Experiments on High-Resolution Digitizer Accuracy in Measuring Voltage Ratio and Phase Difference of Distorted Harmonic Waveforms above 2 kHz

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Abstract: High-resolution multi-channel digitizers are used extensively for precision low voltage measurements in numerous applications and allow the simultaneous measurement of voltage magnitude ratio and phase difference between two different waveforms in power system applications. Delta–sigma-based analog-to-digital conversion enables the use of sampling frequencies in the range of megahertz, which provides accurate measurement bandwidths for transformed high-frequency, high-voltage signals. With the increased use of power electronic converters contributing to high-frequency harmonic emissions in power systems, there is a growing interest in developing calibration systems to measure voltage ratio and phase difference of distorted fundamental frequency waveforms consisting of superimposed, high-frequency harmonics. However, information regarding the accuracy of the high-resolution digitizers in the measurement of distorted voltage waveforms is limited as characterization is typically performed under sinusoidal voltage waveform conditions. This paper presents the details of the accuracy characterization of a 24-bit resolution digitizer under both sinusoidal and distorted waveform conditions for measuring complex voltage ratio and phase error for frequencies up to 10 kHz. The detailed experimental results and the measurement uncertainty evaluations show that increased voltage ratio and phase difference errors should be allocated when these high-resolution digitizers are used to measure distorted voltage waveforms. The estimated expanded uncertainties of complex voltage ratio measurement and phase error measurement for harmonic frequencies up to 10 kHz are ±260 ppm and ±100 µrad, respectively.

Keywords: high-resolution digitizers; voltage ratio error; phase angle error; distorted voltage waveforms; measurement uncertainty

1. Introduction

The modern power system landscape is undergoing a transformative shift due to the increasing penetration of renewable resources, advancements in transmission and distribution infrastructure, and the use of non-linear device-based consumer appliances. As a result of these developments, a significant increase in the use of power electronic converters has been observed. Modern power electronics employ force-commutated semiconductor devices switching at relatively high frequencies, contributing to an increase in harmonic emissions beyond 2 kHz [1,2]. The measurement and assessment of high-frequency harmonics ranging from 2 kHz to 150 kHz have largely been overlooked due to their negligible magnitudes and perceived minimal impact [3]. However, anticipating a future growth in emissions, technical bodies are working to regulate harmonic emissions and to develop measurement techniques covering this range.

Power system voltage and current waveforms, which approximate sinusoidal wave shapes, are subjected to distortion due to the superimposition of harmonic content. With the increasing use of power electronic converters, which use higher switching
frequencies, it is expected that the high-frequency harmonic content superimposed on the fundamental frequency waveforms will continue to increase. The measurement of such high-frequency harmonic content in power systems above 2 kHz has gained significant attention recently. In low voltage (LV) networks, commercial power quality analyzers are available for the measurement of high-frequency harmonic content up to hundreds of kilohertz. However, in medium voltage (MV) and high voltage (HV) networks, an external instrument transducer is required to reduce the primary voltages to measurable LV quantities. The majority of instrument transducers used in power systems are inductive and capacitive voltage transformers, which are designed for accurate voltage transformation at rated fundamental frequency. These transformers have non-linearities at higher frequencies, affecting their frequency response [4]. Such non-linearities of instrument transducers result in harmonic measurement errors, which need to be corrected through accurate calibration of the instrument transducer frequency response.

Conventionally, the HV instrument transducer frequency response calibration is implemented using an LV sinusoidal signal under the assumption that the frequency response is not affected by the application of the HV fundamental signal. However, in recent years, several technical standards have shown that instrument transducer frequency response evaluation should be carried out using a distorted HV waveform consisting of the rated HV fundamental voltage component plus superimposed harmonic components covering the frequency range of interest [5,6]. Figure 1 shows the instrument transducer frequency response calibration procedure suggested in the IEC 61869-103 technical report.

Figure 1. A frequency response evaluation procedure is suggested in the IEC 61869-103 technical report [5].

In adopting the distorted high voltage waveform-based calibration procedure shown in Figure 1, for instrument transducer frequency response calibration above 2 kHz, a measurement system with a wide dynamic range is required. This is due to the high-frequency harmonic content in distorted voltage waveforms generally being significantly smaller in magnitude in comparison to the amplitude of the fundamental frequency.
component. The measurement of such voltage signals requires measurement devices with a large dynamic range, wide frequency bandwidth, and high resolution.

High-speed, high-resolution digitizers such as the PXI-5922 offer the ability to measure distorted voltage waveforms with superimposed high-frequency harmonic content. The PXI-5922 digitizer is based on two 24-bit resolution delta-sigma analog to digital conversion (ADC) digitization channels. The PXI-5922 digitizer provides flexible resolution and sampling frequency combinations to adapt this instrument for the measurement of a wide variety of voltage signals [7]. However, the accuracy specification of these digitizers is provided for sinusoidal voltage signal measurements only. Accordingly, the accuracy characteristics of high-speed digitizers under distorted voltage waveforms are often limited.

When utilizing the digitizer in instrument transformer calibration systems, the voltage ratio error and phase displacement error are calculated for the test transducer by a comparison against a reference transducer. The contribution of measurement digitizer error to the calibration process is considered as an error in voltage ratio between the two digitizing channel signal amplitudes and a difference between phase angles of the two digitized signals. The measurement digitizer error contribution is a direct result of the mismatch of the digitizer channel spectral transmittance function [8,9].

The aim of this paper is to investigate the complex voltage ratio and phase error of the PXI-5922, which contributes to the overall voltage ratio and phase error measurement uncertainty in an instrument transformer calibration system. The evaluation of the digitizer error is performed using both sinusoidal and multi-tone distorted sinewave conditions. Furthermore, the contribution of the use of multi-tone distorted waveforms to the digitizer complex voltage ratio and phase error expanded uncertainty has been evaluated by performing a series of experiments. Section 2 presents the state-of-the-art high-speed digitizer accuracy characterization results published in the literature under sinusoidal and non-sinusoidal conditions and highlights the lack of research that investigates the accuracy of high-speed digitizers when measuring smaller harmonic components that are superimposed on a strong fundamental component, which more closely represents a distorted MV/HV signal consisting of harmonics from real systems. Based on the limitations identified, Section 3 provides the experimental results obtained by characterizing the PXI-5922 digitizer complex voltage ratio and phase error under sinusoidal signals for frequencies from 50 Hz to 10 kHz. This section further evaluates the change of the digitizer error values when determined under distorted sinusoidal conditions and compares the differences. Section 4 provides an extensive uncertainty evaluation based on the experiments performed in Section 3, which allows interested researchers to identify the dominant influencing factors that contribute to error in PXI-5922 digitizer complex voltage ratio and phase angle measurements under non-sinusoidal signal conditions.

2. State-of-the-Art High-Speed, High-Resolution Digitizer Characterization for High-Frequency Measurements

The following section describes the published literature on the characterization of the PXI-5922 digitizer under sinusoidal and distorted waveform conditions.

2.1. Digitizer Characterization under Sinusoidal Waveform Conditions

In the AC amplitude measurement results reported in [10], the performance of a 24-bit resolution PXI-5922 digitizer was evaluated using a thermal voltage converter-based AC-DC transfer standard. The AC amplitude measurement error in the PXI-5922 has been evaluated at different sampling frequencies up to 100 kHz. In addition, the digitizer AC RMS (root mean square) voltage measurement accuracy has been verified using an Agilent 3458A digital multi-meter that uses the Swerlein algorithm. However, this method has only been implemented for three signal frequencies up to a maximum of 200 Hz due to the accuracy limitation of the Swerlein algorithm for frequencies above 200 Hz. These evaluations use sinusoidal voltage test signals, and the results show that the PXI-5922 has variable errors at different sampling frequencies. However, for sampling frequencies
below 1 MHz, a measurement error below ±250 ppm was reported. This type of accuracy evaluation cannot be applied to the measurement of distorted multi-tone waveforms in the presence of a strong fundamental component. In [11], a similar characterization of a PXI-5922 digitizer for AC amplitude measurement error up to 1 MHz has been evaluated using a thermal converter-based AC-DC transfer standard. The presented results align well with the measurement errors presented in [10]. The AC amplitude measurement error up to 100 kHz remains within ±250 ppm. Similar to the experiment presented in [10], the accuracy characterization is based on the use of a sinusoidal test signal and does not consider the effects of distorted waveforms on measurement accuracy. A further evaluation of the voltage linearity of the PXI-5922 digitizer measurement error has been conducted in [12,13]. The presented results show that the digitizer AC amplitude measurement error variation due to voltage linearity is well below ±100 ppm for the entire input voltage range of the PXI-5922 digitizer. However, as these measurements are performed by using sinusoidal test signals, the measurement error variations of the digitizer due to the use of multi-tone distorted waveforms are not considered.

A detailed characterization of the PXI-5922 digitizer magnitude and phase angle measurement errors have been presented in [14]. This research investigated the accuracy of the digitizer under various influencing factors such as frequency flatness, voltage linearity, loading effect due to internal impedance, temperature, and DC offset. The experiments have been carried out up to a frequency of 1 MHz, and the results confirm that the measurement uncertainties of both digitizer channels are below ±400 ppm at 1 MHz. Similar to the studies mentioned above, this study also uses sinusoidal signals for the characterization of the PXI-5922 digitizer, and hence, any impact caused by the use of multi-tone distorted waveforms has not been considered.

2.2. Digitizer Characterization under Distorted Waveform Conditions

Only a few publications report the details of the characterization of high-speed digitizers under distorted waveform conditions. However, such characterization is essential when these digitizers are used for accurate measurement of dynamic signals. In [15], which is an extended version of [10], the AC voltage measurement accuracy of a PXI-5922 digitizer and a PXI-4461 digitizer has been evaluated under both sinusoidal and non-sinusoidal conditions. The non-sinusoidal results present the accuracy characterization of two distorted sinusoidal signals at fundamental frequencies of 50 Hz and 53 Hz. Each signal is added with only three harmonic components at 2nd, 3rd, and 5th harmonic orders in such a way that their harmonic amplitudes would result in a change in total harmonic distortion (THD) of the test signal from 0.01% to 10%. The results show that the PXI-5922 error for the measurement of fundamental signal amplitude for signals with frequencies of 50 Hz and 53 Hz remains well below ±100 µV/V for all THD variations from 0.01% to 10%. It is imperative to highlight that the above research does not consider the measurement of the amplitude of harmonic components in the non-sinusoidal signal in the presence of a strong fundamental component and rather focuses on the measurement of the fundamental amplitude in the presence of harmonic distortion. As stated in Section 1, the aim of this paper is to investigate the accuracy of the PXI-5922 digitizer in measuring the smaller harmonic amplitude and harmonic phase angles, which are present in a distorted multi-tone waveform consisting of a strong fundamental frequency component and multiple harmonic components. Such a signal would be representative of the output of an MV/HV instrument voltage transformer that measures a distorted MV/HV voltage signal, as defined in Figure 1. In [16], the use of a PXI-5922 digitizer combined with a precise current-to-voltage converter for the accuracy testing of current transformers for frequencies up to 20 kHz has been reported. This test arrangement uses two-tone test signals consisting of the fundamental component and a sweeping harmonic component to cover the frequency range up to 20 kHz. The accuracy of the two-channel voltage ratio measurement capability of the PXI-5922 under two-tone signal input conditions has been verified by the application of two-tone test signals consisting of 500 mV fundamental and a 5 mV
harmonic component sweeping from 50 Hz to 10 kHz. However, such two-tone signals do not represent the behavior of dynamic signals, which consist of multiple harmonic components. Furthermore, ref. [17] reports a high-speed digitizer characterization method that uses the Josephson voltage standard for the accurate measurement of currents and voltages up to 100 V and 1 A by adopting precision voltage dividers and current shunts. This research reports that the accuracy evaluation of the PXI-5922 digitizer for frequencies up to 1 kHz using test signals consisting of three harmonic components with different combinations of amplitude and phase angles. The authors highlight the requirements of such a multi-tone signal-based characterization method in cases where the digitizers are used for the measurement of dynamic signals consisting of multiple harmonic components. However, the characterization results reported do not cover the frequencies above 1 kHz, where these errors could increase significantly.

In addition to the above, the recent literature on state-of-the-art metrology investigates low-cost measurement instrument options that can provide voltage and current measurements with acceptable accuracies for traceable power measurements [18]. However, these techniques mainly focus on the measurement of the fundamental frequency power rather than the measurement of harmonic power associated with harmonic components that are superimposed on the fundamental frequency component. In contrast, the field of harmonic measurement of distorted waveforms is more concentrated to the measurement of the harmonic composition of the measured signal.

Based on the above-published literature, the accuracy of the PXI-5922 harmonic amplitude and phase measurements for a distorted signal consisting of multiple harmonics covering the frequency range up to 10 kHz and a strong fundamental harmonic component, which is much greater than the interested harmonic amplitudes, has not been investigated previously. Hence, the aim of this paper is to evaluate the harmonic amplitude and phase angle measurement error of the PXI-5922 digitizer up to 10 kHz using highly distorted multi-tone signals consisting of a strong 50 Hz fundamental component and 200 harmonic components, each having an amplitude of 1% relative to the fundamental. Such an evaluation enables researchers to understand the capabilities of the PXI-5922 digitizer as an accurate measurement tool in MV/HV instrument transformer frequency response calibration for harmonic measurements under actual waveforms representing HV fundamental signals that are superimposed with multiple small harmonic amplitudes.

3. Experimental Results of Digitizer Accuracy Evaluation

The accuracy evaluation of the PXI-5922 digitizer is presented in the following section. A Fluke 5730A precision calibrator was used to generate accurate sinusoidal voltage signals [19]. The calibrator has an absolute AC voltage accuracy of ±42 µV/V for a 1 year stability period from the date of calibration. Since the parameters of interest are the amplitude ratio and phase angle difference between the signals measured by the two digitizer channels, the error in the complex ratio and phase error has been evaluated. These errors can be evaluated by supplying both channels with the same signal through identical Bayonet Neill–Concelman (BNC) cables with identical lengths. The sampling frequency of the digitizer is kept constant at 500 kSa/s, and the record length of the waveform is kept constant at 500 ms for all performed measurements. The evaluation of the digitizer was conducted under two conditions:
1. Evaluation under sinusoidal waveforms;
2. Evaluation under non-sinusoidal waveforms.

3.1. Voltage Ratio and Phase Error under Sinusoidal Input Signals

The complex voltage ratio error ($k_{\text{PXI}_h}$) and phase error ($\phi_{\text{PXI}_h}$) at harmonic order ($h$) can be defined according to Equations (1) and (2), respectively.
\[ k_{PXI_h} = \frac{V_{CH1_h}}{V_{CH2_h}} \]  

(1)

\[ \varphi_{PXI_h} = \theta_{CH1_h} - \theta_{CH2_h} \]  

(2)

where \( V_{CH1_h} \) and \( \theta_{CH1_h} \) are the amplitude and phase angle of the spectral component at harmonic order \((h)\) measured by the PXI-5922 digitizer 1st channel; and \( V_{CH2_h} \) and \( \theta_{CH2_h} \) are the amplitude and phase angle of the spectral component at harmonic order \((h)\) measured by the PXI-5922 digitizer 2nd channel.

3.1.1. Voltage Linearity of Digitizer Complex Voltage Ratio and Phase Error

To evaluate the PXI-5922 digitizer voltage dependence of complex ratio and phase measurement error under sinusoidal signals, sinusoidal test signals at three different voltage levels were generated by an arbitrary waveform generator (AWG). The generated signals were applied to the two channels of the digitizer using identical coaxial measurement cables. For all tests, the sampling rate of the PXI-5922 digitizer has been kept constant at 0.5 Msa/s to ensure that the sampling resolution is fixed at 24 bits. The calculated voltage ratio and phase errors using the digitizer measurements and fast Fourier transform (FFT) algorithm at different frequencies are shown in Table 1 and 2. It can be seen that when the PXI-5922 digitizer is used to measure the complex voltage ratio, the voltage ratio error introduced due to the digitizer was below 100 µV/V for all three test signal levels of 50 mV, 0.5 V, and 1 V. This shows the excellent accuracy of the digitizer in the considered frequency range from 50 Hz to 10 kHz. In terms of the phase difference, the error introduced between the two digitizer channels was well below ±30 µrad.

<table>
<thead>
<tr>
<th>Voltage Ratio Error (µV/V)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal Voltage</td>
<td>50 Hz</td>
</tr>
<tr>
<td>50 mV</td>
<td>70</td>
</tr>
<tr>
<td>500 mV</td>
<td>86</td>
</tr>
<tr>
<td>1000 mV</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 1. Voltage linearity of complex voltage ratio error between PXI-5922 digitizer CH1/CH2.

<table>
<thead>
<tr>
<th>Phase Error Standard Deviation (µrad)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal Voltage</td>
<td>50 Hz</td>
</tr>
<tr>
<td>50 mV</td>
<td>6.065</td>
</tr>
<tr>
<td>500 mV</td>
<td>0.499</td>
</tr>
<tr>
<td>1000 mV</td>
<td>0.268</td>
</tr>
</tbody>
</table>

Table 2. Voltage linearity of phase error between the PXI-5922 variation with frequency (CH1-CH2).

3.1.2. Digitizer Complex Voltage Ratio and Phase Error Variation with Frequency

For this evaluation, a 1 V sinusoidal signal with frequency varying from 50 Hz to 10 kHz was generated using a Fluke 5730A precision calibrator and applied to both channels of the PXI-5922 digitizer. At each harmonic frequency, the complex voltage ratio was calculated by using the digitizer measurements. Figure 2 shows the calculated complex voltage ratio error values for harmonic frequencies from 50 Hz to 10 kHz. According to the results shown in Figure 2, the complex voltage ratio error remains below ±30 µV/V for all harmonic frequencies up to 10 kHz.
Similar to the calculation of complex voltage ratio error described above, at each harmonic frequency phase error was calculated by using the digitizer measurements. Figure 3 shows the calculated phase error values for frequencies from 50 Hz to 10 kHz.

The phase error for frequencies from 50 Hz to 10 kHz was well below ±3 µrad. The phase error measurement results shown in Figure 3 demonstrate a continuous and linear increase in magnitude as the test signal frequency increases. Such linear dependence of digitizer phase error can be attributed to a constant time delay between the measurement channels, which would appear as a continuously increasing phase angle with the increase of frequency in frequency domain measurements.

3.2. Complex Voltage Ratio and Phase Error under Distorted Input Signals

In the case of evaluating the accuracy of the PXI-5922 digitizer complex voltage ratio and phase difference measurement under composite waveforms, a worst-case signal consisting of multiple harmonic components was selected. This signal consisted of a 30 mV fundamental frequency component at 50 Hz and all the harmonic components from 100 Hz to 10 kHz, with each having an amplitude of 1% of the fundamental component. Figure 4 shows the time domain waveform of the test signal.
Figure 4. Test waveform used for the PXI-5922 accuracy verification under distorted signal conditions.

Figure 5 shows the complex voltage ratio of the PXI-5922 digitizer evaluated by the application of the composite signal to the digitizer’s two channels using identical coaxial cables. For all harmonic orders up to the 200th order, the complex voltage ratio measurement error was below ±0.01%. In addition, the standard deviation of the voltage ratio measurement at each harmonic order from 50 Hz to 10 kHz lies below ±0.02%. These values compare well with the amplitude accuracy specifications provided with the PXI-5922 calibration results. However, in comparison to the PXI-5922 complex voltage ratio results determined under sinusoidal signals, the complex voltage ratio under multi-tone distorted waveforms showed increased error magnitudes and increased standard deviation of measurement. In addition, the error under multi-tone signals remained flat with the increase of frequency in contrast to the decreasing error trend, which was observed under sinusoidal signals, as shown in Figure 2.

Figure 5. Complex voltage ratio error of the PXI-5922 digitizer under composite signal.

Figure 6 shows the measured phase error evaluated by the application of the multi-tone signal, as shown in Figure 4. It can be seen that the phase error for all harmonic frequencies from 50 Hz to 10 kHz remains below ±40 µrad. In comparison to the phase error evaluation results under sinusoidal signal conditions shown in Figure 3, a relative increase in phase error values can be observed for all harmonic frequencies up to 10 kHz. However, the linear phase error variation observed under sinusoidal conditions does not occur when the digitizer is used to measure multi-tone distorted waveforms.
3.3. Accuracy Characterization of Fast Fourier Transform (FFT) Algorithm under Multi-Tone Distorted Waveforms

In the determination process of harmonic magnitude ratio and phase angle differences using the PXI-5922 digitizer, the application of an FFT algorithm is required according to the guidelines provided in IEC 61000-4-7 [20]. As the aim of this paper is to investigate the PXI-5922 digitizer measurement error under distorted waveform conditions, the error contribution due to the use of the FFT algorithm was evaluated using multi-tone signals. For this evaluation, two digital signals (Signal 1 and Signal 2) representing the digitized output waveforms of the ADC process were created in LabVIEW software. Each signal was a summation of a fundamental sinusoid at 50 Hz and multiple harmonic sinusoids at frequencies shown in Table 3. The amplitude of all harmonic components is kept constant at 1% relative to the fundamental amplitude. For each harmonic order in the two multi-tone signals, the amplitude ratio of Signal 1 to Signal 2 was fixed at 100 to accommodate any error caused by input signal amplitude level mismatches when the PXI-5922 digitizer is used for calibrating instrument transducers with different voltage transformation ratios. Furthermore, the phase angle between the spectral components of Signal 1 and Signal 2 at each harmonic order is fixed at zero degrees. For each signal, a Gaussian RMS noise of 1% magnitude relative to the fundamental component was added to represent the non-idealities of the analog-to-digital conversion (ADC) process. Table 3 shows the signal ratio calculated by using the FFT algorithm results. It can be observed that the percentage ratio error caused by the application of the FFT algorithm was below ±0.01% for the considered harmonic frequencies up to 10 kHz.

Table 3. Calculated signal ratio results in the evaluation of voltage ratio error due to the FFT algorithm application on the PXI-5922 complex voltage ratio measurements.

<table>
<thead>
<tr>
<th>Harmonic Frequency (Hz)</th>
<th>Calculated Signal Ratio</th>
<th>FFT Amplitude Error (µV/V)</th>
<th>FFT Phase Error (µrad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>99.998933</td>
<td>−10.67</td>
<td>0.406</td>
</tr>
<tr>
<td>150</td>
<td>100.001415</td>
<td>14.15</td>
<td>−0.106</td>
</tr>
<tr>
<td>250</td>
<td>99.999077</td>
<td>−9.23</td>
<td>−11.5</td>
</tr>
<tr>
<td>350</td>
<td>100.005224</td>
<td>52.24</td>
<td>19.5</td>
</tr>
<tr>
<td>450</td>
<td>100.008516</td>
<td>85.16</td>
<td>2.28</td>
</tr>
<tr>
<td>550</td>
<td>100.005281</td>
<td>52.81</td>
<td>11.1</td>
</tr>
<tr>
<td>650</td>
<td>100.000341</td>
<td>3.41</td>
<td>−8.36</td>
</tr>
<tr>
<td>750</td>
<td>99.999951</td>
<td>−0.49</td>
<td>−1.31</td>
</tr>
</tbody>
</table>
4. Uncertainty Evaluation of Complex Voltage Ratio and Phase Error Measurements

Based on the experimental results presented in Section 3, the PXI-5922 digitizer complex voltage ratio and phase error uncertainty can be determined according to the guide on the expression of uncertainty in measurement provided in [21]. Each influencing quantity investigated in the previous section has been considered as an uncertainty contribution to the complex voltage ratio and phase error of the PXI-5922 digitizer. Sections 4.1 and 4.2 present the individual uncertainty evaluation results complex voltage ratio and phase error, respectively.

4.1. Evaluation of Complex Voltage Ratio Expanded Uncertainty

The influencing quantities that contribute to an uncertainty of the PXI-5922 digitizer complex voltage ratio can be defined according to Equation (3).

\[ k_{PXIh} = \left[ \frac{V_{CH1h}}{V_{CH2h,avg}} \right] [\Delta_f \times \Delta_V \times \Delta_MT \times \Delta_FFT \times \Delta_{rep}] \] (3)

The following notations are used for the terms in Equation (3). For all factors, the best estimate is taken as unity for all uncertainty calculations.

- \( \Delta_f \) = correction factor for dynamic variation across frequency.
- \( \Delta_V \) = correction factor for voltage linearity across the channel input voltage range.
- \( \Delta_MT \) = correction factor for intermodulation effects created by residual harmonics in multi-tone input signals
- \( \Delta_FFT \) = correction factor for errors caused by the use of the FFT algorithm and windowing function.
- \( \Delta_{rep} \) = correction factor for repeatability of measurements.

The expanded uncertainty in complex voltage ratio \( \frac{u^2(k_{PXIh})}{|k_{PXIh}|} \) can be expressed according to Equation (4).

\[ \frac{u^2(k_{PXIh})}{|k_{PXIh}|} = \frac{u^2(\Delta_f)}{|\Delta_f|} + \frac{u^2(\Delta_V)}{|\Delta_V|} + \frac{u^2(\Delta_MT)}{|\Delta_MT|} + \frac{u^2(\Delta_FFT)}{|\Delta_FFT|} + \frac{u^2(\Delta_{rep})}{|\Delta_{rep}|} \] (4)

Based on individual standard uncertainty contributions defined in Equation (4), the expanded uncertainty of the PXI-5922 complex voltage ratio measurements can be calculated to the guidelines provided in [21]. The uncertainty calculation is summarized as a budget shown in Table 4.
According to the uncertainty evaluation results above, the expanded uncertainty of the PXI-5922 complex voltage ratio measurements from 50 Hz to 10 kHz can be expressed according to Equation (5).

\[
\frac{u(k_{\text{PXI}})}{|k_{\text{PXI}}|} = \pm 0.026\% \text{, } k = 2
\]

In comparison to the ±100 µV/V complex voltage ratio error of the PXI-5922 digitizer observed under sinusoidal signal conditions, the uncertainty of the complex voltage ratio error under distorted sinusoidal conditions is ±260 µV/V, which is more than double. The main contribution is due to the effect of the multi-tone signal and increased standard deviation of measurements.

4.2. Evaluation of Digitizer Phase Error Expanded Uncertainty

The uncertainty contribution to the phase error of the PXI-5922 digitizer can be defined according to Equation (6).

\[
\varphi_{\text{PXI}} = \left[\varphi_{\text{CH1}} - \varphi_{\text{CH2}}\right]_{\text{avg}} + \theta_f + \theta_V + \theta_{MT} + \theta_{FFT} + \theta_{rep}
\]

where
- \(\theta_f\) = Phase error correction for dynamic variation across frequency;
- \(\theta_V\) = Phase error correction factor for voltage linearity across the channel input voltage range;
- \(\theta_{MT}\) = Phase error correction factor for intermodulation effects created by residual harmonics in multi-tone input signals;
- \(\theta_{FFT}\) = Phase error correction factor for errors caused by the use of the FFT algorithm and windowing function;
- \(\theta_{rep}\) = Phase error correction factor for repeatability of measurements.

The best estimate for each contribution term in Equation (6) can be assumed as zero. Based on this assumption, the combined standard uncertainty of the PXI-5922 phase error difference can be evaluated according to Equation (7).

\[
u_{\varphi,\text{PXI}} = u^2(\theta_f) + u^2(\theta_V) + u^2(\theta_{MT}) + u^2(\theta_{FFT}) + u^2(\theta_{rep})
\]
Based on individual standard uncertainty contributions defined in Equation (7), the expanded uncertainty of the PXI-5922 phase error measurements can be calculated to the guidelines provided in [21]. The uncertainty calculation is summarized as the budget shown in Table 5.

Table 5. Expanded uncertainty budget for PXI-5922 digitizer phase error.

<table>
<thead>
<tr>
<th>Uncertainty Components</th>
<th>Distribution Type</th>
<th>Evaluation Type</th>
<th>Semi-Range, a</th>
<th>Divider, d</th>
<th>Deg of Freedom, νi</th>
<th>Std. Uncertainty, u_i</th>
<th>Sensitivity Factor, c_i</th>
<th>(c_i u_i)^2</th>
<th>(c_i u_i)^4/n_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic variation with frequency</td>
<td>crad</td>
<td>rect</td>
<td>B</td>
<td>0.0003</td>
<td>1.732</td>
<td>199</td>
<td>0.00015</td>
<td>1</td>
<td>1.4953 \times 10^{-4}</td>
</tr>
<tr>
<td>Voltage linearity</td>
<td>crad</td>
<td>rect</td>
<td>B</td>
<td>0.0021</td>
<td>1.732</td>
<td>18</td>
<td>0.00120</td>
<td>1</td>
<td>1.1997 \times 10^{-3}</td>
</tr>
<tr>
<td>Effect of multi-tone signals</td>
<td>crad</td>
<td>rect</td>
<td>B</td>
<td>0.0035</td>
<td>1.732</td>
<td>10</td>
<td>0.00204</td>
<td>1</td>
<td>2.0419 \times 10^{-3}</td>
</tr>
<tr>
<td>Repeatability</td>
<td>crad</td>
<td>norm</td>
<td>A</td>
<td>0.0041</td>
<td>1.000</td>
<td>199</td>
<td>0.00411</td>
<td>1</td>
<td>4.1094 \times 10^{-3}</td>
</tr>
<tr>
<td>Effect of FFT algorithm and window function</td>
<td>crad</td>
<td>rect</td>
<td>B</td>
<td>0.0020</td>
<td>1.732</td>
<td>20</td>
<td>0.00113</td>
<td>1</td>
<td>1.1263 \times 10^{-3}</td>
</tr>
<tr>
<td>Rounding of reported results</td>
<td>crad</td>
<td>rect</td>
<td>B</td>
<td>0.0005</td>
<td>1.732</td>
<td>100</td>
<td>0.00029</td>
<td>1</td>
<td>2.8868 \times 10^{-4}</td>
</tr>
<tr>
<td>Rounding of uncertainty Sums</td>
<td>crad</td>
<td>rect</td>
<td>B</td>
<td>0.0005</td>
<td>1.732</td>
<td>100</td>
<td>0.00029</td>
<td>1</td>
<td>2.8868 \times 10^{-4}</td>
</tr>
<tr>
<td>Combined Standard Uncertainty ((U_c))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.855 \times 10^{-3}</td>
</tr>
<tr>
<td>Effective number of degrees of freedom ((\nu_{eff}))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>170.41</td>
</tr>
<tr>
<td>Coverage factor (k)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.0</td>
</tr>
<tr>
<td>Expanded uncertainty ((U))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.010</td>
</tr>
</tbody>
</table>

According to the uncertainty calculation shown in Table 5, the expanded uncertainty of the PXI-5922 digitizer phase error from 50 Hz to 10 kHz can be expressed according to Equation (8).

\[
U(\varphi_{PXI_{5922}}) = \pm 0.01 \text{ crad, } k = 2
\]  

(8)

In comparison to the ±30 µrad phase error of the PXI-5922 digitizer observed under sinusoidal signal conditions, the uncertainty of the phase error under distorted sinusoidal conditions is ±100 µrad, which is more than three times. The main contribution is due to the effect of the multi-tone signal and increased standard deviation of measurements.

5. Discussion

In this paper, the accuracy of the PXI-5922 digitizer in measuring complex voltage ratio and phase error has been evaluated under multi-tone distorted waveforms consisting of harmonic components up to 10 kHz. The experimental results show that the complex voltage ratio and phase error evaluated under sinusoidal signal conditions are significantly different when the same errors are evaluated using multi-tone distorted waveform conditions. For all harmonic frequencies up to 10 kHz, an increase in the standard deviation of measurements can be observed for both complex voltage ratio and phase error due to the intermodulation effects of multi-tone waveforms. In addition, the error in voltage ratio and phase error measurement in the frequency domain using the FFT algorithm also contribute to the increased errors when used to determine the spectral amplitude and
phase angle values in multi-tone distorted waveforms. Furthermore, the expanded uncertainty of the PXI-5922 digitizer complex voltage ratio measurement is dominated by the contribution due to the use of multi-tone distorted waveforms in comparison to the other conventional uncertainty contributions. Therefore, it is essential to allow increased uncertainty limits when utilizing high-speed, high-resolution digitizers in instrument transformer calibration systems for frequencies up to 10 kHz.

6. Conclusions

It can be stated that the PXI-5922 digitizer can provide high accuracies when measuring both sinusoidal and highly distorted sinusoidal voltage signals. If the measurement quantity of interest is the fundamental frequency component of a highly distorted signal, the PXI-5922 digitizer can provide accurate measurements that are closely aligned with the expanded uncertainties associated with that of sinusoidal signal measurement. However, if the measurement of interest for a distorted signal is the small harmonic amplitudes and phase angles of the harmonic components that are superimposed on a strong fundamental frequency component, relatively higher uncertainty limits should be allocated in terms of both harmonic amplitude and phase angle measurement. In this evaluation, a complex voltage ratio uncertainty of ±260 µV/V and a phase error uncertainty of ±100 µrad were observed for harmonic measurements for frequencies from 50 Hz to 10 kHz. The uncertainty calculations of both harmonic amplitude and phase angle measurements are dominated by the impact of multi-tone signals, and further research should be carried out to investigate the underlying reasons, and extensive experiments should be performed to establish the measurement traceability of amplitude and phase angle measurement for non-sinusoidal signals.

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Data Availability Statement: Data is contained within the article.

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References


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