

Article

Influence of the R -Ratio and the Temperature on the Coefficients of a Novel Exponential Approach for the Formulaic Description of Fatigue Crack Growth Rate Curves

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Abstract

Investigating crack growth in sheet metal originating from clinched joints is a major part of predicting the service life of mechanically joined structures. Clinching allows different materials to be joined together. A key task in this context is to perform crack growth simulations in the vicinity of clinched joints considering different load and environmental conditions. This requires formulaic descriptions of the fatigue crack growth rate curves for the materials used. For this purpose, fatigue crack growth rate curves for different R -ratios and temperatures are determined experimentally. Generally, these fatigue crack growth rate curves can be described very well using a novel two-part exponential approach for the formulaic description of fatigue crack growth rate curves developed at Applied Mechanics of Paderborn University (FAM). In this case, three parameters are sufficient to describe the fatigue crack growth rate curves. For the material HCT590X, which is frequently used in clinched joints, it is shown as an example how the parameters vary with the R -ratio and the temperature. In addition, at high crack growth rates, a significant rise in the fatigue crack growth rate curve sometimes occurs at both low and high temperatures. To account for this feature, the two-part exponential approach can be expanded to include a third part.

Keywords: clinching; crack growth equation; fatigue crack growth rate curves; exponential approach; R influence; temperature influence

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1. Introduction

The thorough investigation of the growth of cracks originating from clinched or riveted joints is the task of the Applied Mechanics of Paderborn University (FAM) in the subproject B04 of the collaborative research center TRR285. The goal is to prevent fracture-mechanical failure caused by the propagation of technically long cracks resulting from operational loads [1]. Since crack growth simulations [2–5] are an important method for estimating the crack growth lifetime of structures containing cracks, one key task in this project is to perform such simulations for cracks emanating from mechanically joined connections, considering different load and environmental conditions. For these simulations, fatigue crack growth rate curves for long cracks are required as material properties [6].

Long cracks are significantly larger than the grain size of the material. The plastic zone is small and limited to the immediate vicinity of the crack tip, and the principles of linear elastic fracture mechanics (LEFM) apply [7]. The fatigue crack growth rate curves for many materials follow an S-shaped course when a double-logarithmic representation is used (Figure 1). In this case, the fatigue crack growth rate curves typically consist of three regions. In region A, the crack growth rate is low. With decreasing range of the stress intensity factor $\Delta K = K_{\max} - K_{\min}$, the crack growth rate tends towards the threshold against crack propagation ΔK_{th} . If $\Delta K \leq \Delta K_{th}$, the propagation of long cracks stops. In region C, the crack growth rate increases significantly and tends towards the cyclic fracture toughness ΔK_c . If $\Delta K \geq \Delta K_c$, unstable crack growth occurs. In region B, stable crack growth occurs, and the course of the crack growth rate curve is nearly linear.

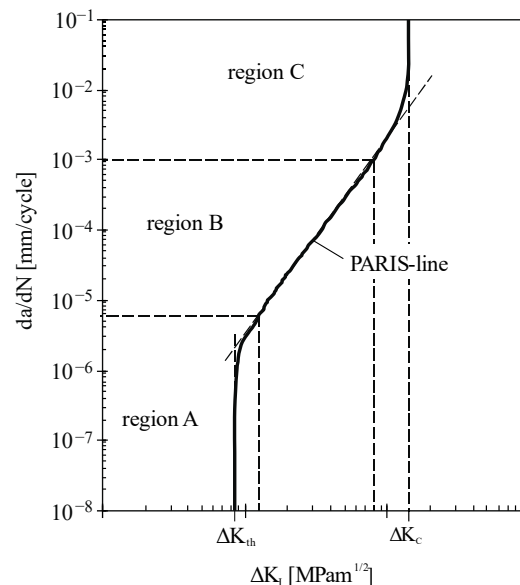


Figure 1. Characteristic course of the fatigue crack growth rate curve and its typical regions. Adapted from Ref. [6].

For their implementation in crack simulation programs, formulaic descriptions of the experimentally determined fatigue crack growth rate curves are required to correlate the calculated stress intensity factors with the respective crack growth rates. A precise description of the fatigue crack growth rate curve is particularly necessary in regions A and B shown in Figure 1, because inaccuracies in these regions can lead to considerable errors in the calculated crack growth lifetime. In contrast, deviations in region C have only a minor effect on the predicted lifetime result because, given the high crack growth rates in this region, only a few additional load cycles are required for the component to fail completely.

A well-known approach to describe the fatigue crack growth rate curve is the so-called Paris law [8,9]

$$\frac{da}{dN} = C(\Delta K)^m. \quad (1)$$

C and m are material parameters that depend on various factors, including temperature and the R -ratio $R = K_{\min}/K_{\max}$. Since the Paris law is only valid in region B of Figure 1, several modifications of the Paris law have been proposed, as discussed in detail in Ref. [10], in which the parameters ΔK_{th} and ΔK_c are included to consider regions A and C

as well. One of these modifications is presented by Forman and Mettu [11] and is known as the NASGRO Equation [12]

$$\frac{da}{dN} = \frac{C \left[\frac{1-\gamma}{1-R} \Delta K \right]^m \left(1 - \frac{\Delta K_{th}}{\Delta K} \right)^p}{\left(1 - \frac{K_{max}}{K_C} \right)^q}. \quad (2)$$

With the exponents p and q , additional adjustment options are available, and γ is used to account for the crack closure effect. Although Equation (2) describes the complete fatigue crack growth rate curve in principle, it is not straightforward to determine the parameters except for ΔK_{th} and ΔK_C . The parameters concurrently influence various characteristics of the fatigue crack growth rate curve, such as the transitions between regions A and B and between B and C, as well as the slope in region B. This problem is avoided with a two-part exponential approach

$$\frac{da}{dN}(\Delta K) = C_1 \exp\left(\lambda_1 (\Delta K)^\kappa\right) - C_2 \exp\left(\lambda_2 (\Delta K)^\kappa\right) \quad (3)$$

developed in Ref. [10] by the FAM. In Ref. [10], it is shown in detail that the slope factor λ_2 of the second exponential function can be set to zero. For the material HCT590X, which is frequently used in mechanically joined connections, the exponent κ can be set to 0.5. Thus, the basic form for this approach is

$$\frac{da}{dN}(\Delta K) = C_1 \exp\left(\lambda_1 (\Delta K)^\kappa\right) - C_2 = C_1 \exp\left(\lambda_1 \left(\sqrt{\Delta K}\right)\right) - C_2 \quad (4)$$

C_1 , λ_1 , and C_2 can be identified using the procedure described in Ref. [10]. With the exponent $\kappa = 0.5$, the slight curvature of the upper region of the fatigue crack growth rate curve, which occurs in steel when plotted on a single logarithmic scale, can be accounted for. Typically, measured values can be described using Equation (4) with a coefficient of determination of $R^2 > 0.9$ [10]. As Figure 2 shows, the first part of Equation (4) describes the increase in the fatigue crack growth rate curve in the upper region when the measured values are plotted over $\sqrt{\Delta K}$. C_1 is the value at which the first exponential function intersects the da/dN axis, and λ_1 is the slope. The first exponential function and the constant term C_2 intersect at the coordinates $\sqrt{\Delta K_{th}}$ and the corner growth rate $\left(\frac{da}{dN}\right)_C = C_2$.

Figure 2 shows that Equation (4) can be used to accurately describe the entire fatigue crack growth rate curve of the steel HCT590X. Since the argument of the exponential function must be dimensionless,

$$k = \left(\frac{1}{\lambda_1} \right)^2 \quad (5)$$

is introduced as a material constant, which has the same dimensions as ΔK . Furthermore, the following applies to $\sqrt{\Delta K_{th}}$

$$\left(\frac{da}{dN} \right)_C = C_1 \exp\left(\lambda_1 \sqrt{\Delta K_{th}}\right) = C_2 \quad (6)$$

and thus

$$\Delta K_{th} = \left[\frac{1}{\lambda_1} \ln \left(\frac{C_2}{C_1} \right) \right]^2 = k \left[\ln \left(\frac{C_2}{C_1} \right) \right]^2. \quad (7)$$

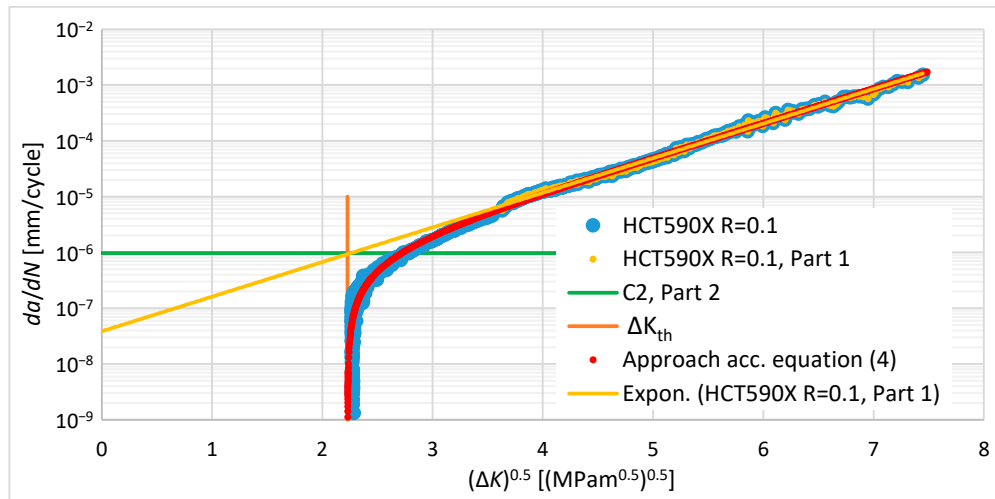


Figure 2. Exemplary fatigue crack growth rate curve for the steel HCT590X over $(\Delta K)^{0.5}$ with exponential regression line.

Because, as described in Ref. [10], the entire measured fatigue crack growth rate curve from ΔK_{th} to ΔK_c is used to identify the parameters C_1 , λ_1 , and C_2 , the threshold ΔK_{th} is not read directly from the measured curve but is calculated from the parameters. However, depending on the density of the experimental data in the lower range of the fatigue crack growth rate curve, it may be advisable to choose C_2 such that the threshold, which can be read from the experimental data, is reproduced with adequate accuracy. Thus, the final formulaic description of the entire fatigue crack growth rate curve is

$$\frac{da}{dN}(\Delta K) = \left(\frac{da}{dN} \right)_c \left[\exp \left(\frac{(\sqrt{\Delta K}) - (\sqrt{\Delta K_{th}})}{\sqrt{k}} \right) - 1 \right]. \quad (8)$$

Here, $\Delta K \geq \Delta K_{th}$ is assumed. As the parameters C_1 , λ_1 , and C_2 depend on the R -ratio and temperature, the objective of this study is to demonstrate how the effects of the R -ratio and the temperature can be considered when describing fatigue crack growth rate curves formulaically with the proposed exponential approach.

2. Experimental Determination of Fatigue Crack Growth Rate Curves

To determine the dependence of the parameters of the exponential approach according to Equation (4) used to describe fatigue crack growth rate curves formulaically on both the type of loading and the temperature, fatigue crack growth rate curves for the material HCT590X are experimentally determined for various R -ratios and temperatures. According to ASTM E 647, two test methods are required to capture the entire range of a fatigue crack growth rate curve [13]. In one test, called the down test, the middle and lower regions of the fatigue crack growth rate curve are determined. In this test, starting from an initial value for ΔK at which the crack grows at a medium rate, ΔK is gradually reduced until crack growth stops and the threshold against crack growth is reached. In a second test, called the up test, the middle and the upper regions of the fatigue crack growth rate curve are determined. In this test, starting with an initial value for ΔK that is clearly

lower than the initial value used in the first test, but which nevertheless results in crack growth at a moderate rate, the specimen is subjected to a constant cyclic loading ΔF until unstable crack growth occurs. In both test configurations, the maximum and minimum forces, the crack length, and the current number of load cycles must be continuously recorded to determine the relationship between the range of stress intensity ΔK and the crack growth rate da/dN after the test. The crack length is measured using the electric potential method [14]. In this method, as shown in Figure 3, a direct current is applied to the specimen, and the electric potential difference between the legs above and below the crack is measured. As the crack grows, the potential difference increases because the electrical resistance rises as the remaining cross-sectional area decreases. Due to the low thickness of the HCT590X sheets under examination, which is only 1.5 mm, the potential measurement points cannot be placed on the lateral end face of the specimen above and below the starter notch, as is usually the case. For this reason, a modified Mini-CT specimen with extended legs is developed [15] so that the potential measuring points can be placed on the front surface of the specimen; see Figure 4.

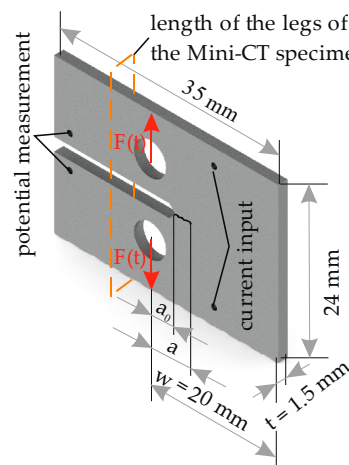


Figure 3. Geometry of the modified CT specimen with extended legs.

The initial crack length is $a_0 = 4 \text{ mm}$. The average accuracy for the specimen dimensions is 0.1 mm . Since both the crack length a and the stress intensity factor K are required for test control during the experiment, in Ref. [15] the calibration curve for converting the potential difference into the crack length and the geometry factor function Y for calculating the stress intensity factor according to the relationship

$$K = \frac{F}{w \cdot t} \sqrt{\pi a} \cdot Y \left(\frac{a}{w} \right) \quad (9)$$

are determined. The geometric factor function Y for the modified CT specimen is the same as that for the standard CT specimen. According to ASTM E 647 [13], this is:

$$Y \left(\frac{a}{w} \right) = \frac{2 + \frac{a}{w}}{\left(1 - \frac{a}{w} \right)^{\frac{3}{2}}} \left[0.886 + 4.64 - 13.32 \left(\frac{a}{w} \right)^2 + 14.72 \left(\frac{a}{w} \right)^3 - 5.6 \left(\frac{a}{w} \right)^4 \right] \quad (10)$$

As shown in Figure 4, a temperature chamber is used for experiments on the temperature dependence. This device allows experiments to be conducted at high or low temperatures using either a built-in heater or liquid nitrogen.

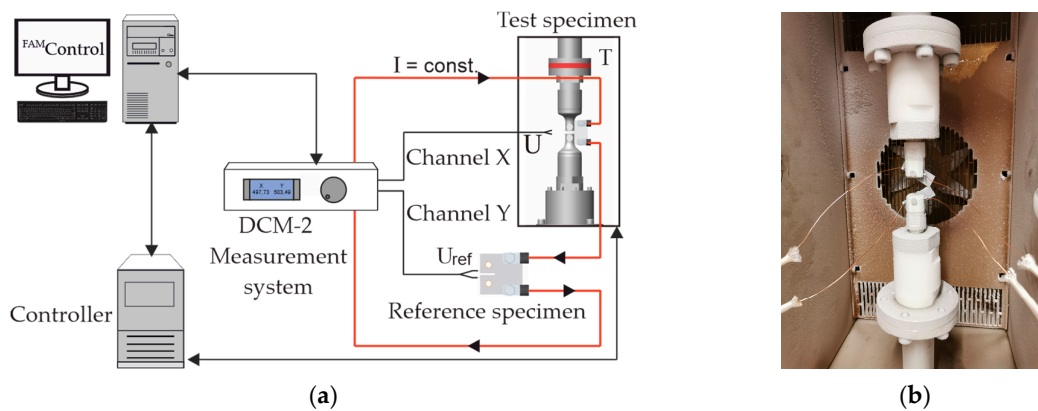


Figure 4. (a) Experimental setup for determining fatigue crack growth rate curves using the electric potential method; (b) Temperature chamber.

The tests are conducted for Mode I loading using the specialized measurement and control software ^{FAM}Control [16], which incorporates the calibration curve for crack length measurement and the geometry factor function for calculating the stress intensity factor. For each R -ratio and each temperature, four to six down tests and two up tests were conducted. Further repetitions of the up tests were not performed because the results were well reproducible. Figure 5 shows the typical fracture surfaces resulting from the down test and the up test. In both tests, the fatigue fracture surface remains flat, and the crack front remains almost straight. Furthermore, no necking is visible in any of the fatigue fracture surfaces except in the up test shortly before the occurrence of residual forced fracture. These results confirm that the conditions of linear fracture mechanics are satisfied during the tests. Only in the upper region of the fatigue crack growth curve are these conditions partially satisfied.

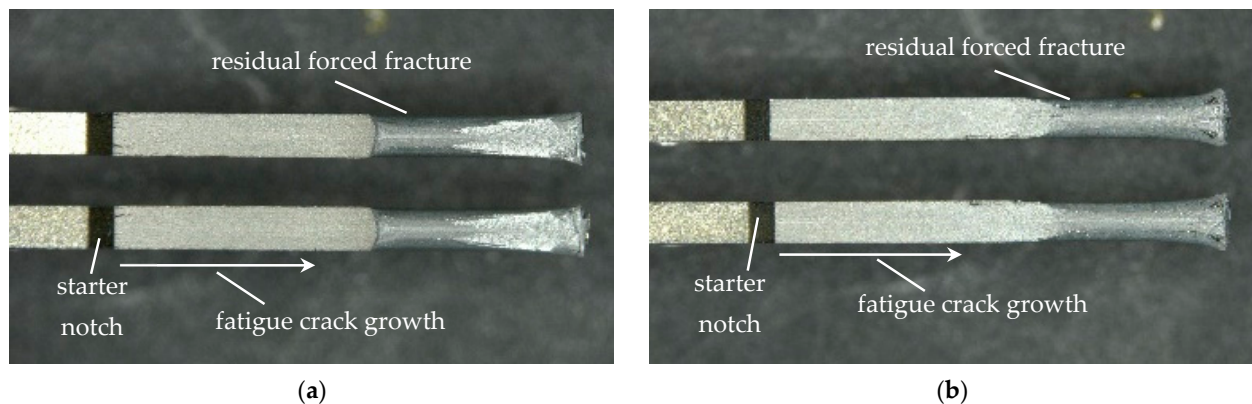


Figure 5. Typical fracture surfaces of the modified CT specimen after the down test (a) and the up test (b). Adapted from Ref. [17].

3. Dependence of the Exponential Approach on the R -Ratio

Figure 6 presents experimentally determined fatigue crack growth rate curves for HCT590X for various R -ratios at room temperature (20 °C). Each data set contains measurement results from several experiments and different rolling directions, both perpendicular and parallel to the crack propagation direction, with the same R -ratio. All measurement results are used to account for typical variations in measured values during parameter identification. The results show that the variations in the measured values are

small, although they are from different experiments. There is no dependence on the rolling direction [15], and stable values for the parameters can be expected.

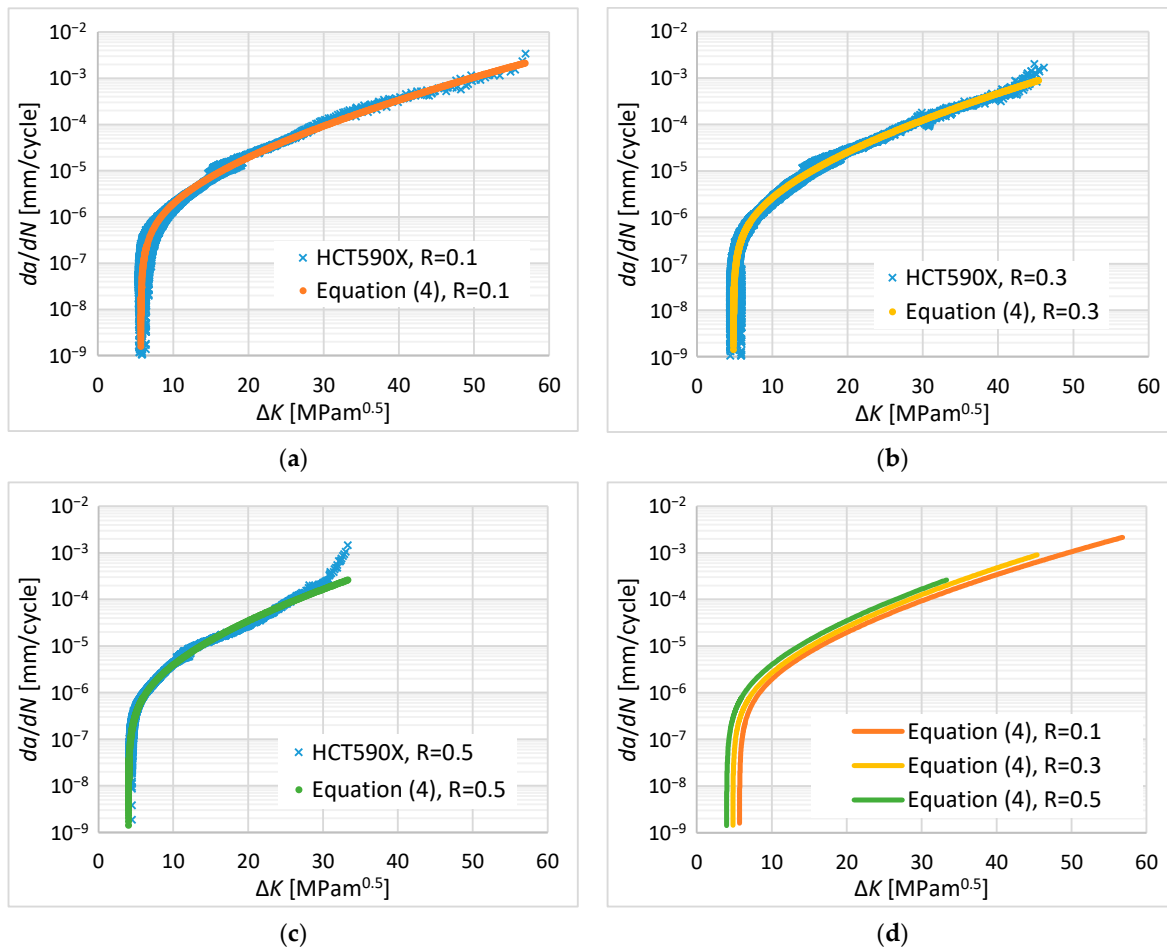


Figure 6. Fatigue crack growth rate curves at $T = 20\text{ }^{\circ}\text{C}$ for HCT590X for different R -ratios in comparison with the adapted fatigue crack growth rate curves according to Equation (4). (a) $R = 0.1$; (b) $R = 0.3$; (c) $R = 0.5$; (d) Comparison of the formulaic descriptions.

The comparison of the experimentally determined fatigue crack growth rate curves with the fitted fatigue crack growth rate curves based on Equation (4) and (8) shows that the exponential approach describes the transition from the threshold region to the middle region of the fatigue crack growth rate curve very well, regardless of the R -ratio, without the need to introduce any further parameters. It is noteworthy that, even when plotted on a single logarithmic scale and when approaching ΔK_c , a slight tendency towards a steeper but not vertical increase in the fatigue crack growth rate curve can be observed at $R = 0.1$ and $R = 0.3$, which is pronounced at $R = 0.5$. This effect did not occur in all experiments carried out to determine the upper part of the fatigue crack growth rate curve for HCT590X, nor is it generally observed, as demonstrated by the fatigue crack growth rate curves for various metals described in Ref. [10]. This steeper increase shortly before ΔK_c cannot be described with Equation (4). How this effect can be accounted for by introducing an additional exponential term is shown in Section 5. Here, the measured values in the region of the steeper increase are not considered when determining the parameters according to Equation (4). However, as discussed in the introduction, this deviation between the measured values and the formulaic description given by Equations (4) and (8),

respectively, does not result in any significant difference in the crack growth lifetime calculated in numerical simulations, because the crack growth rate in this region is so high that only a few more load cycles are required to cause the component to fail. The following example illustrates that an accurate representation of the fatigue crack growth rate curve in the transition region between the threshold and the middle region is significantly more important for predicting the residual crack growth life than an accurate representation of the steeper increase just before unstable crack growth is reached. In the transition region between the threshold and the middle region, the crack growth rate is approximately $da/dN \approx 2 \times 10^{-6}$ mm/cycle. This means that approximately 5×10^5 load cycles are required for a crack to extend by 1 mm. However, if an inaccurate approximation specifies the crack growth rate in this range as $da/dN \approx 1 \times 10^{-6}$ mm/cycle, thereby underestimating it by a factor of 2, then the calculated number of load cycles for 1 mm of crack extension is 1×10^6 . As a result, the residual life is overestimated by 5×10^5 load cycles. As shown in Figure 6c, at the end of the fatigue crack growth rate curve, Equation (4) yields a crack growth rate of approximately $da/dN \approx 2 \times 10^{-4}$ mm/cycle, while the measured crack growth rate is approximately $da/dN \approx 2 \times 10^{-3}$ mm/cycle, thereby underestimating it by a factor of 10. For a crack extension of 1 mm, this results in 5×10^3 load cycles when using the approach based on Equation (4) and 5×10^2 load cycles when using the measured crack growth rate. As a result, the residual life is overestimated by 4.5×10^3 load cycles. This difference is relatively negligible and lower than the typical simulation error if the total crack growth lifetime spans several 10^5 load cycles. Furthermore, this difference becomes smaller as the crack growth rate at the onset of the steeper increase becomes higher.

The identification of the parameters C_1 , λ_1 , and C_2 is shown in Figure 7 as an example for the experimental data in Figure 6c. As described in Ref. [10], according to Equation (4), an estimated value C_{2e} is added to all the measured data except the data from the steeper increase at the end of the fatigue crack growth rate curve. C_{2e} value is iteratively adjusted until the linear trend shown in Figure 7 is achieved. Thus, the following applies:

$$\lim_{C_{2e} \rightarrow C_2} \frac{da}{dN} (\Delta K) + C_{2e} = \lim_{C_{2e} \rightarrow C_2} C_1 \exp(\lambda_1 (\sqrt{\Delta K})) - C_2 + C_{2e} = C_1 \exp(\lambda_1 (\sqrt{\Delta K})). \quad (11)$$

Thus, $C_2 = C_{2e}$ and C_1 , as well as λ_1 are obtained from the exponential regression of the experimental data modified in this way.

The endpoint of each curve corresponds to ΔK_c . Typically, the parameters C_1 , λ_1 , and C_2 , and the material parameters $(da/dN)_c$, k , ΔK_{th} and ΔK_c depend on the R -ratio. These dependencies are summarized in Table 1 and illustrated in Figure 8.

Table 1. Parameters for the formulaic description of the fatigue crack growth rate curves from Figure 6 at $T = 20^\circ \text{C}$ for different R -ratios.

R	C_1 [mm/cycle]	λ_1 [(MPa $\sqrt{\text{m}})^{-0.5}]$	$C_2 = (da/dN)_c$ [mm/cycle]	R^2	ΔK_{th} [MPa $\sqrt{\text{m}}]$	k [MPa $\sqrt{\text{m}}]$	ΔK_c [MPa $\sqrt{\text{m}}]$	K_c [MPa $\sqrt{\text{m}}]$
0.1	2.302×10^{-8}	1.518	8.61×10^{-7}	0.975	5.69	0.434	56.8	63.1
0.3	2.489×10^{-8}	1.558	7.60×10^{-7}	0.931	4.81	0.412	45.3	64.7
0.5	3.689×10^{-8}	1.538	7.90×10^{-7}	0.971	3.97	0.423	33.3	66.6

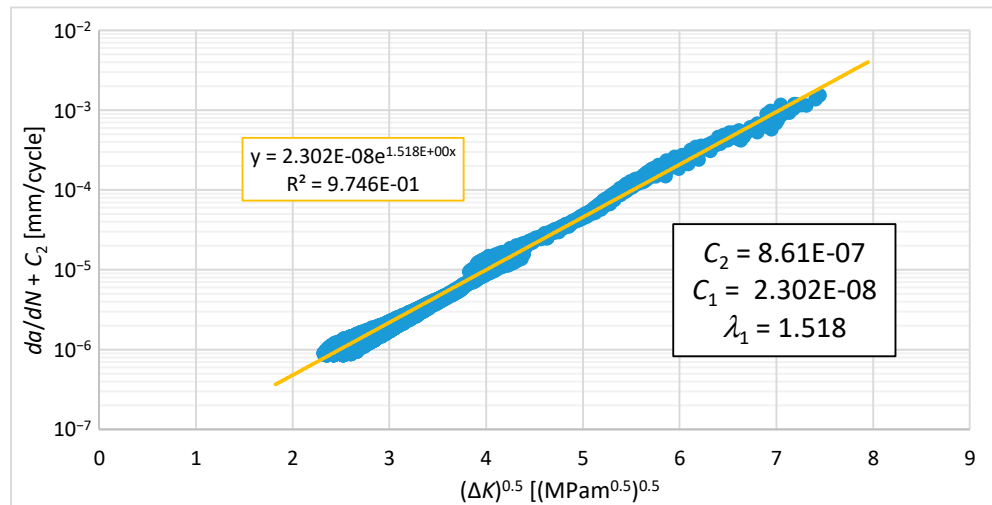


Figure 7. Exemplary modified experimental data for $R = 0.1$ of the steel HCT590X according to Equation (11) over $(\Delta K)^{0.5}$ with an exponential regression line.

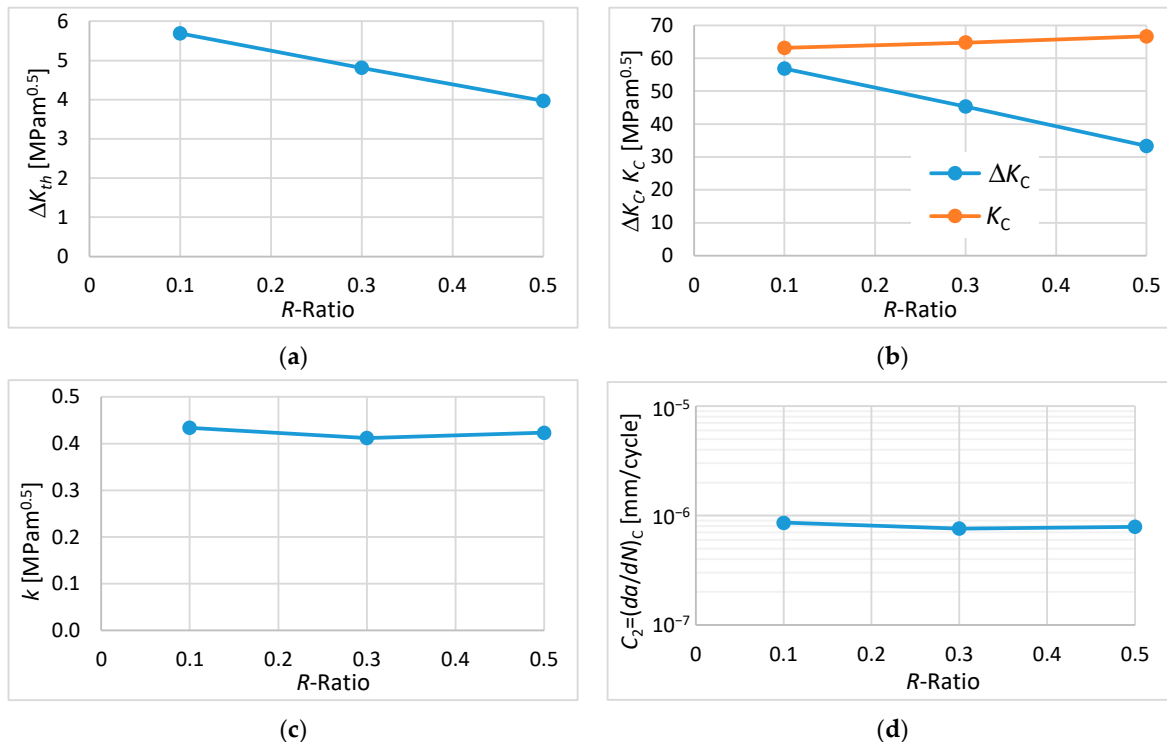


Figure 8. Courses of the material parameters at $T = 20\text{ }^{\circ}\text{C}$ depending on the R -ratio. (a) ΔK_{th} ; (b) ΔK_c and K_c ; (c) k ; (d) $C_2 = (da/dN)_c$.

As expected, the threshold ΔK_{th} and the cyclic fracture toughness ΔK_c decrease as the R -ratio increases. In the R -range under investigation, both parameters decrease almost linearly with the R -ratio. For the threshold, the following applies:

$$\Delta K_{th}(R) = 6.113 \text{ MPa}\sqrt{\text{m}} - 4.294 \text{ MPa}\sqrt{\text{m}} \cdot R. \quad (12)$$

The fracture toughness K_c for the HCT590X sheets can be calculated from ΔK_c using

$$K_c = \frac{\Delta K_c}{1-R}, \quad (13)$$

yielding a constant value essentially $K_c \approx 64.8 \text{ MPa}\sqrt{\text{m}}$. The material parameter $k \approx 0.423 \text{ MPa}\sqrt{\text{m}}$, which is a measure of the gradient of the fatigue crack growth rate curve, and the corner crack growth rate $(da/dN)_c \approx 8.04 \times 10^{-7} \text{ mm/cycle}$ are also almost independent of the R -ratio. As a result, the fatigue crack growth rate curve shifts to the left as the R -ratio increases, leading to higher crack growth rates for the same range of the stress intensity factor.

4. Dependence of the Exponential Approach on the Temperature

Figure 9 shows experimentally determined fatigue crack growth rate curves for HCT590X at various temperatures for $R = 0.1$. Since the tests for different R -ratios show that HCT590X shows no dependence on the rolling direction, the tests for high and low temperatures are only performed with the rolling direction parallel to the crack propagation direction. Test results from multiple experiments conducted at the same temperature are used to account for typical variations in measured values during parameter identification. The test results fall within a narrow range, demonstrating that the experiments are well reproducible. The measured fatigue crack growth rate curves at high and low temperatures show, like the fatigue crack growth rate curve for $R = 0.5$ at $T = 20^\circ\text{C}$ in Figure 6, a pronounced steeper increase shortly before reaching ΔK_c , but do not rise vertically. The measured values at $T = -30^\circ\text{C}$ in Figure 9b show two parallel courses in this region because they originate from two different up tests. The steeper increase occurs when the crack growth rate significantly exceeds 10^{-3} mm/cycle . Such high crack growth rates are not achieved at room temperature because unstable crack growth occurs beforehand. This effect can also be observed when, as the test approaches ΔK_c , the test frequency is significantly reduced to avoid measurement errors caused by the high crack growth rate. The comparison of the experimentally determined fatigue crack growth rate curves with the fitted fatigue crack growth rate curves based on Equation (4) and (8) shows that the exponential approach describes the transition from the threshold region to the middle region of the fatigue crack growth rate curve very well at different temperatures, without the need to introduce any further parameters. Since the steeper increase shortly before reaching ΔK_c cannot be described with this approach, the measured values in the region of the steeper increase are not considered when identifying the parameters. How this effect can be accounted for by introducing an additional exponential term is shown in the following section. However, as discussed in the introduction and discussed for the results depending on the R -ratio, this deviation between the measured values and the formulaic description given by Equations (4) and (8), respectively, does not result in any significant difference in the crack growth lifetime calculated in numerical simulations, because the crack growth rate in this region is so high that only a few more load cycles are required to cause the component to fail. Thus, the presented approach can also be used to describe fatigue crack growth rate curves for both high and low temperatures. Taking the additional increase into account mainly complicates the approach without leading to any significant improvement in lifetime simulation results. By using this approach, the endpoint of each curve corresponds to ΔK_c . Usually, the parameters C_1 , λ_1 , and C_2 , and the material parameters $(da/dN)_c$, k , ΔK_{th} and ΔK_c depend on the temperature. The dependencies are summarized in Table 2 and illustrated in Figure 10.

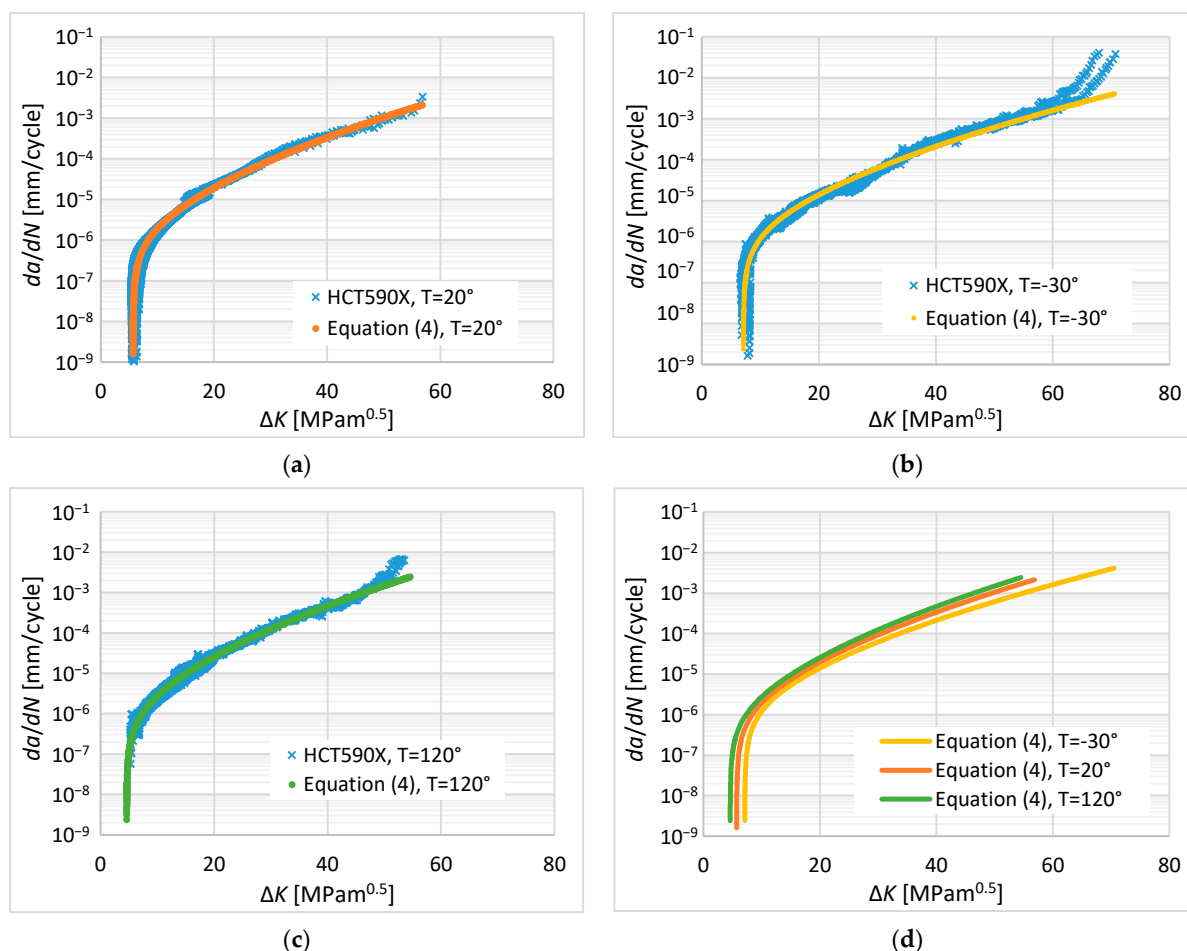


Figure 9. Fatigue crack growth rate curves for $R = 0.1$ for HCT590X for different temperatures in comparison with the adapted fatigue crack growth rate curves according to Equation (4). (a) $T = 20$ °C; (b) $T = -30$ °C; (c) $T = 120$ °C; (d) Comparison of the formulaic descriptions.

Table 2. Parameters for the formulaic description of the fatigue crack growth rate curves from Figure 9 for $R = 0.1$ for different temperatures.

T [°C]	C_1 [mm/cycle]	λ [$(\text{MPa}\sqrt{\text{m}})^{-0.5}$]	$C_2 = (da/dN)_c$ [mm/cycle]	R^2	ΔK_{th} [$\text{MPa}\sqrt{\text{m}}$]	k [$\text{MPa}\sqrt{\text{m}}$]	ΔK_c [$\text{MPa}\sqrt{\text{m}}$]	K_c [$\text{MPa}\sqrt{\text{m}}$]
−30	2.586×10^{-8}	1.427	1.150×10^{-6}	0.964	7.08	0.491	70.7	78.6
20	2.302×10^{-8}	1.518	8.610×10^{-7}	0.975	5.69	0.434	56.8	63.1
120	2.570×10^{-8}	1.552	7.10×10^{-7}	0.976	4.57	0.415	53.5	59.4

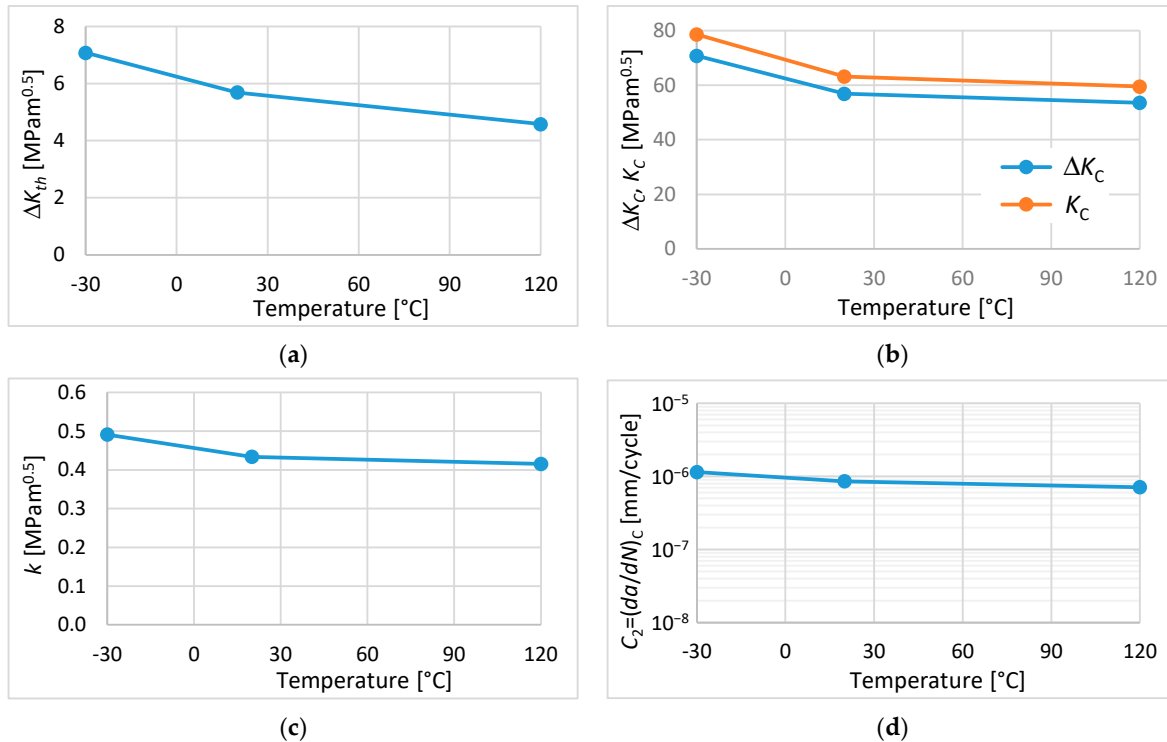


Figure 10. Courses of the material parameters for $R = 0.1$ depending on the temperature. (a) ΔK_{th} ; (b) ΔK_c and K_c ; (c) k ; (d) $C_2 = (da/dN)_c$.

The threshold ΔK_{th} decreases as the temperature rises. The fracture toughness K_c for HCT590X can be calculated from ΔK_c using Equation (13) for $R = 0.1$. ΔK_c and K_c increase with decreasing temperatures. The material parameter k , which is a measure of the gradient of the fatigue crack growth rate curve, and the corner crack growth rate $(da/dN)_c$ slightly decrease with increasing temperature. A formal description of the temperature dependence of these four parameters is deliberately omitted. For this, additional experiments need to be conducted, particularly for temperatures ranging from 20 °C to 120 °C. The comparison of the results for the temperature dependence shown in Figure 9d indicates that the crack growth rate tends to increase as the temperature rises.

5. Extension of the Exponential Approach to Describe the Steeper Increase in the Fatigue Crack Growth Rate Curve at High Crack Growth Rates

Figure 9 shows that, in the experimentally determined fatigue crack growth rate curves for high and low temperatures, an additional steeper increase occurs when the crack growth rate is significantly higher than 10^{-3} mm/cycle. A similar effect shows the fatigue crack growth rate curve for $R = 0.5$ at $T = 20$ °C in Figure 6c. As mentioned earlier, this effect cannot be described using the approach given by Equation (4). Equation (4) is based on the observation that, as shown comprehensively in Ref. [10], the fatigue crack growth rate curves of various metallic materials and different R -ratios can be described as the difference of two exponential functions according to Equation (3). To account for the additional steeper increase, the approach based on Equation (3) can consistently be extended to include an additional exponential function. The extended approach is therefore

$$\frac{da}{dN}(\Delta K) = C_1 \exp\left(\lambda_1 (\Delta K)^\kappa\right) - C_2 \exp\left(\lambda_2 (\Delta K)^\kappa\right) + C_3 \exp\left(\lambda_3 (\Delta K)^\kappa\right). \quad (14)$$

If $\kappa = 0.5$ and $\lambda_2 = 0$ still apply for HCT590X,

$$\frac{da}{dN}(\Delta K) = C_1 \exp\left(\lambda_1 \sqrt{\Delta K}\right) - C_2 + C_3 \exp\left(\lambda_3 \sqrt{\Delta K}\right) \quad (15)$$

is valid. This means that the parameters C_1 , λ_1 , and C_2 , for the description of the fatigue crack growth rate curves using Equation (4), summarized in Tables 1 and 2, remain unchanged. Thus, the parameters C_3 and λ_3 of the additional exponential function can be easily identified based on the experimental results. To do this, the experimental results in the region of the steeper increase are modified accordingly

$$\frac{da}{dN}_{\text{modified}} = \frac{da}{dN}_{\text{measured}} - C_1 \exp\left(\lambda_1 \sqrt{\Delta K}\right) + C_2 = C_3 \exp\left(\lambda_3 \sqrt{\Delta K}\right). \quad (16)$$

As an example, based on one of the two up tests conducted at $T = -30^\circ\text{C}$ and presented in Figure 9c, Figure 11 presents the modified experimental results for the region of the fatigue crack growth rate curve with the steeper increase plotted over $\sqrt{\Delta K}$.

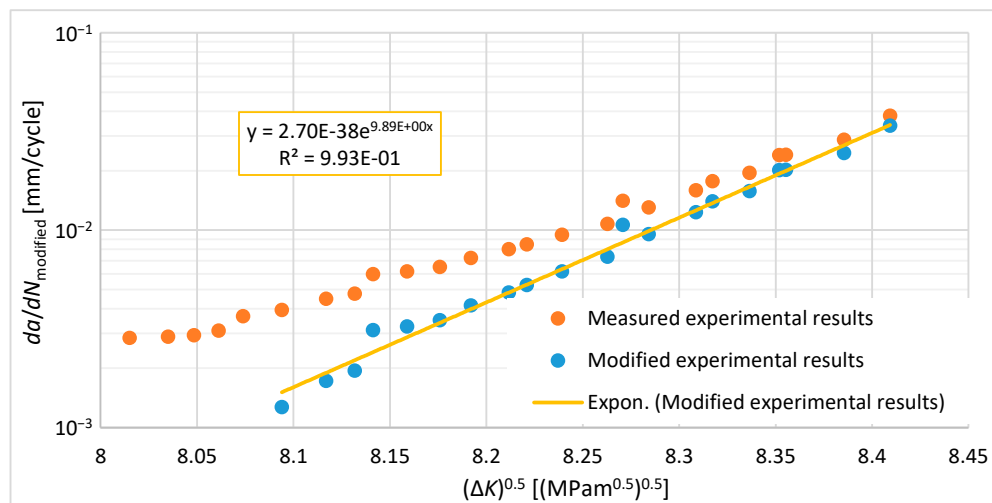


Figure 11. Exemplary modified experimental results for $T = -30^\circ\text{C}$ in the region of the steeper increase for the steel HCT590X over $(\Delta K)^{0.5}$ with an exponential regression line in comparison with the measured experimental results.

The comparison between the experimental results and the modified experimental results in Figure 11 shows that, when plotted against $\sqrt{\Delta K}$, the modified experimental results increase exponentially with a very good approximation. The parameters are $C_3 = 2.70 \times 10^{-38}$ and $\lambda_3 = 9.89$. As with Equation (8), Equation (15) can be transformed into a more intuitive form by introducing the coordinates $\sqrt{\Delta K_s}$ and $(da/dN)_s$ of the point of intersection of the first and the third exponential functions in Equation (15). This intersection point is, essentially, the starting point of the steeper increase of the fatigue crack growth rate curve. Since the argument of the third exponential function must be dimensionless,

$$k_s = \left(\frac{1}{\lambda_3} \right)^2 \quad (17)$$

is defined as a material constant, which has the same dimensions as ΔK . Furthermore, the following applies to $\sqrt{\Delta K_s}$ at the crack growth rate $(da/dN)_s$, where the steeper increase starts

$$\left(\frac{da}{dN} \right)_s = C_1 \exp(\lambda_1 \sqrt{\Delta K_s}) = C_3 \exp(\lambda_3 \sqrt{\Delta K_s}) \quad (18)$$

and thus

$$\Delta K_s = \left[\frac{1}{\lambda_1 - \lambda_3} \ln \left(\frac{C_3}{C_1} \right) \right]^2. \quad (19)$$

For the measured values shown in Figure 11, the values are $\Delta K_s = 66.7 \text{ MPa}\sqrt{\text{m}}$, $(da/dN)_s = 3.20 \times 10^{-3} \text{ mm/cycle}$, and $k_s = 1.02 \times 10^{-2} \text{ MPa}\sqrt{\text{m}}$. Thus, the final formulaic description of the entire fatigue crack growth rate curve is

$$\frac{da}{dN}(\Delta K) = \left(\frac{da}{dN} \right)_c \left[\exp \left(\frac{(\sqrt{\Delta K}) - (\sqrt{\Delta K_{th}})}{\sqrt{k}} \right) - 1 \right] + \left(\frac{da}{dN} \right)_s \left[\exp \left(\frac{(\sqrt{\Delta K}) - (\sqrt{\Delta K_s})}{\sqrt{k_s}} \right) \right]. \quad (20)$$

Here, $\Delta K \geq \Delta K_{th}$ is assumed. Figure 12 shows, as an example, the three exponential functions with the parameters that apply to the experimental results used in Figure 11. Together, they describe the entire fatigue crack growth rate curve, when a steeper rise is additionally observed at very high crack growth rates.

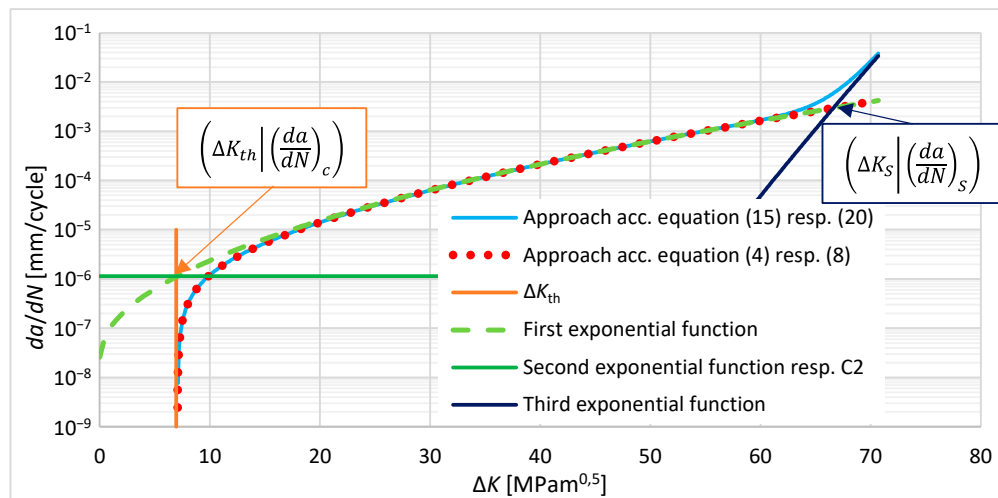


Figure 12. Characteristic course of the fatigue crack growth rate curve described with three exponential functions.

As Figure 10 shows, the third exponential function affects only the upper part of the fatigue crack growth rate curve. The comparison of the approaches with two and three exponential functions, as described by Equations (4) and (15) and experimental results, is shown in Figure 13. The comparison shows that, by introducing a third exponential function, the steeper increase in the fatigue crack growth rate curve, which is sometimes observed at high crack growth rates, can also be described very well.

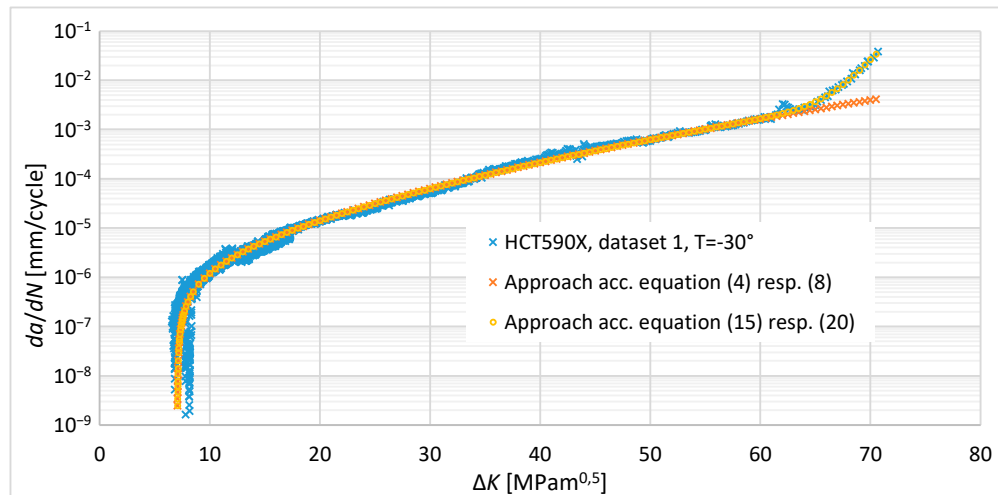


Figure 13. Comparison of the approaches with two and three exponential functions, as described by Equations (4) and (8) and (15), respectively (20), with the fatigue crack growth rate curve for HCT590X at $T = -30\text{ }^{\circ}\text{C}$.

The steeper increase can occur when the crack growth rate reaches very high values. As Figure 9b shows, there can be greater variations between different experiments in the region of the steeper increase, whereas in the middle and lower regions of the fatigue crack growth rate curves, the variations are relatively small.

A hypothesis to explain the steeper increase of the fatigue crack growth rate curve shortly before unstable crack growth occurs is that the crack propagation mechanism probably changes. In the high- and low-temperature tests shown in Figure 9, the steeper increase starts at $(da/dN)_s \approx 1\text{--}3 \times 10^{-3}\text{ mm/cycle}$. Thus, the increase in crack length in this region reaches approximately $1\text{--}3\text{ }\mu\text{m}$ per load cycle. For the dual-phase steel DP600, the grain size ranges from $5\text{--}20\text{ }\mu\text{m}$ [18], and the martensite particle size is about $1.97\text{ }\mu\text{m}$ [19]. In fact, DP600 is another name for HCT590X [20]. This means that, during each load cycle, in the region with the steeper increase, a crack increment is present that is on the order of the grain size and the martensite particle size. Another possible explanation for the steeper increase can be found by examining Figure 5. This figure shows that, during the up test, shortly before the onset of residual force failure, a slight necking can sometimes be observed at the fatigue fracture surface, such that in this region the conditions of linear elastic fracture mechanics are only approximately satisfied.

6. Conclusions

To perform numerical crack growth simulations in the vicinity of mechanical joints, fatigue crack growth rate curves are required for the used material, HCT590X. The method for experimentally determining fatigue crack growth rate curves at different R -ratios and temperatures is presented. Formulaic descriptions of the fatigue crack growth rate curves are typically required for use in crack growth simulation programs. The Paris law or modifications of the Paris law based on a power-law approach are often used for this purpose. Here, it is shown that all fatigue crack growth rate curves can be described very well using a novel approach based on the superposition of two exponential functions. The parameters of the exponential functions can be easily identified based on experimental data and converted into stable material parameters that uniquely characterize specific regions of the fatigue crack growth rate curves. Here, the parameters exhibit easily describable dependencies on the R -ratio and temperature. In particular, in the low- and high-temperature tests, a steeper increase can be observed shortly before the onset of unstable crack

growth, despite the use of a simple logarithmic representation. This may be due to a change in the crack propagation mode, as the crack increment reaches the grain size or the martensite particle size. Another possible explanation is that the conditions of linear elastic fracture mechanics are only partially satisfied. In principle, this additional increase can also be described using the exponential approach by adding a third exponential function. However, for the purpose in a crack growth simulation program, this addition can be neglected because, due to the very high crack growth rates in this region, only a few additional load cycles are required until the component fails. For this reason, using Equation (8) to describe the fatigue crack growth rate curve formulaically is sufficient for applications in crack growth simulations.

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Abbreviations

The following abbreviations are used in this manuscript:

a	crack length
e	$e = 2.7183$ Euler number
k	$k = 1/\lambda_1^2$ material constant describing the slope of the fatigue crack growth rate curve in the middle region with the same dimension as the stress intensity factor
k_s	$k_s = 1/\lambda_3^2$ material constant describing the slope of the fatigue crack growth rate curve in the region with steeper increase with the same dimension as the stress intensity factor
m_{FM}	exponent of the crack growth equation according to Forman/Mettu
m	exponent of the crack growth equation according to Paris
p, q	additional exponents of the crack growth equation according to Forman/Mettu
w	width of the modified CT-specimen
t	thickness of the modified CT-specimen
C_1, C_2, C_3	initial crack growth rates for $\Delta K = 0$ of the exponential crack growth equation with two and with three exponential functions
C_{2e}	estimated value of C_2 of the exponential crack growth equation
C_{FM}	material parameter of the crack growth equation according to Forman/Mettu
C	material parameter of the crack growth equation according to Paris
F	load
ΔF	cyclic load

I	Electric current
K	stress intensity
K_I	stress intensity for Mode I loading
K_C	fracture toughness
K_{max}	maximum stress intensity
K_{min}	minimum stress intensity
ΔK	$\Delta K = K_{max} - K_{min}$ cyclic stress intensity
ΔK_C	$\Delta K_C = (1-R)K_C$ cyclic fracture toughness
ΔK_{th}	threshold against crack growth
ΔK_s	stress intensity at the starting point of the steeper increase of the fatigue crack growth rate curve, cutting point of the first and the third exponential functions of the exponential crack growth equation
N	load cycles
R	$R = K_{min} / K_{max}$ load ratio
T	temperature
U	electric potential difference
Y	geometry factor function
κ	power of ΔK with the exponential crack growth equation
γ	crack closure parameter of the crack growth equation according to Forman/Mettu
$\lambda_1, \lambda_2, \lambda_3$	slope factors of the exponential crack growth equation
da/dN	crack growth rate
$\left(\frac{da}{dN}\right)_C$	corner crack growth rate, cutting point of the first and the second exponential functions of the exponential crack growth equation
$\left(\frac{da}{dN}\right)_s$	crack growth rate at the starting point of the steeper increase of the fatigue crack growth rate curve, cutting point of the first and the third exponential functions of the exponential crack growth equation

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