Design of Aluminum Alloy H-Sections under Minor-Axis Bending

Lin Yuan 1, Qilin Zhang 1,* , Shaoquan Zhang 2 and Hanbin Ge 2

1 College of Civil Engineering, Tongji University, Shanghai 200092, China; yuanlin@tongji.edu.cn
2 Department of Civil Engineering, Meijo University, Nagoya 4688502, Japan
* Correspondence: zhangqilin@tongji.edu.cn

Abstract: Much research has been reported on the global response of aluminum alloy H-sections members, while studies on the local buckling behavior of H-sections under pure bending remain relatively limited. The purpose of the research is to investigate the response of aluminum alloy H-sections subjected to minor axis bending. Using a finite element model, this study analyzed the stress distribution and failure mechanism of aluminum alloy H-sections under minor-axis bending and obtained the ultimate capacities of cross-sections covering a wide range of plate slenderness. The results were compared with the strength predictions based on EN1999-1-1 and the effective width method in AS/NZS 4600. The flange slenderness was found to play the most significant role in determining the normalized capacity. The sections are shown to exhibit an elastic-plastic stress distribution in the tensile flanges. The comparisons given in this study indicate that EN1999-1-1 underestimates the predicted bending strengths. The predictions based on the effective width method are shown to be more accurate than EN1999-1-1. An alternative design method is proposed for treating aluminum H-sections in minor axis bending. This method considers plastic stress distributions in the tensile flanges after the compressed flanges have locally buckled.

Keywords: aluminum alloy H-sections; inelastic behavior; local buckling; minor axis bending; effective width method

1. Introduction

The main properties of aluminum are lightweight and good corrosion resistance, so aluminum alloys are competitive in some typical structural applications, including long-span structures, transmission towers in hardly accessible places, and swimming pool roofs suffering from high humidity [1]. However, compared to carbon steels, the lower Young’s modulus of aluminum alloys may cause structural stability problems, especially for members with slender cross-sections. Aluminum extruded H-section members are often used to form the frames of large-span roof trusses [2].

The flexural resistance of beams is very important in order to ensure the safe transfer of the vertical loads to the foundation. Bartsch et al. [3] investigated the rotation capacity of high-strength steel I-girders. Nassiraei et al. [4,5] studied the static strength of collar plate reinforced tubular T/Y joints subjected to in-plane bending load. Numerous studies were performed on aluminum alloy H-section members in order to examine their ultimate performance and assess the applicability of codified design provisions. Compression tests on aluminum H-section columns have been conducted by many researchers to examine their flexural buckling behavior [6–8]. Wang et al. [9] investigated the lateral-torsional stability of simply supported H-section beams. They found that the modified method provided in [10] provided more accurate predictions compared to EN 1999-1-1 [11]. Wang et al. [12] conducted experimental investigations on H-section beams with and without intermediate stiffeners under concentrated loads.
Yuan et al. [2] carried out similar bending tests to investigate the effects of the web slenderness, the shear aspect ratios and the end post-conditions on the shear resistance of H-section beams. At the cross-sectional level, despite stub column tests that are aimed at studying the local buckling behavior of aluminum H-sections under uniform compression have been reported by [13], investigations on H sections subjected to bending moments are relatively limited.

Despite exhaustive experimental and numerical investigation on the structural response of aluminum alloy columns and beams, research on the local buckling behavior of aluminum alloy cross-section under non-uniform compression is relatively limited and further research is needed. The flanges and the web of an H section are always referred to as unstiffened elements (supported at one longitudinal edge and free at the other edge) and stiffened elements (supported at both longitudinal edges), respectively. A slender element under compression can resist loads at failure which may be considerably higher than those at which local buckling occurs. This is due to the redistribution of longitudinal stresses from the flexible to the stiff parts of the plate [14]. Stiffened elements and unstiffened elements exhibit great differences in post-buckling behavior. Experimental and analytical studies on local buckling of aluminum alloy stiffened elements have been carried out [8,15–17] and the calculation formulas for estimating the effective width or thickness of such elements under uniform compression and under stress gradients were derived. In comparison, experimental data on aluminum alloy unstiffened elements are quite scarce. In the 1990s the applicability of the effective thickness concept to aluminum alloy unstiffened elements made from two tempers of alloy AA6082 was studied by Hopperstad et al. [18], who tested thirty stub columns of cruciform sections in axial compression. Similar tests by Mazzolani et al. [19] were carried out on angle short columns and led to the establishment of an empirical relationship between plastic deformation capacity and plate slenderness parameter. It showed that an effective thickness approach provided a generally safe and relatively accurate prediction of the ultimate loads. New slenderness limits for aluminum alloy cross-sections were proposed by Su et al. [20] to allow for element interaction but were limited for stiffened elements because the available data for unstiffened element parts under stress gradients was rather limited. Georgantzia et al. [21] conducted fourteen channel beam tests made from 6082-T6 alloy subjected to minor axis bending. Their investigations have shown that the combination of employing the reduced thickness of unstiffened plates under stress gradients and estimating the capacity predicted on initiation of yielding in the section provided moment capacities which were up to 50% conservative for slender channel sections under minor axis bending. Besides, all the specimens tested with the flange tip in tension showed curvature at the ultimate that was many times the plastic curvature. The inelastic stress distribution exhibited by such cross-sections was not properly accounted for by existing design methods.

Current design specifications provide different design methods for the local buckling capacity of unstiffened elements under stress gradients. EN1999-1-1 (EC9) [11] adopts an effective thickness concept to predict the ultimate resistance of aluminum alloy elements controlled by local buckling. In 2007, the Australian/New Zealand cold-formed steel specifications modified the elastic effective width formulas for unstiffened elements with stress gradients derived from the plate tests of Bambach and Rasmussen [22]. Besides, Bambach and Rasmussen [23,24] proposed the plastic effective width method for unstiffened elements under stress gradients and inelastic capacity models for channel and I-sections. However, the aluminum alloy channel beam tests by [21] indicate that the current design codes and methods have been shown to be overly conservative. Therefore, a design method is required to accurately calculate the moment capacities of aluminum alloy H-sections under minor-axis bending.

The main objective of this study is to investigate the performance of extruded aluminum alloy H sections bent about the minor axis. Nonlinear finite element analysis was conducted on H-section beams subjected to four-point bending covering a wide
range of plate slenderness ratios. By analyzing the stress distribution process of typical members, the failure mechanism and the factors affecting the bending behavior of such members are discussed. The obtained flexural strengths from the parametric study are used to evaluate the existing design methods in the current EN1999-1-1 [11] and the effective width method in AS/NZS 4600 [25].

2. Numerical Studies

2.1. Finite Element Model

The cross-section of the H-section beam is shown in Figure 1. \( H \) is the cross-section depth. \( B \) is the flange width. \( t_f \) is the flange thickness and \( t_w \) is the web thickness. The flexural behavior of H sections was studied by numerical modeling using ABAQUS [26]. The H-section beams are modeled using the four-node shell element with reduced integration (S4R). Nine integration points were defined through the plate thickness to ensure that the local failure was adequately considered. A mesh convergence study was conducted as tabulated in Table 1. Based on the mesh convergence study a mesh size of 8×8 mm was chosen. The assembled and meshed model of the aluminum alloy H-section beam under four-point bending is shown in Figure 2. Ramberg–Osgood material model [27] is applied to describe the non-linear properties of aluminum alloys. The engineering stress and strain were converted into true stress and true plastic strain according to the ABAQUS material modeling guidelines.

![Figure 1. Definition of symbols in cross-section.](image)

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Ultimate Moment (kN·m)</th>
<th>Maximum Mises Stress (MPa)</th>
<th>Relative CPU Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse (15 mm)</td>
<td>11.63</td>
<td>540.2</td>
<td>0.28</td>
</tr>
<tr>
<td>Normal (10 mm)</td>
<td>11.69</td>
<td>561.8</td>
<td>0.69</td>
</tr>
<tr>
<td>Fine (8 mm)</td>
<td>11.72</td>
<td>562.0</td>
<td>1.00</td>
</tr>
<tr>
<td>Very fine (5 mm)</td>
<td>11.73</td>
<td>562.3</td>
<td>2.56</td>
</tr>
</tbody>
</table>
All the beams were subjected to a four-point bending configuration, which produces a uniform moment between the two loading points in the specimen. The distance between the support and the beam end $L_1$ is equal to the section depth $H$. The pure bending span $L_2$ is chosen to ensure the formation of at least 3 local buckles. The half-wavelength of the full cross-section lies between the half-wavelength envelopes of the isolated critical plates with simply-supported and fixed boundary conditions along the adjoined edges [25]. The half-wavelength of the web in an H section is usually less than the web width. For an H-section beam subjected to a pure minor axis bending moment, the half-wavelength of the flange element is usually less than 2.8 times the element width [28]. Therefore, the half-wavelength of the cross-section under minor axis bending $L_{b,cs}$ satisfies $L_{b,cs} \leq \max (H, 1.4 \, B)$. The bending span $L_2$ is taken as three times the maximum half-wavelength of the cross-section, i.e., $L_2 = 3 \times \max (H, 1.4 \, B)$. A total span of $L = 9 \times \max (H, 1.4 \, B) + 2H$ was adopted.

The cross-sections at the loading and support points are coupled to the corresponding reference points at their centroids. To simulate the simply supported conditions, all degrees of freedom are restrained in reference point RP4 except the rotation about the X axis, and the reference point RP1 is also free in the translation in the Z direction. In the reference points RP2 and RP3, the translations in the X direction and the rotation about Y and Z were restrained in order to prevent their lateral deformation. Concentrated load toward the negative Y direction was applied at the reference points RP2 and RP3. The boundary conditions are shown in Figure 3.

![Figure 2. Meshed model of the H-section beam (8 mm × 8 mm).](image)

![Figure 3. Boundary condition and interaction.](image)
For each modeled beam, linear eigenvalue buckling analysis was conducted first to obtain the lowest buckling mode shape, which was set as an initial geometric imperfection distribution pattern and incorporated into the FE models. The typical buckling mode of H-section beams under minor axis bending is shown in Figure 4. The fabrication and erection specifications [29] provide an initial flatness imperfection tolerance ($w_0$) of 0.8% for ordinary-grade aluminum alloy plates when the plate width $b$ is larger than 25 mm. For high-grade and super high-grade members, the tolerance is 0.6% and 0.4%, respectively. This paper uses $w_0 = 0.8\%b$ as the initial imperfection amplitude. Nonlinear analysis was then performed incorporating material nonlinearity and geometric imperfections. Residual stresses in extruded aluminum alloy profiles have a low value [30] and are neglected in this study. The ultimate bending moment of the beams was hence determined.

![Figure 4. Local buckling mode and initial local imperfection.](image)

### 2.2. Validation of Finite Element Model

The developed finite element model has been validated against the beam test data reported in [31]. Eight H-section beam specimens were tested in four-point flexural tests. For each cross-section, two identical members were tested and the ultimate moment capacities are $M_{u,\text{test_1}}$ and $M_{u,\text{test_2}}$, respectively. The ultimate moments in the FE models $M_{u,\text{FE}}$ were compared with those values from the tests, as shown in Table 2. It can be seen that the FE results were on average within 5% of test results and standard deviations were generally no more than 4%. Therefore, it is deemed that the developed FE models can be applied to predict the bending capacities of H-section beams subjected to minor axis bending with enough accuracy.

### Table 2. Input data of the H-section beam specimens used for validation.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$B$ (mm)</th>
<th>$h_w$ (mm)</th>
<th>$t$ (mm)</th>
<th>$f_y$ (MPa)</th>
<th>$M_{u,\text{test_1}}$ (kN m)</th>
<th>$M_{u,\text{test_2}}$ (kN m)</th>
<th>$M_{u,\text{FE}}$ (kN m)</th>
<th>$\frac{M_{u,\text{test_1}}}{M_{u,\text{FE}}}$</th>
<th>$\frac{M_{u,\text{test_2}}}{M_{u,\text{FE}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B009</td>
<td>100</td>
<td>80</td>
<td>1.9</td>
<td>228</td>
<td>1.75</td>
<td>1.76</td>
<td>1.87</td>
<td>0.94</td>
<td>0.95</td>
</tr>
<tr>
<td>B015</td>
<td>100</td>
<td>80</td>
<td>2.0</td>
<td>368</td>
<td>2.61</td>
<td>2.59</td>
<td>2.91</td>
<td>0.89</td>
<td>0.90</td>
</tr>
<tr>
<td>B005</td>
<td>150</td>
<td>80</td>
<td>1.9</td>
<td>230</td>
<td>3.32</td>
<td>3.34</td>
<td>3.52</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>B007</td>
<td>150</td>
<td>80</td>
<td>1.5</td>
<td>188</td>
<td>2.12</td>
<td>2.2</td>
<td>2.16</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>B013</td>
<td>150</td>
<td>80</td>
<td>2.0</td>
<td>368</td>
<td>5.16</td>
<td>5.13</td>
<td>5.52</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>B003</td>
<td>200</td>
<td>80</td>
<td>1.9</td>
<td>230</td>
<td>5.55</td>
<td>5.46</td>
<td>5.59</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>B001</td>
<td>200</td>
<td>80</td>
<td>1.5</td>
<td>188</td>
<td>3.36</td>
<td>3.36</td>
<td>3.43</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>B011</td>
<td>200</td>
<td>80</td>
<td>2.0</td>
<td>368</td>
<td>8.42</td>
<td>8.36</td>
<td>8.81</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>Mean</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>St.Dev</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.035</td>
<td>0.035</td>
</tr>
</tbody>
</table>
2.3. Parametric Studies

After validating the developed finite element models against available experimental results, the moment capacities of aluminum alloy H sections covering a wide range of plate slenderness were obtained. The key parameters are listed in Table 3. The distance between flange mid-lines $H_0$ was fixed to 200 mm, and the full flange width $B$ was fixed to 200 mm. Three values of web slenderness ratio $R_w$ were considered. At each value of $R_w$, thirteen values of flange slenderness ratio $R_f$ were considered to cover a wide range of cross-section slenderness. $\varepsilon$ is the material parameter and can be calculated by $\varepsilon = \sqrt{\frac{250}{f_{0.2}}}$. Two commonly used aluminum alloys are chosen, 6082-T6 and 6063-T5, representing class A and class B buckling classes, respectively. According to the tensile coupon tests in [21], the mechanical property values are listed in Table 4, including elastic modulus $E$, 0.2% proof stress $f_{0.2}$, ultimate stress of the material $\sigma_u$, the ultimate strain $\varepsilon_u$ and the strain hardening exponent $n$ in the Ramberg-Osgood law. The members are labeled such that the material’s buckling class, the web slenderness ratio and the flange slenderness ratio are shown. For example, the label “HA-40-24” could be interpreted as follows: “H” represents an H-section beam, “A” refers to 6082-T6, the number “40” is the web slenderness ratio $R_w$, and the number “24” is the flange slenderness ratio $R_f$. For each beam analyzed in the parametric study, the peak point of the moment-curvature curve is determined as the ultimate moment value, as exemplified by Figure 5.

![Figure 5. Moment-rotation curve for a typical specimen (HA-10-6).](image)

Table 3. Key parameters in the parametric study.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>13 flange slenderness ratios $R_f = b_f/(t_f \varepsilon)$</td>
<td>3, 4.5, 6, 9, 12, 15, 18, 21, 24, 27, 30, 35, 40</td>
</tr>
<tr>
<td>3 web slenderness ratios $R_w = h_w/(t_w \varepsilon)$</td>
<td>10, 25, 40</td>
</tr>
<tr>
<td>2 aluminum alloys</td>
<td>6063-T5, 6082-T6</td>
</tr>
<tr>
<td>Section depth</td>
<td>$H = H_0 + t_0$ ($H_0 = 200$ mm)</td>
</tr>
<tr>
<td>Section width</td>
<td>$B = 200$ mm</td>
</tr>
</tbody>
</table>

Table 4. Material properties for aluminum alloys considered in parametric studies.

<table>
<thead>
<tr>
<th>Material</th>
<th>$E$ (MPa)</th>
<th>$f_{0.2}$ (MPa)</th>
<th>$\sigma_u$ (MPa)</th>
<th>$n$</th>
<th>$\varepsilon_u$ (%)</th>
<th>Buckling Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>6082-T6</td>
<td>70,000</td>
<td>286</td>
<td>317</td>
<td>32.7</td>
<td>8.8</td>
<td>A</td>
</tr>
<tr>
<td>6063-T5</td>
<td>70,000</td>
<td>164</td>
<td>211</td>
<td>10.0</td>
<td>7.3</td>
<td>B</td>
</tr>
</tbody>
</table>
3. Failure Mechanism for H Sections under Minor Axis Bending

The performance of aluminum alloy H sections bent about the minor axis was analyzed through the longitudinal stress distribution of typical cross-sections. The in-plane longitudinal stresses (z direction in Figure 3) across the cross-section located at the mid-span of the beams were output from the analysis at each load step, and the development progress of the stress redistribution can thus be determined.

Since the flexural response of the aluminum alloy H-sections was observed to vary with section slenderness, comparisons between three typical H-sections are discussed in this study. These sections are HA-10-6, HA-10-27 and HA-40-27. The nonlinear finite element analysis performed in Abaqus allows for the failure response of the aluminum alloy H-sections and the corresponding deformation and stresses to be examined in detail. Figures 6–8 illustrate the deformed shapes and longitudinal membrane stresses at the ultimate moment for the three H-sections of varying section slenderness. For the deformed shape, the displacement is shown by the contour plot. For the longitudinal membrane stress, the local plate stress along the z direction is represented by the contour plot. The stress distribution at the ultimate moment over the flange and half of the web of the three H-sections are provided in Figure 9. In the figures, $M_{el}$ and $M_{pl}$ represent the elastic and fully plastic bending moment of the cross-sections.

![Figure 6. Stress distribution and deformed shape at the ultimate moment for HA-10-6.](image1)

![Figure 7. Stress distribution and deformed shape at the ultimate moment for HA-10-27.](image2)

![Figure 8. Stress distribution and deformed shape at the ultimate moment for HA-40-27.](image3)
Figure 9. Mid-surface longitudinal stress distribution for H-sections (a) HA-10-6, (b) HA-10-27 and (c) HA-40-27.

The ultimate moment of HA-10-6 is greater than the fully plastic moment, i.e., $M_u > M_{pl}$, indicating that the failure of the cross-section goes into the plastic range due to low flange slenderness. The stress at the extreme fiber of the section surpassed the 0.2% proof stress of 286 MPa, as shown in Figure 9a. This is because, at the ultimate moment, the onset of strain hardening has allowed for the stresses at extreme fibers of the flanges to exceed the yield stress. Furthermore, the stress across the web is almost zero, implying the absence of the neutral axis shift.

The ultimate moment of the HA-10-27 and HA-40-27 sections is lower than the corresponding fully plastic moment and is greater than the elastic moment, i.e., $M_{el} < M_u < M_{pl}$. The stresses across the web of HA-10-27 and HA-40-27 are almost zero before local buckling occurs in the flanges, implying that web slenderness has a negligible effect on the performance of the sections in the pre-buckling range. As shown in Figure 9b, the
stresses across the flanges of HA-10-27 are observed to be linear until the moment reaches 1.11 \( M_{ul} \). Then the decrease in the compressive stress at the flange tip indicates the occurrence of local buckling. After local buckling, the compressed flange can be divided into two portions: the tip portion and the root portion. The tip portion exhibits a decrease in stresses due to apparent lateral displacements and it becomes “ineffective” in resisting bending. The root portion is restrained by the adjoined edge of the web and remains effective in resisting bending moment. This causes the effective neutral axis of the section to shift towards the tension flanges and compressive stresses to develop in the web.

As shown in Figure 9c, the cross-section HA-40-27 has a very slender web and develops a great extent of stress redistribution after the local buckling of the flanges so that tensile stresses develop at the tip of the compressed flanges. With the onset of local buckling, the neutral axis position shifts into the parts of the cross-section that were initially in tension. In order to maintain the balance of force, the web needs to carry additional compressive stresses. The tensile flanges remain nearly flat at the ultimate condition and yield in tension. The stress distribution and yielding in the tensile flanges provide a significant post-buckling reserve. By comparing the stress distributions of HA-10-27 and HA-40-27, it can be seen that with the increase in the web slenderness, the compressive stresses of the web are higher and the restraint provided to the flanges is lower. Consequently, the flange-web interaction has a significant impact on the post-buckling behavior of the member.

4. Assessment of Design Predictions

In this section, the ultimate bending moment capacities of aluminum alloy H-sections subjected to minor axis obtained from the parametric studies are used to assess the applicability and accuracy of the design rules specified in EN1999-1-1 (EC9) [11]. The elastic effective width method in AS/NZS 4600 [25] is also evaluated. The partial safety factors were set equal to unity.

4.1. EN1999-1-1

EC9 defines four classes of cross-sections [11]. Class 1 and 2 cross-sections can develop their plastic moment resistance. Class 3 cross-sections can reach their elastic moment resistance and cannot develop full plastic moment resistance due to local buckling. Class 4 cross-sections cannot develop elastic moment resistance because local buckling will occur in slender parts.

\[
M_{Ed} = \begin{cases} 
M_{pl} & \beta \leq \beta_2 \\
M_{el} & \beta_2 < \beta \leq \beta_3 \\
M_{el} & \beta_3 < \beta \leq \beta_4 
\end{cases} \tag{1}
\]

In Equation (1), \( \beta \) is the element slenderness parameter

\[
\beta = \eta \frac{b}{t}
\tag{2}
\]

where \( \eta \) is the stress gradient factor. For internal elements under stress gradient and outstand elements with peak compression at the supported edge, \( \eta \) is calculated by Equation (2). For outstand elements with peak compression at the free edge, \( \eta \) is equal to unity.

\[
\eta = \begin{cases} 
0.7 + 0.3\psi & -1 \leq \psi \leq 1 \\
0.8/(1 - \psi) & \psi < -1 
\end{cases} \tag{3}
\]

In Equation (3), \( \psi = \sigma_2/\sigma_1 \) is the stress ratio, where \( \sigma_1 \) is the maximum compressive stress (with compression taken as positive) and \( \sigma_2 \) is the minimum compressive or maximum tensile stress.
EC9 uses the effective thickness concept to consider the local buckling effect of slender elements. For flat elements without welds, the effective thickness reduction factor of a compressed element is expressed as

$$\rho_e = \frac{t_e}{t} = \frac{C_1}{\beta_2}\left(\frac{C_2}{\beta_1}\right)^2 \leq 1 \quad \beta \geq \beta_3$$

where $t_e$ is the reduced thickness. $C_1$ and $C_2$ are the calculation constants related to the material buckling class and boundary condition of the element.

4.1.1. Class 2 and Class 3 Slenderness Limits for Outstand Elements

The values of ultimate bending moments obtained from the FE results $M_u$ are used to evaluate the Class 2 and Class 3 slenderness limits for outstand elements under a stress gradient with maximum compression at the free edge. The $M_u$ values were firstly normalized by the corresponding fully plastic moment $M_{pl}$ and are plotted against the flange slenderness parameter in Figure 10. It can be seen that the flange’s slenderness has a significant effect on the ultimate bearing capacity. With the flange’s slenderness increasing, the normalized ultimate bearing capacity of the cross-section decreases. The web’s slenderness has little effect on the ultimate bearing capacity of such members.

![Figure 10](image1.png)

The Class 2 slenderness limit of $\beta_2/\varepsilon = 4.5$ for material Class A and Class B materials defined in EC9 is also plotted in this figure. It is shown that the Class 2 limit of EC9 is slightly conservative and can be relaxed to $\beta_2/\varepsilon = 7$ to obtain more economical prediction results. Then the $M_u$ values were normalized by the corresponding elastic moment $M_{el}$ in Figure 11. The Class 3 slenderness limits of $\beta_3/\varepsilon = 6$ for material Class A and $\beta_3/\varepsilon = 5$ for Class B materials defined in EC9 are also plotted in the figures. It is shown that the Class 3 limits of EC9 are overly conservative for aluminum alloy H-sections under minor-axis bending since most data points have a $M_u/M_{el}$ value larger than unity. It indicates that the inelastic stress distribution in the tension part of the flanges significantly raises the bending capacity of the cross-section, and counteracts the effect of the reduction in the effective area caused by local buckling of the compressed outstand flanges.
4.1.2. Strength Predictions

The effective section for a Class 4 H-section considering the reduced thickness of the compressed flanges is illustrated in Figure 12. In the figure, $t_{fe}$ is the effective thickness of the compressed flanges. If the effective centroid of the cross-section is located in the tensile flanges, the reduced thickness of the web ($t_{we}$) under compression also needs to be calculated. The cross-sectional thickness ranges from 2 mm to 12.5 mm, keeping the section depth $H$ fixed to 100 mm and the flange width $B$ fixed to 100 mm. The Class A aluminum alloy 6082-T6 was investigated. The ultimate moment normalized by the corresponding fully plastic moment for each beam is plotted against the flange slenderness parameter in Figure 13. Additionally, shown in the figure are the design strength prediction curves calculated from Equation (1) based on EC9. It can be seen that the slenderness parameter limits specified from the current EC9 for the flange element of H-sections bent about the minor axis are quite conservative. The numerical-to-predicted ratio $M_{u,FE}/M_{u,EC9}$ is up to 2.27 for slender H-sections, indicating that the effective thickness formula of the current EC9 provides overly conservative predictions for the bending capacities of such members. This is due to the fact that (i) when the section is in the ultimate state, the elastic-plastic stress distribution occurs on the tension flanges, and (ii) in the compression-loaded part of the cross-section, the effective thickness is calculated as if it were uniformly compressed (due to the used of stress gradient factor of 1), and the interaction between the individual plate elements is not accounted for.

![Figure 11. $M_u/M_d$ results for aluminum alloy H-section beams.](image)

**Figure 12.** Effective thickness of a slender H-section beam under minor axis bending.
4.2. AS/NZS 4600

The effective width method specified in the design standard AS/NZS 4600 [25] for cold-formed steel structures is also assessed herein. The effective width method is based on the assumption of elastic effective widths situated adjacent to the supported edge [24]. The effective width factor ($\rho$) multiplied by the gross element width $b$ gives the effective width of the element $b_e$. The elastic effective width equations are presented here.

1. Outstand elements under stress gradient causing compression at both longitudinal edges:

$$
\rho = \begin{cases} 
1 & \text{for } \bar{\lambda} \leq 0.673 \\
1 - \frac{0.22}{\frac{\lambda}{\bar{\lambda}}} & \text{for } \bar{\lambda} > 0.673
\end{cases}
$$

(5)

$\bar{\lambda}$ is the normalized slenderness of the plate, as given in Equation (6).

$$
\bar{\lambda} = \frac{f_{02}}{f_{\alpha}} = \sqrt{\frac{12(1 - \nu^2)}{k_o \pi^2 E}} \frac{b}{t}
$$

(6)

$k_o$ is the buckling coefficient. When the maximum compression is at the supported edge

$$
k_o = \frac{0.578}{\nu + 0.34}
$$

(7)

When the maximum compression is at the free edge

$$
k_o = 0.07\psi^2 - 0.21\psi + 0.57
$$

(8)

2. Outstand elements under stress gradient causing compression at the free edge and tension at the supported edge:
An effective section may be established using the effective width Equations (6)–(13). According to AS/NZS 4600, inelastic stress distributions can be applied for sections with fully effective elements by taking the maximum compression strain in the section as $C_y e_y$. $C_y$ is the compression strain factor between one to three, and $e_y$ is the yield strain. It is suggested that $C_y = 3$ for unstiffened elements that form part of an H-section in minor axis bending [32]. Based on the effective width concept, the effective section for a slender H-section considering the reduced width of slender parts is illustrated in Figure 14. The obtained numerical results were used to evaluate the suitability of the effective width method (EWM) in AS/NZS 4600 [25] for aluminum alloy H-sections bent about the minor axis. For a fully effective section, inelastic stress distributions are considered by using $C_y = 3$. Figure 13 also depicts the normalized prediction strength curve of the effective width method. For cross-sections with $R_f$ between 4.5 and 8, the EWM design strength predictions are quite improved compared with EC9, as they are able to take into account the strain-hardening effect of the flanges. It is shown that for cross-sections with $R_f$ larger than 8, the EWM design strength predictions are slightly improved compared with EC9, as they are able to take into account the effect of the stress ratio on the buckling coefficient of the locally buckled flanges (involving $\psi$ in Equation (10)).

![Figure 14. Elastic effective width of H-section beams under minor axis bending.](image)

5. Design Proposals

This section presents a method for calculating the bending capacity of H-sections about the minor axis considering the inelastic capacity reserve of the flanges. It is assumed that the effect of the effective centroid shift is ignored. For an H-section under minor axis bending, the stresses across the cross-section exhibit nonlinear distributions,
as illustrated by Figure 15. The bending capacity of the effective section \( (M_d) \) is calculated using the sum of the compressed outstand flanges and the tensile outstand flanges as shown in Equation (14), where \( M_{cL} \) and \( M_{tL} \) is the capacity for a single outstand flange under compression and tension, respectively.

\[
M_d = 2M_{cL} + 2M_{tL}
\]  

(14)

![Figure 15. Inelastic reserve capacity for slender H-sections in minor-axis bending.](image)

The compressed flanges under stress gradient with maximum compression at the tips may locally buckle. The effective thickness \( (t_{f,e}) \) can be determined using the effective thickness factor \( (\rho_c) \) given in EC9. The capacity of the compressed slender flange is calculated as the first yield capacity, i.e., the stress distribution at ultimate is assumed to be linear elastic with the maximum stress, the 0.2% proof stress, at the extreme fiber, as shown in Figure 15b. \( M_{cL} \) can be calculated by Equation (15), where \( M_{yf} \) is the yield moment of the outstand flange under such a load case.

\[
M_{cL} = \frac{2}{3}b_i(t_{f,e}f_{yf}) = \rho_cM_{yf}
\]  

(15)

The tensile flanges under non-uniform tension can always develop inelastic stress distributions. \( M_{tL} \) can be calculated by Equation (16), where \( \alpha_u \) is the ultimate shape factor. Kim [33] proposed a fitted curve of the ultimate shape factor for an aluminum alloy plate with its neutral axis not at mid-depth, as given by Equation (17). The coefficients are fitted based on alloy-temper combinations of EC9. In this equation, \( y_{NA} \) is the distance from the neutral axis to the mid-depth, and \( h \) is the plate width. It is known that the ultimate shape factor for a plate is the same when its neutral axis is at mid-depth and at the plate edge, \( \alpha_u\left|_{y_{NA}=0} = \alpha_u\left|_{y_{NA}=0.5} \right. \right) \). Therefore, the ultimate shape factor \( \alpha_u \) in Equation (17) can be simplified as Equation (18) and introduced into Equation (16).

\[
M_{tL} = \alpha_u M_{yf}
\]  

(16)

\[
\alpha_u = (14f_u/f_{yf} + 13)(y_{NA}/h)^3(0.5 - y_{NA}/h) + (1.22f_u/f_{yf} + 0.26)
\]  

(17)

\[
\alpha_u = (1.22f_u/f_{yf} + 0.26)
\]  

(18)

By introducing Equations (15) and (16) into (14), the ultimate bending capacity for an H-section beam bent about the minor axis can finally be expressed as Equation (19).

\[
M_d = 2(\rho_c + \alpha_u)M_{yf} = 0.5(\rho_c + \alpha_u)M_{yf}
\]  

(19)

When the thickness reduction factor of the compressed flanges \( \rho_c \) satisfies

\[
\rho_c = 2 - \alpha_u
\]  

(20)

The cross-section can reach its elastic moment capacity. The Class 3 slenderness limit of the flanges can be derived by introducing Equation (4) into Equation (20), and
the results are shown in Figure 16. It can be seen from Figure 16 and Table 5 that when the ultimate-to-yield ratio $f_u/f_{0.2}$ is larger than 1.3, the corresponding slenderness limit exceeds 50, which is beyond practical limits for aluminum alloy unstiffened elements. In this case, aluminum alloy H-sections under minor-axis bending are generally classified as non-slender sections. The $\beta_3/\varepsilon$ limits obtained from Equation (20) for 6082-T6 and 6063-T5 alloys are shown to be well agreed with the FE results (Figure 11). It is recommended that the limits for Class A and Class B aluminum alloy beams can be taken as $\beta_3/\varepsilon = 16.4$ and $\beta_3/\varepsilon = 14.7$, respectively, which are the results when $f_u/f_{0.2}$ is equal to 1 (Table 6).

Table 5. Class 3 flange slenderness limits of aluminum alloy H-sections under minor axis bending.

<table>
<thead>
<tr>
<th>$f_u/f_{0.2}$</th>
<th>Class A Alloys $\beta_3/\varepsilon$</th>
<th>Class B Alloys $\beta_3/\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>16.4</td>
<td>14.7</td>
</tr>
<tr>
<td>1.05</td>
<td>19.0</td>
<td>17.1</td>
</tr>
<tr>
<td>1.10</td>
<td>22.4</td>
<td>20.1</td>
</tr>
<tr>
<td>1.15</td>
<td>27.0</td>
<td>24.3</td>
</tr>
<tr>
<td>1.20</td>
<td>33.6</td>
<td>30.2</td>
</tr>
<tr>
<td>1.25</td>
<td>44.0</td>
<td>39.5</td>
</tr>
<tr>
<td>1.30</td>
<td>62.4</td>
<td>56.1</td>
</tr>
</tbody>
</table>

Figure 16. Class 3 flange slenderness limit vs. $f_u/f_{0.2}$ for aluminum alloy H-sections under minor axis bending.

Table 6. Proposal of Class 2 and Class 3 flange slenderness limits of aluminum alloy H-sections under minor axis bending.

<table>
<thead>
<tr>
<th>Design Method</th>
<th>Class A Alloys $\beta_2/\varepsilon$</th>
<th>Class A Alloys $\beta_3/\varepsilon$</th>
<th>Class B Alloys $\beta_2/\varepsilon$</th>
<th>Class B Alloys $\beta_3/\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC9</td>
<td>4.5</td>
<td>6</td>
<td>4.5</td>
<td>5</td>
</tr>
<tr>
<td>Proposal</td>
<td>7</td>
<td>16.4</td>
<td>7</td>
<td>14.7</td>
</tr>
</tbody>
</table>

The final expression for the calculation of the bending capacity of aluminum alloy H-sections under minor-axis bending is given by Equation (21). For Class 3 cross-sections, the moment capacity can be taken as the linear interpolation between $M_\theta$ and $M_{\theta t}$. For slender cross-sections, the strain-hardening effect of the tensile flanges is
ignored and the ultimate shape factor in Equation (19) is taken as $\alpha_u = 1.48$. The proposed method is verified against the FE results (Figure 13). The numerical-to-predicted ratio $M_{\text{d,FE}}/M_d$ is 1.16, indicating that the proposals produce more accurate estimates of the bending capacity than EC9 and EWM.

$$
M_d = \begin{cases} 
M_{pl} & \beta_1 \leq \beta_2 \\
M_{pl} + \frac{\beta_3 - \beta_1}{\beta_3 - \beta_2} \left( M_{pl} - M_{el} \right) & \beta_2 < \beta_1 \leq \beta_3 \\
0.5M_{el}(1.48 + \rho_f) & \beta_1 > \beta_3 
\end{cases}
$$

The presented method can also be used for designing beams under minor-axis bending made of steel. The moment capacity predictions of the specimens in [31] are calculated based on the effective width method and the proposed method. The comparison results are listed in Table 7. Since the values $M_d/M_{\text{d,Fe}}$ are close to 1.0 and have very low scatter, it may be concluded that the proposed method provides fairly good ultimate bending strength estimates for the steel beams.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$M_{\text{d,Fe}}$ (kN m)</th>
<th>$M_{d,\text{EWM}}$ (kN m)</th>
<th>$M_{u,\text{EWM}}$ (kN m)</th>
<th>$M_{\text{d,Fe}}/M_{d,\text{Fe}}$</th>
<th>$M_{\text{d,Fe}}/M_{u,\text{EWM}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B009</td>
<td>1.76</td>
<td>1.76</td>
<td>1.87</td>
<td>1.31</td>
<td>1.994</td>
</tr>
<tr>
<td>B015</td>
<td>2.60</td>
<td>2.59</td>
<td>2.91</td>
<td>1.22</td>
<td>1.887</td>
</tr>
<tr>
<td>B005</td>
<td>3.33</td>
<td>3.34</td>
<td>3.52</td>
<td>1.17</td>
<td>1.916</td>
</tr>
<tr>
<td>B007</td>
<td>2.16</td>
<td>2.2</td>
<td>2.16</td>
<td>1.18</td>
<td>1.965</td>
</tr>
<tr>
<td>B013</td>
<td>5.15</td>
<td>5.13</td>
<td>5.52</td>
<td>1.13</td>
<td>1.862</td>
</tr>
<tr>
<td>B003</td>
<td>5.51</td>
<td>5.46</td>
<td>5.59</td>
<td>1.13</td>
<td>1.909</td>
</tr>
<tr>
<td>B001</td>
<td>3.36</td>
<td>3.36</td>
<td>3.43</td>
<td>1.07</td>
<td>1.890</td>
</tr>
<tr>
<td>B011</td>
<td>8.39</td>
<td>8.36</td>
<td>8.81</td>
<td>1.06</td>
<td>1.854</td>
</tr>
<tr>
<td>Mean</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.157</td>
<td>1.910</td>
</tr>
<tr>
<td>CoV</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.070</td>
<td>0.025</td>
</tr>
</tbody>
</table>

6. Conclusions
This paper investigated the bending behavior of extruded aluminum alloy H-sections under minor-axis bending and provided moment capacity evaluation approaches. The flexural response of typical aluminum alloy H-sections was characterized by means of finite element analyses considering the material and geometric nonlinearities. The following conclusions are made from this study:

1. Even though EN 1999-1-1 adopted the classification framework for aluminum alloy cross-sections, an apparent underestimation in resistance predictions was observed for all analyzed sections. The highest discrepancies with numerical strengths were mostly obtained for the Class 3 and Class 4 sections. The flange slenderness was found to govern the behavior and non-dimensional strength of H-sections under this load case.

2. The plate stress gradient factor $\eta = 1.0$ is used by the effective thickness method in EN 1999-1-1 for an outstand element under stress gradient with peak compression at the tip, which is equivalent to a buckling coefficient of $k = 0.425$ in AS/NZS 4600. This was found conservative since the stress gradient effect is ignored.

3. The EWM design strength predictions are quite improved compared with EC9 because they are able to take into account the effects of strain hardening and the stress gradient for the Class 3 and Class 4 sections, respectively. In this paper, a design method for calculating the bending capacity of H-sections under minor-axis bending considering the inelastic response is presented. The proposed method
provides accurate strength determination and is shown to compare well the numerical data.

(4) The strength predictions based on the proposed method can also be extended to design steel H-beams under minor axis bending. Comparisons with the experimental data in [31] show that compared with the effective width method, the proposed method provides more accurate resistance predictions.

(5) Since aluminum alloy is a relatively more expensive material than steel, it is necessary to achieve significant material savings. This could be achieved through more economical design solutions such as plastic design. Current design rules in EC9 are shown to result in bending capacities that were overly conservative for H-sections in minor-axis bending. This paper provides a design approach based on inelastic reserve capacity for such members, which achieves a cost-efficient design solution.

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