Mathematical and Computational Applications, Vol. 16, No. 2, pp. 370-381, 2011.

AN IMPROVED VOGEL'S APPROXIMATION METHOD FOR THE TRANSPORTATION PROBLEM

Serdar Korukoğlu[1] and Serkan Ballı[2]
[1]Department of Computer Engineering, Ege University, 35000, Bornova, İzmir, Turkey
serdar.korukoglu@ege.edu.tr
[2]Department of Statistics, Muğla University, 48187, Muğla, Turkey
serkan@mu.edu.tr

Abstract- Determining efficient solutions for large scale transportation problems is an important task in operations research. In this study, Vogel's Approximation Method (VAM) which is one of well-known transportation methods in the literature was investigated to obtain more efficient initial solutions. A variant of VAM was proposed by using total opportunity cost and regarding alternative allocation costs. Computational experiments were carried out to evaluate VAM and improved version of VAM (IVAM). It was seen that IVAM conspicuously obtains more efficient initial solutions for large scale transportation problems. Performance of IVAM over VAM was discussed in terms of iteration numbers and CPU times required to reach the optimal solutions.

Keywords- Transportation Problem, Integer Programming, Vogel's Approximation Method, Total Opportunity Cost, Simulation Experiments

1. INTRODUCTION

The transportation problem is a special kind of the network optimization problems. It has the special data structure in solution characterized as a transportation graph. Transportation models play an important role in logistics and supply chains. The problem basically deals with the determination of a cost plan for transporting a single commodity from a number of sources to a number of destinations [16]. The purpose is to minimize the cost of shipping goods from one location to another so that the needs of each arrival area are met and every shipping location operates within its capacity [10]. Network model of the transportation problem is shown in Figure 1 [17]. It aims to find the best way to fulfill the demand of n demand points using the capacities of m supply points.

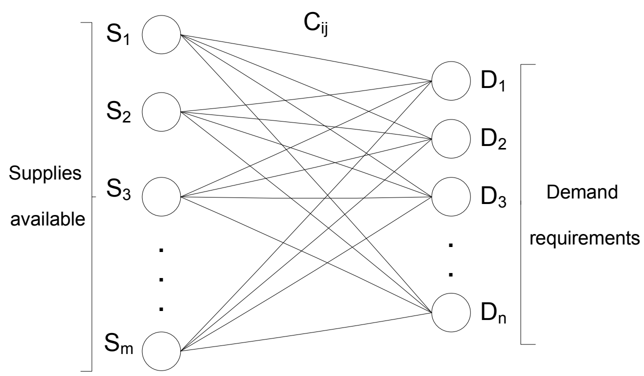

Figure 1. Network model of the transportation problem

In Figure 1, S_1-S_m are sources and D_1-D_n are destinations. c_{ij} is cost and x_{ij} is number of units shipped from supply point i to demand point j then the general linear programming representation of a transportation problem is :

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} \le S_i \quad (i=1,2,...,m) \quad \text{Supply constraints} \qquad (1)$$

$$\sum_{j=1}^{n} x_{ij} \ge D_j \quad (j=1,2,...,n) \quad \text{Demand constraints} \qquad (2)$$

$$x_{ij} \ge 0 \text{ and integer for all } i,j$$

If total supply equals total demand then the problem is said to be a balanced transportation problem. The reader may refer to Wagner [17] and Taha [16] for detailed coverage of transportation problem.

Transportation problems can be solved by using general simplex based integer programming methods, however it involves time-consuming computations. There are specialized algorithms for transportation problem that are much more efficient than the simplex algorithm [18]. The basic steps to solve transportation problem are:
Step 1. Determination the initial feasible solution,
Step 2. Determination optimal solution using the initial solution.

In this study, basic idea is to get better initial solutions for the transportation problem. Therefore, study focused on Step 1 above. Several heuristic methods are available to get an initial basic feasible solution. Although some heuristics can find an initial feasible solution very quickly, oftentimes the solution they find is not very good in terms of minimizing total cost. On the other hand, some heuristics may not find an initial solution as quickly, but the solution they find is often very good in terms of minimizing total cost [2]. Well-known heuristics methods are North West Corner [4], Best Cell Method, Vogel's Approximation Method (VAM) [11], Shimshak et. al.'s version of VAM [14], Goyal's version of VAM [6], Ramakrishnan's version of VAM [9] etc. Kirca and Satir [7] developed a heuristic to obtain efficient initial basic feasible solutions, called Total Opportunity-cost Method (TOM). Balakrishnan [3] proposed a modified version of VAM for unbalanced transportation problems. Gass [5] reviewed various methods and discussed on solving the transportation problem. Sharma and Sharma [12] proposed a new procedure to solve the dual of the well-known uncapacitated transportation problem. Sharma and Prasad [13] proposed heuristic gives significantly better solutions than the well-known VAM. This is a best heuristic method than Vogel's to get initial solution to uncapacitated transportation problem. Adlakha and Kowalski [1] presented a simple heuristic algorithm for the solution of small fixed-charge transportation problems. Mathirajan and Meenakshi [8] were extended TOM using the VAM procedure. They coupled VAM with total opportunity cost and achieved very efficient initial solutions.

In this paper, VAM was improved by using total opportunity cost and regarding alternative allocation costs. Mathirajan and Meenakshi [8] applied VAM on the total opportunity cost matrix. In addition to this method, improved VAM (IVAM) considers highest three penalty costs and calculates alternative allocation costs in VAM procedure. Then it selects minimum one of them.

Paper is organized as follows. VAM is summarized and illustrated with solving a sample transportation problem in the following section. IVAM is explained in the third section. In the fourth section, simulation experiments are given and performance of IVAM over VAM is discussed. Results are clarified in fifth section.

2. VOGEL'S APPROXIMATION METHOD (VAM)

VAM is a heuristic and usually provides a better starting solution than other methods. Application of VAM to a given problem does not guarantee that an optimal solution will result. However, a very good solution is invariably obtained with comparatively little effort [15]. In fact, VAM generally yields an optimum or close to optimum starting solution for small sized transportation problems [16].

VAM is based on the concept of penalty cost or regret. A penalty cost is the difference between the largest and next largest cell cost in a row or column. VAM allocates as much as possible to the minimum cost cell in the row or column with the largest penalty cost. Detailed processes of VAM are given below:

Step 1:	Balance the given transportation problem if either (total supply>total demand) or (total supply<total demand)
Step 2:	Determine the penalty cost for each row and column by subtracting the lowest cell cost in the row or column from the next lowest cell cost in the same row or column.
Step 3:	Select the row or column with the highest penalty cost (breaking ties arbitrarily or choosing the lowest-cost cell).
Step 4:	Allocate as much as possible to the feasible cell with the lowest transportation cost in the row or column with the highest penalty cost.
Step 5:	Repeat steps 2, 3 and 4 until all requirements have been meet.
Step 6:	Compute total transportation cost for the feasible allocations.

An example transportation problem is given in Table 1. In the example matrix size is 5x5. S1-S5 are source points and D1-D5 are destination points. Each box in the left of the columns represents constant costs (c_{ij}) and each empty box in the right of the columns represents allocation quantities (x_{ij}) which is number of units shipped from supply point i to demand point j.

Table 1. An example of 5x5 transportation problem

From/To	D1		D2		D3		D4		D5		Supply
S1	46		74		9		28		99		**461**
S2	12		75		6		36		48		**277**
S3	35		199		4		5		71		**356**
S4	61		81		44		88		9		**488**
S5	85		60		14		25		79		**393**
Demand	**278**		**60**		**461**		**116**		**1060**		

Initial basic solution for this problem was obtained using VAM and given in Table 2. Using the values in Table 2, initial cost was calculated as 68804.

Table 2. Initial solution tableau of VAM

From/To	D1		D2		D3		D4		D5		Supply
S1	46	1	74	60	9	68	28		99	332	**461**
S2	12	277	75		6		36		48		**277**
S3	35		199		4		5	116	71	240	**356**
S4	61		81		44		88		9	488	**488**
S5	85		60		14	393	25		79		**393**
Demand	**278**		**60**		**461**		**116**		**1060**		

Optimal solution was achieved using transportation simplex algorithm [4] within five iterations and final cost was found as 59356. Optimal solution tableau is given in Table 3. Solution of proposed method (IVAM) for the same problem is illustrated in the following section.

Table 3. Optimal solution tableau

From/To	D1		D2		D3		D4		D5		Supply
S1	46		74		9	461	28		99		**461**
S2	12	277	75		6		36		48		**277**
S3	35	1	199		4		5	116	71	239	**356**
S4	61		81		44		88		9	488	**488**
S5	85		60	60	14		25		79	333	**393**
Demand	**278**		**60**		**461**		**116**		**1060**		

3. IMPROVED VAM (IVAM)

VAM was improved by using total opportunity cost (TOC) matrix and regarding alternative allocation costs. The TOC matrix is obtained by adding the "row opportunity cost matrix" (row opportunity cost matrix: for each row, the smallest cost of that row is subtracted from each element of the same row) and the "column opportunity cost matrix" (column opportunity cost matrix: for each column of the original transportation cost matrix the smallest cost of that column is subtracted from each element of the same column) [8]. Proposed algorithm is applied on the TOC matrix and it considers highest three penalty costs and calculates alternative allocation costs in VAM procedure. Then it selects minimum one of them. Detailed processes are given below:

Step 1: Balance the given transportation problem if either (total supply>total demand) or (total supply<total demand).
Step 2: Obtain the TOC matrix.
Step 3: Determine the penalty cost for each row and column by subtracting the lowest cell cost in the row or column from the next lowest cell cost in the same row or column.
Step 4: Select the rows or columns with the highest three penalty costs (breaking ties arbitrarily or choosing the lowest-cost cell).
Step 5: Compute three transportation costs for selected three rows or columns in step 4 by allocating as much as possible to the feasible cell with the lowest transportation cost.

Step 6: Select minimum transportation cost of three allocations in step 5 (breaking ties arbitrarily or choosing the lowest-cost cell).
Step 7: Repeat steps 3-6 until all requirements have been meet.
Step 8: Compute total transportation cost for the feasible allocations using the original balanced-transportation cost matrix.

For the transportation problem given in Table 1, initial solution of VAM requires five additional iterations to reach the optimal solution. The problem was resolved using IVAM and initial basic solution for this problem is given in Table 4. Initial solution of IVAM is the optimal solution of the given example problem without additional iterations. Initial cost from Table 4 is 59356 and this also is the optimal value of the considered problem.

Table 4. Initial solution tableau of IVAM

From/To	**D1**		**D2**		**D3**		**D4**		**D5**		**Supply**
S1	46		74		9	461	28		99		**461**
S2	12	277	75		6		36		48		**277**
S3	35	1	199		4		5	116	71	239	**356**
S4	61		81		44		88		9	488	**488**
S5	85		60	60	14		25		79	333	**393**
Demand	**278**		**60**		**461**		**116**		**1060**		

This paper focuses on the iteration numbers to the optimal solution and computation times of the VAM and improved version IVAM. Simulation experiments of VAM and IVAM for different sized transportation problems are presented in the following section.

4. SIMULATION EXPERIMENTS

For evaluating the performance of the VAM and its variant IVAM, simulation experiments were carried out on a 2.13 GHz Intel Core 2 Duo machine with 4096 MB RAM. The main goal of the experiment was to evaluate the effectiveness of the initial solutions obtained by VAM and IVAM by comparing them with optimal solutions. Effectiveness indicates closeness degree which is the lowest iteration number between initial solution and the optimal solution. Measures of effectiveness are explained below.

4.1 Measure of effectiveness

The performances of VAM and IVAM are compared using the following measures:
Average Iteration (AI): Mean of iteration numbers to obtain optimal solutions using the initial solutions of VAM and IVAM over various sized problem instances.
Number of best solutions (NBS): A frequency which indicates the number of instances VAM and IVAM yielded optimal solution with lower iteration over the total of problem instances. NBS does not contain case of equal iteration between VAM and IVAM.
Computation Time: The CPU time is represented by three variables: T_1, T_2 and T_3. T_1 is the time to reach initial solution. T_2 is the time to reach optimal solution from initial solution and T_3 is the total time from the beginning that is sum of T_1 and T_2.

4.2 Experimental design

The transportation problems were randomly generated with twelve different sizes (row x column): 5x5, 10x10, 10x20, 10x30, 10x40, 20x20, 10x60, 30x30, 10x100, 40x40, 50x50 and 100x100, respectively. The performance of the VAM and IVAM were compared over 1000 problem instances for each different sized problem. Total problem instances were 12000. Costs were generated as uniformly discrete in the range of (0,1000) and supplies and demands were generated in the range of (0,100) uniformly discrete. All the 12000 problem instances were balanced. The experimental design was implemented using ANSI C.

4.3 Comparison of VAM and IVAM

The experiments and the analysis of the experimental data are presented in this section. For each problem instance, a linear programming model was implemented and solved. In order to get a linear programming model for each problem instance, a matrix generator procedure and VAM and IVAM were implemented using ANSI C. For each problem instance, the heuristic solutions were obtained using VAM and IVAM. The performance of the VAM and IVAM in comparison with the optimal solution is presented below.

Performance measure - AI: AI and other statistics for the iteration numbers of VAM and IVAM over various sized problems are given in Table 5. For each different sized problem, statistics are calculated using 1000 problem instances. From Table 5, it is seen that AI of VAM is better than IVAM for small sized transportation problems but it deteriorates when problem size increases. IVAM yields more efficient results than VAM for large sized problems.

Table 5. Statistical indicators for the iteration numbers of VAM and IVAM

Problem size	AI		Standard Error		Median		Range	
	VAM	IVAM	VAM	IVAM	VAM	IVAM	VAM	IVAM
5x5	2.198	2.676	0.034	0.035	2	3	5	5
10x10	6.155	6.359	0.069	0.066	6	6	14	14
10x20	10.063	9.913	0.094	0.092	10	10	18	18
10x30	13.264	12.951	0.120	0.116	13	13	24	27
10x40	17.761	17.495	0.141	0.144	17	17	28	30
20x20	17.007	16.337	0.146	0.135	17	16	31	31
10x60	22.869	22.118	0.173	0.166	23	22	36	32
30x30	29.912	28.363	0.210	0.200	30	28	56	39
10x100	32.561	31.139	0.221	0.220	32	31	40	43
40x40	44.801	42.603	0.291	0.269	44	42	55	59
50x50	60.505	57.651	0.346	0.328	60	57	74	60
100x100	158.890	149.53	0.657	0.610	158	149	137	112

Both parametric and nonparametric statistical tests were performed using MINITAB-15 Statistical Package for comparing iteration numbers of VAM and IVAM. Firstly, Student's t-test was used for testing the mean of differences of the iteration numbers between VAM and IVAM based on 1000 samples. Secondly Wilcoxon test was used for testing the median of differences of the iteration numbers between VAM and IVAM on the same samples. The formal test is H_o: No statistically significant difference in the amount of mean (median) to complete the transportation optimization between the VAM and IVAM initial methods. Table 6 gives a summary of the results of two-sided statistical tests.

Table 6. Statistical tests for difference of iteration numbers between VAM and IVAM

Problem size	Student's t-test				Wilcoxon Test		
	Mean ± Standard Error	Confidence Interval %95	t	P	Estimated Median	Wilcoxon Statistic	P
5x5	-0.478±0.039	-0.555 ; -0.401	-12.190	0.000	-0.500	52276.000	0.000
10x10	-0.204±0.074	-0.348 ; -0.060	-2.770	0.006	0.000	148409.000	0.008
10x20	0.150±0.102	-0.051 ; 0.351	1.470	0.143	0.000	203812.000	0.103
10x30	0.313±0.125	0.068 ; 0.558	2.510	0.012	0.500	218149.500	0.022
10x40	0.266±0.159	-0.047 ; 0.579	1.670	0.096	0.500	229297.500	0.104
20x20	0.670±0.157	0.362 ; 0.978	4.270	0.000	0.500	245585.500	0.000
10x60	0.751±0.191	0.376 ; 1.126	3.930	0.000	1.000	248683.500	0.000
30x30	1.549±0.229	1.099 ; 1.999	6.760	0.000	1.500	281480.500	0.000
10x100	1.422±0.249	0.934 ; 1.910	5.720	0.000	1.500	276839.500	0.000
40x40	2.198±0.313	1.584 ; 2.812	7.030	0.000	2.500	290830.500	0.000
50x50	2.854±0.375	2.118 ; 3.590	7.600	0.000	3.000	293811.500	0.000
100x100	9.360±0.725	7.937 ; 10.783	12.900	0.000	9.000	349416.000	0.000

It is seen from Table 6 that, VAM has better results at the 0.01 significance level for the cases 5x5 and 10x10 problem sizes regarding both mean and median test. There is no difference at the 0.05 significance level between VAM and IVAM in the cases 10x20 and 10x30 problem sizes, regarding both mean and median test. On the other hand, in the rest of all the cases 10x40, 20x20, 10x60, 30x30, 10x100, 40x40, 50x50 and 100x100, both Student's t-test and Wilcoxon test show the same result: the method of IVAM is statistically significantly different from the method of VAM. This indicates that, in the most of the cases IVAM has better performance in terms of iteration numbers than VAM; because of the given statistics in Table 6 are significant at the 0.01 significance level. As a result of the performed tests above, it can be said that IVAM is significantly better than VAM for large sized problems such as having greater than 400 arcs or cells.

Performance measure - NBS: VAM and IVAM yield optimal solutions with different iteration numbers for different sized 1000 problem instances. These values are given in Table 7. From Table 7, it is clear that the NBS of VAM and IVAM significantly vary for different sized problems. Graphical representation of these values is shown in Figure 2. VAM gets efficient initial solutions for small sized transportation problems but it is insufficient for large sized transportation problems. VAM is better for the problem sizes: 5x5, 10x10, 10x20. But for other sized problems, IVAM yields more efficient results than VAM. In 100x100 sized transportation problems, IVAM determines more effective initial solutions than VAM for 664 of 1000 problem instances.

Table 7. Number of best solutions

Matrix size		**NBS**	
mxn	**Size, mxn**	**VAM**	**IVAM**
5x5	25	460	183
10x10	100	450	365
10x20	200	420	455
10x30	300	413	482
10x40	400	430	499
20x20	500	414	511
10x60	600	406	523
30x30	900	395	557
10x100	1000	394	564
40x40	1600	382	578
50x50	2500	375	587
100x100	10000	321	664

NBS does not include case of equal iteration between VAM and IVAM. Success rate is the ratio of NBS to total of problem instances. For each sized problem, success rates (including case of equality) are shown in Figure 3. It shows that IVAM conspicuously obtains more efficient initial solutions than VAM.

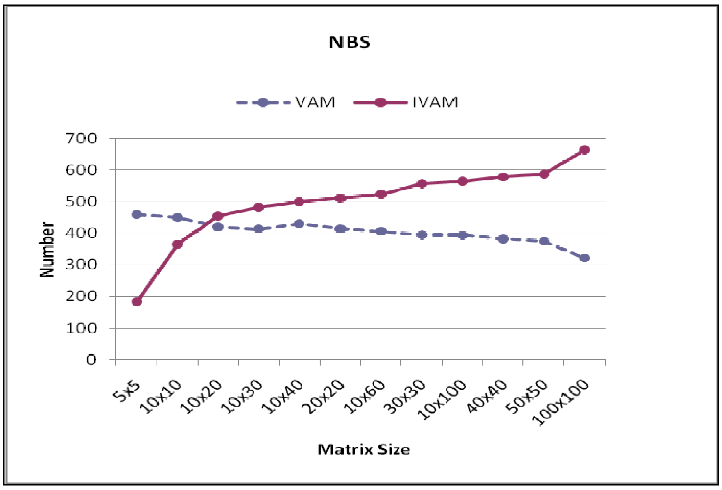

Figure 2. Number of best solutions

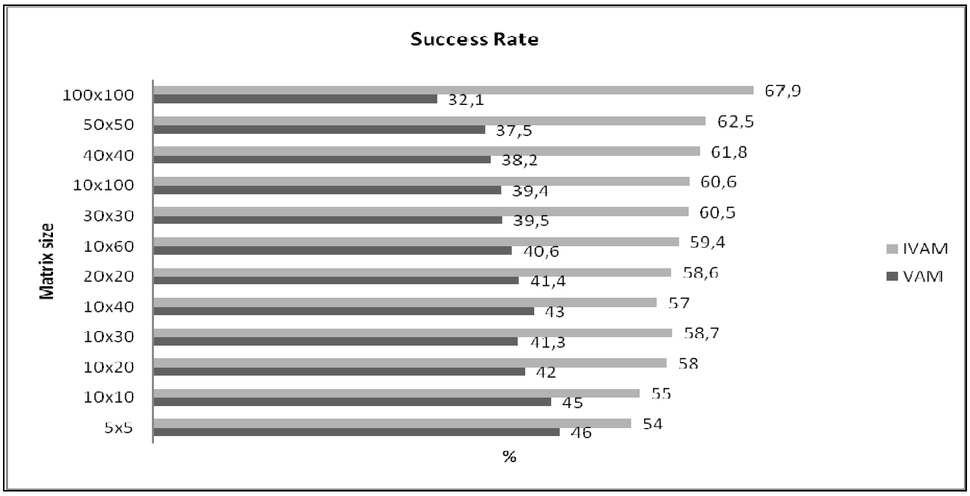

Figure 3. Success rate of VAM and IVAM for different sized problems

Performance measure -CPU Time: T_1, T_2 and T_3 times for VAM and IVAM over various sized problem instances are given in Table 8. For each different sized problem; mean, standard error, coefficient of variation and range of times are calculated based on 1000 samples.

Statistical tests were also performed for comparing total CPU times T_3 of VAM and IVAM. Student's t-test was used for testing the mean of differences of the total CPU times between VAM and IVAM based on 1000 samples. And also, one sample Wilcoxon test was used for testing the median of differences of total CPU times between VAM and IVAM based on 1000 samples. The formal test now is H_o: No statistically significant difference in the amount of total CPU time mean (median) to complete the transportation optimization between VAM and IVAM initial methods. Table 9 gives a summary of the results of two-sided statistical tests.

Table 8. T_1, T_2 and T_3 times for VAM and IVAM

Problem size	Time	Mean ± Standard Error		Coefficient of Variation		Range	
		VAM	IVAM	VAM	IVAM	VAM	IVAM
5x5	T_1	0.923±0.006	1.335±0.003	20.060	6,720	0.969	2.456
	T_2	0.490±0.004	0.532±0.023	23.500	140.460	1.507	23.811
	T_3	1.413±0.007	1.867±0.024	15.960	41.230	1.795	24.410
10x10	T_1	3.914±0.007	4.395±0.255	5.83	183.730	1.004	255.404
	T_2	1.591±0.021	1.599±0.012	41.65	23.510	14.416	3.447
	T_3	5.505±0.022	5.995±0.256	13.05	134.910	12.805	255.960
10x20	T_1	8.188±0.017	9.115±0.059	6.85	20.540	13.777	47.398
	T_2	5.421±0.368	4.853±0.054	214.84	35.390	264.654	21.039
	T_3	13.610±0.371	13.969±0.090	86.26	20.450	267.549	68.193
10x30	T_1	14.427±0.023	14.700±0.046	5.15	9.860	16.248	22.308
	T_2	9.646±0.0778	9.509±0.082	25.54	27.540	24.748	35.524
	T_3	24.074±0.815	24.209±0.094	10.71	12.270	26.586	35.934
10x40	T_1	23.599±0.027	23.908±0.046	3.64	6.110	16.223	17.899
	T_2	23.198±0.167	22.872±0.175	22.74	24.160	38.456	47.612
	T_3	46.798±0.170	46.780±0.185	11.49	12.480	47.656	48.769
20x20	T_1	20.604±0.018	20.790±0.049	2.84	7.460	12.533	24.137
	T_2	20.393±0.194	19.415±0.178	30.10	28.920	42.207	50.310
	T_3	40.997±0.196	40.205±0.185	15.11	14.540	44.453	50.462
10x60	T_1	37.521±0.272	37.134±0.044	2.290	3.750	20.643	15.402
	T_2	50.587±0.361	49.185±0.345	22.570	22.210	74.967	66.375
	T_3	88.108±0.363	86.319±0.347	13.020	12.730	75.154	66.355
30x30	T_1	64.438±0.098	60.469±0.147	4.830	7.690	51.111	73.508
	T_2	126.01±0.900	119.450±0.826	22.600	21.860	271.920	165.460
	T_3	190.45±0.917	179.920±0.858	15.230	26.080	275.680	224.840
10x100	T_1	82.871±0.122	75.701±0.144	4.670	6.000	72.1770	49.895
	T_2	182.630±1.230	175.180±1.220	21.290	22.080	275.440	243.270
	T_3	265.500±1.250	250.880±1.250	14.870	15.690	278.990	256.180
40x40	T_1	156.980±0.138	143.22±0.341	2.780	7.52	70.390	273.43
	T_2	552.460±3.560	526.340±3.290	20.400	19.76	678.010	717.07
	T_3	709.403±3.570	669.550±3.310	15.930	15.63	688.250	732.43
50x50	T_1	321.800±0.040	276.580±0.075	0.390	0.860	20.630	41.410
	T_2	1662.900±9.440	1585.700±8.960	17.950	17.870	2029.700	1622.700
	T_3	1984.700±9.440	1862.300±8.960	15.040	15.220	2029.300	1620.800
100x100	T_1	4737.7±1.440	2625.400±0.646	0.960	0.780	799.4	278.400
	T_2	51811±214.000	48760±199	13.060	12.900	44226	37299
	T_3	56549±214.000	51386±199	11.960	12.240	44226	37332

Table 9. Statistical tests for difference of total CPU times between VAM and IVAM

Problem size	Student's t-test				Wilcoxon Test		
	Mean ± Standard Error	Confidence Interval %95	t	P	Estimated Median	Wilcoxon Statistic	P
5x5	-0.454±0.024	-0.501 ; -0.406	-18.890	0.000	-0.426	3700.500	0.000
10x10	-0.490±0.257	-0.993 ; 0.014	-1.910	0.057	-0.246	114172.000	0.000
10x20	-0.359±0.380	-1.104 ; 0.385	-0.950	0.344	-0.745	146494.500	0.000
10x30	-0.135±0.103	-0.337 ; 0.066	-1.320	0.189	-0.035	246697.000	0.697
10x40	0.018±0.204	-0.381 ; 0.418	0.090	0.929	0.066	253332.000	0.736
20x20	0.792±0.254	0.294 ; 1.291	3.120	0.002	0.750	278891.000	0.002
10x60	1.789±0.401	1.003 ; 2.576	4.460	0.000	1.816	291998.500	0.000
30x30	10.525±0.975	8.611 ; 12.439	10.790	0.000	10.370	343213.000	0.000
10x100	14.620±1.400	11.880 ; 17.360	10.480	0.000	14.600	341338.500	0.000
40x40	39.880±3.860	32.310 ; 47.450	10.340	0.000	40.920	342112.000	0.000
50x50	122.400±10.300	102.300;142.600	11.930	0.000	121.100	351714.000	0.000
100x100	5163.000±236	4701 ; 5626	21.910	0.000	5112	419545.000	0.000

It is seen from Table 9 that, VAM method has better result in the Student's t-test for only the case 5x5 at the 0.01 significance level. VAM has also better results in Wilcoxon test for the cases 5x5, 10x10 and 20x20 problem sizes. There is no difference at the 0.05 significance level between means of VAM and IVAM for the cases 10x10, 10x20 and 10x30. Wilcoxon tests show the result: VAM method is not statistically significantly different from IVAM in the cases 10x30 and 10x40. Both the Student t-test and the Wilcoxon test show the same result in all the cases 20x20, 10x60, 30x30, 10x100, 40x40, 50x50 and 100x100: IVAM method is statistically significantly different from VAM. This indicates that IVAM has better performance in terms of CPU time to complete the optimization problems than VAM in the following cases, 20x20, 10x60, 30x30, 10x100, 40x40, 50x50 and 100x100. All these results are significant at the 0.01 significance level.

5. CONCLUSION

In this study, Vogel's Approximation Method which is one of well-known transportation methods for getting initial solution was investigated to obtain more efficient initial solutions. VAM was improved by using total opportunity cost and regarding alternative allocation costs. Proposed method considers highest three penalty costs and calculates additional two alternative allocation costs in VAM procedure. For more penalty costs, alternative allocation costs can be calculated but it increases computational complexity and time too much. Therefore, IVAM consider only two additional costs. Simulation experiments showed that VAM gets efficient initial solutions for small sized transportation problems but it is insufficient for large sized transportation problems. IVAM conspicuously obtains more efficient initial solutions for large scale transportation problems and it reduces total iteration number, CPU times and computational difficulty for the optimal solution.

6. REFERENCES

1. V. Adlakha and K. Kowalski, A simple heuristic for solving small fixed-charge transportation problems. *Omega* **31**(3), 205-211, 2003.
2. D. R. Anderson, D. J. Sweeney and T. A. Williams, *An introduction to management science: quantitative approaches to decision making*, Minnesota: West Publishing Company, 1991.
3. N. Balakrishnan, Modified Vogel's approximation method for unbalanced transportation problem. *Applied Mathematics Letters* **3**(2), 9–11, 1990.
4. G. B. Dantzig, *Linear Programming and Extensions*, New Jersey: Princeton University Press, 1963.
5. S. I. Gass, On solving the transportation problem, *Journal of Operational Research Society* **41**(4), 291-297, 1990.
6. S. K. Goyal, Improving VAM for unbalanced transportation problems, *Journal of Operational Research Society* **35**(12), 1113-1114, 1984.
7. O. Kirca and A. Satir, A heuristic for obtaining an initial solution for the transportation problem, *Journal of Operational Research Society* **41**(9), 865-871, 1990.
8. M. Mathirajan and B. Meenakshi, Experimental Analysis of Some Variants of Vogel's Approximation Method, *Asia-Pacific Journal of Operational Research* **21**(4), 447-462, 2004.
9. G. S. Ramakrishnan, An improvement to Goyal's modified VAM for the unbalanced transportation problem, *Journal of Operational Research Society* **39**(6), 609-610, 1988.
10. J. E. Reeb and S. Leavengood, *Transportation problem: A special case for linear programming,* Oregon : Oregon State University Extension Service Publications EM 8779, 2002.
11. N. V. Reinfeld and W. R. Vogel, *Mathematical Programming*, New Jersey: Prentice-Hall, Englewood Cliffs, 1958.
12. R. R. K. Sharma and K. D. Sharma, A new dual based procedure for the transportation problem, *European Journal of Operational Research* **122**, 611-624, 2000.
13. R. R. K. Sharma and S. Prasad, Obtaining a good primal solution to the uncapacitated transportation problem, *European Journal of Operations Research* **144**, 560-564, 2003.
14. D. G. Shimshak, J. A. Kaslik and T. D. Barclay, A modification of Vogel's approximation method through the use of heuristics, *Infor* **19**, 259-263, 1981.
15. H. H. Shore, The Transportation Problem and the Vogel Approximation Method, *Decision Sciences* **1**(3-4), 441–457, 1970.
16. H. A. Taha, *Operations Research: An Introduction,* New York: Macmillan Publishing Company, 1987.
17. H. Wagner, *Principles of Operations Research,* New Jersey: Prentice-Hall, Englewood Cliffs, 1969.
18. W. L. Winston, *Operations Research Applications and Algorithms*, California: Wadsworth Publishing, 1991.