Mathematical and Computational Applications, Vol. 16, No. 1, pp. 77-86, 2011.

OPTIMIZATION OF OPEN CANAL CROSS SECTIONS BY DIFFERENTIAL EVOLUTION ALGORITHM

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Abstract- Open canals are important water transfer structures used in water resources systems. As such, they may require substantial amount of investment depending on its length and cross section. Therefore, cross section design should be carried out on an optimization basis. Traditionally, optimal sizing of open canal cross sections are undertaken by nonlinear optimization techniques such as Lagrange Multipliers. In this study, optimum cross sections of different canal geometries are obtained using differential evolution algorithm and the findings of these exercises are compared with those of given in related literature. It is observed that differential evolution algorithm can be well applicable to the problem and capable of giving the global optima.

Key Words- Open canal, differential evolution algorithm, optimization.

1. INTRODUCTION

Open canals are used in water resources systems to transfer large quantity of water from a river or another source to where it is used. They are essential elements of irrigation and waterpower systems. They are free surface structures, which carry water by gravity. An open canal may require substantial amount of investment depending on its length and cross section, making the optimal sizing essential. Optimal sizing is to find the optimal cross section dimensions at minimum construction cost.

Nonlinear optimization techniques such as Lagrange Multipliers have traditionally been used to undertake optimal sizing of open canal cross sections. Several researches under different conditions have been performed into optimal cross section design. Majority of this research deals with assessing optimal canal sections of different geometries for uniform flow conditions [1], [2], [3], [4]. There are relatively fewer number of studies for this issue considering non-uniform flow conditions [5][6]. Swamee et al. (2000) have obtained the parameters of an optimal canal cross section based on the minimum cost of earthwork, which increases with an increase in excavation depth [6].

Differential evolution algorithm (DEA) is one of the evolutionary algorithms that can be used for an optimization process. There are several DEA studies for different optimization problems including the identification of structural system parameters [7], multiple objective reservoir operation problem [8] and mass minimization truss problem [9].

In this study, optimum cross sections of different canal geometries are obtained using differential evolution algorithm. The different geometries considered include triangular, circular, rectangular and trapezoidal canal sections.

The objective function is in the form of cost minimization. The cost figures taken into account are the costs of canal covering and excavation. The findings of optimization exercises carried out by differential evolution algorithm are compared with those of given in related literature. Based on the findings obtained, it can be stated DEA is capable of finding the optimal dimensions for canal cross-sections.

2. DEFINITION OF THE PROBLEM

The solution to an optimization problem aims to find global optima. In the particular problem of canal cross section problem, the optimization problem comprises an objective function in the form of minimum cost subject to the flow requirements to convey a specific discharge in the canal considered. The decision variables, the dimensions of canal cross sections, are side slope, bottom width, flow depth, and radius. Geometric properties of a generic canal cross section are given in Figure 1.

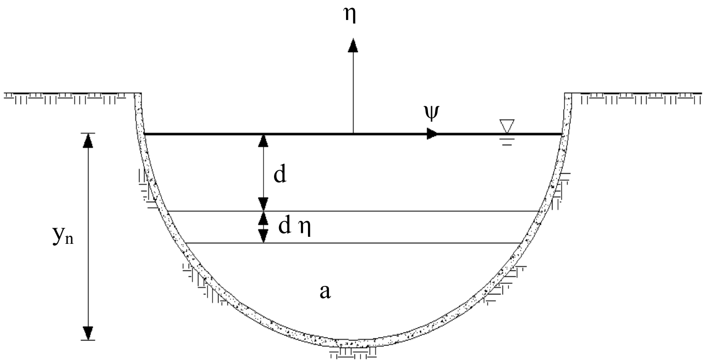


Figure 1. Generic Type of Canal Section

Given the type of canal lining and slope, the cost function for a lined canal can be written as follows:

$$C = \beta_L P + \beta_E A + \beta_A \int_0^A a\, d\eta \tag{1}$$

where β_L : unit lining cost in TL/L^2, β_E : unit excavation cost in TL/L^3, β_A : additional cost of excavation per unit depth in TL/L^4, a: flow area at height η, P: wetted perimeter, A: flow area and L: length. Such a total cost expression for a canal assumes that the canal is constructed for the same soil conditions and cost of excavation increases with the increase in excavation depth.

Assuming the uniform flow conditions apply in the canal, Manning's uniform flow equation is used to define uniform flow as follows:

$$Q = \frac{1}{n} A R^{2/3} S^{1/2} \tag{2}$$

where n: Manning roughness coefficient, A: flow area, R: hydraulic radius and S: bottom slope of canal

The optimization problem for any canal cross section is as follows:

minimize

$$C = \beta_L P + \beta_E A + \beta_A \int_0^A a\, d\eta \tag{3}$$

subject to

$$Q - \frac{1}{n} A R^{2/3} S^{1/2} = 0 \tag{4}$$

Several dimensionless parameters can be defined to examine the effects of the variables and to compare with the results given in related literature using the length scale

$$\lambda = \left(\frac{Q n}{S^{1/2}} \right)^{3/8} \tag{5}$$

The dimensionless parameters defined are as follows:

$$C_* = C / \beta_E \lambda^2 \tag{6}$$

$$\beta_{L*} = \beta_L / \beta_E \lambda^2 \tag{7}$$

$$\beta_{A*} = \beta_A \lambda / \beta_E \tag{8}$$

$$A_* = A / \lambda^2 \tag{9}$$

$$P_* = P / \lambda \tag{10}$$

$$b_* = b / \lambda \tag{11}$$

$$y_{n*} = y_n / \lambda \tag{12}$$

$$r_* = r / \lambda \tag{13}$$

Using these dimensionless parameters, the optimization problems stated in (3) and (4) can be rewritten as follows:

minimize

$$C_* = \beta_{L*} P_* + A_* + \frac{\beta_{A*} \int_0^{y_n} a\, d\eta}{\lambda^3} \tag{14}$$

subject to

$$1 - A_*^{5/3} P_*^{-2/3} = 0 \tag{15}$$

The optimization problem given in (14) and (15) is arranged for triangular, rectangular, trapezoidal and circular canal cross sections, of which geometric dimensions are given in Figure 2, 3, 4 and 5 as follows:

Optimization problem for triangular cross section:

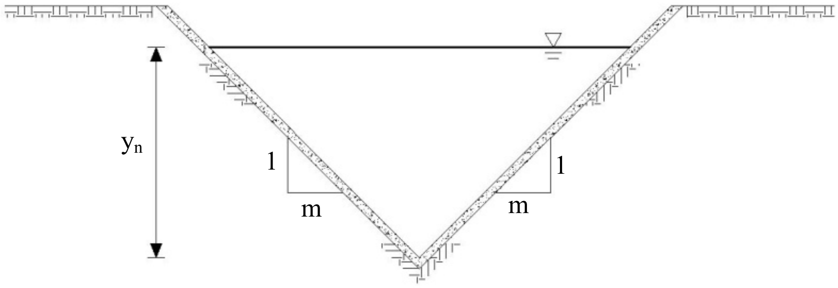


Figure 2. Geometric Dimensions for Triangular Cross Section

minimize

$$C_* = 2\,\beta_{L*}\,y_{n*}\sqrt{1+m^2} + m\,y_{n*}^2 + \frac{\beta_{A*}\,y_{n*}^3}{3} \tag{16}$$

subject to

$$1 - \frac{\left(m\,y_{n*}^2\right)^{5/3}}{\left(2\,y_{n*}\sqrt{1+m^2}\right)^{2/3}} = 0 \tag{17}$$

Optimization problem for rectangular cross section:

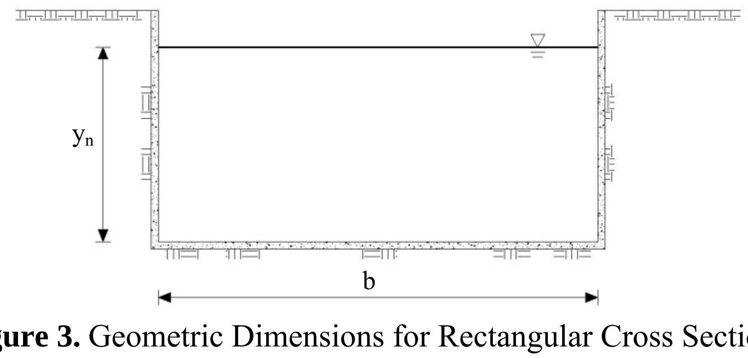


Figure 3. Geometric Dimensions for Rectangular Cross Section

minimize

$$C_* = \beta_{L*}\left(2\,y_{n*} + b_*\right) + b_*\,y_{n*} + \frac{\beta_{A*}\,b_*\,y_{n*}^2}{2} \tag{18}$$

subject to

$$1-\frac{\left(b_{*}\, y_{n*}\right)^{5/3}}{\left(2\, y_{n*}+b_{*}\right)^{2/3}}=0 \tag{19}$$

Optimization problem for trapezoidal cross section:

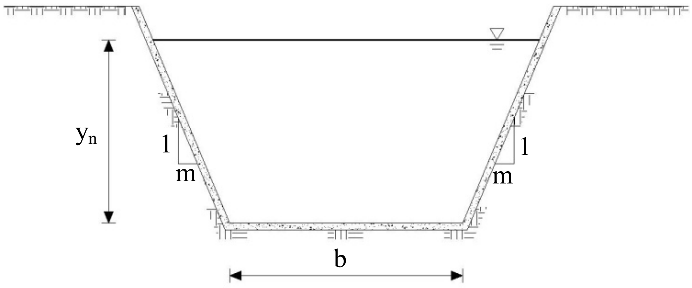


Figure 4. Geometric Dimensions for Trapezoidal Cross Section

minimize

$$C_{*}=\beta_{L*}\left(2\, y_{n*}\sqrt{1+m^{2}}+b_{*}\right)+\left(b_{*}\, y_{n*}+m\, y_{n*}^{2}\right)+\beta_{A*}\left(\frac{b_{*}y_{n*}^{2}}{2}+\frac{m\, y_{n*}^{3}}{3}\right) \tag{20}$$

subject to

$$1-\frac{\left(b_{*}\, y_{n*}+m\, y_{n*}^{2}\right)^{5/3}}{\left(2\, y_{n*}\sqrt{1+m^{2}}+b_{*}\right)^{2/3}}=0 \tag{21}$$

Optimization problem for circular cross section:

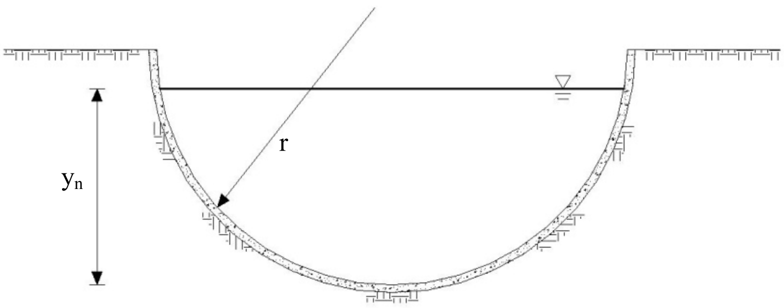


Figure 5. Geometric Dimensions for Circular Cross Section

minimize

$$C_* = \beta_{L*}\left\{ r_* \left[\pi - 2\arcsin\left(\frac{r_* - y_{n*}}{r_*} \right) \right] \right\} + \frac{r_*^2}{2}\left[\pi - 2\arcsin\left(\frac{r_* - y_{n*}}{r_*} \right) - 2\left(\frac{r_* - y_{n*}}{r_*} \right) \frac{\sqrt{r_*^2 - (r_* - y_{n*})^2}}{r_*} \right]$$
$$+ \beta_{A*}\left(\frac{r_*}{2} \right)\left\{ -(r_* - y_{n*})\left[\pi - 2\arcsin\left(\frac{r_* - y_{n*}}{r_*} \right) \right] + 2\sqrt{r_*^2 - (r_* - y_{n*})^2}\left[\frac{2r_*^2 + (r_* - y_{n*})^2}{3r_*^2} \right] \right\} \quad (22)$$

subject to

$$1 - \frac{\left\{ \frac{r_*^2}{2}\left[\pi - 2\arcsin\left(\frac{r_* - y_{n*}}{r_*} \right) - 2\left(\frac{r_* - y_{n*}}{r_*} \right) \frac{\sqrt{r_*^2 - (r_* - y_{n*})^2}}{r_*} \right] \right\}^{5/3}}{\left\{ r_* \left[\pi - 2\arcsin\left(\frac{r_* - y_{n*}}{r_*} \right) \right] \right\}^{2/3}} = 0 \quad (23)$$

3. THE DIFFERENTIAL EVOLUTION ALGORITHM (DEA)

The differential evolution algorithm (DEA), which was proposed by Storn and Price [10], is an evolutionary optimization algorithm like genetic algorithms.

DEA executes population basis. The individuals constituting population generate new populations through mutation, crossover and selection operators. Each individual in the population is a vector of D dimension. The dimension of the vector, D, is the same number as the number of variables in optimization problem [11].

DEA reaches to optima by the following steps;

1. *Initialization*

Lower ($b_{j,L}$) and upper ($b_{j,U}$) bounds of each variables are determined. The initial value of the jth parameter of the ith vector is calculated by uniformly distributed via

$$x_{j,i,0} = rand_j(0,1)(b_{j,U} - b_{j,L}) + b_{j,L} \quad (24)$$

2. *Mutation*

The solution point that the vector represents moves in solution space by this operator. This process can be realized by 3 randomly selected vectors. The mutation vector can be obtained by;

$$v_{i,g} = x_{r3,g} + F(x_{r1,g} - x_{r2,g}) \quad (25)$$

F, a positive reel number, is called scale factor.

3. *Crossover*

A trial vector is obtained through the crossover of two vectors, mutant vector and target vector, as follows:

$$u_{i,g} = u_{j,i,g} = \begin{cases} v_{j,i,g} & \text{if } rand_j(0,1) \leq Cr \\ x_{j,i,g} & \text{otherwise} \end{cases} \tag{26}$$

C_r is called crossover probability and takes the values between 0 and 1.

4. *Selection*

The individuals to form the new generation are selected at this stage. Each trial vector obtained through crossover is compared with the target vector. The vector with the lowest objective function value survives into the next generation g+1. This process is expressed as follows:

$$x_{i,g+1} = \begin{cases} u_{i,g} & \text{if } f(u_{i,g}) \leq f(u_{i,g}) \\ x_{i,g} & \text{otherwise} \end{cases} \tag{27}$$

5. *Termination Criteria*

Since the process is iterative, the termination criteria are normally required to stop the process. For this purpose, the sufficiently small difference, namely error, between the values obtained or a specific number of iteration are used [12].

4. APPLICATIONS

The optimization problems set forth in Section 2 for four different shapes of canal cross sections have been solved by differential evolution algorithm and the section variables obtained. During the operation of DEA, some user-defined parameters are required. These parameters include the number of individuals in population, NP, scale rate, F, and crossover probability Cr. These parameters should be selected in such a way that the solution process is speeded up. In this study, following values were used: Np=10, F=0.85 and Cr=0.5.

The results obtained in this study by DEA have been compared with those given in Aksoy and Sakarya for the same section types.

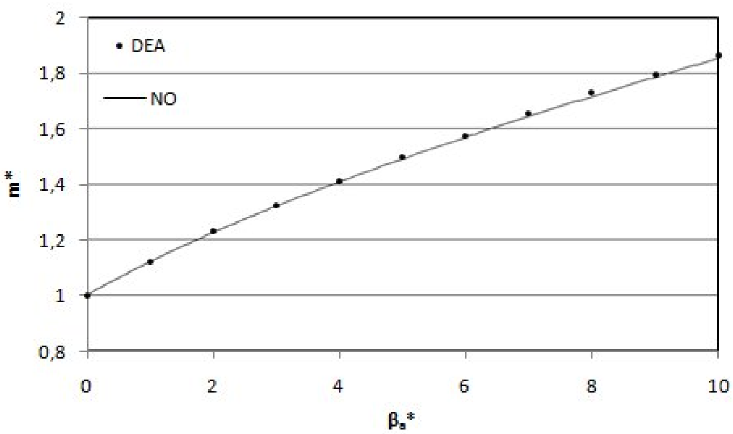
First of all, classical optimum cross section problem where excavation cost does not change with excavation depth, e.g. β_{a*}=0, was solved. This problem was solved in Literature by Langrange Multipliers (LM) and the results of such solution are taken into account for comparing the results of DEA as shown in Table (1) and (2).

Table 1. The results of LM and DEA for optimum nondimensional section variables for $\beta_{A*} = 0$

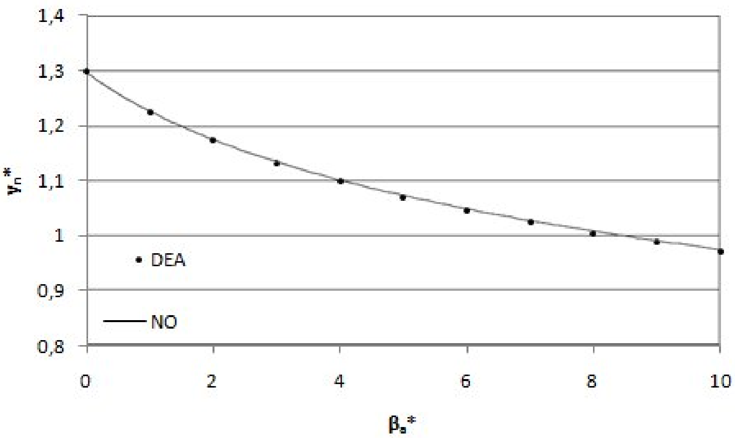
Section Variables	Triangular section		Rectangular section	
	LM	DEA	LM	DEA
Side slope, m*	1.000	1.000	-	-
Bottom width, b*	-	-	1.834	1.83358
Flow depth, y_{n*}	1.297	1,297	0.917	0.91721

Section Variables	Trapezoidal section		Circular section	
	LM	DEA	LM	DEA
Side slope, m*	0.577	0.577	-	-
Bottom width, b*	1.118	1.117	-	-
Flow depth, y_{n*}	0.968	0.968	1.004	1.004
Radius, r*	-	-	1.004	1.00387

As a second exercise, the problem is solved by DEA for the different values of β_{A*} and β_{L*} and section variables obtained. Aksoy and Sakarya solved the same problem by a numerical optimization (NO) technique. For triangular cross section, the results of both DEA and NO have been presented in Figure 6 and 7 as the results for other types are not traceable in Aksoy and Sakarya.

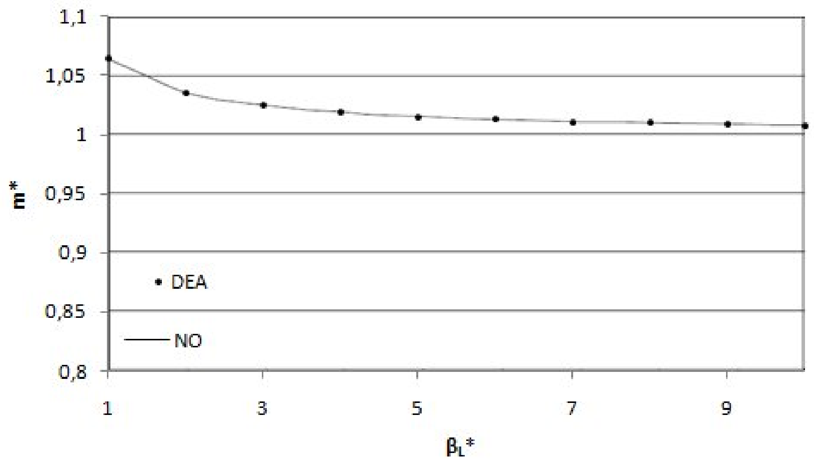


(a)

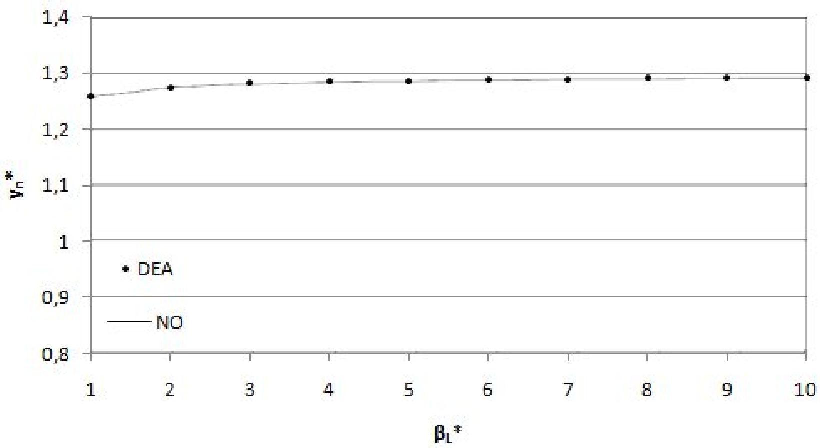


(b)

Figure 6. Variation of (a) optimum nondimensional side slope and (b) normal depth of a triangular section with β_{L*} = 1.0



(a)



(b)

Figure 7. Variation of (a) optimum nondimensional side slope and (b) normal depth of a triangular section with $\beta_{A*} = 0.5$

6. CONCLUSION

The classical problem of optimal canal cross section design has been revisited. The problem is solved by Differential Evolutionary Algorithm for the different types of section types including circular triangular, rectangular and trapezoidal shapes. The problem is considered the same as that formulated in related literature so as to make one-to-one comparisons to see the capability of DEA results of DEA have confirmed the results obtained in the literature. Therefore, it has been concluded that DEA can successfully be applied to the problem and also is capable of satisfactory results.

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