellites (D_1,D_2,D_3). Then the airplane is located using the Newton Rhapson method of intersecting three spheres. Predetermined variations of random GPS errors are added to satellite coordinates and to distance measurements (D_1,D_2,D_3) between satellite and airplane, when the location of the airplane is being calculated. The results are compared and contrasted with each other.

The horizontal flight simulation has been made according to dynamic flight equations [6] with a Matlab program. The movement of satellites has been simulated according to the Gravitational Law. [7]

$$\frac{d^2x}{dt^2} = -\gamma M \frac{x}{r^3}$$

$$\frac{d^2y}{dt^2} = -\gamma M \frac{y}{r^3}$$

$$\frac{d^2z}{dt^2} = -\gamma M \frac{z}{r^3}$$

$$\gamma = 6{,}668x10^{-3}\ cm^3 / grsc^2$$

$$M = 59778494.3x10^3\ kg$$

$$r = r_0 + h$$

The movement equations of the satellite are as follows:

$$x_i = x_{i-1} + u_{i-1}\,\Delta t$$

$$y_i = y_{i-1} + v_{i-1}\,\Delta t$$

$$z_i = z_{i-1} + w_{i-1}\,\Delta t$$

$$u_i = u_{i-1} - \left(\gamma M \frac{x_{i-1}}{r^3}\right)\Delta t$$

$$v_i = v_{i-1} - \left(\gamma M \frac{y_{i-1}}{r^3}\right)\Delta t$$

$$w_i = w_{i-1} - \left(\gamma M \frac{z_{i-1}}{r^3}\right)\Delta t$$

A main program has been devised from the combination of these Matlab programs. Random errors have been added to error variations of satellite-airplane distances and the position of the airplane has been found with the three sphere's intersection point using the Newton Rhapson method.

According to simulation results, graphs have been drawn to demonstrate absolute errors and relative errors when finding the position of the plane with GPS. With the help of statistical testing method (Monte Carlo), the statistical characteristics of these errors have been demonstrated. As can be seen in Figure 3 and Figure 4 these errors vary. The studies have shown that this variation is related to the distances between the airplane and the satellites, thus related to the coordinates of the satellites and the airplane.

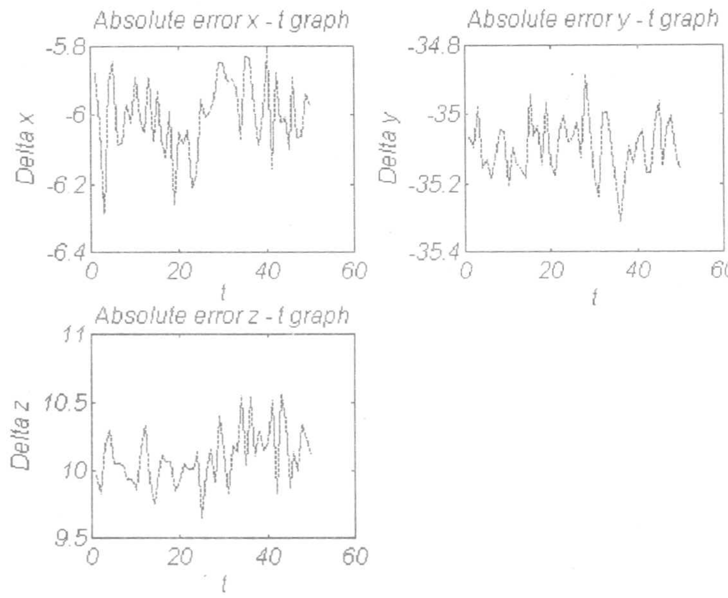

Figure 3.1

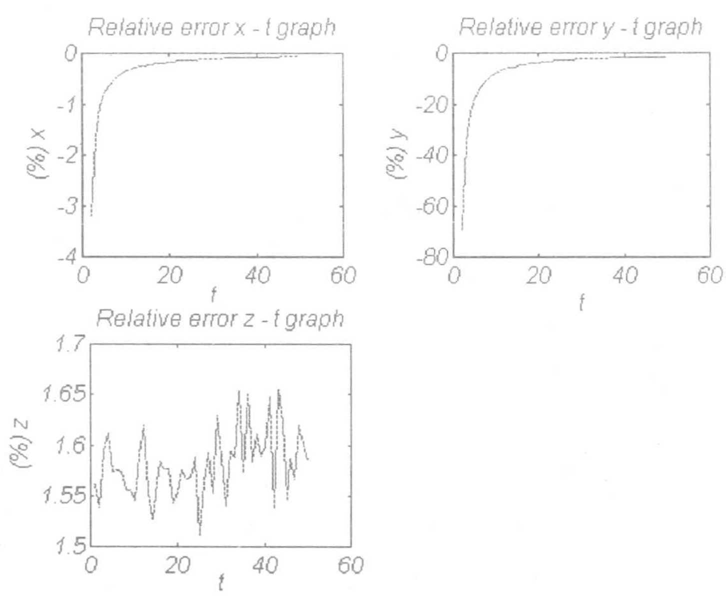

Figure 3.2

standard deviation of x coordinate	:	0.10833283202363
standard deviation of y coordinate	:	0.06926680732545
standard deviation of z coordinate	:	0.28195551766476

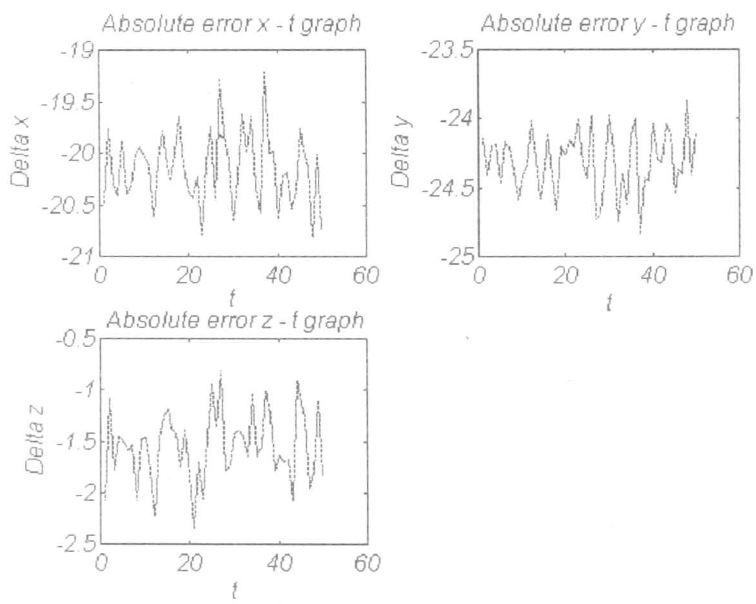

Figure 4.1

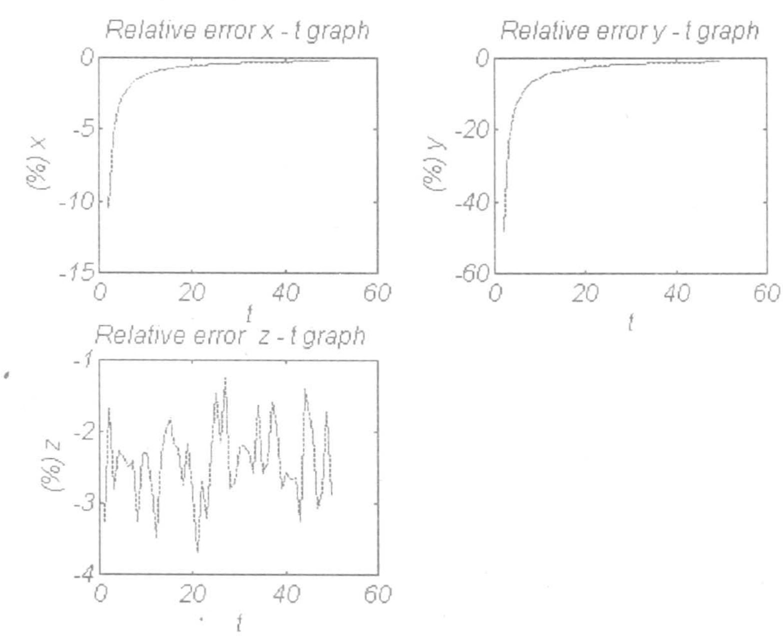

Figure 4.2

Standard deviation of x coordinate : 0.39157726541

Standard deviation of y coordinate : 0.25043341101

Standard deviation of z coordinate : 0.31073458697

CONCLUSION

The accuracy of locating the coordinates of an airplane by GPS has been analyzed using the statistical testing method. The resulting error has a dynamic character and varies with the airplane's and satellites' coordinates.

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