



Article

Quantum Statistical Behaviors of Carriers in Strong Inversion Layers Associated with Mobility and Threshold Voltage in FinFET Transistors

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Abstract

Gated transistors in the form of MOSFET, FinFET, or IGBT are capable of controlling and transferring either signals or powers. These capabilities are closely associated with applied biases on Gates, which surpass the respective threshold voltages. Source/Drain bias, V_{DS} , then establishes the electric field, E_{DS} , driving carriers to flow with a speed which is proportional to E_{DS} with the proportionality, termed mobility, μ . The mobility somewhat addresses the electrical performances of the specific transistor, and is V_{GS} -dependent, where the generated electric field is perpendicular to the interface in between the Gate and the Gate oxide and is directed across the channel. The mobility may be treated as the collective quantum statistical behaviors of carriers, i.e., electrons or fermions. It is worth analyzing the electrical performances by way of quantum statistics. Nevertheless, the threshold voltages are surprisingly negative on FinFETs as the fitting is performed, which means that I_{DS} would flow even without applied voltage on the Gate. I_{DS} - V_{DS} characteristic curves with negative threshold voltage intriguingly perform just like the other ones with positive threshold voltages. Therefore, there might exist some kind of mechanism enhancing strong inversion layers that is responsible for the characteristics. In this paper, characteristic curves of FinFETs may be well fitted by using both modified characteristic formulas and the proposed kink effects. The extracted parameters (k_N , V_{th} , λ) thus provide information on mobility, concentration of p ($1/\text{cm}^3$), or even leakage current. Also, the mobility, μ , here is analyzed by using Fermion statistics. Furthermore, trivial solutions for the specific boundary conditions, $V_{GS} = 0$ V, surrounding the channel are presented, where one of the possibilities proposed is the mass plasma oscillation of electrons, which might be an option for addressing the negative threshold voltage.



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Keywords: MOSFET; FinFET; negative threshold voltage; radiation by accelerated charges; strong inversion layer; kink effects

1. Introduction

1.1. Preparation of FinFET Devices

By ionic dry etching the epi-silicon layer which is pregrown by flowing silane (SiH_4) accompanied with slightly adulterated HCl, three sizes of fin width (110 nm, 115 nm, 120 nm) of 3D “I” are formed with aspect ratios of 9, with the ends named the Source and Drain. The epi-silicon fin channel is grown with the ultra-thin gate oxide of 14 angstroms and

covered with arsenic heavily doped poly-silicon. After SF_6 dry poly-silicon etching, various channel lengths (e.g., 240 nm, 160 nm, 120 nm, 100 nm) are characterized with probe-testing on $I_{\text{DS}}\text{-}V_{\text{DS}}$ [1–3]. FinFETs with 110 nm fin widths are ruled out because of flattened and suppressed characteristic curves in the triode region, which are not acceptably normal FETs. FinFETs with 115 nm fin widths are not feasible because of non-linear dependence of k_{N} verse the inverse of channel length [4–7]. The only reliable transistors are those with channel fin widths of 120 nm, which demonstrate the feasibility and set up the design rule in Mask Layout referred to in reference [1]. k_{N} is linearly dependent of $(1/L)$ with a fin width of 120 nm, where three FinFETs (W120L240, W120L160, and W120L100) are analyzed in the following graph, as shown in Figure 1.

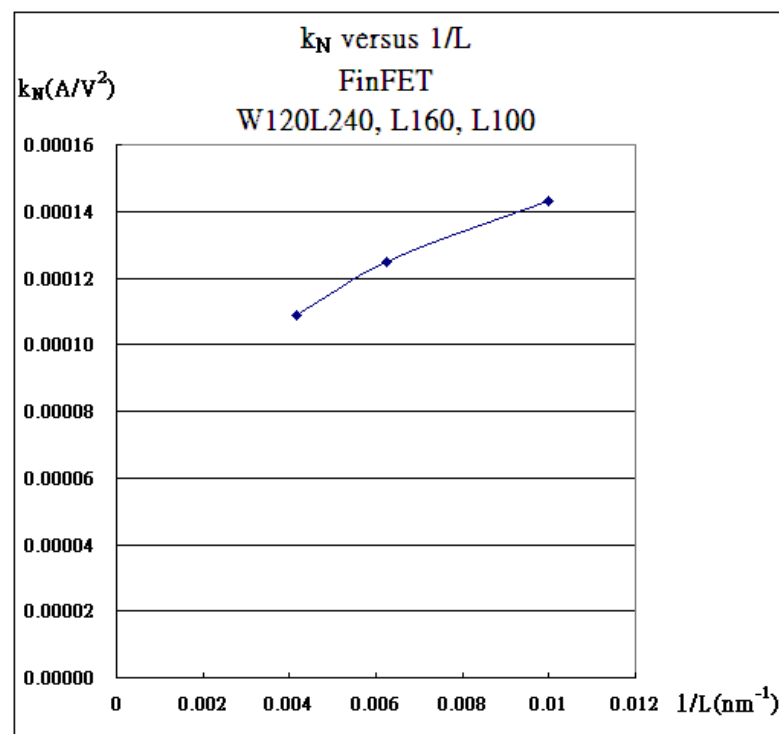


Figure 1. k_{N} is supposed to be proportional to $1/L$.

1.2. A Conclusive Model Addressing Characteristic Curves of Field-Effect Transistors

A newly generated model addressing characteristics of field-effect transistors (FETs) is due to a little modification over the conventional formulas, which extract the mobility (μ) from the integration over the channel variable from 0 to V_{DS} at constant V_{GS} [8]. The characteristic curves always become saturated at $V_{\text{DS}} = (V_{\text{GS}} - V_{\text{th}})$ and have an almost straight saturation line with the slope of λ . With the determined V_{th} and λ , k_{N} can be easily found and a discrepancy always occurs between the as-measured curve and the as-formulated one. The discrepancy is easily eliminated by introducing a Gaussian-like kink effect such that as-measured curves are well fitted. This conclusive model is presented in Equation (1) as follows:

$$\begin{cases} I_{DS}(triode) = k_N[(V_{GS} - V_{th})V_{DS} - \frac{V_{DS}^2}{2}](1 + \lambda V_{DS}) \\ \quad - I_{kink} \exp[-W_{kink}(V_{DS} - V_{kink})^2] \\ I_{DS}(saturation) = k_N[\frac{(V_{GS} - V_{th})^2}{2}](1 + \lambda V_{DS}) \\ \quad - I_{kink} \exp[-W_{kink}(V_{DS} - V_{kink})^2] \end{cases} \quad (1)$$

where

$$k_N = \frac{C_{ox}^{(1)} W_{eff} \mu_N}{L_o},$$

$$V_{th} = (-0.56 - |\Phi_p|)_{FB} - \frac{Q'_{ox}}{C'_{ox}} + 2|\Phi_p| + \frac{Q'_{dep}}{C'_{ox}}$$

with

$$V_{GS} \geq 2|\Phi_p| = 2\frac{k_B T}{e} \ln\left(\frac{p}{n_i}\right) \quad (\text{deepstronginversion}),$$

and fitting by minimizing root-mean-square formulas as follows:

$$\delta_{rms} = \sqrt{\frac{\sum_{i=1}^N (I_{DS_fitting} - I_{DS_as-measured})^2}{N}}$$

and (V_{kink}, I_{kink}) is the maximum discrepancy with W_{kink} for the effective Gaussian width. Normally, concerning non-zero bias, the model is applied to the Gate such that the channel is first depleted and then strongly inversed. The strong inversion layer is responsible for conduction of current in between the Source and Drain [9–12].

1.3. Potential Energy Associated with Depletion Region and Strong Inversion Layer

By solving the divergence of electric field in Maxwell's equations, the electrical potential energy as applied with a Gate bias (V_{GS}) is expressed as follows:

$$U(x) = -eV(x) = -\frac{pe^2}{2\epsilon_{Si}}(x - D)^2 + \frac{pe^2}{2\epsilon_{Si}}D^2 - eV_{GS}.$$

with boundary conditions

$$\begin{cases} U(0) = -eV_{GS} \\ U(D) = 0 \end{cases}.$$

The thickness of depletion region, $D = \sqrt{\frac{2\epsilon_{Si}}{pe} V_{GS}}$, is to be derived as the boundary condition $U(D) = 0$ is applied. Also, the thickness of the strong inversion layer is to be resolved by setting $U(d_\Phi) = -k_B T \ln(p/n_i)$. Graphs showing the depletion region and the strong inversion layer are presented in Figures 2 and 3. In addition, the effective width of the FinFET is set to 19 times the thickness of the depletion region at $V_{GS} = 1.0$ V, and the concentration of the channel is thus identified as 3.7×10^{17} ($1/\text{cm}^3$), whose breakdown voltage of the channel is 3.6 V [10].

$$\begin{cases} d_\Phi = x_{S_\Phi} = D_{dep}(1 - \sqrt{\frac{\Phi}{V_{GS}}}) \\ d_{2\Phi} = x_{S_{2\Phi}} = D_{dep}(1 - \sqrt{\frac{2\Phi}{V_{GS}}}) \end{cases} \quad (2)$$

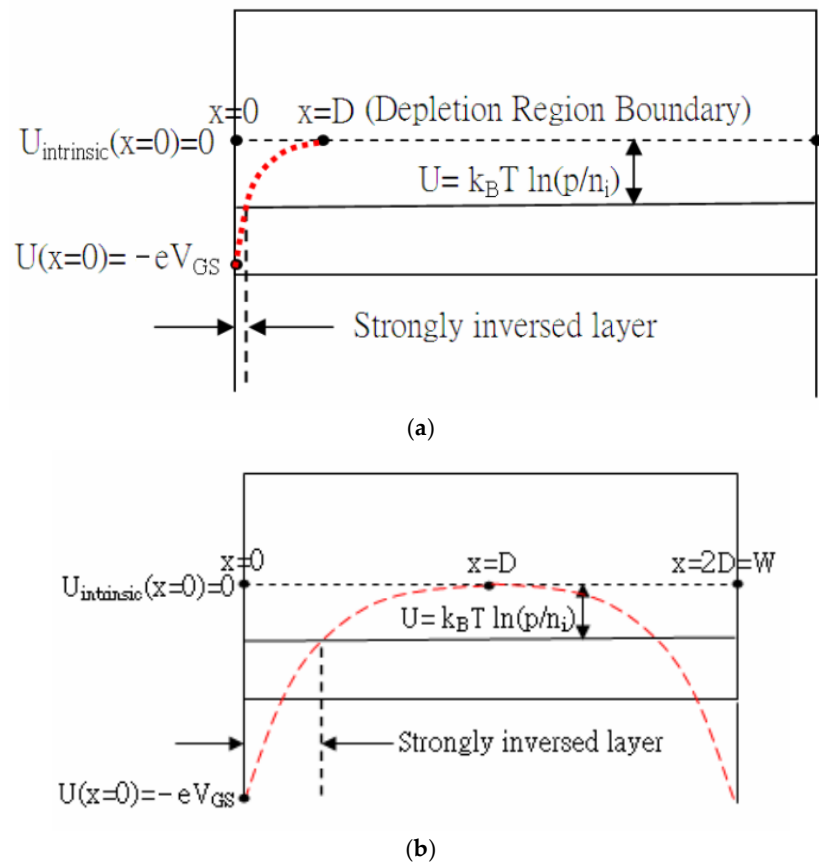


Figure 2. Red detached lines indicate the potential energy. (a) Constant p-type Fermi energy and silicon energy gap with intrinsic Fermi energy that may bend down as positive bias is applied to the Gate. (b) FinFETs (W120L100, W120L160, and W120L240) are supposed to have the fin width $2D = 120$ nm at $V_{GS} = 1$ V, which corresponds to $p(1/\text{cm}^3) = 3.7 \times 10^{17}/\text{cm}^3$.

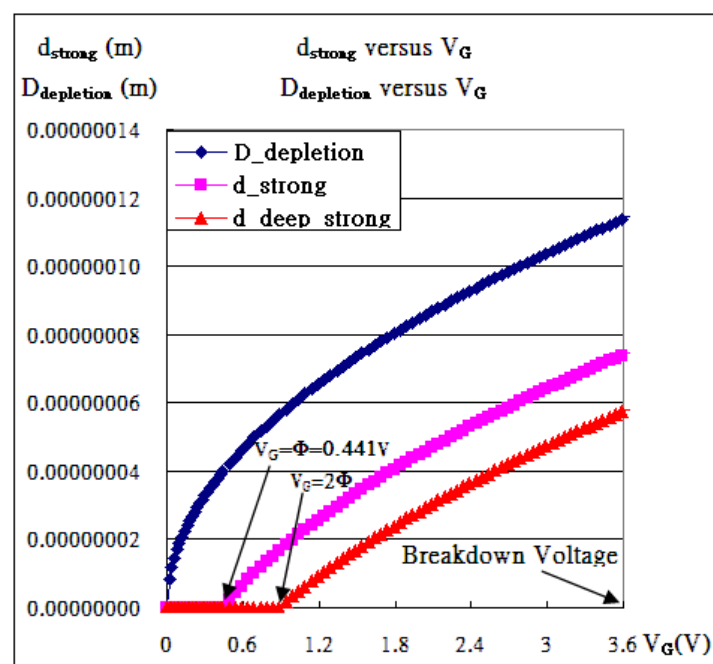


Figure 3. Depletion region and strong inversion layer at non-zero V_{GS} , and breakdown voltage (3.6 V) along channel length with $p(1/\text{cm}^3) = 3.7 \times 10^{17}/\text{m}^3$.

1.4. NFinFETs with Negative Threshold Voltages

An idea arises regarding the semiconductor that there are negative threshold voltages being applied to n-channel transistors. That is, transistors without strong inversion layers within depletion regions can even become conductive at $V_{GS} = 0.0$ V as non-zero V_{DS} is applied to the Source and Drain. Intriguingly at $V_{GS} = 0.0$ V, I_{DS} - V_{DS} characteristic curves with negative V_{th} look like the others with positive V_{th} , which have the features of characteristic curves containing a triode region and saturation region. Zero V_{GS} is thought not to build up a depletion region or even a strong inversion layer. Meanwhile, a depletion region helps contribute positive quantity to threshold voltage. An example is shown in Figure 2. But why is the channel conductive and why does the Drain/Source current reach satisfaction at $V_{DS} = (V_{GS} - V_{th})$. The explanation might be quantum effects or something else. An idea came from the bulk or collective plasma oscillation making instantaneous positive and negative biases on either side of the slim channel of the FinFET, which cause an instantaneous strong inversion layer on either side of the fin channel based on the plasma frequency. This creates electromagnetic wave radiation from all FinFETs in IC, even though its intensity might be weak due to non-coherent motions. Nevertheless, characteristic curves are demanded to fit the modified conventional formulas (one is in the triode region and the other in the saturation region).

Solving the field solution, $V(x, y, z)$, is important for addressing the whole field effects inside the transistor. The uniqueness theorem may provide solid assurance of solutions with fixed boundary conditions, which may give information about how the transistor has negative threshold voltage if the applied Gate bias is zero with non-zero V_{DS} .

1.5. Statistical Behaviors of Carriers

There are two kinds of carriers, electrons and holes, in a semiconductor. Both carry either electrical power or electrical signals, which statistically need large quantities. Intriguingly, they are called “Fermions” and ought somewhat to obey a certain statistical regulation, Fermi–Dirac statistics, which requires the order of trillion times trillion to make it effective at lower temperature. In principle, no two identical fermions are allowed to exist in the same state [13]. Fermi–Dirac Statistics with $p_o = 3.7 \times 10^{17}/\text{cm}^3$ whose Fermi-energy is $U(d)$ is as follows:

$$p(x) = \frac{p_o}{e^{\frac{U(x)-U(d)}{k_B T}} + 1}$$

and

$$p(x) = \frac{p_o}{e^{\frac{U(x)-U(d)}{k_B T}} + 1} = \frac{p_o}{e^{\frac{-e[V(x)-V(d)]}{k_B T}} + 1}$$

For $e[V(x) - V(d)] \ll k_B T$

$$p(x) \xrightarrow{T=300K} \frac{p_o}{e^{\frac{-[V(x)-V(d)]}{0.0259}} + 1}$$

$$p(x) \rightarrow p_o \frac{e^{\frac{V(x)-V(d)}{0.0259}}}{0.0259} \leq 0.1 p_o$$

Their statistical behavior may approach Maxwell–Boltzmann statistics if the temperature is above 1000 K. As the ambient temperature in the circumstances is at room temperature corresponding to $(k_B T/e) = 0.0259$ V, the voltage difference, $(V(x) - V(d))$, around the strong inversion layer may be reasonably less than one tenth of 0.0259 V. CMOS-FET, CFinFET, and CGAAFET mainly control or convey signals and thus have collective carriers move around according to some specific circuit design. Those collective behaviors of carriers may explain I–V characteristic curves of transistors, kink effects, mobility, etc.

1.6. Mobility

Collective carriers move along the Source to the Drain depending on the transverse electric field (E). The stronger E is, the lower the mobility (μ) is, which is reasonably

explained as attracted carriers hitting the interfacial oxide and causing retardation. For planar MOSFET, FinFET, and GAAFET, E is actually proportional to $(V_{GS} - V_{th})$ at non-zero V_{GS} . Nevertheless, it still does not address how the mobility depends on the electric field as $V_{GS} = 0$ V.

In this paper, three FinFET transistors (W120L100, W120L160, and W120L240) at various applied Gate biases are fitted with three parameters (k_N , V_{th} , λ), followed and finalized by kink effects. Mobility is statistically analyzed and discussed. Fitting of characteristic curves of three kinds of transistors at $V_{GS} = 0$ V is feasibly presented. Laplace's differential equation with boundary conditions at $V_{GS} = 0$ V is solved, and the solution, $V(x,y,z)$, resulting in plasma oscillation being introduced physically, addressing the negative threshold voltage.

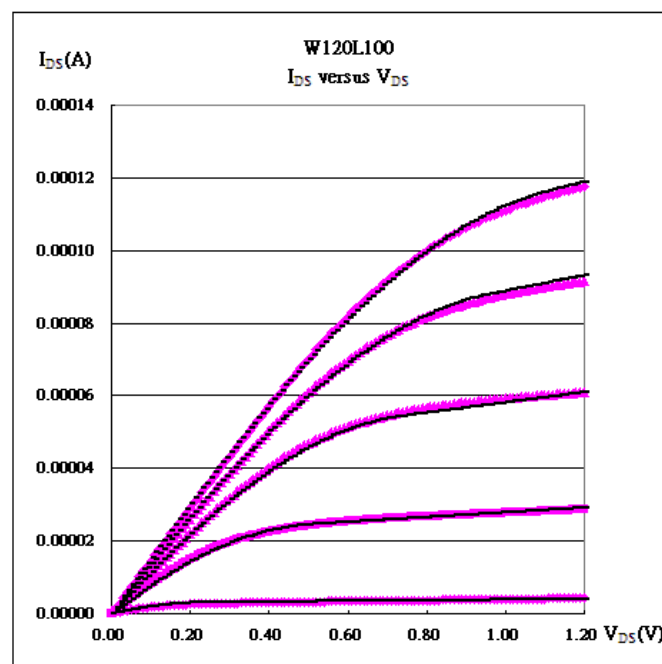
2. Characteristics Curves Fitted with Kink Effects

2.1. Extraction of Three Parameters (k_N , V_{th} , λ) at Non-Zero-Biased V_{GS}

FinFET transistors with three various channel lengths are fitted with the modified conventional formula. k_N is extracted for analysis of mobility as in Table 1. As demonstrated in Figure 4, the kink effect is also added to make the fitting as perfect as possible. The fitting procedures include (1) λ for the slope, (2) V_{th} for the turning point at $V_{DS} = (V_{GS} - V_{th})$, and (3) k_N for current level adjustment. The above fitting can be concluded in a peak-like δ - V_{DS} graph, which may be flattened by the kink effect.

Table 1. (k_N , V_{th} , λ).

V_{GS} (V)	k_N (A/V ²) (L100)	V_{th} (V) (L100)	λ (1/V) (L100)	k_N (A/V ²) (L160)	V_{th} (V) (L160)	λ (1/V) (L160)	k_N (A/V ²) (L240)	V_{th} (V) (L240)	λ (1/V) (L240)
0.00	1.60×10^{-4}	−0.1790	0.55	1.20×10^{-4}	−0.1160	0.55	7.00×10^{-5}	−0.0800	0.55
0.25	1.70×10^{-4}	−0.2498	0.31	1.42×10^{-4}	−0.1682	0.17	1.61×10^{-4}	−0.0302	0.13
0.50	1.63×10^{-4}	−0.2350	0.32	1.31×10^{-4}	−0.2227	0.17	1.31×10^{-4}	−0.0233	0.13
0.75	1.46×10^{-4}	−0.2088	0.32	1.31×10^{-4}	−0.1804	0.17	1.18×10^{-4}	−0.0200	0.13
1.00	1.42×10^{-4}	−0.2000	0.32	1.25×10^{-4}	−0.1065	0.17	1.09×10^{-4}	−0.0969	0.13



(a)

Figure 4. Cont.

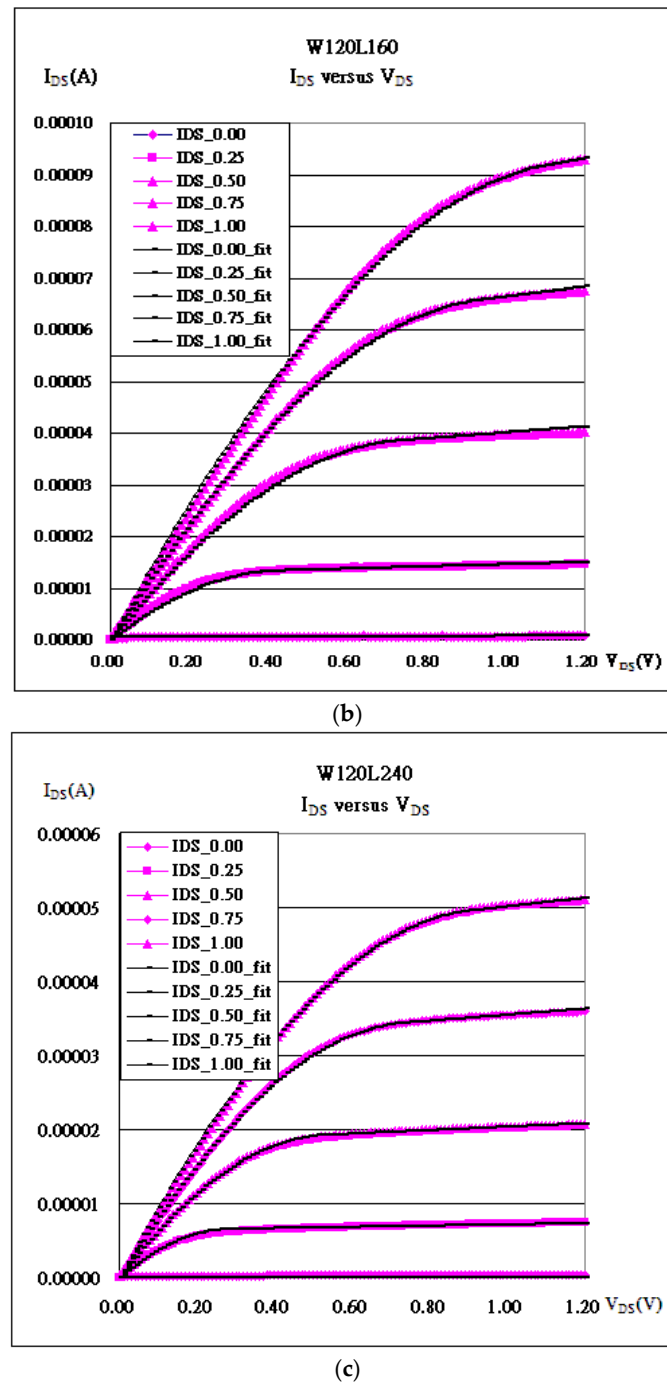
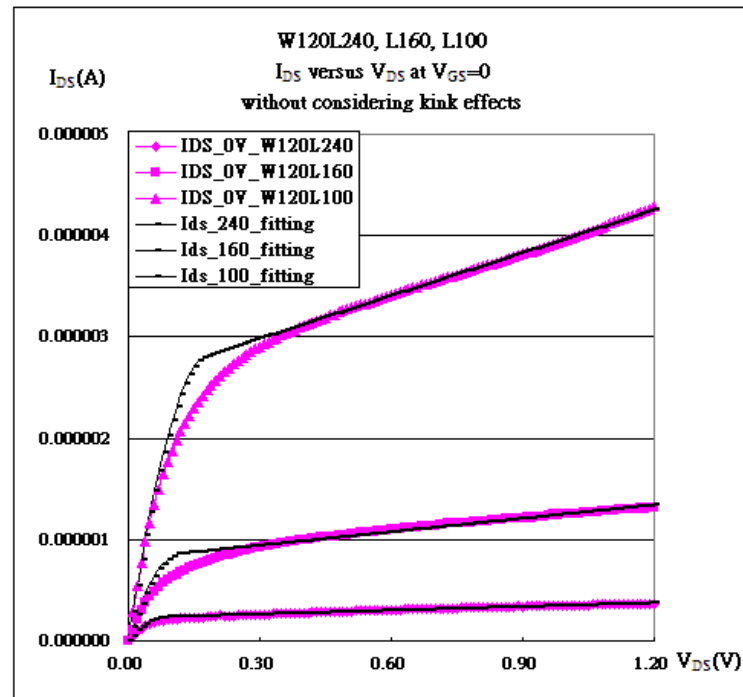


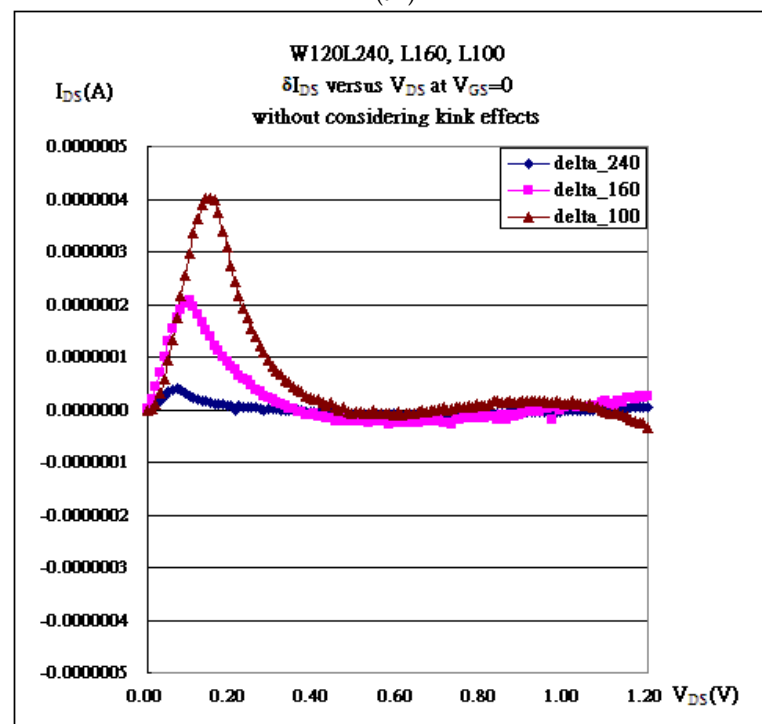
Figure 4. I_{DS} - V_{DS} fitting of FinFET transistors with channel width 120 nm: (a) W120L100; (b) W120L160; (c) W120L240.

2.2. Extraction of Three Parameters (k_N , V_{th} , λ) at Zero-Biased V_{GS}

FinFET transistors with three various channel lengths are fitted with the modified conventional formula. k_N is extracted for analysis of mobility as in Table 1 and as demonstrated in Figure 5.



(a1)



(a2)

Figure 5. Cont.

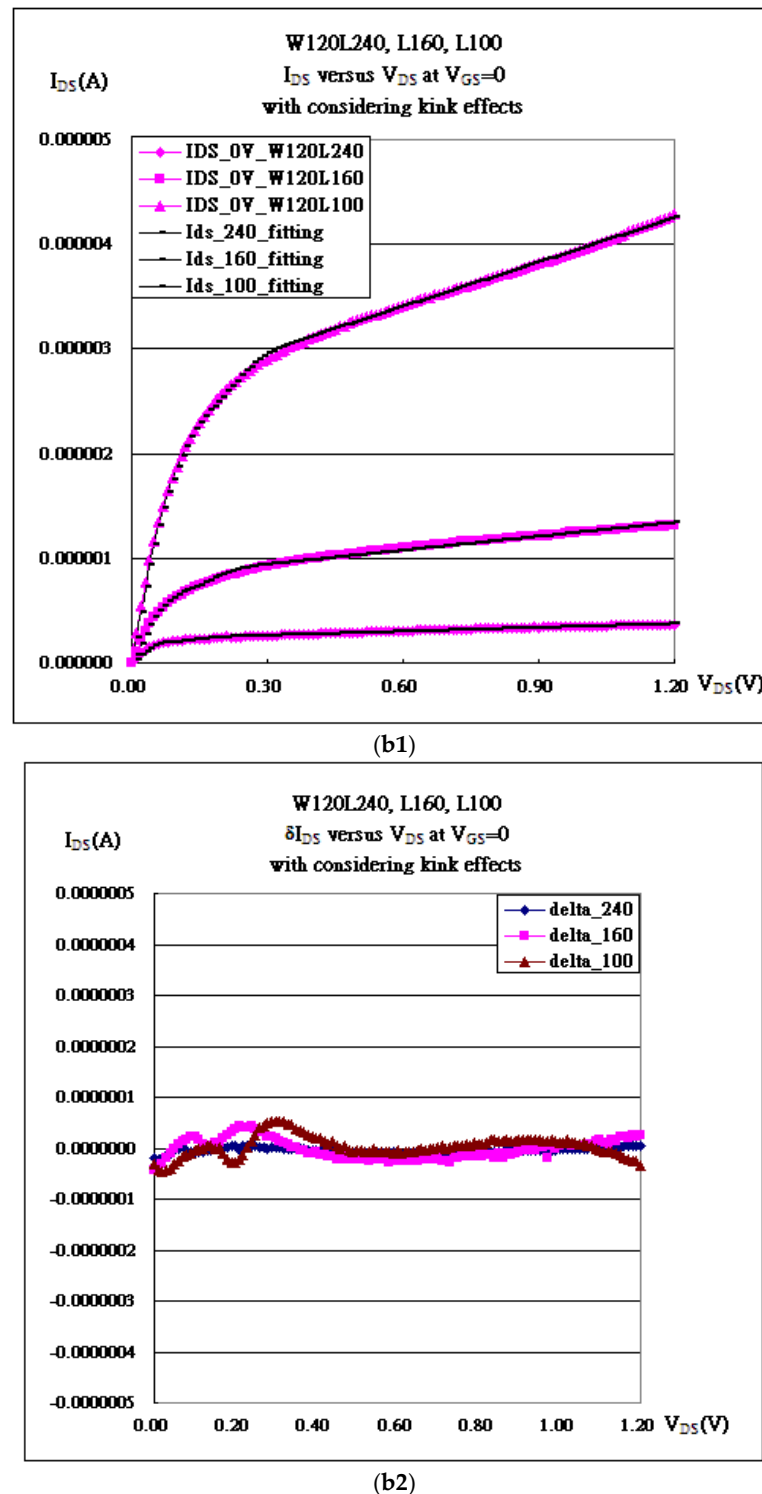


Figure 5. I_{DS} - V_{DS} fitting of FinFET transistors with channel length 100 nm, 160 nm and 240 nm at $V_{GS} = 0$ V (a1,a2) with no kink effect; (b1,b2) with kink effect.

3. Results and Discussion

3.1. Leakage Current at Various V_{GS}

Instead of defining I_{off} , the whole characteristic curve at $V_{GS} = 0$ somewhat equivalently demonstrates the leakage current, which may be expressed by the term, “ λV_{DS} ” in Equation (1). And “ λV_{DS} ” is the ratio of leakage current ($I_{leakage}$) to saturation current (I_o

at $V_{DS} = V_{GS} - V_{th}$), and an equivalent r_o is internally inserted in between the Drain and Source as follows:

$$\lambda V_{DS} \equiv \frac{I_{leakage}}{I_o} = \frac{(V_{DS}/r_o)}{I_o} = \frac{V_{DS}}{I_o r_o} = \frac{V_{DS}}{|V_A|},$$

where V_A is Early voltage.

In Table 1, the larger the channel length is, the less leaky the transistor is at non-zero V_{GS} . At $V_{GS} = 0$, the leakage current associated with the same $\lambda = 0.55$ is not even particularly controllable because there is no depletion region (in general, directly generated from the Gate) to help block leakage across the Drain and Source. For all three different channel lengths, the same λ (i.e., the same early voltage) indicates that r_o was proportional to channel length with the same channel width if the mobility was the same. Actually, the corresponding equivalent internal resistances are 9.32×10^6 Ohm (W120L240), 2.67×10^6 Ohm (W120L160), and 8.78×10^5 Ohm (W120L100).

3.2. Mobility Proportional to Channel Length

The three FinFET transistors are all 120 nm in fin width, which at least helps prevent the issue of over etching. Actually, the aspect ratio is 9, making the effective width 19 times that of 120 nm. On the other hand, the channel length is the length that carriers move across in between the Source and Drain. The shorter the channel length is, the faster the carriers transmit the signals. Preferably, shorter channels are very competitive. But why is the mobility lower for shorter channel, such as 5 nm FinFETs or 2 nm GAAFETs? Originally, carriers are accelerated by the electric field and drift across the channel confronting viscosity obstacles from the vibrating lattice, which is presumably proportional to the power of N of the speed. Promptly, carriers reach the saturation of speed in a very short distance, which is mainly contributed to by near-zero denominators in the integration, as below. N needs setting to 1 if the speed of carriers is presumably proportional to the electric field with the mobility as the proportionality. Then mobility, μ , must be inversely proportional to the viscosity, ζ , and $\zeta L = \text{constant}$ for fixed V_{DS} . This means that ζ is inversely proportional to the channel length, L , at fixed V_{DS} , while μ is inversely proportional to ζ . Therefore, μ is proportional to L , as shown in Figure 6. So, why is viscosity inversely proportional to the channel length? It is because of the corresponding standing sound wave at a longer channel length having a longer wavelength and lower vibration frequency, which causes less viscosity. The derivation is presented in detail as follows:

$$\begin{aligned} F_{DS} &= m_e a = m_e \frac{dv}{dl} v = -eE_{DS} + \zeta v^N \\ \frac{dv}{dl} v &= -\frac{e}{m_e L} V_{DS} + \frac{\zeta}{m_e} v^N \\ l &= \int_0^{v_{term}} \frac{m_e v dv}{-eE_{DS} + \zeta v^N} \\ l &= \int \frac{v dv}{-\frac{eV_{DS}}{m_e L} + \frac{\zeta}{m_e} v^N} \\ -\frac{eV_{DS}}{L} + \zeta v^N &\cong 0 \\ v &= \sqrt[N]{\frac{eV_{DS}}{L\zeta}} \xrightarrow{N=1} v = \frac{eV_{DS}}{L\zeta} = \frac{e}{\zeta} E_{DS} \\ \Rightarrow \begin{cases} \mu = \frac{e}{\zeta} \\ L\zeta = \text{const}, \text{ for fixed } V_{DS} \end{cases} \end{aligned}$$

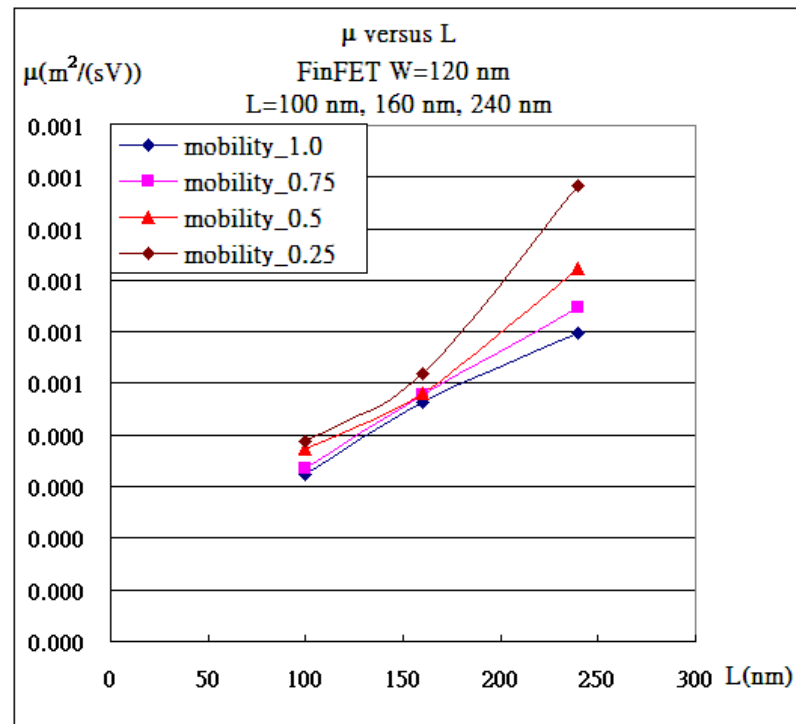


Figure 6. Mobility is sensitive to channel length, and mobility is linearly proportional to channel length (L) (μ/L is almost the same.)

3.3. Mobility Inversely Proportional to $E^{-1/3}$

As seen in Figure 2, the strong inversion layer is so narrow that the conductive carriers can only move in between the Source and Drain and squeeze in the Δx narrow aisle, which is perpendicular to the channel length. The following proof first takes Fermion statistics into account with the potential energy to verify the concentration distribution, which is substituted in the divergence of electric field (the first equation of Maxwell's equations). The differential equation is integrated and the scope, Δx , of the strong inversion layer is discovered. There are actually two parts after the integration. One is concerning the depletion region where no free carriers exist. The other one is concerning strong inversion layers where free carriers carry signals back and forth in between the Source and Drain. The electric field due to V_{GS} depletes the channel with the boundary D beyond which $E(x > D)$ is 0, i.e., $E(x < D) = (-e p_0 / \epsilon_{Si})(x - D) = (-e p_0 / \epsilon_{Si})X$. There is no free carrier in the depletion region and $E(x)$ is linearly proportional to X as follows:

$$\begin{aligned} \nabla \cdot \vec{E} &= \frac{\rho}{\epsilon_{Si}} = \frac{-ep(x)}{\epsilon_{Si}} \\ \Rightarrow \frac{dE}{dx} &= \frac{-ep_0}{\epsilon_{Si}} \\ \Rightarrow E(x) &= \frac{-ep_0}{\epsilon_{Si}}(x - D) \end{aligned}$$

with boundary condition $E(D) = 0$ for $x \geq D$

Besides this, $E(x)$ generates both the depletion region and a free statistical carrier part, where $E(x) \sim X^3$ (i.e., $\Delta x = \Delta X = E^{(-1/3)}$) addresses the uncertain region. In addition, Δp_x is about the same order of $\Delta p_L \sim \Delta(m_e v) \sim \mu$ along the channel. The uncertainty principle, $\Delta x \Delta p_x \sim E^{(1/3)} \mu \sim h$, verifies that μ is inversely proportional to $\Delta x \sim E^{(1/3)}$, which means $\mu \sim E^{(-1/3)}$ (Figure 7). Physically, the stronger the electric field (V_{GS}) is, the harder carriers

move because E may attract carriers toward the interface to bump into the interface and result in extra friction [14].

$$\begin{aligned}\nabla \cdot \vec{E} &= \frac{\rho}{\epsilon_{Si}} = \frac{-ep(x)}{\epsilon_{Si}} \\ \Rightarrow \frac{dE}{dx} &= \frac{-e}{\epsilon_{Si}} \left(p_0 \frac{V(x)}{0.0259} \right) = \frac{p_0}{0.0259\epsilon_{Si}} U(x) \\ \Rightarrow \frac{dE}{dx} &= \frac{-p_0 e}{0.0259\epsilon_{Si}} \left[\frac{p_0 e}{2\epsilon_{Si}} (x - D)^2 \right] \\ \Rightarrow \frac{dE}{dX} &= \frac{-p_0 e}{0.0259\epsilon_{Si}} \left[\frac{p_0 e}{2\epsilon_{Si}} X^2 \right] \\ E(X) &= \frac{-p_0 e}{0.0259\epsilon_{Si}} \frac{p_0 e}{6\epsilon_{Si}} X^3 \\ \text{with } X &\equiv x - D \\ \Rightarrow \Delta x_{Str} &= \Delta X = (\zeta E)^{\frac{1}{3}}\end{aligned}$$

$$\begin{cases} \Delta x_{\Delta p_fixed} = (\zeta E)^{\frac{1}{3}} \\ \Delta p_{\Delta x_fixed} = m_e \Delta v = \gamma \mu \\ \Delta x \Delta p \cong h \Rightarrow \mu E^{\frac{1}{3}} = const \end{cases}$$

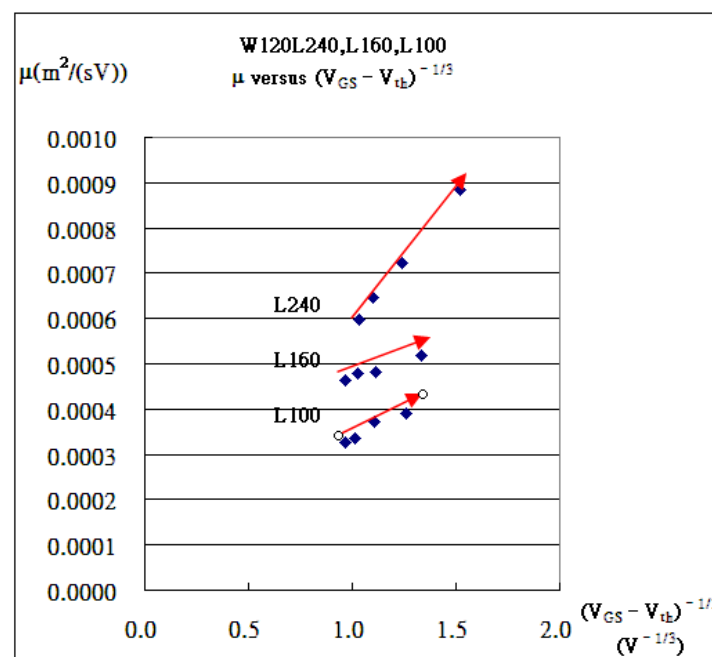


Figure 7. Mobility tends to be inversely proportional to $E^{1/3} \sim (V_{GS} - V_{th})^{1/3}$ (for the collected 3 devices, W120L100, W120L160, and W120L240).

3.4. Boundary Problems with the Zero-Biased V_{GS}

Surprisingly, the depletion region and strong inversion layer might have appeared even though no Gate bias was applied. It is speculated that there must have existed a mechanism making it so. V_{DS} may be applied to the Drain either suddenly or gradually from zero to non-zero V_{DS} . Therefore, for both fields, it is mandatory to introduce the boundary value problems. One is Laplace's differential equation as in Equation (3), where no depletion region is taken into account, and the other is Poisson's differential equation, as in Equation (4). Along the x -axis or y -axis, as in Figure 8, there exist complete sets to fit the boundary value problem. Along the z -axis, the solutions are trivial [15]. The pinch-off near the Drain ($z = L$) can be observed in Equation (4).

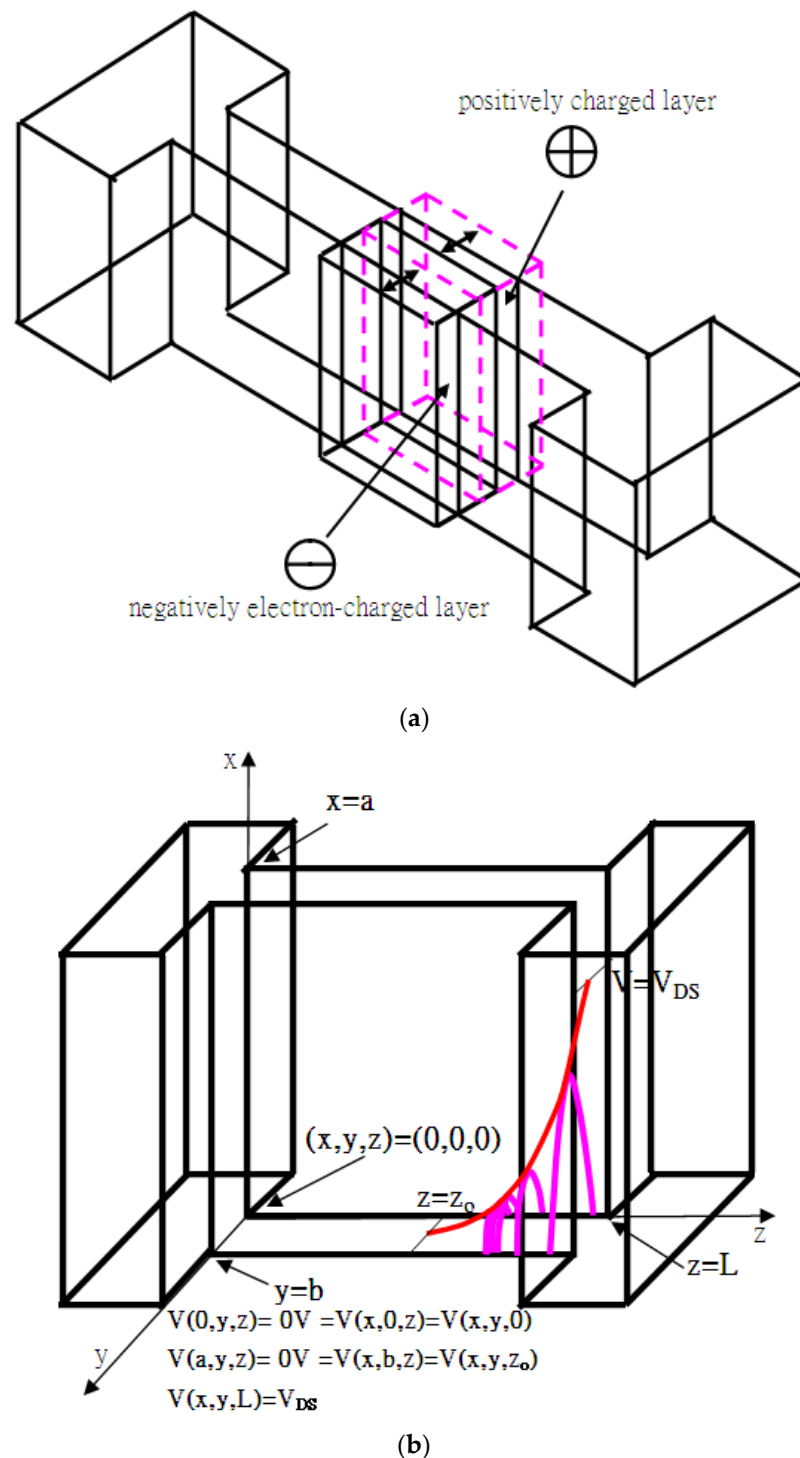


Figure 8. (a) Laplace's differential equation giving the possibility of plasma oscillation; (b) the asymptotical solution of the combination of Laplace's differential equation and Poisson's differential equation (it is a naïve expression of potential function because the solution is a kind of field that is spatially at the same level).

(1) Laplace equation:

$$\begin{aligned}
 \nabla^2 V &= 0 \\
 V(x, y, z) &= \sum_{m,n} V_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \sinh(\gamma_{mn} z) \\
 \begin{cases} \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \gamma_{mn}^2 \\ V_{DS} = \sum_{m,n} V_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \sinh \gamma_{mn} L \end{cases} \\
 V_{DS} \frac{2a}{m\pi} \frac{2b}{n\pi} &= V_{mn} \frac{a}{2} \frac{b}{2} \sinh \gamma_{mn} L \\
 \Rightarrow V_{mn} &= \frac{16V_{DS}}{mn\pi^2 \sinh \gamma_{mn} L} \\
 \text{where} \\
 m \& n &= \text{positive_odd}
 \end{aligned} \tag{3}$$

(2) Poisson Equation:

$$\begin{aligned}
 \nabla^2 V &= \frac{ep}{\epsilon_{Si}} \text{ or } \nabla \cdot \vec{E} = -\frac{ep}{\epsilon_{Si}} \text{ or } \frac{dE}{dz} = -\frac{ep}{\epsilon_{Si}} \\
 E_z &= -\frac{ep}{\epsilon_{Si}}(z - z_0) \quad \text{as } z > z_0 > 0 \\
 V(x, y, z) &= \sum_{m,n} V_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \left(\frac{V_{DS}}{(L - z_0)^2}\right) (z - z_0)^2 \\
 V_{DS} \frac{2a}{m\pi} \frac{2b}{n\pi} &= V_{mn} \frac{a}{2} \frac{b}{2} V_{DS} \\
 \Rightarrow \begin{cases} V_{mn} = \frac{16}{mn\pi^2} \\ z_0 = D < L \end{cases}
 \end{aligned} \tag{4}$$

Without considering the depletion region, the p-type semiconductor is neutral and plasma oscillation may happen due to the thermal agitation of the collective electrons as shown in Figure 8a. As seen in Figure 8a, a sudden collective thin layer of electrons extensively appears on one side of the slim channel due to thermal agitation. The sinusoidal potential forms an attracting force to draw the positively or negatively charged layer back such that the oscillation collectively occurs, as derived in Equation (5). It happens even for the solution of Poisson's equation, as shown in Figure 8b. This sudden layered charge may generate an electric field and deplete the slim channel, making it strongly inversed. In the meantime, positive/negative may switch to negative/positive according to the plasma oscillation frequency, and so on [16].

$$\begin{aligned}
 \nabla^2 V &= 0 (\text{thin_film_plasma_oscillation}) \\
 F_e &= m_e a = eE_x = -e \frac{\sigma_e}{\epsilon_{Si}} = -\frac{e^2 p}{\epsilon_{Si}} x \\
 \frac{d^2 x}{dt^2} &= -\frac{e^2 p}{m_e \epsilon_{Si}} x = -\omega_{\text{plasma}}^2 x \\
 \omega_{\text{plasma}} &= \sqrt{\frac{e^2 p}{m_e \epsilon_{Si}}} = 9.88 \times 10^{12} / \text{sec}
 \end{aligned} \tag{5}$$

3.5. Threshold Voltage

The characteristic curves with negative threshold voltage behave like the traditional ones. That is, conventional formulas including the triode region and saturation region may apply in addition to Ioff or leakage current alone. For more precise fitting, the kink effect may be introduced like others. As has long been well known, initial parabolic I_{DS} - V_{DS} characteristics curves concerning saturated flats are much associated with strong inversion layers, which are the result of surpassing the limits of threshold voltages. There are several conceptual steps to achieve this goal. (1) Flat-band voltage (work function difference) makes the transistor return to the starting point. The work function difference between the poly-

silicon gate and the channel is $V_{Flat} = (-\Phi_F) - 0.55 = -0.991$ V. (2) Conventionally, threshold voltage is mostly due to V_{GS} being larger than or equal to $2\Phi_F = 2(k_B T/e) \ln[p/n_i] = 0.881$ V, which bends down intrinsic Fermi energy to obtain a strong inversion layer. (3) With zero V_{GS} , it would be impossible for channel doping, fixed oxide charge, interface charge, charge sharing effects, etc., to have to take care of at least the $2\Phi_F = 0.882$ V, which statistically allows extra free electron carriers to move in the strong inversion layer. Even if it could, the reliability is worrying. (4) There might be another option like plasma oscillation in which the mechanism might propose the possibility of raising $2\Phi_F$ that builds up a strong inversion layer. Unfortunately, attempts to verify and test the plasma oscillation are still ongoing.

4. Conclusions

In this paper, (1) three 120 nm wide FinFET transistors (W120L100, W120L160, W120L240) with a channel length of 100 nm, 160 nm, and 240 nm are presented, which are well fitted by using the minimum of the root-mean-square in Equation (1). The kink effect is also demonstrated as V_{GE} is set to zero. (2) The mobility is sometimes zero such that carriers do not move even though they are in the strong inversion layer, especially in the case of Gate bias which is less than the threshold voltage. The less-than- V_{th} carriers in the strong inversion layer are realized to be subjected to Quantum Confinement. (3) For these moving carriers, the associated $(k_N V_{th}, \lambda)$ values are listed in Table 1, and the corresponding mobility (μ) is calculated in Table 2. Mobility (μ) is then found to be inversely proportional to viscosity (ζ), which is inversely proportional to channel length (L). This means μ is proportional to L . Physically in between the Source and Drain, there should exist standing waves with wavelengths of reasonable fractions of L . The longer L is, the longer the wavelength of the standing wave is. The frequency of the standing wave is then lower, reducing the obstacles. In other words, the quantized phonons are less energetic, allowing the viscosity to decrease. (4) Mobility (μ) is proved to be proportional to $E^{(-1/3)}$, as shown in Figure 7. By imposing the generated carriers from Fermi–Dirac statistics, $p(x)$ is substituted into Coulomb’s Law in Maxwell’s equations. Δx is then proportional to $E^{(1/3)}$. The uncertainty principle then helps demonstrate that the linearity in Figure 7 is true. (5) Zero-biased V_{GS} over FinFET channels makes the channel a boundary condition problem. With or without considering the depletion region, the potential in 3D has been solved, leaving the sinusoidal function on the boundary, which exerts attractive forces on these collective free carriers. Massive collective electrons move back and forth with a frequency of plasma oscillation in between two walls of the fin. The plasma frequency might be detected electrically as applied with EM waves on the IC, which could lead to resonant stimulating EM waves in the spectrum.

Table 2. k_N (A/V²) corresponding to μ (m²/Vsec) with $C^{(1)} = 1.92 \times 10^{-2}$ F/m².

V_{GS}	k_N (A/V ²) (L100)	μ (m ² /Vsec) (L100)	k_N (A/V ²) (L160)	μ (m ² /Vsec) (L160)	k_N (A/V ²) (L240)	μ (m ² /Vsec) (L240)
0.00	1.60×10^{-4}	3.66×10^{-4}	1.20×10^{-4}	4.39×10^{-4}	7.00×10^{-5}	8.78×10^{-4}
0.25	1.70×10^{-4}	3.89×10^{-4}	1.42×10^{-4}	5.20×10^{-4}	1.61×10^{-4}	8.84×10^{-4}
0.50	1.63×10^{-4}	3.73×10^{-4}	1.31×10^{-4}	4.80×10^{-4}	1.31×10^{-4}	7.22×10^{-4}
0.75	1.46×10^{-4}	3.34×10^{-4}	1.31×10^{-4}	4.79×10^{-4}	1.18×10^{-4}	6.48×10^{-4}
1.00	1.42×10^{-4}	3.25×10^{-4}	1.26×10^{-4}	4.62×10^{-4}	1.09×10^{-4}	5.97×10^{-4}

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