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Wavefront-Based Error Calibration for Coordinate Measuring Machines in the Surface Metrology of Large Optics

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Abstract

Precise measurement of large mirrors is essential for high-performance space-based optics. Traditionally, laser interferometry provides high accuracy but faces challenges related to the requirements of computer-generated holograms (CGHs) and the transformation of standard wavefronts to match complex test surfaces. This paper presents a proof-of-concept contact-based surface metrology method for large mirrors. To reduce drift errors caused by stylus instability or contact loss, a wavefront-based calibration method was developed to suppress measurement artifacts while preserving intrinsic surface errors. The method was tested on a concave aspherical mirror using a tactile coordinate measuring machine (CMM), and its accuracy was evaluated by comparing deviations between calibrated CMM error maps and laser-based measurements. Experimental results demonstrated that the XENO CMM achieved a global metric accuracy comparable to interferometric systems for relevant geometric parameters. These findings suggest that ultra-precision CMMs combined with focus point calibration can provide practical preliminary surface verification and serve as a flexible alternative to laser interferometry when global geometric accuracy is the primary concern. Future work will enhance robustness by addressing higher-order aberrations.

Keywords: optical metrology; surface measurement; coordinate measuring machine; wavefront aberrations



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1. Introduction

Accurate metrology of large optics, especially those with large radii of curvature (RoCs), is a critical prerequisite for streamlining their fabrication. High-precision, large aspheric mirrors are essential to meet the growing demand for space-based systems of capturing deep-space imagery. As the demand for high-performance optical systems increases, large aspheric mirrors are increasingly incorporated into instruments such as astronomical telescopes. In general, larger apertures improve imaging spatial resolution, while appropriately selected radii of curvature optimize the focal length and optical layout. Such mirrors are widely deployed in spaceborne applications, including deep-space exploration, astronomical observatories, and Earth-observation platforms [1–3]. The primary mirror typically serves as the core element of a telescope: it is the largest optical component in diameter and often has a highly aspheric profile and a large RoC. A well-known example

is the Ritchey–Chrétien (R–C) Cassegrain configuration used in the Hubble Space Telescope (HST), which employs surfaces with large radii of curvature for both the primary and secondary mirrors to achieve 28' field of view (FOV). Similar design principles apply to multifunctional, miniaturized systems, which integrate complex freeform surfaces to enhance flexibility when precise aberration correction and wider FOVs are required.

However, even small irregularities in the mirror surface profile can significantly degrade an optical system's imaging performance, placing stringent demands on metrology [3–6]. Consequently, optical metrology has become an integral part of the manufacturing workflow, ensuring that each fabrication stage adheres to design specifications.

Metrology techniques for large-mirror surface verification can be broadly categorized according to two criteria. The first is interaction type: contact (tactile) methods versus non-contact (optical) techniques. The second is measurement coverage—full-aperture acquisition versus partial scanning—together with the data-merging methodology employed [7–18]. A more detailed classification of metrology systems for large complex-surface verification is provided in Table 1.

Table 1. Classification of metrology systems for surface quality verifications.

Operational Characteristic	Type	Advantages	Limitations
Structured light [15]	Deflectometry (triangulation principles/pattern projection) [13,16]	Less sensitive to specular surfaces (reflective surfaces)	Requires rigorous calibration and susceptible to slope ambiguity
	Confocal microscopy (spotlight projection) [14]	High-resolution (sub-micron), areal-based resolution	Susceptible to light-of-sight challenges and steep slope measuring problems
Tactile metrology	Coordinate measuring machines (CMMs) [17–20]	Higher dynamic measurement range	Susceptible to stylus deflection (geometric measuring errors)
	Stylus profilometry [15]	Measuring surface features at the micro- or nanoscale	Limited measuring coverage and requires stitching algorithms for data merging
Wavefront measurement	Laser interferometry	Highly accurate measurement (nanoscale precision)	Requires tailor-made computer-generated holograms (CGHs)

As a rule of thumb, any geometric verification system should achieve a measurement accuracy at least one order of magnitude better than the specified surface tolerance. For precision CNC-manufactured spherical surfaces, this typically requires metrology systems capable of assessing form deviations with uncertainties below 0.010 mm.

Laser interferometry [4–6] remains the standard for the metrology of space-based optics. It is a mature technique that delivers three-dimensional surface maps with height-measurement repeatability below 1 nm. However, higher accuracy traditionally comes at the expense of measurement range. The fundamental limitations of conventional interferometry include restricted dynamic range and limited flexibility. When a surface departs significantly from a reference sphere, high fringe densities can introduce retrace errors that compromise measurement fidelity.

Computer-generated holograms (CGHs) [7,8] are commonly employed to compensate for aspheric departure in null testing. However, each test surface generally requires a

custom-designated CGH to transform a standard reference wavefront into one that matches the target geometry. Because CGHs must be uniquely fabricated for each optic, the process is both costly and time-consuming. Moreover, fabrication constraints limit CGH applicability, particularly for surfaces with steep local gradients [13].

Traditional interferometric setups typically require the test beam to be focused at the optical center of curvature of the surface under test. This constraint makes measurements highly surface-specific and imposes strict layout requirements. Evaluating optics with large radii of curvature often necessitates long optical paths and makes the setup highly sensitive to misalignment, which can introduce significant off-axis aberrations. To accommodate larger apertures [9–12], stitching interferometry techniques have been developed. However, reconstructing the full surface wavefront from multiple sub-aperture measurements requires substantial computational overhead, particularly as the number of sub-apertures increases. Collectively, these factors can significantly reduce the efficiency and cost-effectiveness of testing large optical surfaces.

Furthermore, in configurations that rely on optical auto-collimation, structural tilt between the optical surface and its mounting substrate may remain undetected. As a result, the optical axis of the measured surface may not align with the mechanical axis of its mount. To minimize the influence of alignment artifacts and low-frequency noise during wavefront reconstruction, it is standard practice to filter low-order Zernike terms: piston and tilt are typically subtracted for spherical surfaces, whereas piston, tilt, and defocus are generally removed when characterizing planar features.

Recent advances in measurement precision have expanded the adoption of non-null-optic metrology techniques, including coordinate measuring machines (CMMs), structured light deflectometry, confocal microscopy, and stylus profilometers. These approaches are complemented by multi-sensor integration and multi-scan stitching algorithms, which significantly enhance measurement versatility and spatial resolution. Among these techniques, CMMs stand out for large-surface metrology because of their operational flexibility and high dynamic range. Since Y. Li et al. [21] first demonstrated micron-level accuracy on complex surfaces in 2005, CMMs have been widely adopted in optical metrology [19,20,22–24], on-machine inspection [25], and swing-arm profilometry [26,27].

However, tactile CMMs are inherently prone to systematic errors. By recording discrete probe coordinates relative to a machine reference frame, they are highly sensitive to probe arm flexure, which introduces geometric inaccuracies. This limitation is exacerbated on large, gently curved surfaces, where minor deflections can lead to significant measurement deviations. In addition, contact scanning of large mirrors often suffers from drift errors caused by stylus instability or intermittent loss of surface contact. Furthermore, optical components in common spaceborne systems typically feature large RoC values or large f-numbers (focal length-to-aperture ratios), making them highly susceptible to tilt during precision grinding in machining centers. As a result, their manufacturing precision is frequently limited by these metrological challenges [27–29]. Addressing these systematic and drift errors is therefore critical for advancing large-aperture optical fabrication.

To this end, this paper introduces a novel focus point calibration framework integrated with advanced CMM operation. The proposed method circumvents the scalability constraints of laser interferometry for large surfaces while actively suppressing the inherent drift and systematic inaccuracies of CMMs, thereby enabling high-precision metrology for complex, large-scale optical components.

2. Materials and Methods

This study evaluates the feasibility of a wavefront-based coordinate measuring machine (CMM) method for the optical metrology of large mirrors. As noted above, CMM

accuracy is affected by several error sources, including rigid-body misalignment, probe arm flexure, and mechanical vibrations, such as stylus instability [30–33]. In addition, thermal errors can arise from the thermal expansion of the machine structure, probe system, and test piece. The magnitude of thermal drift is directly correlated with measurement duration and ambient temperature fluctuations, making it particularly significant during prolonged measurement sequences [30–34]. To mitigate these thermal effects, measurements were conducted in a temperature-controlled environment of 20 ± 0.5 °C after an adequate stabilization period before data acquisition. Furthermore, because the XENOS CMM (ZEISS, Oberkochen, Germany) system uses linear scales with near-zero thermal expansion, thermal drift was considered negligible.

Wavefront aberration analysis was used to evaluate figure errors and polishing defects derived from the CMM measurements. Zernike polynomials [34], which are widely used in optical metrology, decompose surface figure errors into orthogonal aberration modes. This decomposition helps isolate non-intrinsic error components, enabling their removal from the reconstructed surface profile. However, accurate characterization of spherical surfaces requires genuine surface irregularities to be distinguished from alignment-induced artifacts. To address this issue, a context-dependent filtering strategy was adopted, in which selected Zernike terms were retained or removed according to the measurement objective while remaining insensitivity to focus position.

2.1. Wavefront Analysis

Wavefront analysis is an optical metrology technique used to characterize light propagation by quantifying phase deviations introduced by optical imperfections. These deviations are commonly expressed as a linear combination of Zernike polynomials [34], which form an orthogonal basis over a circular pupil. The measured wavefront $W(\rho, \theta)$ can be written as

$$W(\rho, \theta) = \sum_{j=0}^J a_j Z_j(\rho, \theta) \quad (1)$$

where a_j is the coefficients of the j -th Zernike term Z_j and (ρ, θ) is normalized polar coordinates on the unit disk ($0 \leq \rho \leq 1$). Because the Zernike basis is orthogonal, it provides an effective means of separating low-order systematic errors from higher-order surface figure deviations. In the proposed wavefront-analysis workflow, selected Zernike coefficients were filtered to suppress artifacts arising from rigid-body misalignment and probe deflection while preserving intrinsic surface errors. This strategy enables accurate characterization of spherical surfaces relative to the mirror's nominal radius of curvature.

The Zernike polynomials $Z_n^m(\rho, \theta)$ are defined as the product of a normalization constant, a radial polynomial $R_n^{|m|}(\rho)$, and an angular function. Using a common normalization convention,

$$\text{For } m > 0 : Z_n^m(\rho, \theta) = \sqrt{2(n+1)} R_n^m(\rho) \cos(m\theta) \quad (2)$$

$$\text{For } m < 0 : Z_n^m(\rho, \theta) = \sqrt{2(n+1)} R_n^{|m|}(\rho) \sin(|m|\theta) \quad (3)$$

$$\text{For } m = 0 : Z_n^0(\rho, \theta) = \sqrt{(n+1)} R_n^0(\rho) \quad (4)$$

where n is the radial degree and m is the azimuthal frequency, subject to $n \geq 0$, $|m| \leq n$, and even $n - |m|$. The radial polynomial $R_n^{|m|}(\rho)$ is given by

$$R_n^{|m|}(\rho) = \sum_{k=0}^{\frac{n-|m|}{2}} \frac{(-1)^k (n-k)!}{k! \left(\frac{n+|m|}{2} - k\right)! \left(\frac{n-|m|}{2} - k\right)!} \rho^{n-2k} \quad (5)$$

where the upper summation index $(n - |m|)/2$ is an integer.

2.2. Measured Object

The test specimen is a weakly aspherical optical mirror developed by the Chinese Academy of Sciences for meteorological satellite applications. As shown in Figure 1, the mirror has a diameter of 190.8 mm and a radius of curvature of 1430 mm. This mirror was selected to validate the proposed wavefront-based CMM method against high-precision laser interferometry. Its small aspheric departure allows for conventional interferometric measurement without a custom null lens, thereby simplifying the validation process and reducing both experimental complexity and cost.



Figure 1. Test specimen—optical mirror.

This experiment serves as a proof-of-concept demonstration of the focus point calibration (FPC) framework. An interferometrically accessible surface was chosen to isolate the effect of FPC on systematic CMM geometric errors. In this work, “large optics” refers to surfaces with large radii of curvature and large f-numbers rather than necessarily large physical dimensions. The selected mirror is representative of this class and is sensitive to low-order form errors, such as defocus, which FPC is designed to address. Large-aperture-specific effects, including environmental drift, long-travel stage errors, and gravity-induced deformation, are not considered here.

2.3. Surface Measurement

Two instruments were used in this study (Figure 2): a German-made ZEISS XENOS high-precision tactile coordinate measuring machine (CMM) and a US-made PhaseCam 6000 dynamic laser interferometer from 4D Technology (Tucson, USA). The XENOS CMM has a measuring volume of $900 \times 1500 \times 700 \text{ mm}^3$ and a certified volumetric accuracy of $\pm (0.2 + L/1000) \text{ }\mu\text{m}$ over $19.5\text{--}20.5 \text{ }^\circ\text{C}$. The PhaseCam 6000 served as the reference instrument, providing a measurement accuracy of approximately 2% of the He–Ne laser wavelength ($\lambda = 632.8 \text{ nm}$). Although optical measurement systems can exhibit a lower signal-to-noise ratio (SNR) compared to contact methods, this limitation was mitigated in this study by using the high-performance PhaseCam 6000 laser system in a controlled laboratory environment. Its dynamic phase-shifting capability, high-speed data acquisition, and vibration-insensitive design make it suitable for characterizing large optical surfaces and long optical paths. All CMM measurements were performed in a temperature-controlled laboratory.



Figure 2. Testing instruments ((left) ZEISS XENOS CMM, (right) PhaseCam 6000 Dynamic Laser Interferometer).

The CMM data acquisition process consisted of workpiece alignment and surface scanning. Accurate alignment is essential because it establishes a consistent spatial relationship between the optical surface and the machine coordinate system. To ensure uniform coverage of the aperture, a concentric circular scanning strategy was adopted. The surface was divided into multiple rings of increasing radii, and the probe acquired data points sequentially along each ring. After one ring was completed, the CMM stage moved to the next radial position. The measurement parameters are listed in detail in Table 2.

Table 2. Measurement parameters.

	Category	Parameter	Value/Description
1	CMM Sampling	Measurement Mode	Touch-trigger mode
		Sampling pattern	10 circular rings
		Point spacing	0.1° angular spacing
		Total scan time	2 h
2	Probe Specification	Probe type	VAST Gold
		Stylus diameter & material	5 mm diameter, Ruby
		Probing force	50 mN
		Probe-radius compensation	Gaussian filtration with least-squares fitted sphere
3	Interferometric Reference	Configuration	Confocal test at the center of curvature
		Cavity length	1430 mm
		Number of average frames	10
		Alignment terms removed	Piston, tip, tilt, and power
4	Data Processing	Grid resampling	500 × 500
5	Mechanical Setup	Fixturing	3-point support system

Measurement artifacts and erroneous data points can significantly degrade surface fitting accuracy [26–28]. To mitigate these effects, post-acquisition processing was performed to separate the true surface form from measurement artifacts.

To define the wavefront, a best-fit sphere (BFS) was first computed from the measured data to serve as a reference surface. This involved calculating the coordinates of the sphere center, denoted as (x_0, y_0, z_0) , using a least-squares fitting algorithm [35]. This method minimizes the sum of the squared radial deviations between the measured CMM surface

points and the theoretical spherical model. The objective function minimizes the distance deviations from each measured point to the sphere center, thereby providing a robust estimate of the BFS parameters even in the presence of minor measurement noise.

$$F = \sum_{i=1}^n \left(\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} - R \right)^2 \quad (6)$$

where (x_i, y_i, z_i) are the Cartesian coordinates of the i -th measured point, (x_0, y_0, z_0) represents the center of the BFS, R is the radius of the BFS, and the fitting process determines the parameters that minimize the objective function F .

Residuals relative to the BFS were then statistically analyzed to identify and remove outliers and high-frequency noise while preserving the mid- and low-spatial-frequency surface errors. Contact CMM data may be acquired using either the touch-trigger method or continuous scanning. The touch-trigger measurement typically produces errors that follow an approximately Gaussian distribution, whereas continuous scanning often introduces non-Gaussian noise. Accordingly, a 2σ threshold was applied to touch-trigger data, while a more conservative threshold was used for continuous-scanning data to avoid rejecting valid surface features.

2.4. Data Processing and Wavefront Reconstruction

To facilitate error elimination and cross-platform comparison, all datasets were normalized to ensure consistency in aperture, sampling format, and wavefront representation.

(1) Data normalization

To facilitate accurate wavefront evaluation and mitigate the non-uniform angular distribution inherent to tactile CMM measurements, a data normalization procedure was applied. This process involved aperture standardization and spatial resampling. First, the analysis region was restricted to a common circular aperture corresponding to the maximum shared diameter across all datasets (e.g., 180 mm diameter aperture) to isolate the overlapping surface and minimize edge artifacts. Subsequently, the best-fit sphere (BFS), the CMM-measured surface, and the laser-measured surface were resampled onto a uniform Cartesian grid (e.g., 500×500). This ensured all datasets shared a consistent spatial topology, eliminated irregular sampling and enabled direct comparative wavefront analysis.

(2) Wavefront reconstruction

The wavefront was defined as the difference between the measured CMM surface and the reference BFS:

$$\Delta W = Z_{CMM} - Z_{BFS} \quad (7)$$

where ΔW is the wavefront, Z_{CMM} is the gridded CMM surface, and Z_{BFS} is the gridded BFS surface.

The sampled points (x, y, z) were then transformed from Cartesian coordinates to normalized cylindrical coordinates (ρ, θ, z) according to

$$\rho = \frac{\sqrt{(x^2 + y^2)}}{r} \quad (8)$$

$$\theta = \text{atan2}(y, x) \quad (9)$$

where r is the aperture radius.

The wavefront was subsequently expanded in the Zernike basis as

$$\Delta W(\rho, \theta) = \sum_{j=0}^J a_j Z_j(\rho, \theta) \quad (10)$$

where a_j are the Zernike coefficients, Z_j are the Zernike polynomials, and J is the total number of fitted terms.

2.5. Focus Point Calibration

Focus point calibration (FPC) is proposed to improve the measurement accuracy of spherical surfaces by calibration of the focus point location while explicitly incorporating information from the radius-of-curvature approximation. This enables more reliable retrieval of the true surface profile. In wavefront analysis, low-order aberrations are commonly removed from the measured wavefront data to suppress artifacts caused by reference shifts and sensor misalignment. However, for spherical surface approximations, this practice may inadvertently remove aberration components associated with intrinsic surface figure errors, particularly when a focus shift is present.

To address this issue, the proposed FPC method employs an iterative procedure to evaluate the defocus error and compensate for wavefront aberrations using curvature information. Specifically, calibration is achieved by systematically relating the measured wavefront aberrations to the radius of curvature of the test mirror.

To quantify the measurement accuracy, a defocus error is introduced based on curvature-radius analysis, as illustrated in Figure 3. This parameter represents the deviation between the reference spherical surface approximations and the measured surface approximations in terms of their sag values at a given aperture radius. More specifically, the defocus error is evaluated by comparing the sag of the reference spherical surface associated with the ideal focus position, F_1 , with the sag of the measured surface approximations associated with the defocused position, F_2 . The calibration procedure is repeated iteratively until the defocus error is minimized or a predefined threshold is met.

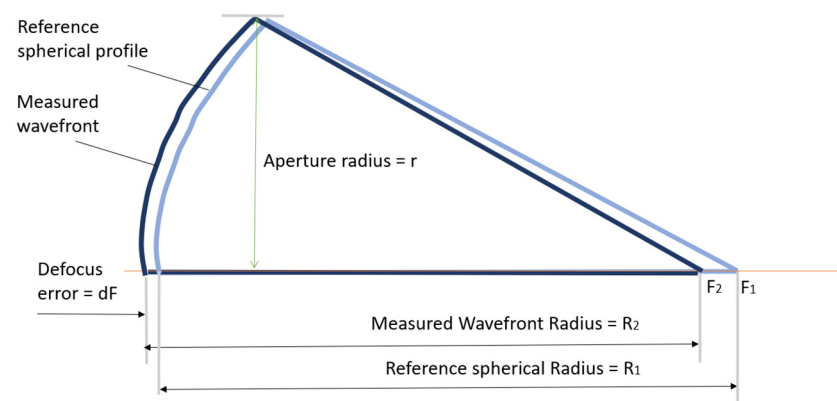


Figure 3. A schematic diagram of the defocus error, illustrating the sagittal difference between the reference spherical surface and measured approximations.

The defocus error, dF , is defined as the difference between the sag values of the measured and reference spherical approximations:

$$dF = s(R_2, r) - s(R_1, r) \quad (11)$$

where

$$s(R, r) = R - \sqrt{(R^2 - r^2)} \quad (12)$$

R_1 is the radius of curvature of the reference surface approximations, R_2 is the radius of curvature obtained from the measured surface approximations, and r is the aperture radius.

The FPC model iteratively decouples the true aspheric surface form from alignment and fitting errors, as shown in Algorithm 1. The model is initialized with the design radius. The reference radius is then updated via a fixed step size and the iteration stops when convergence is achieved or when the maximum number of iterations is reached.

Algorithm 1 Iterative FPC Model

Initialization:

R_1 = design value, 1430 mm

Initial wavefront W_0 derived from CMM minus its BFS

Iteration k:

1. **Remove low-order aberrations** from current wavefront W_k :
 - Fit and subtract Zernike terms Z_1 – Z_4 (piston, tilt-x, tilt-y, defocus)
 2. **Obtain the residual wavefront:**
 - Result: W_k'
 3. **Calculate R_2** from updated wavefront W_k'
 4. **Compute defocus error:**
 - $dF = s(R_2, r) - s(R_1, r)$
 - where $s(R, r) = R - \sqrt{(R^2 - r^2)}$
 5. **Update R_1 for next iteration:**
 - $R_1^{(k+1)} = R_1^{(k)} + \Delta R$, where ΔR = fixed step (0.001 mm)
 6. **Check convergence:**
 - If $|dF| \leq \varepsilon$, 0, or iterations ≥ 10 , stop
-

In summary, the FPC model provides a focus point calibration framework that integrates defocus error estimation, wavefront error analysis, and iterative correction to improve the accuracy of spherical surface measurements. This approach is particularly valuable for high-precision optical metrology applications, such as the characterization of high-performance mirrors for astronomical instruments.

3. Results

To evaluate the performance of the wavefront-based coordinate measuring machine (CMM) approach and the FPC framework in correcting focus errors, a 36-term Zernike polynomial decomposition was applied to represent surface form errors as wavefront aberrations. For consistency, the reconstructed surface profiles were cropped to an aperture diameter of 180 mm and resampled onto a 500×500 grid to ensure a uniform data distribution. The normalized data were then fitted to a best-fit sphere (BFS), yielding a diameter of 2860.0241 mm, and subsequently calibrated using the FPC framework. The focus error was corrected based on the sag deviations between the BFS and measured spherical approximations.

3.1. Qualitative Analysis

Before calibration, the CMM measurements exhibited greater deviations from the reference interferometer in the surface error maps, as shown in Figure 4, indicating the presence of systematic measurement errors. Furthermore, the deviation maps revealed high-frequency “chatter” in the CMM data, which may be attributable to structural instability or

stylus contamination. A pronounced angular offset was also observed between the two datasets, likely resulting from misregistration of the azimuthal reference between systems.

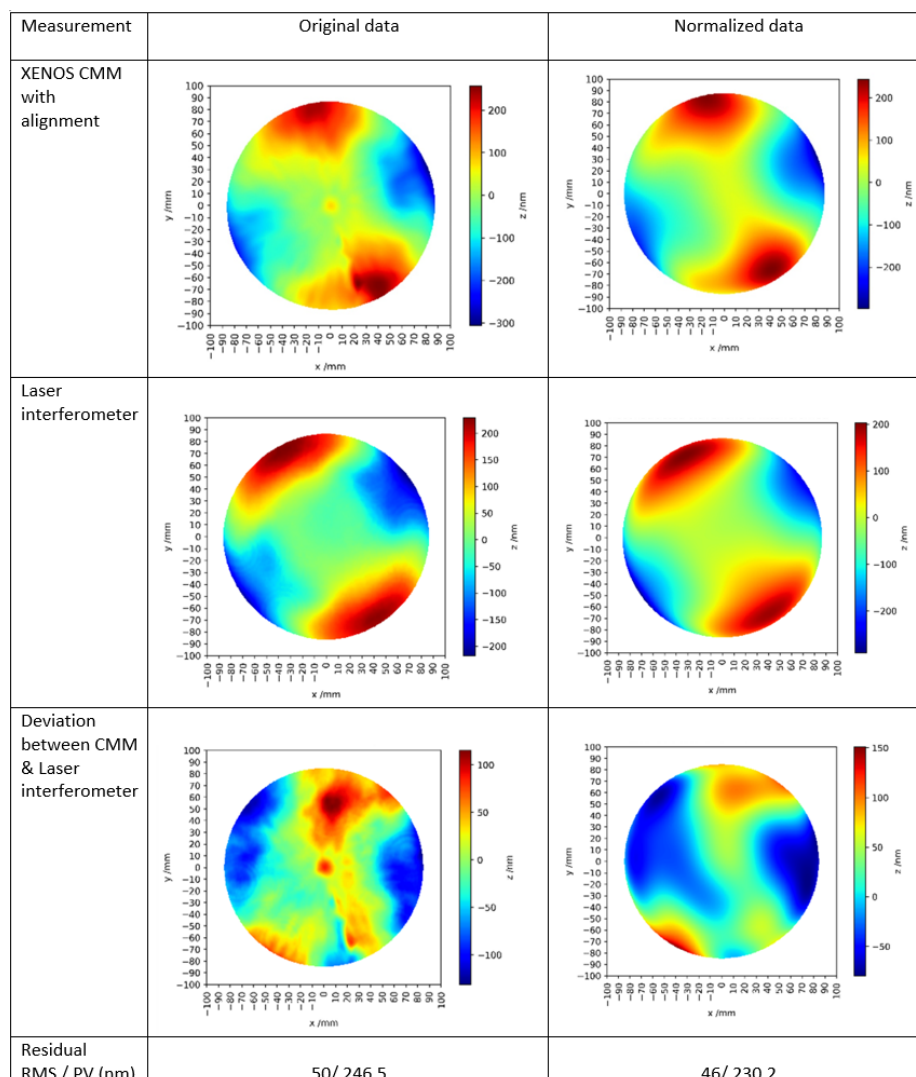


Figure 4. Surface error maps: original and normalized data from the CMM and laser interferometer, and the discrepancy between the two methods.

To enable direct comparison, the CMM surface map was rotated to align with the interferometer reference. The deviation between the surface error maps obtained from the two methods is presented in the bottom rows of Figure 4. After FPC, the agreement between the two methods improved, with residual differences reaching 46 nm RMS and 230.2 nm PV, corresponding to improvements of 8% and 6.6%, respectively. These results indicate enhanced consistency between the measurements in global surface form metrics.

3.2. Quantitative Analysis Using Zernike Terms

Table 3 compares optical mirror surface error measurements before and after focus point calibration (FPC) against a high-precision laser interferometer reference. Zernike coefficients, using Noll indexing, were compared after aligning the azimuthal zero, or clocking angle, between the CMM and interferometer frames.

Table 3. Zernike Noll index surface error comparison (nm).

Zernike Terms	Laser Interferometer			XENOS CMM			XENOS CMM (Calibrated)			Reduction (%)	
	RMS	PV	Coefficient (λ)	RMS	PV	Coefficient (λ)	RMS	PV	Coefficient (λ)	RMS	PV
Piston	0.0	0.0	0	0	0	0.012	0	0	0.015	—	—
Tilt	0.0	0.0	0	6.8	27.2	0.011	1.1	4.5	0.002	83.82	83.46
Defocus	0.0	0.0	0	9.5	32.8	0.015	0.3	0.9	0.000	96.84	97.26
Astigmatism	93.6	458.7	0.148	102.1	500.3	0.161	101.2	496.0	0.160	0.88	0.86
Coma	6.7	38.1	0.011	4.0	22.7	0.006	3.7	21.1	0.006	7.50	7.05
Trefoil	5.8	32.7	0.009	18.8	106.4	0.030	18.8	106.6	0.030	0.00	0.19
Spherical	6.6	22.2	−0.010	1.7	5.8	−0.003	2.1	7.1	−0.003	−47.06	22.41

Before-calibration CMM systematic errors in tilt (6.8 nm RMS) and defocus (9.5 nm RMS), attributed to mechanical deflection or misalignment, were reduced by 83.8% to 1.1 nm RMS and 96.8% to 0.3 nm RMS, after FPC. These values closely match the interferometer baseline of 0.0 nm, confirming the effectiveness of FPC in suppressing dominant low-order systematic errors.

Astigmatism remains high, approximately 100 nm RMS, across all methods after clocking registration, indicating that it represents a true surface figure error. The observed increase in piston, caused by defocus compensation, is excluded from this analysis because piston depends on an arbitrary height reference and does not affect measurement accuracy.

Discrepancies in trefoil likely arise from sensitivity to high-frequency structures and instrument-specific systematic errors. In Figure 4, a distinct trefoil pattern, also visible in the surface error map, is attributed to CMM measurement error.

Minor variations in coma and spherical terms reflect residual calibration errors or localized surface features not fully captured by the model. Overall, FPC significantly reduces systematic geometric errors in tactile CMM measurements, particularly defocus. However, higher-order aberrations such as trefoil require further refinement in the FPC implementation.

It is worth noting that the nanometer- and sub-nanometer-level differences reported in Table 3, such as 0.3 nm RMS in defocus after calibration, refer to the estimated coefficients of low-order Zernike modes derived from global surface data, rather than to the direct measurement resolution. This spatial averaging reduces the statistical uncertainty of low-order terms, enabling their estimation with higher precision than the underlying point-wise measurement accuracy.

3.3. Comparative Analysis

Direct removal of low-order Zernike terms can inadvertently eliminate components of the true surface figure. The proposed FPC method addresses this issue by separating rigid-body misalignment from the true surface figure. Table 4 compares FPC against direct low-order removal and free-radius BFS fitting.

Table 4. Method comparison: direct low-order removal, free-radius BFS fitting, and FPC.

Method	Residual RMS (nm)	Residual PV (nm)
(i) Direct low-order removal	50.59	256.25
(ii) Free-radius BFS fitting	48.86	226.07
(iii) Proposed FPC	46.50	230.20

As shown in Table 4, FPC yields the lowest residual RMS (46.50 nm) and a reduced PV compared to direct removal. It should be noted that a full uncertainty budget or repeatability analysis is not included at this proof-of-concept stage. Therefore, the re-

sults are based on representative measurements of a single sample using a traceable interferometric reference.

4. Discussion

This study demonstrates the feasibility of a wavefront-based coordinate measuring machine (CMM) approach for the optical metrology optics with a large radius of curvature. Spaceborne optical components with large f-numbers are highly susceptible to misalignment and tilt during machining. This susceptibility, combined with systematic errors inherent to tactile probing, limits both machining precision and the inspection accuracy of spherical mirrors. To address these challenges, this paper proposes a wavefront-based method that enables tactile CMMs to decouple and remove piston, x-tilt, y-tilt, and defocus components, thereby mitigating alignment-induced systematic geometric errors while preserving the true surface figure.

Furthermore, a novel FPC framework utilizing Zernike polynomials corrects focus offsets during spherical surface analysis. Through an iterative process that resolves defocus errors and compensates for wavefront aberrations, the FPC framework reconstructs the true surface profile from discrete tactile data into a wavefront-based representation.

The results obtained in this study can be contextualized through comparison with the existing literature in optical metrology. For instance, Machkour-Deshayes et al. [36] reported a 6 nm RMS/22 nm PV difference between calibrated Geometry Measuring Machine (GEMM) and NIST M-48 CMM profiles, with 3 nm RMS repeatability. Similarly, Arnold et al. [37] reported a 107 nm (2σ) measurement uncertainty for the ISARA 400 CMM and less than 10 nm uncertainty for a stitched D100 interferometer. Furthermore, Wang et al. [38] demonstrated a 9 nm RMS difference between a swing-arm optical CMM and full-aperture interferometry on a 1.4 m aspheric mirror.

Because these systems employ different probes, calibration chains, apertures, sampling densities, and support conditions, this comparison is contextual rather than a direct performance ranking. The accuracy achieved in the present study is not intended to match that of laser interferometry or purpose-built optical metrology systems. Instead, the results serve as a proof of concept demonstrating that a commercial tactile CMM, when combined with wavefront-based calibration, can be effectively used for optical surface measurement.

Therefore, the proposed method is particularly relevant in scenarios where global geometric dimensions, such as radius of curvature and form deviation, are of greater concern than full-field nanometer-scale surface characterization. These scenarios include in-process inspection, pre-polishing evaluation, and machining feedback, as well as cases in which traditional interferometry is constrained by limited dynamic range, large RoC, or complex setup requirements. In such applications, the wavefront-based CMM method provides a practical alternative to laser interferometry for large-RoC mirror metrology. It offers greater operational flexibility, a wider dynamic range, and lower implementation cost, while demonstrating the feasibility of extending CMM-based inspection into optical metrology.

Limitations and Future Work

Despite these advancements, certain limitations remain:

- (i) **Residual Errors:** Although the high astigmatism of approximately 100 nm RMS observed across instruments represents a true surface figure defect rather than a calibration artifact, minor higher-order aberrations, including coma, trefoil, and spherical terms, persist. This occurs because the calibration model relies on a low-order truncated Zernike representation, limiting its ability to compensate for spatially localized errors.

- (ii) **Spatial Resolution and Contact Constraints:** The discrete point-cloud sampling of the CMM provides lower spatial resolution than full-field interferometry, making it less effective for high-frequency or mid-spatial frequency errors. Furthermore, as a tactile technique, it is unsuitable for delicate or soft optical coatings susceptible to probe-induced damage.

These limitations, together with the scope of the current validation, directly motivate future research. To accommodate higher-order aberrations, such as trefoil, future work will expand the wavefront model beyond the current low-order Zernike truncation. To improve spatial resolution and capture mid-spatial frequency features, measurement paths will be optimized for denser point-cloud sampling. Finally, to overcome tactile constraints and enable measurement of delicate coatings and support for scaling to physically larger mirrors where long-travel geometric errors and gravity-induced deformations occur, non-contact optical probes will be integrated into the system.

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Abbreviations

The following abbreviations are used in this manuscript:

CMM	Coordinate measuring machine
CGH	Computer generated hologram
CNC	Computer numerical control
BFS	Best-fit sphere
FPC	Focus point calibration
RMS	Root mean square
PV	Peak to valley

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