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EASY AND IMPROVED ALGORITHMS TO JOINT DETERMINATION OF THE REPLENISHMENT LOT SIZE AND NUMBER OF SHIPMENTS FOR AN EPQ MODEL WITH REWORK

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Abstract- Recently, in Mathematical and Computational Applications Journal Chiu *et al*. [1] and Chen and Chiu [2] propose an inventory model based on EPQ with rework to determine the replenishment lot size and the number of shipments for a vendor-buyer integrated production-inventory system. They solve the inventory problem by considering both variables as continuous. However, the number of shipments must be considered as discrete variable. In this direction, this paper revisits and solves the inventory problem of [1-2] considering the decision variables according to their nature. Two easy and improved algorithms are proposed which simplify and complement the research works of [1-2].

Key Words- Replenishment lot size, Multiple shipments, EPQ model, Random defective rate, Rework.

1.INTRODUCTION

The first inventory model called EOQ was derived by Harris in 1913 [3] and five years later the EPQ inventory model was proposed by Taft in 1918 [4]. Since then exist an enormous interest of researchers in developing extensions of both inventory models. Recently, Mathematical and Computational Applications Journal have published some extensions of EOQ and EPQ inventory models i.e. [1-2, 5-12]. Also other journals do the same, i.e. see the research works of [13-19]; just to name a few papers related to EOQ/EPQ inventory models.

The research works of Chiu *et al.* [1] and Chen and Chiu [2] deal with the same type of inventory problem which is the joint determination of the replenishment lot size and the number of shipments for a vendor-buyer integrated production-inventory system based on an EPQ model with rework. We read [1-2] with a considerable interest. We found that in both papers the authors considered the number of shipments (n) as continuous variable and then they round off it in order to obtain the discrete value. This action could carry us to a non-optimal solution. In this direction, our paper revisits and solves the inventory model of [1-2] considering the decision variables according their nature. Two cases are considered: Case 1) the replenishment lot size (Q) continuous and the number of shipments (n) discrete, and Case 2) the replenishment lot size (Q) discrete and the number of shipments (n) discrete. For each case we propose an easy to apply algorithm.

2. ALGORITHMS TO THE JOINT DETERMINATION OF THE REPLENISHMENT LOT SIZE AND NUMBER OF SHIPMENTS

We refer to the readers to see in Chiu *et al.* [1] and Chen and Chiu [2] the nomenclature, assumptions and full derivation of the total lung-run average costs of the inventory system. Chiu *et al.* [1] and Chen and Chiu [2] develop the following total lung-run average costs, here rewritten as follows:

$$E[TCU(Q,n)] = Z_0 + \left(Z_1 + \frac{Z_4}{n}\right)Q + \frac{Z_2 + Z_3 n}{Q} \tag{1}$$

Where;

$$Z_0 = C\lambda + C_R E[x]\lambda + C_T\lambda > 0 \tag{2}$$

$$Z_1 = \left(\frac{h}{2}\right)\left[1 + \frac{\lambda E[x]}{P_1}\left[1 - E[x]\right]\right] + \left(\frac{h_2}{2}\right)\left[\frac{\lambda}{P} + \frac{\lambda E[x]}{P_1}\right] + \left(\frac{h_1}{2}\right)\left(\frac{\lambda\left(E[x]\right)^2}{P_1}\right) > 0 \tag{3}$$

$$Z_2 = K\lambda > 0 \tag{4}$$

$$Z_3 = K_1\lambda > 0 \tag{5}$$

$$Z_4 = \left[1 - \frac{\lambda}{P} - \frac{E[x]\lambda}{P_1}\right]\left(\frac{h_2}{2} - \frac{h}{2}\right) \tag{6}$$

Applying the algebraic method of completing perfect squares [20-23] or the well-known differential calculus, one can show that when Q is considered as continuous variable Equation (1) is minimized at

$$Q = \sqrt{\frac{Z_2 + Z_3 n}{\left(Z_1 + \frac{Z_4}{n}\right)}} \tag{7}$$

Substituting Equation (7) into Equation (1) and simplifying one obtains total lung-run average costs,

$$E[TCU(n)] = Z_0 + 2\sqrt{f_n + Z_1 Z_2 + Z_3 Z_4} \tag{8}$$

where,

$$f_n = Z_1 Z_3 n + \frac{Z_2 Z_4}{n} \tag{9}$$

According to García-Laguna *et al.* [24] the function f_n is minimized when

$$n = \left\lceil -0.5 + \sqrt{0.25 + \frac{Z_2 Z_4}{Z_1 Z_3}} \right\rceil \tag{10}$$

or

$$n = \left\lfloor 0.5 + \sqrt{0.25 + \frac{Z_2 Z_4}{Z_1 Z_3}} \right\rfloor \tag{11}$$

Remember that $\lceil \psi \rceil$ is the smallest integer greater than or equal to ψ; and $\lfloor \psi \rfloor$ is the largest integer less than or equal to ψ, respectively. Certainly, it is easy to show that $\lceil \psi \rceil = \lfloor \psi + 1 \rfloor$ if and only if ψ is not an integer value. In this situation the optimization problem has a unique solution for n which is $n^* = n$ (given by either of the two expressions of Equations (10) and (11). Otherwise, the optimization problem has two solutions for n that are $n^* = n$ and $n^* = n+1$. This result was also applied in Cárdenas-Barrón [25] for developing of the close form for computing the number of shipments for the inventory models of Chang [26] and Lin [27]. Clearly, we know that $Z_1 Z_3 > 0$. But, $Z_2 Z_4$ can be positive, zero or negative since the term Z_4 can be positive, zero or negative. If Z_4 takes positive values then the optimal solution for n is given by Equation (10) or (11). On the other hand, When Z_4 takes zero or negative values, evidently f_n achieves its global minimum value at *n=1.*

Now, if Q is limited to be a discrete variable then Equation (1) reaches its minimum when

$$Q = \left\lceil -0.5 + \sqrt{0.25 + \frac{Z_2 + Z_3 n}{\left(Z_1 + \frac{Z_4}{n} \right)}} \right\rceil \tag{13}$$

or

$$Q = \left\lfloor 0.5 + \sqrt{0.25 + \frac{Z_2 + Z_3 n}{\left(Z_1 + \frac{Z_4}{n} \right)}} \right\rfloor \tag{14}$$

A lower bound for the total lung-run average costs can be developed easily just considering both variables as continuous and it is as follows

$$LB = Z_0 + 2\left(\sqrt{Z_1 Z_2} + \sqrt{Z_3 Z_4}\right) \tag{15}$$

Considering Equations (1), (7), (8), (10), (11), (13) and (14) we propose two algorithms for solving the two cases previously mentioned.

Case 1. The replenishment lot size (*Q*) continuous and the number of shipments (*n*) discrete.

Algorithm for the Case 1

Step 1. Calculate Z_4,

If $Z_4 \le 0$ then set *n=1* and calculate *Q* using Equation (7) and go to ***Step 4***, else go to ***Step 2.***

Step 2. Calculate the integral value for *n*

If $-0.5 + \sqrt{0.25 + \dfrac{Z_2 Z_4}{Z_1 Z_3}}$ is an integer value then *n** = *n* and *n** = *n* + *1*.

Otherwise, *n** = *n*.

Where *n* is calculated using Equation (10) or (11).

Step 3. Given the discrete value of n then calculate the continuous value for the lot size Q using Equation (7).

Step 4. Calculate the total cost using Equation (1) or Equation (8).

If exist two solutions for *n* then there are two optimal solutions. For each solution of *n* do Steps 3 and 4 and report the two optimal solutions.

Case 2. The replenishment lot size (*Q*) discrete and the number of shipments (*n*) discrete.

Algorithm for Case 2

Step 1. Calculate Z_4,

If $Z_4 \le 0$ then set *n=1* and calculate *Q* using Equation (13) or (14) and go to ***Step 4,*** else go to ***Step 2.***

Step 2. Compute the integral value for *n*

If $-0.5 + \sqrt{0.25 + \dfrac{Z_2 Z_4}{Z_1 Z_3}}$ is an integer value then *n** = *n* and *n** = *n* + *1*.

Otherwise, *n** = *n*.

Where *n* is calculated using Equation (10) or (11).

Step 3. Given the discrete value of n then calculate the discrete value for the lot size Q using Equations (13) or (14).

Step 4. Calculate total cost using Equation (1).

If exist two solutions for *n* then there are two solutions. For each solution of *n* do Steps 3 and 4 and select the solution with the minimal total cost.

Instance	*Q* continuous and *n* discrete Algorithm for Case 1	*Q* and *n* discrete Algorithm for Case 2	Lower bound
Instance 1 from [1-2]	*Q=1672.647522* *n=2* *E[TCU]= 487617.3019*	*Q=1673* *n=2* *E[TCU]= 487617.3045*	*LB=487616.7365*
Special Case x=0 then E[x]=0	*Q=2275.596848* *n=3* *E[TCU]= 439100.9032*	*Q=2276* *n=3* *E[TCU]= 439100.9047*	*LB=439020.0644*

3. CONCLUDING REMARKS

This paper develops two algorithms to determine jointly both the optimal replenishment lot size and the optimal number of shipments for the inventory model proposed by Chiu *et al*. [1] and Chen and Chiu [2]. The proposed algorithms are easy to apply and implement. It is important to remark that Chiu *et al*. [1] and Chen and Chiu [2] do not consider the situation when $Z_4 \le 0$. Also, they do not solve the inventory problem when both variables are considered as discrete variables. These are important characteristics that our paper dealt. Our paper simplifies, improves and complements Chiu *et al*. [1] and Chen and Chiu [2] research works. Interested readers in this topic also can see the research work of Cárdenas-Barrón *et al.* [28].

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