Mathematical and Computational Applications, Vol. 9, No. 3, pp. 435-442, 2004.

FLOW CALCULATIONS IN STRAIGHT-THROUGH LABYRINTH SEALS BY USING MOODY'S FRICTION-FACTOR MODEL

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Abstract - In this work, the gas flow in the straight-through labyrinth seal is studied. Leakage flowrate and pressure distributions are calculated by using Neumann Modifed Method and circumferential velocity distributions are calculated by using Moody's Friction-Factor Model. Results are compared to the other papers.

Keywords- Labyrinth Seal, Leakage Flowrate, Pressure, Velocity, Moody's Friction-Factor Model

1. INTRODUCTION

Labyrinth seals are commonly found in turbines and compressors. They separate high pressure region from the low ones and minimize the leakage of the high pressure gas. This leakage, which depends on a great variety of parameters such as geometry of the teeth, number of cavities, pressure differences, temperature and type of gas, etc., is inevitably present even in the case of abradable seals. There are a few types of labyrinth seals which are being used. The most common one is the straight through labyrinth seals which were studied by [1], [2], [5], [6] etc. The correct prediction and control of this leakage is crucial for efficient and economic operation of turbomachinery.

The gas flow through a labyrinth seal may be briefly described as follows. Swirling gas at the high pressure enters through the clearance between the first tooth of the labyrinth seal and wall opposite to it to first cavity of labyrinth seal, expanding somewhat and altering its rotational momentum by the first friction of cavity walls which may rotate at speeds quite different from the inlet swirl. This rotation is in general non-axisymmetric and time dependent due to small but nevertheless important vibration of the rotor. Once the gas crosses several such cavities it emerges at the other end of the labyrinth seal at significantly reduced pressure. A significant assumption which facilities the semi analytic treatment of this very complex three dimensional unsteady flow is that the gas pressure in each labyrinth cavity as well as the circumferential velocity in each cavity are independent of the radial and axial coordinates within the cavity. Of course, appropriate boundary layers are utilized in estimating the circumferential momentum transfer from the walls to the gas. In addition, when the zeroth order approximation to this flow is considered, i.e. the one for a perfectly centric rotor rotation, the flow can be taken to be axisymmetric and of steady state.

2. LABYRINTH SEAL GEOMETRIES AND NOTATIONS

Labyrinth seal geometries used in this paper are given in Figure 1. Here, NT is the tooth number which varies from 5 to 18, Rs is the shaft radius, Cr is the clearance between teeth and rotor surface. The labyrinth seal pitch L_i is taken equal of labyrinth seal height B_i.

In this work, we assume that the seal geometry is axially symmetric when there is no motion of the rotor axis. The pressure and the circumferential velocity in each cavity are assumed to be uniform and are indicated by P_i and V_i. In front of the first tooth the inlet pressure P_{IN} of the fluid is indicated by P_0 and the outlet pressure P_{OUT} of the fluid beyond the NT 'th tooth is indicated by P_{NT}. Similarly, V_0 denotes the inlet swirl velocity. The leakage at each tooth i is indicated by $\dot{m}_i$ and represents the mass flowrate over the entire circumference of the gap created by the clearance. This gap is denoted by $ANAR_{ri}$ when the teeth are on the rotor and by $ANAR_{si}$ when they are on the stator.

These annular flow areas are defined by

$$ANAR_{si} = \pi(2Rs_i + Cr_i)Cr_i \quad , \quad \text{teeth on stator}$$

$$ANAR_{ri} = \pi(2Rs_i + 2B_i + Cr_i)Cr_i \quad , \quad \text{teeth on rotor.}$$

The gas flow results in viscous shear stresses τ_{si} at the stator wall surfaces and τ_{ri} on the rotor surface. Therefore, we need to define the stress area to each labyrinth cavity. The rotor shear are is defined by

$$RSA = 2\pi Rs_i L_i a_{ri}$$

and the stator shear area is defined by

$$SSA = 2\pi Rs_i L_i a_{si}.$$

Here the dimensionless rotor shear area is

$$a_{ri} = \begin{cases} (2B_i + L_i)/L_i & , \text{ for teeth on the rotor} \\ 1 & , \text{ for teeth on the stator} \end{cases}$$

and the dimensionless staror shear area is

$$a_{si} = \begin{cases} (2B_i + L_i)/L_i & , \text{ for teeth on the stator} \\ 1 & , \text{ for teeth on the rotor.} \end{cases}$$

These quantities are given Figure 2.

3. LEAKAGE FLOWRATE CALCULATIONS AND PRESSURE DISTRIBUTIONS

When the rotor rotates with a constant speed, with no eccentricity present, the flow is time independent. In this steady state situation the continuity equation implies that

$$\dot{m}_1 = \dot{m}_2 = \cdots \dot{m}_{NT} = \dot{m}.$$

The flowrate $\dot{m}$ depends on the geometry of the seals, the pressure difference $P_{IN} - P_{OUT}$, and the inlet temperature T_{IN}. In this work we assume that the gas in each cavity obeys the perfect gas law

$$P_i = \rho R T_i$$

where P_i is the pressure, ρ is the density, T_i is the temperature in the cavity i, R is the gas constant. The flow is taken to be isothermal with $T_i = T_{IN}$ for, $i = 1,\ldots,NT$. In this work, the leakage flowrate and the related pressure distribution were computed using Modified Neumann Method. In this method, $\dot{m}$ is given as

$$\dot{m}_i = \mu_{1i} ANAR_i \frac{P_{i-1}}{\sqrt{RT_{IN}}} \frac{\sqrt{1-\left(\frac{P_i}{P_{i-1}}\right)^2}}{\sqrt{1-\alpha}}.$$

Here, α is the Vermes' residual kinetic energy carry-over factor which is given by

$$\alpha = \frac{8.52}{\frac{L_i - TIPLEN}{Cr_i} + 7.23},$$

μ_{1i} is the discharge flow coefficient and defined as

$$\mu_{1i} = \frac{\pi}{\pi + 2 - 5S_i + 2S_i^2}, \quad S_i = \left(\frac{P_{i-1}}{P_i}\right)^{\frac{\gamma-1}{\gamma}} - 1.$$

This flow coefficient μ_{1i} is same as in [2].

Here, pressure ratio is defined as

$$\frac{P_{i-1}}{P_i} = \left[1 + \frac{\gamma-1}{2\gamma} \frac{U_i^2}{RT_{IN}}\right]^{\frac{\gamma}{\gamma-1}}$$

where U_i is the axial gas velocity at the tooth i and γ is the specific heat ratio.

The formulas given above are valid for subsonic flow assuming that choking does not occur at a particular restriction. Since, the possibility of critical flow at the last tooth of the seals is always present, we must check for critical conditions at the output before proceeding with the pressure distribution calculation.

Choked flow of the gas in the last restriction will occur if

$$\frac{P_{NT}}{P_{NT-1}} \leq \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}, \quad (= 0.528 \text{ for air}).$$

When a particular restriction is choked, leakage flowrate equation at last tooth must be replaced with

$$\dot{m}_{NT} = \mu_{NT} ANAR_{NT} \frac{P_{NT-1}}{\sqrt{RT_{IN}}} \frac{\sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{2\gamma}{\gamma-1}}}}{\sqrt{1-\alpha}}.$$

4. CIRCUMFERENTIAL VELOCITY DISTRIBUTIONS

The bulk circumferential velocity V_i in the i-th labyrinth cavity results in viscous shear stress τ_{si} and τ_{ri} at the stator and rotor surfaces. These stresses influence the momentum balance.

The circumferential momentum of the gas is equal to the mass of the gas times circumferential velocity in the same cavity. The rate of change of circumferential momentum entering the control volume minus the circumferential momentum exiting the control volume is equal to the sum of the forces acting on the control volume.

The steady state circumferential momentum equation is

$$\dot{m}_i V_i - \dot{m}_{i-1} V_{i-1} = RSF - SSF .$$

Here RSF is the rotor shear force and defined as

$$RSF = \tau_{ri}\left(2\pi Rs_i a_{ri} L_i\right)$$

and SSF is the stator shear force and defined as

$$SSF = \tau_{si}\left(2\pi Rs_i a_{si} L_i\right).$$

Using these in the circumferential momentum equation we obtain

$$\dot{m}\left(V_i - V_{i-1}\right) = 2\pi Rs_i L_i\left(\tau_{ri} a_{ri} - \tau_{si} a_{si}\right).$$

Using this formula, we calculate the circumferential velocities in the labyrinth cavities, once the shearing stress have been calculated.

Moody has produced the following approximate representation for pipe-friction factor

$$f = a_1\left[1 + \left(\frac{b_1 e}{Dh} + \frac{b_2}{Re}\right)^{1/3}\right], Re = \frac{VDh\rho}{\mu}$$

where

$$a_1 = 1.375\times10^{-3},\ b_1 = 2\times10^{4},\ b_2 = 10^{6}$$

and e/Dh is the relative roughness. This formula gives values which are within %5 of the Moody diagram for $4000 \le Re \le 10^{7}$ and $e/Dh \le 0.01$. For $e/Dh \ge 0.01$, it significantly underestimates f, [3].

Blasius determined that the shearing stress for turbulent flow in a smooth pipe can be written as

$$\tau = 0.5 f\rho V^{2}.$$

τ_{si} and τ_{ri} for smooth stator and rotor surface can be defined using the Moody's wall friction-factor model.

τ_{si} is given by

$$\tau_{si} = \frac{1}{2}\rho_i V_i^{2} a_1\left[1 + \left(b_1\frac{e}{Dh} + \frac{b_2}{Re}\right)^{1/3}\right].$$

Relative to the rotating rotor surface, the bulk circumferential flow is moving with velocity $\left|Rs_i w - V_i\right|$. Therefore, the shear stress of the rotor surface of the i-th cavity is

$$\tau_{ri} = \frac{1}{2}\rho_i\left(Rs_i w - V_i^2\right)a_1\left[1+\left(b_1\frac{e}{Dh}+\frac{b_2}{Re}\right)^{1/3}\right].$$

Here Re is Reynolds number and defined by

$$Re = \frac{|V_i|Dh\rho}{\mu} \qquad \text{teeth on stator}$$

$$Re = \frac{|Rs_i w - V_i|Dh\rho}{\mu} \qquad \text{teeth on rotor.}$$

Since the kinematic viscosity $\nu = \mu/\rho$, we can write the Reynolds number as

$$Re = \frac{|V_i|Dh}{\nu} \qquad \text{teeth on stator}$$

$$Re = \frac{|Rs_i w - V_i|Dh}{\nu} \qquad \text{teeth on rotor.}$$

Here Dh_i is the hydraulic diameter and defined by

$$Dh = 4\frac{\text{Cross Sectional Flow Area}}{\text{Wetted Perimeter}}$$

$$= \frac{2(Cr_i + B_i)L_i}{Cr_i + B_i + L_i}.$$

5. RESULTS AND CONCLUSIONS

The geometry and the operating conditions used here are given in Table 1. The number of teeth, the pitch of the teeth, the radial clearance, the step height and radius are kept constant in all cases. The operating conditions are also kept costant for all cases. These geometry and conditions are taken from [1], [4] and [7]. We use two different seal types here. These seal types are given Figure 1. The labyrinth seal types we have here are as follows.

LSTYPE 1= Straight-through teeth on stator

LSTYPE 2= Straight-through teeth on rotor.

We compare our results to the results of [4] and [7]. In Figure 1 we compared our leakage flowrate results to the results of [4]. This comparison shows that our leakage is lower then his leakage. Our circumferential velocity results are compared to results of [7] in Figure 3. Our results are in between his results, and it is satisfactory.

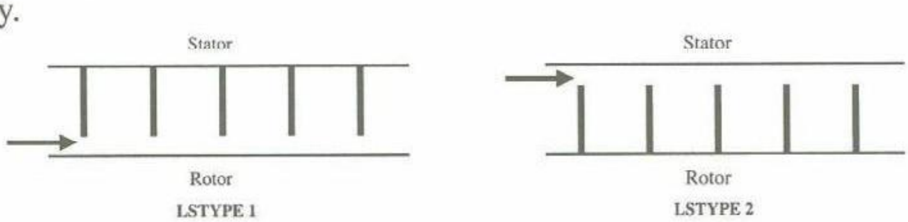


Figure 1. Labyrinth seal types.

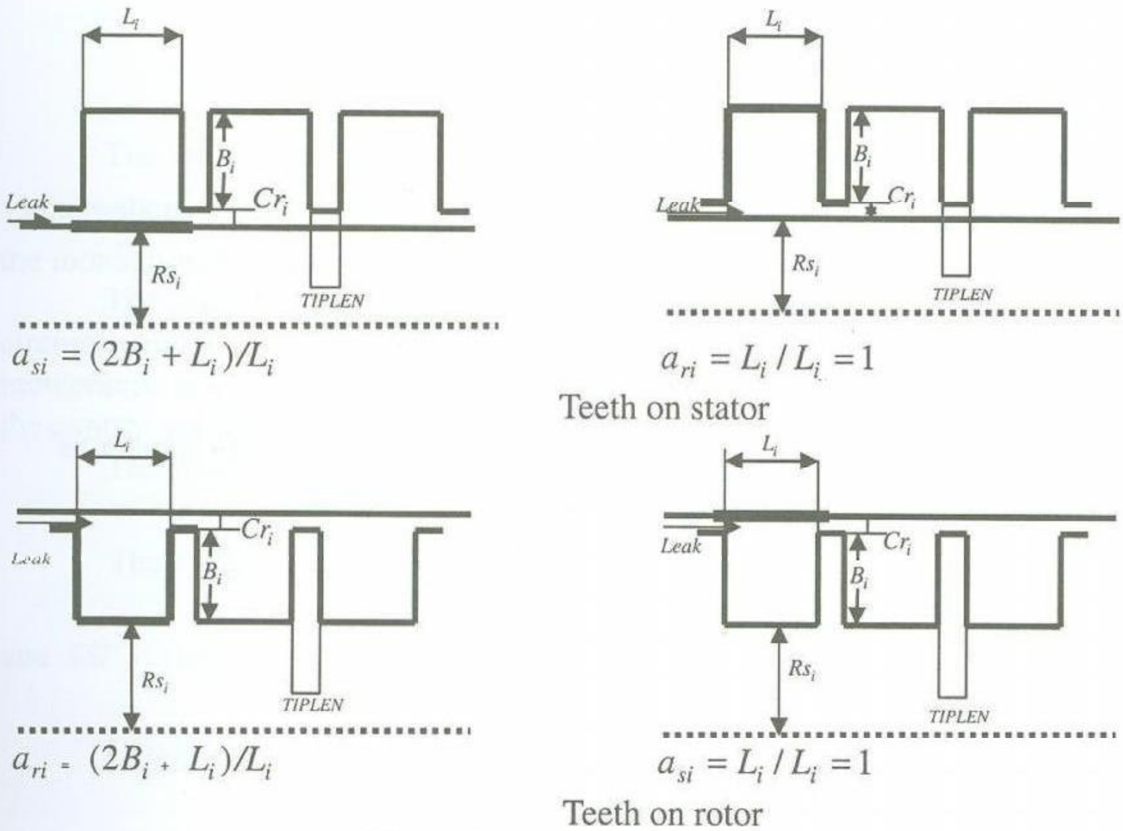


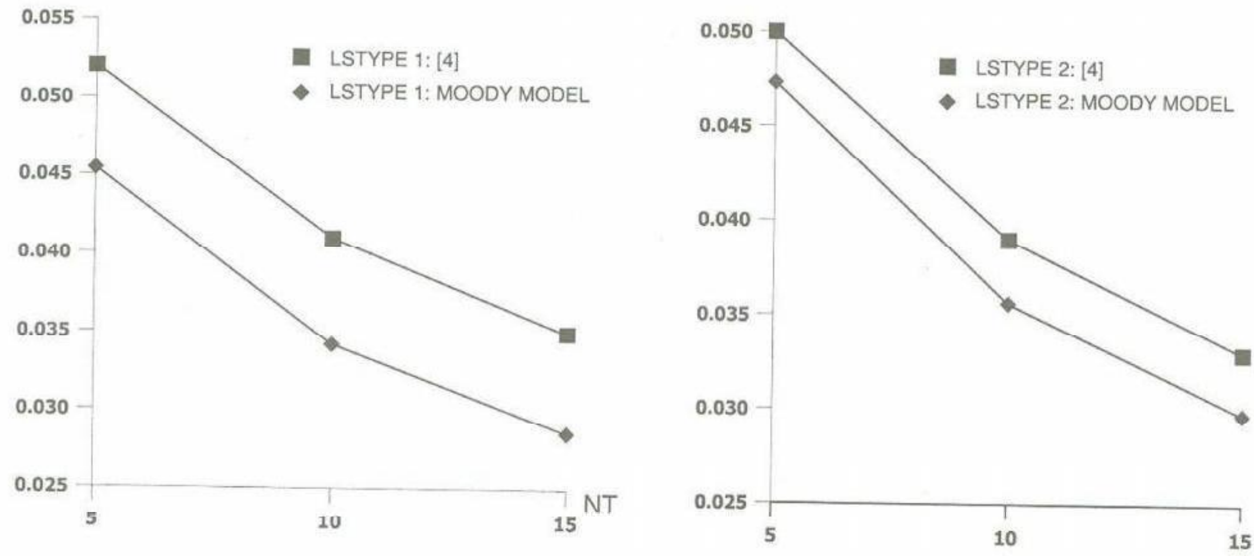
Figure 2. The dimensionless shear areas.

Table 1. Seal geometries and operating conditions.

$NT = 5,10,15,16$	$P_{IN} = 7.00E+5, 3.08E+5\ N/m^2$
$V(0) = 60\ m/s$	$P_{OUT} = 1.01E+5\ N/m^2$
$Rs = 0.0756\ m$	$WRPM = 16000, 20000\ rpm$
$B = 0.003175\ m$	$L = 0.002175, 0.003175\ m$
$Cr = 0.000127,$ 0.00033 m	$T = 300.\ ^oK$
$TIPLEN = .2E-4\ m$	$R = 287.06\ Nm/kg^o K$
	Fluid: Air

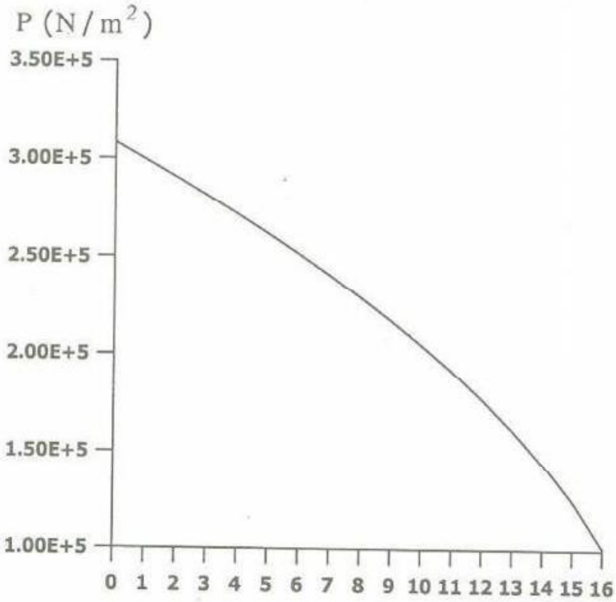
LSTYPE 1: straight-through labyrinth seal teeth on stator
LSTYPE 2: straight-through labyrinth seal teeth on rotor

$\dot{m}$ (kg / s) $\dot{m}$ (kg / s)



$P_{IN} = 7.00E+5,\ WRPM = 20000,\ Cr = 0.000127,\ L = 0.002175$

Figure 3. Comparison of leakage flowrate to the results of [4].



$NT = 16,\ P_{IN} = 3.08E+5,\ WRPM = 16000,$
$Cr = 0.00033,\ L = 0.003175$

Figure 4. Pressure distribution.

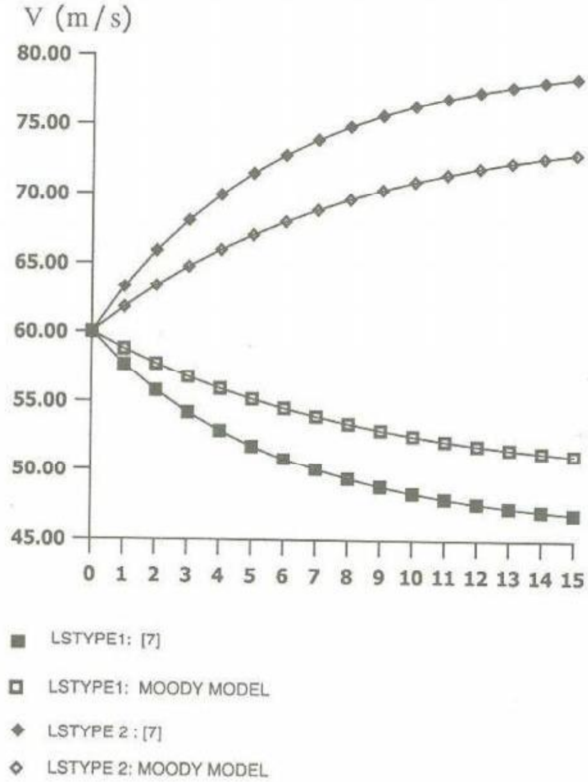


Figure 5. Comparison of circumferential velocity to the results of [7].

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